COSMOLOGICAL GRAVITATIONAL WAVE BACKGROUNDS

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OUTLINE

- Introduction
  - Stochastic backgrounds
  - Characteristic frequency
  - Observational bounds
- Cosmological sources
  - Inflation
  - Preheating
  - Cosmic strings
  - Primordial phase transitions

GW: unique probe of the very early universe

- as the universe expands, particles can get out of thermal equilibrium
- the weakest the interaction, the earlier in the history of the universe the particles decouple

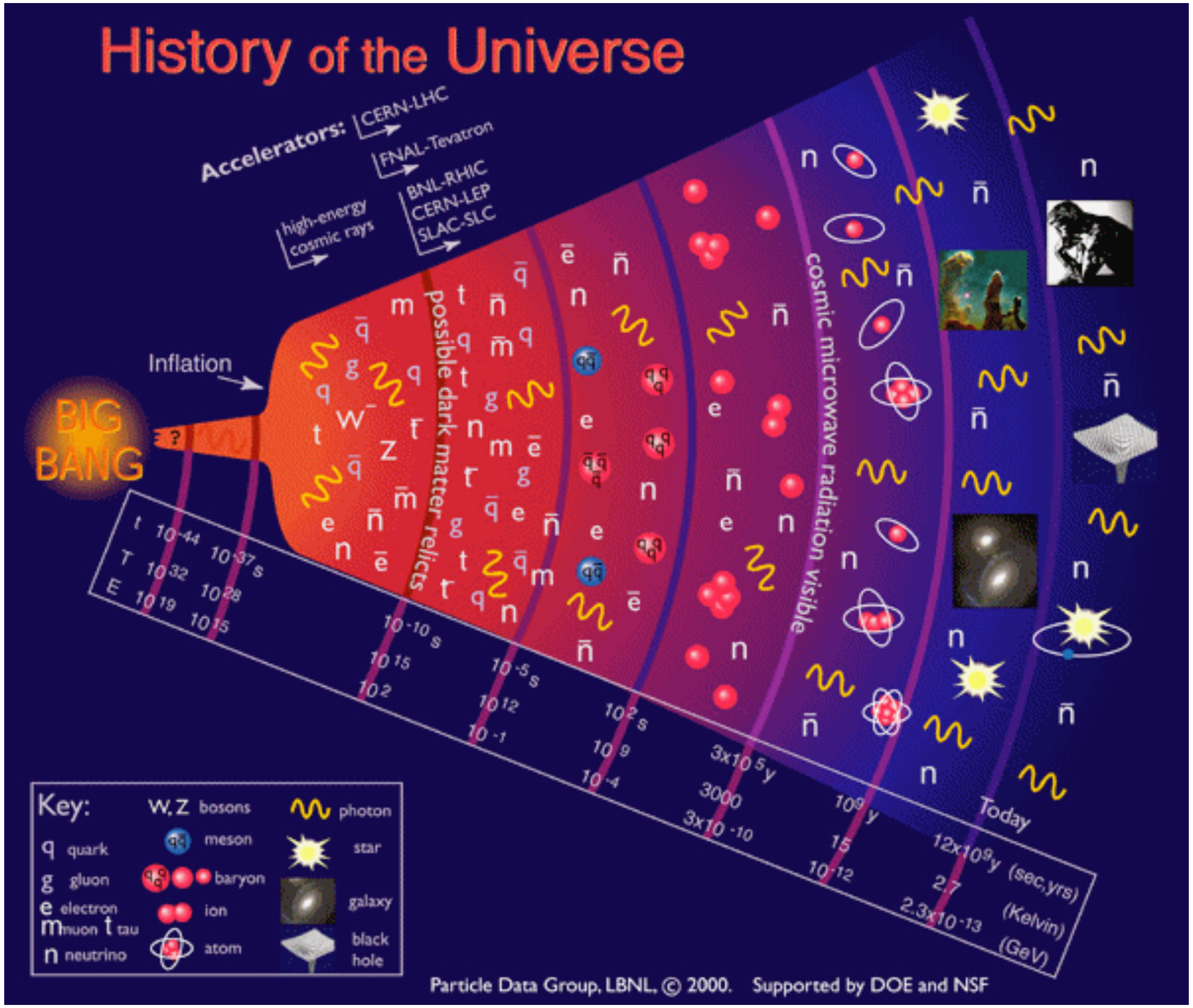
rate of the interaction maintaining thermal equilibrium ← $\Gamma(T)$

$$\frac{\Gamma(T)}{H(T)} < 1$$

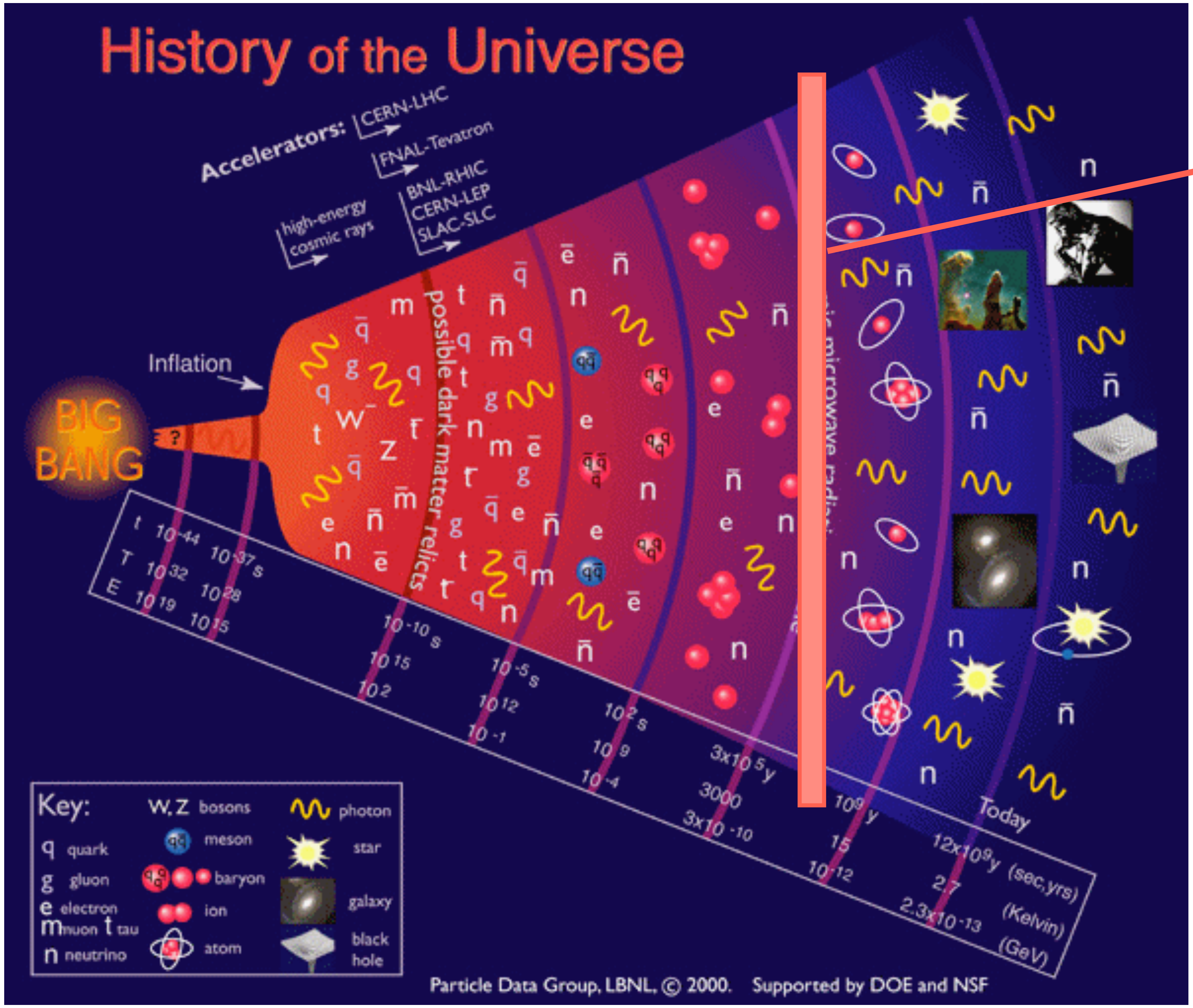
$H(T)$ → rate of expansion of the universe

- particles that decouple at temperature $T_{\rm dec}$ give a snapshot of the universe at that temperature
- propagate freely after decoupling, carry direct information about the early universe

GW: unique probe of the very early universe



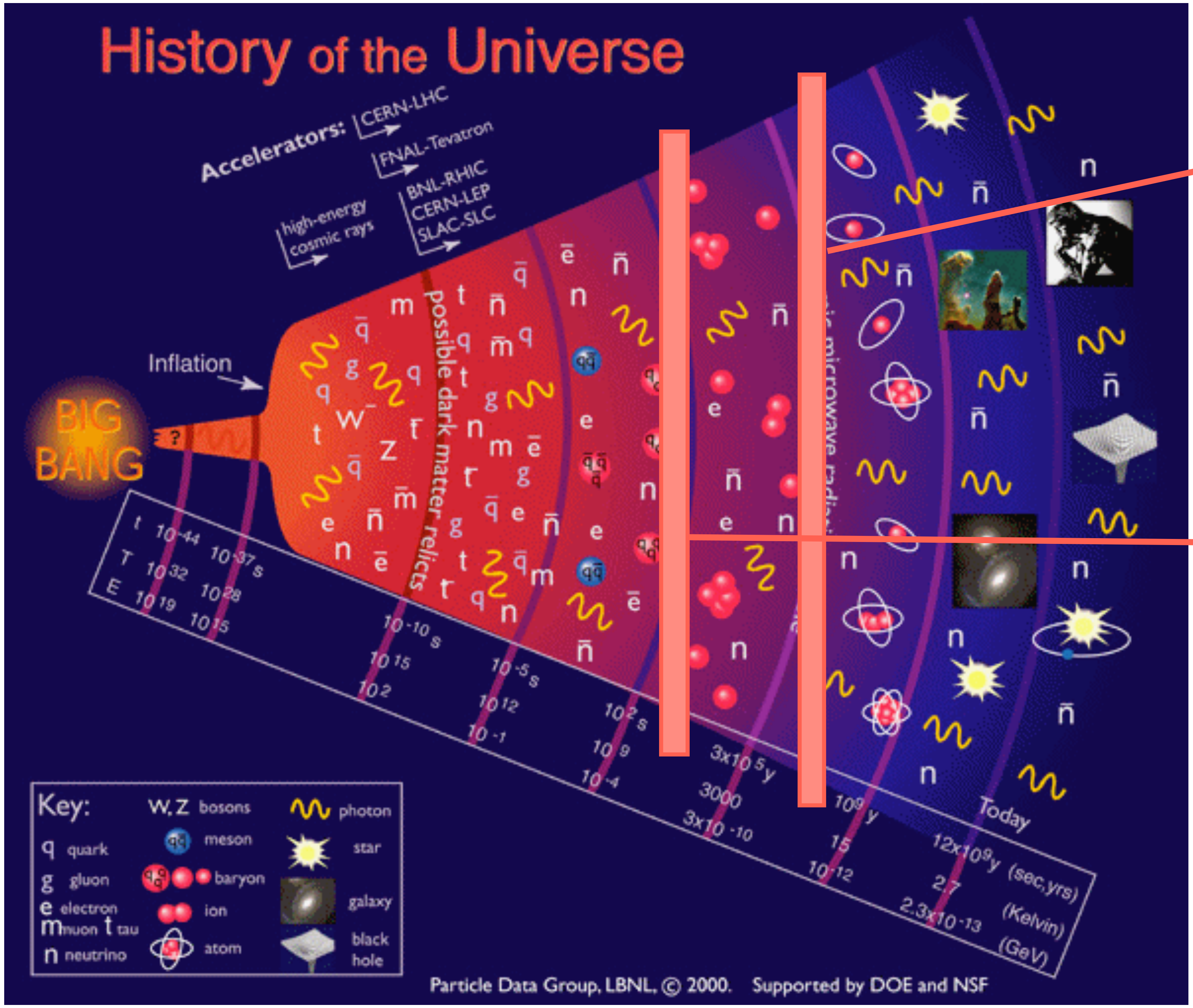
GW: unique probe of the very early universe



