

Exact Non-Perturbative Results in Gauge Theories in the Near-SUSY Limit

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UCLouvain, May 13, 2025

Main point

- very few methods to study strongly coupled systems, e.g., QCD
- non-perturbative effects are essential
- supersymmetry makes exact analytic studies of non-perturbative effects possible
- small anomaly-mediated SUSY breaking still allows for exact solutions

$$e^{-8\pi^2/g^2\hbar}$$



Main point

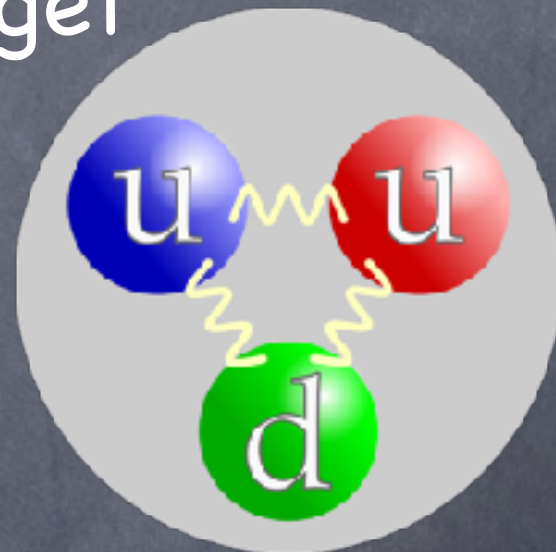
- very few methods to study strongly coupled systems, e.g., QCD
- non-perturbative effects are essential
- supersymmetry makes exact analytic studies of non-perturbative effects possible
- small anomaly-mediated SUSY breaking still allows for exact solutions



QCD: theory of
strong nuclear force

Can we solve QCD?

- When we first learn about quarks, we get told we can never see them
 - Internet Scam?



Dear friend,

I am Andre Ouedraogo, a banker by profession from Burkina Faso in West Africa and currently holding the post of Director Auditing and Accounting unit of the bank. It's my urgent need for a foreign partner that made me to contact you for this business. I have the opportunity of transferring the left over funds (\$11.5 million) of one of my bank clients who died along with his entire family on 31 July 2000 in a plane crash. You can confirm the genuineness of the deceased death by clicking on this website.
<http://news.bbc.co.uk/1/hi/world/europe/859479.stm>

I need a foreign partner who will support me because i can not claim this money alone without a foreign partner since the deceased client (the owner of the fund) was a foreigner.

This fund (\$11.5 million) will be shared between us in the ratio of 60/40. I agreed that 40% of this money will be for you as a respect to the provision of a foreign account while 60% will be for me and I want to assure you that this transaction is absolutely legal and risk free since i work in this bank and i have all the necessary information that might be needed. Before we proceed, i would like to know your ability to handle this over there in your country.

Please tell me more about the political/economic stability/monetary policy of your country. I need to know all these because i don't want to have problem with the Government of your country.

Kindly update me with the following information because i want to know you more before we proceed on this transaction. Hope you will understand the importance of this request.

- 1. Your full name.....
- 2. Your age/sex
- 3. your occupation
- 4. Your residential address
- 5. Your nationality
- 6. Your private phone number
- 7. Your fax number

I will be waiting for your response.

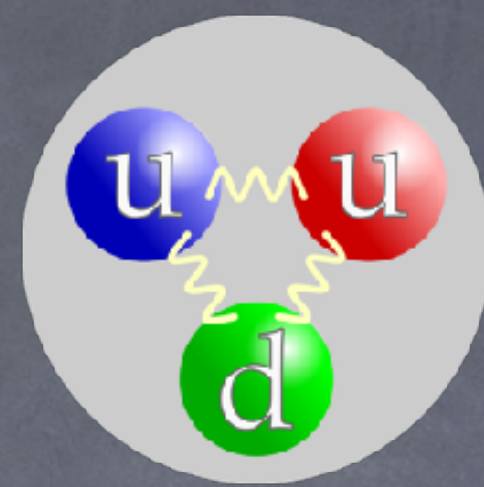
Thanks for your understanding.

Have a great day.

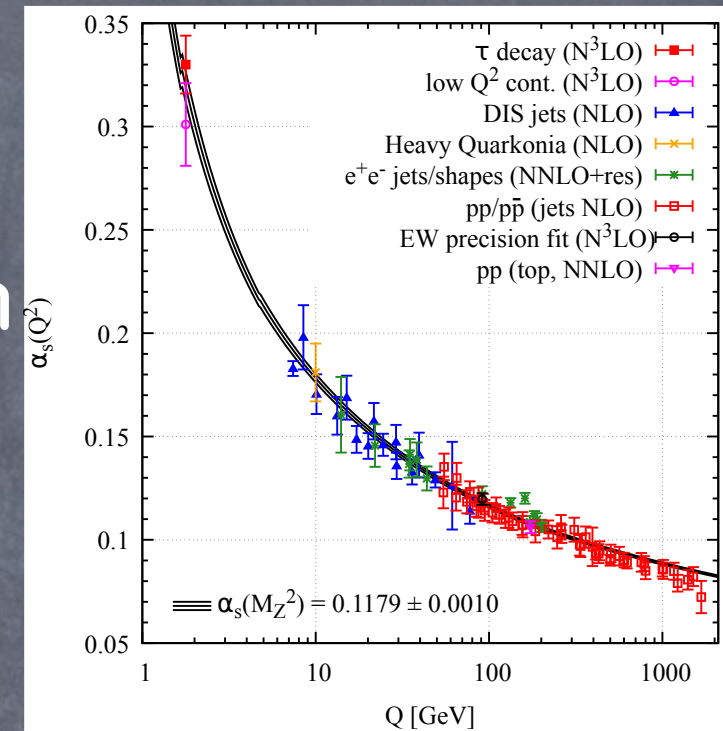
Yours.

