Electroweak precision predictions in the Standard Model

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UCLouvain, Louvain-la-Neuve, 03/11/2026

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  - Motivations;
  - Precision physics @LHC.
- A phenomenological example
  - Relevance of EW corrections;
  - Mixed QCDxEW corrections to neutral-current Drell-Yan.
- The challenges
  - Treatment of γ_5;
  - Multiscale loop integrals;
  - Development of the framework OceaNN.
- Towards NNLO EW
  - QED corrections to Drell-Yan;
  - Fermionic contribution to NNLO EW for the Drell-Yan.



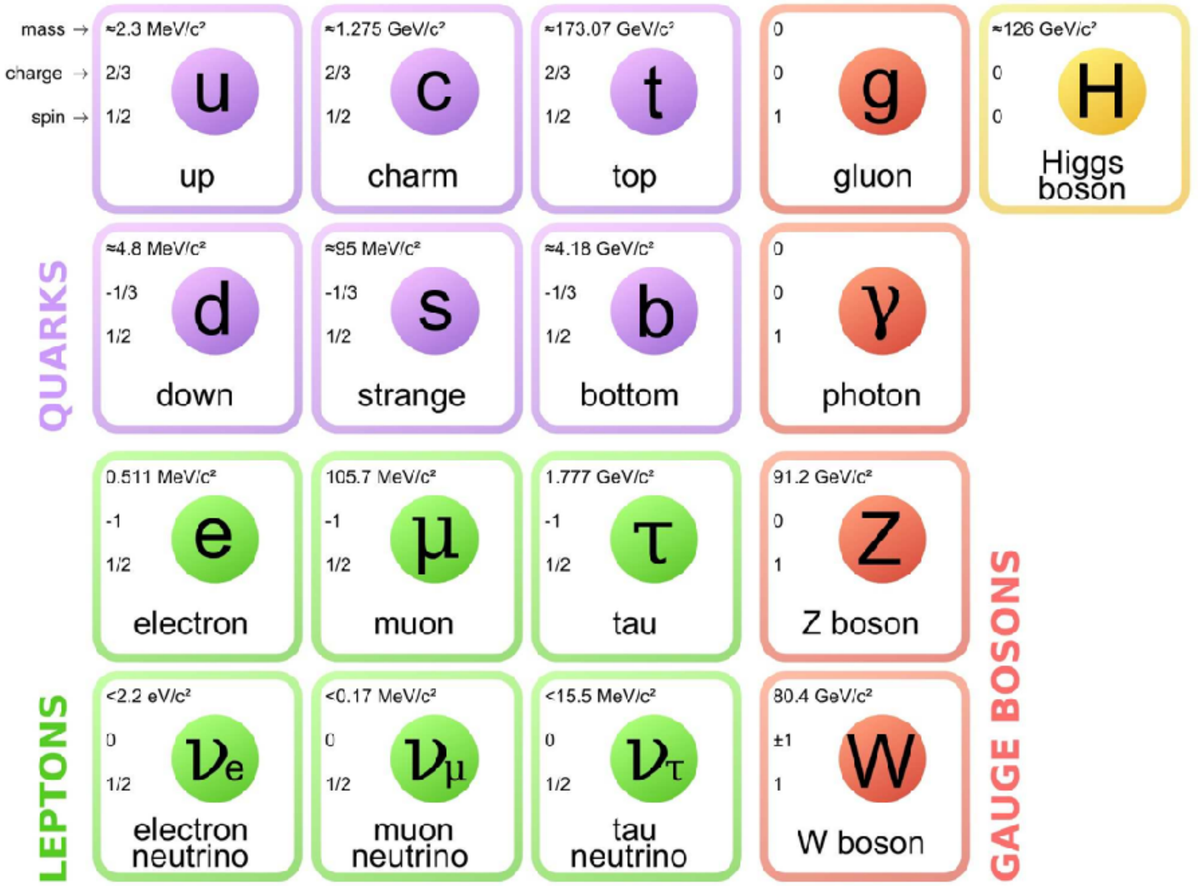
Introduction



The Standard Model

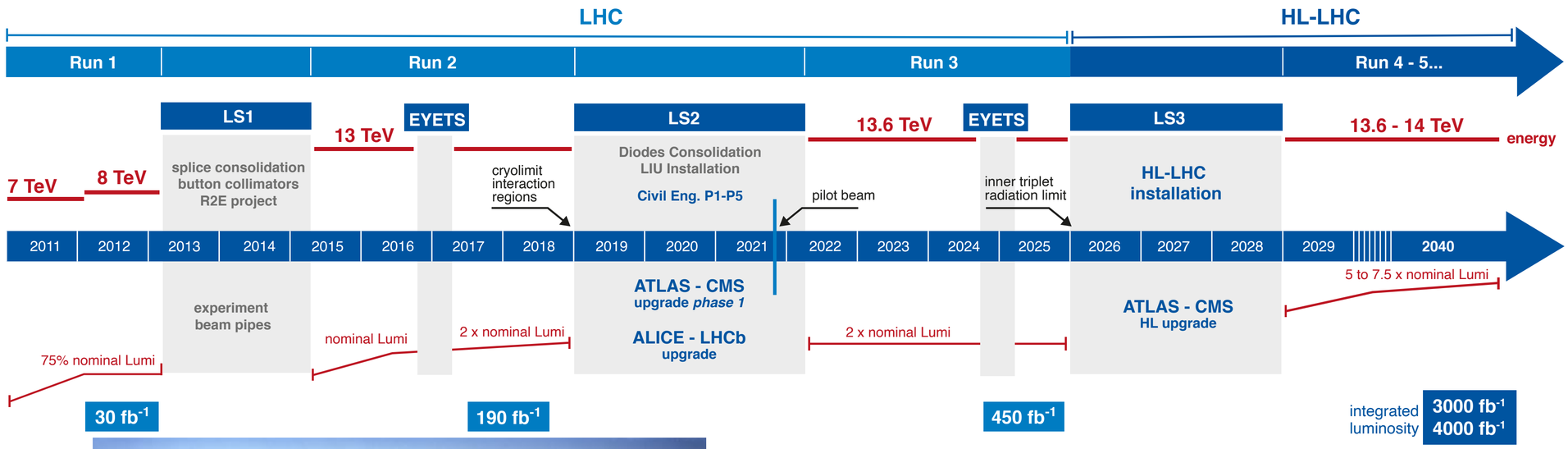
Our (incomplete) understanding of the particle world

- Open questions:
  - How can we accommodate Gravity?
  - How can we accomodate Neutrinos masses?
  - What is the Dark Matter?
  - What is the Dark Energy?
  - Where is all the antimatter (origin of the matter-antimatter asymmetry)?
  - ...
- The EW sector comes from a broken symmetry: can the answer lie in its rich structure?

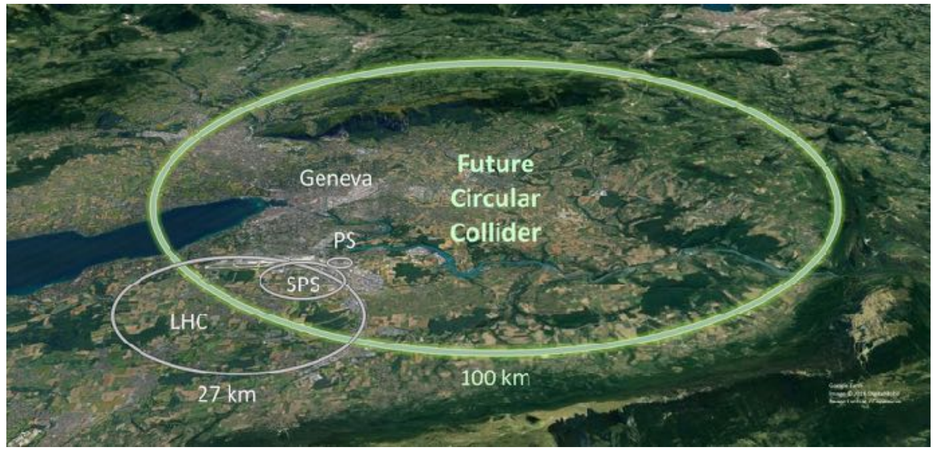


The Large Hadron Collider

Probing the Standard Model



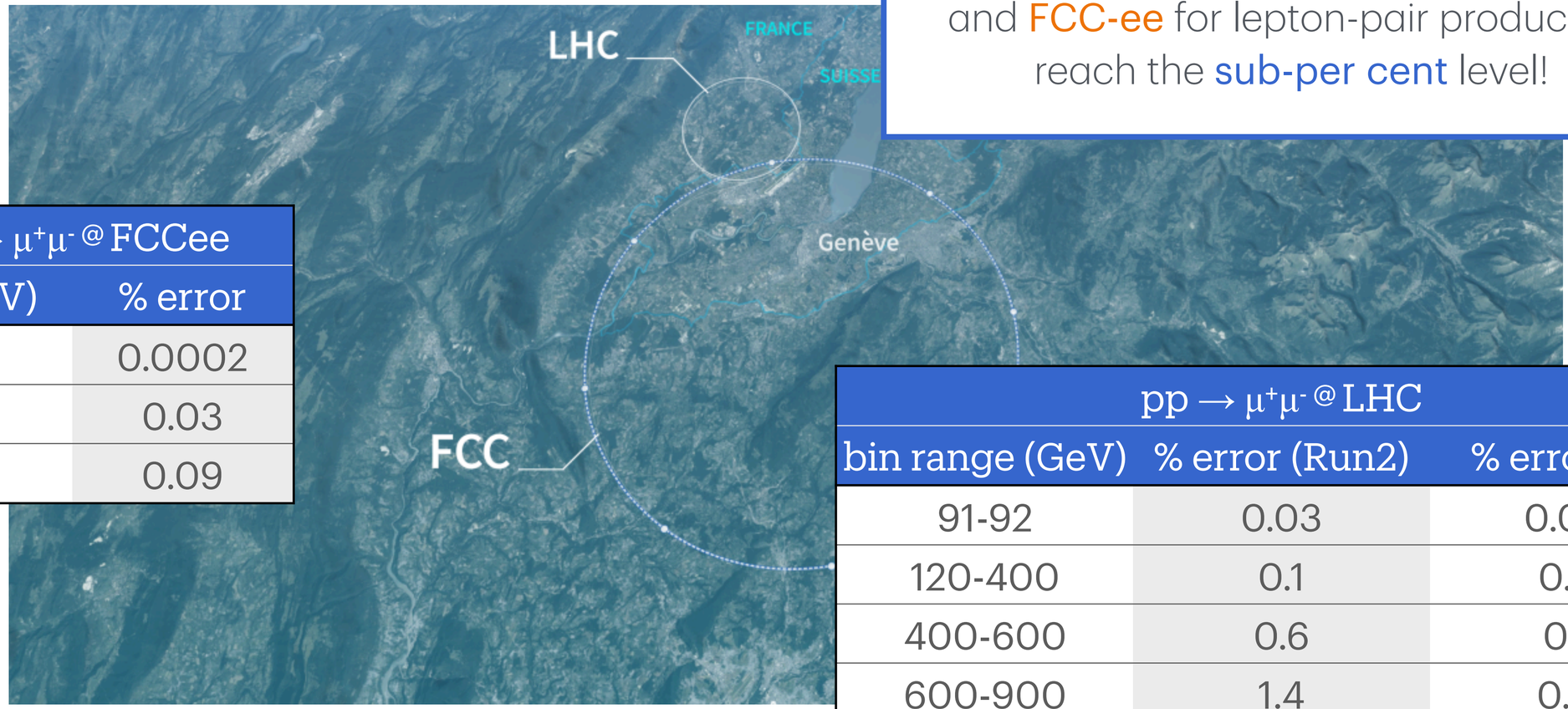




The Large Hadron Collider

Precision

The projected statistical errors at HL-LHC and FCC-ee for lepton-pair production reach the sub-per cent level!



