

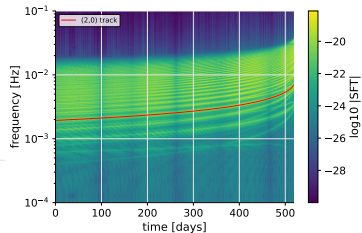
# A track search for EMRIs in LISA using template banks

**Bert Depoorter**

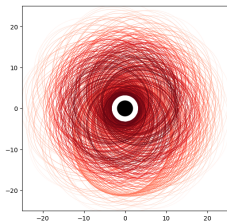
Royal Observatory of Belgium · KU Leuven

Supervisors: J. Janquart (ROB), H. Inchauspé, T. Li (KUL)

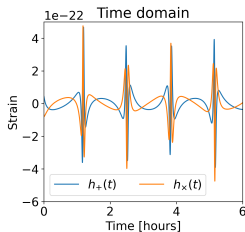
Collaborators: L. Speri (ESA), F. Fantoccoli (AEI)



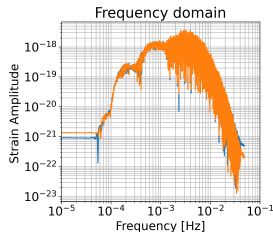
# Extreme mass-ratio inspirals in LISA



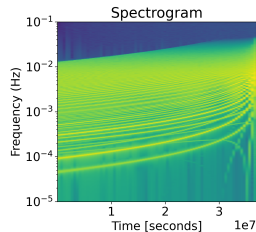
$\sim 10^4$  orbits  
close to the SMBH



Complicated waveform:  
eccentric, inclined,  
precessing



Multi-harmonic:  
Power spread over many  
frequencies

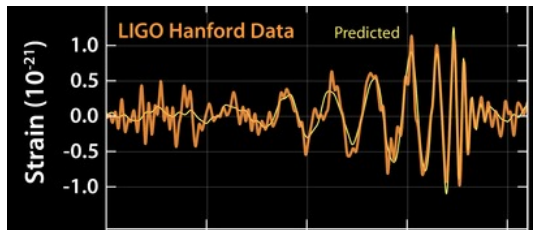


long-lived: in-band  
over years

Promise: high precision measurements of intrinsic parameters  
Relative uncertainty  $10^{-5} - 10^{-6}$

# Searching for GWs: matched filtering

$$\rho^2 \sim \langle d | h(\vec{\theta}) \rangle = 4 \operatorname{Re} \int_0^\infty \frac{\tilde{d}(f) \tilde{h}^*(f; \vec{\theta})}{S_n(f)} df$$



Standard CBC approach: template bank over  $\vec{\theta}$

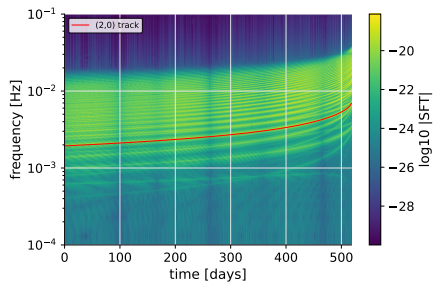
→ evaluate every template on the data, rank by maximal detection statistic

# Why not for EMRIs?

| EMRI-specific bottleneck  |                                                                |
|---------------------------|----------------------------------------------------------------|
| <b>Signal model</b>       | waveform + response + TDI<br>~ 200 ms per full template on GPU |
| <b>Prior-to-posterior</b> | ~ $10^{40}$ templates for a coherent bank                      |
| <b>Secondary maxima</b>   | Multi-harmonic structure introduces secondary likelihood peaks |

Finding the weak EMRI signals in gaussian, stationary noise is a major open challenge.  
Finding EMRIs in Global Fit is uncharted territory

# A different angle: continuous-wave searches



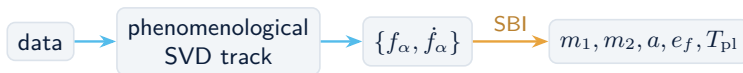
Chop the data in short Fourier transforms (SFTs), add power incoherently

⇒ wider likelihood peaks → smaller prior-to-posterior ratio

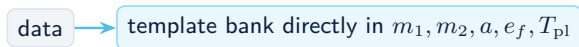
→ template banks feasible (at the cost of some sensitivity)

# Motivation

## Earlier track search



## This work



$\{f_\alpha, \dot{f}_\alpha\} \mapsto$  EMRI parameters is non-trivial  $\rightarrow$  needs simulation-based inference

Can we find EMRIs with a template bank of single-harmonic tracks directly in the intrinsic parameters, using a semi-coherent statistic that treats the track as a linear chirp within each SFT?

