

Tidal Deformations of Neutron Stars

Role of Superfluidity and Temperature

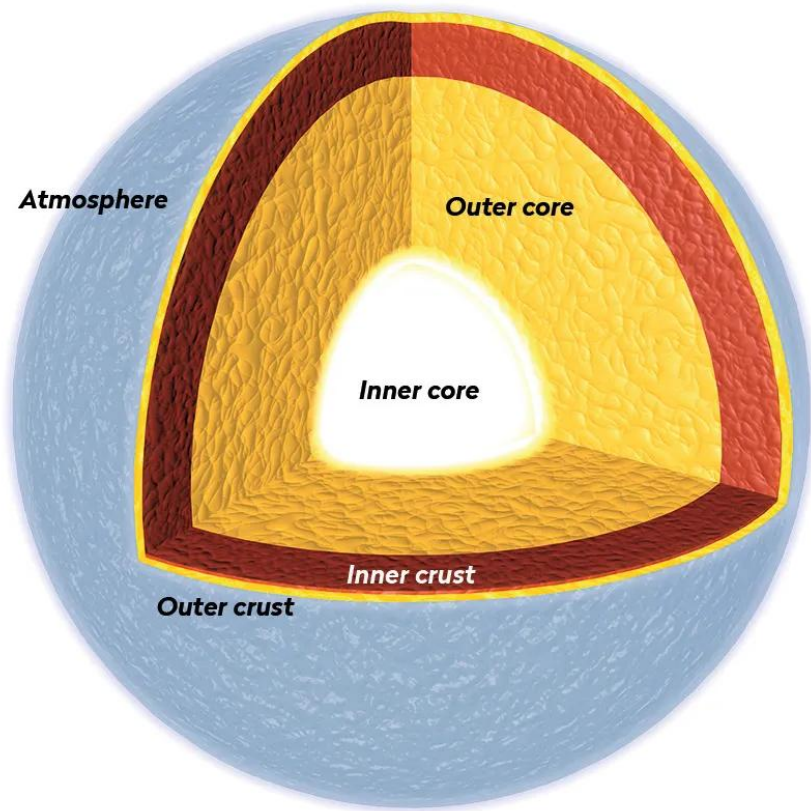
Ethan Carlier

Belgian-Dutch Gravitational Wave Meeting 2026

October 8, 2026



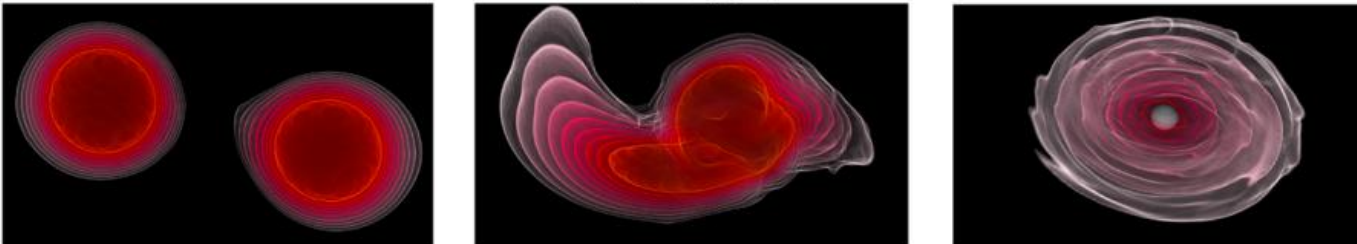
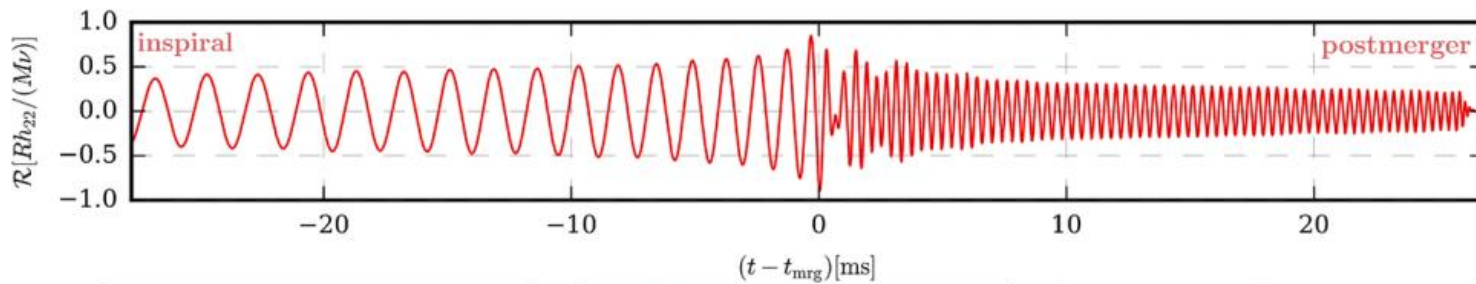
Neutron stars: laboratories of extreme physics