$T_{\rm dec} = 0.3\,{\rm eV}$

photons decouple: CMB

GW: unique probe of the very early universe



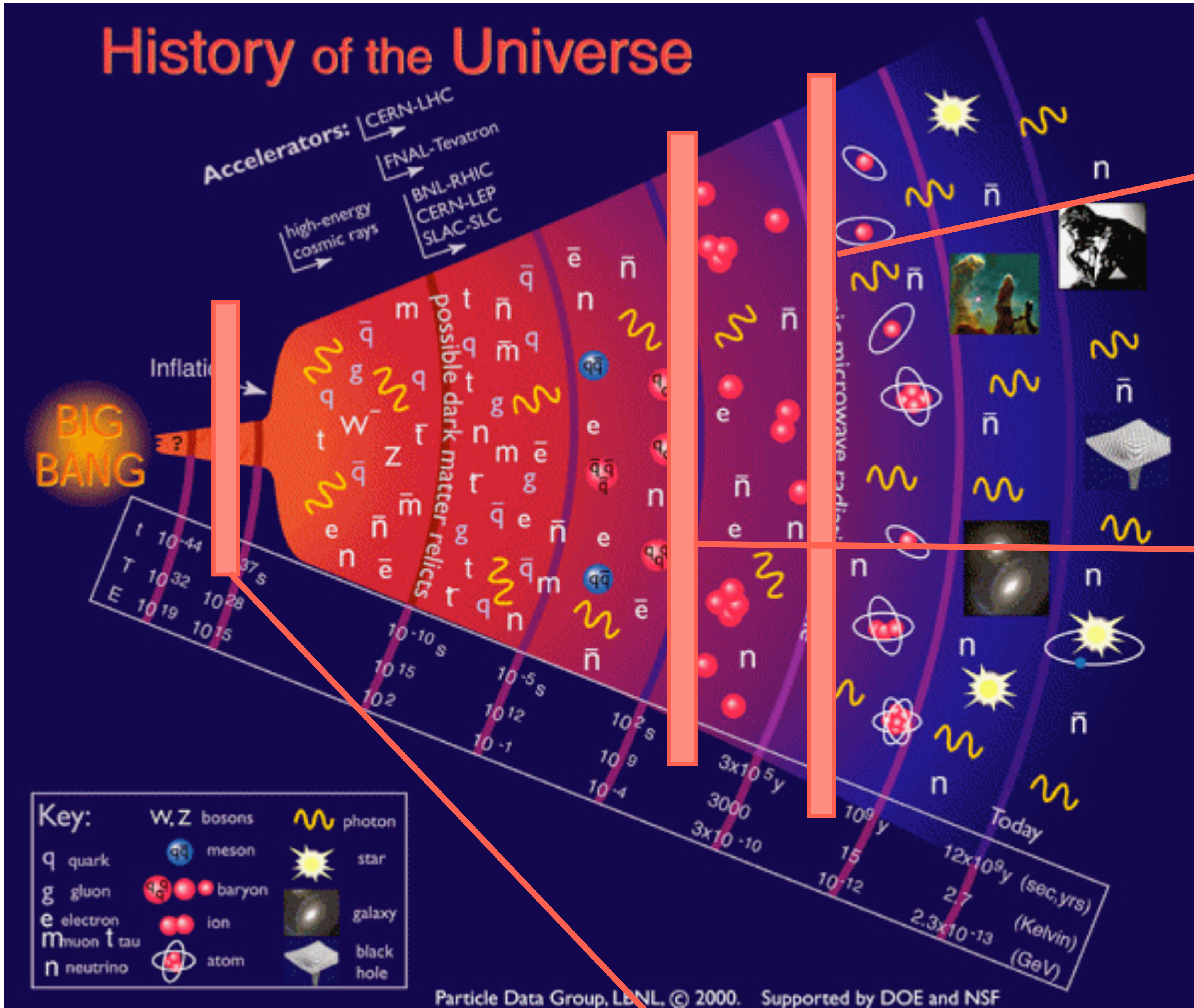
$T_{\rm dec} = 0.3\,{\rm eV}$

photons decouple: CMB

$T_{\rm dec} = 1\,{\rm MeV}$

neutrinos decouple: ν bckg

GW: unique probe of the very early universe



$T_{\rm dec} = 0.3\,{\rm eV}$

photons decouple:
CMB

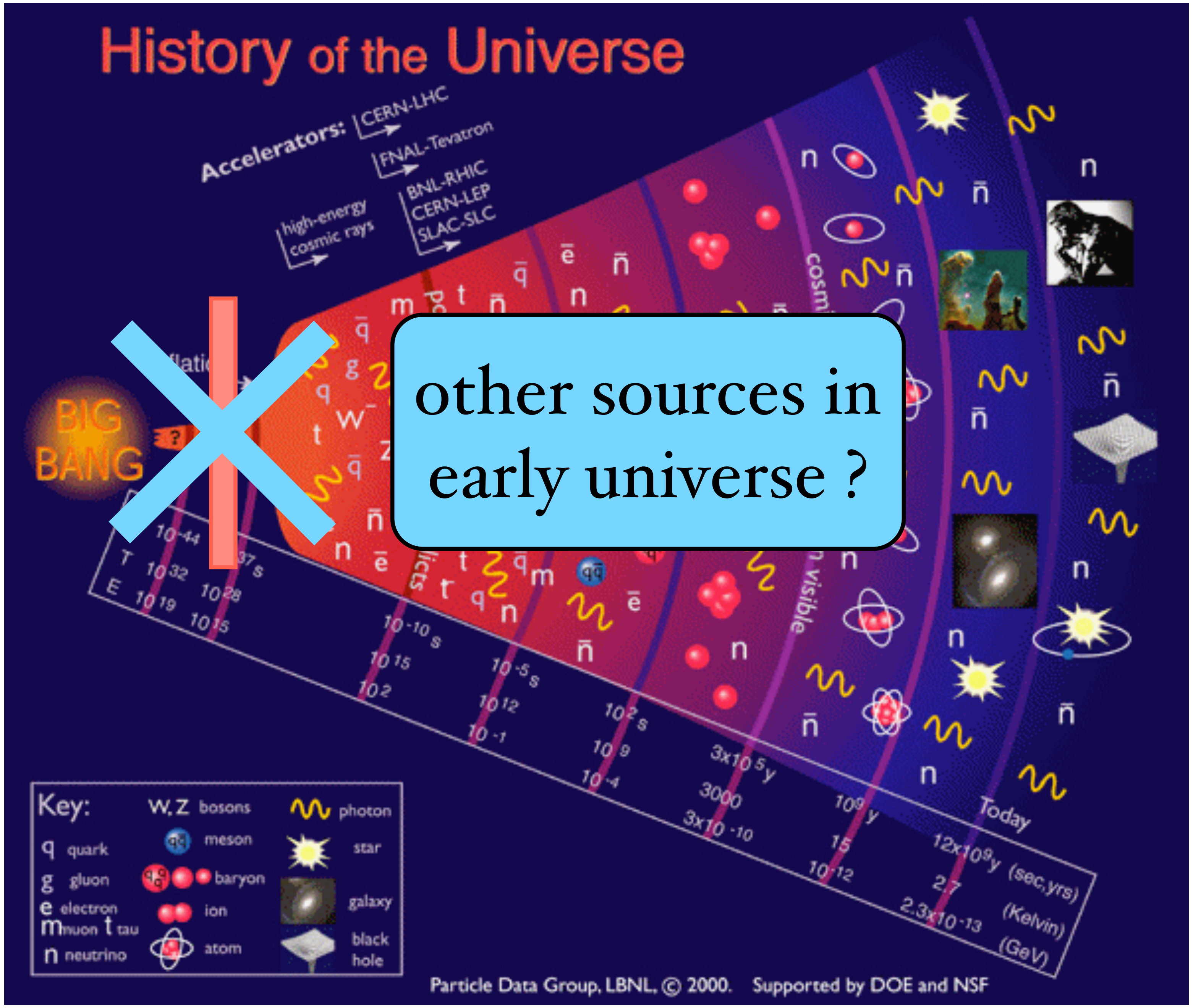
$T_{\rm dec} = 1\,{\rm MeV}$

neutrinos decouple:
ν bckg

$T_{\rm dec} = 10^{19}\,{\rm GeV}$ gravitons decouple: bckg of GW ??

big step forward in knowledge of the very early universe

GW: unique probe of the very early universe



because of the weakness of the gravitational interaction the universe is transparent to GWs

GWs from any generation process in the early universe carry direct information on the process itself

big step forward in knowledge of the very early universe

GW from cosmological sources

- tensor perturbations of FRW metric: $(h_i{}^i = h_i{}^j{}_{|j} = 0)$

$$ds^2 = a^2(t)[-dt^2 + (\delta_{ij} + 2h_{ij})dx^i dx^j]$$

$$G_{\mu\nu} = 8\pi G\, T_{\mu\nu} \qquad \ddot{h}_{ij} + 2\frac{\dot{a}}{a}\dot{h}_{ij} + k^2 h_{ij} = 8\pi G\, a^2\, \Pi_{ij}$$

- source: Π_{ij} tensor anisotropic stress

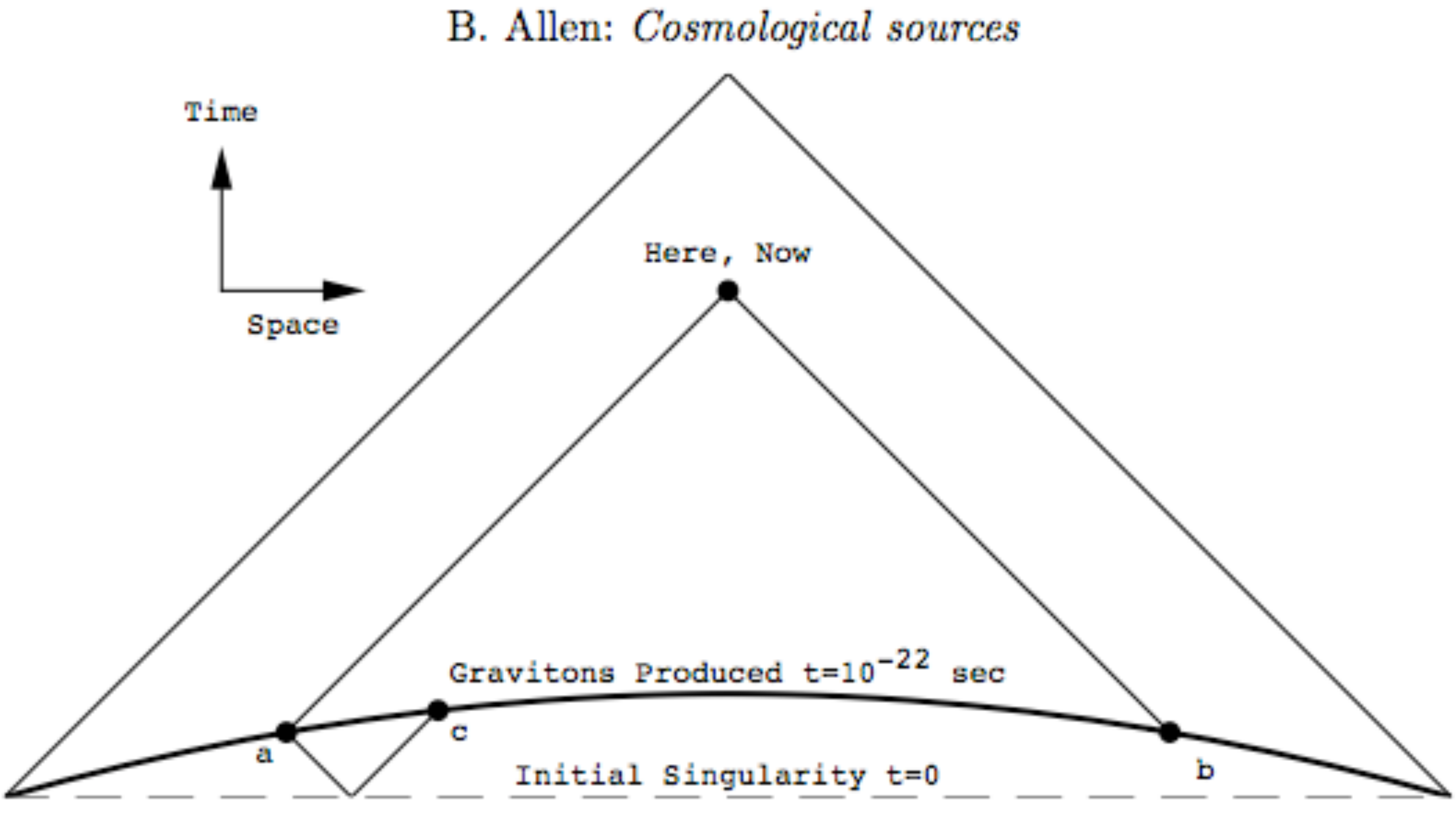
- GW energy density:

$$\Omega_{\rm GW} = \frac{\rho_{\rm GW}}{\rho_c} = \frac{\langle \dot{h}_{ij}\dot{h}_{ij}\rangle}{8\pi G\, a^2\, \rho_c} = \int \frac{{\rm d}k}{k}\frac{{\rm d}\Omega_{\rm GW}}{{\rm d}\ln k}$$

→ power spectrum

GW from cosmological sources

- stochastic background of GW, statistically homogenous, isotropic and Gaussian



GW from cosmological sources

- stochastic background of GW, statistically homogenous, isotropic and Gaussian

$$\langle \dot{h}_{ij}(\mathbf{k})\dot{h}^*_{ij}(\mathbf{q})\rangle = (2\pi)^3\delta(\mathbf{k}-\mathbf{q})|\dot{h}(k)|^2$$

statistical homogeneity and isotropy ← $\delta(\mathbf{k}-\mathbf{q})$

$|\dot{h}(k)|^2$ → power spectrum

$$\frac{\mathrm{d}\Omega_{\mathrm{GW}}}{\mathrm{d}\ln k} = \frac{k^3|\dot{h}(k)|^2}{2(2\pi)^3\,G\,a^2\,\rho_c}$$