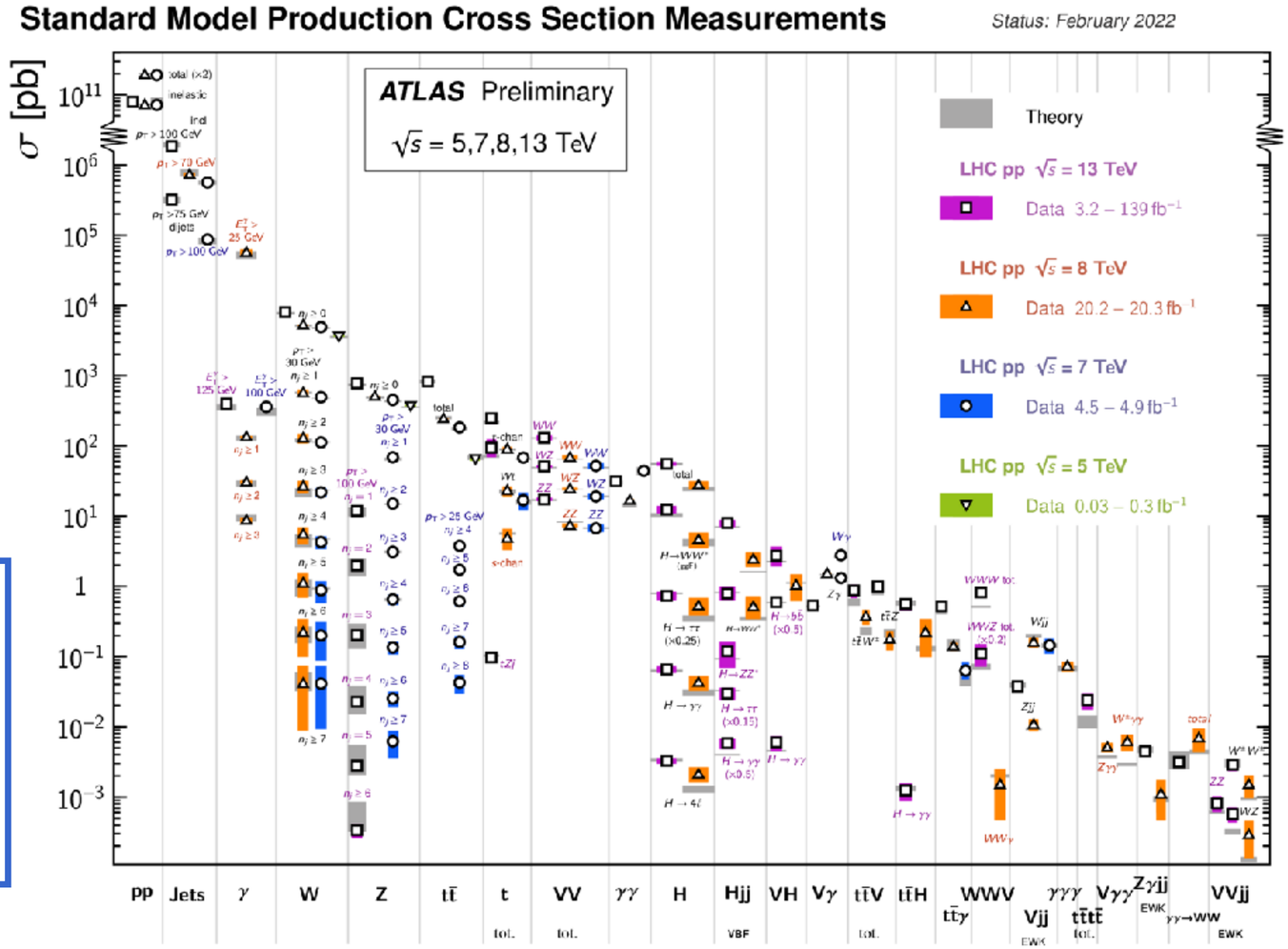
$ee \to \mu^+\mu^-$ @ FCCee	
$\sqrt{s}$ (GeV)	% error
91	0.0002
240	0.03
365	0.09

$pp \to \mu^+\mu^-$ @ LHC		
bin range (GeV)	% error (Run2)	% error (HL)
91-92	0.03	0.006
120-400	0.1	0.02
400-600	0.6	0.13
600-900	1.4	0.30
900-1300	3.2	0.69

The Large Hadron Collider

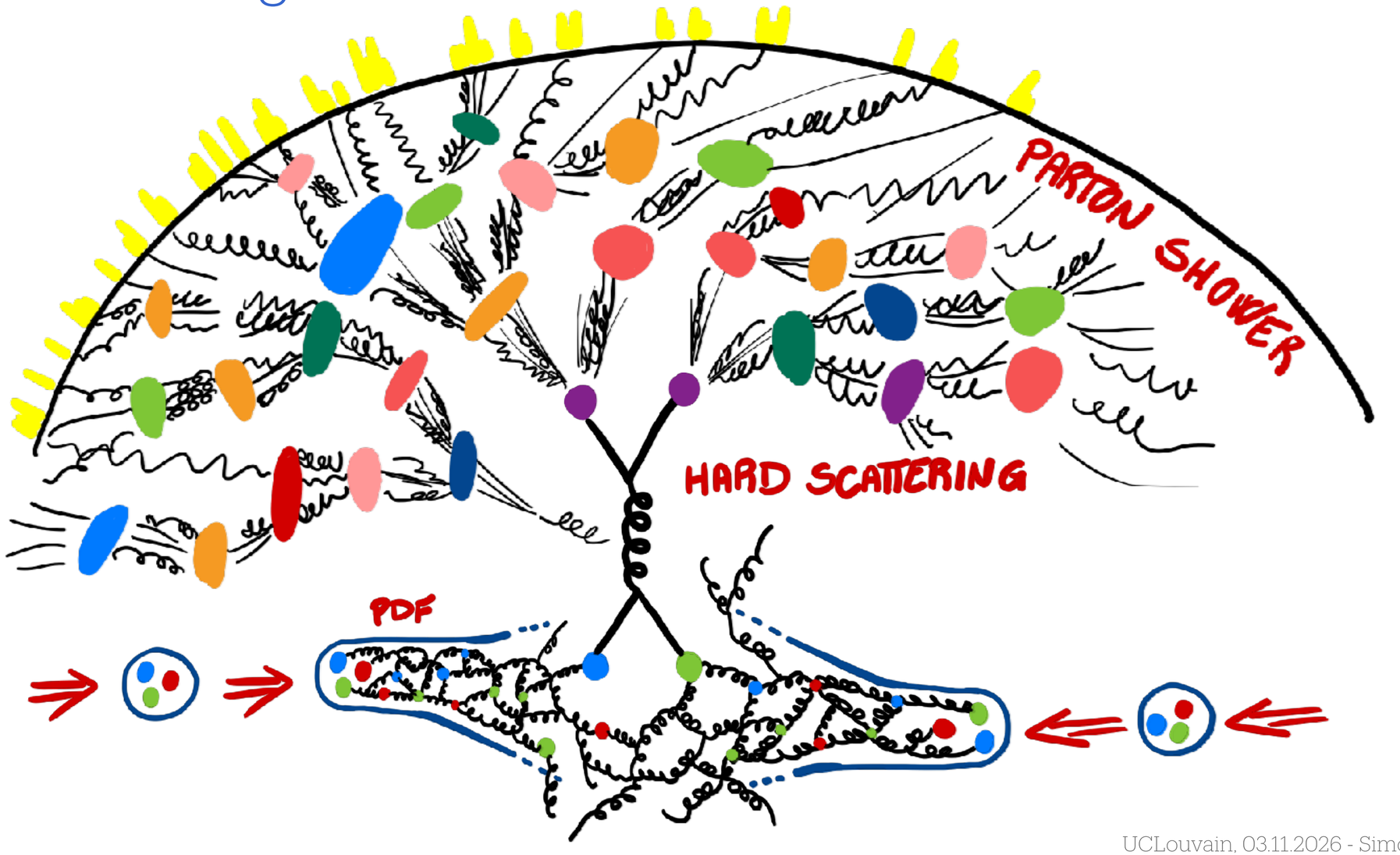
What are we learning?

- LHC has proven itself to be:
  - A discovery machine: Higgs boson discovery;
  - A precision machine: precise measurement of SM parameters.
- Model Building (BSM): Create new models that go beyond the Standard Model (BSM) trying to explain the open questions.
- Precision Studies: Search for small deviations from the predictions of the Standard Model by systematically comparing the best theoretical predictions with data.



A hadronic collision

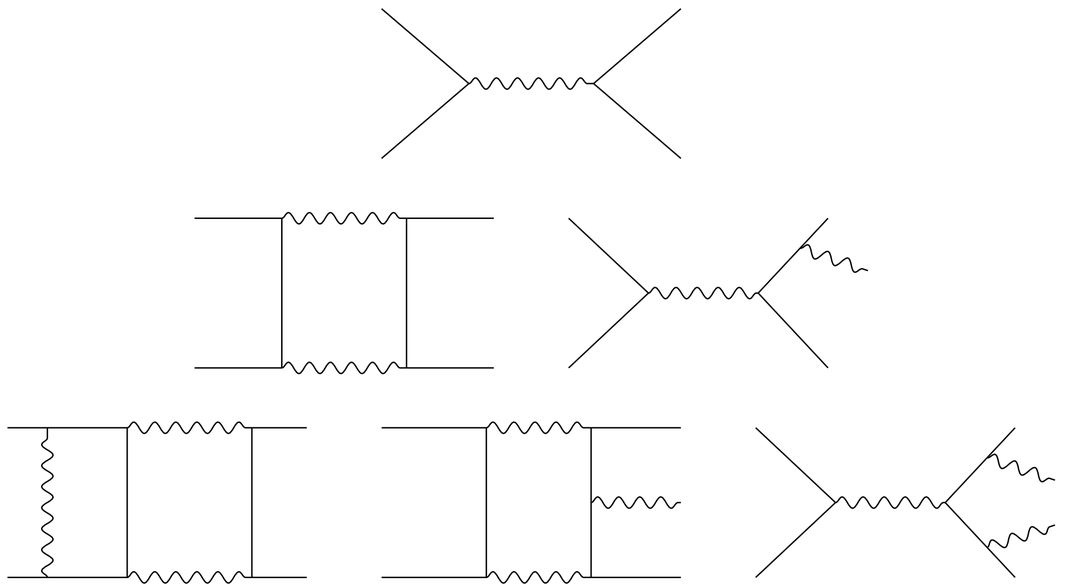
The theoretical ingredients



The hard scattering

The theory bottlenecks

- Leading Order (LO)
- Next-to-Leading Order (NLO)
- Next-to-Next-to-Leading Order (NNLO)



What makes higher-order corrections challenging?