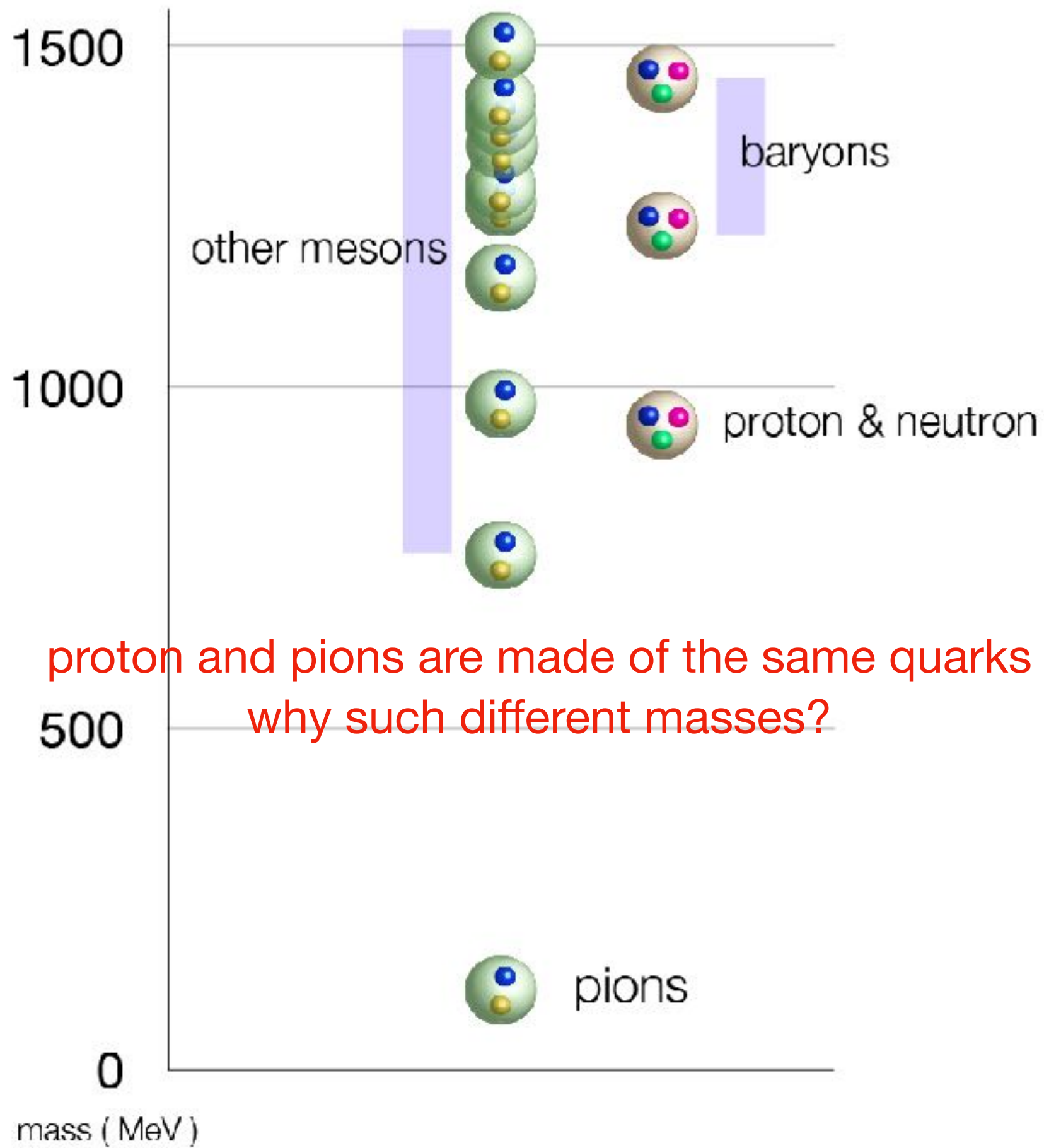
Andre Ouedraogo

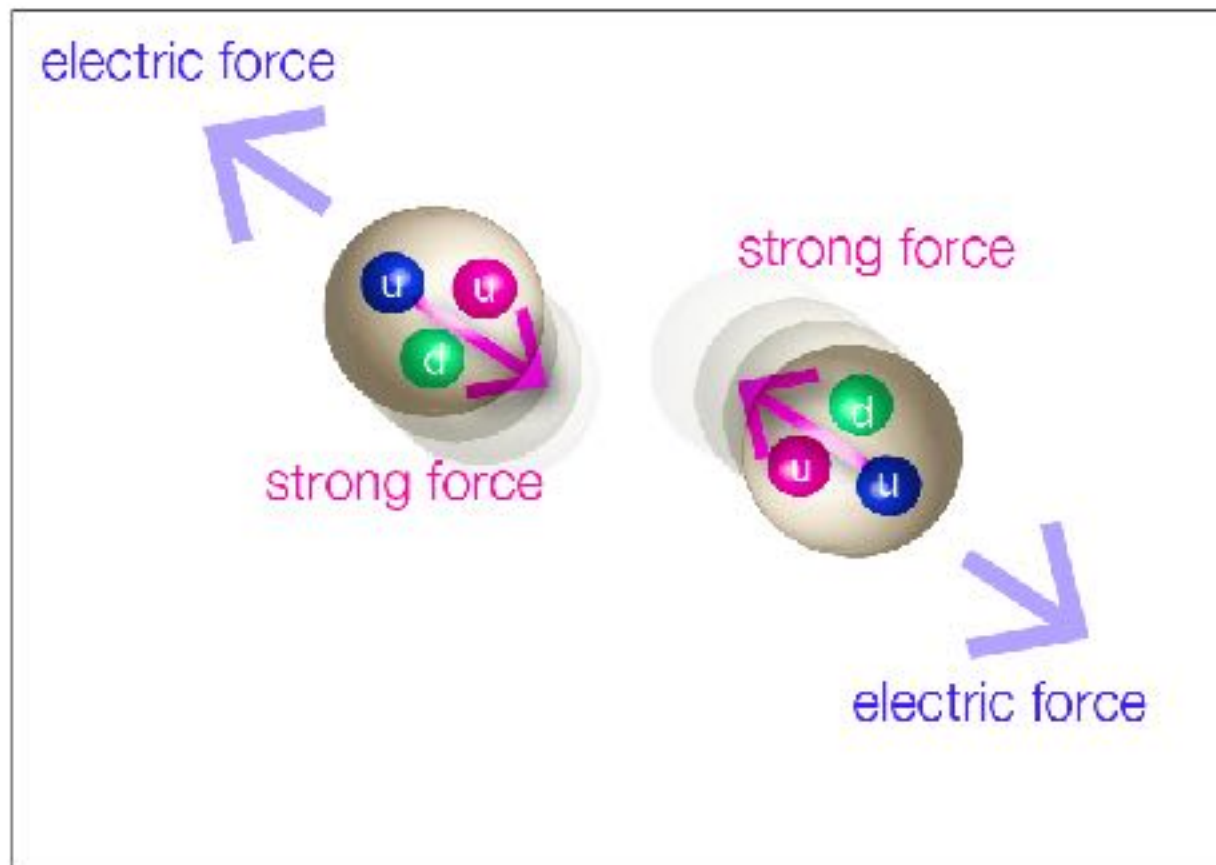
Can we understand QCD?



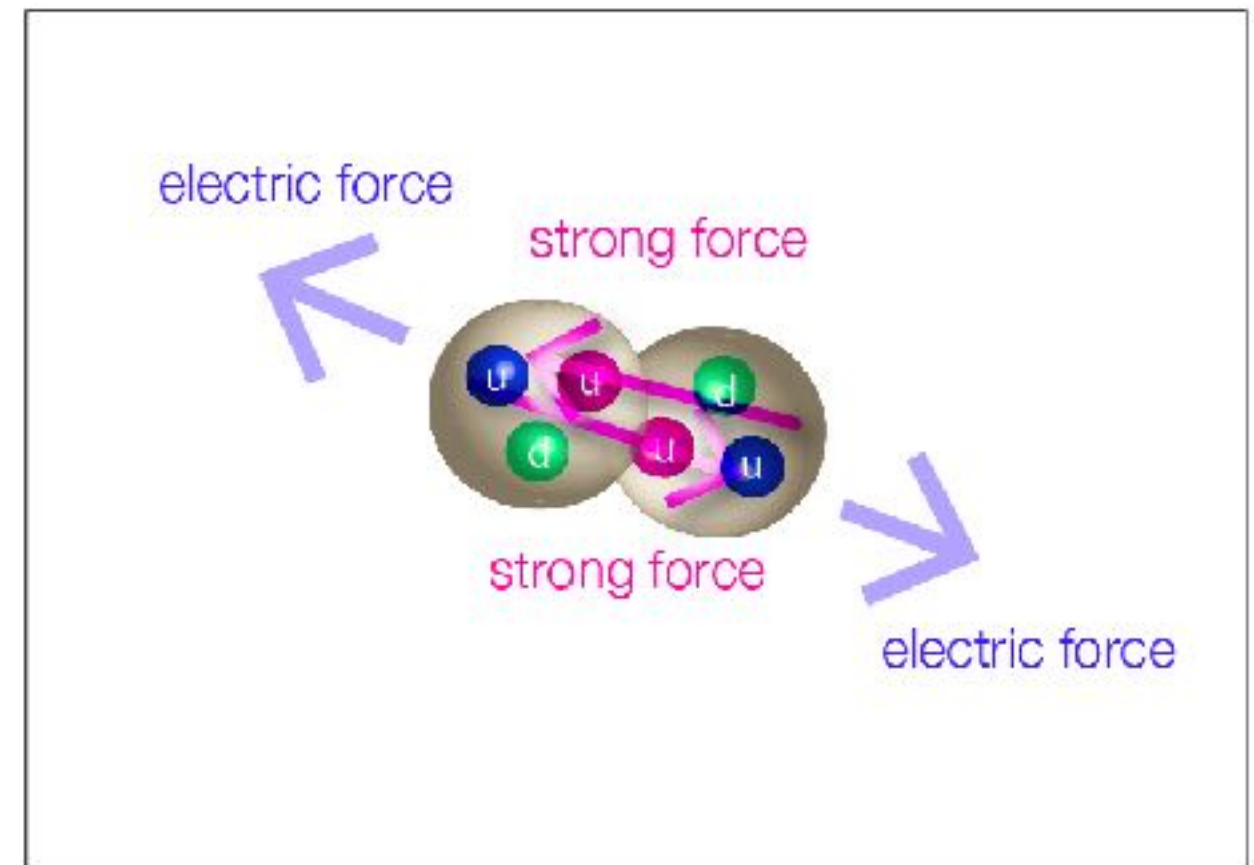
- When we first learn about quarks, we get told we can never see them
 - Internet Scam?
 - Confinement!
 - $\beta < 0$ and asymptotic freedom
 - only suggestive, doesn't prove confinement
- Another puzzle: proton and pion are made of same quarks
 - why pion $\approx$ massless $\ll$ proton?
- very mysterious!







If pions are heavy



With real light-weight pions

elements in the Universe

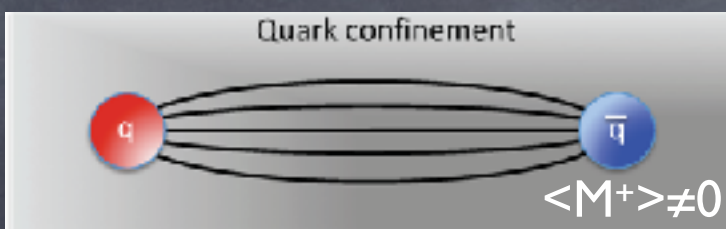
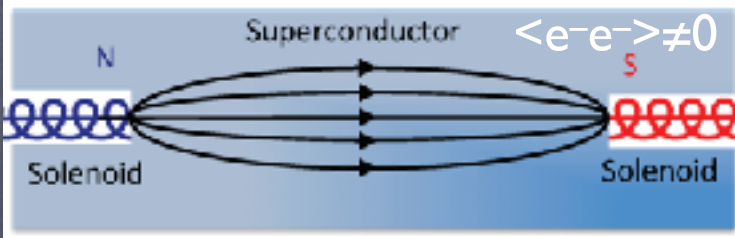
元素の周期表
The Periodic Table

