The effective route to new physics

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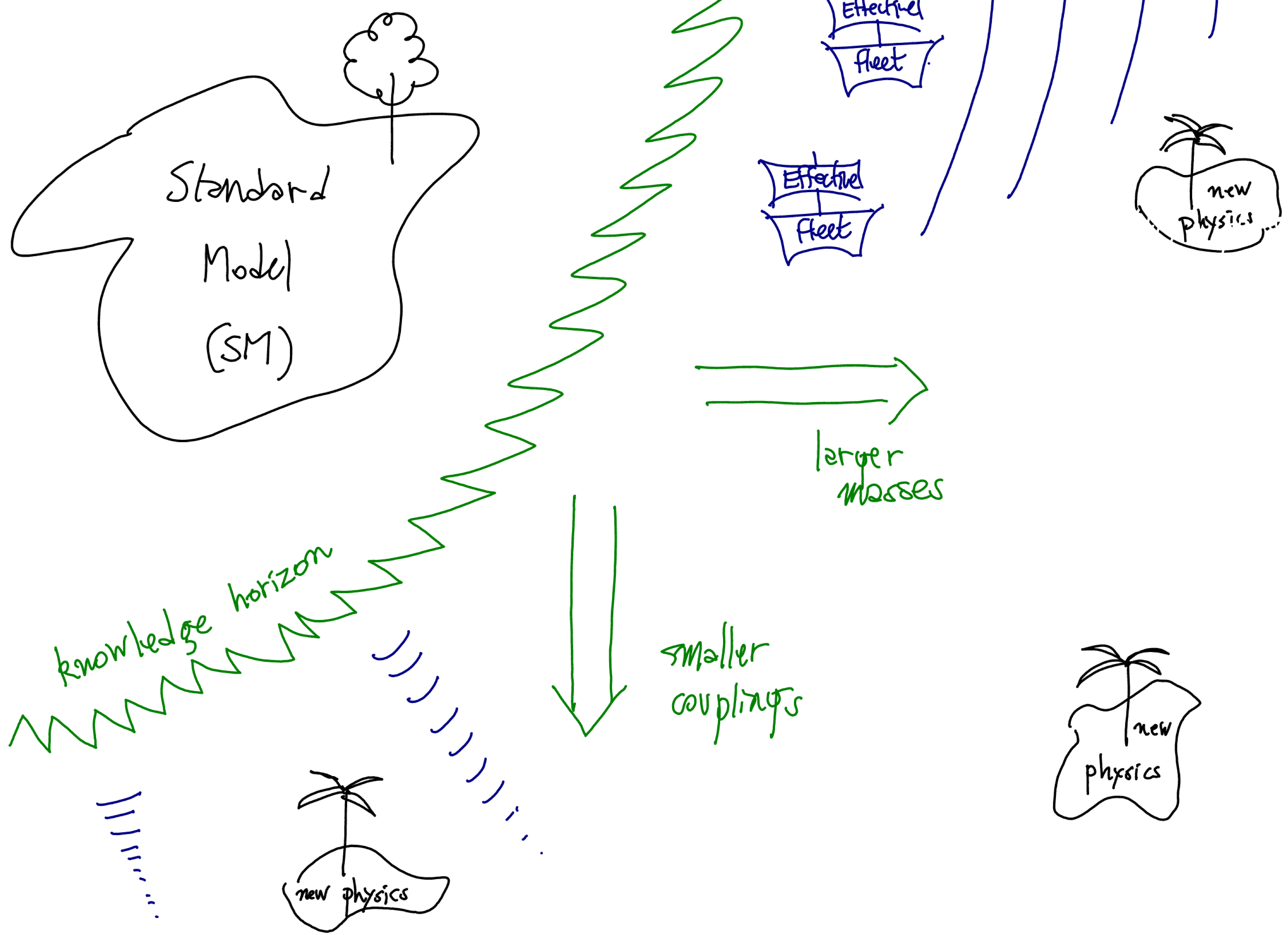
The reductionist ladder

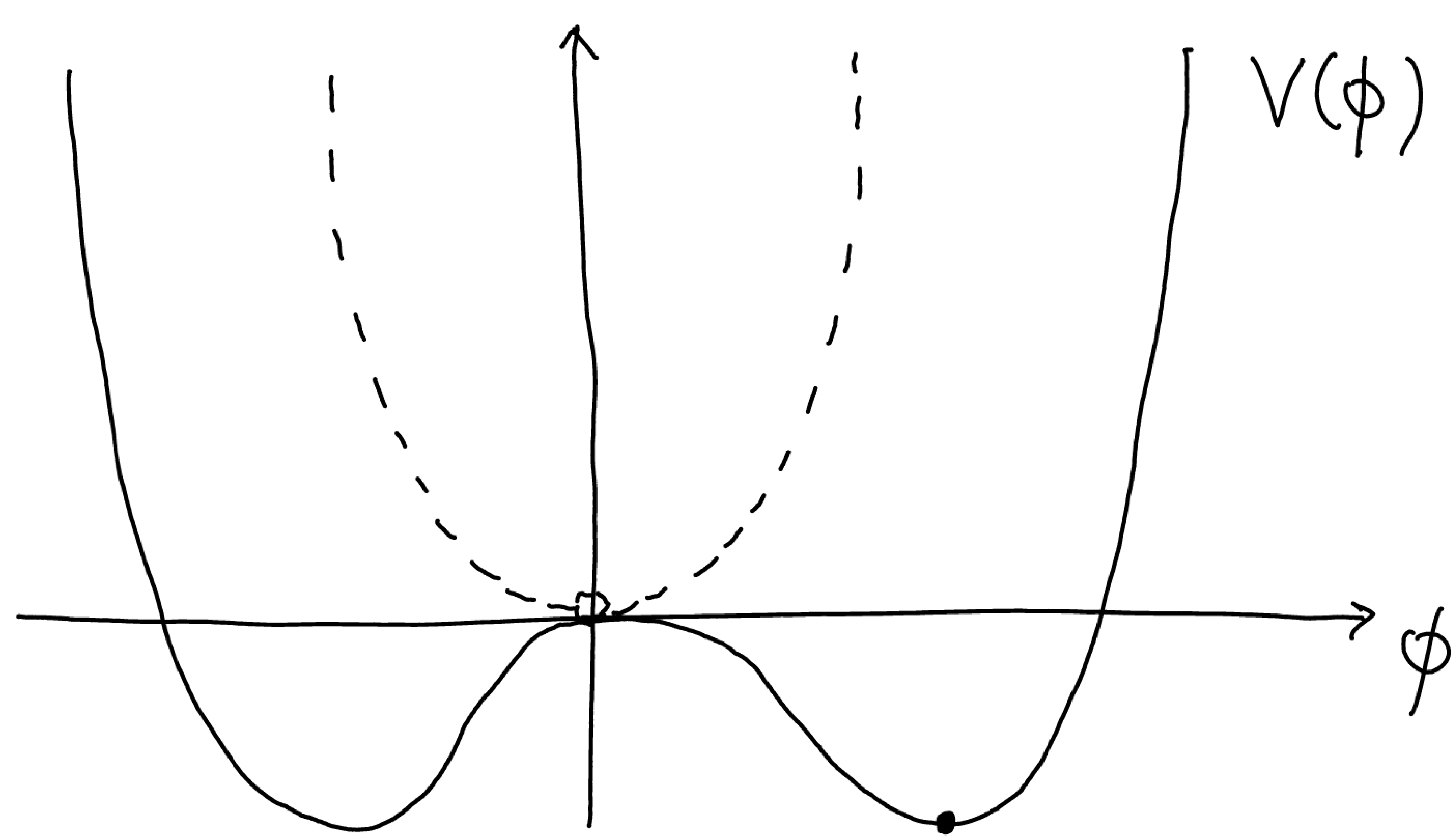
Particle physics landscape



An unsatisfactory aspect of the SM

Mass generation mechanism of Brout, Englert, Higgs

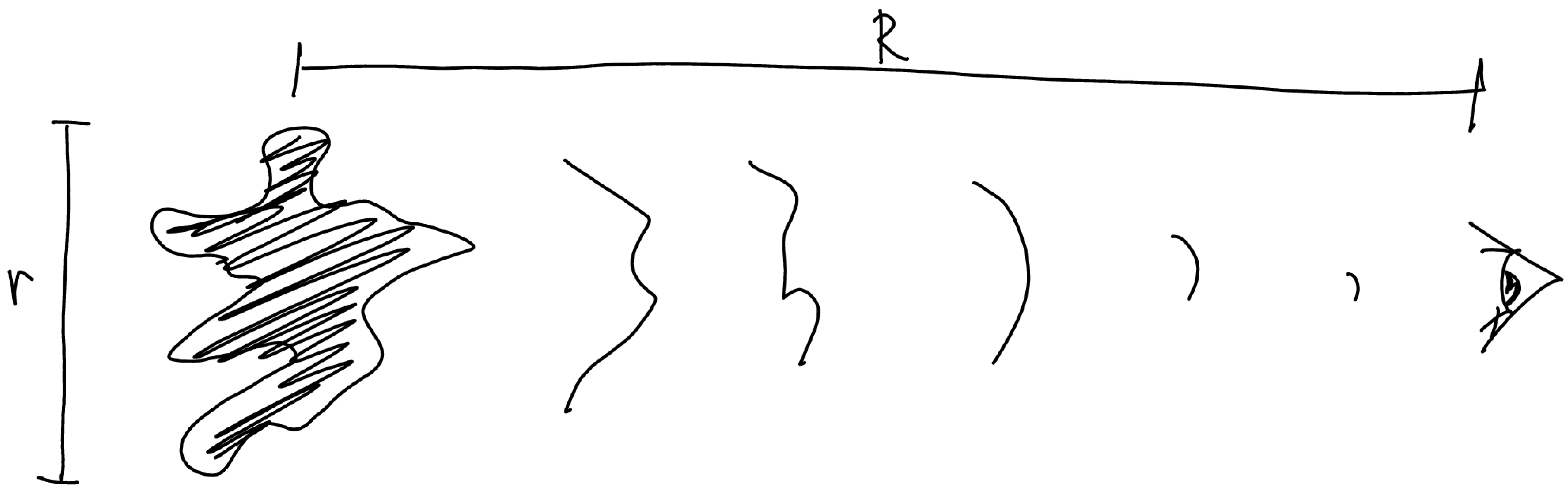
Phenomenological description of a more microscopic dynamics like the Ginzburg-Landau theory of superconductivity?



Effective approach

Long-distance / low-energy description of (unknown) microphysics.

