Measuring of mass and spin of Dark Matter particles at ILC

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Dark matter. Candidates

There are many models in which Dark Matter (DM) consists of particles similar to those in SM. Discovery of such a candidate for Dark Matter particle (DMP) and measurement of its properties is one of the most important problems for collider physics. The LHC program solves this problem for some specific DM models. It is difficult to expect high precision in the DM mass measurement at the LHC, the value of spin of DMP is fixed by the choice of model.

The experiments at ILC/ CLIC allow to detect unambiguously the DMP candidate, to measure accurately its mass and spin for a wide class of DMP models.

Main advantages of ILC

1. Well fixed 4-momentum of elementary initial state
2. Well fixed coupling constants dark/known particles ($\equiv \gamma$ or Z) – constants of EW theory

In the considered models (like MSSM or IDM – inert doublet model)
I. DMP D with mass M_D has a new conserved discrete quantum number, which I denote as D-parity. All known particles are D-even, while the DM particle is D-odd.
II. In addition to the neutral DMP D, another D-odd particles exist, a neutral D^A and a charged $D^\pm$, with the same spin $s_D = 0$ or $1/2$ as D and with masses M_A and $M_\pm$ larger than M_D.
The D-parity conservation ensures stability of the lightest D-odd particle and restricts possible decay modes of D^A and $D^\pm$.
III. D-particles interact with the SM particles only via the covariant derivative in the kinetic term of the Lagrangian – gauge interactions with the standard electroweak gauge couplings e, g and g':

$$D^+D^-\gamma, \quad D^+D^-Z, \quad D^+DW^-, \quad D^+D^AW^-, \quad D^ADZ\,.$$

The cosmology and LEP constraints references

$$M_D < 60\ GeV, \quad M_A,\, M_\pm > 80\ GeV, \qquad |M_A - M_\pm| \ll M_A.$$

Other regions of M_D also possible

All numerical examples below — for $M_D = 50$ GeV.

Processes

We assume that the ILC/ CLIC beam energy $E = \sqrt{s}/2$ is sufficient for production of DD^A or (and) D^+D^- pairs but the heavier D-particles, if they exist, cannot be produced $\Rightarrow$ we consider production of D-particles in the processes

$$e^+e^- \to Z \to DD^A, \quad e^+e^- \to (\gamma, Z) \to D^+D^-$$

$+$ subsequent decay of $D^\pm \to DW^\pm$ or $D^A \to DZ$, etc. with either on-shell or off-shell W's and Z's.

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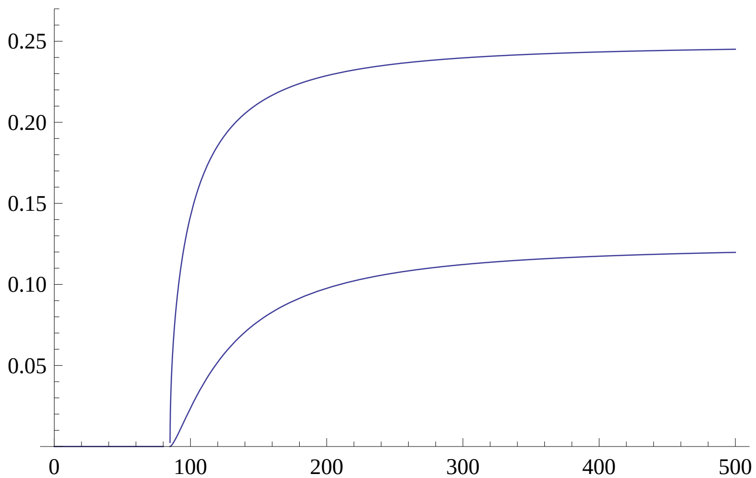
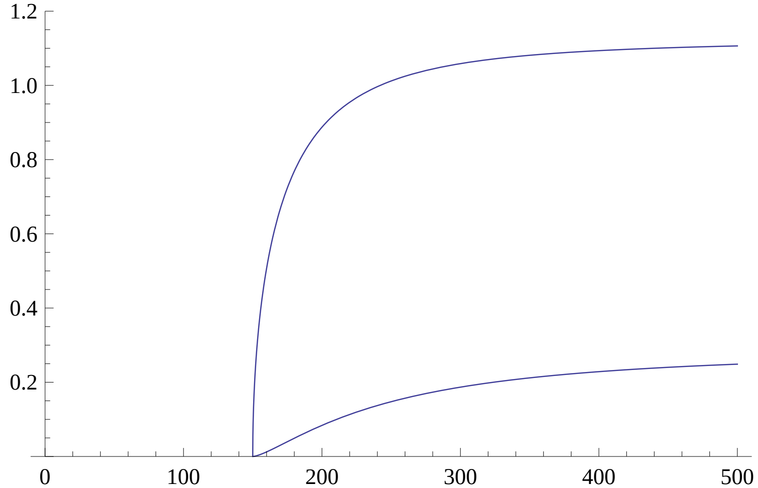
The cross sections of these processes are calculated easily, they are

$$\sim \sigma_0 \equiv \sigma(e^+e^- \to \gamma \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}.$$