周期\族	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H 水素 Hydrogen 1.00798																	2 He ヘリウム Helium 4.0026
2	3 Li リチウム Lithium 6.968	4 Be ベリリウム Beryllium 9.01218											5 B 硼(ホウ)素 Boron 10.814	6 C 炭素 Carbon 12.0106	7 N 窒素 Nitrogen 14.0069	8 O 酸素 Oxygen 15.9994	9 F 弗(フッ)素 Fluorine 18.9984	10 Ne ネオン Neon 20.1797
3	11 Na ナトリウム Sodium 22.9898	12 Mg マグネシウム Magnesium 24.306											13 Al アルミニウム Aluminum 26.9815	14 Si 珪(ケイ)素 Silicon 28.085	15 P 燐(リン)素 Phosphorus 30.9738	16 S 硫黄 Sulfur 32.068	17 Cl 塩素 Chlorine 35.452	18 Ar アルゴン Argon 39.948
4	19 K カリウム Potassium 39.0983	20 Ca カルシウム Calcium 40.078	21 Sc スカンジウム Scandium 44.9559	22 Ti チタン Titanium 47.867	23 V バナジウム Vanadium 50.9415	24 Cr クロム Chromium 51.9961	25 Mn マンガン Manganese 54.938	26 Fe 鉄 Iron 55.845	27 Co コバルト Cobalt 58.9332	28 Ni ニッケル Nickel 58.6934	29 Cu 銅 Copper 63.546	30 Zn 亜鉛 Zinc 65.38	31 Ga ガリウム Gallium 69.723	32 Ge ゲルマニウム Germanium 72.630	33 As 砒(ヒ)素 Arsenic 74.9216	34 Se セレン Selenium 78.971	35 Br 臭素 Bromine 79.904	36 Kr クリプトン Krypton 83.798
5	37 Rb ルビジウム Rubidium 85.4678	38 Sr ストロンチウム Strontium 87.62	39 Y イットリウム Yttrium 88.9058	40 Zr ジルコニウム Zirconium 91.224	41 Nb ニオブ Niobium 92.9064	42 Mo モリブデン Molybdenum 95.95	43 Tc テクネチウム Technetium [99]	44 Ru ルテチウム Ruthenium 101.07	45 Rh ロジウム Rhodium 102.906	46 Pd パラジウム Palladium 106.42	47 Ag 銀 Silver 107.868	48 Cd カドミウム Cadmium 112.414	49 In インジウム Indium 114.818	50 Sn 錫(スズ) Tin 118.710	51 Sb アンチモン Antimony 121.760	52 Te テルル Tellurium 127.60	53 I 沃(ヨウ)素 Iodine 126.904	54 Xe キセノン Xenon 131.293
6	55 Cs セシウム Cesium 132.905	56 Ba バリウム Barium 137.327	※1	72 Hf ハフニウム Hafnium 178.49	73 Ta タンタル Tantalum 180.948	74 W タングステン Tungsten 183.84	75 Re レーニウム Rhenium 186.207	76 Os オスミウム Osmium 190.23	77 Ir イリジウム Iridium 192.217	78 Pt 白金(プラチナ) Platinum 195.084	79 Au 金 Gold 196.967	80 Hg 水銀 Mercury 200.592	81 Tl タリウム Thallium 204.384	82 Pb 鉛 Lead 207.2	83 Bi ビスマス Bismuth 208.980	84 Po ポロニウム Polonium [210]	85 At アスタチン Astatine [210]	86 Rn ラドン Radon [222]
7	87 Fr フランシウム Francium [223]	88 Ra ラジウム Radium [226]	※2	104 Rf ラザホージウム Rutherfordium [267]	105 Db ドブニウム Dubnium [268]	106 Sg シーボーギウム Seaborgium [271]	107 Bh ボーリウム Bohrium [272]	108 Hs ハッシウム Hassium [277]	109 Mt マイタネリウム Meitnerium [276]	110 Ds ダームスタテウム Darmstadtium [281]	111 Rg レントジニウム Roentgenium [280]	112 Cn コペルニシウム Copernicium [285]	113 Nh ニホニウム Nihonium [278]	114 Fl フレロビウム Flerovium [289]	115 Mc モスコビウム Moscovium [289]	116 Lv リバモリウム Livermorium [293]	117 Ts テネシン Tennessine [293]	118 Og オガネソン Oganesson [294]
※1 ランタノイド系	57 La ランタン Lanthanum 138.905	58 Ce セリウム Cerium 140.116	59 Pr プラセオジウム Praseodymium 140.908	60 Nd ネオジム Neodymium 144.242	61 Pm プロメチウム Promethium [145]	62 Sm サマリウム Samarium 150.36	63 Eu ユウロピウム Europium 151.964	64 Gd ガドリニウム Gadolinium 157.25	65 Tb テルビウム Terbium 158.925	66 Dy ジスプロシウム Dysprosium 162.500	67 Ho ホルミウム Holmium 164.930	68 Er エルビウム Erbium 167.259	69 Tm ツリウム Thulium 168.934	70 Yb イットリビウム Ytterbium 173.045	71 Lu ルアテウム Lutetium 174.967			
※2 アクチノイド系	89 Ac アクチニウム Actinium [227]	90 Th トリウム Thorium 232.038	91 Pa プロト アクチニウム Protactinium 231.036	92 U ウラン Uranium 238.029	93 Np ネプツニウム Neptunium [237]	94 Pu プルトニウム Plutonium [239]	95 Am アメリシウム Americium [243]	96 Cm キュリウム Curium [247]	97 Bk バーケリウム Berkelium [247]	98 Cf カリホルニウム Californium [252]	99 Es アインスタイニウム Einsteinium [252]	100 Fm フェルミウム Fermium [257]	101 Md メンデレービウム Mendelevium [258]	102 No ノーベリウム Nobelium [259]	103 Lr ローレンシウム Lawrencium [262]			

elements in the Universe

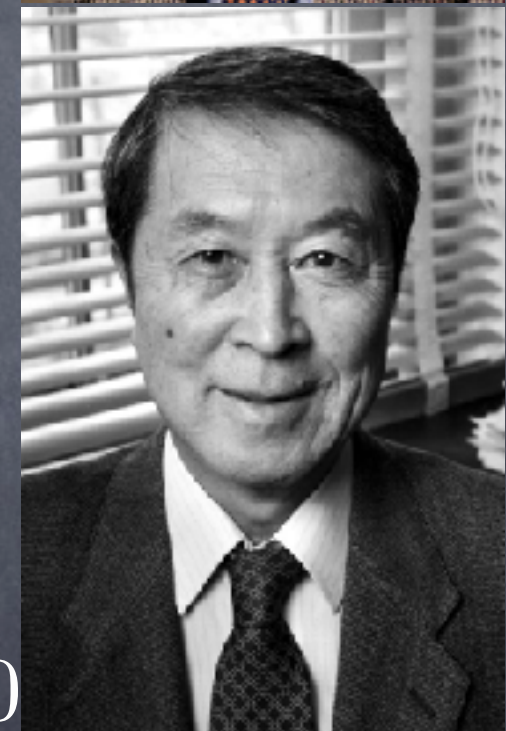
1	H
水素	
Hydrogen	
1.00798	

Unless pions are light, we wouldn't exist!