Analogy: unknown charge distribution $\sim r$
examined from distance $R \gg r$

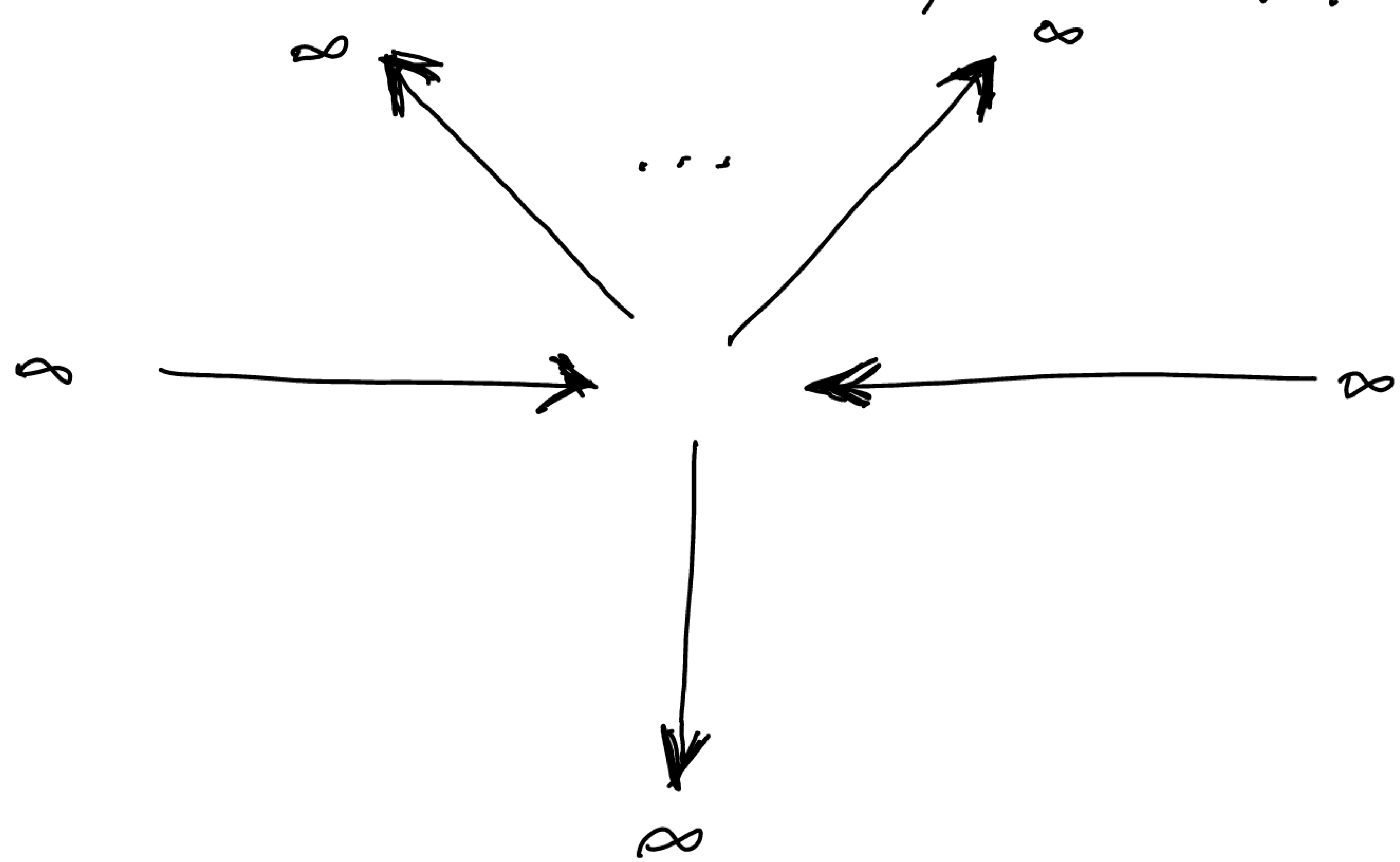


$\Rightarrow$ multipole expansion:

$$V(R) = \frac{1}{R}\left[\overset{\text{charge}}{c_0} + \overset{\text{dipole}}{c_1}\left(\frac{r}{R}\right) Y_1^m(\theta,\varphi) + \overset{\text{quadrupole}}{c_2}\left(\frac{r}{R}\right)^2 Y_2^m(\theta,\varphi) + \dots\right]$$

Scattering amplitudes

Main observables of interest in particle physics



($1\ \text{TeV} \sim 10^{-19}\ m$)

Computed perturbatively in a small coupling expansion of our QFTs

traditionally from

Feynman diagrams obtains using

Lagrangian containing operators

made of fields.

unphysical objects

$\Longrightarrow$ redundant description

Standard Model effective theory (SMEFT)

Interactions described by Lagrangian terms = operators :

- made of SM fields :

$$\left.\begin{array}{lll} q, d, u & \text{spin } 1/2 & \text{"quarks"} \\ l, e & \text{spin } 1/2 & \text{"leptons"} \end{array}\right\} \text{fermions} \times 3 \text{ generations/Families}$$

$$\left.\begin{array}{lll} G, W, B & \text{spin } 1 & \text{gauge} \\ H & \text{spin } 0 & \text{Higgs} \end{array}\right\} \text{bosons}$$

- satisfying known symmetries : [before spontaneous breaking by the Higgs vacuum]

  spacetime : Lorentz

  internal gauge : $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\Rightarrow \mathcal{L}_{SMEFT} = \sum_{d=0}^{\infty} \frac{c_i^{(d)}}{\Lambda^{d-4}} O_i^{(d)} \quad \begin{cases} \cdot\ \mathcal{L}^{(d\leq 4)} \equiv \mathcal{L}_{SM} \\ \cdot\ \mathcal{L}^{(d>4)} : \left(\frac{E}{\Lambda}\right)^{d-4} \text{ suppressed} \end{cases}$$

Naturalness puzzles for $d < 4$

special !

The <u>mass operator of a fundamental scalar</u> has $d = 2$:

$$\mathcal{L}_{SM} \ni c^{(2)}_{|H|^2} \Lambda^2 |H|^2 .$$

If Λ is large $\gg$ TeV ,
$c^{(2)}_{|H|^2}$ would need to be surprisingly small
to realise $m_H \sim 0.1$ TeV .

- actually $\Lambda \sim m_H$ and we'll soon see signs of it
- $c^{(2)}_{|H|^2}$ is just an input parameter which happen to be small
- $c^{(2)}_{|H|^2}$ is calculable and small for a good reason (e.g. symmetry)