Speri, Tenorio, Chapman-Bird & Gerosa (2025)

# A cheap detection statistic

$$h_{\alpha}(t) \simeq A_{\alpha} \exp\left(i\phi_{\alpha} + 2\pi i f_{\alpha} \tau + i\pi \dot{f}_{\alpha} \tau^2\right)$$

linear chirp in each SFT

$$\Lambda(\vec{\theta}) = \sum_{\alpha=1}^{N_{\text{SFT}}} \frac{(\max_{\phi_{\alpha}} \langle h_{\alpha} | d_{\alpha} \rangle)^2}{\langle h_{\alpha} | h_{\alpha} \rangle}$$

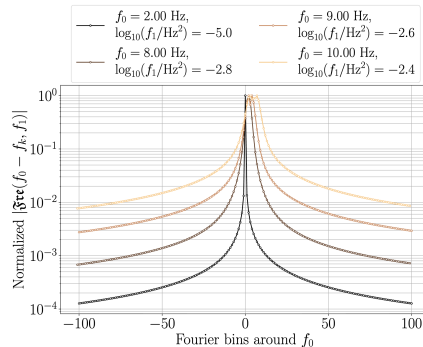
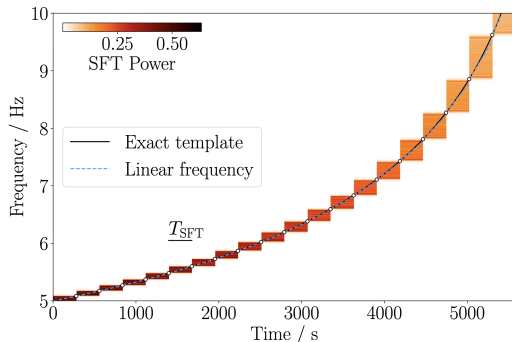
max over  $A_{\alpha}$ ,  $\phi_{\alpha}$   
→ semi-coherent

$$\max_{\phi_{\alpha}} \langle h_{\alpha} | d_{\alpha} \rangle = 4 \left| \sum_{k=-P}^P \frac{\tilde{d}_{\alpha,k}^* \tilde{h}_{\alpha,k}(f_{\alpha}, \dot{f}_{\alpha})}{S_n(f_k)} \Delta f \right|$$

only  $2P + 1$  bins per SFT

$N_{\text{SFT}} (2P + 1)$  terms instead of  $N_{\text{SFT}} N_f$

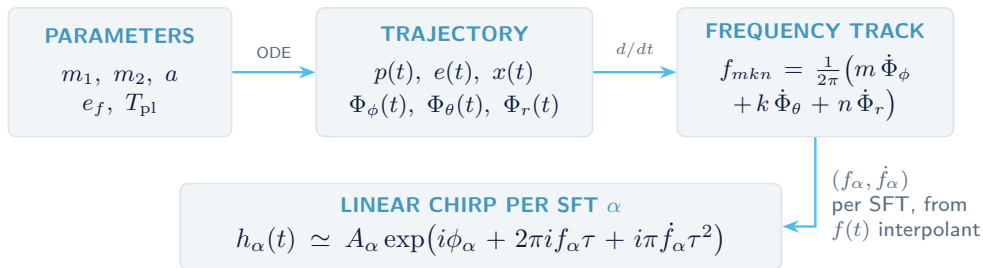
# Linear chirps in SFTs



chirp power spreads over a few bins → analytic Fresnel kernel

Tenorio & Gerosa (2025) · So far: existing work [refs]