- Gaussian: many independent horizon volumes visible today

Characteristic frequency

- causal source of GW cannot operate beyond the cosmological horizon:

$$k_* \geq \mathcal{H}_*$$

Characteristic frequency

- causal source of GW cannot operate beyond the cosmological horizon:

$$k_* \geq \mathcal{H}_*$$

$$k_* = 1.6 \cdot 10^{-7}\, \epsilon^{-1} \left(\frac{T_*}{1\,\mathrm{GeV}}\right) \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \mathrm{Hz}$$

characteristic wavenumber

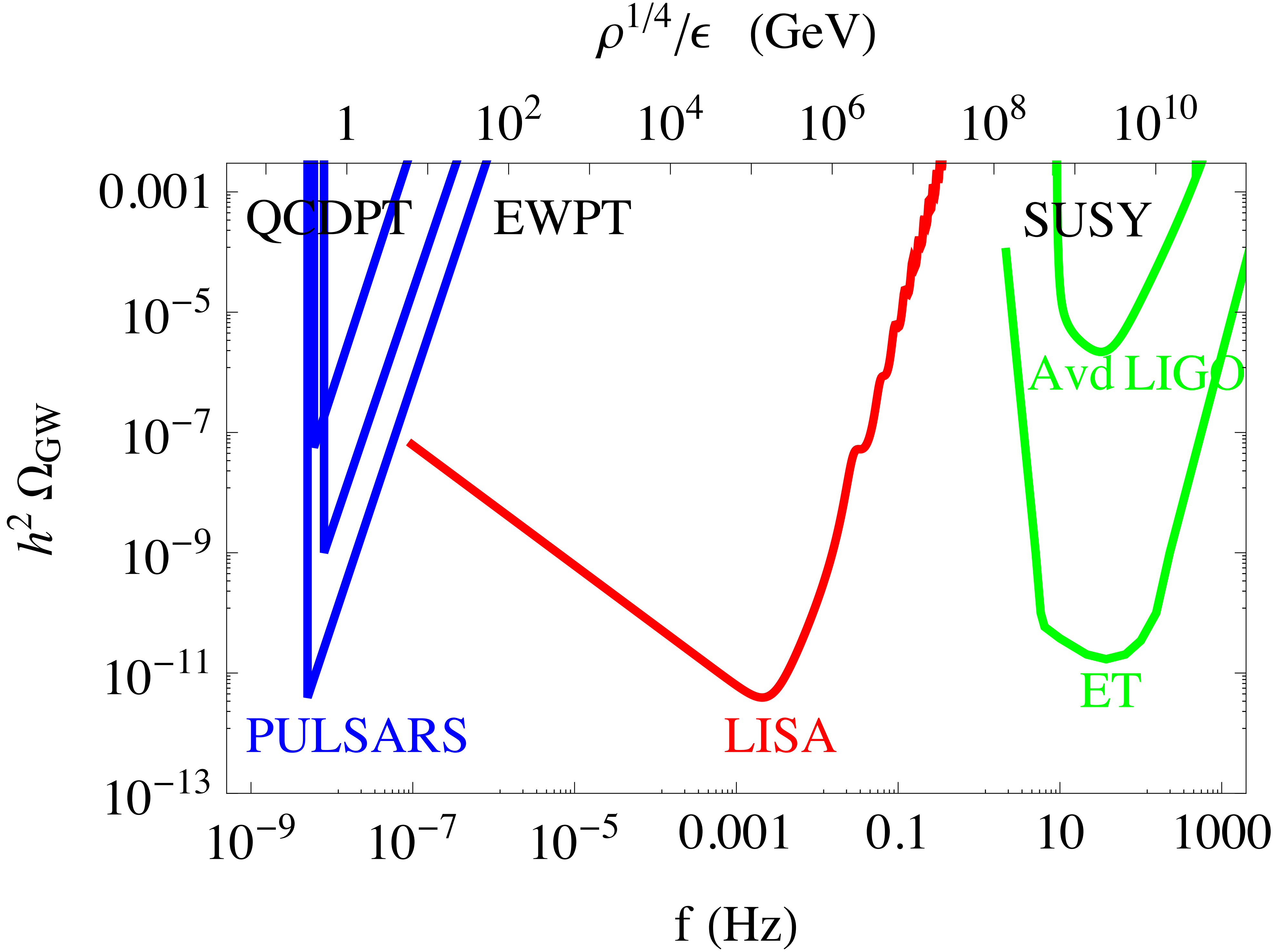
$\epsilon \leq 1$

dynamics of the source

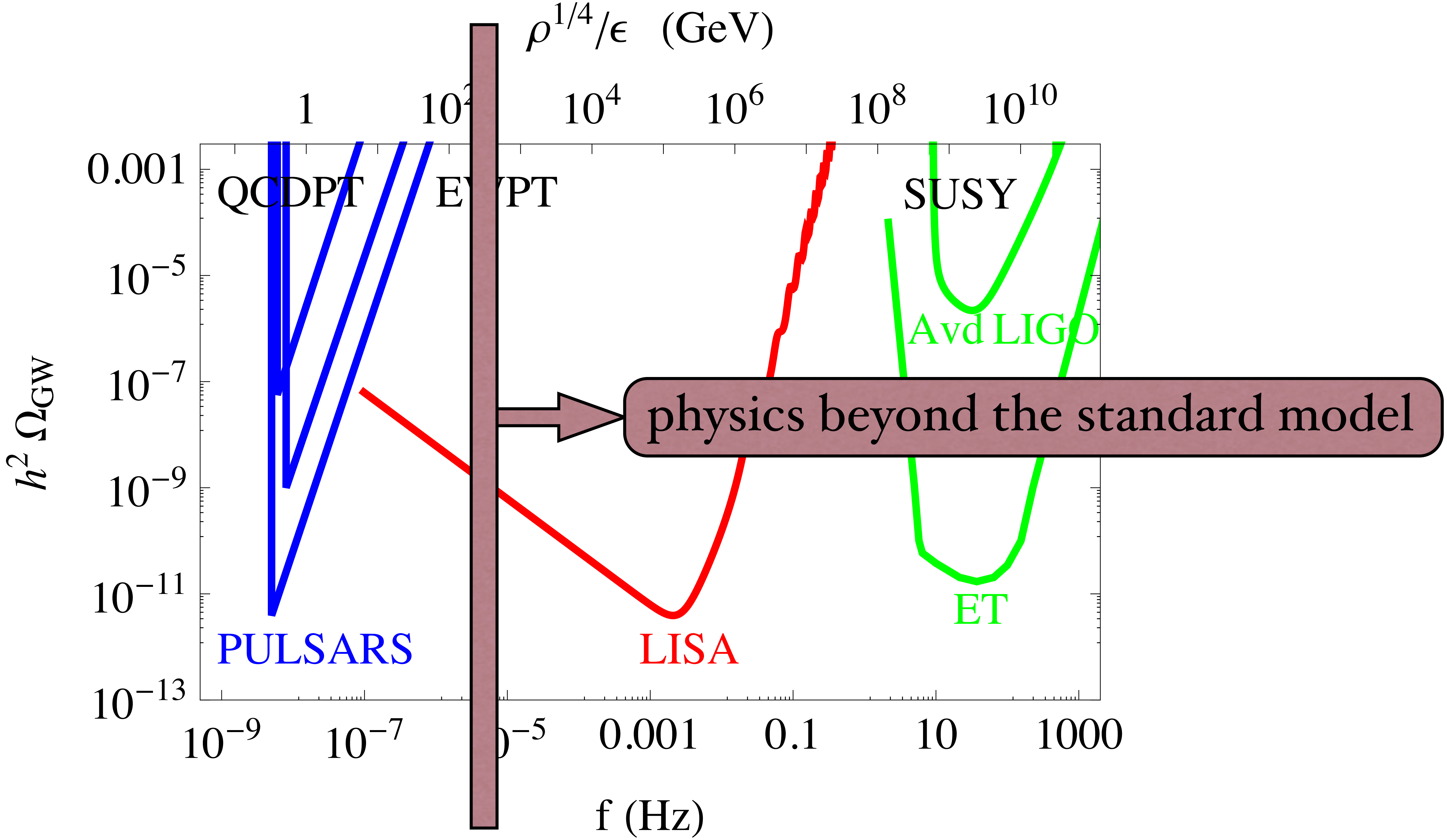
temperature (energy density) of the universe at the source time

(standard thermal history)

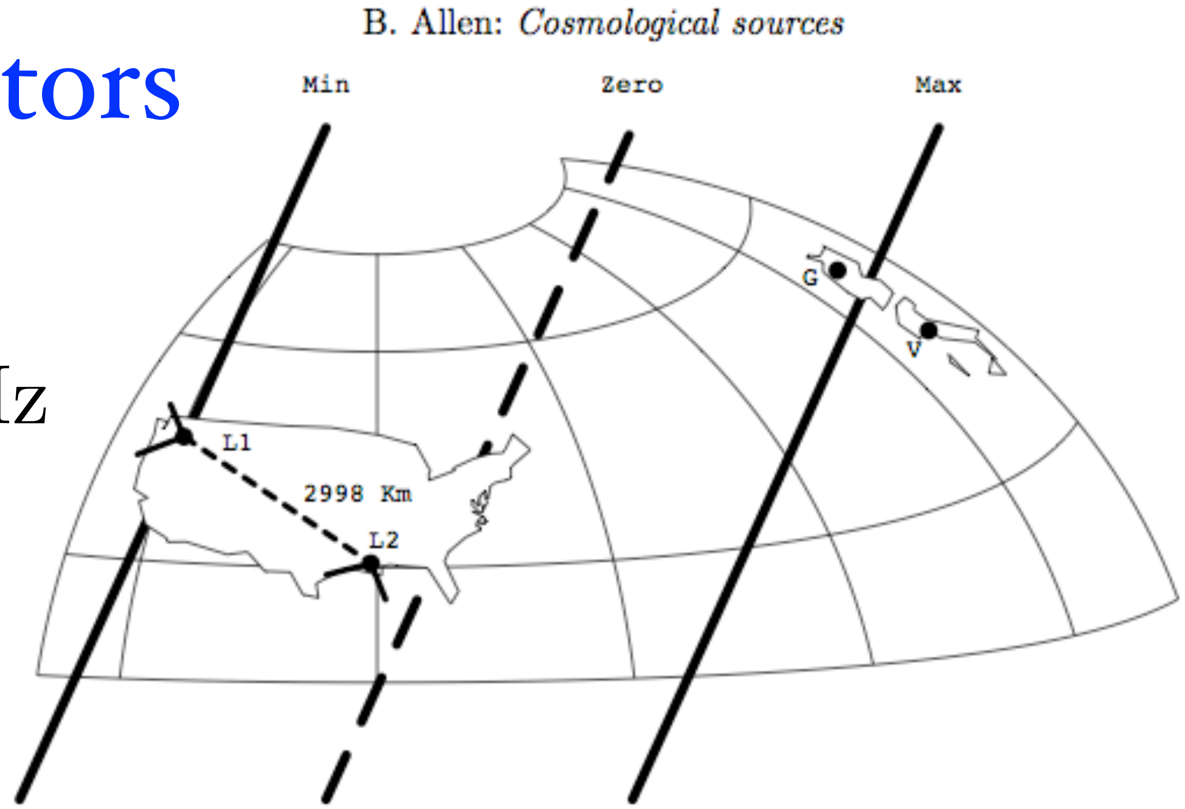
Characteristic frequency



Characteristic frequency

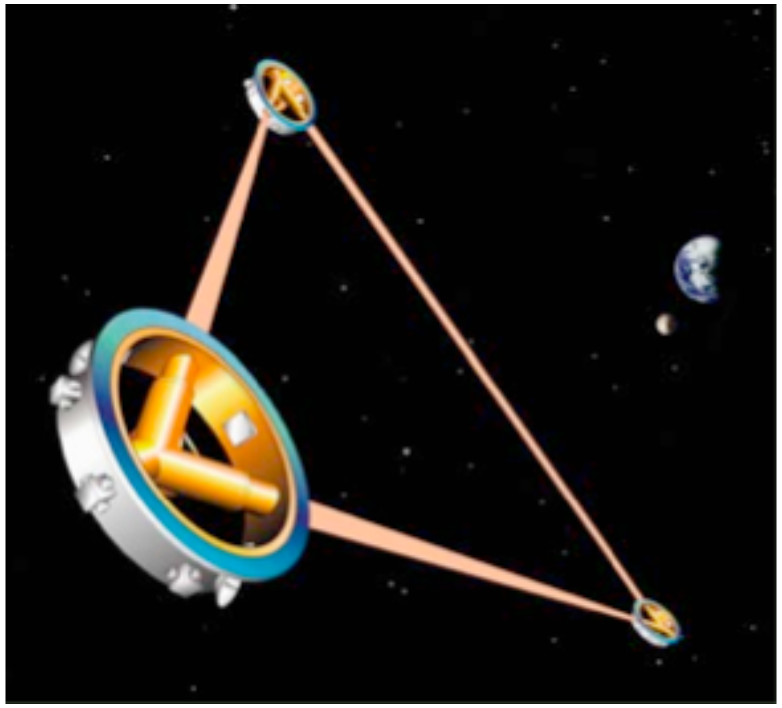


Stochastic background detectors



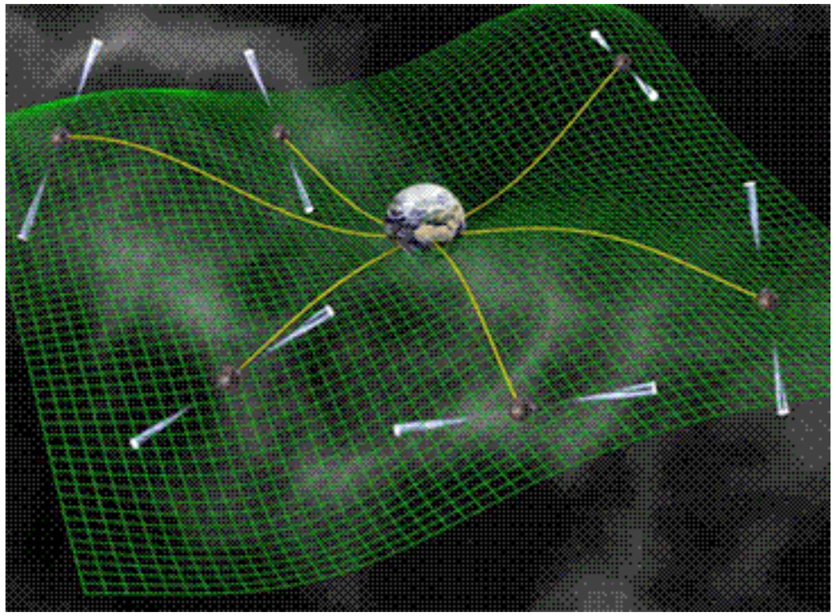
- correlation of earth based detectors:
  LIGO, VIRGO, GEO $1\,\mathrm{Hz} < f < 1\,\mathrm{kHz}$