DD_A at $M_A = 120$ GeV, D^+D^- at $M_\pm = 120$ GeV

Cross sections σ/σ_0 in dependence on beam energy E

Signature

Observable states: (**I.**) Decay products of W or Z. (**II.**) Large missing E_T, carried away by the neutral, stable D-particle + (**III.**) *nothing*

$$e^+e^- \to DD_A \to DDZ$$

The quark dijet or e^+e^- or $\mu^+\mu^-$ pair with large missing E_T + *nothing*. The effective mass of this pair or dijet is either M_Z or lower than M_Z with identical distribution for all modes; its total energy is lower than E. They move in one hemisphere

$$e^+e^- \to D^+D^- \to DDW^+W^-$$

Two dijets or one dijet + e or μ with large missing E_T + *nothing* with total energy for each dijet lower than E. The effective mass of each dijet is either M_W or lower than M_W. Typically these dijets (or dijet and lepton) move in the opposite hemispheres.

At $M_A > M_\pm$ one more channel is added into the decay of D_A, $D_A \to D^\pm W^\mp \to DW^+W^-$, with small BR and simple signature for $e^+e^- \to DD_A$, different from that for $e^+e^- \to D^+D^- \to DDW^+W^-$.

At $M_A < M_\pm$ one more channel is added into the decay of $D_\pm$, $D_\pm \to D_A W^\pm \to DW^+W^-$, with small BR and simple signature for $e^+e^- \to D^+D^-$.

(Reason for small BR: smaller final phase space in these decays in comparison with main decays since $|M_A - M^\pm| \ll M_A$ at similar couplings)

Complete rate of processes with mentioned signature at known BR's gives

total cross sections $e^+e^- \to DD_A$ and $e^+e^- \to D^+D^-$

with reasonable accuracy.

Energy distributions of products

• Processes $e^+e^- \to DD_A$ and $e^+e^- \to D^+D^-$ are two-body decays of well defined initial state energy 2E, momentum 0. $\Rightarrow$. The energy E_A or $E_\pm$, momentum p_A or p_+, γ-factor $\gamma_A = E_A/M_A$ or $\gamma_\pm = E_\pm/M_\pm$ and velocity $\beta_A = p_A/E_A$ or $\beta_\pm = p_\pm/E_\pm$ of the produced particles D_A and $D_|pm$ are calculated easily and unambiguously.

• The decays $D_A \to DZ$ and $D^\pm \to DW^\pm$ in the rest frame are two-body decays with easily and unambiguously calculated parameters of produced Z or W, energy $E^r_Z(M^*)$ or $E^r_W(M^*)$, momentum $p^r_Z(M^*)$ or $p^r_W(M^*)$. Here M^* is effective mass of decay products of Z or W. For on-shell Z and W we have $M^* = M_Z$ or $M^* = M_W$. For off shell Z and W quantity M^* varies from 0 to $M_A - M_D < M_Z$ or $M_\pm - M_D < M_W$ respectively.

• The angular distributions of W or Z in the rest frame of $D^{\pm}$ or D_A are uniform (at least after averaging over intermediate spin state for spinor D-particles).

For off shell W and Z at $M^* \gtrsim$ 10 GeV the branching ratios for different channels are roughly the same as for mass shell W and Z.

To measure masses with reasonable precision, one can use only leptons from Z or W.

Dilepton energy spectrum for $e^+e^- \to DD_A \to DDZ$ and measuring M_D and M_A

The dilepton (e^+e^- or $\mu^+\mu^-$) represents Z completely, its energy, momentum and effective mass can be measured with high precision. At each fixed M^* possible energies of dimuon in the lab system E_Z^L are distributed uniformly within interval with end points $E_{Z\pm}^L$

$$E_{Z+}^L = \gamma_A(E_Z^r + \beta_A p_Z^r) \geqslant E_Z^L \geqslant E_{Z-}^L = \gamma_A(E_Z^r - \beta_A p_Z^r); \quad (E_{Z+}^L < E).$$

These relations provide 2 equations allowing to determine two unknown quantities M_D and M_A from measured end points $E_{Z\pm}^L$.

