Generation of high–energy photons with large orbital angular momentum by Compton backscattering

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Plan:

1. Introduction

2. Twisted photons

3. Compton backscattering for twisted photons
   in the initial state

4. Compton backscattering for twisted photons
   in the initial and final states

5. Conclusion

This report is based mainly on recent papers:

[1] U.D. Jentschura, V.G. Serbo "Generation of High–Energy Photons with Large Orbital Angular Momentum by Compton Backscattering", Phys. Rev. Lett. 106 (2011) 013001

[2] U.D. Jentschura, V.G. Serbo "Compton Upconversion of Twisted Photons: Backscattering of Particles with Non-Planar Wave Functions", Eur. Phys. Journ. C 71 (2011) 1571

1. Introduction

An interesting research direction in modern optics is related to experiments with so-called "twisted photons."

These are states of the laser beam whose photons have a defined value $\hbar m$ of the angular momentum projection on the beam propagation axis where m is a (large) integer

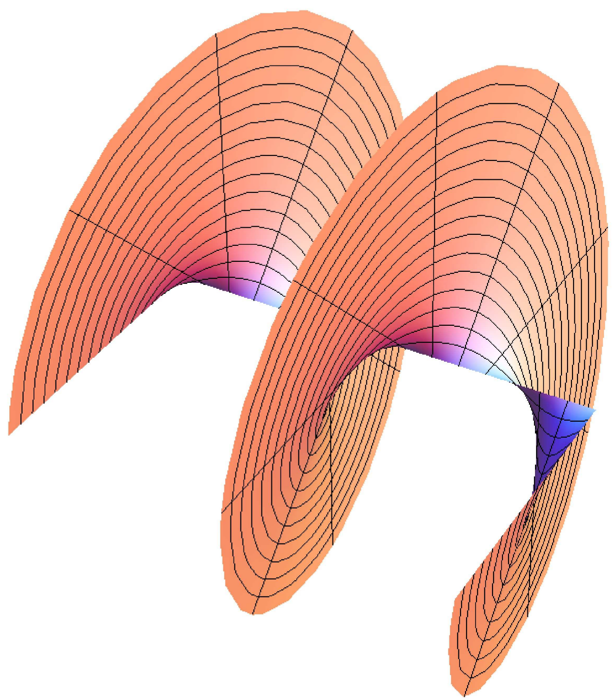
L. Allen et al., Phys. Rev. A 45, 8185 (1992);

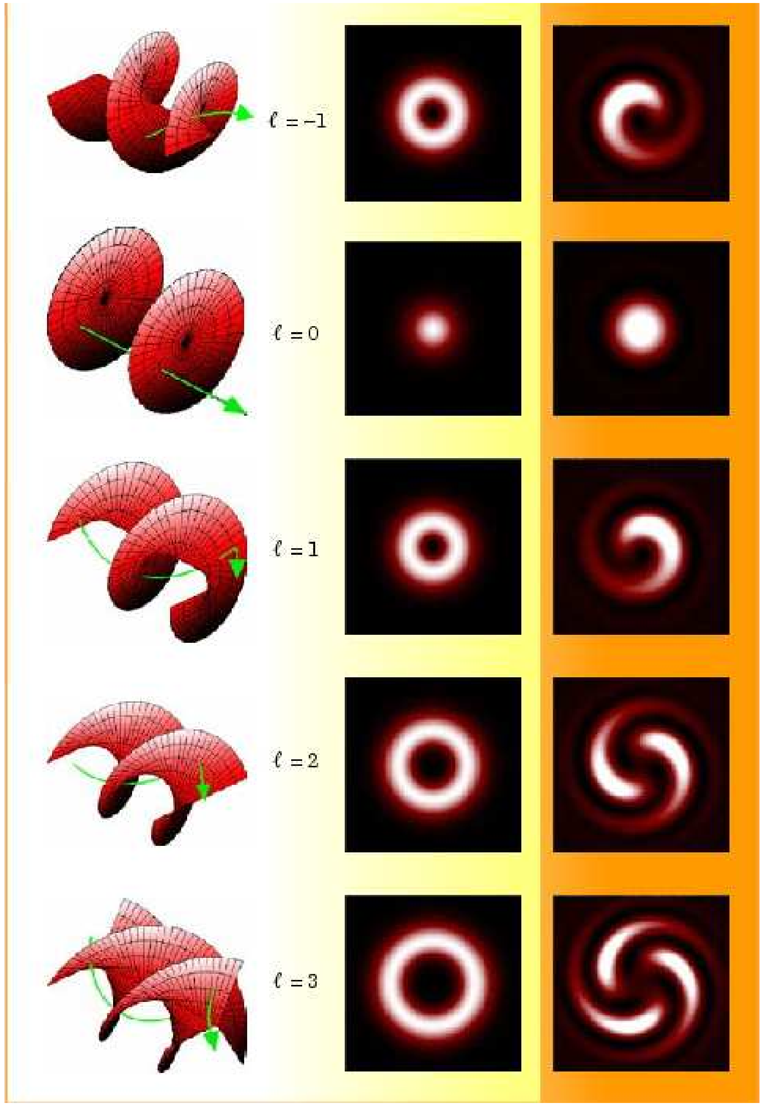
S. Franke-Arnold, L. Allen, M. Padgett, Laser and Photonics Reviews 2, 299 (2008).