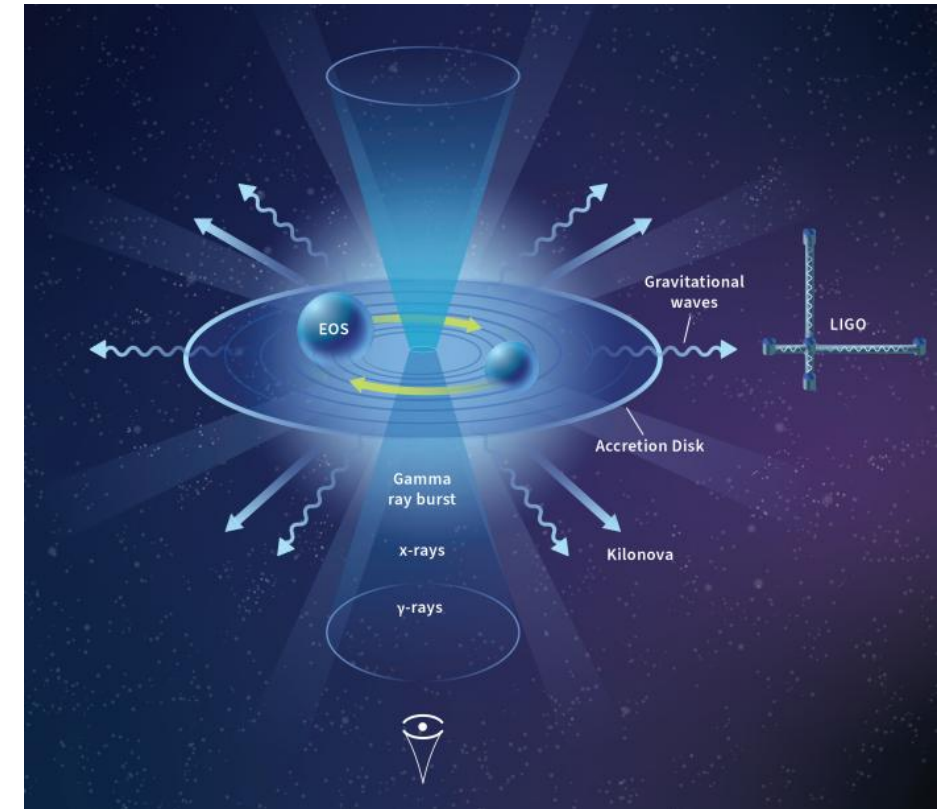
- One of the most **extreme** laboratories in the universe
→ $R \sim 10 \text{ km}$ & $M = 1 - 2 M_{\odot}$
- Density up to $\sim 10^{15} \text{ g/cm}^3$
→ **Uncertainties** about the properties of matter
- How to probe dense matter and its **equation of state**?

Gravitational-wave astronomy

- Binary neutron star mergers $\Rightarrow$ gravitational-wave emission
- Matter leaves an imprint on the gravitational-wave signal



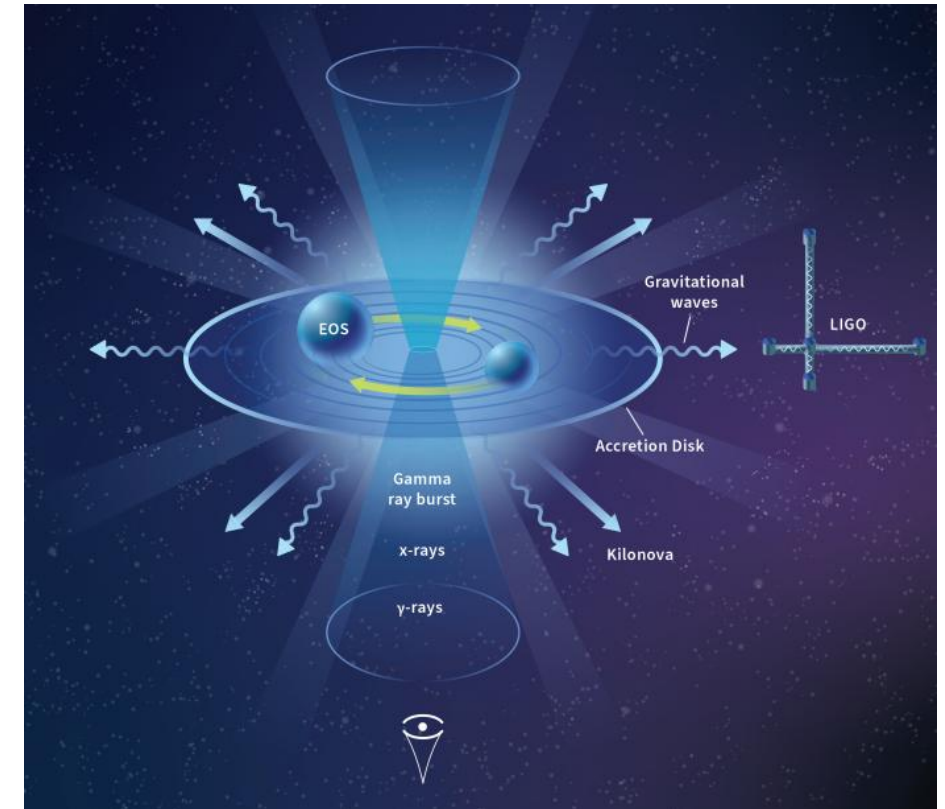
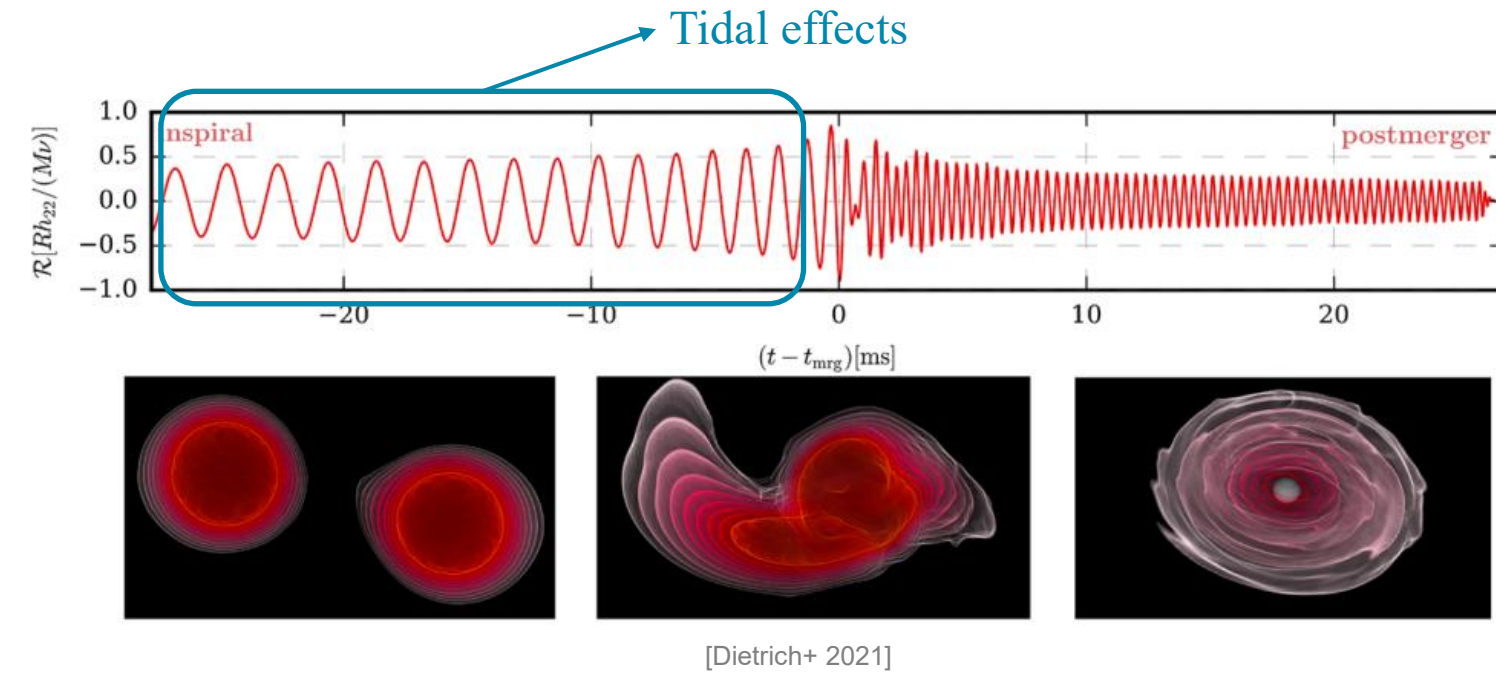
[Dietrich+ 2021]