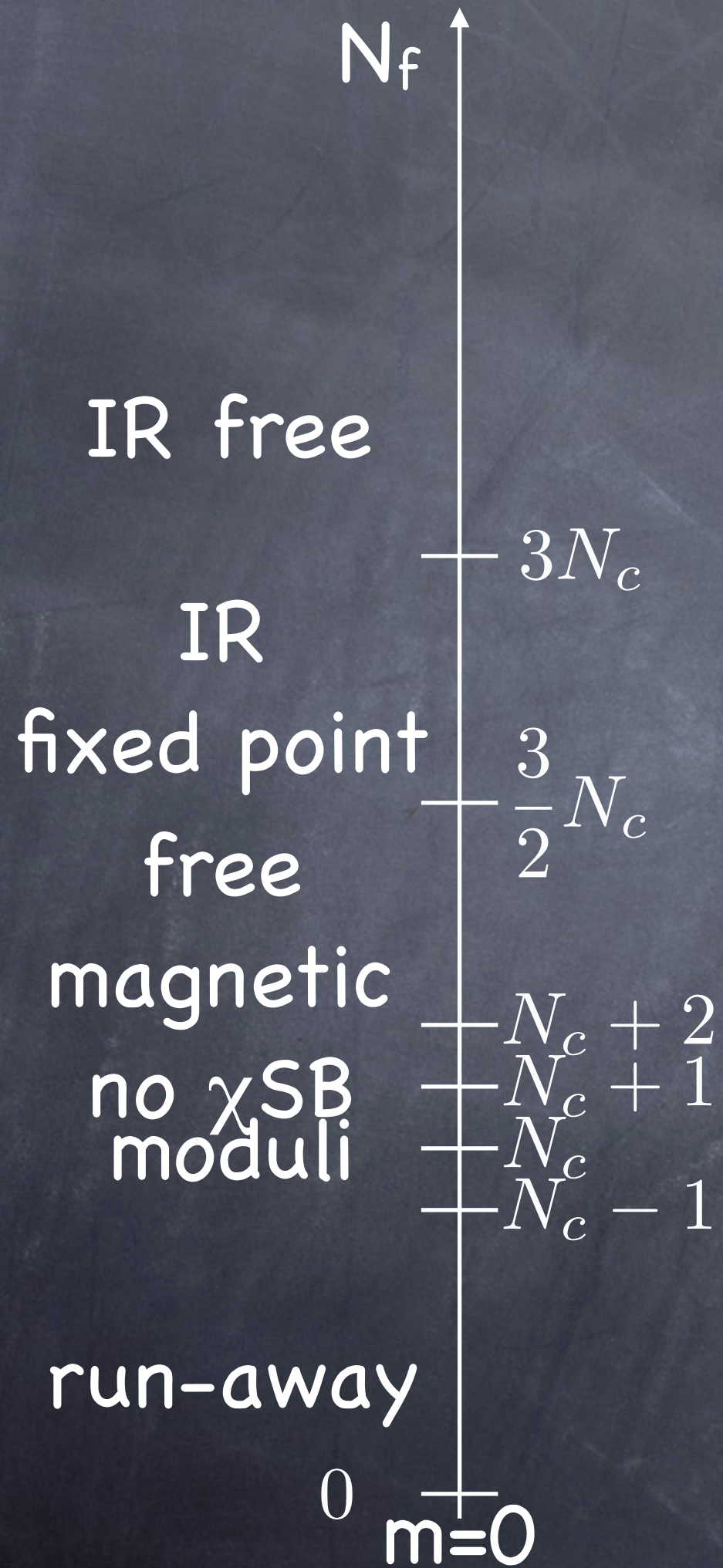
Feeling better

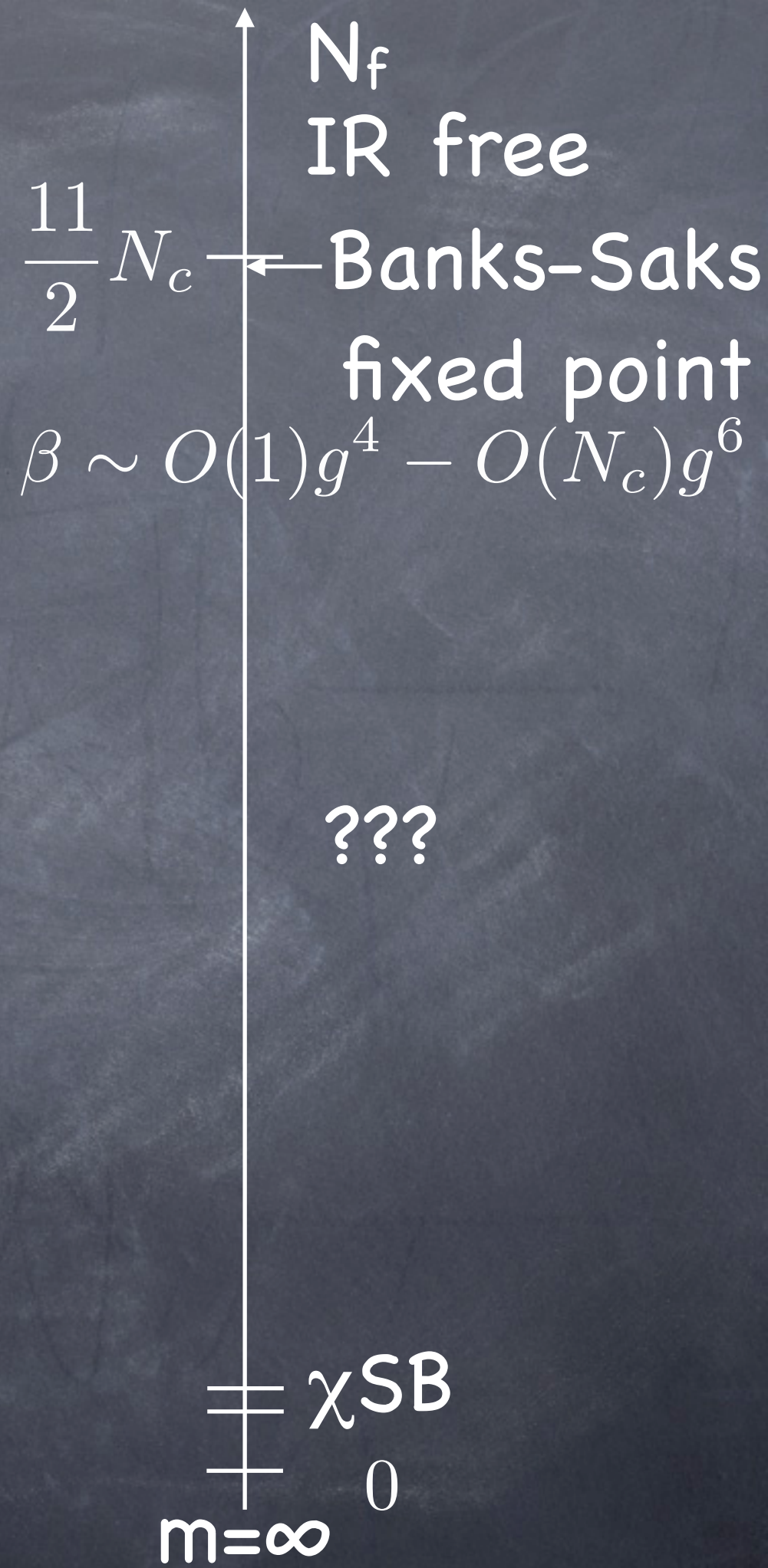
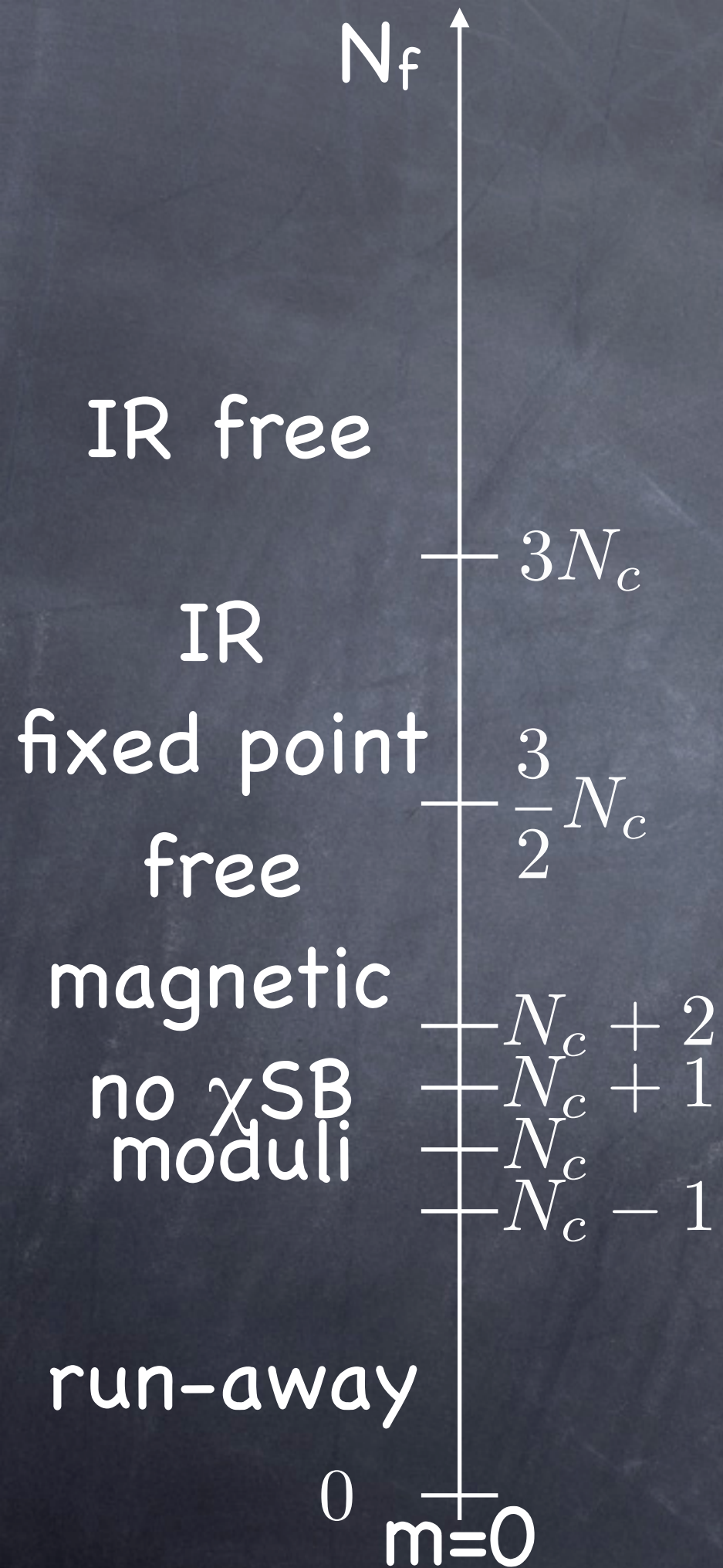
- Qualitative picture makes us feel better
- **Confinement**
 - dual Meißner effect (Mandelstam)
 - **assume** monopole condensation
 - quarks confined by electric flux tube
- **Chiral symmetry breaking** (Nambu)
 - massless QCD invariant under $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$
 - **assume** broken to $SU(N_f)_V \times U(1)_B$ $\langle \bar{q}q \rangle \neq 0$
 - pion = Nambu-Goldstone boson = massless
- but still **not derived from QCD!**