An experimental realization exists for states with projections as large as $m = 200$

J. E. Curtis, B. A. Koss, and D. G. Gries, Opt. Commun. **207**, 169 (2002).

The wavefront of such states rotates around the propagation axis, and their Poynting vector looks like a corkscrew:




$\ell = -1$
$\ell = 0$
$\ell = 1$
$\ell = 2$
$\ell = 3$

Such photons can be created, for example, from usual laser beams by means of numerically computed hologram.

It was demonstrated that micron-sized Teflon and calcite "particles" start to rotate after absorbing twisted photons

N. B. Simpson, K. Dholakia, L. Allen, and M. J. Padgett, Opt. Lett. **22**, 52 (1997)

The observation of orbital angular momentum of light scattered by black holes could be very instructive

M. Harwit, Astrophysical J. **597**, 1266 (2003)

We show that it is possible to convert twisted photons from an energy range of about 1 eV to a higher energies of up to a hundred GeV using Compton backscattering off ultra-relativistic electrons.

In principle, Compton backscattering is an established method for the creation of high-energy photons and is used successfully in various application areas from the study of photo-nuclear reactions to colliding photon beams of high energy.

However, the central question is how to treat Compton backscattering of twisted photons, whose field configuration is manifestly different from plane waves.

2. Twisted photons

Twisted scalar particle

Usual plane-wave state of a scalar particle with mass equal to zero has a defined 3-momentum $\boldsymbol{k}$, energy $\omega = |\boldsymbol{k}|$ and

$$\Psi_{\boldsymbol{k}}(t, \boldsymbol{r}) = \frac{\mathrm{e}^{-\mathrm{i}(\omega\, t - \boldsymbol{k}\boldsymbol{r})}}{\sqrt{2\omega}}, \tag{1}$$

(here and below $\hbar = 1$ and $c = 1$).

A twisted scalar particle has the following quantum numbers:
longitudinal momentum k_z,
absolute value of the transverse momentum $\varkappa$,
energy $\omega = |\boldsymbol{k}| = \sqrt{\varkappa^2 + k_z^2}$
and projection m of the orbital angular momentum onto the z axis:

$$\partial_\mu \partial^\mu \Psi_{\varkappa m k_z}(t, \boldsymbol{r}) = 0\,, \tag{2}$$

$$\check{p}_z \Psi_{\varkappa m k_z} = k_z \Psi_{\varkappa m k_z}\,, \tag{3}$$

$$\check{L}_z \Psi_{\varkappa m k_z} = m \Psi_{\varkappa m k_z}\,. \tag{4}$$

where $\check{p}_z = -\mathrm{i}\partial/\partial z$ and $\check{L}_z = -\mathrm{i}\partial/\partial \varphi_r$.

Its evident form in cylindrical coordinates r, φ_r, z is

$$\Psi_{\varkappa m k_z}(r, \varphi_r, z, t) = \frac{\mathrm{e}^{-\mathrm{i}(\omega t - k_z z)}}{\sqrt{2\omega}}\, \psi_{\varkappa m}(r, \varphi_r)\,,$$

$$\psi_{\varkappa m}(r, \varphi_r) = \frac{\mathrm{e}^{\mathrm{i} m \varphi_r}}{\sqrt{2\pi}} \sqrt{\varkappa}\, J_m(\varkappa\, r)\,, \qquad (5)$$

where $J_m(x)$ is the Bessel function.