- solar system scale detector: LISA
  $10^{-4}\,\mathrm{Hz} < f < 1\,\mathrm{Hz}$



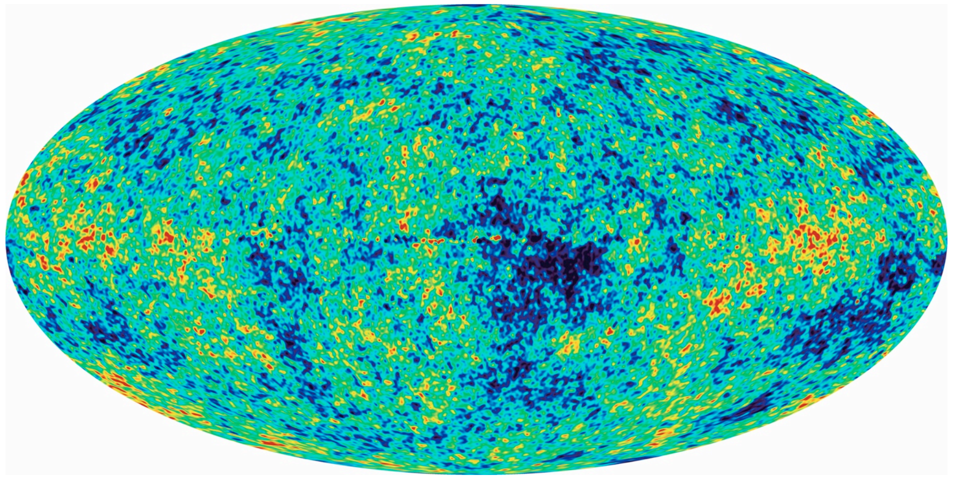
- galactic scale detector:
  msec pulsars

  $10^{-9}\,\mathrm{Hz} < f < 10^{-7}\,\mathrm{Hz}$



- Hubble scale detector: CMB

  $f > 10^{-18}\,\mathrm{Hz}$



Current limits on a stochastic background

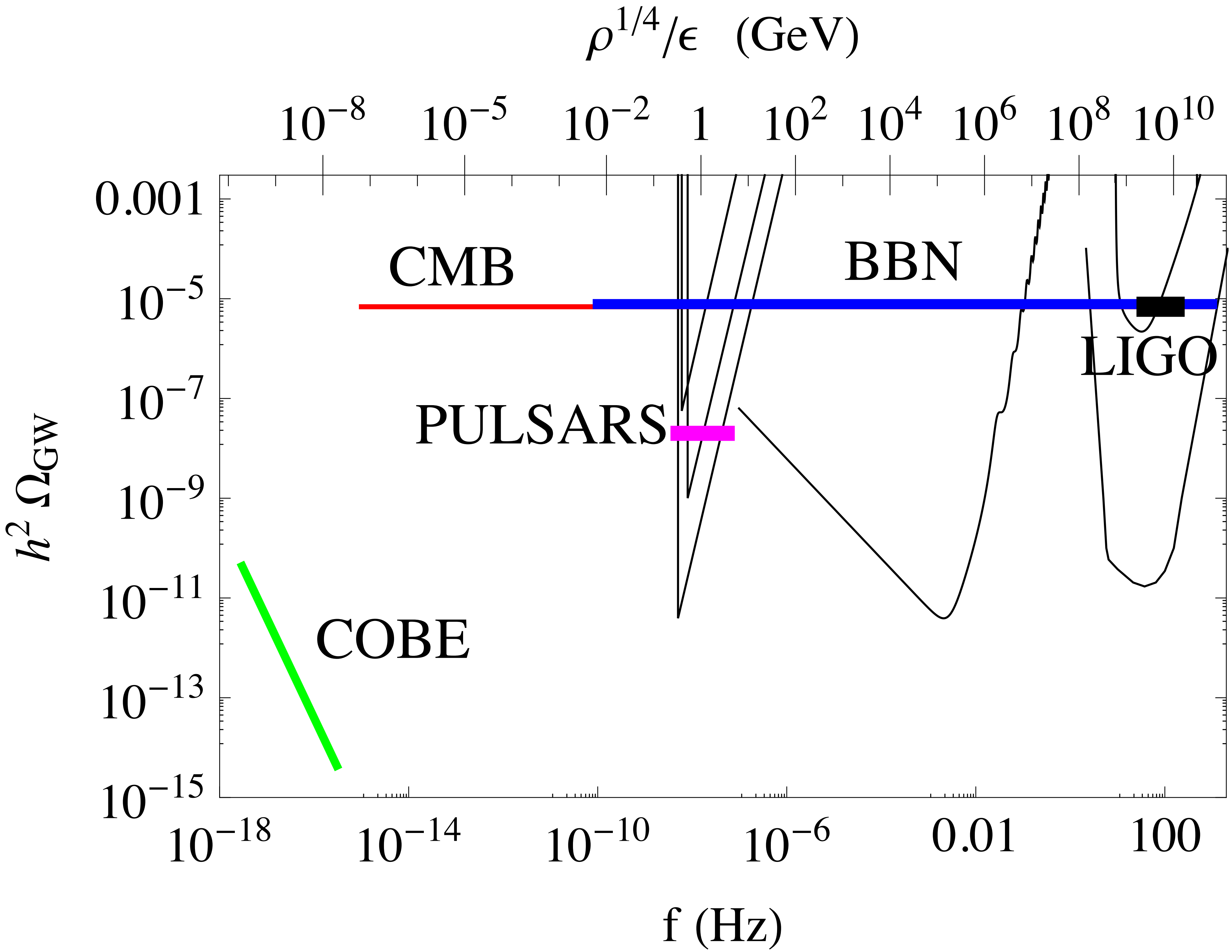
- Nucleosynthesis and CMB: measure of the relativistic energy density in the universe

$$h^2\Omega_{\rm GW} \lesssim 7.8 \cdot 10^{-6} \qquad f > 10^{-10}\,{\rm Hz}$$

$$h^2\Omega_{\rm GW} \lesssim 6.9 \cdot 10^{-6} \qquad f > 10^{-16}\,{\rm Hz}$$

(Smith et al, astro-ph/0603144)

Current limits on a stochastic background



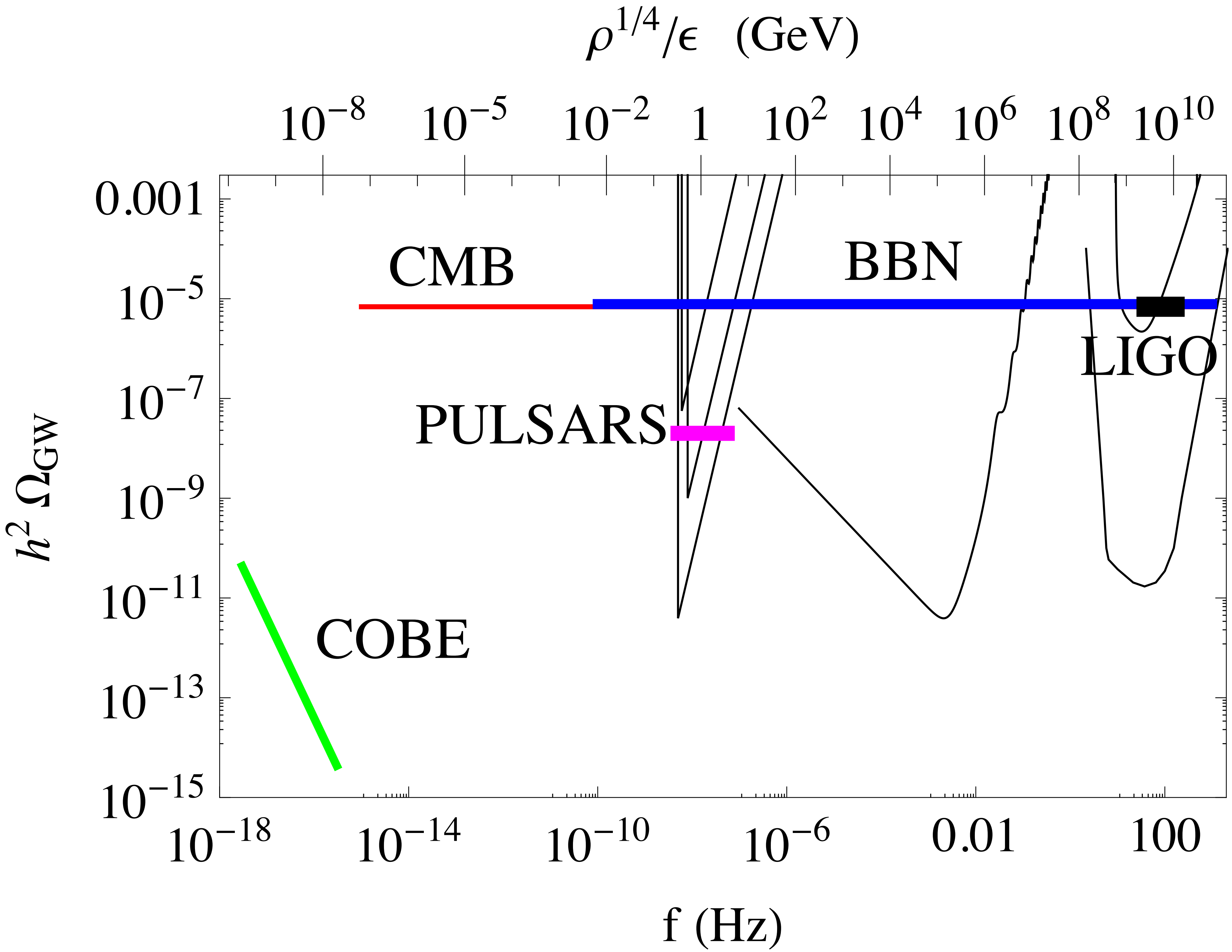
Current limits on a stochastic background

- Nucleosynthesis and CMB: measure of the relativistic energy density in the universe
- LIGO science run 2005-2007 : cross-correlation of LIGO interferometers

$$h^2\Omega_{\rm GW} < 6.9 \cdot 10^{-6} \qquad 41\,{\rm Hz} < f < 169\,{\rm Hz}$$

(Abbott et al, 0910.5772)

Current limits on a stochastic background



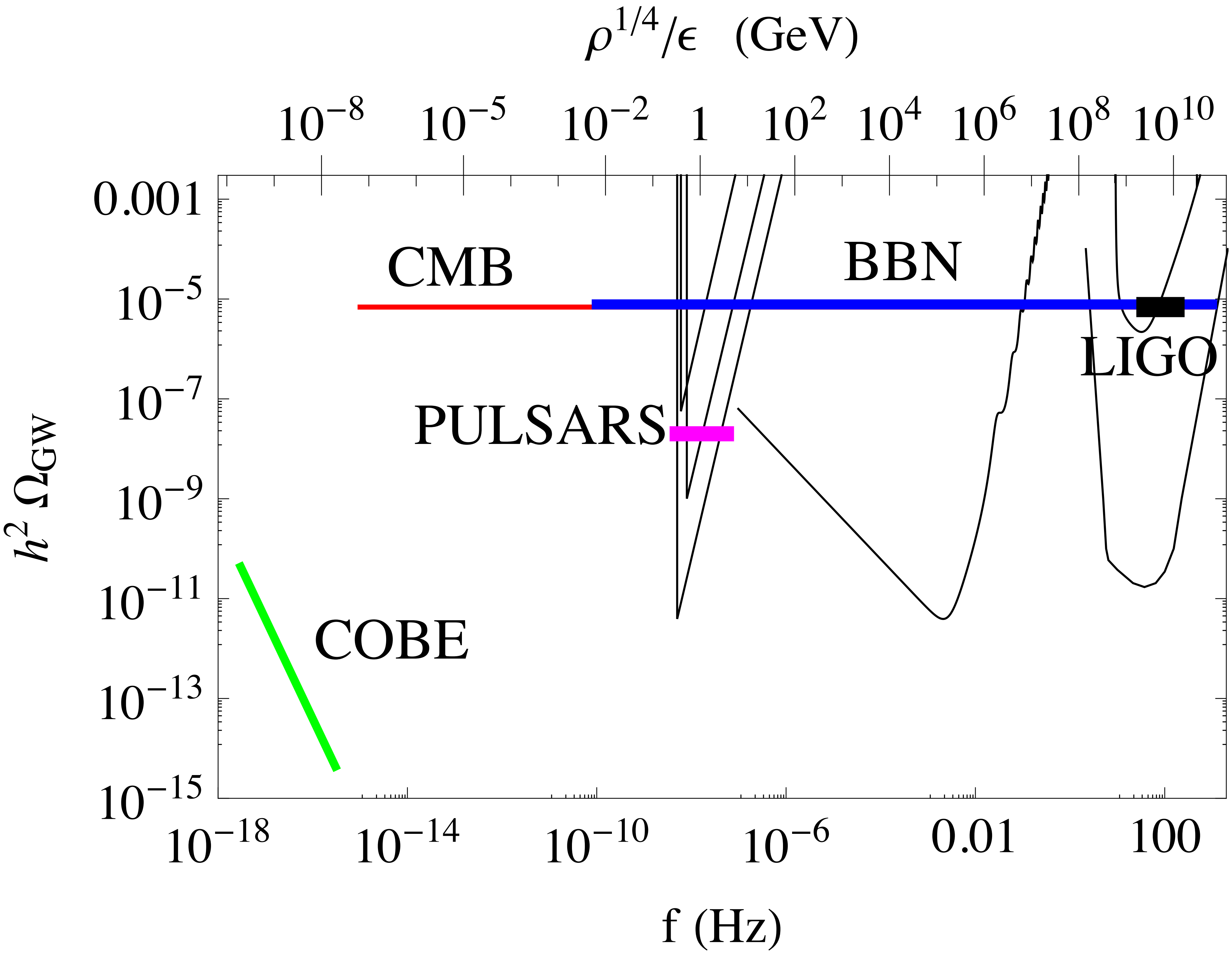
Current limits on a stochastic background