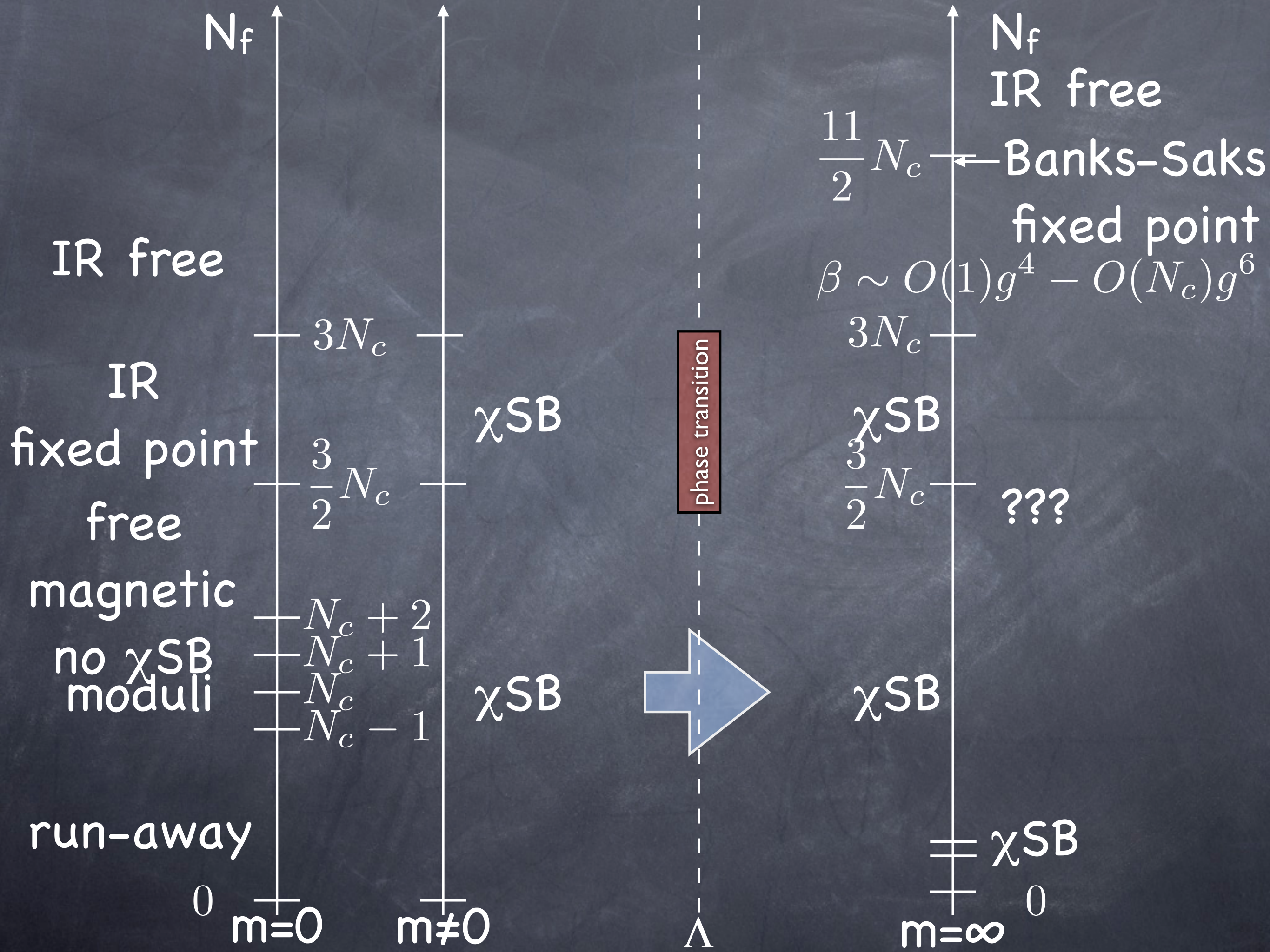


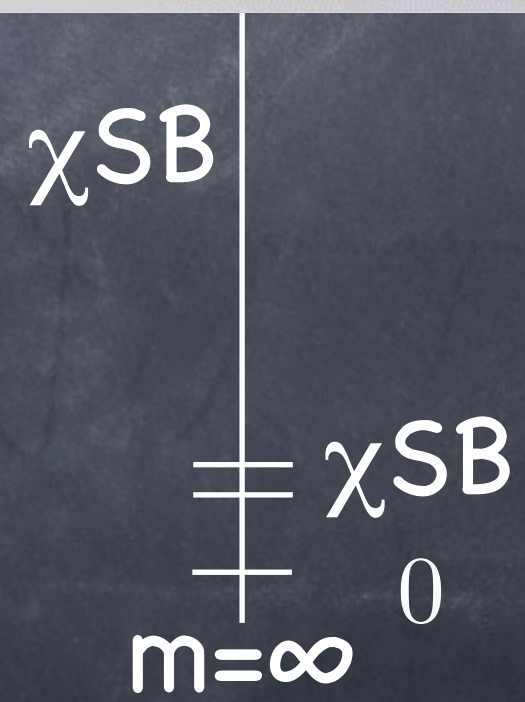
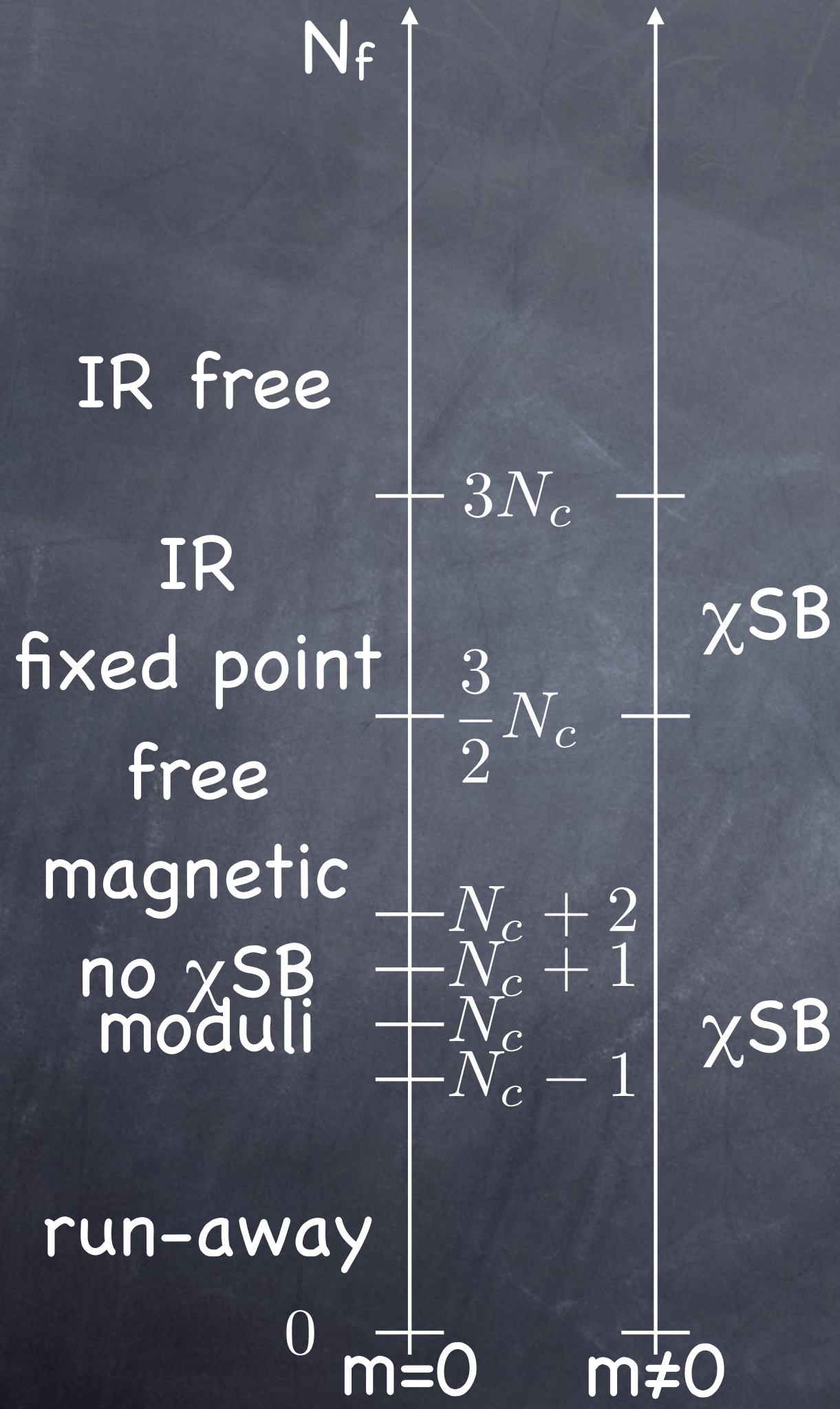
Feeling even better but not there yet

- Progress in understanding QCD
- Confinement** (Seiberg-Witten)
 - $N=2$ $SU(2)$ SYM a triplet field = vacua
 - $SU(2) \rightarrow U(1)$: magnetic monopoles!
 - special points = massless monopole/dyon
 - $N=1$ perturbation: monopole condensation!
- Chiral symmetry breaking**
 - $N=2$ doesn't have the chiral symmetry
 - $N=1$ (Seiberg) has too unusual phases

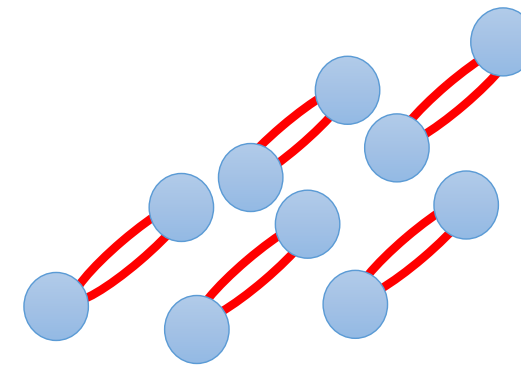
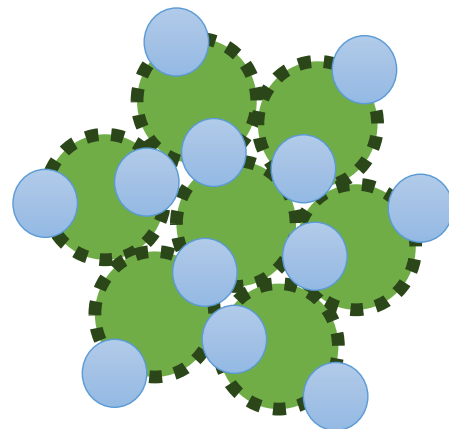
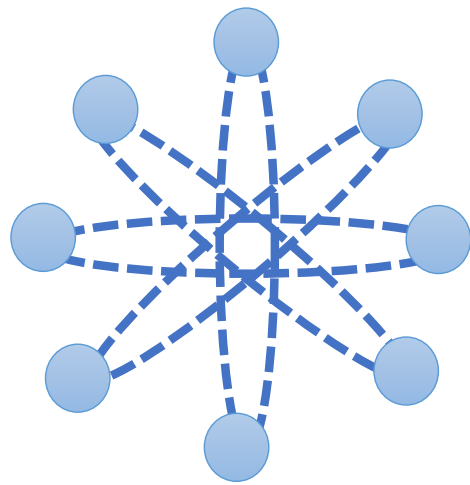








BCS-BEC cross over



Attractive interaction

g

SQCD&AMSB (ASQCD)

SQCD+deformation

QCD?