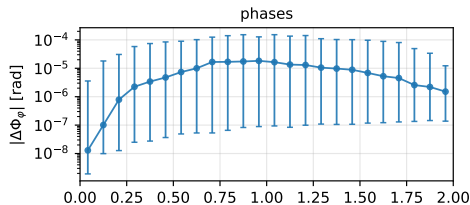
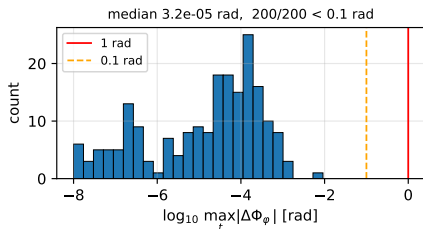
# A track model



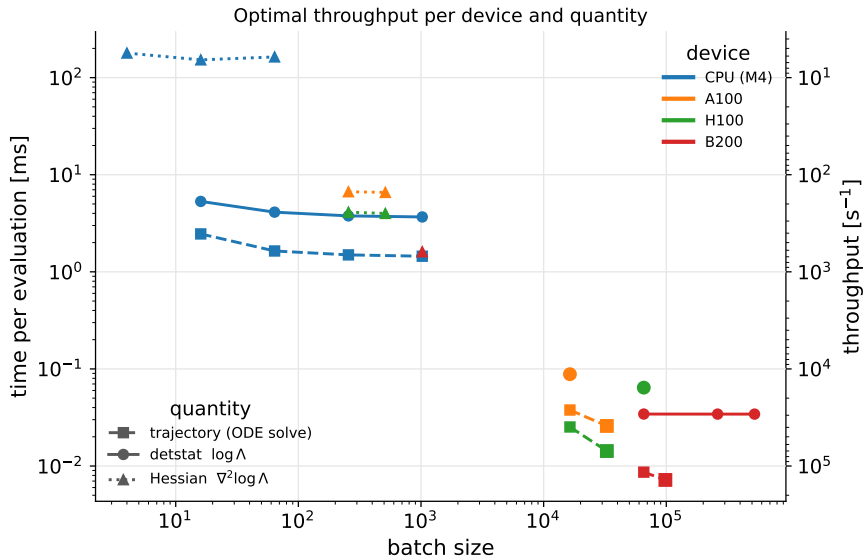
- Ignore response and TDI for the track search
- Focus on a single harmonic,  $(m, n) = (2, 0)$
- Full coherent template: **13**  $\rightarrow$  **5 parameters**

# Frequency tracks in batches: fewtrax

- JAX implementation of the FEW trajectory module
- Orbital evolution from flux balance laws on precomputed flux grids
- `jit · vmap · autodiff`
- Matches FEW: median  $\max_t |\Delta\Phi_\phi| \approx 3 \times 10^{-5}$  rad over 2 yr



fewtrax vs FEW, 200 systems over 2 yr



# Single-harmonic search: starting point

## Earlier work

Speri et al. (2025)

- Dominant  $(2, 0)$  harmonic only
- Semi-coherent SFT statistic
- Mildly eccentric, equatorial EMRIs
- 94% detection at SNR 30  
(false-alarm prob.  $10^{-2}$ )

## Prior box

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|                                |                                |
|--------------------------------|--------------------------------|
| primary mass $m_1$             | $[0.5, 5] \times 10^6 M_\odot$ |
| secondary mass $m_2$           | $[1, 100] M_\odot$             |
| spin $a$                       | $[0, 0.998]$                   |
| final eccentricity $e_f$       | $[0, 0.1]$                     |
| time to plunge $T_{\text{pl}}$ | $[0, 2] \text{ yr}$            |

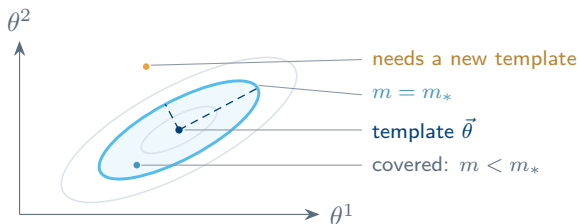
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same box as Speri et al. → direct comparison