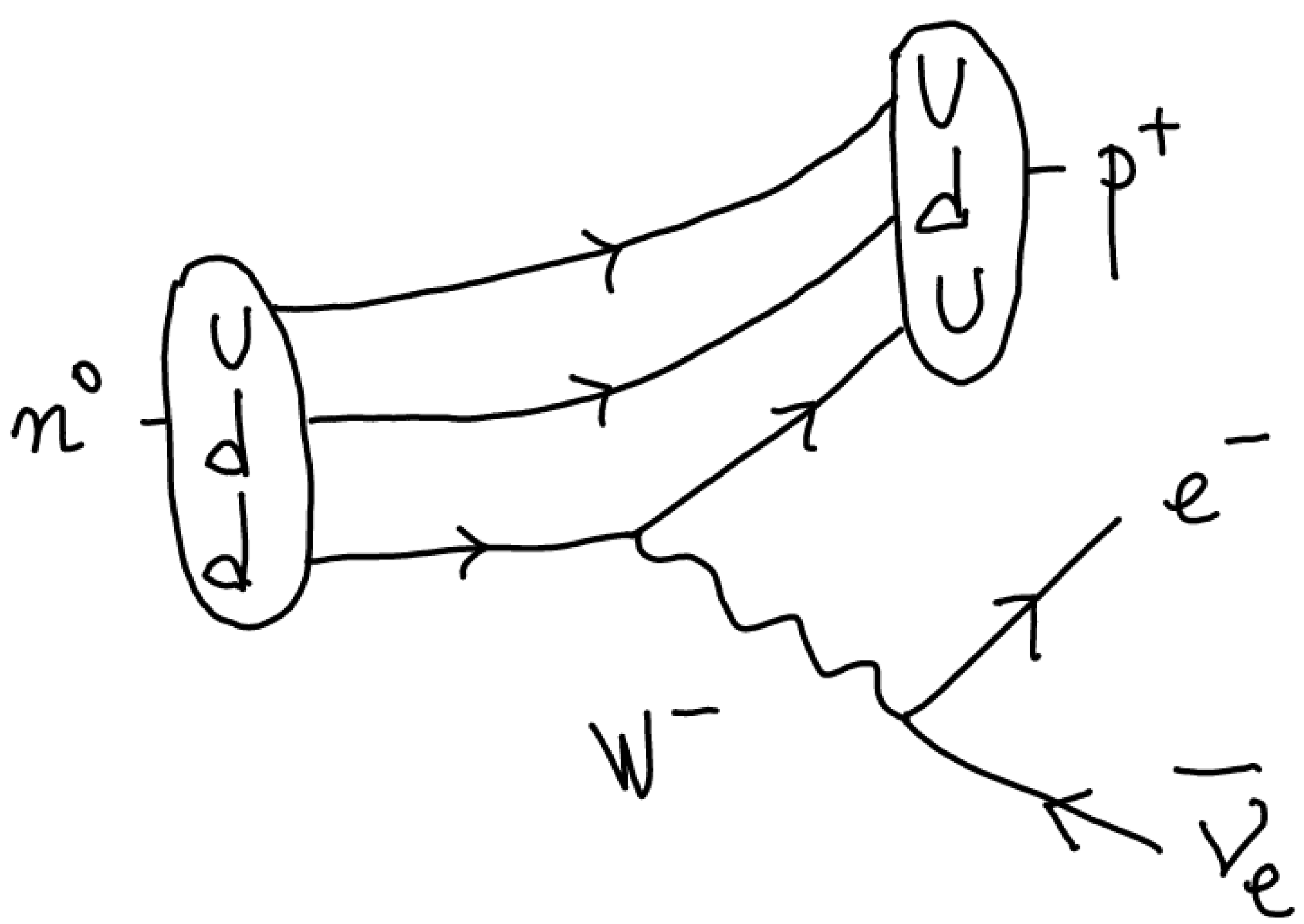
+ cosmo. constant at $d = 0$

+ strong CP at $d = 4$

Local operators capture heavy physics

Thanks to the uncertaincy principle of quantum mechanics

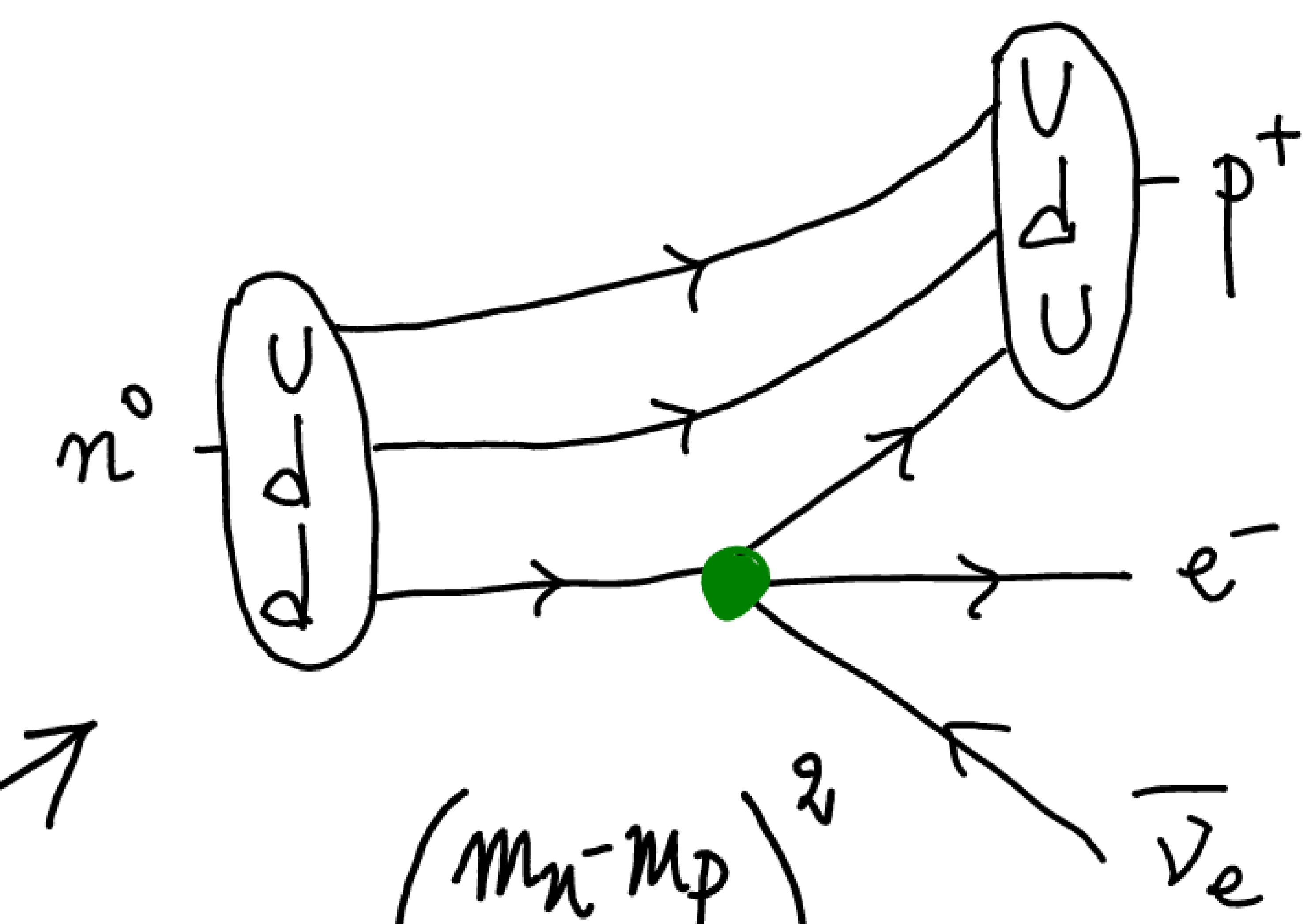
Even at $E \ll M$ precise measurements can probe virtual effects of heavy states :



$$E \sim m_n - m_p \sim 10^{-6}\,\text{TeV}$$

$$\wedge\!\wedge$$

$$M_W \sim 10^{-1}\,\text{TeV}$$



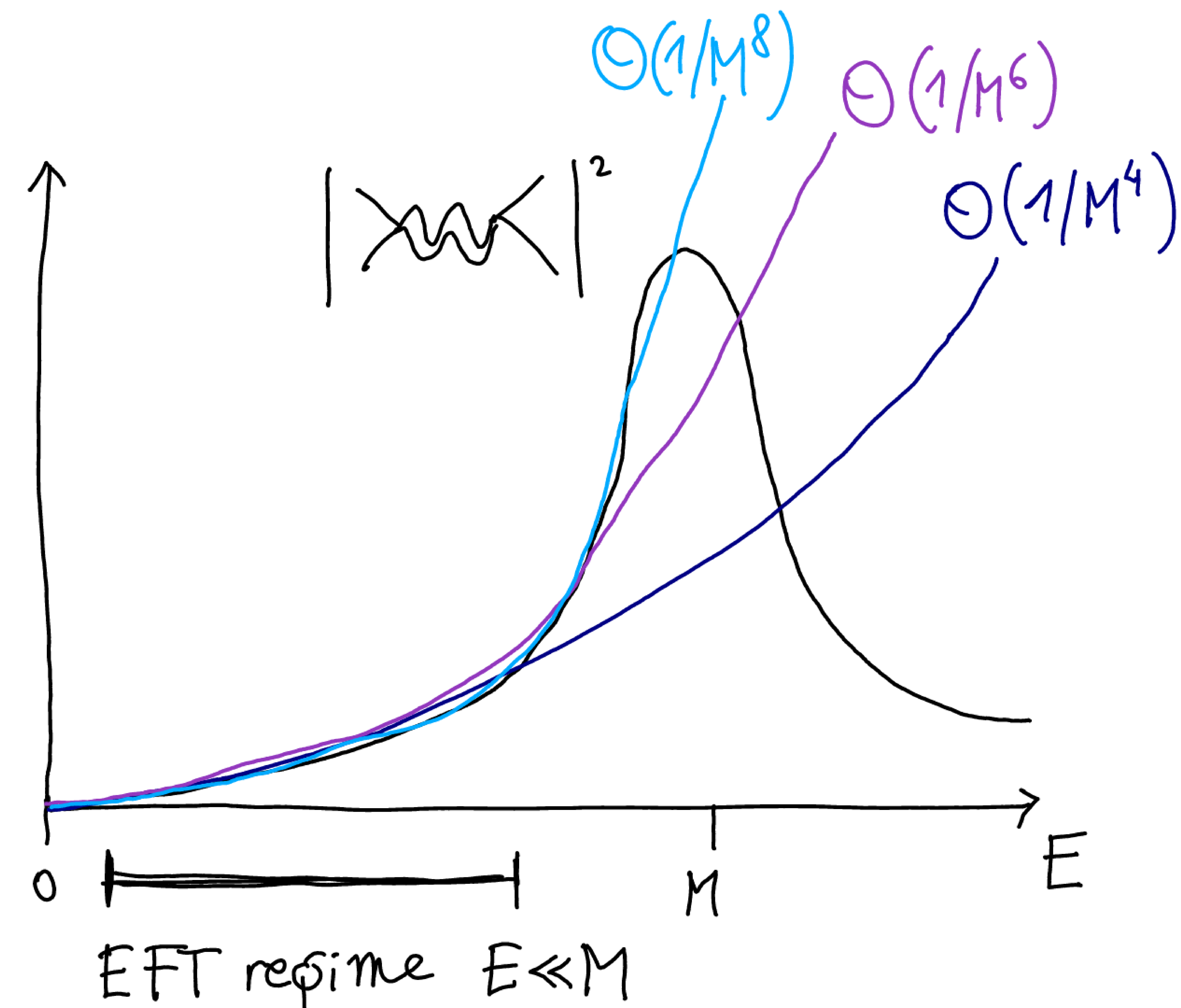
$$\sim \left(\frac{m_n - m_p}{M_W}\right)^2$$

Very loosely speaking : One can "borrow" energy beyond E to reach M but only for a short amount of time, i.e. almost no propagation, i.e. essentially local

Local operators capture heavy physics

$$\sim -\frac{g^2E^2}{E^2-M^2+iM\Gamma} \underset{E\ll M}{\sim} \underset{\uparrow\, d=6}{g^2\left(\frac{E}{M}\right)^2} + \underset{\uparrow\, d=8}{g^2\left(\frac{E}{M}\right)^4} + \cdots$$

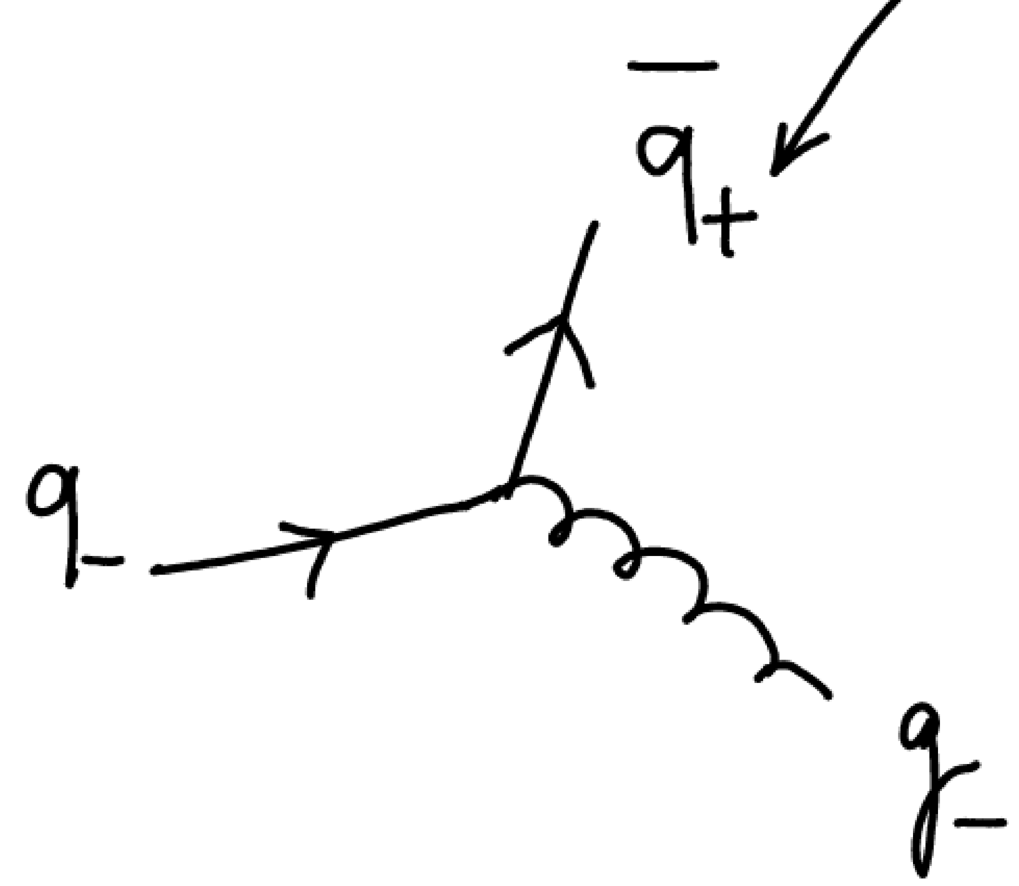
$$\frac{c^{(6)}}{\Lambda^2} = \frac{g^2}{M^2} \qquad \frac{c^{(8)}}{\Lambda^4} = \frac{g^2}{M^4} \qquad \cdots$$



"matching" $\equiv$ computing the operator coefficient values generated by specific heavy physics