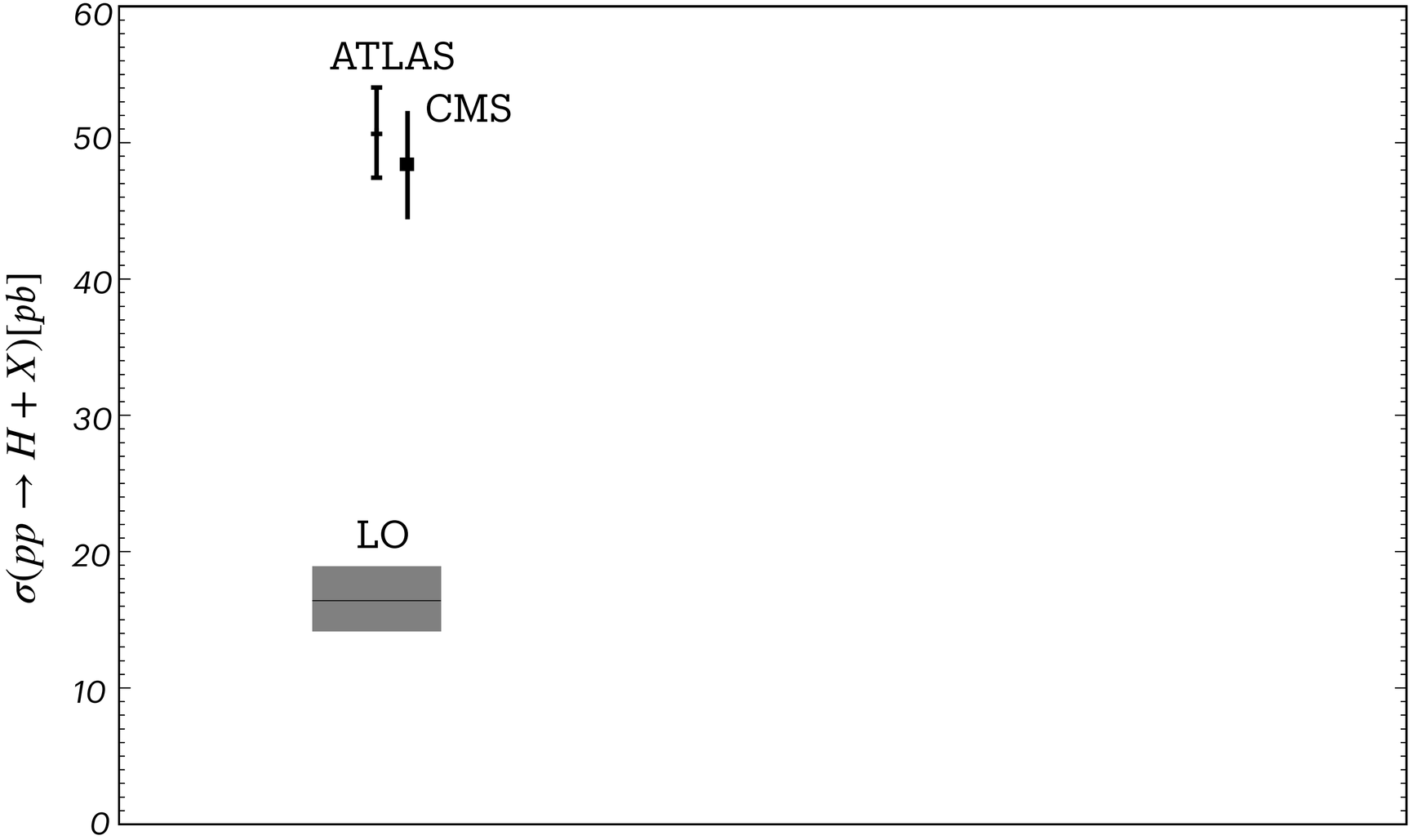
Multi loop amplitudes

Subtraction procedure

The hard scattering

Slide from M.Grazzini

The importance of precision

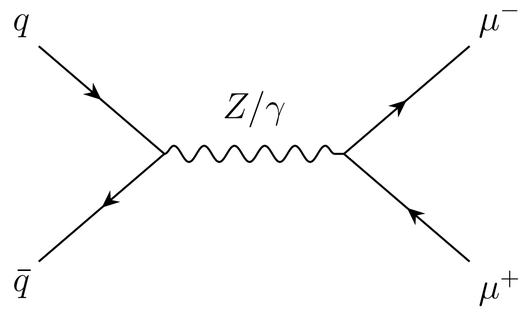


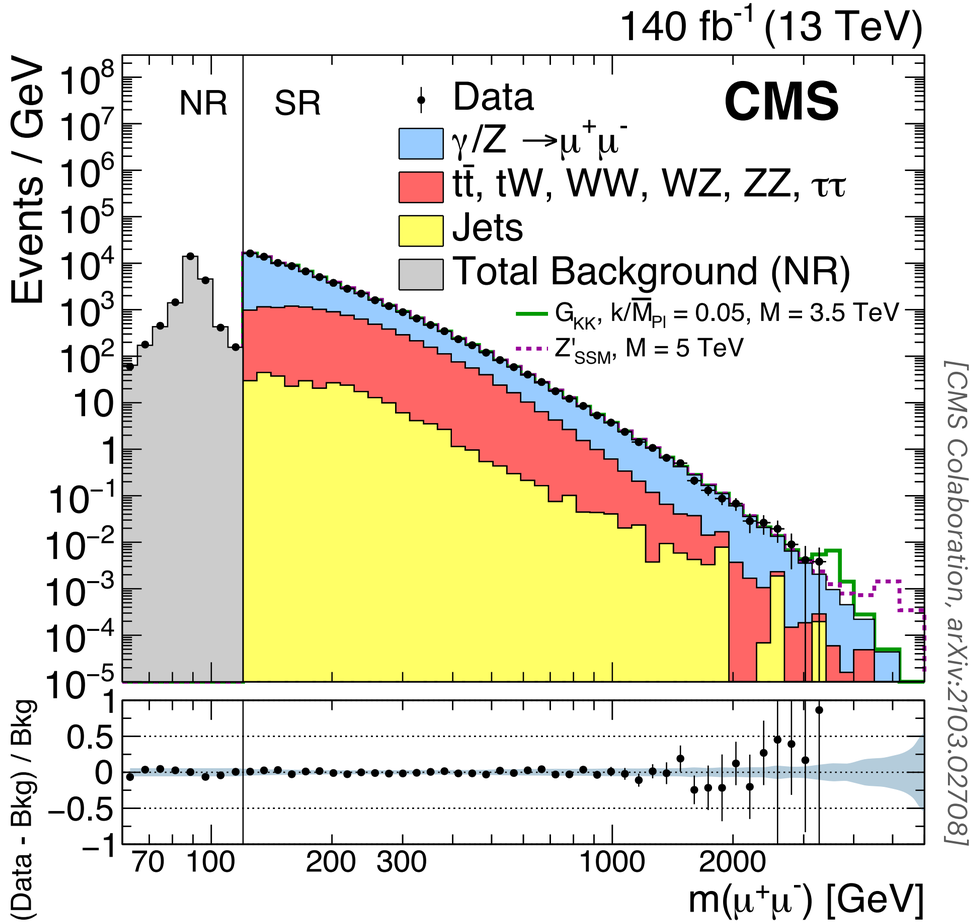
Phenomenology



The Drell-Yan Process

A standard candle





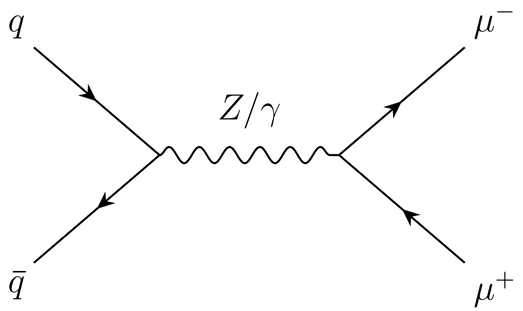
[CMS Collaboration, arXiv:2103.02708]

- **Lepton-pair invariant mass:** at Z resonance: Z properties; AFB: $\sin^2\theta^{\ell}_{eff}$;
- **Lepton-pair transverse mass:** W resonance: m_W;
- **Charged lepton transverse momentum:** m_Z, m_W;
- **Lepton-pair (pseudo-)rapidity:** PDF constraints;
- **Lepton-pair transverse momentum:** low transverse momentum - α_S; large transverse momentum - PDF constraints;
- **Large invariant mass tail:** BSM Signal, PDF constraint.

Higher order corrections

Expansion in the coupling

$$q(p_1) + \bar{q}(p_2) \to l^-(p_3) + l^+(p_4) \quad = \quad \sigma^{(0,0)} \quad \textit{Drell-Yan (1970)}$$



$$+\ \alpha_S\,\sigma^{(1,0)} \quad + \ \alpha\,\sigma^{(0,1)}$$

$$+\ \alpha_S^2\,\sigma^{(2,0)} \quad + \ \alpha\,\alpha_S\,\sigma^{(1,1)} \quad + \ \alpha^2\,\sigma^{(0,2)}$$

$$+\ \alpha_S^3\,\sigma^{(3,0)} \quad + \ \ldots$$

QCD MIXED EW

Higher order corrections

QCD corrections

$$q(p_1) + \bar{q}(p_2) \to l^-(p_3) + l^+(p_4) \quad = \quad \sigma^{(0,0)} \quad \textit{Drell-Yan (1970)}$$

$$+ \alpha_S \sigma^{(1,0)} + \alpha \sigma^{(0,1)}$$

$$+ \alpha_S^2 \sigma^{(2,0)} + \alpha \alpha_S \sigma^{(1,1)} + \alpha^2 \sigma^{(0,2)}$$

$$+ \alpha_S^3 \sigma^{(3,0)} + \ldots$$

- Dominant effect;
- Known up to N3LO.

NLO:
[G.Altarelli, R.Ellis, G.Martinelli Nucl.Phys.B 157 (1979)];

NNLO:
[R.Hamberg, T.Matsuura, W.van Nerveen, Nucl. Phys. B 359 (1991)];
[C.Anastasiou, L.J.Dixon, K.Melnikov, F.Petriello, hep-ph:0306192];
[S.Catani, L.Cieri, G.Ferrera, D.de Florian, M.Grazzini arXiv:0903.2120];

N3LO:
[C.Duhr, F.Dulat, B.Mistlberger arXiv:2007.13313];
[X.Chen, T.Gehrmann, N.Glover, A.Huss, T.Yang, and H.Zhu arXiv:2107.09085];
[S.Camarda, L.Cieri, G.Ferrera arXiv:2103.04974];
[X.Chen, T.Gehrmann, N.Glover, A.Huss, P.Monni, E.Re, L.Rottoli, P.Torrielli arXiv:2203.01565];.

Higher order corrections

EW corrections

$$q(p_1) + \bar{q}(p_2) \to l^-(p_3) + l^+(p_4) \quad = \quad \sigma^{(0,0)} \quad \textit{Drell-Yan (1970)}$$

$$+ \alpha_S \sigma^{(1,0)} + \alpha \sigma^{(0,1)}$$

$$+ \alpha_S^2 \sigma^{(2,0)} + \alpha \alpha_S \sigma^{(1,1)} + \alpha^2 \sigma^{(0,2)}$$

$$+ \alpha_S^3 \sigma^{(3,0)} + \ldots$$

- Subdominant with respect to QCD: physical counting $\alpha_S^2 \simeq \alpha$;

NLO:

[U.Baur, O.Brein, W.Hollik, C.Schappacher, D.Wackeroth, hep-ph:0108274]; [S.Dittmaier, M.Kramer, hep-ph:0109062];[U.Baur, D.Wackeroth, hep-ph:0405191];

Ongoing work towards NNLO:

[B. Jantzen, J.H.Kühn. A.A.Penin, V.A.Smirnov, hep-ph:0509157]; [R. Bonciani, A. Broggio, S. Di Vita, A. Ferroglia, M. K. Mandal, P. Mastrolia, L. Mattiazzi, A. Primo, J. Ronca, U. Schubert, W. J. Torres Bobadilla, F. Tramontano, arXiv:2106.13179]; [A. Freitas, E.J.Wallace, arXiv:2512.15700]; [T. Armadillo, SD, M. Dradi, A.Vicini, arXiv:2511.20365].

Higher order corrections

Mixed corrections

$$q(p_1) + \bar{q}(p_2) \to l^-(p_3) + l^+(p_4) \quad = \quad \sigma^{(0,0)} \quad \textit{Drell-Yan (1970)}$$

$$+ \alpha_S\, \sigma^{(1,0)} + \alpha\, \sigma^{(0,1)}$$

$$+ \alpha_S^2\, \sigma^{(2,0)} + \alpha\, \alpha_S\, \sigma^{(1,1)} + \alpha^2\, \sigma^{(0,2)}$$

$$+ \alpha_S^3\, \sigma^{(3,0)} + \ldots$$

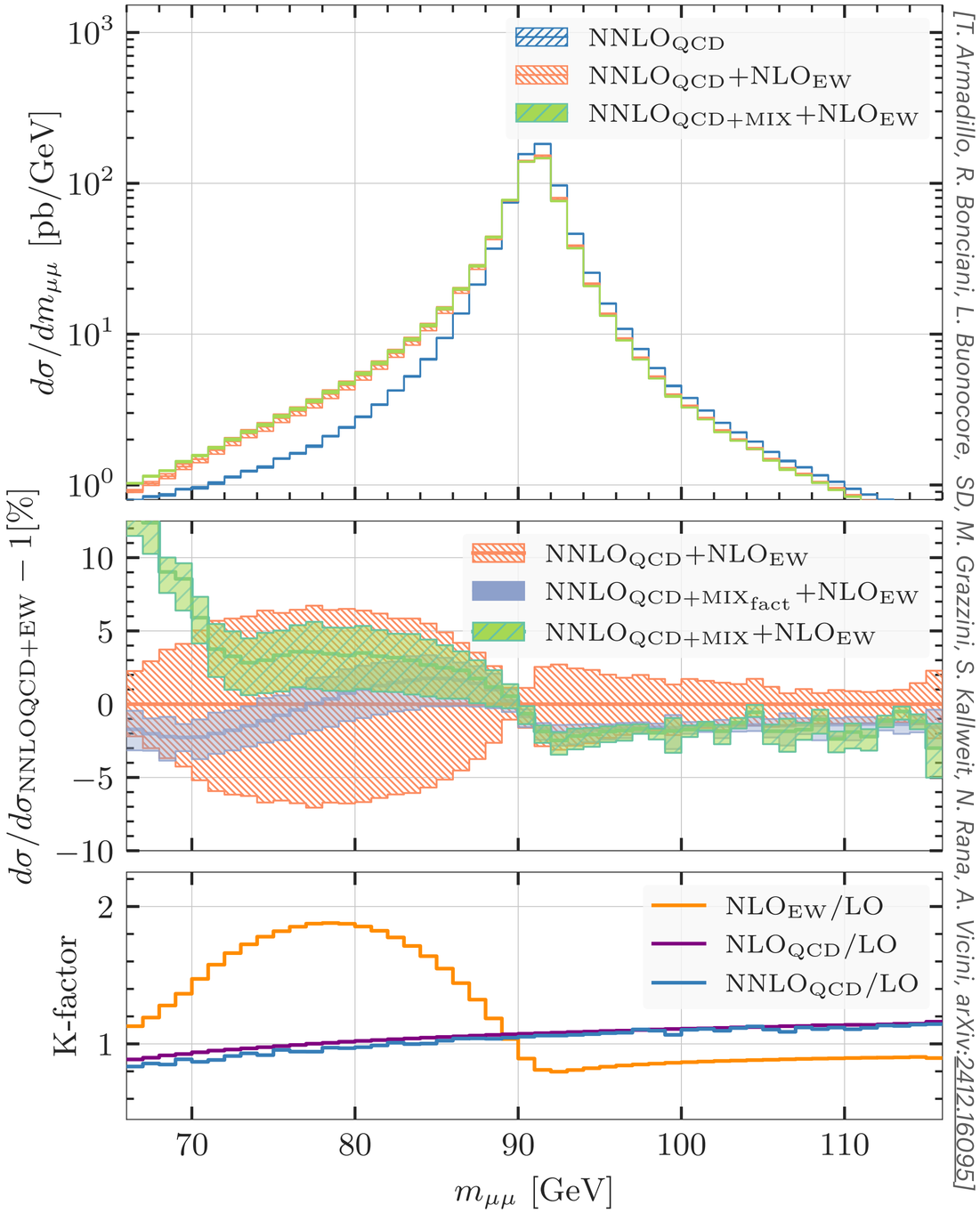
- By physical counting, the expected size is comparable to N^{3}LO QCD.