Soft mass

m

References

- Important related works:
 - Konishi, 1996
 - Evans, Hsu, Schwetz, 1996
 - Cheng, Shadmi, 1998
 - Martin, Wells, 1998
 - Luty, Rattazzi, 1999
 - Abel, Buican, Komargodski, 2011
 - Córdova, Dumitrescu, 2018 ...
- What we do is consistent with previous works
 - Focus on anomaly-mediated SUSY breaking
 - Pushed the method further
- Exact solutions to Chiral gauge theories

Global SUSY

SUSY algebra

- Poincaré: Momentum P_μ and Lorentz generators $M_{\mu\nu}$. Add spinorial charges Q_α

- Specialize to $P_\mu = E(1, 0, 0, 1)$

- **chiral multiplet:**

$$b|h = -\frac{1}{2}\rangle = 0, \quad |h = 0\rangle = b^\dagger|h = -\frac{1}{2}\rangle$$

i.e., **a Weyl fermion and a complex scalar**

- **vector multiplet:**

$$b|h = \frac{1}{2}\rangle = 0, \quad |h = 1\rangle = b^\dagger|h = \frac{1}{2}\rangle$$

i.e., **a Weyl fermion and a gauge field**

$$[P_\mu, Q_\alpha] = 0$$

$$[M^{\mu\nu}, Q_\alpha] = \frac{1}{2}(\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta$$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2P_\mu(\sigma^\mu)_{\alpha\dot{\beta}}$$

$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

$$P^\mu = E(1, 0, 0, 1)$$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2E \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

$$b^i = \frac{1}{\sqrt{4E}} Q_1, b^{i\dagger} = \frac{1}{\sqrt{4E}} \bar{Q}_i$$

$$\{b, b^\dagger\} = 1$$

$$\{h, b\} = -\frac{1}{2}b$$

$$\{h, b^\dagger\} = \frac{1}{2}b$$

SUSY algebra

- Poincaré: Momentum P_μ and Lorentz generators $M_{\mu\nu}$. Add spinorial charges Q_α
- Specialize to $P_\mu = E(1, 0, 0, 1)$



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$$\{b, b^\dagger\} = 1$$

$$\{h, b\} = -\frac{1}{2}b$$

$$\{h, b^\dagger\} = \frac{1}{2}b$$

N=1 superspace chiral superfield

- introduce Grassman-odd coordinates θ_α
- superspace: $(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$

$$P_\mu = i\partial_\mu$$

$$M^{\mu\nu} = i(x^\mu\partial^\nu - x^\nu\partial^\mu)$$

$$Q_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\bar{\theta}^{\dot{\beta}}\sigma^\mu_{\dot{\beta}\alpha}\partial_\mu$$
- Covariant derivative

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\bar{\theta}^{\dot{\beta}}\sigma^\mu_{\dot{\beta}\alpha}\partial_\mu$$

$$\{D_\alpha, Q_\beta\} = \{D_\alpha, \bar{Q}_{\dot{\beta}}\} = 0$$

$$\{D_\alpha, \bar{D}_{\dot{\beta}}\} = -2i\partial_\mu(\sigma^\mu)_{\alpha\dot{\beta}}$$
- Can place a constraint $\bar{D}_{\dot{\alpha}}\phi(x, \theta, \bar{\theta}) = 0$
- solution: $\phi(x, \theta, \bar{\theta}) = \phi(y, \theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$
- where $y^\mu = x^\mu - i\theta\sigma^\mu\bar{\theta}$, $\bar{D}_{\dot{\alpha}}y^\mu = 0$

vector superfield

- introduce Grassman-odd coordinates θ_α

$$(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}})$$

- real (vector) superfield:

$$V(x, \theta, \bar{\theta}) = C + \theta\chi + \bar{\theta}\bar{\chi} + \theta^2 M + \bar{\theta}^2 M^* + \theta\sigma^\mu\bar{\theta}A_\mu + \theta^2\bar{\theta}\bar{\lambda} + \bar{\theta}^2\theta\lambda + \theta^2\bar{\theta}^2 D$$

- gauge transformation:
- Λ is a chiral superfield
- Can eliminate C, χ, M :
Wess-Zumino gauge
- remaining dof: A_μ, λ, D
- Field strength chiral superfield:

$$V = V^a T^a, \quad \Lambda = \Lambda^a T^a$$

$$e^V \rightarrow e^{-i\bar{\Lambda}} e^V e^{i\Lambda}$$

$$\phi \rightarrow e^{-i\Lambda} \phi$$

$$\phi^\dagger e^V \phi : \text{invariant}$$

$$W_\alpha = \bar{D}^2 D_\alpha V = \lambda_\alpha(y) + \theta^\beta \sigma_{\beta\alpha}^{\mu\nu} F_{\mu\nu} + \theta_\alpha D$$

Kähler and superpotentials

- Two ways to construct invariants (up to total derivatives)

- full superspace integral of general superfield
"Kähler potential"

$$\int d^4\theta \phi^\dagger e^V \phi = |D_\mu A|^2 + \bar{\psi} i \not{D} \psi + F^\dagger F + D^a A^\dagger T^a A + (\sqrt{2} A^\dagger \lambda^a T^a \psi + h.c.)$$

- chiral superspace integral of chiral superfield
"superpotential": holomorphic

$$\int d^2\theta W(\phi) = \frac{\partial W}{\partial \phi^i} F^i + \frac{1}{2} \frac{\partial^2 W}{\partial \phi^i \partial \phi^j} \psi^i \psi^j$$

- gauge kinetic term

$$\int d^2\theta \frac{1}{g^2} W_\alpha^a W^{a\alpha} = -\frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu a} + \frac{1}{g^2} \bar{\lambda}^a i \not{D} \lambda^a + \frac{1}{2g^2} D^a D^a$$

- solve auxiliary fields $D^a = g^2 A^\dagger T^a A, \quad F^{i*} = \frac{\partial W}{\partial \phi^i}$

$$V = \frac{1}{2g^2} D^a D^a + |F^i|^2 = \frac{g^2}{2} (A^\dagger T^a A)^2 + \left| \frac{\partial W}{\partial \phi^i} \right|^2$$

Renormalization

- Start with the Wilsonian action at scale μ

$$\int d^4\theta \sum_i \phi_i^* e^V \phi_i + \int d^2\theta \left(\frac{1}{g_0^2} W_\alpha W^\alpha + \lambda_0^{ijk} \phi_i \phi_j \phi_k \right)$$

- non-renormalization theorem: only renormalization is Z , $b_0 = 3C_A - \sum_i T_F^i$
- holomorphy, and transitivity at scale $\mu' = \mu e^{-t}$

$$\int d^4\theta \sum_i Z_i \phi_i^* e^V \phi_i + \int d^2\theta \left(\left(\frac{1}{g_0^2} - \frac{b_0}{8\pi^2} t \right) W_\alpha W^\alpha + \lambda_0^{ijk} \phi_i \phi_j \phi_k \right)$$

- to identify coupling constants, need to rescale fields to canonical normalization
- However, rescaling fields yield anomalous Jacobians

Renormalization

Nima Arkani-Hamed and HM, arXiv:hep-th/9707133

$$\int d^4\theta \sum_i Z_i \phi_i^* e^V \phi_i + \int d^2\theta \left(\left(\frac{1}{g_0^2} - \frac{b_0}{8\pi^2} t \right) W_\alpha W^\alpha + \lambda_0^{ijk} \phi_i \phi_j \phi_k \right)$$

• Konishi anomaly $\int \mathcal{D}\phi_i = \int \mathcal{D}(e^\sigma \phi_i) e^{-\int d^2\theta T_F^i \frac{1}{8\pi^2} 2\sigma W_\alpha W^\alpha}$

• rescaling anomaly $\int \mathcal{D}V = \int \mathcal{D}(e^\sigma V) e^{+\int d^2\theta C_A \frac{1}{8\pi^2} 2\sigma W_\alpha W^\alpha}$

• first rescale matter fields

$$\int d^4\theta \sum_i \phi_i^* e^V \phi_i + \int d^2\theta \left(\left(\frac{1}{g_0^2} - \frac{b_0}{8\pi^2} t - \sum_i T_F^i \frac{1}{8\pi^2} \ln Z_i \right) W_\alpha W^\alpha + Z_i^{-1/2} Z_j^{-1/2} Z_k^{-1/2} \lambda_0^{ijk} \phi_i \phi_j \phi_k \right)$$

• then rescale the gauge field $V \rightarrow g_c V$

$$\frac{1}{g_c^2} = \frac{1}{g_0^2} - \frac{b_0}{8\pi^2} t - \sum_i T_F^i \frac{1}{8\pi^2} \ln Z_i - C_A \frac{1}{8\pi^2} \ln g_c^2$$

$$\int d^2\theta \frac{1}{g_c^2} (\bar{D}^2 e^{-g_c V} D_\alpha e^{g_c V})^2 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \dots$$

Anomaly-Mediated Supersymmetry Breaking

Randall, Sundrum (1998)

Giudice, Luty, HM, Rattazzi (1998)