$e^+e^- \to D^+D^- \to DDW^+W^-$. Lepton energy spectra and measuring M_D and $M_\pm$

We study energy distribution of single muon from W decay in the events where the first W decays to $\mu\nu$, and the second one – to $q\bar{q}$ (dijet). The observable leptons from W decay don't represent W completely. The muon energy distribution is the convolution of energy distribution of W in the Lab system and distribution of μ in the W rest frame.

At each fixed M^* possible energies of W in the lab system E_W^L are distributed uniformly within interval with end points $E_{W\pm}^L$

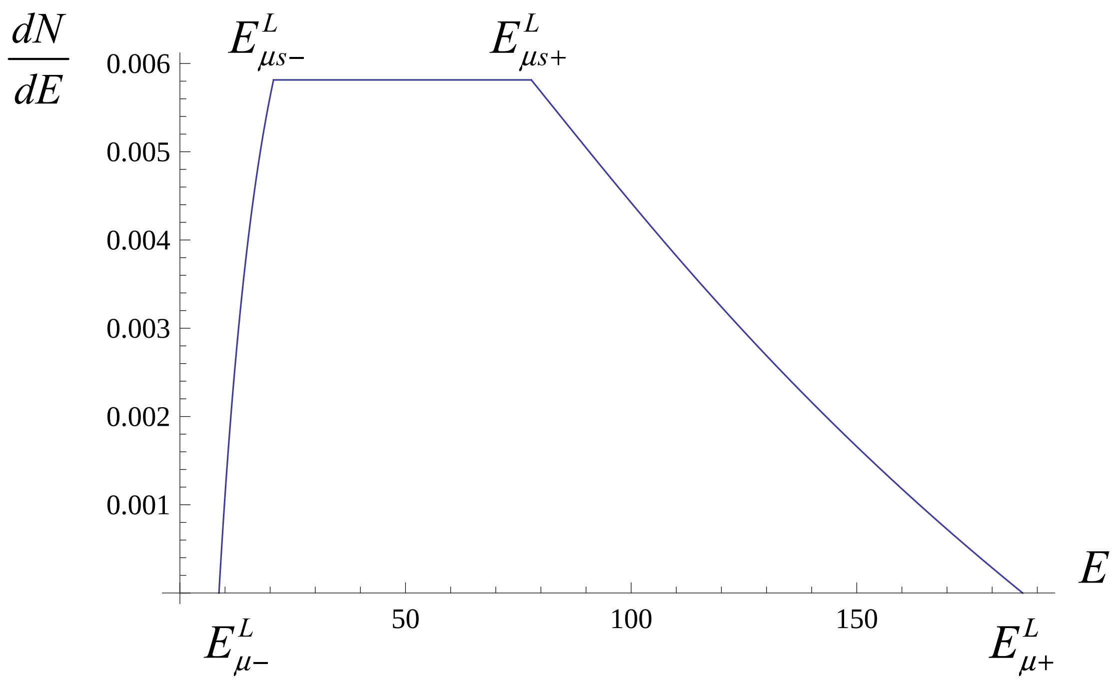
$$E_{W+}^L = \gamma_\pm (E_W^r + \beta_\pm p_W^r) \geqslant E_W^L \geqslant E_{W-}^L = \gamma_\pm (E_W^r - \beta_\pm p_W^r); \quad (E_{W+}^L < E).$$

• On shell W. In the W rest frame energy and momentum of μ are $M_W/2$. It is easy to find that the muon energies lie within the interval

$$E^L_{\mu+} \geqslant E_\mu \geqslant E^L_{\mu-}, \;\; where \;\; E^L_{\mu\pm} = \frac{1}{2}\left(E^L_{W+} \pm \sqrt{(E_{W+}LW)^2 - M_W^2}\right) .$$

The total density of states within this interval increases monotonically from outer limits up to the energies

$$E^L_{\mu s\pm} = \frac{1}{2}\left(E^L_{W-} \pm \sqrt{(E^L_{W-})^2 - M_W^2}\;\right).$$



The energy distribution of single μ from $e^+e^- \to D^+D^- \to DDW^+W^-$ at $M_\pm = 150$ GeV, $M_D = 50$ GeV, $E = 250$ GeV

• Off shell W has effective mass M^*, varying in the interval $(0, M_\pm - M_D$. AT $M^* \gtrsim 10$ GeV the distribution in M^* for each particular channel is given by the spin dependent factor $R(s_D)dM^*$:

$$R(0) = \frac{(p^r_W(M^*))^3\, M^*}{(M_W^2 - M^{*2})^2}\,,$$

$$R(1/2) = \frac{\left[2(M_\pm^2 + M_D^2 - M^{*2}) - \dfrac{(M_\pm^2 + M_D^2)M^{*2} - (M_\pm^2 - M_D^2)^2}{M_W^2}\right] p^r_{Z*}\, M^*}{(M_W^2 - M^{*2})^2}.$$