[L. Cieri, D. de Florian, M.Der, J.Mazzitelli, arXiv:2005.01315]; [L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539]; [R. Bonciani, L. Buonocore, M. Grazzini, S. Kallweit, N. Rana, F. Tramontano, A.Vicini, arXiv:2106.11953]; [T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A.Vicini, arXiv:2412.16095]; [F. Buccioni, F. Caola, H. Chawdhry, F. Devoto, M.Heller, A. von Manteuffel, K. Melnikov, R. Röntsch, C. Signorile-Signorile, arXiv:2203.11237]

Differential results

Invariant mass distribution

- Mild shape distortion from QCD corrections;
- Strong shape distortion from EW corrections, which generate a radiative tail before the peak;
- The mixed corrections provide a non-trivial shape distortion;
- The factorised approximation does not give a good description of the resonant region.

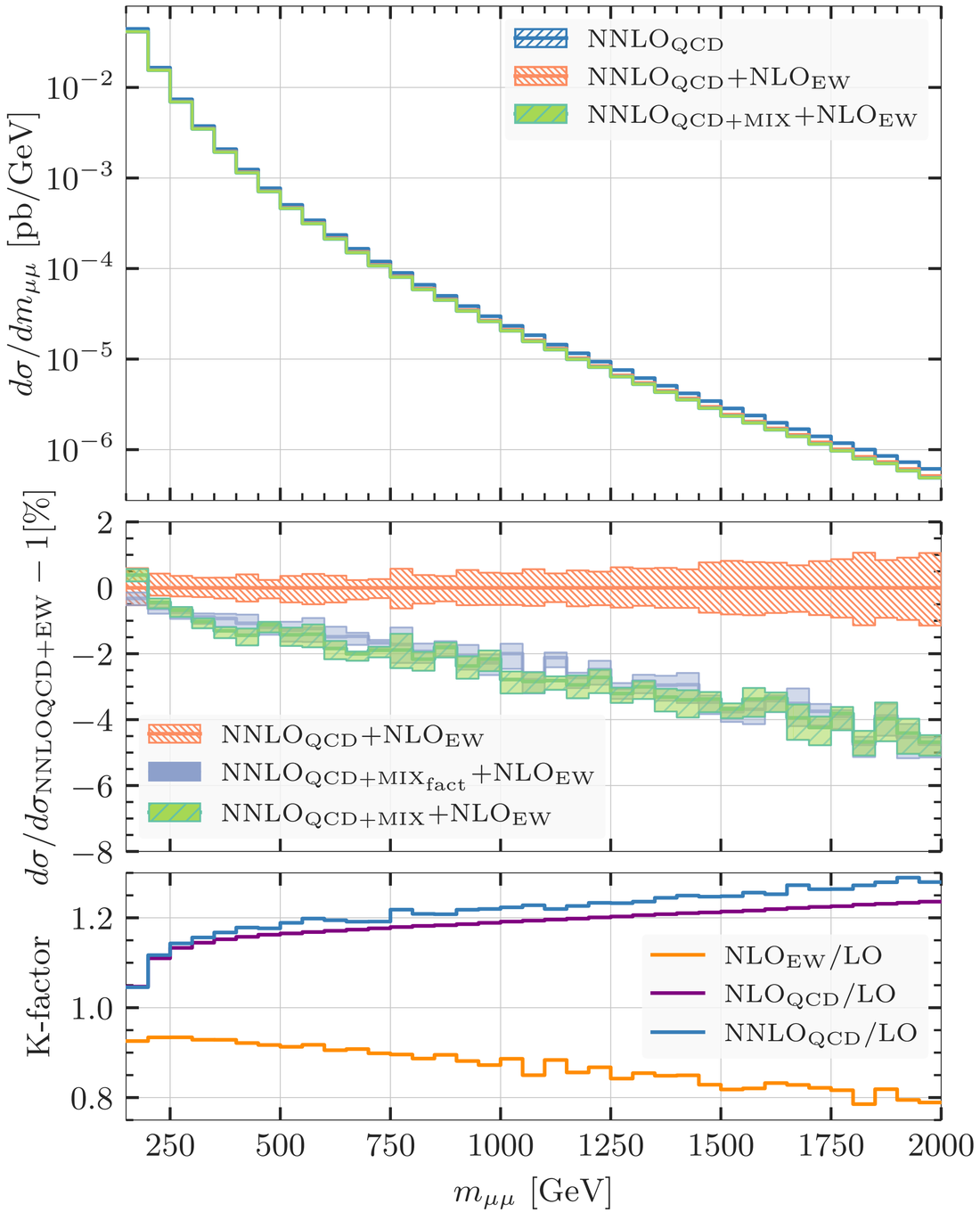


[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]

Differential results

Invariant mass distribution

- We examine the high invariant-mass region, following cuts from *[CMS collaboration, arXiv:2103.02708]*:
  $p_{T,\mu^\pm} > 53\,\text{GeV},\ |y_\mu| < 2.4,\ m_{\mu\mu} > 150\,\text{GeV}$
- The mixed corrections provide a relevant contribution, increasing at higher invariant mass and larger than the statistical error expected by the end of HL-LHC;
- The factorised approximation gives a good description of the high-invariant mass region.



[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]

Forward-backward asymmetry

Measuring the weak mixing angle

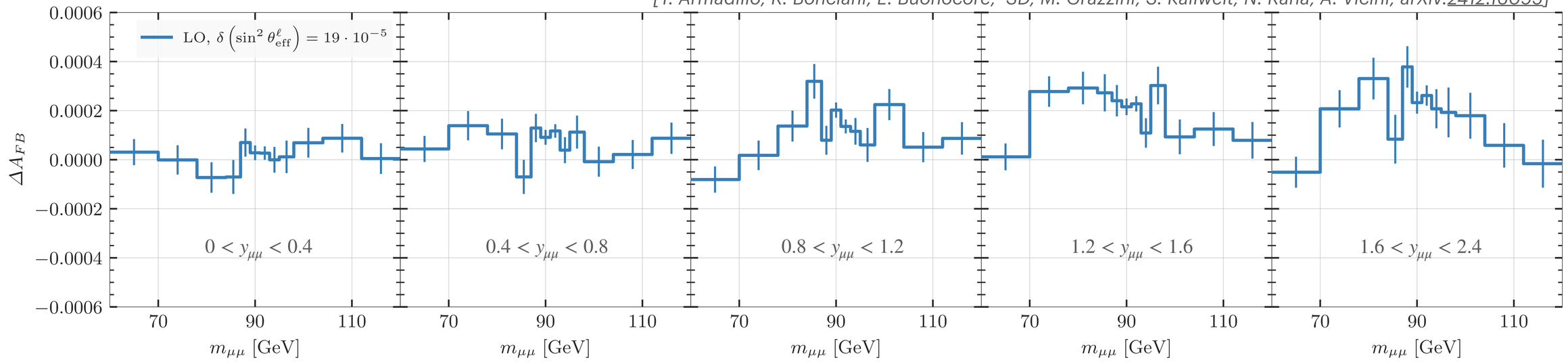
$$A_{FB}(m_{\mu\mu}) = \frac{F(m_{\mu\mu}) - B(m_{\mu\mu})}{F(m_{\mu\mu}) + B(m_{\mu\mu})}$$

$$B(m_{\ell\ell}) = \int_{-1}^{0} d\cos\theta^* \frac{d\sigma}{d\cos\theta^* dm_{\ell\ell}}(m_{\ell\ell})$$

Around the Z resonance, sensitivity to $\sin^2\theta^{\ell}_{eff}$!

$$F(m_{\ell\ell}) = \int_{0}^{1} d\cos\theta^* \frac{d\sigma}{d\cos\theta^* dm_{\ell\ell}}(m_{\ell\ell})$$

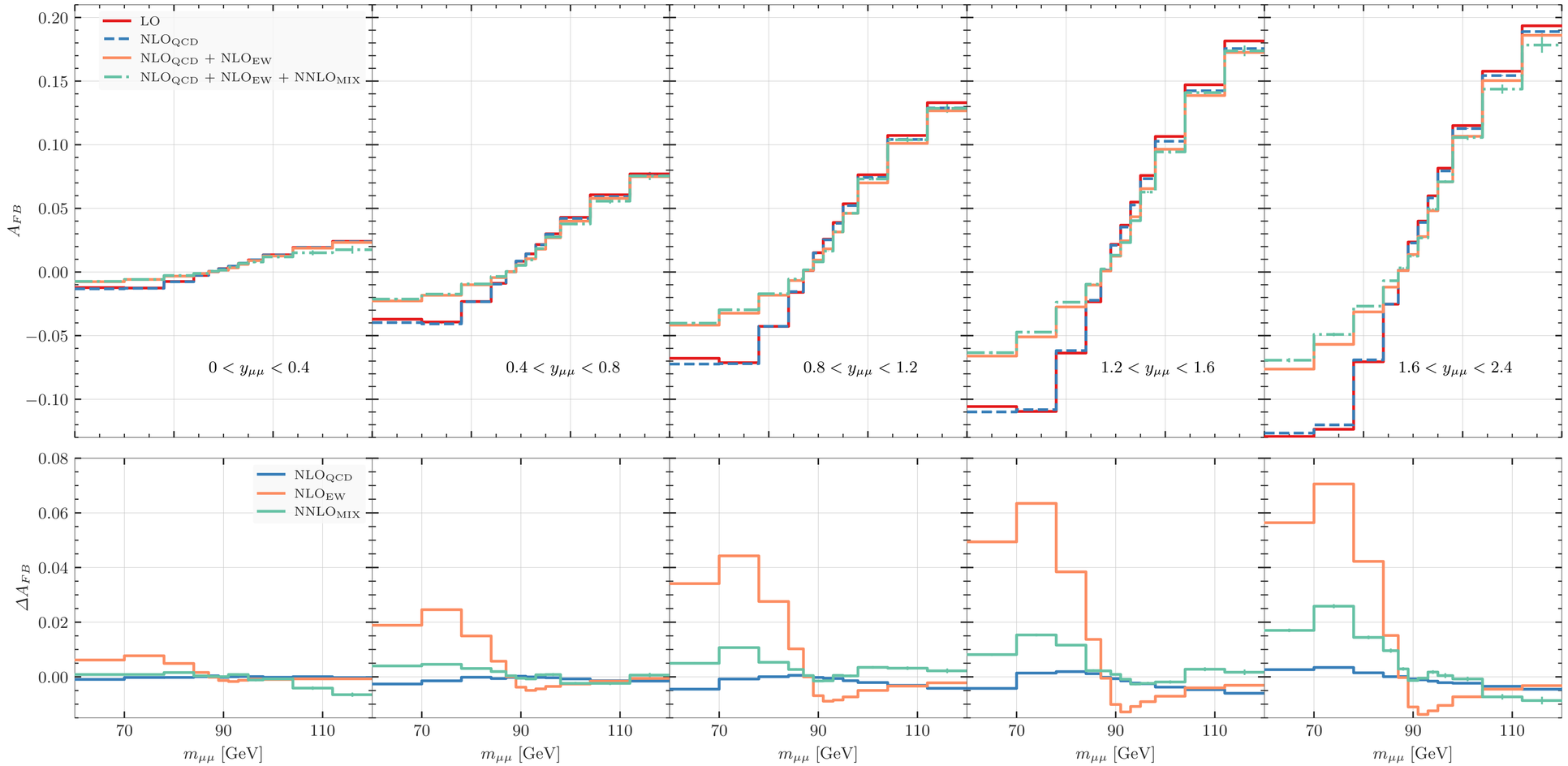
[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]



Forward-backward asymmetry

The effect of mixed corrections

[T. Armadillo, R. Bonciani, L. Buonocore, SD, M. Grazzini, S. Kallweit, N. Rana, A. Vicini, arXiv:2412.16095]



The challenges



Computing an amplitude

Coefficients and integrals

What makes higher-order corrections challenging?