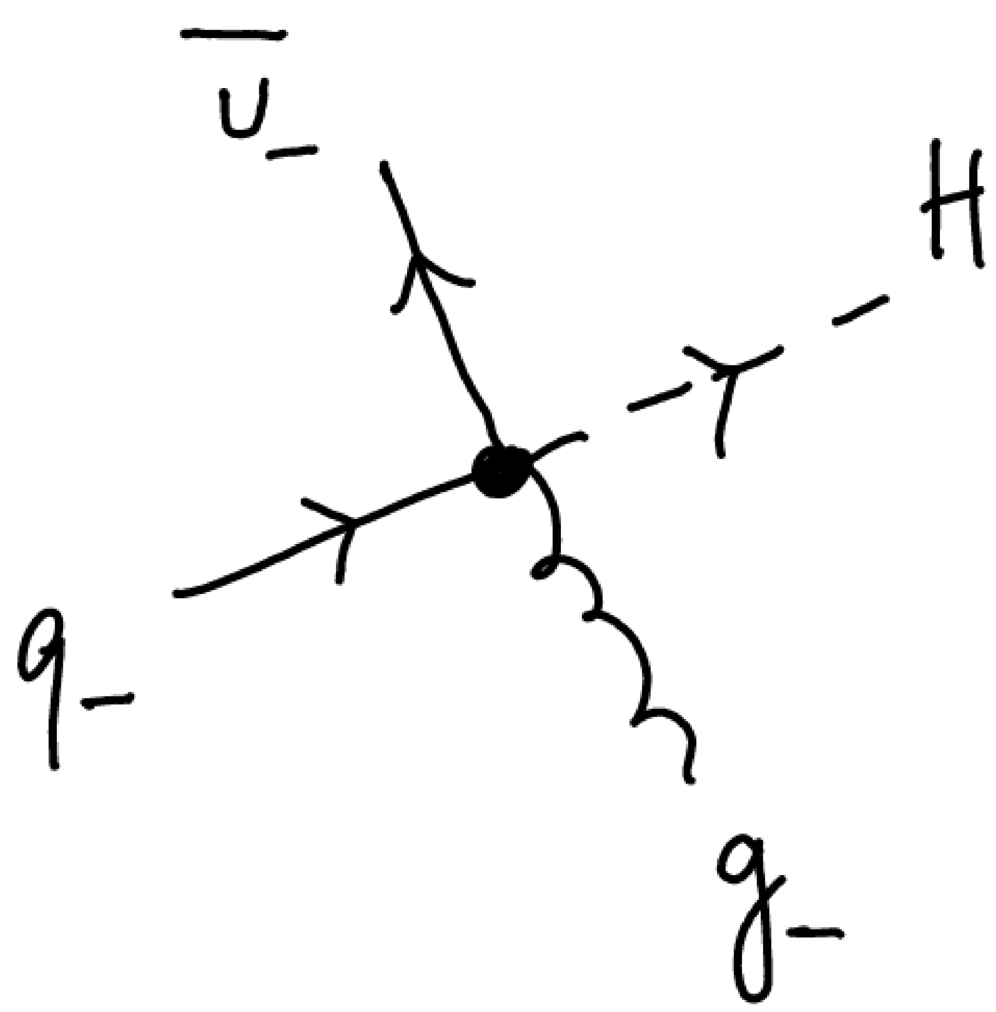
SMEFT interaction examples

helicity ≡ spin projection along momentum



$\sim \dfrac{\langle q g\rangle^2}{\langle q\bar{q}\rangle}$

amplitude generated by operator

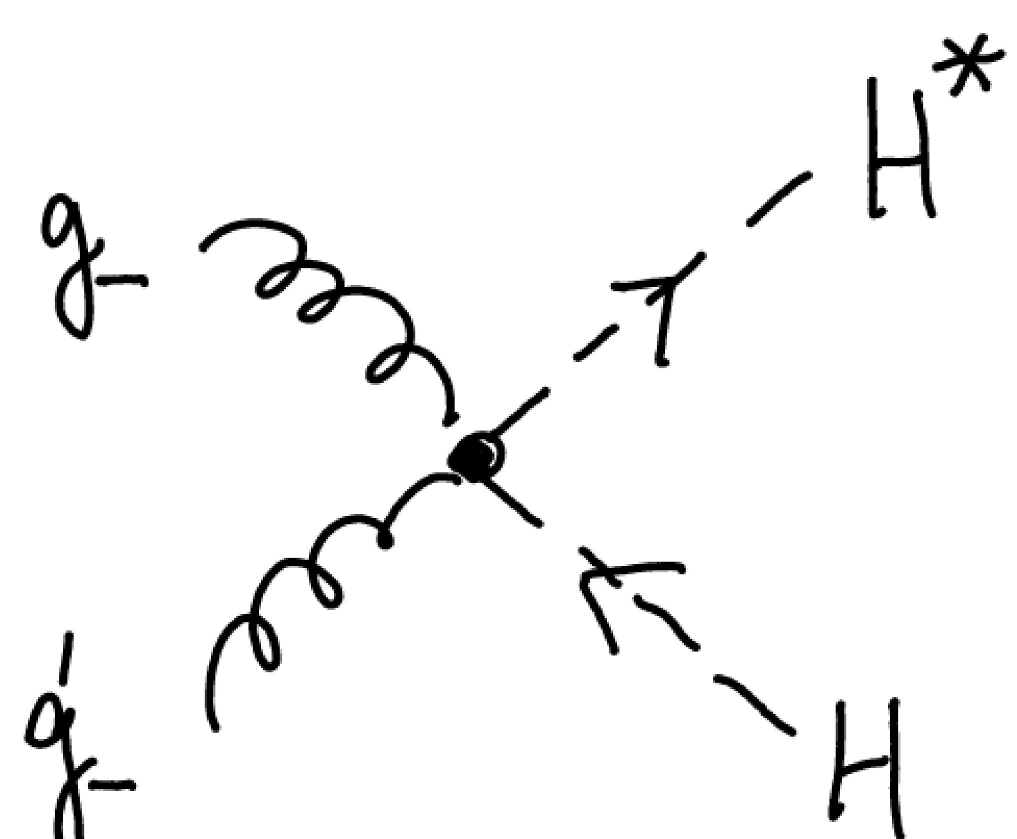


spinors:

$\sim \dfrac{\langle q g\rangle\langle \bar{u} g\rangle}{\Lambda^2}$

dictated by dimensionality

$\begin{cases} \dim(A_n) = 4-n \\ \dim(\langle ij\rangle, [ij]) = 1 \end{cases}$



$\sim \dfrac{\langle g g'\rangle^2}{\Lambda^2}$

Lorentz

$SO(3,1) \sim$

$SU(2)_- \times SU(2)_+$

$\downarrow \qquad\qquad \downarrow$

$\langle i_\alpha \qquad [i_{\dot\alpha}$

spinors encoding kinematics

e.g. massless momentum:

$p_\mu \sigma^\mu_{\alpha\dot\alpha} = {}_\alpha p\rangle [p_{\dot\alpha}$

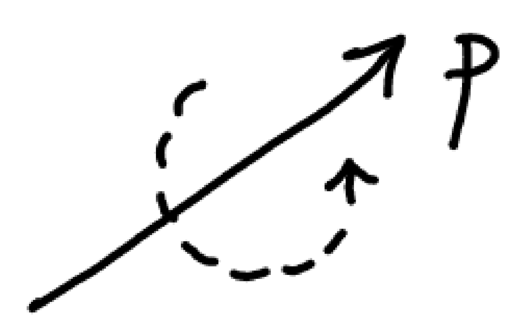
trivialises

$p^2 = \langle pp\rangle[pp] = 0$

Little-group covariance of amplitudes

[massless case]

Little-group Lorentz transformations leave a given momentum invariant.

e.g. 

$U(1)$ rotation around a massless momentum p

Quantum states pick up a phase dictated by their spin under such rotations.

So do spinors: $\quad p] \to e^{+i\theta/2}\, p] \quad \& \quad p\rangle \to e^{-i\theta/2}\, p\rangle$

$\left(\quad p_\mu \sigma^\mu = p\rangle[p \quad \text{indeed invariant} \quad \right)$

and amplitudes: $\quad A(\{p,h\}) \longrightarrow e^{ih\theta} A(\{p,h\})$

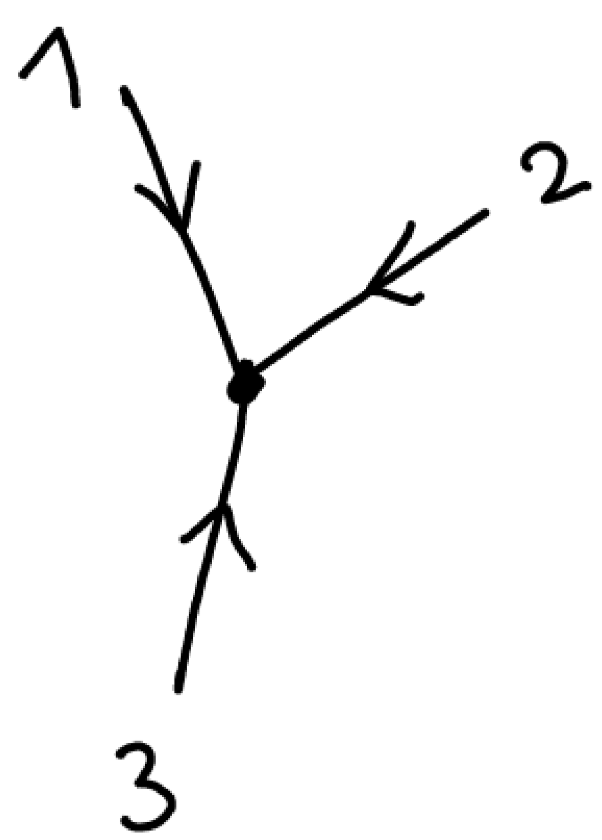
$\Rightarrow$ h fixes the net number of spinors of particle p in A

Massless three-point amplitudes are unique

The restricted kinematics forces only ·⟩ or ·] to appear.

They are only 3 possible spinor contractions : (12), (23), (13).