For small $r \ll 1/\varkappa$, the function $\psi_{\varkappa m}(r, \varphi_r)$ is of order of r^m, has a maximum at $r \sim m/\varkappa$ and then drops at large values $r \gg 1/\varkappa$

$$\psi_{\varkappa m}(r, \varphi_r) \approx \frac{\mathrm{e}^{\mathrm{i} m \varphi_r}}{\pi \sqrt{r}} \cos\left(\varkappa\, r - \frac{m\,\pi}{2} - \frac{\pi}{4}\right). \qquad (6)$$

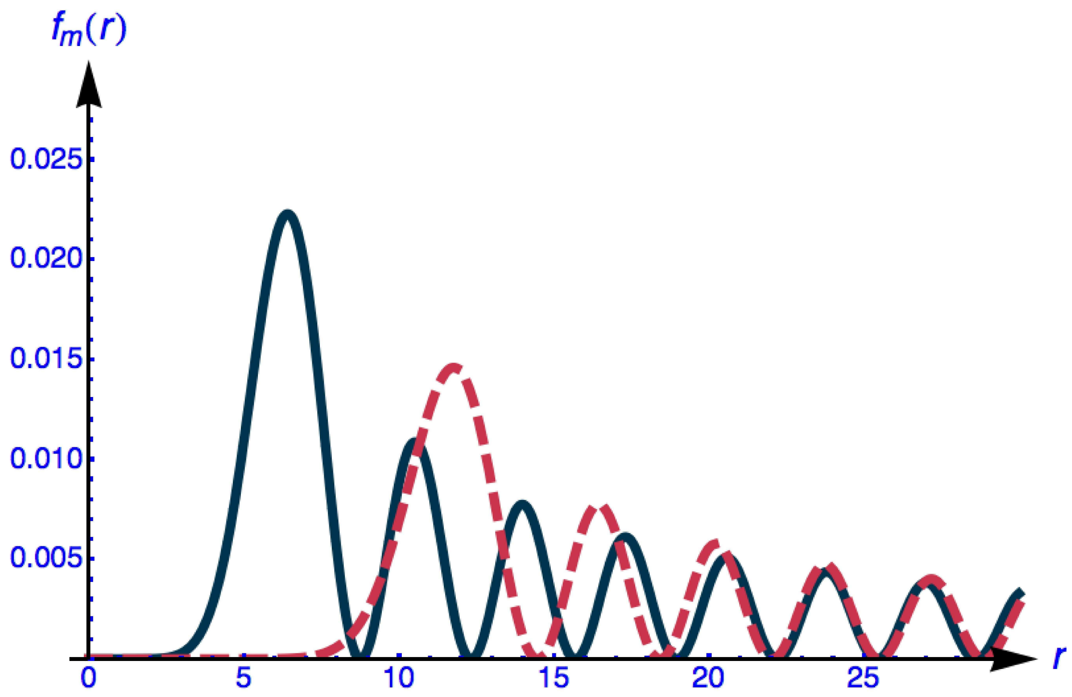


Fig. 1 Plot of the radial probability density $f_m(r) = |\psi_{\varkappa m}(r,\varphi)|^2$ for $m = 5$ (solid line) and $m = 10$ (dashed line) at $\varkappa = 1$.

The function $\psi_{\varkappa m}(r, \varphi)$ may be expressed as a superposition of plane waves in the xy plane,

$$\psi_{\varkappa m}(r, \varphi) = \int a_{\varkappa m}(\boldsymbol{k}_\perp)\, \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\perp \boldsymbol{r}}\, \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2}\,, \tag{7}$$

where the Fourier amplitude $a_{\varkappa m}(\boldsymbol{k}_\perp)$ is concentrated on the circle with $k_\perp \equiv |\boldsymbol{k}_\perp| = \varkappa$,

$$a_{\varkappa m}(\boldsymbol{k}_\perp) = (-\mathrm{i})^m\, \mathrm{e}^{\mathrm{i}m\varphi_k}\, \sqrt{\frac{2\pi}{\varkappa}}\, \delta(k_\perp - \varkappa)\,. \tag{8}$$

Therefore, the function $\Psi_{\varkappa m k_z}(r, \varphi_r, z, t)$ can be regarded as a superposition of plane waves with defined longitudinal momentum k_z, absolute value of transverse momentum $\varkappa$, energy $\omega = \sqrt{\varkappa^2 + k_z^2}$ and different directions of the vector $\boldsymbol{k}_\perp$ given by the angle φ_k.

