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The Unit of Electric Charge and the Mass Hierarchy of Heavy Particles

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$\underline{e}$ @ $\underline{W}$, $\underline{Z}$, $\underline{t}$, $\underline{H}$

PHYSICAL INPUTS:

① Sommerfeld fine-structure constant
$\underline{1916}$

$$\alpha = \frac{q^2}{4\pi\varepsilon_0 \hbar c} = \frac{e^2}{4\pi}$$

$$\alpha^{-1} = 137.035\,999\,710\,(96)$$

α and "partial" masses.

Examples:

Hydrogen atom

$$m_H = m_e + m_p - \frac{1}{2}\mu_e \alpha^2$$

with $\frac{1}{\mu_e} = \frac{1}{m_e} + \frac{1}{m_p}$

$$\Rightarrow m_H = (m_e + m_p) - \frac{1}{2}\left(\frac{1}{\frac{1}{m_e}+\frac{1}{m_p}}\right)\alpha^2$$

E.M. Self mass of pion (Das et al. PRL 18 (1967) 759)

$$\delta^\gamma m_{\pi^+} = \frac{\alpha\, m_\rho^2}{m_\pi}\,\frac{3\ln 2}{4\pi} \,/\, \delta m_{\pi^0} = 0$$

E.M. Self mass of rho

$$\delta^\gamma m_{\rho^+} = \frac{\alpha\, m_{\rho^+}}{8\pi}\left(2 + \pi\sqrt{3} - \frac{2}{3}\right) = 1.49\ \mathrm{MeV}$$

$$\delta^\gamma m_{\rho^0} = \frac{2\pi\alpha\, m_{\rho^0}}{f_\rho^2} = 1.47\ \mathrm{MeV}$$
$$\pm 0.02$$

Nambu's proposal (1951)

$$m_\mu = \frac{3}{2}\,\frac{1}{\alpha}\,m_e = 105.04\ \text{MeV}$$

$$m_\mu = 105.66\ \text{MeV}\quad(\text{exp})$$

② Fermi Constant 1933

$$G_F = 1.16637\,(1) \times 10^{-5}\ \text{GeV}^{-2}$$

$$\tau_\mu^{-1} = \frac{G_F^2\, m_\mu^5}{192\,\pi^3}\, F\left(\frac{m_e^2}{m_\mu^2}\right)\cdot$$

$$\left(1 + \frac{3}{5}\,\frac{m_\mu^2}{M_W^2}\right)\cdot(1+C_1)$$

□ 1958 - 1959 Berman
Kinoshita + Sirlin

$$C_1 = \frac{1}{2}\left(\frac{25}{4} - \pi^2\right)\cdot \alpha/\pi + \cdots$$

□ 1971 Sirlin + Roos

$$C_1 = \frac{1}{2}\left(\frac{25}{4} - \pi^2\right)\cdot \frac{\alpha(m_\mu)}{\pi} + \cdots$$

□ 1999 van Ritbergen + Stuart

$$C = \frac{1}{2}\left(\frac{25}{4} - \pi^2\right) \cdot \frac{\alpha(m_\mu)}{\pi}$$

$$+ \underline{6.700} \cdot \left(\frac{\alpha(m_\mu)}{\pi}\right)^2 + \dots$$

$$\alpha(m_\mu) = \frac{\alpha}{1 - \left(\frac{2\alpha}{3\pi} + \frac{\alpha^2}{2\pi^2}\right) \ln \frac{m_\mu}{m_e}}$$

But (Ferroglia, Ossola + Sirlin)

$$C = \frac{1}{2}\left(\frac{25}{4} - \pi^2\right) \cdot \frac{\alpha}{\pi} +$$

$$\underline{0.26724} \cdot \left(\frac{\alpha}{\pi}\right)^2$$

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vacuum expectation value of the Higgs field $H(x)$