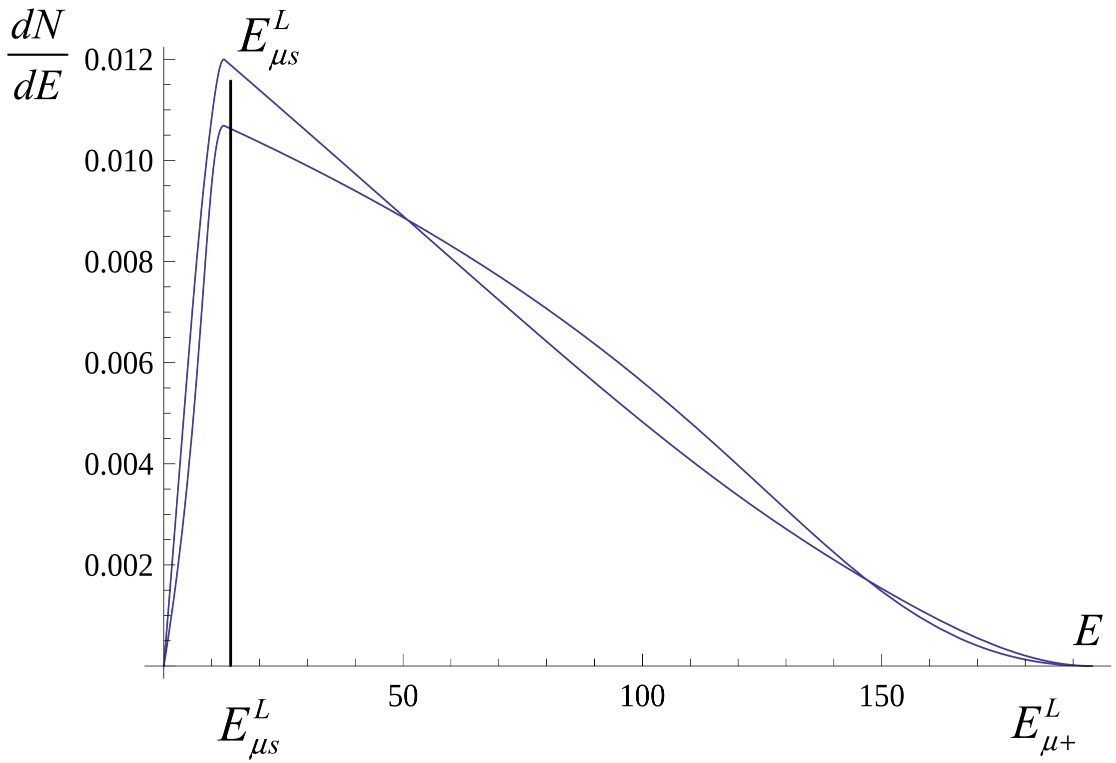
At each value M^* the energy distribution of muon is described by the same equation as for on shell W with the change $M_W \to M^*$. Complete energy distribution is obtained by integrated with weight $R(s_D)dM^*$. As a result the muon energies lie within the interval

$$\left(0,\, E^L_{\mu+} = \gamma_\pm(1+\beta_\pm)\frac{M_\pm^2 - M_D^2}{2M_\pm}\right).$$

The density of states in muon energy has maximum at

$$E^L_{\mu s} = \gamma_\pm(1+\beta_\pm)(M_+ - M_D)/2.$$

The relations for end point $E^L_{\mu+}$ and singular point $E_{\mu s}L$ or $E_{\mu s+}L$ provide two equations for finding two masses M_D and $M_\pm$.



The energy distribution of single μ from $e^+e^- \to D^+D^- \to DDW^+W^-$ at $M_\pm = 120$ GeV, $M_D = 50$ GeV, $E = 250$ GeV. Upper peak corresponds $s_D = 0$, lower – $s_D = 1/2$.

The known values of M_D, M_A, $M_\pm$ allow to calculate cross sections of processes $e^+e^- \to DD^A$ and $e^+e^- \to D^+D^-$. The results for spins $s_D = 0$ and $s_D = 1/2$ differ from each other by the factor about 4 for $e^+e^- \to D^+D^-$ and factor more than 2 for $e^+e^- \to DD^A$. $\Rightarrow$

Measured values of these cross sections give value of spin of DM s_D.

Additional signature of s_D – very different energy dependence for $s_D = 0$ and $s_D = 1/2$.

The background can be eliminated by suitable choice of cuts and by subtraction of other contributions

The end