Our Needs

- We'd like to connect N=1 SUSY results by Seiberg to non-SUSY gauge theories
 - decouple gauginos and squarks!
 - SUSY breaking m_λ and $m_{\tilde{Q}}$
- But we need to deal with composites such as mesons and baryons
- In particular, we need to know signs of their mass-squared to understand symmetry breaking patterns and universality classes
- possible with AMSB or $U(1)_R$ D-term
 - but the latter does not have gauging mass

background spacetime

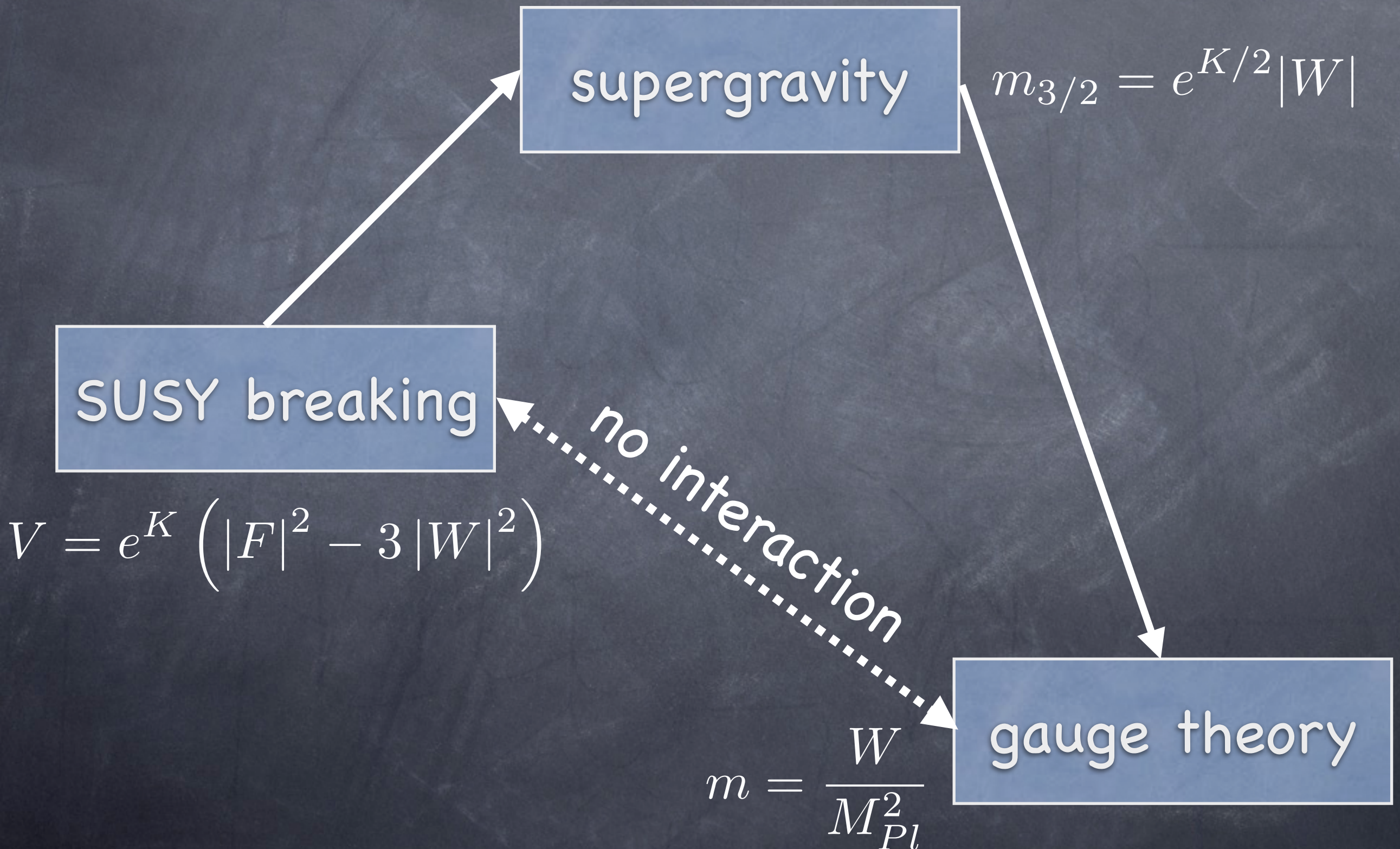
- Good example of external symmetry is general coordinate invariance
- consider QCD in a background metric

$$\mathcal{L}_{\text{QCD}} = \sqrt{-g} \left(-\frac{1}{2} g^{\mu\rho} g^{\nu\sigma} \text{Tr} G_{\mu\nu} G_{\rho\sigma} + \bar{Q} e_a^\mu \gamma^a i D_\mu Q \right)$$

- switch from UV to IR description
- we know how chiral Lagrangian couples to the background spacetime!

$$\mathcal{L}_\chi = \sqrt{-g} \left(\frac{f_\pi^2}{4} g^{\mu\nu} \text{Tr} \partial_\mu U^\dagger \partial_\nu U + f_\pi \mu^2 \text{Tr} M^\dagger U + c.c. \right)$$

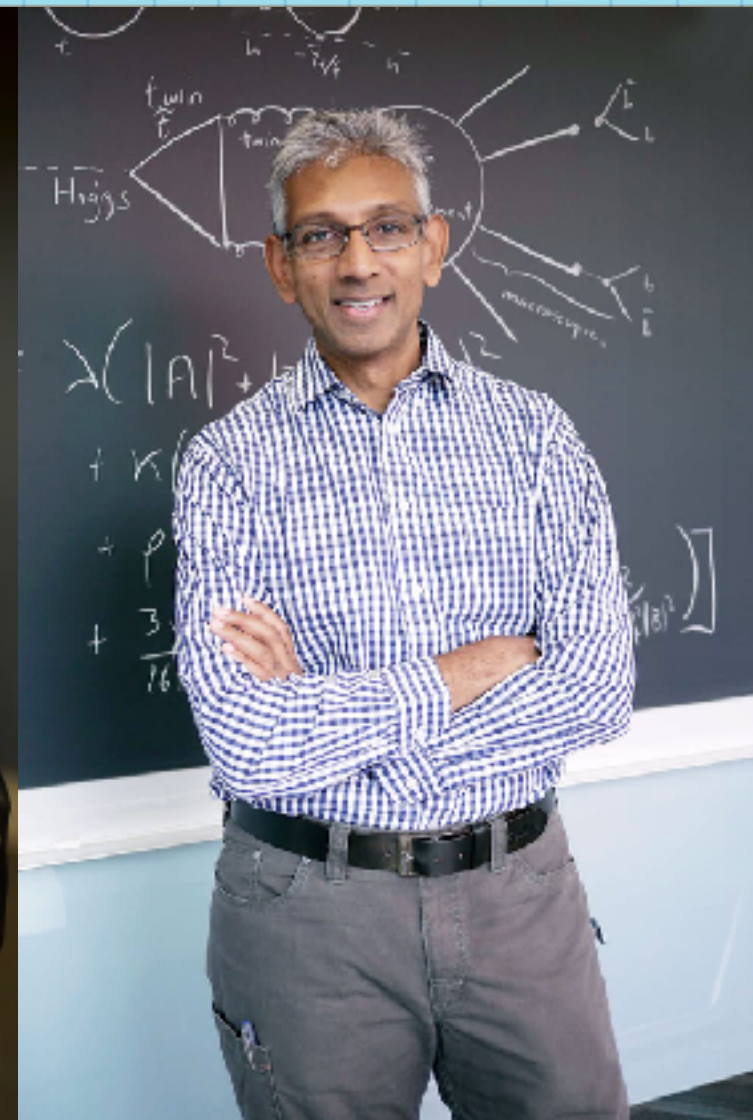
Sequestering



Saved by time diff

- worked in summer 1998
at CERN
- left CERN with half-
finished draft
- Giudice, Rattazzi@CERN
- Luty@Maryland

1630 citations



Weyl compensator

- “superspacetime background”

$$\int d^4\theta \mathcal{E}^* \mathcal{E} K - \int d^2\theta \mathcal{E}^3 W$$
$$\mathcal{E} = 1 + \theta^2 m$$

- all effects of SUSY breaking encoded in $\mathcal{E}$
- it can be removed by conformal transformation $\phi \rightarrow \phi/\mathcal{E}$ if no mass params

$$\mathcal{L} = \int d^4\theta \mathcal{E}^* \mathcal{E} \phi^* \phi - \int d^2\theta \mathcal{E}^3 \lambda \phi^3$$
$$\rightarrow \mathcal{L} = \int d^4\theta \phi^* \phi - \int d^2\theta \lambda \phi^3$$

Weyl compensator

- “superspacetime background”

$$\int d^4\theta \mathcal{E}^* \mathcal{E} K - \int d^2\theta \mathcal{E}^3 W$$
$$\mathcal{E} = 1 + \theta^2 m \qquad \phi \rightarrow \phi/\mathcal{E}$$

- dimensionful parameters receive SUSY breaking

$$\mathcal{L} = \int d^4\theta \mathcal{E}^* \mathcal{E} \phi^* \phi - \int d^2\theta \mathcal{E}^3 \left(\frac{1}{2} M \phi^2 + \lambda \phi^3 \right)$$

$$\rightarrow \mathcal{L} = \int d^4\theta \phi^* \phi - \int d^2\theta \left(\frac{1}{2} M \mathcal{E} \phi^2 + \lambda \phi^3 \right)$$

$$V_{\text{AMSB}} = -m \left(\phi \frac{\partial W}{\partial \phi} - 3W \right)$$

Superconformal Anomaly

- Cutoff scale also acquires SUSY breaking

$$\frac{1}{g^2} \left(\frac{\mu}{M} \right) \rightarrow \frac{1}{g^2} \left(\frac{\mu}{M\mathcal{E}} \right) = \frac{1}{g^2} - \theta^2 \frac{\beta(g^2)}{g^4} m$$

$$\int d^2\theta \frac{1}{g^2} \left(\frac{\mu}{M\mathcal{E}} \right) W_\alpha W^\alpha \supset -\frac{\beta(g^2)}{4g^4} m \lambda\lambda$$