[Chatziioannou+ 2024]

Gravitational-wave astronomy

- Binary neutron star mergers $\Rightarrow$ gravitational-wave emission
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[Chatziioannou+ 2024]

Tidal effects in binary neutron star systems

- Tides are totally defined by the **Love numbers** k_ℓ and j_ℓ (in the adiabatic limit):

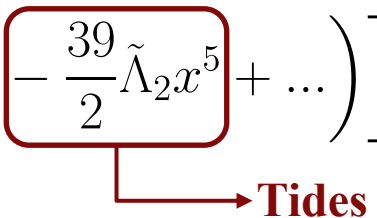
$$\mathcal{Q}_L = \frac{2R^{2\ell+1}}{(2\ell-1)!!} k_\ell \mathcal{E}_L \quad \& \quad \mathcal{S}_L = \frac{R^{2\ell+1}}{4(2\ell-1)!!} j_\ell \mathcal{M}_L$$

$$\Lambda_\ell = \frac{2}{(2\ell-1)!!} k_\ell \left(\frac{R}{M}\right)^{2\ell+1}$$

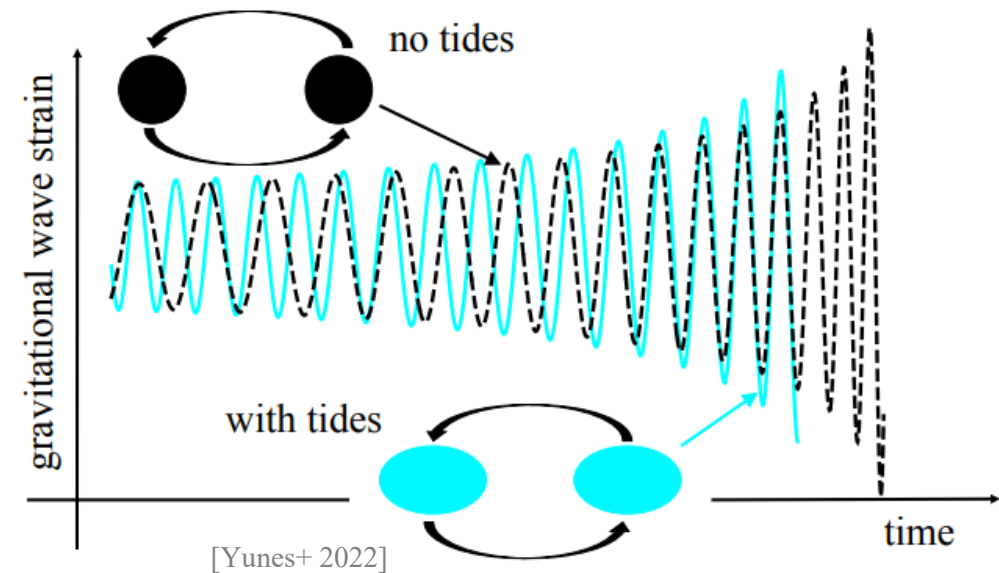
$$\Sigma_\ell = \frac{1}{4(2\ell-1)!!} j_\ell \left(\frac{R}{M}\right)^{2\ell+1}$$

- k_ℓ and j_ℓ depend on the properties of matter
- Love numbers appear at orders **(2ℓ + 1)PN** and **(2ℓ + 2)PN** in the waveforms

$$h(f) = \mathcal{A} f^{-7/6} \exp \left[\psi_{\text{pm}} \left(1 - \frac{39}{2} \tilde{\Lambda}_2 x^5 + \dots \right) \right]$$