- Nucleosynthesis and CMB: measure of the relativistic energy density in the universe
- LIGO science run 2005-2007 : cross-correlation of three interferometers
- combined data sets of observation of 7 pulsars for 8 years (Parkes pulsars timing array)

$$h^2\Omega_{\rm GW} < 2 \cdot 10^{-8} \qquad\qquad f \simeq 10^{-9}\,{\rm Hz}$$

(Jenet et al, astro-ph/0609013)

Current limits on a stochastic background

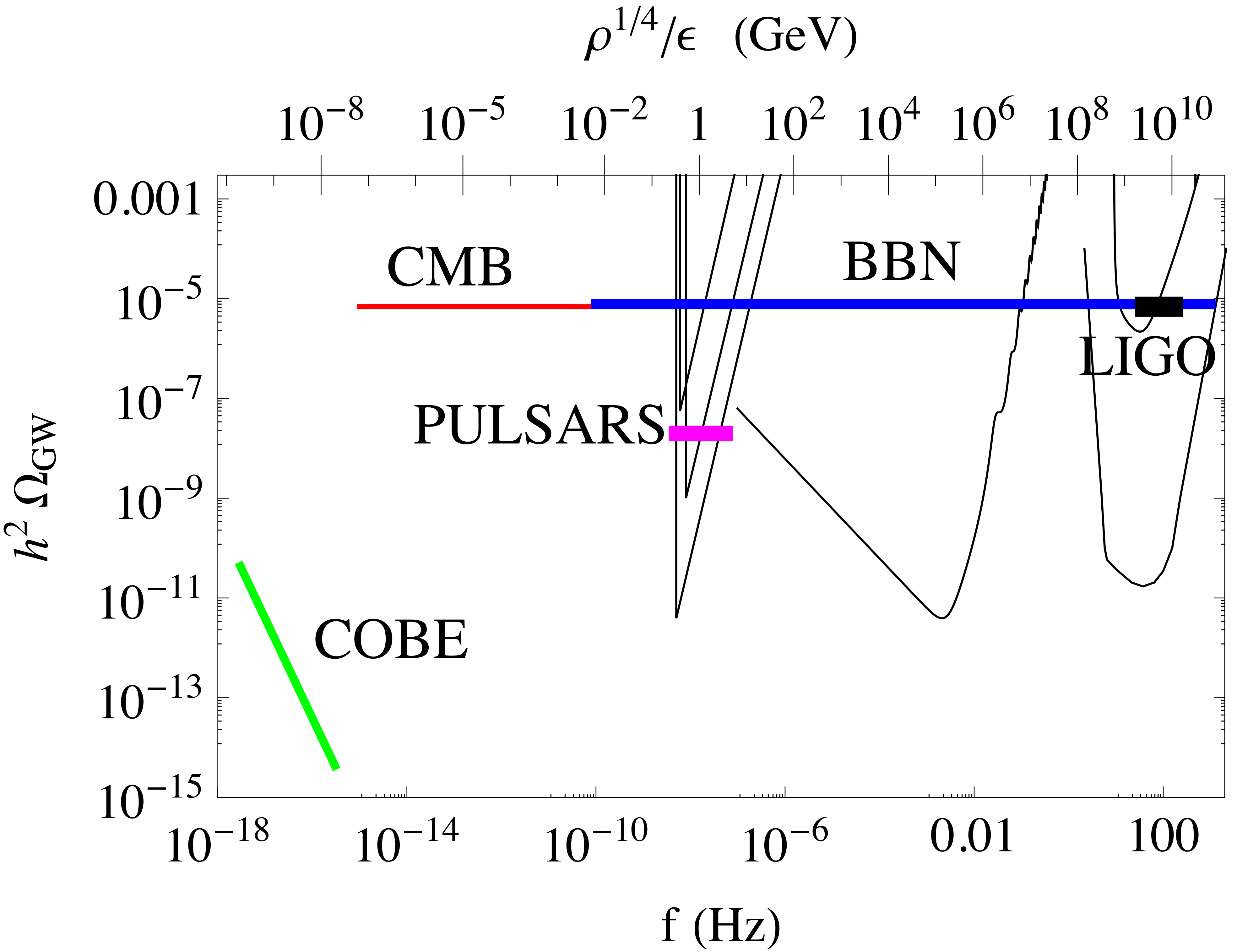


Current limits on a stochastic background

- Nucleosynthesis and CMB: measure of the relativistic energy density in the universe
- LIGO science run 2005-2007 : cross-correlation of three interferometers
- combined data sets of observation of 7 pulsars for 8 years (Parkes pulsars timing array)
- COBE measurements of temperature fluctuations in CMB

$$h^2\Omega_{\rm GW} < 7 \cdot 10^{-11} \left(\frac{H_0}{f}\right)^2 \qquad 10^{-18}\,{\rm Hz} < f < 10^{-16}\,{\rm Hz}$$

Current limits on a stochastic background



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GW background from inflation

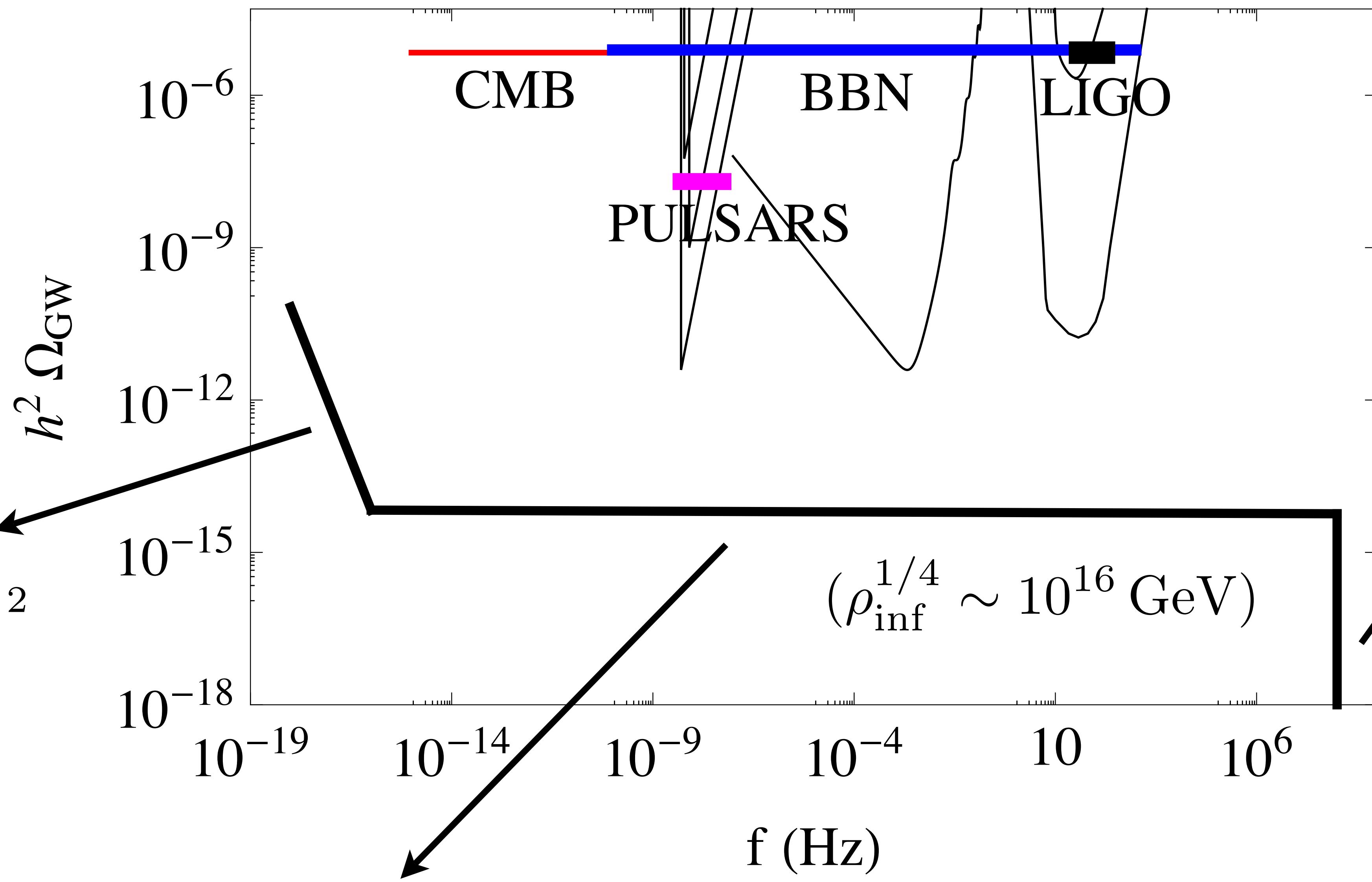
- amplification of vacuum fluctuations at the transition between inflationary and RD phase (followed by MD)

$$\ddot{h}_{ij} + \left(k^2 - \frac{\ddot{a}}{a}\right) h_{ij} = 0$$

$H_0 < k < H_{\rm eq}$

enter the horizon in the MD era

$$\frac{d\,\Omega_{\rm GW}}{d\ln k} \sim 10^{-5} \times \left(\frac{H_{\rm inf}}{m_{\rm Pl}}\right)^2 \left(\frac{k_{\rm eq}}{k}\right)^2$$



$k > H_{\rm inf}$

smooth transition no particle creation

$H_{\rm eq} < k < H_{\rm inf}$

abrupt transition particle creation

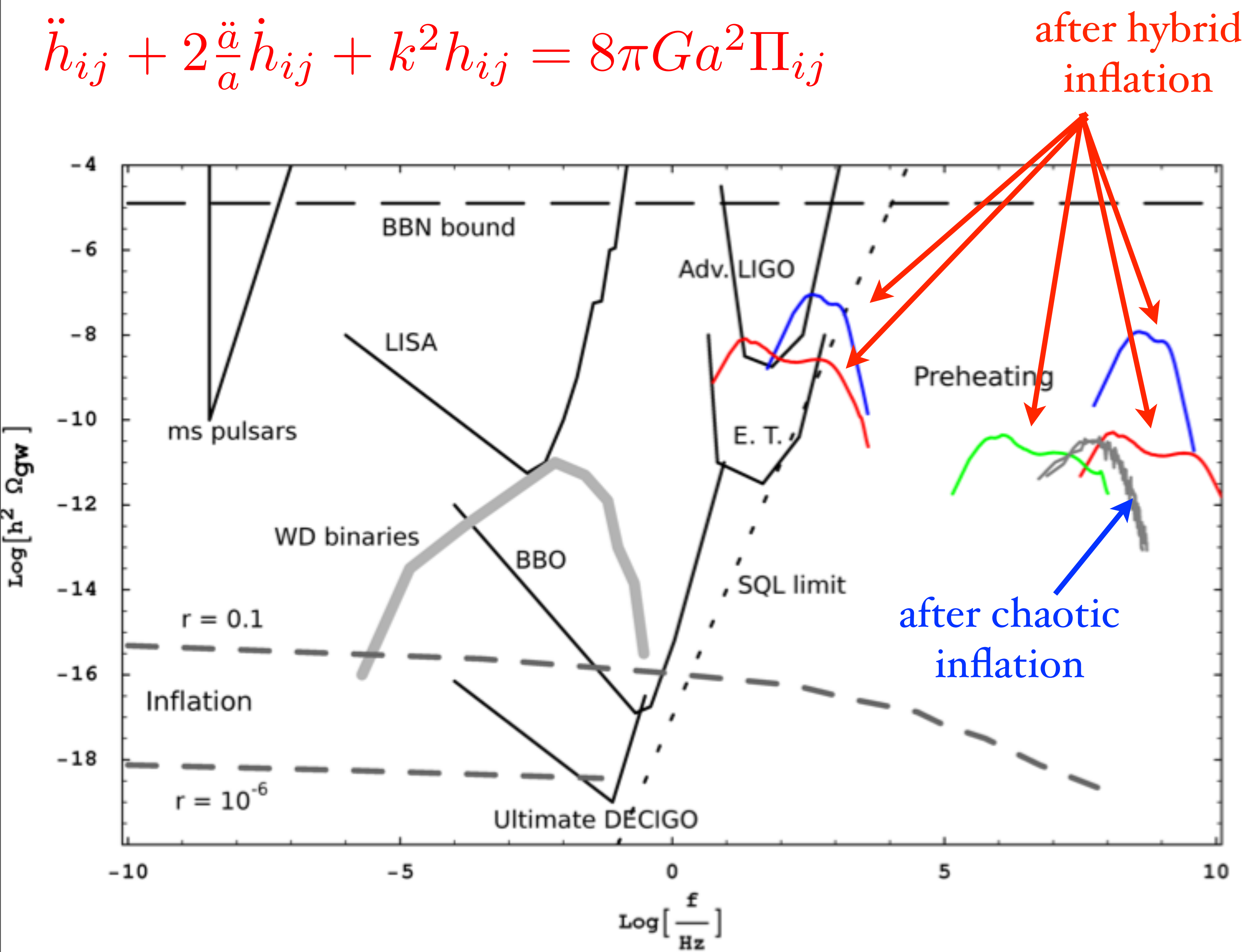
$$\frac{d\,\Omega_{\rm GW}}{d\ln k} \sim 10^{-5} \left(\frac{H_{\rm inf}}{m_{\rm Pl}}\right)^2 \left(\frac{k}{k_{\rm eq}}\right)^{n_T}$$

(Grishchuk 1974, Starobinsky 1979, Abbott and Harari 1986, ...)