Their 3 powers are fixed by the 3 particle helicities, to satisfy little-group covariance.



$$\sim \frac{1}{\Lambda^{|h_1+h_2+h_3|-1}} \begin{cases} [12]^{h_1+h_2-h_3}\,[23]^{h_2+h_3-h_1}\,[13]^{h_1+h_3-h_2} & \underline{\text{for}}\ h_1+h_2+h_3>0 \\ \langle 12\rangle^{h_3-h_1-h_2}\,\langle 23\rangle^{h_1-h_2-h_3}\,\langle 13\rangle^{h_2-h_1-h_3} & \underline{\text{for}}\ h_1+h_2+h_3<0 \end{cases}$$

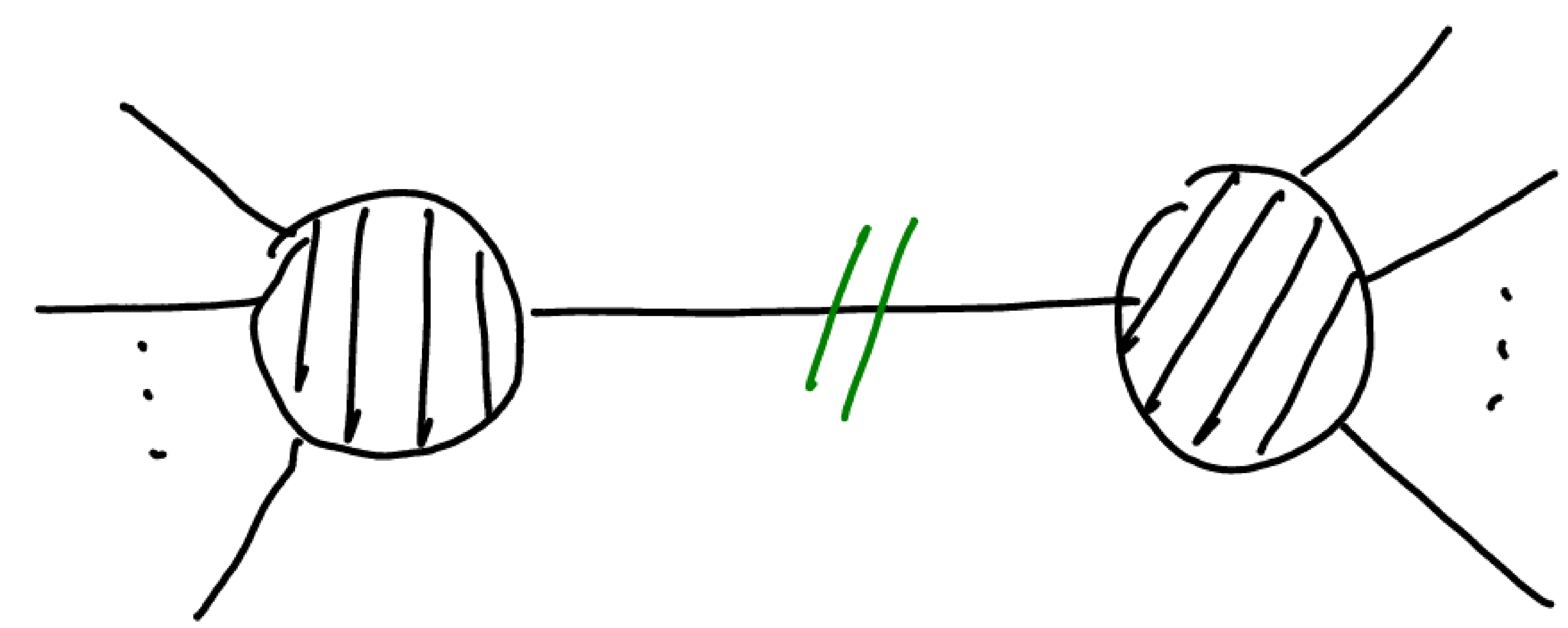
	h_1	h_2	h_3				
e.g.	0	$\pm\frac{1}{2}$	$\pm\frac{1}{2}$	:	Yukawa coupling	:	$[23]$
	$+1$	$+\frac{1}{2}$	$-\frac{1}{2}$	:	gauge-fermion coupling	:	$[12]^2/[23]$
	$+1$	0	0	:	gauge-scalar coupling	:	$[12][13]/[23]$
	$+1$	$+1$	-1	:	gauge self-coupling	:	$[12]^3/[13][23]$

Recursive construction of amplitudes

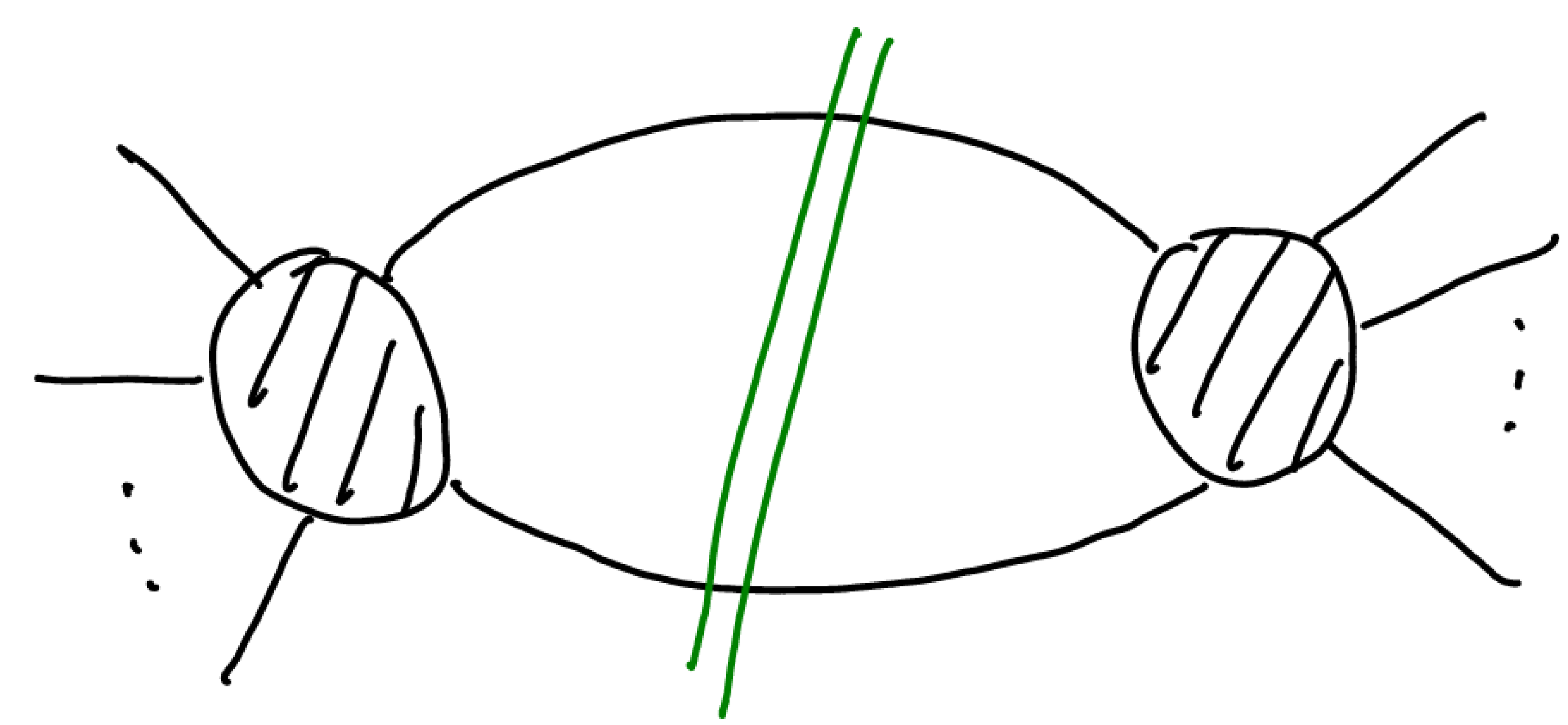
Amplitudes are analytic functions of the kinematics except on propagators [causality].

There, they factorise into simpler amplitudes [unitarity]:

"trees $\overset{\text{residue}}{\text{cut}}$ in smaller trees"
+ local contact terms



"loops $\overset{\text{discontinuity}}{\text{cut}}$ in lower loops"

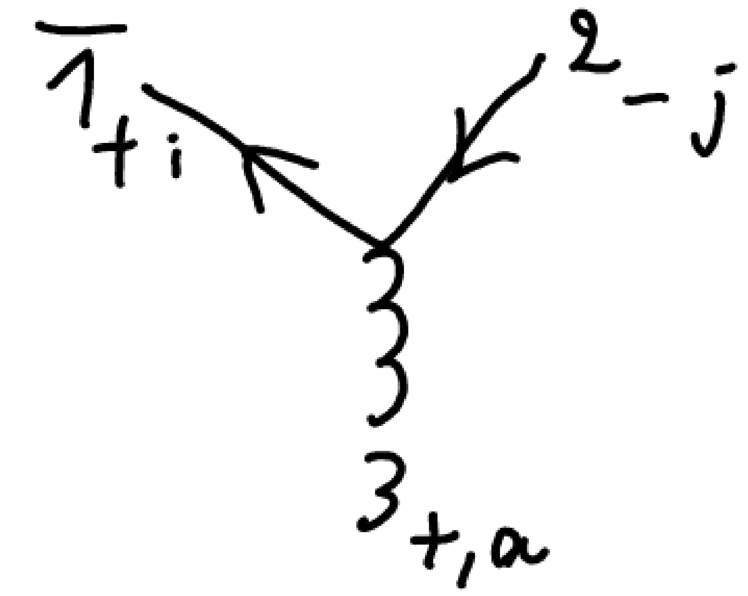


Conversely, one can reconstruct all amplitudes from the simplest ones.

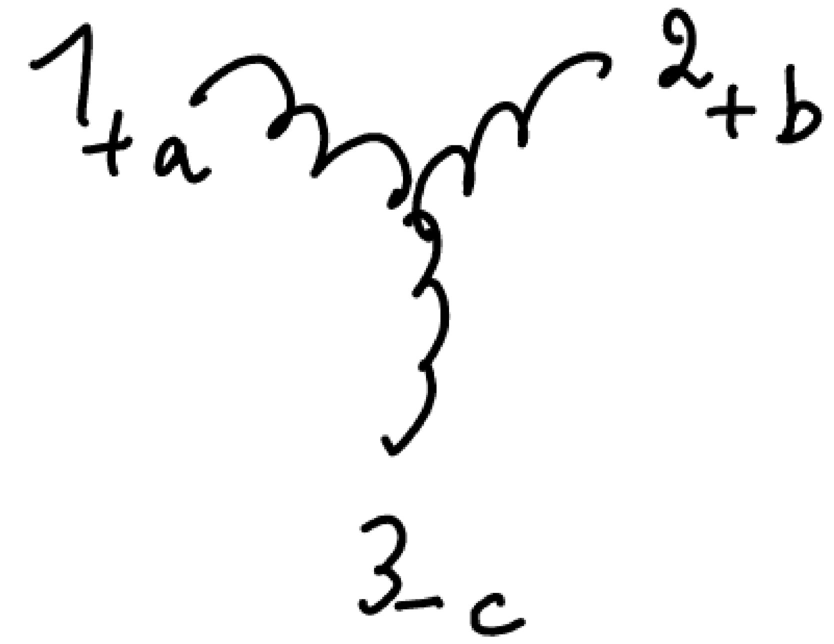
$\Rightarrow$ define the theory from these, bypassing field & gauge redundancies.