2 yr of data ·  $(2, 0)$  harmonic ·  $T_{\text{pl}}$  rolled, not a bank axis

# A metric on track space

$$\langle h_1 | h_2 \rangle = \sum_{\alpha=1}^{N_{\text{SFT}}} \left( 4 \left| \sum_{k=-P}^P \frac{\tilde{h}_{1,\alpha k}^* \tilde{h}_{2,\alpha k}}{S_n(f_k)} \Delta f \right|^2 \right), \quad [h_1 | h_2] = \frac{\langle h_1 | h_2 \rangle}{\sqrt{\langle h_1 | h_1 \rangle \langle h_2 | h_2 \rangle}},$$
$$m(\vec{\theta}, \vec{\theta} + \delta\vec{\theta}) = 1 - [h(\vec{\theta}) | h(\vec{\theta} + \delta\vec{\theta})] \simeq g_{ij} \delta\theta^i \delta\theta^j$$



$g$ : hessian, by autodiff

$T_{\text{pl}}$  projected out  $\rightarrow$  4D bank

# How many templates?

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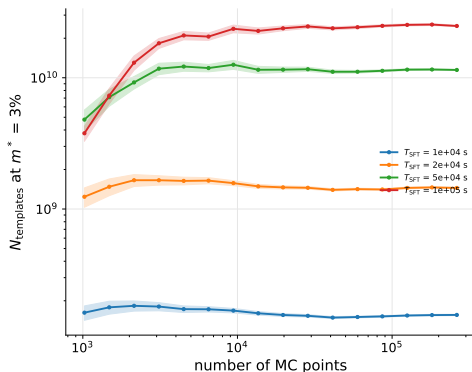
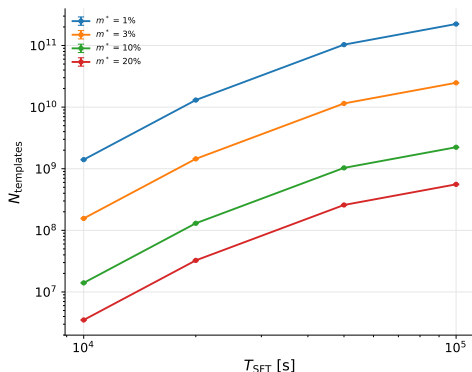
$$N_{\text{templates}} \propto m_*^{-d/2} \int_{\Theta} \sqrt{\det g(\vec{\theta})} d\vec{\theta} \stackrel{d=4}{\propto} m_*^{-2}$$

Monte Carlo integral over the prior box, in the bank coordinates

- $m_*$ : mismatch we accept before adding a template
- $T_{\text{SFT}}$ : longer coherence  $\rightarrow$  more templates
- $T_{\text{data}}$ : length of the data set
- $P$ : bins per SFT  $\rightarrow$  better recovery, but cost  $\propto P$   
(longer SFTs: the chirp drifts over more bins)

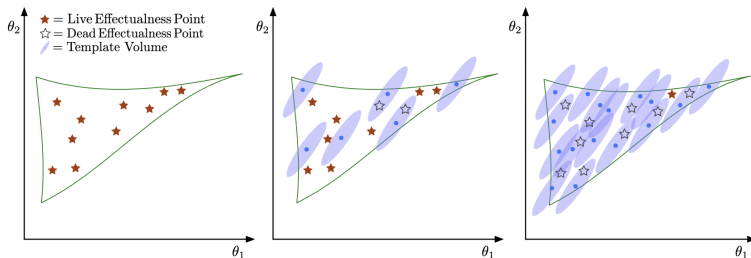
# First bank estimates

B200 run, quadratic mismatch 4D bank, LPA strain PSD (2 yr,  $P = 25$ )



at  $m_* = 3\%$ :  $\sim 10^8$  templates ( $T_{\text{SFT}} = 10^4$  s) up to  $\sim 10^{10}$  ( $T_{\text{SFT}} = 10^5$  s)

# Next step: building the bank



Coogan et al. 2022 (diffbank; fork compatible with fewtrax)

