Erased loop size distribution of the loop-erased random walk on networks

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Abstract

Random walks provide a natural framework for studying exploration and transport on networks. However, a random-walk trajectory generally contains many redundant excursions, as the walker may repeatedly return to previously visited vertices before reaching its destination. Loop-erased random walks provide a way to remove these redundancies by chronologically erasing the loops formed along the trajectory, leaving a self-avoiding path. In this talk, I will investigate the distribution of the sizes of the loops removed during the exploration.

The analysis will rely on the fundamental connection between loop-erased random walks and uniform spanning trees. Using the transfer-current kernel, I will show how probabilities of loop-erased paths can be expressed through determinantal formulas. This framework provides a way to determine the distribution of erased-loop sizes. The general results will be illustrated on finite networks and specialized to representative graph families, including complete graphs, cycle graphs, and motif-based networks. Numerical simulations will be compared with the theoretical predictions to show the validity of the proposed approach.

Keywords: Loop-erased random walk, Spanning trees, Networks.