From three to higher points

<u>E.g.</u> $d \leq 4$ theory of massless fermion and vector multiplets

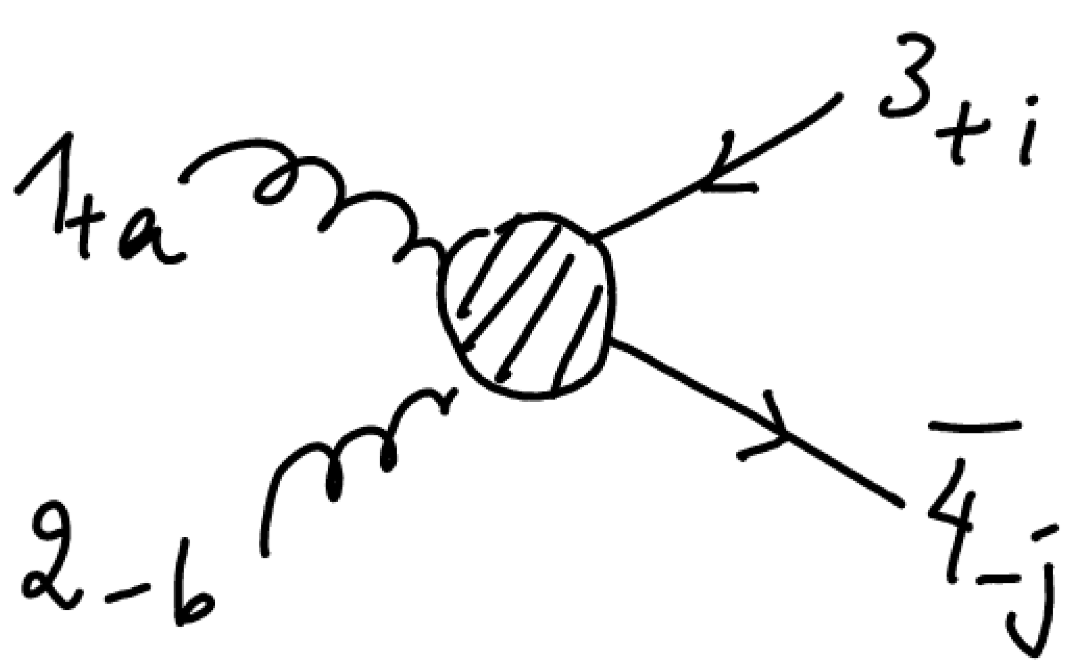


$$\sim (T^a)^i{}_j \; [13]^2/[12]$$



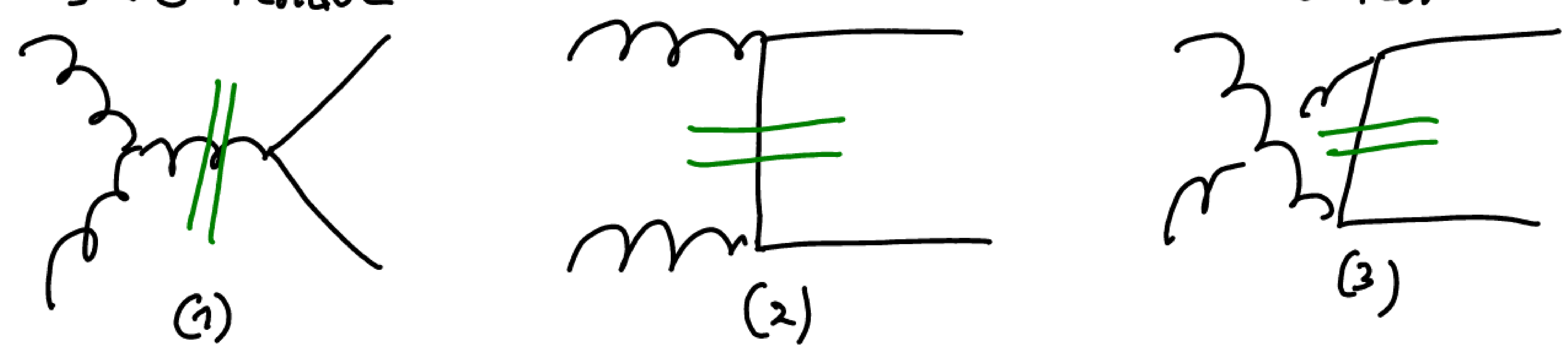
$$\sim f^{abc}\,[12]^3/[13][23]$$

From which:



with

$s \to 0$ residue $\qquad$ $t \to 0$ residue $\qquad$ $u \to 0$ residue



$$= [13]\langle 24\rangle [142\rangle \left(\frac{a}{tu} + \frac{b}{su} + \frac{c}{st}\right)$$

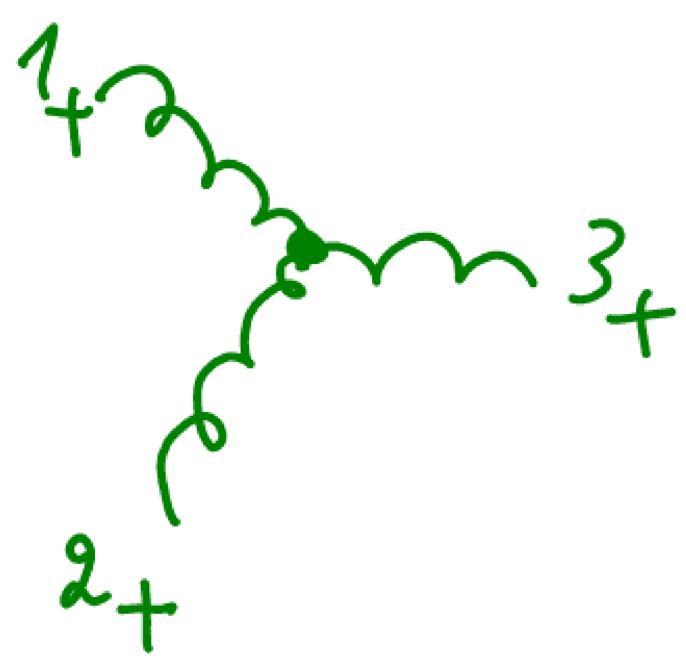
$$\left[\, s \equiv (p_1+p_2)^2 ,\; t \equiv (p_1+p_3)^2 ,\; u \equiv (p_1+p_4)^2 \,\right]$$

$$\left.\begin{aligned} &(1)\; c-b = f^{abc}T^c \\ &(2)\; c-a = T^aT^b \\ &(3)\; b-a = T^bT^a \end{aligned}\right\} \; [T^a, T^b] = f^{abc}T^c$$

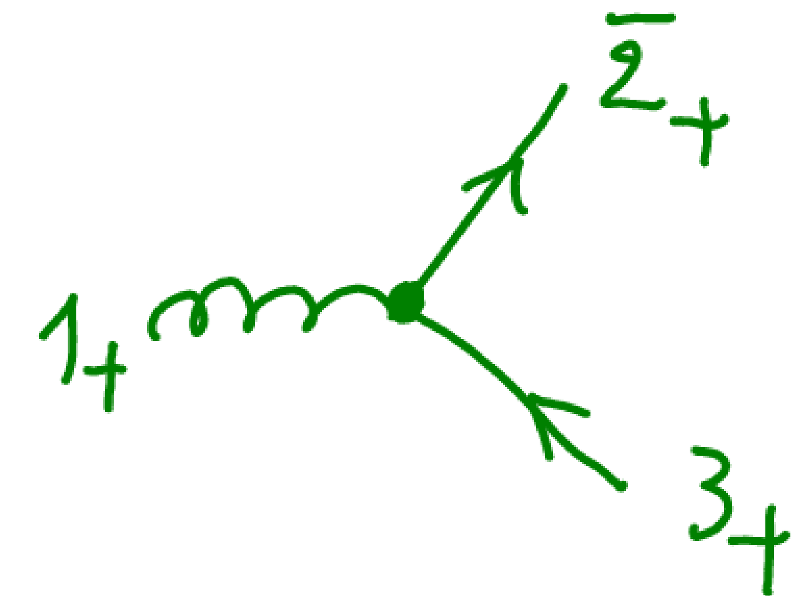
$\Rightarrow$ Lie algebra of a gauge theory

$d > 4$ operators

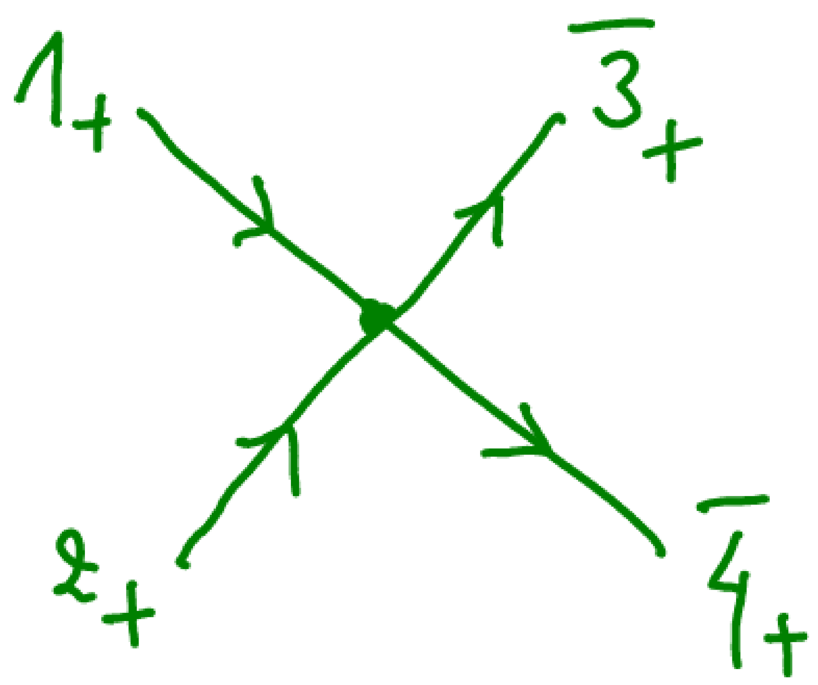
More local building blocks / contact-term amplitudes:



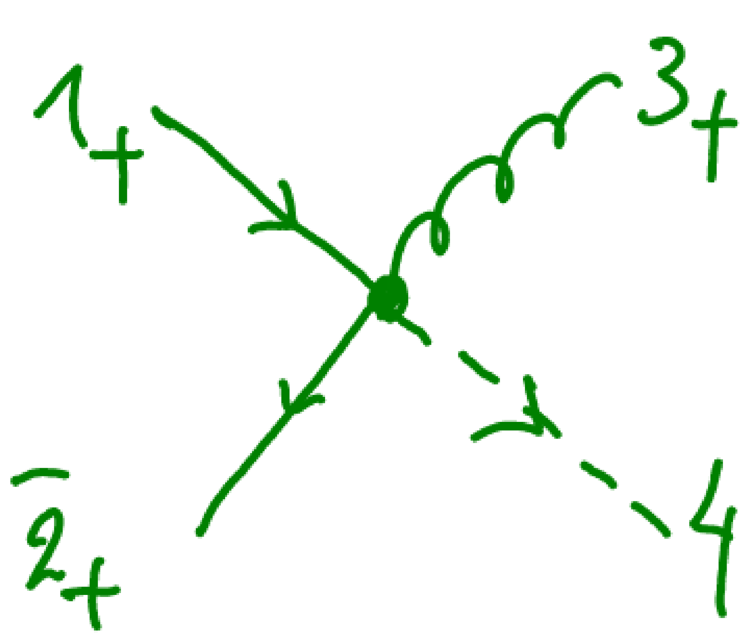
$\sim \dfrac{[12][23][13]}{\Lambda^2}$,



$\sim \dfrac{[12][13]}{\Lambda}$



$\sim \dfrac{[12][34]}{\Lambda^2}$ or $\dfrac{[13][24]}{\Lambda^2}$ or $\cancel{\dfrac{[14][23]}{\Lambda^2}}$ redundant