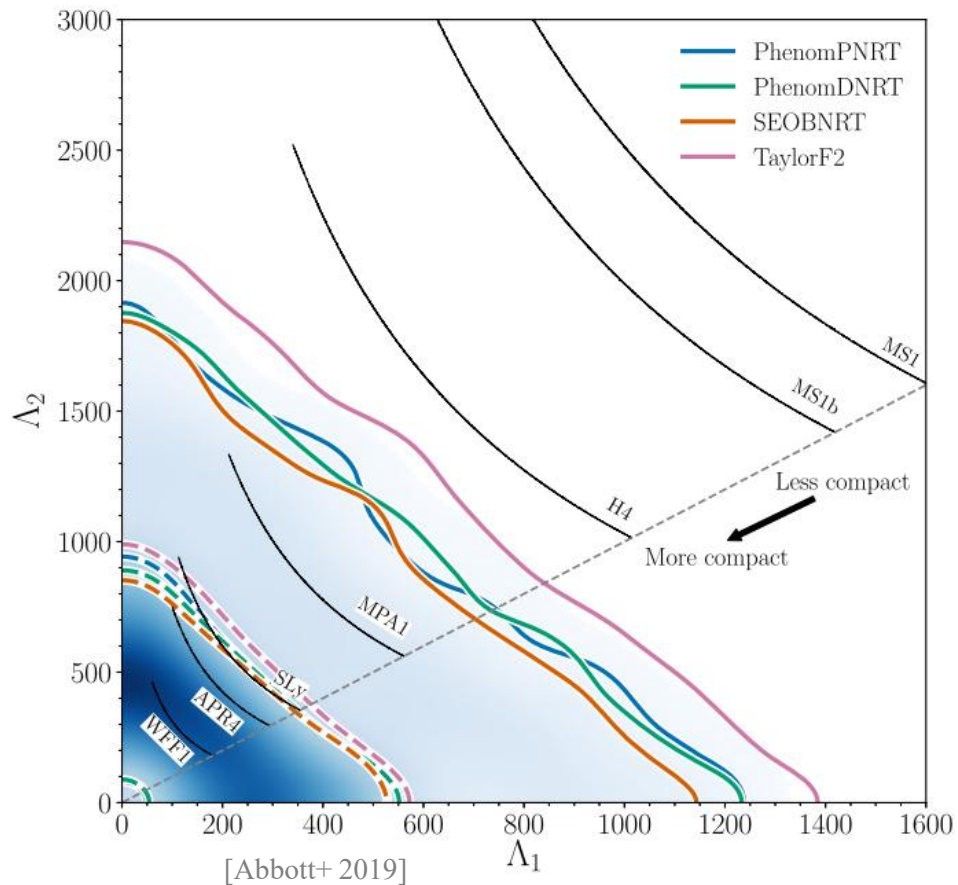

Tides

$$\tilde{\Lambda}_2 = \frac{16}{13} \left[\left(1 + \frac{12m_B}{m_A} \right) \frac{\Lambda_{2,A}}{M^5} + \left(1 + \frac{12m_A}{m_B} \right) \frac{\Lambda_{2,B}}{M^5} \right]$$