Multi loop amplitudes

Subtraction procedure

- The amplitude can be cast in terms of a sum of algebraic coefficients and integrals:
  - Scalar integrals;
  - Master integrals.

$$\sum_{i=1}^{N} c^{(S,i)}\, I_{S,\vec{\beta}^{(i)}} \qquad I_{S,\vec{\beta}^{(i)}} = \int \prod_{j=1}^{L} d^n q_j \, \frac{1}{\mathscr{P}_1^{\beta_1^{(i)}} \dots \mathscr{P}_t^{\beta_t^{(i)}} \mathscr{P}_{t+1}^{\beta_{t+1}^{(i)}} \dots \mathscr{P}_N^{\beta_N^{(i)}}}$$

$$\sum_{i=1}^{n} c^{(M,i)}\, I_{M,\vec{\beta}^{(i)}} \qquad n \ll N$$

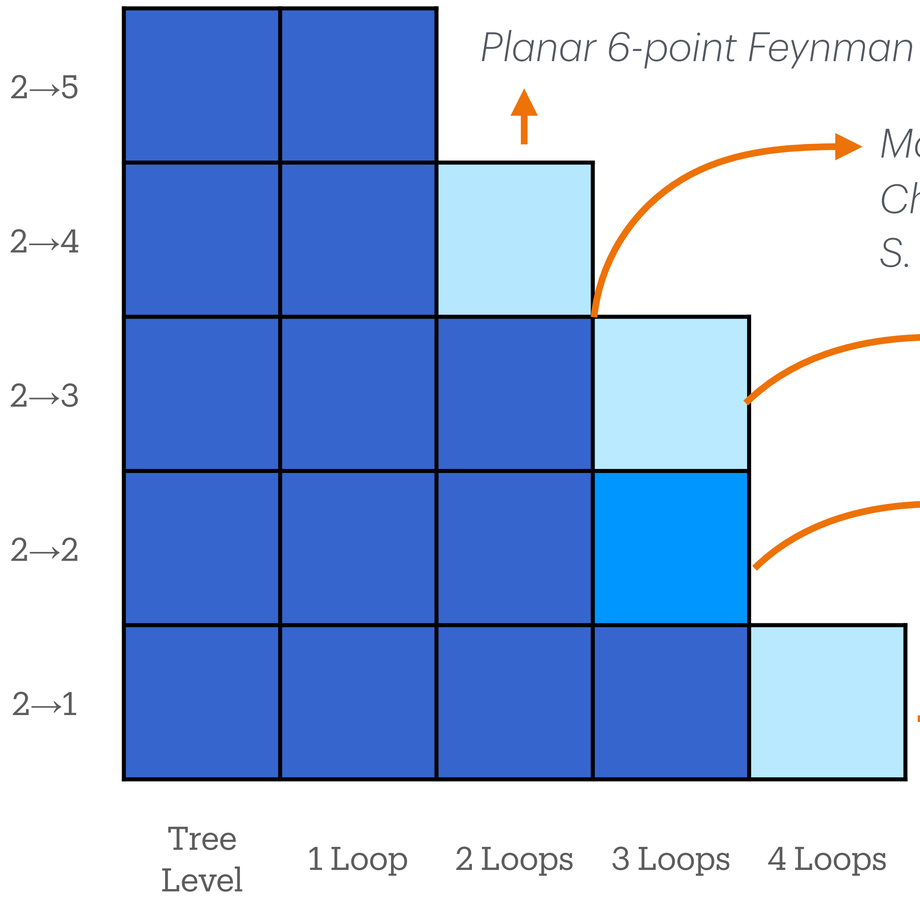
$$\mathscr{A} = \sum_k c_k\, I_k$$

- Master Integrals are closed under differentiation: deriving with respect to one of the kinematic variables one of the master integrals, we obtain a linear combination of master integrals;
- We can write a system of differential equations satisfied by the master integrals!

$$dI_k = A_{kj} I_j$$

State of the Art

QCD Corrections



Planar 6-point Feynman Integrals: e.g. [S. Abreu, P.F. Monni, B. Page, J. Usovitsch: 2412.19884]; …

Massless available, massive ongoing: e.g. $t\bar{t}W$ *[M. Becchetti, D. Canko, X. Cheng, V. Chestnov, M. Delto, S. Ditsch, T. Peraro, M. Pozzoli, C. Savoini, S. Zoia: 2608.03746]; …*

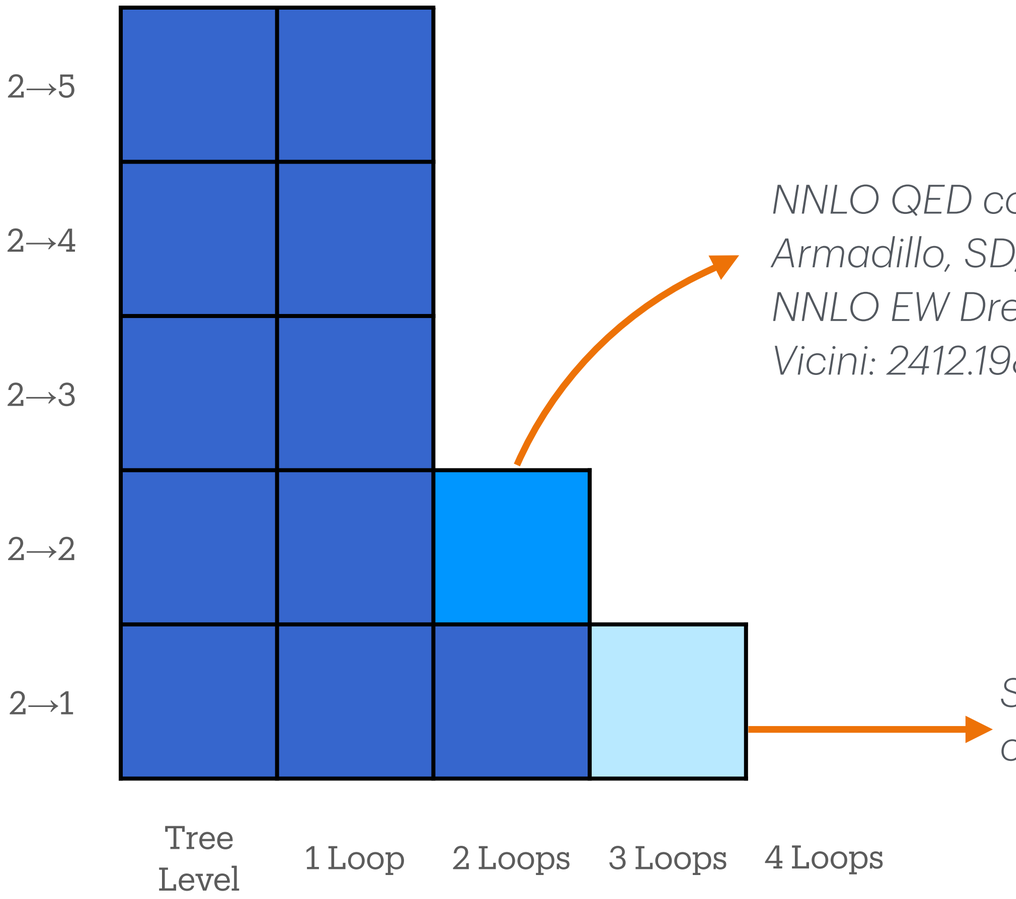
First results for Feynman Integrals [Y. Liu , A. Matijašić, J. Miczajka, Y. Xu, Y. Xu, Y. Zhang: 2411.18697]; …

Selected amplitudes in Leading Colour approximation: e.g. $H+$ *jet [X. Chen, X. Guan, B. Mistlberger: 2504.06490]*

RVVxV contribution to Higgs production [B. Mistlberger, A. Suresh: 2504.10574]

State of the Art

EW Corrections



NNLO QED corrections to Drell-Yan [R. Bonciani et al., arXiv:2106.13179], [T. Armadillo, SD, M. Dradi, A.Vicini, arXiv:2511.20365] ; Fermionic contributions to NNLO EW Drell-Yan [A. Freitas, J. Wallace: 2512.15700],[T. Armadillo, SD, M. Dradi, A. Vicini: 2412.19884].

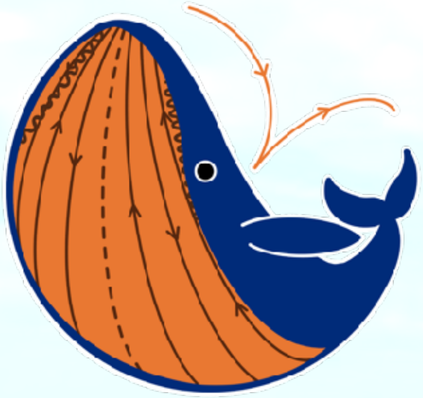
Selected contributions from the leading fermionic contributions [L. Chen, A. Freitas: 2002.05845]

The Challenges

What makes EW corrections challenging?

- Large computations ($\mathcal{O}(10^4)$ diagrams);
  systematic bookeeping is a priority;
- Treatment of γ_5;
  how can γ_5 be consistently used in dimensional regularisation?
- Additional internal massive lines;
  additional scales in the problem (m_Z, m_W, m_H...) bring additional complications, in particular elliptic integrals;
- Need for the complex mass scheme;
  requires to analytically continue the master integrals on the complex plane!





OceaNN

Organised **C**alculation of **E**lectroweak **A**mplitudes at **NNLO**

Process definition

$$|\mathscr{M}|^2 = \sum_k c_k I_k$$

c_k=coefficients; I_k=Master Integrals

Numerical grid

FeynArts
Public Mathematica package

Abiss
Private Mathematica package

Kira
With C++ Firefly library

Evaluation of I_k

Differential Equations

Boundaries

Evolution

LiteRed
Public Mathematica package

AMFlow
Public Mathematica package

SeaSyde
Public Mathematica package

ABISS

Evaluating c_k



- ABISS is a MATHEMATICA package to perform Dirac and Lorentz algebra;



- Output of ABISS:
  - Unpolarized virtual amplitude expressed as a sum of scalar integrals.
- Main Strengths of ABISS:
  - Automation, keeping bookkeeping manageable;
  - Consistent handling of γ_5, allowing for checking independence from the conventions.

Public release coming soon!
STAY TUNED!

Handling γ_5

Prescriptions in dimensional regularisation



	't Hooft and Veltmann Nucl. Phys. B 44 (1972) 189–213	Kreimer et al. Phys. Lett. B 237 (1990) 59–62
Anticommutation	✘	✔
Cyclicity	✔	✘
Implementation	• $\bar{\gamma}_\mu \rightarrow \mu = 0,\ldots,3$ • $\hat{\gamma}_\mu \rightarrow (-2\epsilon)$ dimensional • $\{\bar{\gamma}_\mu, \gamma_5\} = 0$; $[\hat{\gamma}_\mu, \gamma_5] = 0$;	• Define a consistent reading point as the "start" of each dirac trace: fixed point prescription;
Comments	• The most mathematically sound approach; • Increased computational load; • Chiral symmetry needs to be restored with ad hoc counterterms; • Non-trivial implementation at higher orders.	• More streamlined computational implementation; • Single traces give different results for different choices of the fixed point; • Sanity check: verify that the final result does not depend on the choice of the fixed point.

HANDLING γ_5

Prescriptions in dimensional regularisation



- For neutral-current Drell-Yan, proven *[M. Heller, A. von Manteuffel, R. M. Schabinger and H. Spiesberger, arXiv:hep-ph/2012.05918]* that at two loops the two prescriptions yield:
  - different bare scattering amplitudes;
  - same finite UV-renormalised and IR-subtracted amplitudes.

- ABISS automates Kreimer's prescription;
- A consistent implementation of Kreimer's scheme requires:
  - treating $\epsilon_{\mu\nu\rho\sigma}$ as a pure 4-dimensional object;
  - computing higher-dimensional integrals.

$$k^\mu = \bar{k}^\mu + \hat{k}^\mu \qquad g^{\mu\nu} = \bar{g}^{\mu\nu} + \hat{g}^{\mu\nu} \qquad \bar{g}^{\mu\nu}\bar{g}_{\mu\nu} = 4$$

- Successful automated treatment of 2→2 processes;
- Next Steps: Generalisation to arbitrary processes? Implementation of 't Hooft-Veltmann prescription?

Computing an amplitude

Evaluating the integrals

- The integrals can be computed by following different strategies:

Analytic methods

- Canonical basis: differential equation in dlog form, solution in terms of special functions;
- Internal masses complicate the picture: elliptic functions, Calabi-Yau, …

Numerical methods

- Numerical integration via sector decomposition: pySecDec *[G. Heinrich et al. 2305.19768]* ;
- Numerical integration of a parametric representation (Feynman parameter, Baikov variables, …);
- Numerical solution of the differential equation via series expansion.