GW background from preheating

- reheating: the energy density driving inflation is converted in radiation and matter
- preheating: possible first stage of this conversion, the inflaton decays in an explosive and highly inhomogeneous way

$$\ddot{h}_{ij} + 2\frac{\ddot{a}}{a}\dot{h}_{ij} + k^2 h_{ij} = 8\pi G a^2 \Pi_{ij}$$



R_* size of the inhomogeneities

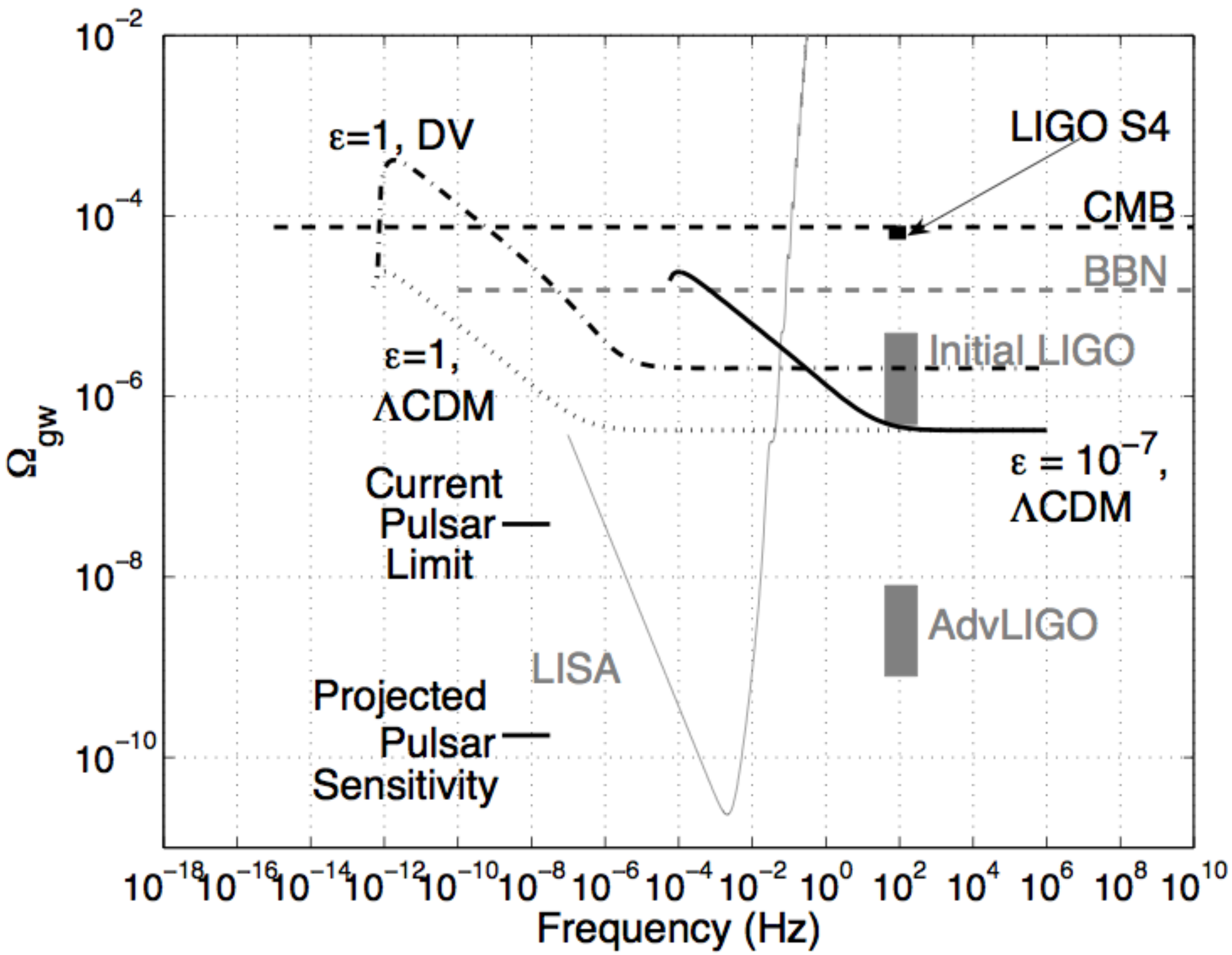
$$f_* \sim \frac{4 \cdot 10^{10}\,\mathrm{Hz}}{R_*\,\rho_p^{1/4}}$$

$$\Omega^*_{\rm GW} \sim 10^{-6}(R_*\,H_p)^2$$

(Kofman et al 1994, Khlebnikov and Tkachev 1997, ..., Dufaux et al 2009)

GW background from cosmic strings

- one dimensional topological defects formed during symmetry breaking phase transitions or in the context of string theory at the end of brane inflation
- form a cosmological network which reach a scaling regime where the only relevant scale is the Hubble length and decay by GW emission from string loops



STOCHASTIC BACKGROUND FROM CUSPS

mass per unit length

$$G\mu = 10^{-7}$$

reconnection probability

$$p = 5 \cdot 10^{-3}$$

size of the loops at formation

$$\varepsilon = \alpha/(50G\mu)$$

(Kibble 1976, Vilenkin 1985, Damour and Vilekin 2000, Sarangi and Tye 2002 ..., Siemens et al 2009)

GW from first order phase transitions

- universe expands and temperature decreases : PT
- nature of PT depends on the particle theory model
- if it is first order it can lead to the production of GW

(Hogan '83, Witten '84, Hogan '86...)

(Turner et al '92, Kosowsky et al '92, Kosowsky and Turner '93, Kamionkowski et al '94, Kosowsky et al '02, Dolgov et al '02, Gogoberidze et al '08, Kahniashvili et al '08...)

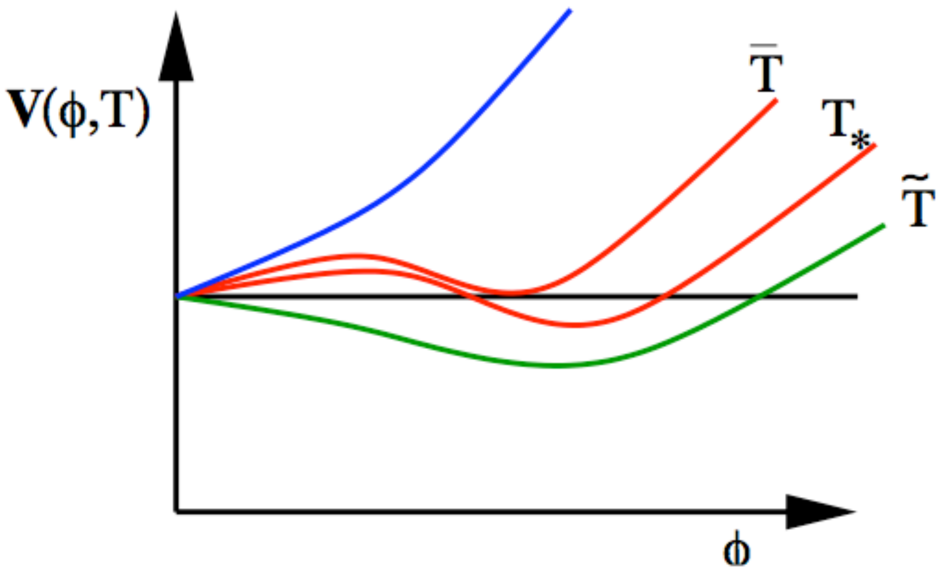
- EWPT : beyond the standard model (baryogenesis)

(Apreda et al '01, Nicolis '04, Grojean et al '05, Huber and Konstandin '08, Kahniashvili et al '09, Kehayias and Profumo '09, Chung and Long '10...)

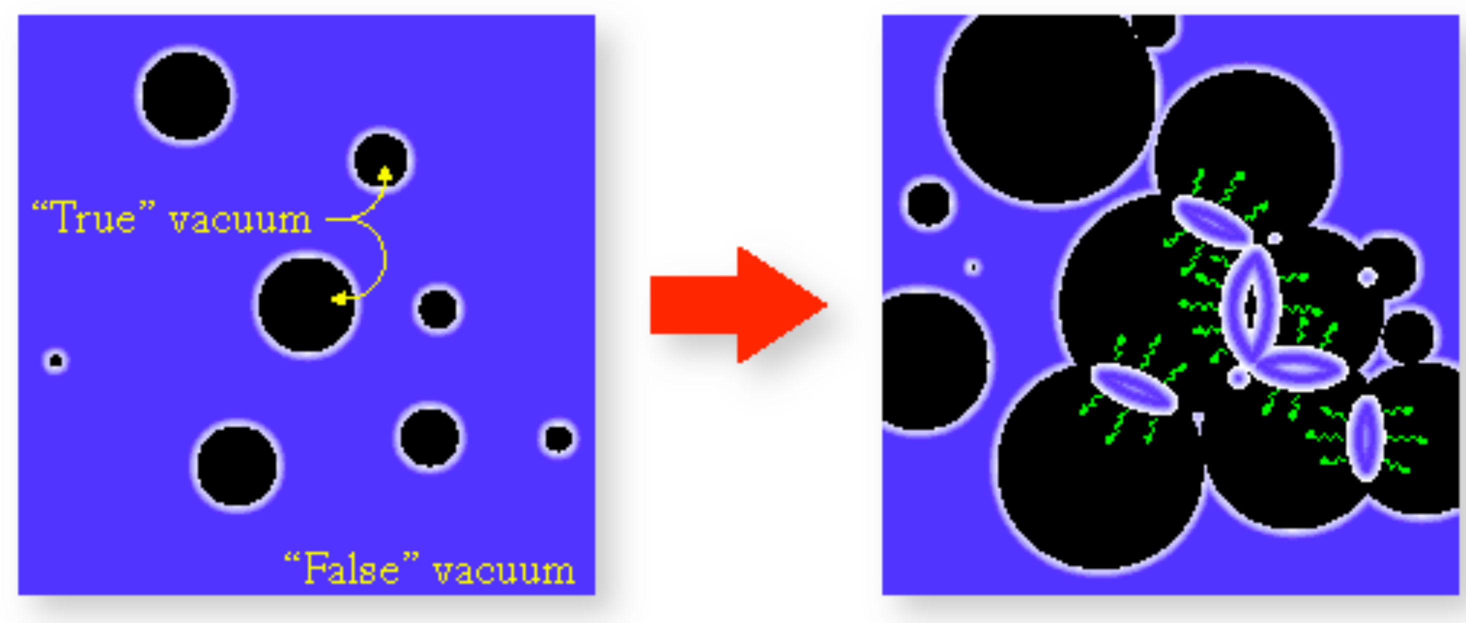
- QCDPT : if lepton asymmetry is large (Schwarz and Stuke '09)

GW from first order phase transitions

potential barrier separates true and false vacua



quantum tunneling across the barrier : nucleation of bubbles of true vacuum



- collisions of bubble walls
- MHD turbulence in the primordial fluid
- primordial magnetic fields

GW from PT : characteristic frequency

GW generation processes related to size of the bubbles
towards the end of the PT

$$k_* = 1.6 \cdot 10^{-7} \epsilon^{-1} \left(\frac{T_*}{1\,\mathrm{GeV}} \right) \left(\frac{g_*}{100} \right)^{\frac{1}{6}} \mathrm{Hz}$$

$\epsilon \leq 1$

dynamics of the source

β^{-1} duration of the PT

$R_* = v_b \, \beta^{-1}$ size of bubbles at collision

$v_b \leq 1$ speed of bubble walls

GW from PT : characteristic frequency

$$\epsilon \simeq \mathcal{H}_* \, \beta^{-1} \, , \; \mathcal{H}_* \, R_*$$

- corresponding to the source characteristic time or scale depending on the source properties: space and time correlations

- value : $\dfrac{\beta}{\mathcal{H}_*} \sim 4 \ln \left(\dfrac{m_{\rm Pl}}{T_*} \right)$ ⟶ $\epsilon \simeq \; 0.01$

(Hogan '83)

EWPT

$$k_{100\rm GeV} \sim 10^{-3}{\rm Hz} \quad \text{LISA}$$

QCDPT

$$k_{100\rm MeV} \sim 10^{-7}{\rm Hz} \quad \text{pulsars}$$

($k_{10^7\rm GeV} \sim 100\,{\rm Hz}$ LIGO)