Twisted photons

The wave function of a twisted photon (vector particle) can be constructed as a generalization of the scalar wave function. We start from the plane-wave photon state with a defined 4-momentum $k = (\omega, \boldsymbol{k})$ and helicity $\Lambda = \pm 1$,

$$A^{\mu}_{k\Lambda}(t, \boldsymbol{r}) = \sqrt{4\pi}\, e^{\mu}_{k\Lambda}\, \frac{\mathrm{e}^{-\mathrm{i}(\omega\, t - \boldsymbol{k}\boldsymbol{r})}}{\sqrt{2\omega}}\,, \tag{9a}$$

$$e_{k\Lambda} \cdot k = 0\,, \quad e^{*}_{k\Lambda} \cdot e_{k\Lambda'} = -\delta_{\Lambda\Lambda'}\,, \tag{9b}$$

where $e^{\mu}_{k\Lambda}$ is the polarization four-vector of the photon.

The twisted photon vector potential

$$\mathcal{A}^{\mu}_{\varkappa m k_z \Lambda}(r, \varphi_r, z, t) = \int a_{\varkappa m}(\boldsymbol{k}_\perp)\, A^{\mu}_{k\Lambda}(t, \boldsymbol{r})\, \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \tag{10}$$

$$= (-\mathrm{i})^m \sqrt{2\pi\varkappa} \int_0^{2\pi} \mathrm{d}\varphi_k \int_0^{\infty} \mathrm{d}k_\perp\, \delta(k_\perp - \varkappa)\, \frac{e^{\mathrm{i}m\varphi_k}}{(2\pi)^2}\, A^{\mu}_{k\Lambda}(t, \boldsymbol{r})$$

is given as a two-fold integral over the perpendicular components $\boldsymbol{k}_\perp = (k_x, k_y, 0)$ of the wave vector $\boldsymbol{k} = (k_x,\, k_y,\, k_z)$.