How to compute integrals

Strengths and weaknesses of the methods

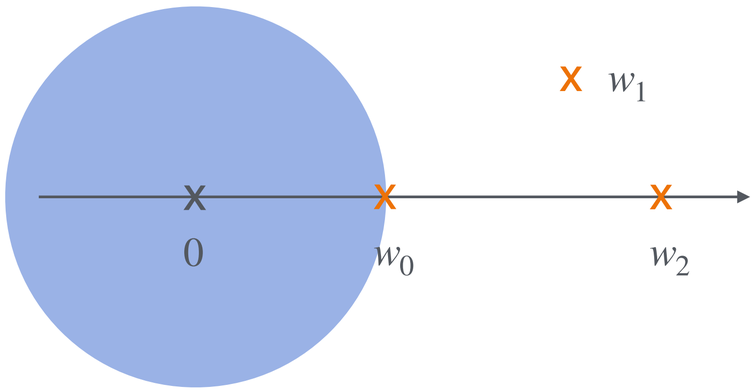
	Analytical Methods	Numerical Methods	Semi-Analytical methods
Functional relations	✔	✘	✘
Analytic properties *(poles location)*	✔ *Mathematical study of function properties.*	✘ *In pySecDec, the pole structure is exploited at the integrand level.*	✔ *Pole structure in the coefficients of the differential equations.*
Numerical evaluation	*Series expansion of analytical functions.*	• *Monte Carlo integration;* • *Numerical quadrature of parametric representations (Feynman, Baikov, ...).*	*Result expressed as series expansion.*
Error esitmation	Estimation through the truncation of the series.	• Monte Carlo: 1/√n; • Numerical quadrature: comparison between different number of nodes.	Estimation through the truncation of the series.

Series Expansion

Taylor and logarithmic expansion

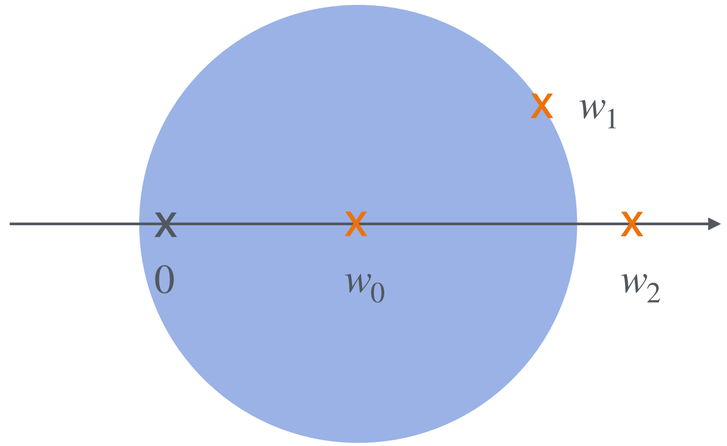
Taylor Ansatz:

$$f(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$



Logarithmic ansatz:

$$f(x) = x^r \sum_{k,j=0}^{\infty} c_{k,j} (x - w_0)^k \log^j(x - w_0)$$

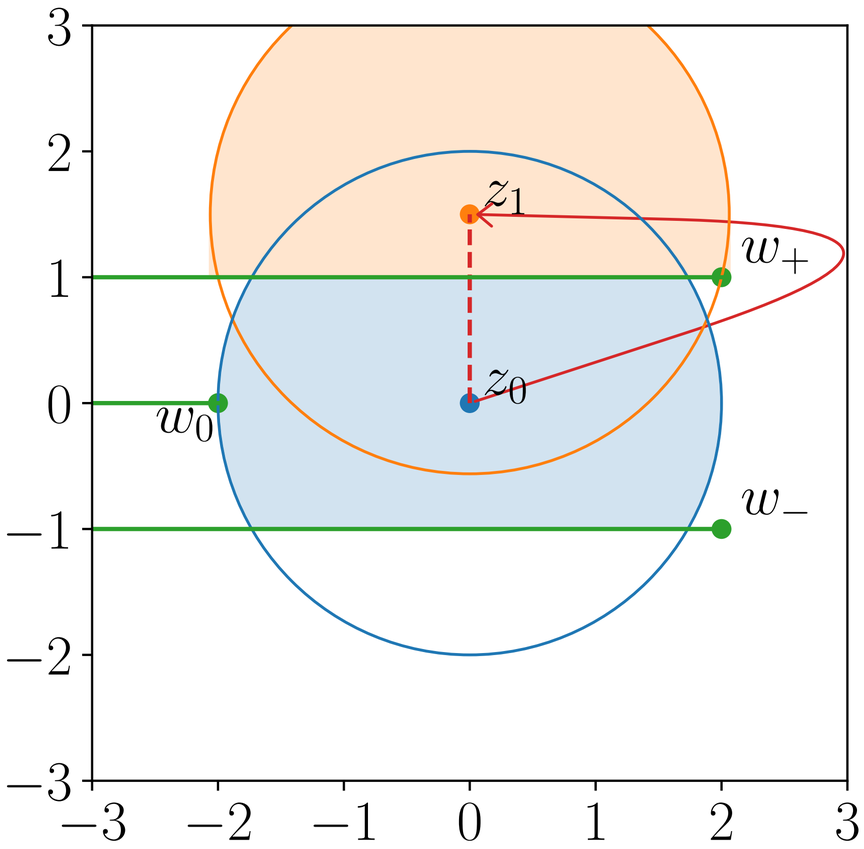


- Original implementation *[F.Moriello, arXiv:1907.13234]* in DIFFEXP, *[M.Hidding, arXiv:2006.05510]*;
- Now, multiple codes are available (SEASYDE *[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2205.03345]*, AMFLOW *[R. Huang, X. Liu, Y. Ma , arXiv: 2607.08477]*, LINE *[R. Prisco, J. Ronca, F. Tramontano, arXiv: 2501.01943]*, ...).

SeaSyde

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2205.03345]

Moving into the complex plane



- A **gauge-invariant** definition of the mass can be obtained through the **complex mass scheme**;
- Complex kinematical variables require analytically continuing the solution over the complex plane;
- SeaSyde is a Mathematica package implementing the series expansion approach on **the complex plane**.
- The radius of convergence of the series is limited by the presence of **poles**;
- "Transport" of the boundary conditions needs to consider **branch cuts**.

Towards NNLO EW

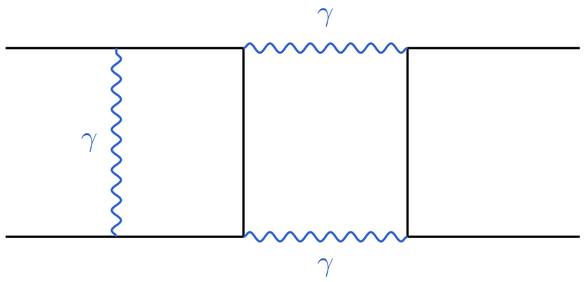


Towards NNLO EW

[R. Bonciani et al., arXiv:2106.13179]

NNLO QED

- QED: easiest subset of EW corrections: no γ_5, no massive gauge bosons;
- Analytic results have been computed in terms of special functions;
- IR structure shared with the full EW calculation;
- IR subtraction operator can be obtained via abelianisation of the corresponding QCD operator:



$$C_F \to Q_{q/e}^2 \qquad C_A \to 0 \qquad T_R n_{l/h} \to \sum_{l/h} N_{C,i} Q_i^2$$

Expansion in the couplings:

$$|\hat{\mathcal{M}}\rangle = 4\pi\hat{\alpha} \sum_{i,j} \left(\frac{\hat{\alpha}_S}{4\pi}\right)^i \left(\frac{\hat{\alpha}}{4\pi}\right)^j |\hat{\mathcal{M}}^{(i,j)}\rangle$$

$$|\mathcal{M}^{(0,0),fin}\rangle = |\mathcal{M}^{(0,0)}\rangle$$

$$|\mathcal{M}^{(0,1),fin}\rangle = |\mathcal{M}^{(0,1)}\rangle - \mathcal{I}^{(0,1)} |\mathcal{M}^{(0,0)}\rangle$$

$$|\mathcal{M}^{(0,2),fin}\rangle = |\mathcal{M}^{(0,2)}\rangle - \mathcal{I}^{(0,2)} |\mathcal{M}^{(0,0)}\rangle - \mathcal{I}^{(0,1)} |\mathcal{M}^{(0,1),fin}\rangle$$

Towards NNLO EW

[T. Armadillo, SD, M. Dradi, A.Vicini, arXiv:2511.20365]

NNLO QED

$s=10000\ \text{GeV}^2$; $t=-2500\ \text{GeV}^2$; $\mu_R=m_Z$.		
ε^{-4}	0.43895747601285225040892043215096	0
	-0.43895747601285225040892043215096	
	0	
ε^{-3}	-50.507936715422813794673578353212	0
	50.507936715422813794673578353212	
	0	
ε^{-2}	729.67339746440682380540184060883	0
	-1334.17941387822635237175428142592	
	604.50601641381952856635244081709	
ε^{-1}	26528.662344660489566812058589673 5	0
	-463.09023418725937944723920907369	
	-26065.572110473230187364819380599 86	
ε^{0}	401847.44734513081272965403938263 52	97243.87157762092035589431304130 70
	4376.81276799416282425402258394 02	
	-308980.38853550405519801374892526 8	

- Semi-analytic calculation to prove automation OceaNN;
- Main methodological choice: arbitrary numerical precision at each step of the calculation;
- Control of the cancellation of the poles with 35-digit precision;
- Successful comparison with analytic results.

$$\langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,2)} \rangle - \mathcal{I}^{(0,1)} \langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,1)} \rangle - \mathcal{I}^{(0,2)} \langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,0)} \rangle$$

ABISS SEASYDE

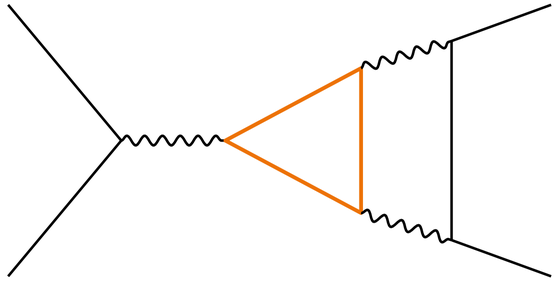
Towards NNLO EW

[T. Armadillo, SD, M. Dradi, A. Vicini: 2412.19884]

NNLO EW: Fermionic Contribution



s=10000 GeV2; t=-2500 GeV2; μR=mZ.		
ε^{-3}	-35.4668	0
	35.4672	
	0	
ε^{-2}	1856.03 + 374.962 i	0
	-1207.81 + 241.417 i	
	-648.21 - 616.378 i	
ε^{-1}	-35083.52 - 4774.08	0
	9463.56 - 5495.62 i	
	25620.07 + 10269.72 i	
ε^{0}	-523472.8 + 82010.2 i	-454654.8 + 118286.9 i
	32550.2 - 16686.4 i	
	36267.7 + 52963.0 i	



- Fermionic contribution: gauge-invariant subset with at least one closed fermion sub-loop;
- Recently computed with numerical methods *[A. Freitas, J. Wallace: 2512.15700]*
- Alternative calculation within OceaNN;
- Calculation performed with the muon mass regulating collinear divergences;
- Cancellation of the poles up to m_μ^2 power corrections

$$\langle \mathcal{M}^{(0,0)} \,|\, \mathcal{M}^{(0,2)} \rangle - \mathcal{F}^{(0,1)} \langle \mathcal{M}^{(0,0)} \,|\, \mathcal{M}^{(0,1)} \rangle - \mathcal{F}^{(0,2)} \langle \mathcal{M}^{(0,0)} \,|\, \mathcal{M}^{(0,0)} \rangle$$

Summary

- Higher-order EW corrections are needed to fully exploit the scientific potential of HL-LHC and FCC-ee;
- Multiloop EW corrections are the main challenge;
- We developed OceaNN, a framework which relies on Abiss for the evaluation of rational coefficients and SeaSyde for the evaluation of the Master Integrals;
- We successfully applied our framework to the computation of QED corrections and the fermionic contribution to the Drell-Yan process.
- On our way to meeting the requirements of HL-LHC?