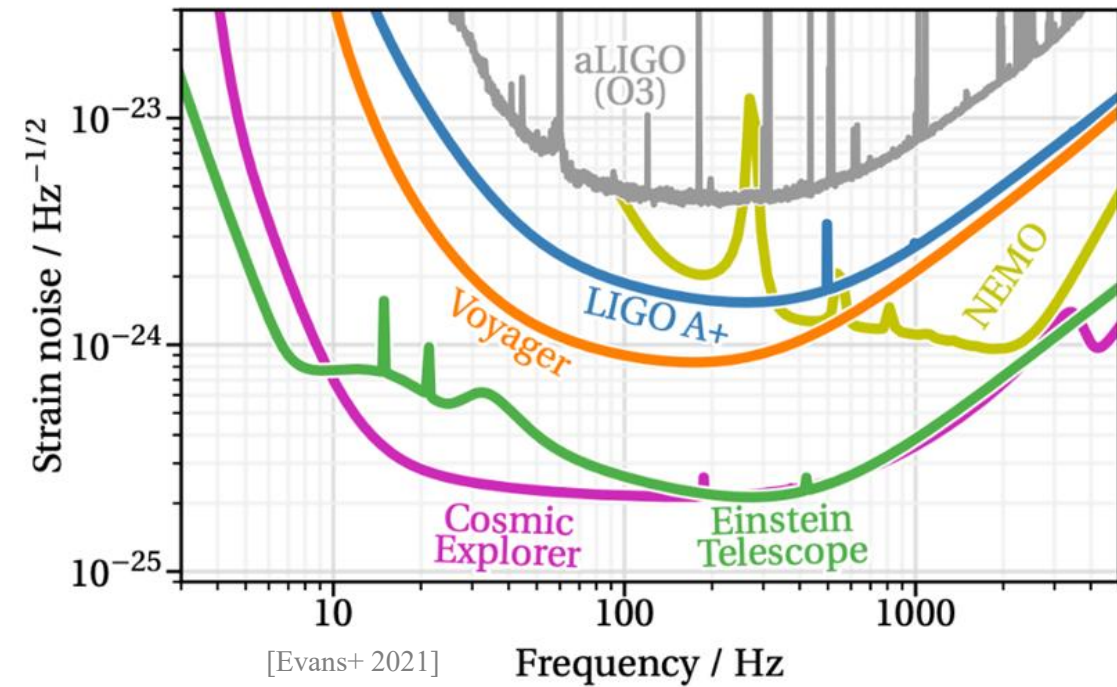


Tidal effects & observations

- Event GW170817: first **constraint** on the tidal deformability Λ_2



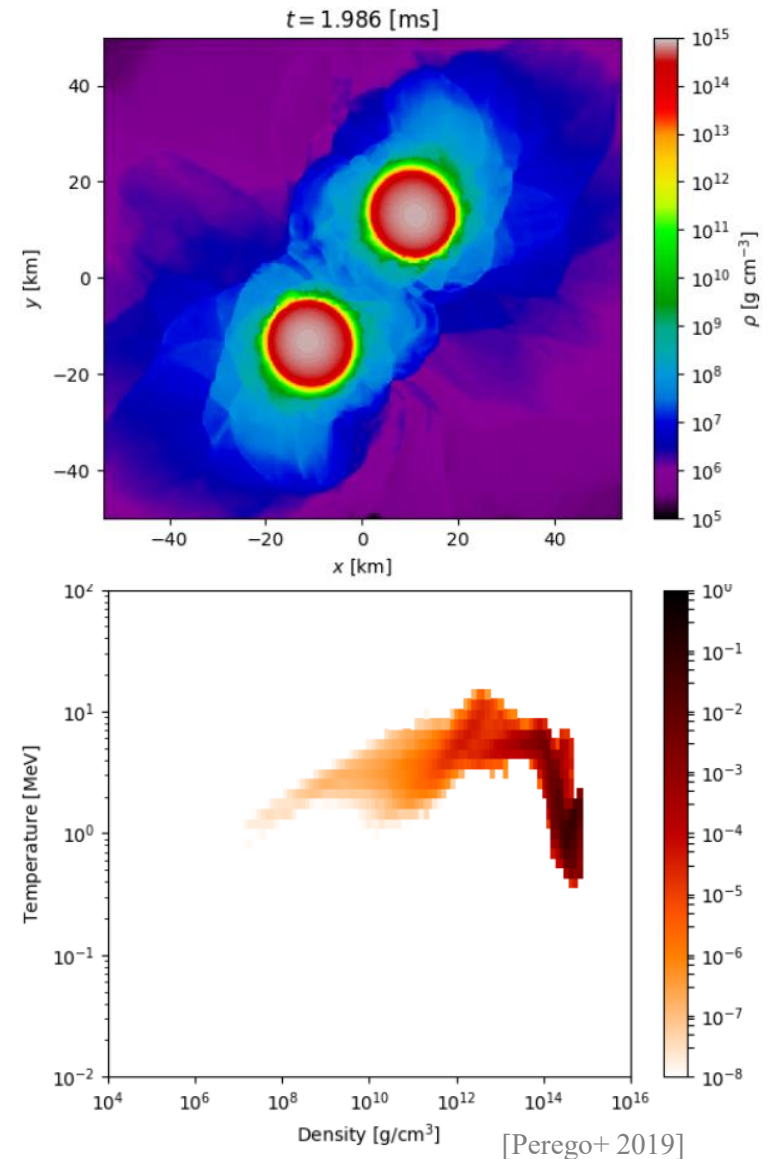
- ET and CE designed to be way more **sensitive**
- A major source of error will be the **modeling** $\Rightarrow$ need more **accurate** models with **richer physics**



Thermal effects and superfluidity in the inspiral

- Early inspiral: **cold catalyzed matter** at $T = 0$ MeV
⇒ Neutrons become **superfluid** in the core and inner crust [Sedrakian+ 2021]
- Late inspiral: matter at $T > 0$ MeV
→ Mechanical energy transformed into **heat** due to tidal friction
- $T = 0.01 - 3$ MeV in the late inspiral
- **Critical temperature** of neutron superfluidity up to $T_c \sim 1$ MeV
⇒ Superfluidity can **survive** in the late inspiral

Finite-temperature effects and superfluidity **cannot be neglected** *a priori*



State of the art

- Tidal deformations of **finite-temperature superfluid** neutron stars have never been explored
- **At zero temperature**, superfluidity has been included, but there is a discrepancy in the literature:
 - In [Char+ 2018, Datta+ 2020], they found **a non-negligible** effect of superfluidity on tidal deformations ($\sim 20\%$)
 - In [Passamonti+ 2022], they found that superfluidity has a **negligible** impact
- **Thermal effects** have been considered in [Kanakis-Pegios+ 2022], but not in a fully consistent way:
 - Temperature is included only in the equation of state, **not** in the equations of motion
 - The temperature profile is **fixed by hand**

How can we include these effects?

Well-known **cold** treatment of tidal deformations: [Hinderer 2008]

- **Static** and **spherically** symmetric background **perturbed** by an external adiabatic tidal field
- **Barotropic** equation of state: $\mathcal{E}(n_b), P(n_b), \dots$
- **Perfect fluid** model for matter: $T^\mu{}_\nu = P\delta^\mu_\nu + (\mathcal{E} + P)u^\mu u_\nu$
- Solve $G_{\mu\nu} = 8\pi T_{\mu\nu}$ & $\nabla_\mu T^{\mu\nu} = 0$

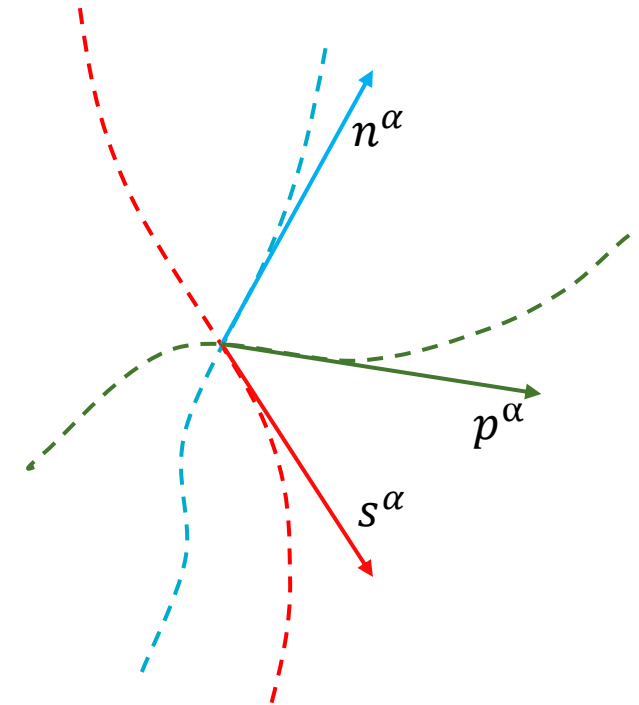
How can we include these effects?