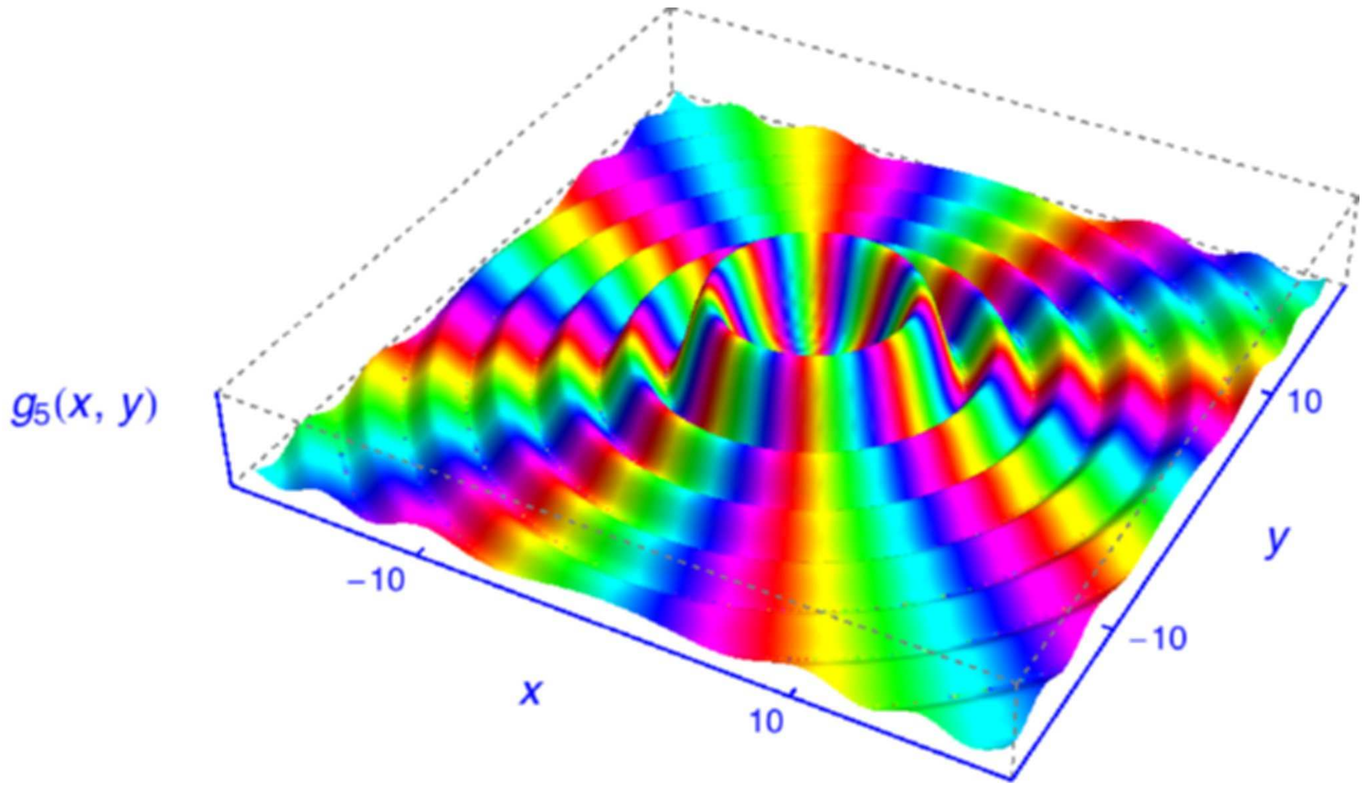


Fig. 2A

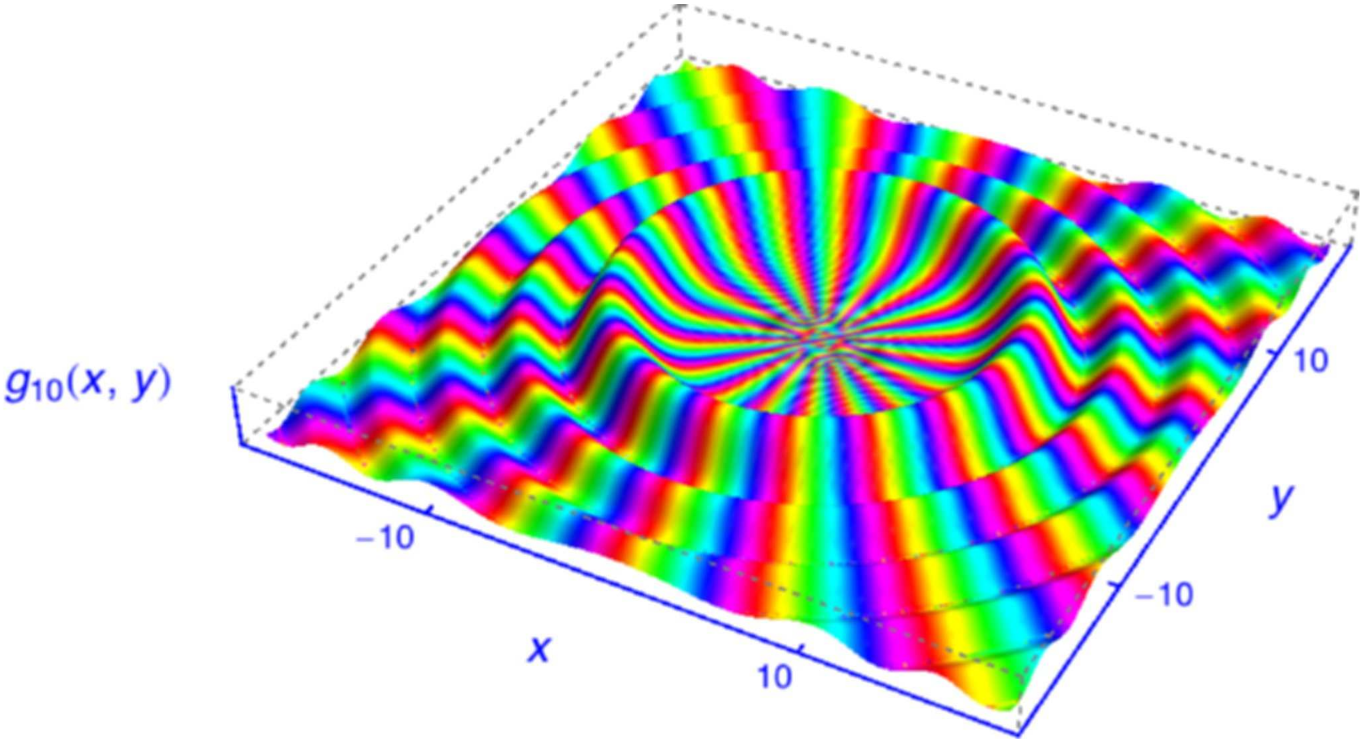


Fig. 2B Vector potential $\mathcal{A}^{\mu}_{\varkappa m k_z \Lambda}$ of a twisted photon: $g_m(x,y) = |\mathcal{A}^{\mu}_{\varkappa m k_z \Lambda}(0,x,y,0)|^2$ as a function of x and y; upper plot $m=5$, lower plot $m=10$. The parameters are $\mu=1$ (x component), $\varkappa=1$.