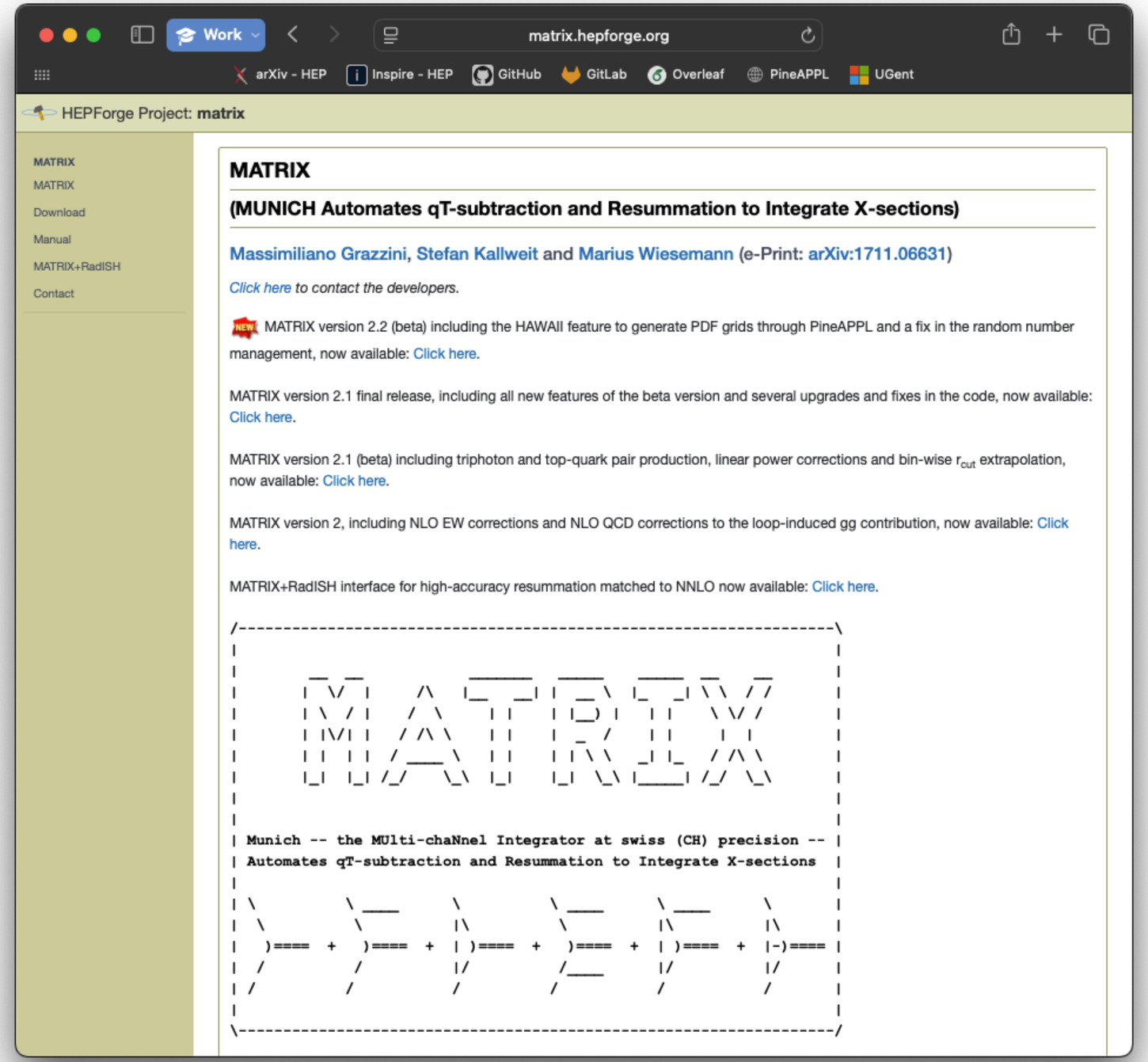
Thank you!



Backup slides

MATRIX

- MATRIX *[M. Grazzini, S. Kallweit, M. Wiesemann: 1711.06631]* allows the user to evaluate fully differential cross sections for a wide class of processes at hadron colliders in NNLO QCD and NLO EW;
- New: support for the generation of interpolation grids for the processes via PINEAPPL (MATRIX HAWAII) *[SD, T. Ježo, S. Kallweit, C. Schwan: 2506.14486]*



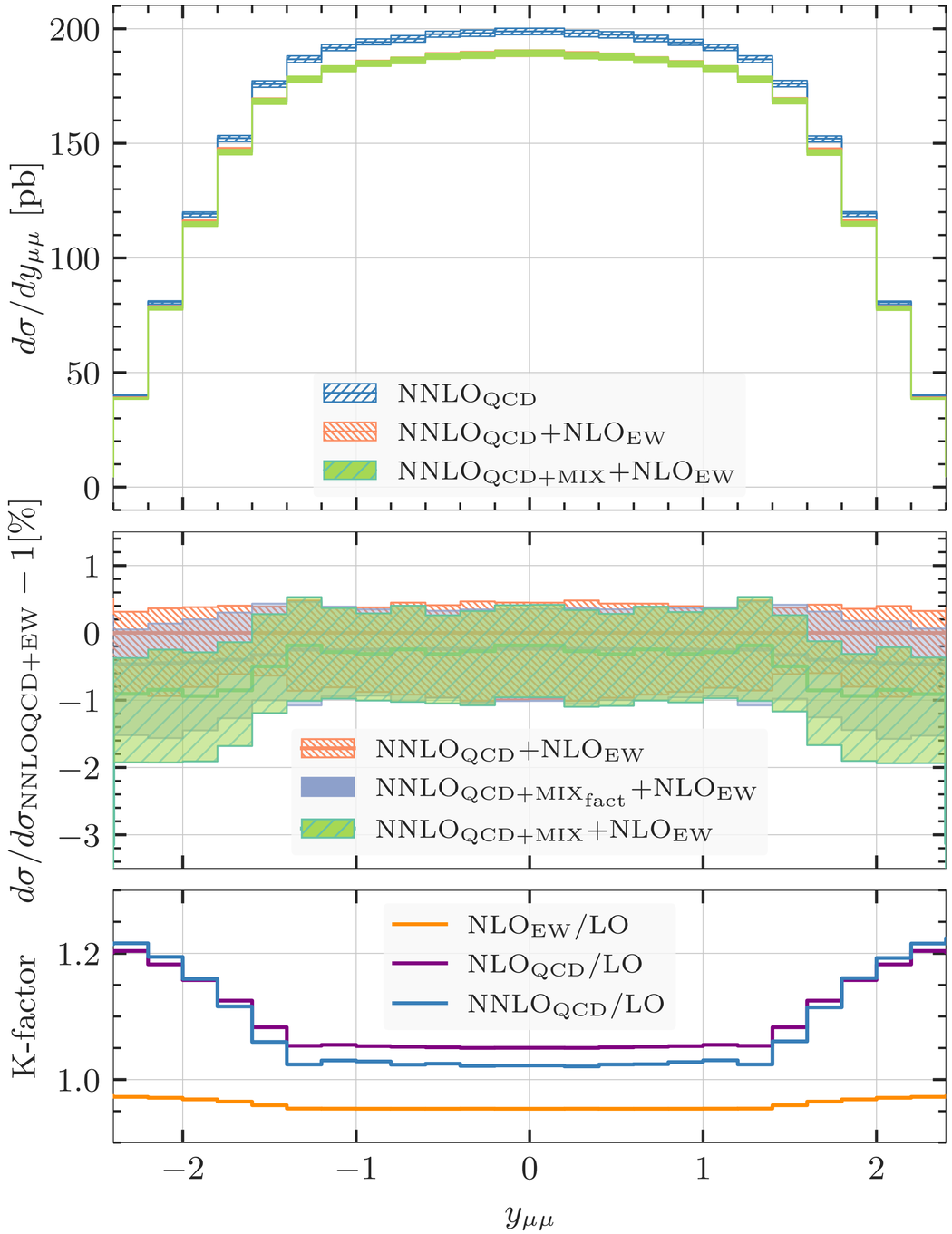
Differential results

Rapidity distribution

- We compare exact results MIX with the factorised approximation $\mathrm{MIX}_{\mathrm{fact}}$

$$\frac{d\sigma^{(1,1)}_{\mathrm{fact}}}{dX} = \left(\frac{d\sigma^{(1,0)}}{dX}\right) \times \left(\frac{d\sigma^{(0,1)}_{q\bar{q}}}{dX}\right) \times \left(\frac{d\sigma_{\mathrm{LO}}}{dX}\right)^{-1} ;$$

- We observe a shape distortion: the strongest effect comes from NNLO QCD, but it is emphasised by the mixed corrections;
- Effect of the mixed corrections up to the $\mathcal{O}(1\%)$ level;
- Shape distortions are relevant for PDF determinations.



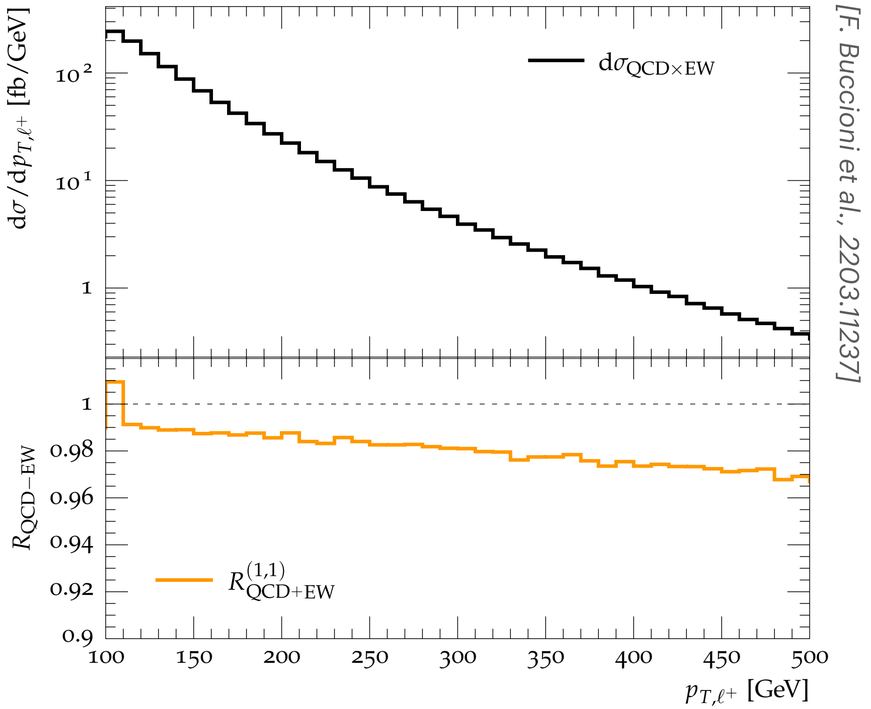
Mixed QCD-EW Corrections

[T. Armadillo, et al, arXiv:2412.16095]
[F. Buccioni et al., arXiv:2203.11237]

Neutral Current Drell-Yan

- Two fully independent calculations: massive bare leptons and semi-analytic amplitudes *[T. Armadillo, et al., arXiv:2412.16095]*; massless dressed leptons and analytic amplitudes *[F. Buccioni et al., arXiv:2203.11237]*.
- Dressing the massive leptons allowed for comparison, finding excellent agreement: differences are due to the lepton mass and are negligible for fiducial cross sections.

	T. Armadillo et al.	F. Buccioni et al.
Final State Leptons	Massive	Massless
Subtraction Method	q_T subtraction	Nested Subtraction
2-loop Amplitudes	Semi-Analytic	Analytic
$\sigma^{(1,1)}(qq)$ [pb]	-15.7(7)	-16.80
$\sigma^{(1,1)}(qg)$ [pb]	2.15(7)	2.07
$\sigma^{(1,1)}(q\gamma)$ [pb]	-0.231(9)	-0.207
$\sigma^{(1,1)}(g\gamma)$ [pb]	0.266(1)	0.2668

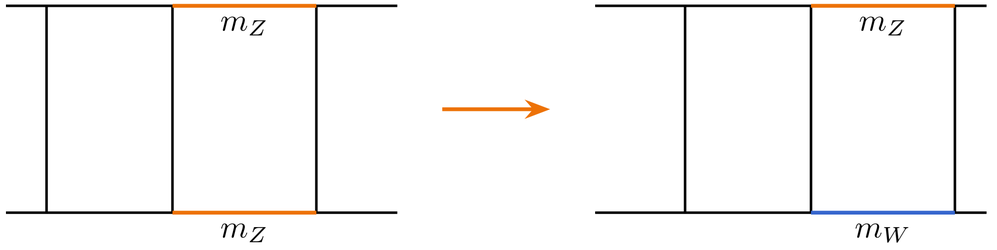


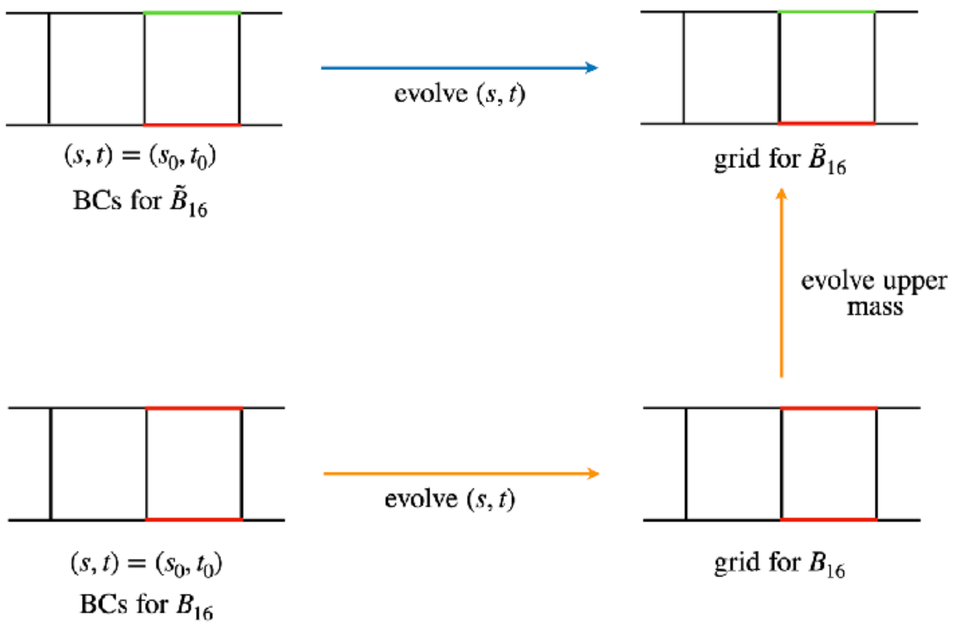
Mixed QCD-EW Corrections

[T. Armadillo, R. Bonciani, SD, N.Rana, A.Vicini, arXiv:2405.00612]

Charged Current Drell-Yan

- Computationally similar to neutral-current Drell-Yan;
- Extra complexity comes from new diagrams with two different internal massive lines;
- At the moment, only semi-analytical results are available.