GW from PT : characteristic amplitude

$$\Omega_{\rm GW} \sim \Omega_{\rm rad} \left(\frac{\mathcal{H}_*}{\beta}\right)^2 (\Omega_{\rm s}^*)^2$$

radiation parameter

10^{-5}

duration of the source with respect to Hubble time

10^{-4}

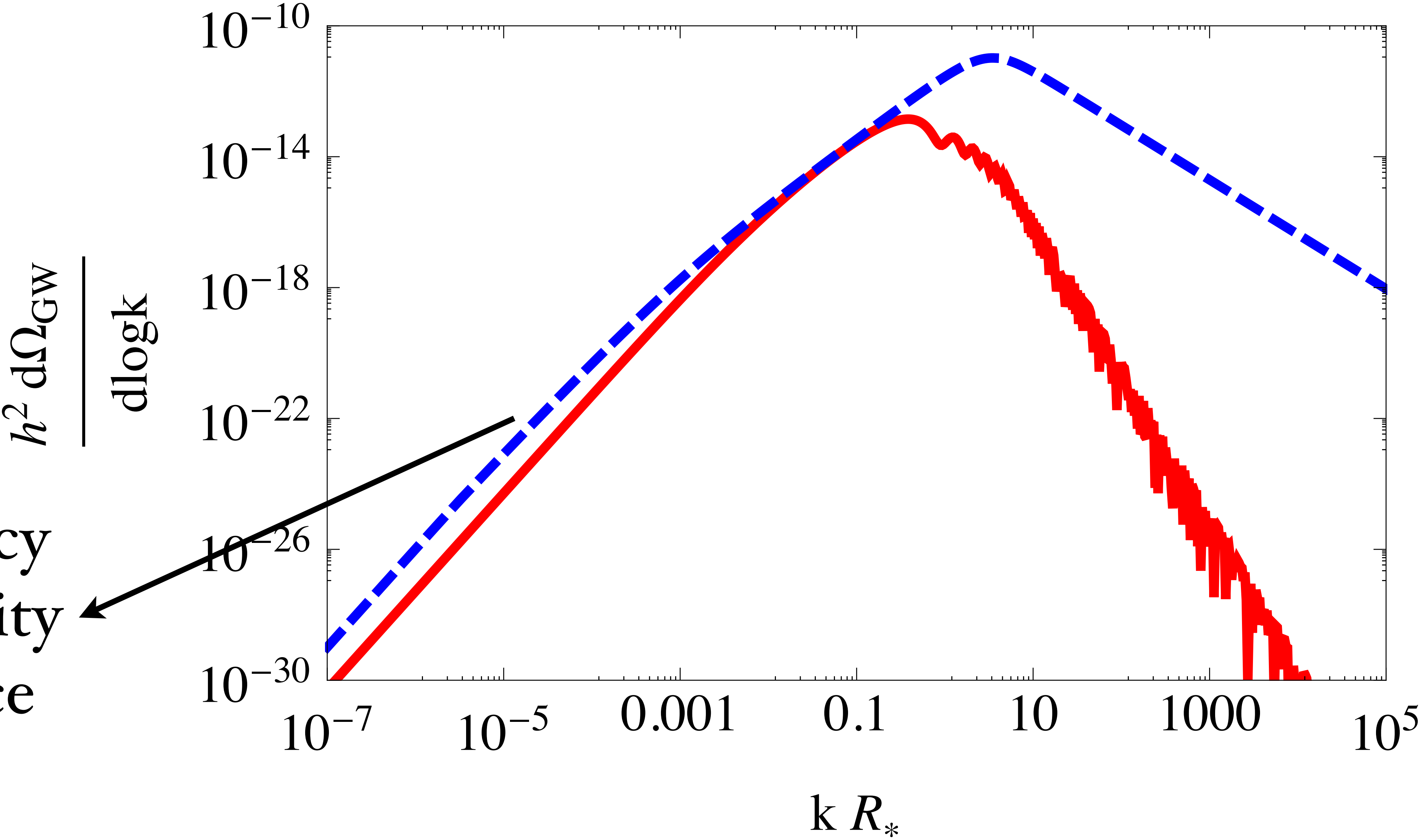
relative energy density available in the (radiation like) source for the GW generation

$$T \sim \rho_* \, \Omega_{\rm s}^*$$

10^{-2} for very strong PT

amplitude OK for LISA and future PTA

Characteristic shape of the GW power spectrum



low frequency slope: causality of the source

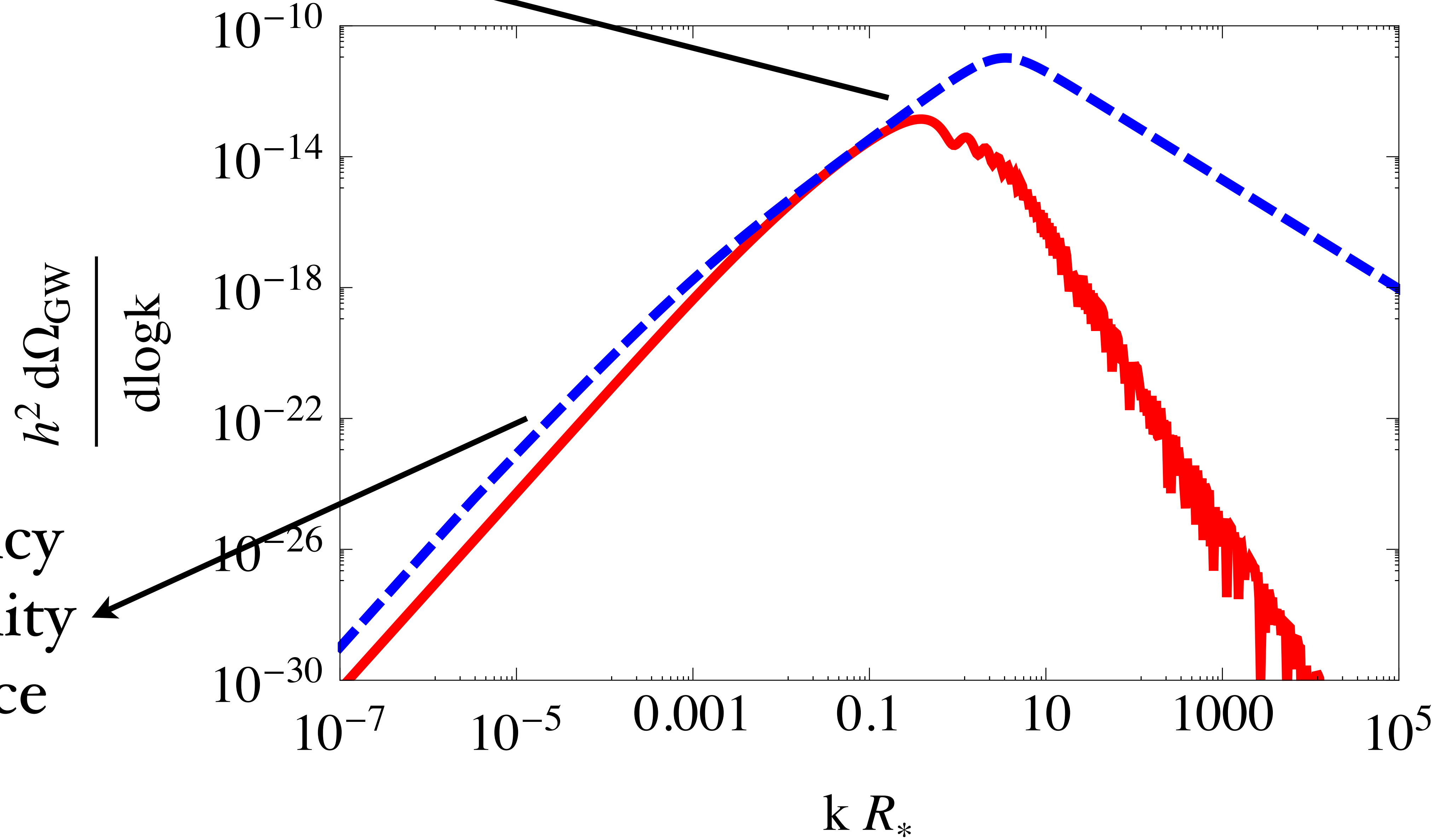
k^3

Characteristic shape of the GW power spectrum

peak position: depending on the source properties (time correlation)

$k_* \sim \beta \quad k_* \sim R_*^{-1}$

$R_* = v_b/\beta$



low frequency slope: causality of the source

k^3

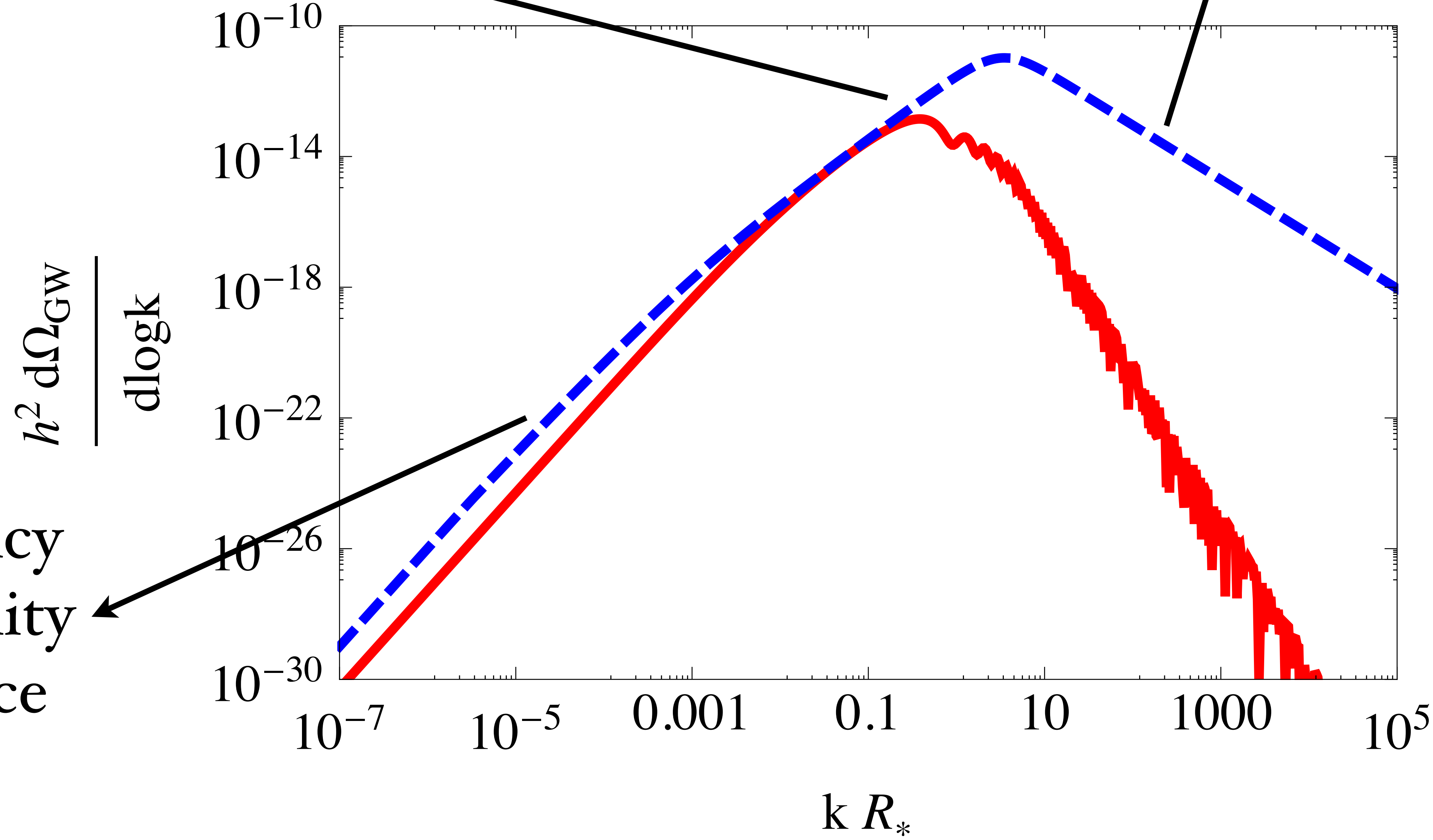
Characteristic shape of the GW power spectrum

peak position: depending on the source properties (time correlation)

$k_* \sim \beta$ $\quad$ $k_* \sim R_*^{-1}$

$R_* = v_b/\beta$

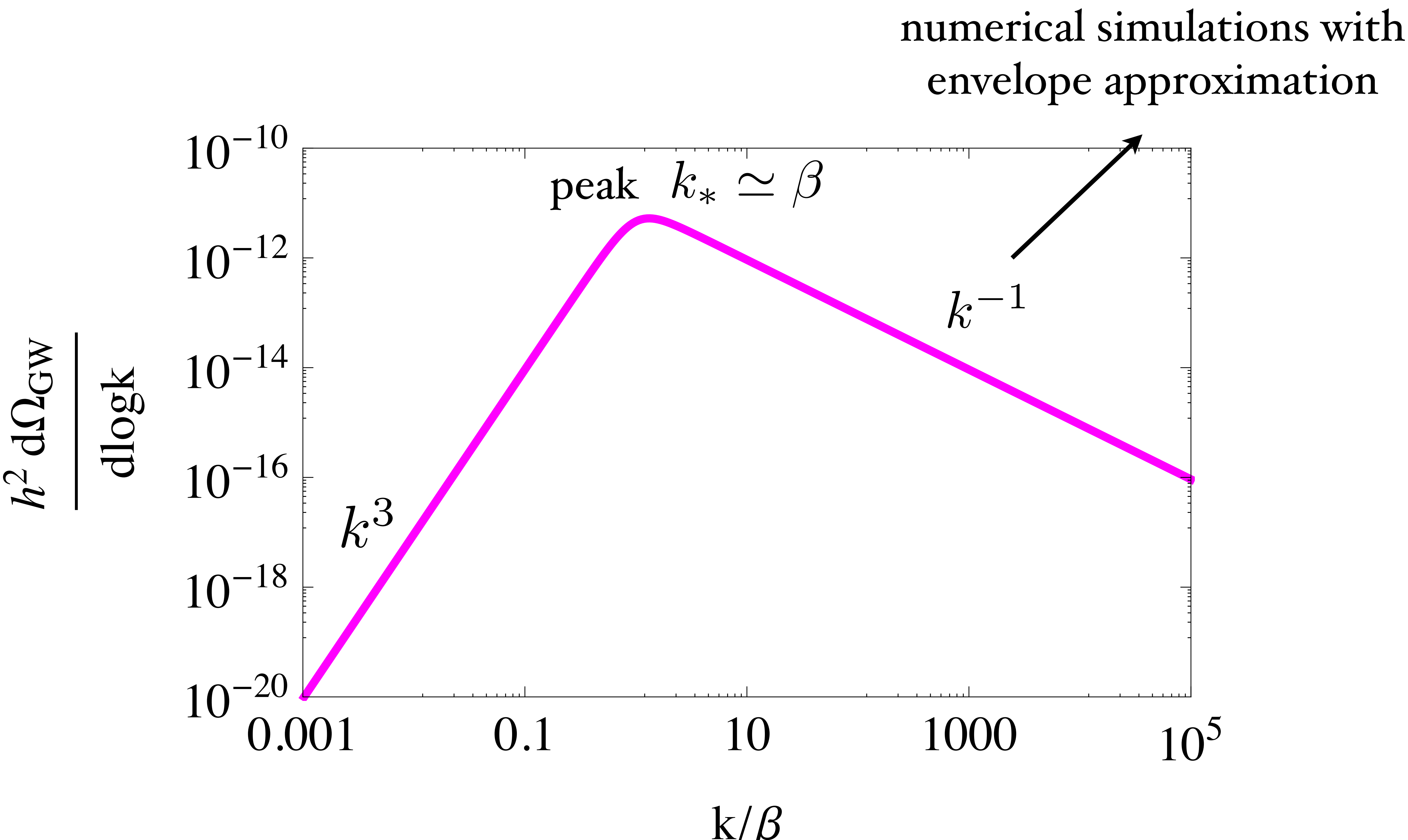
high frequency slope: depends on both power spectrum and time correlation of the source