1. Propose a point from  $p(\vec{\theta}) \propto \sqrt{\det g}$
2. Inside the  $m_*$  ellipsoid of a template?  $\rightarrow$  reject; else  $\rightarrow$  new template
3. Effectualness points track the coverage  $\rightarrow$  stop at fraction  $\eta$

# Conclusions and outlook

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- fewtrax: up to  $10^5$  trajectories per second on GPU
- Differentiable semi-coherent detection statistic
- First estimates:  $N_{\text{templates}} \sim 10^7 - 10^{11}$ ; bank construction in progress

## Next steps

- Significance and sensitivity
- Best templates as starting points for PE
- Multiple harmonics → wider priors

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# Backup

# Single-harmonic search: Speri et al. (2025)

*Ab uno disce omnes*, arXiv:2510.20891

- Semi-coherent time-frequency search for one harmonic
- Frequency evolution modelled phenomenologically with an SVD basis
- Mildly eccentric, equatorial EMRIs in stationary Gaussian noise
- 1 yr of data searched in  $\sim 1$  h on a single GPU
- 94% detection probability at SNR 30, false-alarm probability  $10^{-2}$
- Dominant harmonic's frequency evolution recovered to  $\sim 1\%$
- Mapped back: sub-percent precision on the intrinsic parameters

Output of the search is a track  $\{f_\alpha, \dot{f}_\alpha\}$ , not EMRI parameters  $\rightarrow$  extra inference step to get to  $m_1, m_2, a, e_f, T_{\text{pl}}$

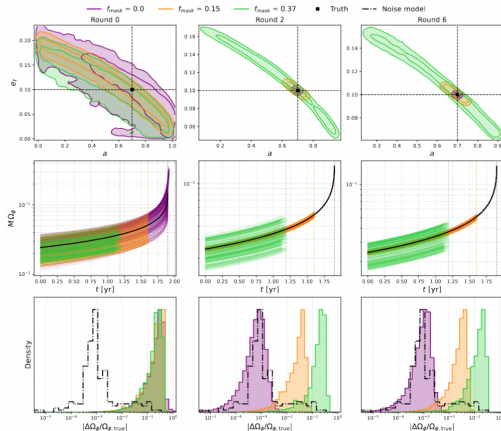
# Linear chirps and the Fresnel kernel

Tenorio & Gerosa (2025)

$$\tilde{h}_{\alpha,k} = A_{\alpha} e^{i\phi_{\alpha}} \int_{-T_{\text{SFT}}/2}^{T_{\text{SFT}}/2} e^{2\pi i (f_{\alpha} - f_k) \tau + i\pi \dot{f}_{\alpha} \tau^2} d\tau \equiv A_{\alpha} e^{i\phi_{\alpha}} T_{\text{SFT}} \text{Fre}(f_{\alpha} - f_k, \dot{f}_{\alpha})$$
$$\text{Fre} \propto \frac{e^{-i\pi(f_{\alpha} - f_k)^2 / \dot{f}_{\alpha}}}{\sqrt{2\dot{f}_{\alpha}}} \left[ C(u) + i S(u) \right]_{u_-}^{u_+}, \quad u = \sqrt{2\dot{f}_{\alpha}} \left( \tau + \frac{f_{\alpha} - f_k}{\dot{f}_{\alpha}} \right)$$

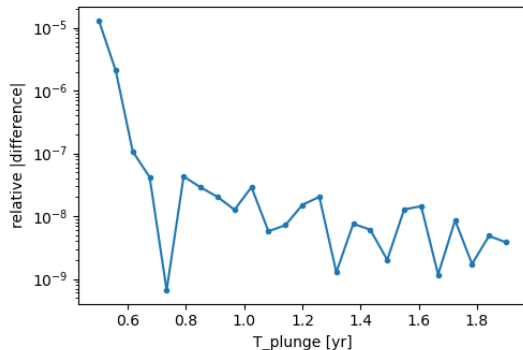
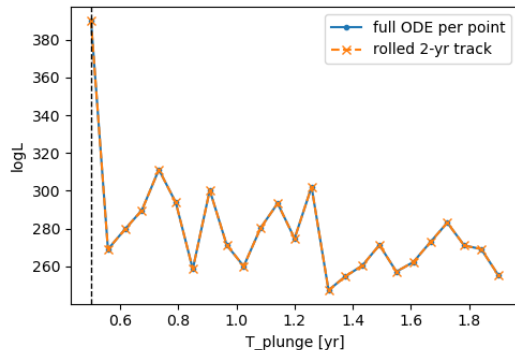
- Complete the square  $\rightarrow$  closed form in Fresnel integrals  $C, S$ : cheap, differentiable
- $\dot{f}_{\alpha} \rightarrow 0$ : back to the usual Dirichlet (sinc) kernel
- Power within  $\pm P$  bins of  $f_{\alpha} \rightarrow$  truncate the sum
- Valid while  $\ddot{f}_{\alpha} T_{\text{SFT}}^3 \lesssim 6$