Finite-temperature superfluid model of tidal deformations:

- Static and spherically symmetric background perturbed by an external adiabatic tidal field
- Non-barotropic equation of state: $\mathcal{E}(T, n_b, Y_e), P(T, n_b, Y_e), \dots$
- Carter's three-fluid model for matter: $T^\mu_\nu = \Psi \delta^\mu_\nu + \sum_x n^\mu_x \pi^\nu_x$ [Carter 1989]
 - 1) Neutron superfluid
 - 2) Charged-neutral fluid containing all other species
 - 3) Entropy fluid

Currents and momenta are generally not aligned: $\pi^\alpha_x = \sum_y \mathcal{A}_{xy} n^\alpha_y$
⇒ Entrainment effect

- Solve $G_{\mu\nu} = 8\pi T_{\mu\nu}$ & $n^\mu_x \nabla_{[\mu} \pi^\nu_x] = 0$



Tidal deformations of multifluid neutron stars

To be as general as possible, let's consider a neutron star composed of N independent fluid species [Carlier+ 2026]

- **Hydrostatic** equations:

$$\left\{ \begin{array}{l} m' = -4\pi r^2 \Lambda \\ \nu' = 2 \frac{m + 4\pi r^3 \Psi}{r(r - 2m)} \\ 0 = \sum_y \mathcal{B}^{xy} n'_y + \frac{\nu'}{2} \pi_x \end{array} \right.$$

Λ = generalized energy density

Ψ = generalized pressure

- **Perturbation** equations:

$$H''_\ell + H'_\ell \left[4\pi r e^\lambda (\Psi + \Lambda) + \frac{1 + e^\lambda}{r} \right] + H_\ell \left[4\pi e^\lambda (9\Psi - 5\Lambda + \mathcal{J}_N) - (\nu')^2 - \frac{e^\lambda}{r^2} \ell(\ell + 1) \right] = 0$$

$$\mathcal{J}_N = N \frac{\sum_{x,y} \sum_{x_i,y_i} \pi_x \pi_y \delta_{yy_1 \dots y_{N-1}}^{\text{xx}_1 \dots \text{x}_{N-1}} \mathcal{B}_{x_1}^{y_1} \dots \mathcal{B}_{x_{N-1}}^{y_{N-1}}}{\sum_{x_i,y_i} \delta_{y_1 \dots y_N}^{\text{x}_1 \dots \text{x}_N} \mathcal{B}_{x_1}^{y_1} \dots \mathcal{B}_{x_N}^{y_N}}$$

$$h''_\ell - 4\pi r e^\lambda (\Psi - \Lambda) h'_\ell + e^\lambda h_\ell \left[\frac{4m}{r^3} \pm 8\pi (\Psi - \Lambda) - \frac{\ell(\ell + 1)}{r^2} \right] = 0$$

All these equations *a priori* depend on the **entrainment matrix** $\mathcal{A}$, and thus on the superfluid nature of matter

Superfluidity does not affect adiabatic tides

- In principle, solving these equations requires a **microphysical model** for entrainment, but some simplifications are possible...
- Assuming only the analyticity of Λ , we have demonstrated that all the equations are **independent of entrainment!**
⇒ They only depend on the energy density, the pressure, and their partial derivatives

⇒ Superfluidity leaves no imprint on the adiabatic tidal responses

- This is true for both **electric** and **magnetic** tidal responses, and at any multipolar order!
- This result also applies to **any multifluid compact star**, including those containing dark matter

The resulting equations

- Since superfluidity has no impact, we can neglect it from the outset and instead consider a simpler **two-fluid** description (**matter** and **entropy** fluids)
- Hydrostatic** equations at finite temperature:

$$\left\{ \begin{array}{l} m' = 4\pi r^2 \mathcal{E} \\ \nu' = 2 \frac{m + 4\pi r^3 P}{r(r - 2m)} \\ n' = -n \frac{\nu'}{2} \frac{(\partial \mathcal{E} / \partial n)_T}{(\partial P / \partial n)_T} \\ T' = -\frac{\nu'}{2} T \end{array} \right.$$

- Perturbation** equations at finite temperature:

$$H_\ell'' + H_\ell' \left[4\pi r e^\lambda (P - \mathcal{E}) + \frac{1 + e^\lambda}{r} \right] + H_\ell \left[4\pi e^\lambda (9P + 5\mathcal{E} + \mathcal{J}) - (\nu')^2 - \frac{e^\lambda}{r^2} \ell(\ell + 1) \right] = 0$$

$$\mathcal{J} = \frac{n}{(\partial P / \partial n)_T} \left[\left(\frac{\partial \mathcal{E}}{\partial n} \right)_T \right]^2 + T \left(\frac{\partial \mathcal{E}}{\partial T} \right)_n$$