3. Compton backscattering for twisted photons in the initial state

1. Compton scattering of plane-wave photons

The S-matrix element for plane waves is

$$S_{fi}^{(\mathsf{PW})} = \mathrm{i}\,(2\pi)^4\,\delta(p+k-p'-k')\,\frac{M_{fi}}{4\sqrt{E\,E'\,\omega\,\omega'}}\,, \qquad (11)$$

where the amplitude M_{fi} in the Feynman gauge is

$$
\begin{aligned}
M_{fi} &= 4\pi\alpha\left(\frac{A}{s-m_e^2}+\frac{B}{u-m_e^2}\right), &\text{(12a)}\\
A &= \bar{u}_{p'\lambda'}\,\widehat{e}^{*}_{k'\Lambda'}\left(\widehat{p}+\widehat{k}+m_e\right)\widehat{e}_{k\Lambda}\,u_{p\lambda}\,, &\text{(12b)}\\
B &= \bar{u}_{p'\lambda'}\,\widehat{e}_{k\Lambda}\left(\widehat{p}'-\widehat{k}+m_e\right)\widehat{e}^{*}_{k'\Lambda'}\,u_{p\lambda}\,. &\text{(12c)}
\end{aligned}
$$

The bispinors $u_{p\lambda}$ and $u_{p'\lambda'}$ describe the initial and final electrons, and $e_{k\Lambda}$ and $e_{k'\Lambda'}$ are the polarization vectors of the initial and final photon.

For **a head-on collision of a plane-wave photon and electron**, the differential cross section reads

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega'} = \frac{2\alpha^2\gamma^2}{m_e^2} F(x,n)\,, \qquad n \equiv \gamma\,\theta'\,, \qquad x = \frac{4\omega E}{m_e^2}\,, \tag{13}$$

$$F(x,n) = \frac{1}{(1+x+n^2)^2}\left[\frac{1+x+n^2}{1+n^2} + \frac{1+n^2}{1+x+n^2} - 4\frac{n^2}{(1+n^2)^2}\right]$$

It is seen that the differential cross section has a maximum in the backscattering region, where **the photon propagates almost along the direction of momentum of the initial electron**.