# From tracks back to parameters: SBI



F. Fantoccoli *et al.* [In prep.]: sequential SBI on noisy time-frequency tracks

# Rolling the track: accuracy



rolled 2-yr track vs. full ODE solve per  $T_{\text{pl}}$ : relative difference in  $\log \mathcal{L} \lesssim 10^{-5}$

# Counting templates: Monte Carlo

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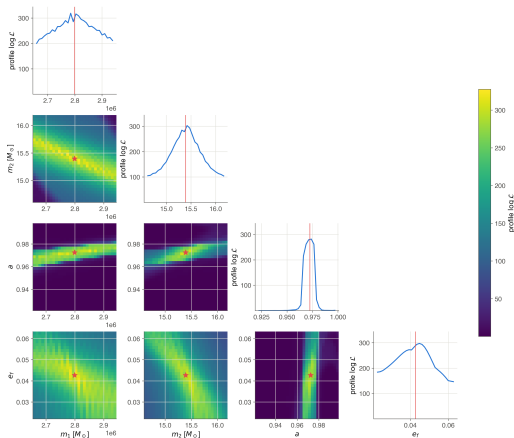
$$N_{\text{templates}} = \Theta_d \frac{V_{\text{proper}}}{V_d(\sqrt{2m_*})}, \quad V_{\text{proper}} = \int_{\text{box}} \sqrt{\det g(\vec{\theta})} d^d \theta$$

1. Draw  $\vec{\theta}_n$  uniformly in the box, in the bank coordinates
2. Hessian of  $\Lambda$  by autodiff  $\rightarrow g$ , with  $T_{\text{pl}}$  projected out
3. Walk out along each eigenvector until  $\Lambda$  drops by  $m_*$   $\rightarrow \lambda_{\text{eff},i} = 2m_*/r_i^2$
4.  $f_n = \sqrt{\prod_i \lambda_{\text{eff},i}}$ ; invalid tracks ( $\Lambda = 0$ ) count as  $f_n = 0$

$$\hat{V}_{\text{proper}} = \frac{V_{\text{box}}}{N} \sum_n f_n, \quad \text{ESS} = \frac{(\sum_n f_n)^2}{\sum_n f_n^2}$$

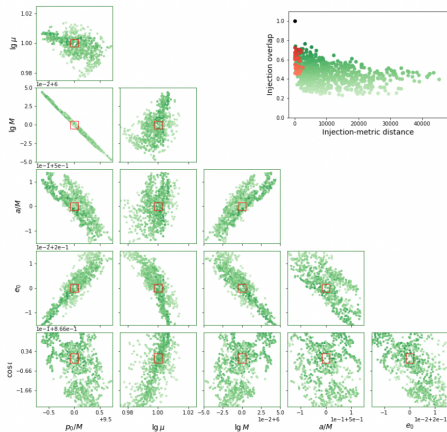
$\Theta_4 \approx 1.77$ : thinnest lattice covering · small ESS  $\rightarrow$  a few points dominate, error bar not reliable

# Strong correlations in track space



profile  $\log \mathcal{L}$  on a grid around Mojito EMRI source 1,  $T_{\text{SFT}} = 5 \times 10^4$  s

## Secondary maxima



Chua & Cutler (2022) · templates far from the truth still overlap  $\sim 0.4\text{--}0.7$  with the injection