$$|\langle H(x) \rangle| = \frac{1}{\sqrt{2}} v \quad \text{One definition:}$$

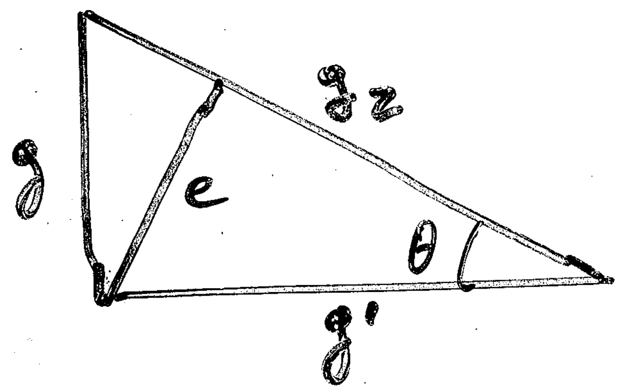
$$\Longrightarrow \quad v_F \equiv \left(\sqrt{2}\, G_F\right)^{-1/2} = \underline{246.221 \pm 0.001 \text{ GeV}}$$

Mathematical prerequisites

① right angle triangle

$$g_Z^2 = g^2 + g'^2 \qquad \text{(Pythagoras)}$$

$$\frac{1}{e^2} = \frac{1}{g'^2} + \frac{1}{g^2} \qquad \text{(dual relation)}$$



② trigonometry

(a) $$2 \tan 2\theta = \frac{1}{\tan(\frac{\pi}{4}-\theta)} - \tan(\frac{\pi}{4}-\theta)$$

$$\tan(\frac{\pi}{4}-\theta) = \frac{1-\tan\theta}{1+\tan\theta}.$$

(b) $$\frac{1}{2}\tan 2\theta = \left(\frac{1}{\tan\theta} - \tan\theta\right)^{-1}$$

6. Heavy Particles: experimental data.

$m_Z = 91.1876 \pm 0.0021$ GeV (PDG 2006)

$m_W = 80.403 \pm 0.029$ GeV (PDG 2006)

$ 80.392 \pm 0.029$ GeV

$m_t = 174.2 \pm 3.3$ GeV (PDG 2006)

$ 171.4 \pm 2.1$ GeV (hep-ex/0608032)

$(m_H < 199 \text{ GeV})$

With

$$\cos\theta \equiv \frac{m_W}{m_Z} = 0.88174 \pm 0.00033$$

Then

$$\tan 2\theta = 1.4991 \pm 0.0045 \approx \frac{3}{2}$$

$$\Rightarrow 2\tan 2\theta = \frac{1}{\tan\left(\frac{\pi}{4}-\theta\right)} - \tan\left(\frac{\pi}{4}-\theta\right) \simeq 3$$

7.

$$\alpha \Rightarrow e = 0.30282212$$

$$\frac{1}{e} - e = 2.99944654$$

$$= 3\left(1 - \frac{\alpha}{4\pi^2} + \dots\right)$$

In first (very good) approximation

$$\frac{1}{e} - e = 3.$$

We assume

$$\boxed{e = \tan\left(\frac{\pi}{4} - \theta\right) = \frac{1-\tan\theta}{1+\tan\theta}}$$

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$$\tan\theta = \frac{1-e}{1+e} \qquad g = \frac{e}{\sin\theta}$$

$$\sin\theta = \frac{1-e}{\sqrt{2(1+e^2)}} \qquad g' = \frac{e}{\cos\theta}$$

$$\cos\theta = \frac{1+e}{\sqrt{2(1+e^2)}}$$

8. WEAK BOSON MASS MATRIX

$$\begin{pmatrix} +\frac{1}{4}\bar{g}^2 v^2 & -\frac{1}{4}\bar{g}\bar{g}' v^2 \\ -\frac{1}{4}\bar{g}\bar{g}' v^2 & +\frac{1}{4}\bar{g}'^2 v^2 \end{pmatrix} =$$

$$\begin{pmatrix} m_W^2 & -m_W m_B \\ -m_W m_B & m_B^2 \end{pmatrix} \longrightarrow$$

$$\begin{pmatrix} m_Z^2 & 0 \\ 0 & 0 \end{pmatrix}.$$

$$m_W^2 + m_B^2 = m_Z^2$$

$$e = \frac{m_W - m_B}{m_W + m_B} = \frac{1 - tg\,\theta}{1 + tg\,\theta} = tg\left(\frac{\pi}{4} - \theta\right)$$