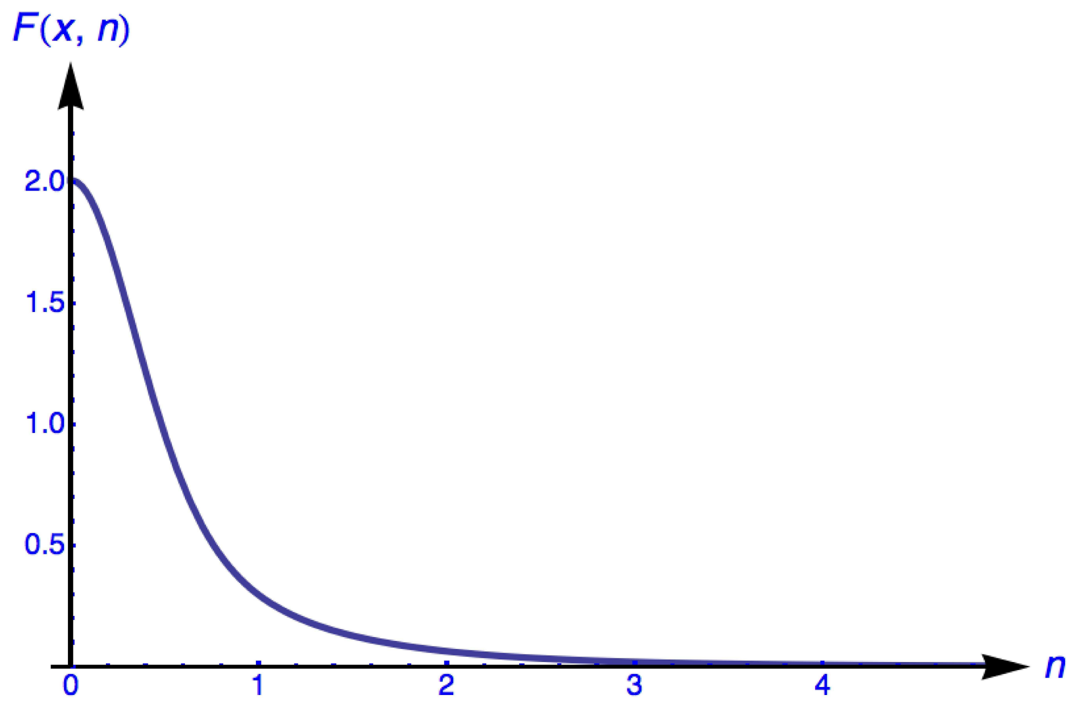


Fig. 3. The angular distribution $\mathrm{d}\sigma/\mathrm{d}\Omega' = 2\alpha^2\gamma^2 m_e^{-2} F(x,n)$ of the final photons in the Compton scattering in dependence on $n = \gamma\,\theta'$. The value $x = 0.092$ corresponds to the VEPP-4M collider.

2. Compton scattering with the twisted photon in the initial state

Since a twisted photon is a superposition of plane-wave photons, for the case where the initial photon is twisted m-photon, but the outgoing one is a plane-wave photon, we have

$$
\begin{aligned}
S_{fi}^{(\mathrm{m})} &\equiv \langle k', \Lambda'; p', \lambda' | S | \varkappa, m, k_z, \Lambda; p, \lambda \rangle \\
&= \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \, S_{fi}^{(\mathrm{PW})} \, a_{\varkappa m}(\boldsymbol{k}_\perp) \,. \qquad (14)
\end{aligned}
$$

Simple detailed consideration shows that the corresponding cross section is given by Eq. (13) with the only replacement

$$
x = \frac{4\omega E}{m_e^2} \to \frac{4\omega E \, \cos^2 \alpha_0}{m_e^2} \,, \qquad (15)
$$

where α_0 is the conical angle of the initial photon.

It is important since it proves that the cross section for

twisted initial photons has no additional smallness

as compare with the ordinary Compton scattering.

This result looks very natural since the initial photon state is nothing else but a superposition of plane waves with the same absolute value of their transverse momentums.

4. Compton backscattering for twisted photons in the initial and final states

1. Compton strict backward scattering of twisted photons

For twisted photons, the final m' photon is a superposition of plane waves with small transverse momentum $\boldsymbol{k}'_\perp = \boldsymbol{k}_\perp$ and very small scattering angle $\theta' = k'_\perp/\omega' \lesssim (1+x)/(4\gamma^2)$:

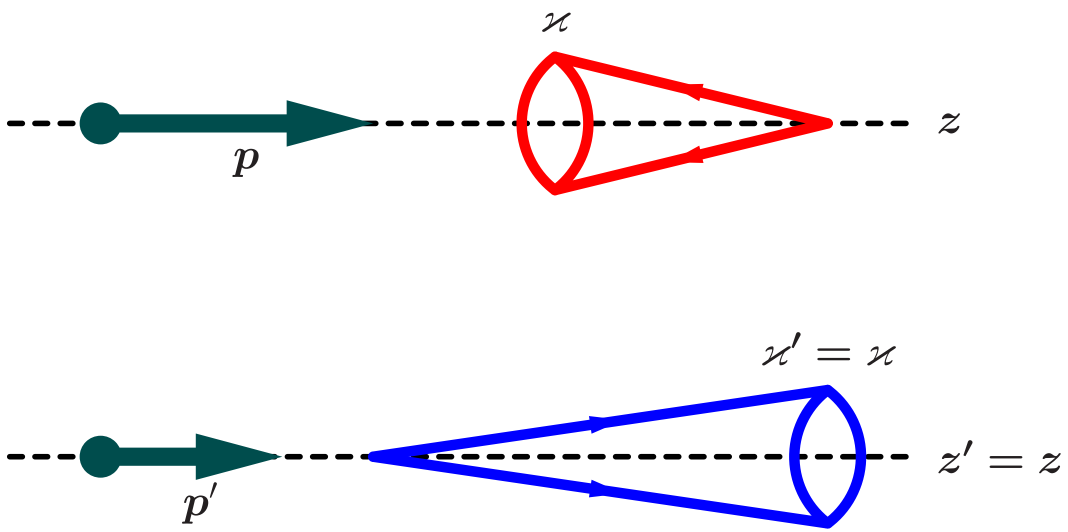


Fig. 4. Initial (above) and final (below) states for the head-on Compton backscattering geometry of a twisted photon.