low frequency slope: causality of the source

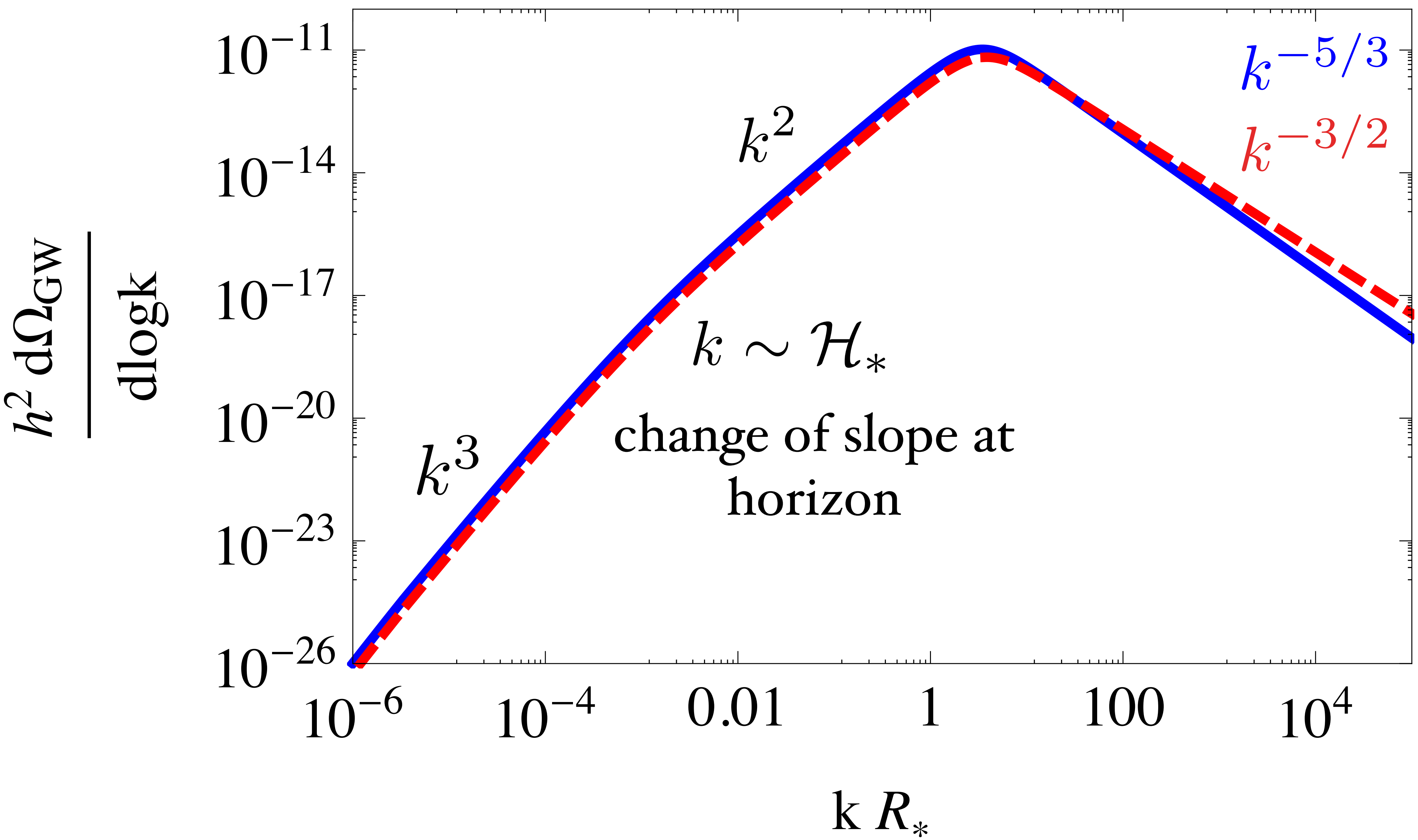
k^3

GW spectrum from bubble collisions



GW spectrum from MHD turbulence

peak $k_* \sim R_*^{-1}$

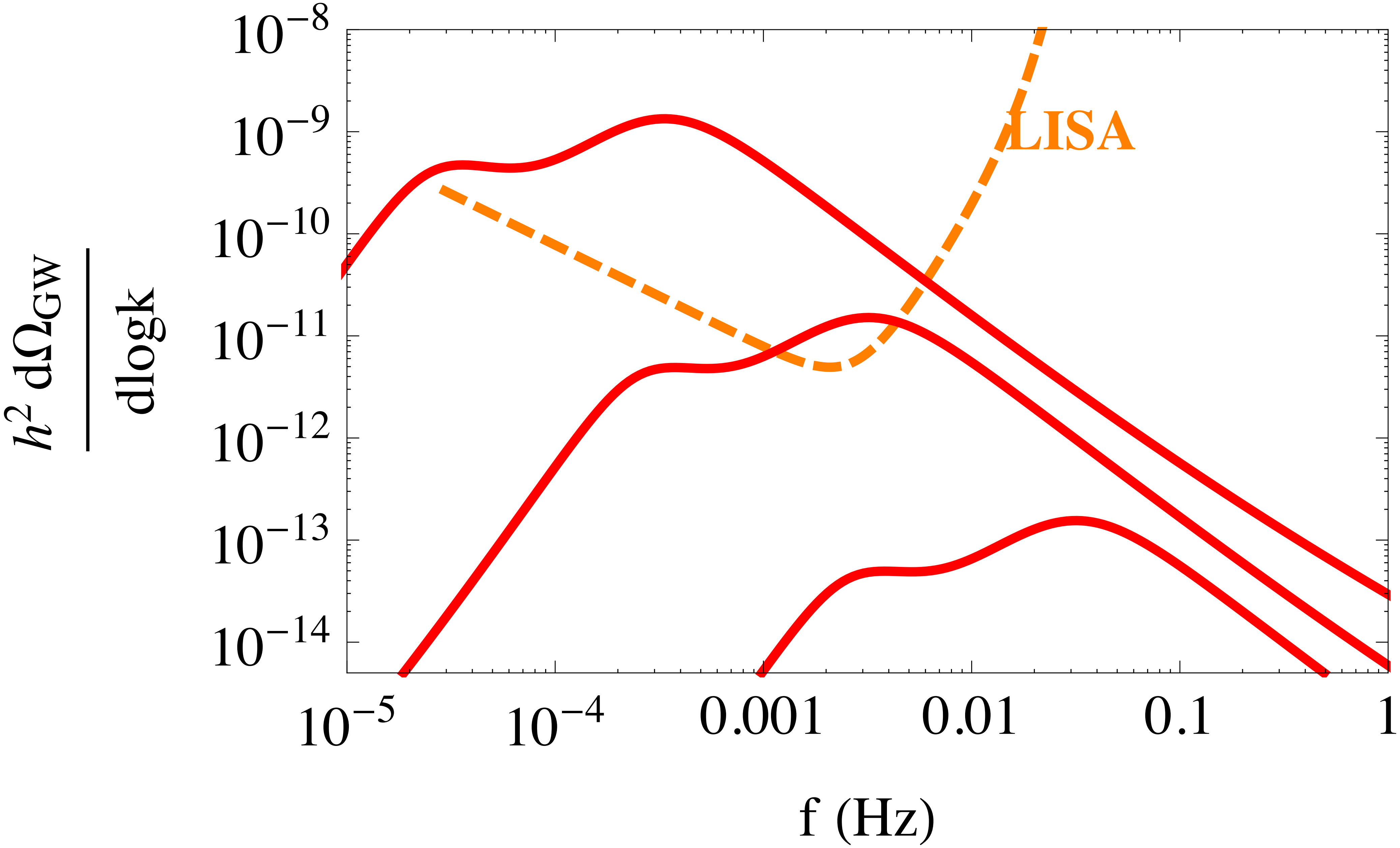


Kolmogorov or Iroshnikov Kraichnan spectrum

total GW spectrum for the EWPT

$$T_* = 100\,\text{GeV} \qquad \Omega_s^* = 0.2 \qquad v_b = 0.87$$

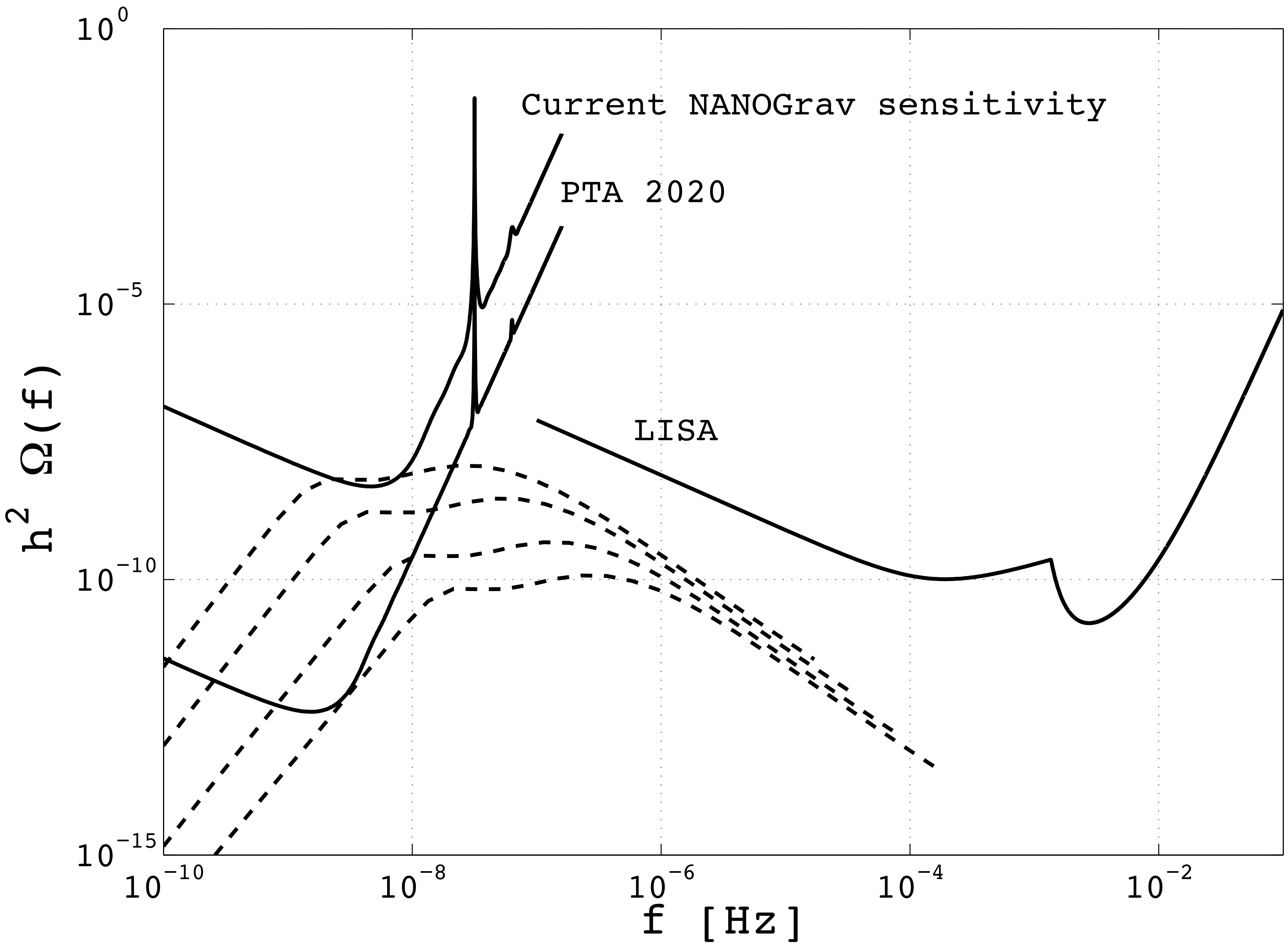
$$\frac{\beta}{\mathcal{H}_*} = 10\,,\ 100\,,\ 1000$$



total GW spectrum for the QCDPT

$$T_* = 100\,\mathrm{MeV} \qquad \Omega_s^* = 0.1 \qquad v_b = 0.7$$

$$\frac{\beta}{\mathcal{H}_*} = 1\,,\ 2\,,\ 5\,,\ 10$$



Other possible GW sources in the early universe

- phase transitions at the end of inflation
  (Turner and Wilczeck 1990...)

- phase transitions in braneworld scenarios
  (Randall and Servant 2007...)

- pre big bang and bouncing universe scenarios
  (Gasperini and Veneziano 1993..., Khoury et al 2001...)

- decay of SUSY flat directions (Dufaux 2009)

- scalar field relaxation (Krauss 1992...)

- ...

Conclusions

we have little information about the physics and the processes operating in the very early universe

it could be in principle possible to generate a stochastic background of GWs detectable by the next generation interferometers or by PTA
(ultimate sensitivity 10^{-12} in energy density)

GWs are a powerful mean to learn about the early universe and high energy physics:
detection extremely difficult but great payoff

Conclusions