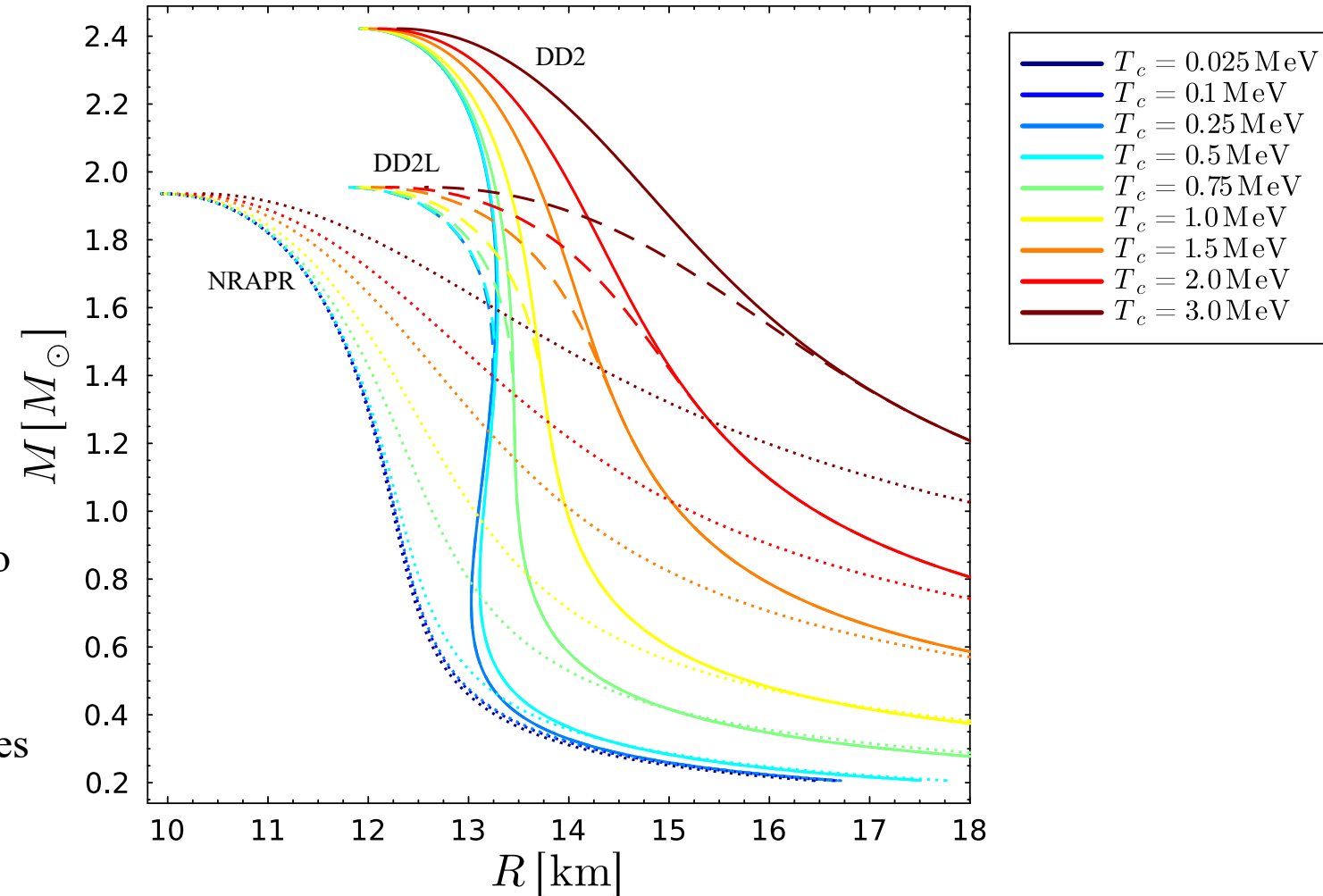
$$h_\ell'' - 4\pi r e^\lambda (P + \mathcal{E}) h_\ell' + e^\lambda h_\ell \left[\frac{4m}{r^3} \pm 8\pi (P + \mathcal{E}) - \frac{\ell(\ell + 1)}{r^2} \right] = 0$$

————→ Solving these equations allows one to compute k_ℓ and j_ℓ for hot neutron stars

Thermal effects on the structure of neutron stars

Impact of T on the radius:

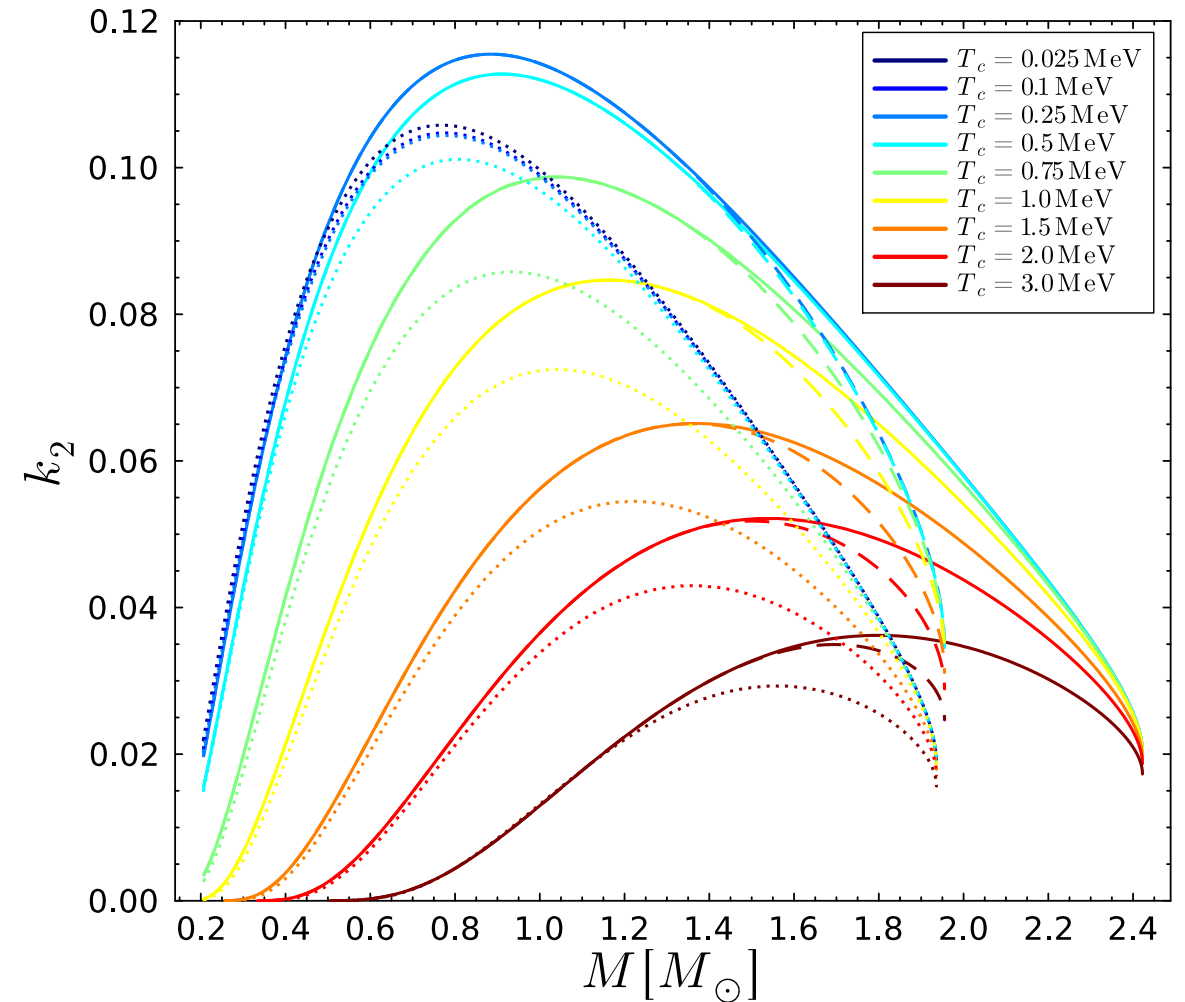
- Equations of state from CompOSE
- For a given M , R increases with T
- More pronounced effect for lighter masses
- Next-generation observatories are expected to achieve a radius accuracy of about 100 m
- Non-negligible effect for realistic temperatures