The S matrix element $S_{fi}^{(\mathsf{TW})}$ for the scattering of a twisted (TW) photon $|\varkappa, m, k_z, \Lambda\rangle$ into the state $|\varkappa', m', k_z', \Lambda'\rangle$ needs to be integrated as follows,

$$
\begin{aligned}
S_{fi}^{(\mathsf{TW})} &\equiv \langle \varkappa', m', k_z', \Lambda'; p', \lambda' | S | \varkappa, m, k_z, \Lambda; p, \lambda \rangle \\
&= \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \frac{\mathrm{d}^2 k'_\perp}{(2\pi)^2} \, a^*_{\varkappa' m'}(\boldsymbol{k}'_\perp) \, S_{fi}^{(\mathsf{PW})} \, a_{\varkappa m}(\boldsymbol{k}_\perp) \, .
\end{aligned} \tag{16}
$$

Using Dirac algebra, we calculated the amplitude M_{fi} which enters in $S_{fi}^{(\mathsf{PW})}$:

$$
M_{fi} = 4\pi\alpha \left(\frac{A}{s - m_e^2} + \frac{B}{u - m_e^2} \right) \tag{17}
$$

with

$$
\begin{aligned}
A = 2\omega \sqrt{EE'} \Big[& (1 - \Lambda \Lambda' \cos\alpha_0)\,(1 + \cos\alpha_0) \\
& + 2\lambda \Lambda \sin^2\alpha_0 \Big]\, \delta_{\lambda\lambda'}\, \delta_{2\lambda,-\Lambda'}\,, && \text{(18a)} \\
B = \; - 2\omega \sqrt{EE'} \Big[& (1 - \Lambda \Lambda' \cos\alpha_0)\,(1 + \cos\alpha_0) \\
& - 2\lambda \Lambda \sin^2\alpha_0 \Big]\, \delta_{\lambda\lambda'}\, \delta_{2\lambda,\Lambda'}\,. && \text{(18b)}
\end{aligned}
$$

As a result, the S matrix element reads

$$
\begin{aligned}
S_{fi}^{(\mathrm{TW})} = \mathrm{i}\,(2\pi)^2\, \delta_{mm'}\, \delta(\varkappa - \varkappa')\, \delta(E + \omega - E' - \omega') \\
\times\;\; \delta(p_z + k_z - p'_z - k'_z) \frac{M_{fi}}{4\sqrt{EE'\omega\omega'}}\,. && \text{(19)}
\end{aligned}
$$

This result states that **for strict backscattering**, the angular momentum projection $m' = m$ and the conical momentum spread $\varkappa' = \varkappa$ of the twisted photons **are conserved**.

A technique for the registration of electrons scattered at small **(even zero)** angles after the loss of energy in the Compton process is implemented, for example, in the device for backscattered Compton photons installed on the VEPP-4M collider (Novosibirsk)

V. G. Nedorezov, A. A. Turinge, and Y. M. Shatunov, Phys. Usp. **47**, 341 (2004).

Two further theoretical problems:

2. How to convert $S_{fi}^{(\mathrm{TW})}$ to the cross section?

This problem is considered in detail in our second paper:

[2] U.D. Jentschura, V.G. Serbo "Compton Upconversion of Twisted Photons: Backscattering of Particles with Non-Planar Wave Functions", Eur. Phys. Journ. C 71 (2011) 1571

3. What happens for not strict backward scattering of twisted photons?

This problem is studied in detail in the paper of I.P. Ivanov and V.G. Serbo which is now under preparation.

5. Conclusion

1. We have investigated the scattering of a twisted photon by an incoming ultra-relativistic electron, in the Compton backscattering geometry.
2. We found out that the magnetic quantum number $m' = m$ and the conical momentum spread $\varkappa' = \varkappa$ are preserved, but the energy of the final twisted photon is increased dramatically ($\omega'/\omega \sim \gamma^2 \gg 1$).
3. As a result, we prove the principal possibility to create high-energy photons with large orbital angular momenta projections.
4. Such photons may be useful for experimental studies regarding the excitation of atoms into circular Rydberg states, and for studying the photo-effect and the ionization of atoms, as well as the pair production off nuclei.