$$\frac{m_W}{m_Z} = \cos\theta \qquad \frac{m_B}{m_Z} = \sin\theta$$

9. Masses of gauge bosons.

$$m_W + m_B = 1 \; \frac{v}{2}$$

$$m_W - m_B = e \; \frac{v}{2}$$

$$m_W = \frac{v}{4}(1+e)$$

$$m_B = \frac{v}{4}(1-e)$$

$$m_Z^2 = \frac{v^2}{8}(1+e^2)$$

$$\longrightarrow v = 246.8476 \pm 0.0057 \text{ GeV}$$

$$(v_F = 246.221 \pm 0.001 \text{ GeV}).$$

$$m_W = 80.400 \pm 0.002 \text{ GeV}$$

$$(m_W^{exp} = 80.403 \pm 0.029 \text{ GeV}).$$

10.

$$m_W = \frac{e}{\sin\theta_W} \frac{1}{(1-\Delta r)^{1/2}} \frac{v_F}{2}$$

$$\left(= \frac{e\,(M \simeq 10\,\mathrm{GeV})}{\sin\theta} \frac{v_F}{2} \right).$$

$$\Updownarrow$$

$$m_W = (1+e)\frac{v}{4} = \frac{e x}{1-e}\frac{v}{4}$$

$$x = \left(\frac{1}{e} - e\right) = 2\sqrt{\frac{2(1+e^2)}{(1-\Delta r)}}\,\frac{v_F}{v}$$

$$\overset{\shortparallel}{2.99944654}$$

$$\Downarrow$$

$$\Delta r = 0.03416$$

to be compared with
standard Model

$$\Delta r = 0.03630 \mp 0.0011 \pm 0.00014$$

Using the approximation

$$\frac{1}{e} - e = 3$$

$$m_W = \frac{1+e}{2}\frac{v}{2} = \frac{3e}{1-e}\frac{v}{2}$$

$$m_Z = \sqrt{2(1+e^2)}\,\frac{v}{4} = \sqrt{2(1+e^2)}\,\frac{3e}{1-e^2}\,\frac{v}{4}$$

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Top Quark + Higgs boson(s)

$$2\, m_W^2 + m_Z^2 = (1 + e^2 + e)\, \frac{v^2}{4}$$

$$= (1 - e)\, \frac{v^2}{2} = \left(4 - \frac{1}{e}\right) \frac{v^2}{2}$$

to be compared with

$$2\, m_W^2 + m_Z^2 = 4\, m_t^2 - m_H^2$$

$$m_t^2 = (2 + X)\, \frac{v^2}{4}$$

$$m_H^2 = \left(\frac{1}{e} + 2X\right) \frac{v^2}{2}$$

With $X = 0$

$$m_t = \frac{v}{\sqrt{2}} = 174.5 \text{ GeV}$$

$$m_H = \frac{v}{\sqrt{2e}} = 317.2 \text{ GeV}$$

With $X = -(1+e)^2/24$

$m_t = 171.43$ GeV

$m_H = 310.35$ GeV

A particular 2-Higgs-doublets-model

[illegible]:

$$2 m_W^2 + m_Z^2 = 4 m_t^2 - (5 m_+^2 + m_-^2)/6$$

In that case:

$$m_t^2 = (2+X) \frac{v^2}{4}$$

$$\frac{5 m_+^2 + m_-^2}{6} = \left(\frac{1}{2} + 2 X\right) \frac{v^2}{2}$$

CONCLUSION

Our model is an effective EW standard model with new physics above a cut-off.

See $\left| \begin{array}{c} \Delta r. \\ m_H \end{array} \right|$ values.

Results.

$$e = \operatorname{tg}\left(\frac{\pi}{4} - \theta\right)$$

$$\frac{1}{e} - e \simeq 3\left(1 - \frac{\alpha}{4\pi^2} + \dots\right)$$

$$m_W = \frac{v}{4}(1+e)$$

$$m_Z = \frac{v}{4}\sqrt{2(1+e^2)}$$

$$m_t \simeq \frac{v}{\sqrt{2}} \simeq 174.5 \text{ GeV}$$

$$m_H \simeq \frac{v}{\sqrt{2e}} \simeq 317.2 \text{ GeV}$$