- Numerical grid for the amplitude computed with SEASYDE;
- Possible approaches for the evolution:
  - Compute the boundary conditions with AMFLOW; use the differential equations in s and t to build the grid;
  - Use the neutral-current Drell-Yan grid as a boundary condition; use the differential equation in one of the masses;
- The differential equation in the mass allows keeping a dependence on the W boson mass.

HANDLING γ_5

Evaluation of (-2ϵ)-dimensional integrals



$$k^\mu = \bar{k}^\mu + \hat{k}^\mu \qquad g^{\mu\nu} = \bar{g}^{\mu\nu} + \hat{g}^{\mu\nu} \qquad \bar{g}^{\mu\nu}\bar{g}_{\mu\nu} = 4$$

- The introduction in the numerator of a (-2ϵ)-dimensional scalar product can be obtained by introducing the Schwinger parametrisation and differentiating with respect to the appropriate Schwinger parameter.

  [M. Heller, A. von Manteuffel, R. M. Schabinger and H. Spiesberger, arXiv:hep-ph/2012.05918]

$$J_N^{(L)} = (\mu^{D-4})^L \int \prod_{r=1}^{L} d^D q_r \prod_{j=1}^{N} \frac{1}{(k_j^2 - m_j^2)^{\nu_j}}$$

$$= \frac{(i\pi^{D/2}\mu^{D-4})^L}{\prod_{j=1}^{N}\Gamma(\nu_j)} \int_{\alpha_j \geq 0} d^N\vec{\alpha} \left(\prod_{j=1}^{N} (-1)^{\nu_j} \alpha_j^{\nu_j - 1} \right) [\mathcal{U}(\vec{\alpha})]^{-D/2} \exp - \frac{\mathcal{F}(\vec{\alpha})}{\mathcal{U}(\vec{\alpha})}$$

$$\frac{1}{A^\nu} = \frac{1}{\Gamma(\nu)} \int_0^\infty d\alpha\, \alpha^{\nu-1} e^{-\alpha A}$$

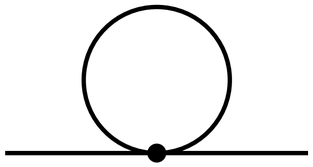
How to compute integrals

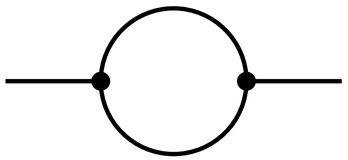
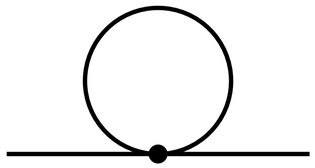
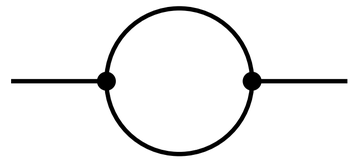
Integration through differentiation

- Master Integrals are **closed under differentiation**: by deriving with respect to one of the kinematic variables one of the master integrals, we obtain alinear combination of master integrals;
- We can write a **system of differential equations** satisfied by the master integrals!

An example: 

; Masters: {  ,  } .

$$\frac{d}{dp_1^2}$$  $$= 0$$

$$\frac{d}{dp_1^2}$$  $$= \frac{d-2}{p_1^2(4m^2-p_1^2)}$$  $$-\frac{(d-4)p_1^2+4m^2}{2p_1^2(4m^2-p_1^2)}$$ 

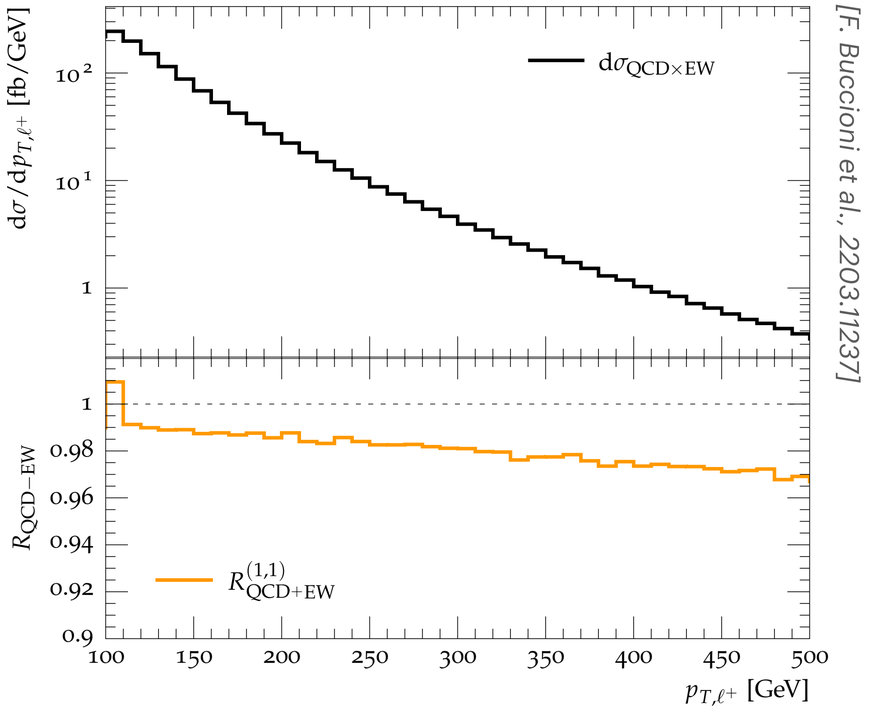
Neutral current Drell-Yan

[T. Armadillo, et al, arXiv:2412.16095]
[F. Buccioni et al., arXiv:2203.11237]

A comparison

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AMFlow

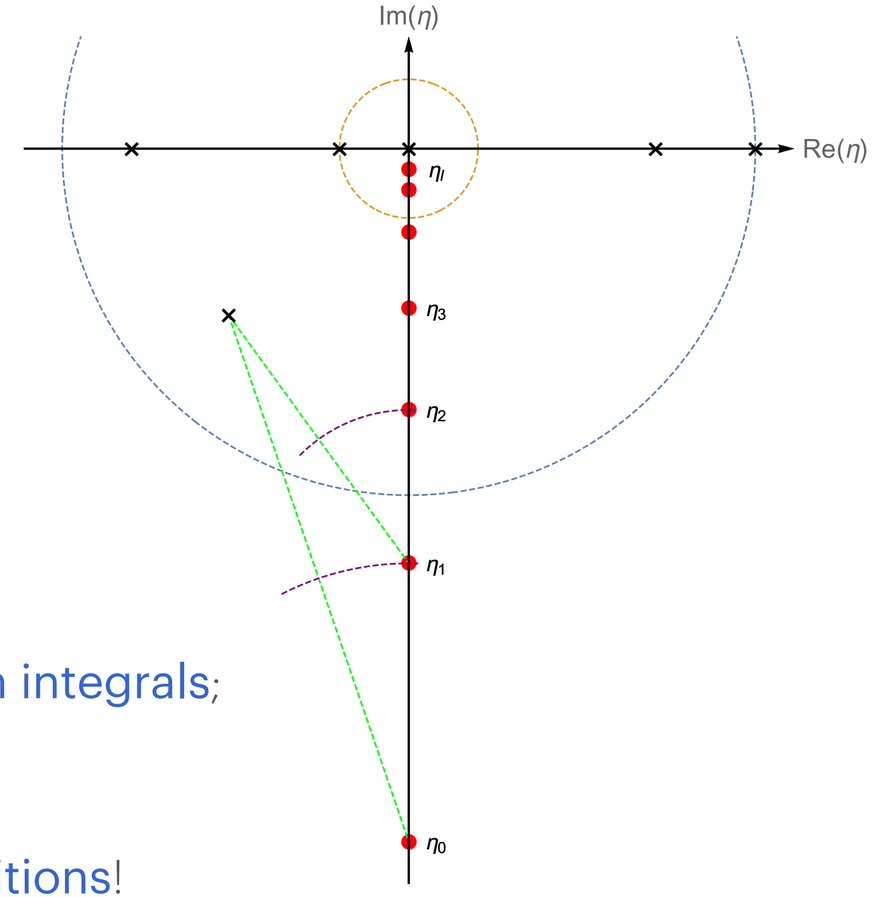
[R. Huang, X. Liu, Y. Ma , arXiv: 2607.08477]
[X. Liu and Y.-Q. Ma, arXiv: 2201.11669]

No need to compute boundaries

- Introduce an auxiliary mass parameter η:

$$I(\vec{\nu},\vec{s},\epsilon)=\int\prod_{i=1}^{L}\frac{\mathrm{d}^D\ell_i}{\mathrm{i}\pi^{D/2}}\frac{\mathscr{D}_{K+1}^{-\nu_{K+1}}\cdots\mathscr{D}_N^{-\nu_N}}{(\mathscr{D}_1+\mathrm{i}0^+)^{\nu_1}\cdots(\mathscr{D}_K+\mathrm{i}0^+)^{\nu_K}}$$

$$I_{\text{aux}}(\vec{\nu},\vec{s},\epsilon,\eta)=\int\prod_{i=1}^{L}\frac{\mathrm{d}^D\ell_i}{\mathrm{i}\pi^{D/2}}\frac{\mathscr{D}_{K+1}^{-\nu_{K+1}}\cdots\mathscr{D}_N^{-\nu_N}}{(\mathscr{D}_1-\eta)^{\nu_1}\cdots(\mathscr{D}_K-\eta)^{\nu_K}}$$



- The limit $\eta\to\infty$ can be computed analytically in terms of vacuum integrals;
- The limit $\eta\to 0$ provides the target integrals;
- The auxiliary mass flow approach does not require boundary conditions!

NEW: AMFlow2

Alternate recursion mode, High-performance differential equation solver, improved IBP interfaces, ...

Series Expansion

A simple example

$$\begin{cases} f'(x) + \frac{1}{x^2 - 4x + 5} f(x) = \frac{1}{x+2} \\ f(0) = 1 \end{cases}$$

Ansatz:

$$f_{hom}(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$

$$f'_{hom}(x) = \sum_{k=0}^{\infty} (k+r)\, c_k\, x^{(k+r-1)}$$

$$\begin{cases} rc_0 = 0 \\ \frac{1}{5}c_0 + c_1(r+1) = 0 \\ \frac{4}{25}c_0 + \frac{1}{5}c_1 + c_2(2+r) = 0 \\ \dots \end{cases}$$

$$f_{hom}(x) = 5 - x - \frac{3}{10}x^2 + \frac{11}{150}x^3 + \dots$$

Expanded around $x' = 0$

$$f_{part}(x) = f_{hom}(x) \int_0^x dx' \frac{1}{(x'+2)} f_{hom}^{-1}(x')$$

$$= \frac{1}{2}x - \frac{7}{40}x^2 + \frac{2}{75}x^3 + \dots$$

$$f(x) = f_{part}(x) + C f_{hom}(x)$$

$$f(0) = 1 \rightarrow C = \frac{1}{5}$$

Towards NNLO EW

[A. Freitas, J. Wallace: 2512.15700]

NNLO EW: Fermionic Contribution

- Fermionic contribution: at least one closed fermion sub-loop;
- Gauge-invariant subset which is expected to be relatively large due to the top quark mass;
- Direct numerical integration of each contribution (no IBP):
  - Vertex and box diagrams with a sub-loop fermion self-energy: dispersion relation for the self-energy (numerical integration over σ);
  - Vertex diagrams with a triangle fermion sub-loop: Feynman parametrisation followed by dispersion relation (numerical integration over x and σ)
- Numerical integration performed in C++ (GNU Scientific Library, QUADPACK)