Thermal effects on Love numbers of neutron stars

Impact of T on Love numbers:

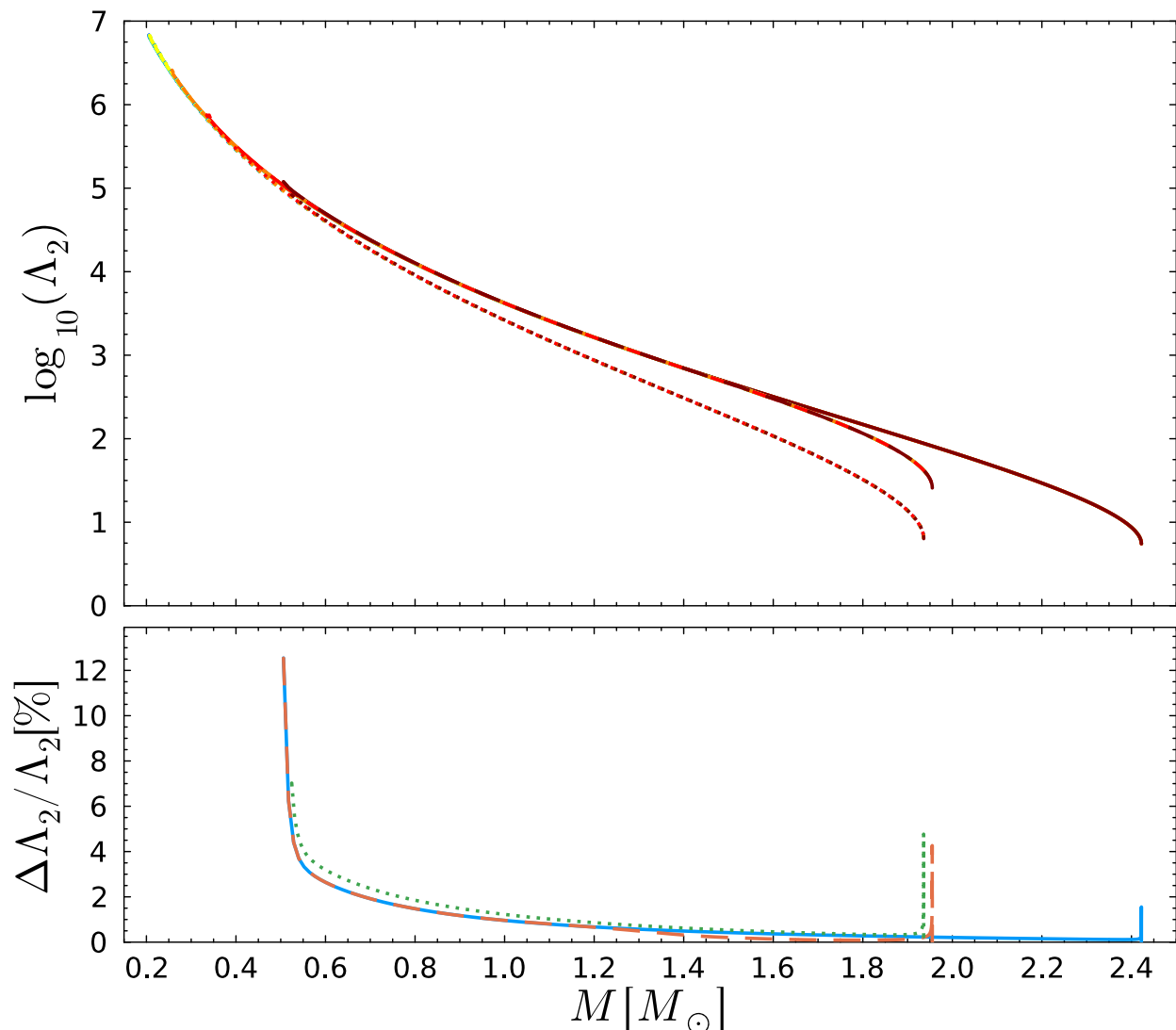
- For a given M , k_2 decreases with T
- More pronounced effect for lighter masses
- Non-negligible effect for realistic temperatures
- Same qualitative behavior for higher-order and magnetic Love numbers



Thermal effects on tidal deformabilities of neutron stars

Impact of T on Λ_2 :

- Within the **expected mass** range, Λ_2 is weakly sensitive to T
- Relative difference: $\Delta\Lambda_2/\Lambda_2 \sim 2\%$
- R and k_2 are very **sensitive to T** but the combination $\Lambda_2 \sim k_2 R^5$ is **not**
- Next-generation observatories are expected to achieve a precision of a few % at best
- Same qualitative behavior for higher-order and magnetic tidal deformabilities



Conclusion

- During the late inspiral, neutron stars are expected to **heat up**, while **superfluidity** may persist
- We have shown **exactly** that superfluidity has **no impact** on adiabatic tides
- The mass-radius relation and the Love numbers are very **sensitive** to temperature
- Neutron-star tidal deformabilities are **weakly sensitive** to temperature
- Neglecting temperature may seem reasonable when computing tidal deformabilities, but thermal effects can become important when **inferring radii** from them