$\sim \dfrac{[13][23]}{\Lambda^2}$

↳ fully characterise the EFT

↳ easier to enumerate and manipulate than operators

Operator running and selection rules

Caron-Huot, Wilhelm '16
Cheung, Shen '15

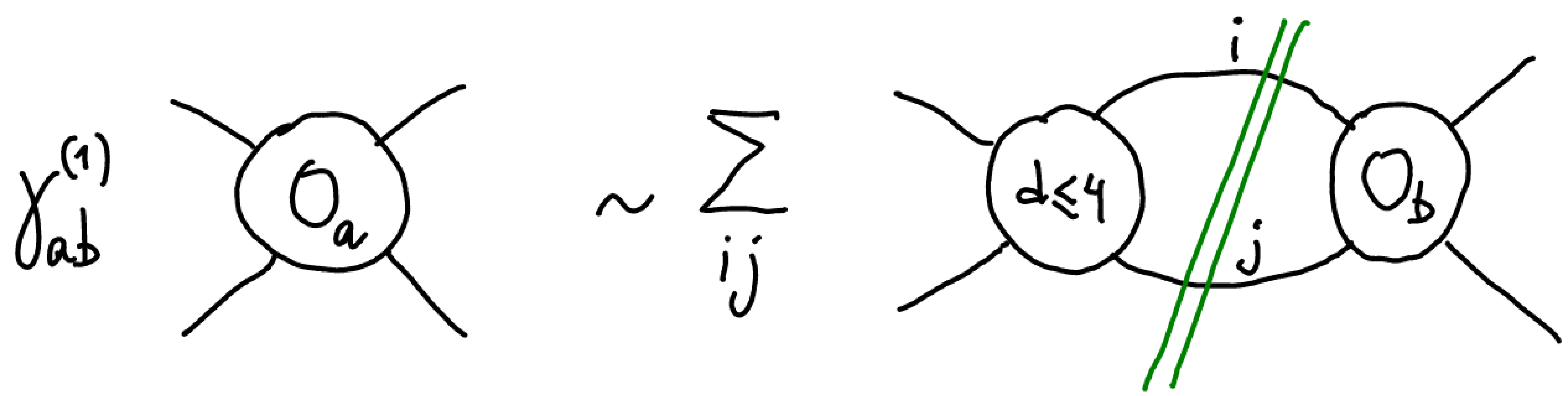
The "most effective" theory depends on the scale of the process studied. Operator coefficients "run" and "mix":

$$c_a(\mu) = \gamma^{(1)}_{ab} \frac{d c_b}{d \log \mu}(\mu)$$

pace of running and pattern of mixing

$(p_i + \dots + p_j)^2 \equiv$

This is mirrored in $\log \frac{s_{i \dots j}}{\mu^2}$ dependence of their (one-) loop amplitudes and is thus encoded in their discontinuities:



→ efficient computation with one fewer loop (cut)

→ structural understanding (e.g. vanishing  imply γ_{ab} zeros)

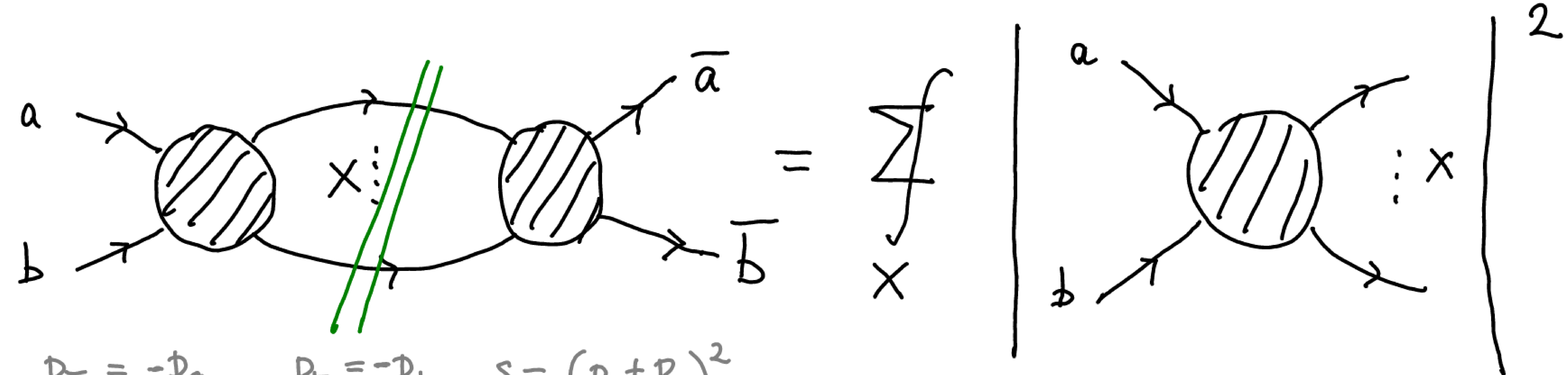
Positivity constraints

Not all op. coefficient values are acceptable!

Adams et al. '06

The discontinuities of elastic forward 4-point amplitudes are positive:

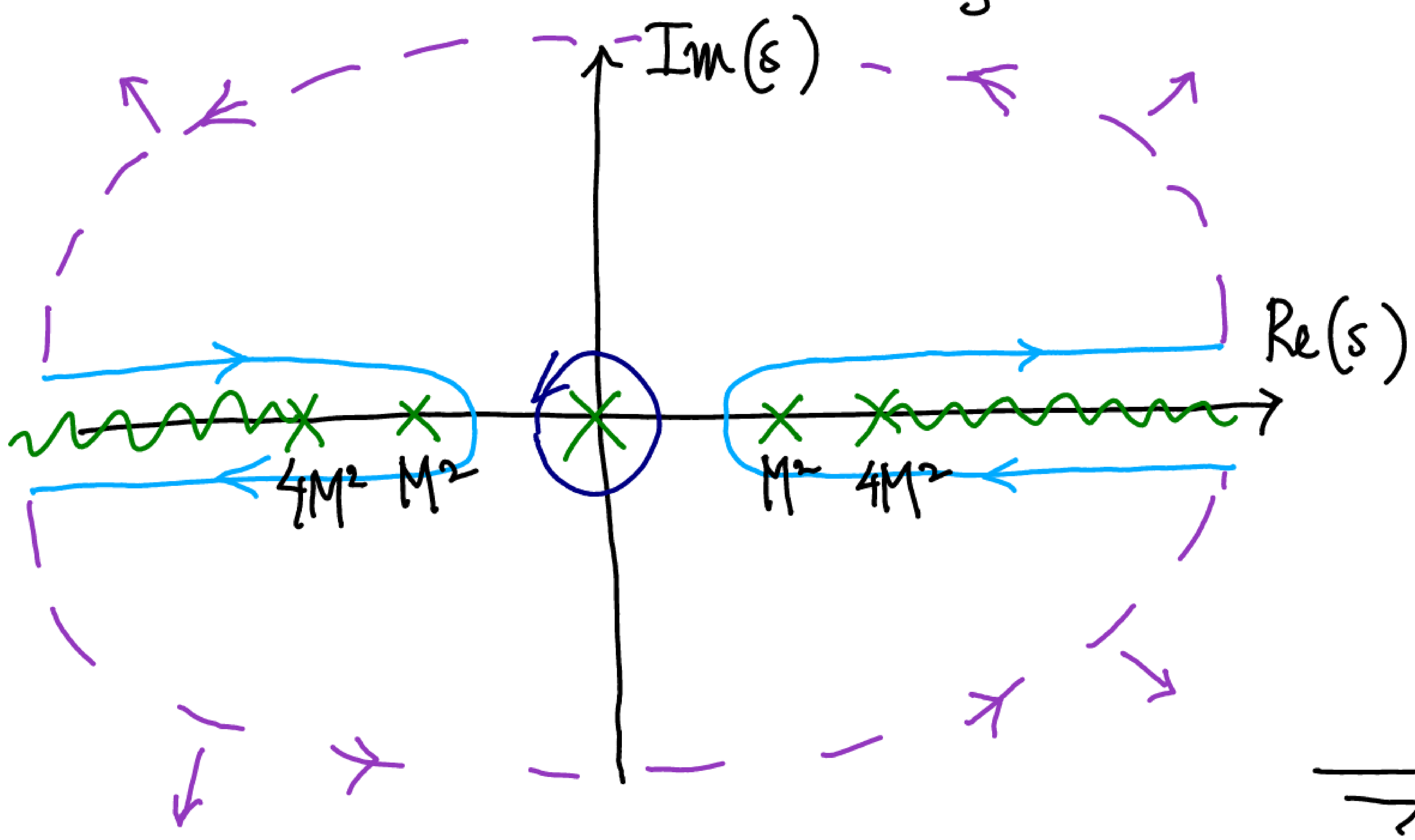
Disc $A(s)$ =



≥ 0

$p_{\bar a} = -p_a \; , \; p_{\bar b} = -p_b \, , \; s = (p_a + p_b)^2$

Analytic continuation of $\dfrac{A(s)}{s^{n+1}}$ in the s complex plane:



$= [1+(-1)^n]$ $+$

c_n coefficient of s^n in the expansion of $A(s)$, $s \to 0$

≥ 0 see above

$= 0$ for $n \geq 2$

$\Longrightarrow \quad c_n \geq 0 \quad$ for $n \geq 2$, even

EFT matching from cuts

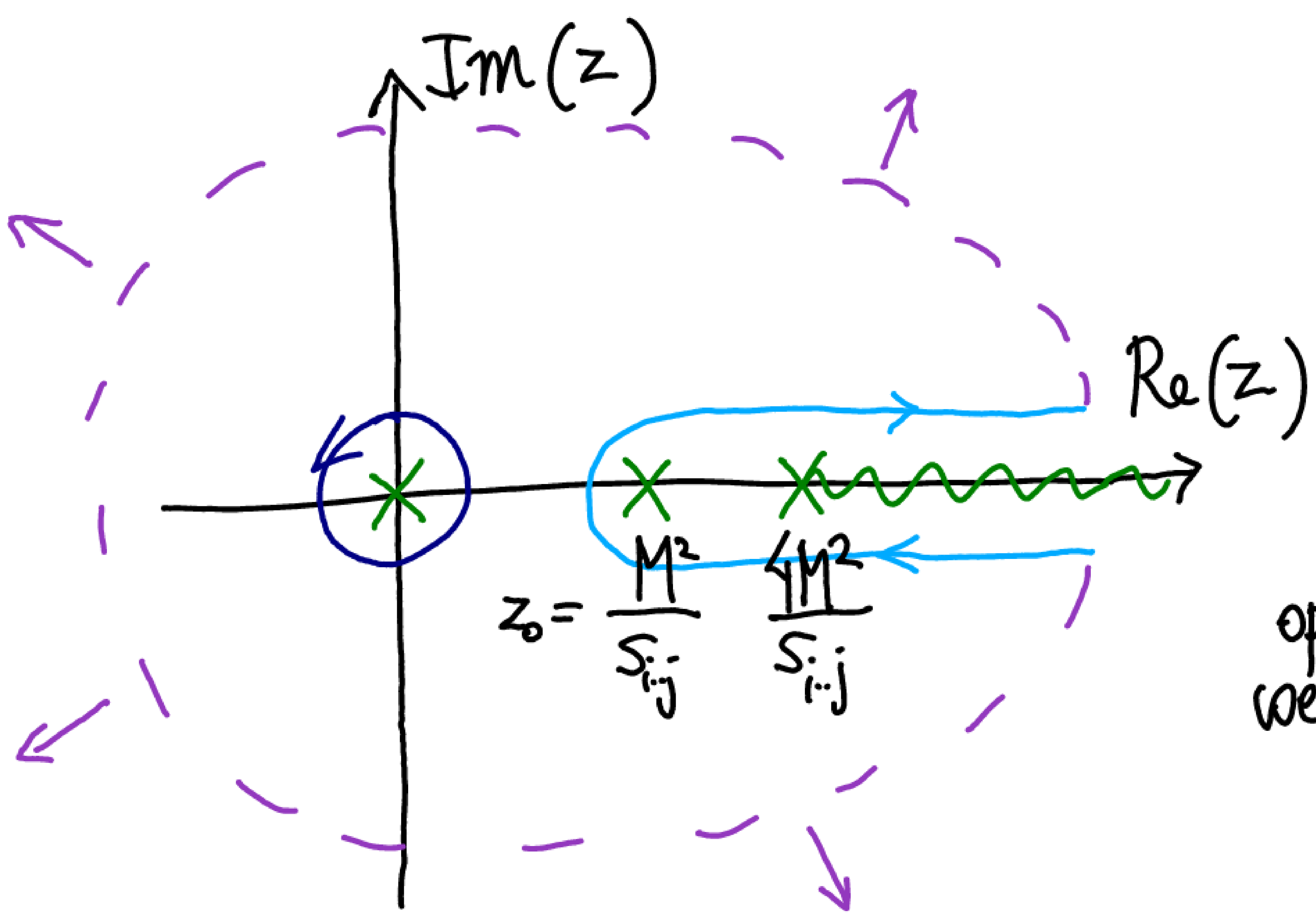
De Angelis, GD '23

Similar technique to compute operator coefficient values if a specific theory is assumed at high energies : "matching"

Difficulties: analytic structure of amplitudes beyond 4-point is unknown in many $s_{i\dots j} \equiv (p_i + \dots + p_j)^2$ variables

→ dilate all $s_{i\dots j}$ by a single complex variable z

→ use a deformed amplitude $F(z)$ with all singularities at $z > 0$



Taking the analytic continuation of $\frac{F(z)}{z^{n+1}}$:

$$c_n P_n(s_{i\dots j}) = \int_{z_0}^{\infty} dz \, \frac{\mathrm{Disc}\, F(z)}{z^{n+1}} + C_\infty$$

operator coefficient

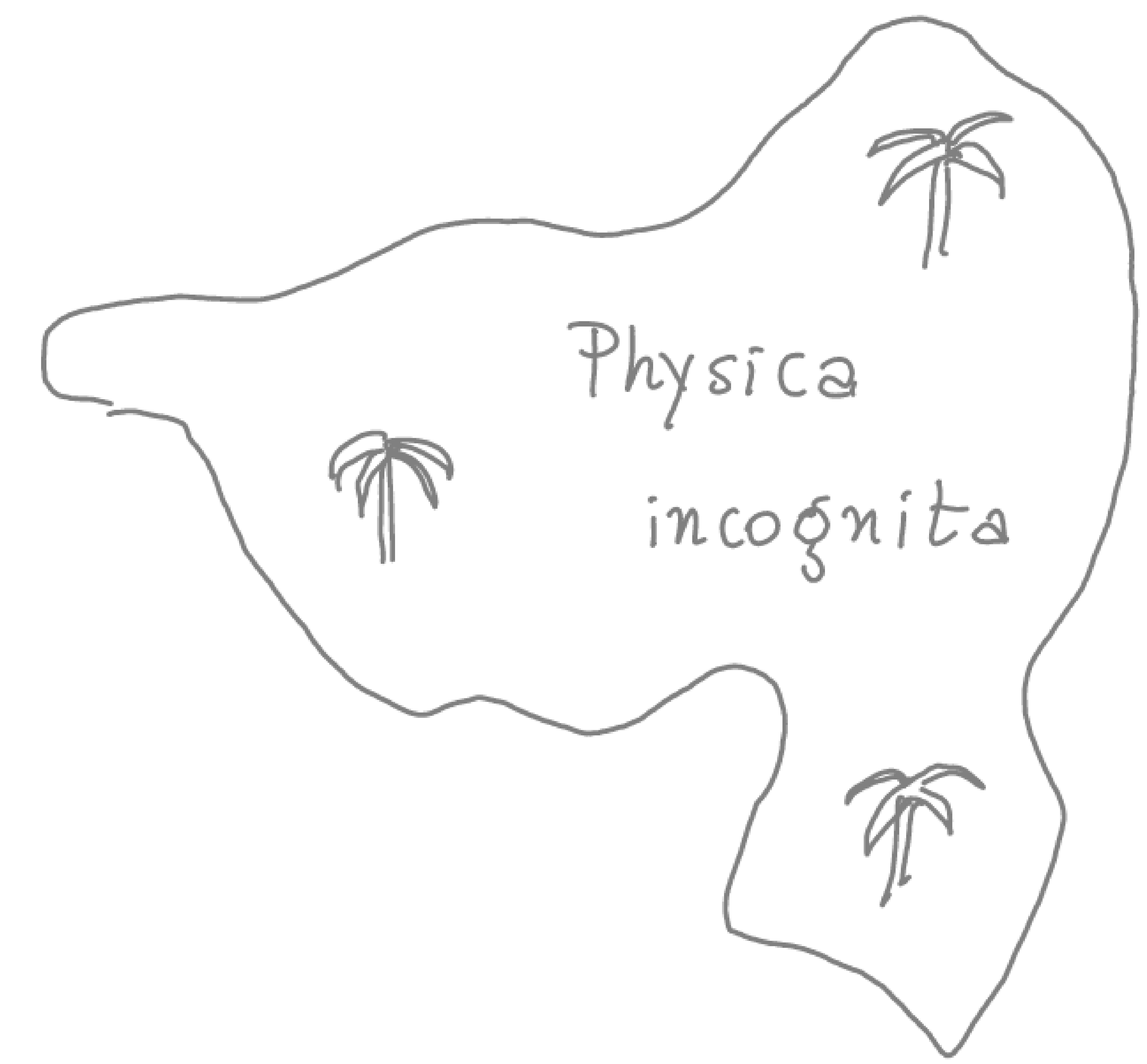
homogeneous polynomial of degree n

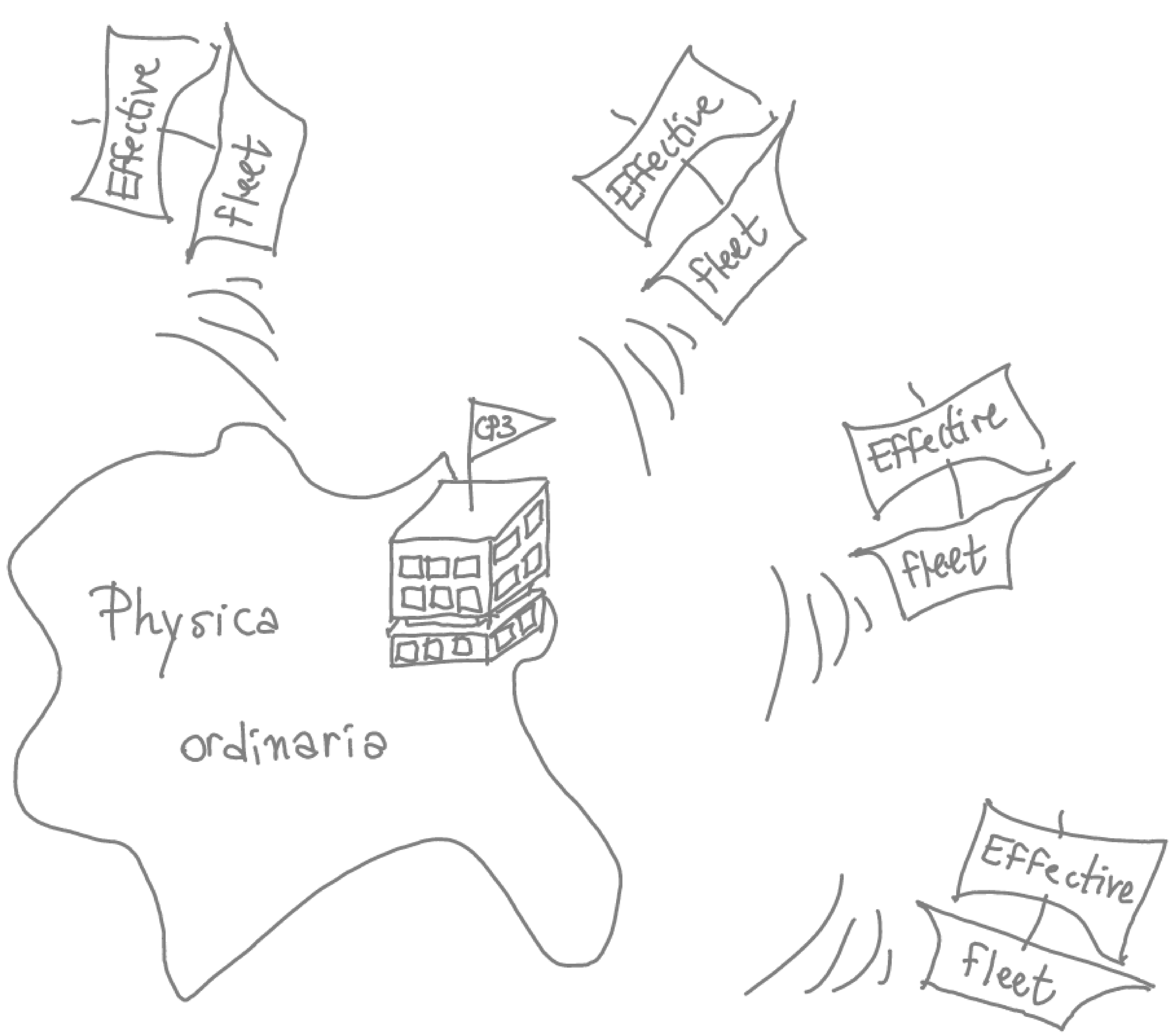
fewer loops & fewer legs

mostly for $d=4$ op.

The effective route to new physics

Effective theories enable systematic searches for heavy physics beyond the Standard Model.





Exploiting amplitude properties to bypass redundancies opens a new effective route to study them.