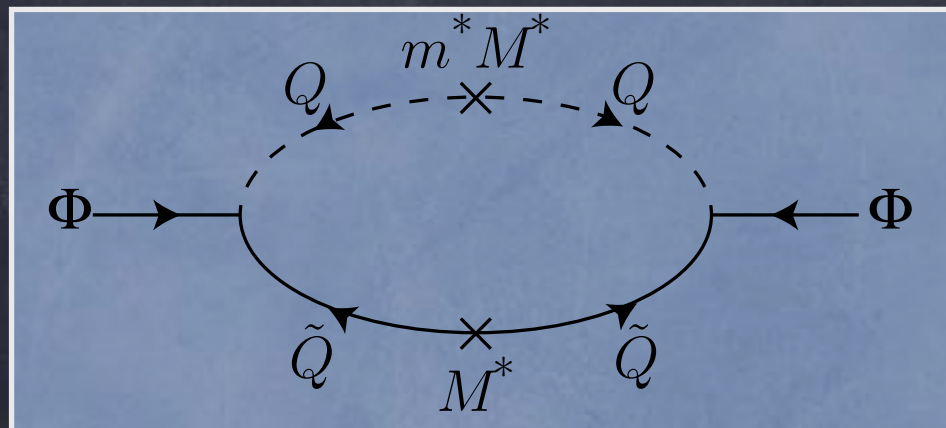
$$m_\lambda = -\frac{\beta(g^2)}{2g^2} m$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

UV insensitivity

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2}m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4}m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)m_{3/2}$$

- Surprising result: **AMSB depends only on physics at the energy scale of interest**
- No matter how complicated the UV physics is, they all disappear from low-energy soft SUSY breaking
- e.g., decouple a massive matter field:
 - Changes the beta function
 - one-loop threshold correction precisely account for the change in gaugino mass



Superconformal Anomaly

- Cutoff scale also acquires SUSY breaking

$$Z\left(\frac{\mu}{M}\right) \rightarrow \mathcal{Z}\left(\frac{\mu}{M\mathcal{E}}\right) = Z\left(1 + \gamma\frac{1}{2}\ln\frac{\mu^2}{M^*\mathcal{E}^*M\mathcal{E}} + \frac{1}{2}\dot{\gamma}\frac{1}{4}\ln^2\frac{\mu^2}{M^*\mathcal{E}^*M\mathcal{E}} + \dots\right)$$

$$\int d^4\theta \mathcal{Z} \phi^* \phi = Z \left(F^* F + \gamma \frac{1}{2} (m^* \phi^* F + m \phi F^*) + \frac{1}{2} \dot{\gamma} m^* m \phi^* \phi \right)$$

$$V = -\frac{1}{4} \dot{\gamma}_i m^* m \phi_i^* \phi_i - \frac{1}{2} (\gamma_i + \gamma_j + \gamma_k) m \lambda_{ijk} \phi_i \phi_j \phi_k + c.c.$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

UV insensitivity cont.

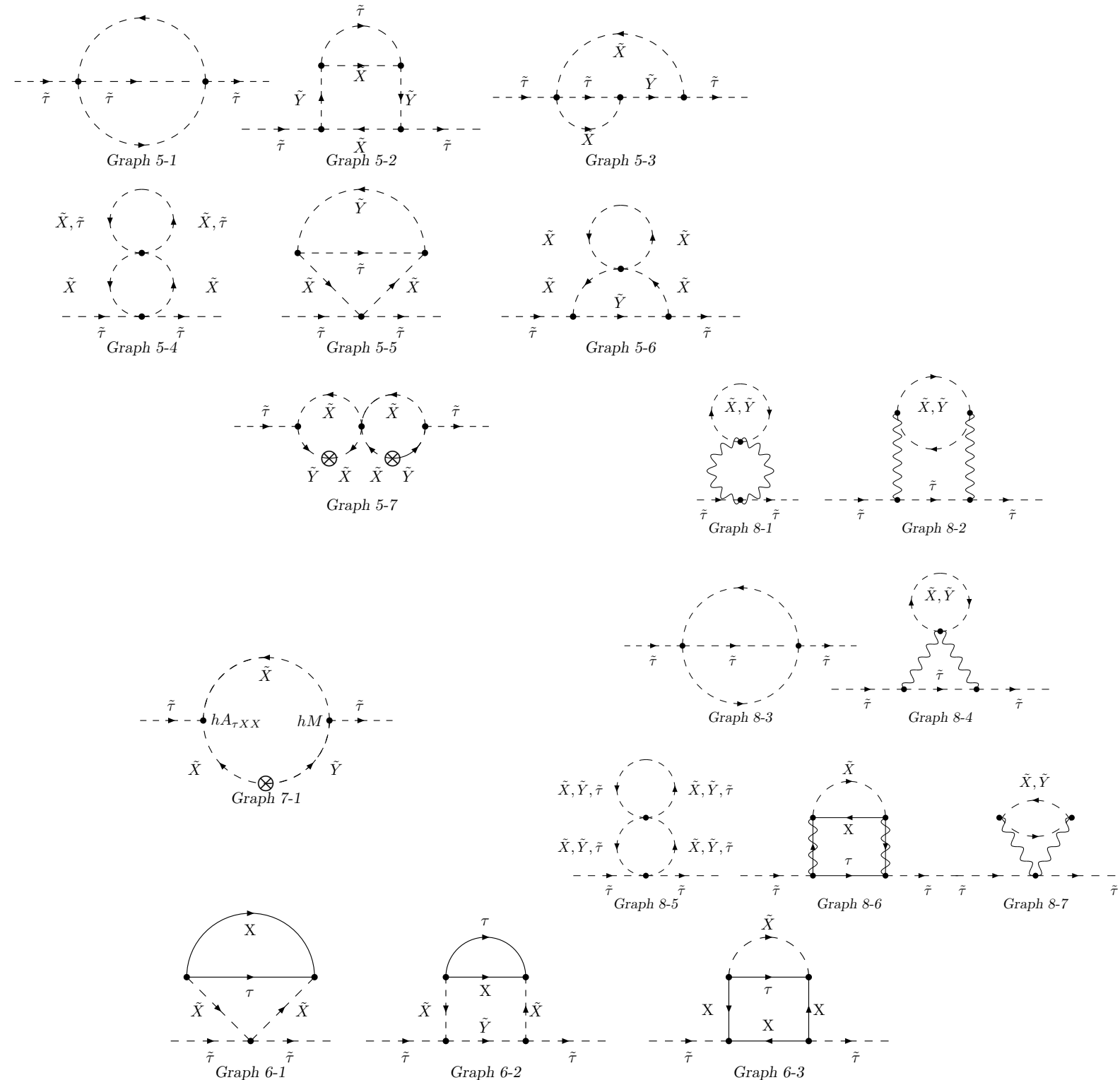
- decouple a massive matter field
- two-loop threshold correction precisely account for the change in the anomalous dimension and hence the scalar mass

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}$$

$$m_i^2 = -\frac{\dot{\gamma}_i}{4} m_{3/2}^2,$$

$$A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

Boyda, HM, Pierce 2001



Anomaly Mediation of SUSY Breaking (AMSB)

- Tree-level piece on dimensionful parameters

$$V_{\text{AMSB}} = -m \left(\phi \frac{\partial W}{\partial \phi} - 3W \right)$$

- loop-level piece from running

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4} m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

It is sometimes convenient to introduce a complex gravitino mass, defined as

$$\tilde{m}_g \equiv \frac{2\kappa}{3} \left(\langle s \rangle + i \langle p \rangle \right), \quad (31.3.20)$$

whose absolute magnitude is the physical gravitino mass (31.3.19).

31.4 Anomaly-Mediated Supersymmetry Breaking

In Section 28.3 the possibility was raised that supersymmetry may be broken in some sort of hidden sector of superfields that do not carry the $SU(3) \times SU(2) \times U(1)$ quantum numbers of the standard model, and communicated to observable particles gravitationally. In this section we will deal with one class of supersymmetry-breaking effects in the minimum supersymmetric standard model, those of first order in $\kappa \equiv \sqrt{8\pi G}$. This includes the gaugino masses and the parameters A_{ij} and B in the Lagrangian density (28.4.1). Other supersymmetry-breaking effects such as squark and slepton squared masses are of second order in κ , and will be taken up in Section 31.7, when we consider gravity-mediated supersymmetry breaking using the general supergravity formalism described in Section 31.6.

We can find the effects of gravity-mediated supersymmetry breaking to first order in κ by simply replacing the component fields of the gravitational supermultiplet in the interaction (31.1.34) with their expectation values. The only ones of these component fields that can acquire non-vanishing vacuum expectation values from the spontaneous breakdown



I must acknowledge my special intellectual debt to colleagues at the University of Texas, notably Luis Boya, Phil Candelas, Bryce and Cecile De Witt, Willy Fischler, Daniel Freed, Joaquim Gomis, Vadim Kaplunovsky, and especially Jacques Distler. Also, Sally Dawson, Michael Dine, Michael Duff, Lawrence Hall, Hitoshi Murayama, Joe Polchinski, Edward Witten, and Bruno Zumino gave valuable help with special topics. Jonathan Evans read through the manuscript of this volume, and made many valuable suggestions. For pointing out various errors in the first printing of this book, I am greatly indebted to Stephen Adler, Jose Espinora, Tony Gherghetta, and San Fu Tuan. For corrections to the first printing of this volume I am indebted to several colleagues, especially Stephen Adler. Thanks are due to Alyce Wilson, who prepared the illustrations, to Terry

Volume III Supersymmetry

THE QUANTUM THEORY OF FIELDS

STEVEN WEINBERG

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Preface

xix

Riley for finding countless books and articles, and to Jan Duffy for many helps. I am grateful to Maureen Storey of Cambridge University Press for working to ready this book for publication, and especially to my editor, Rufus Neal, for his continued friendly good advice.

STEVEN WEINBERG

Austin, Texas
May, 1999

References

- Important related works:
 - Konishi, 1996
 - Evans, Hsu, Schwetz, 1996
 - Cheng, Shadmi, 1998
 - Luty, Rattazzi, 1999
 - Abel, Buican, Komargodski, 2011
 - Córdova, Dumitrescu, 2018 ...
- What we do is consistent with previous works
 - Focus on anomaly-mediated SUSY breaking
 - Pushed the method further
- Exact solutions to Chiral gauge theories

$SU(N_c)$ QCD ($N_c \geq 3$)

HM, 2104.01179

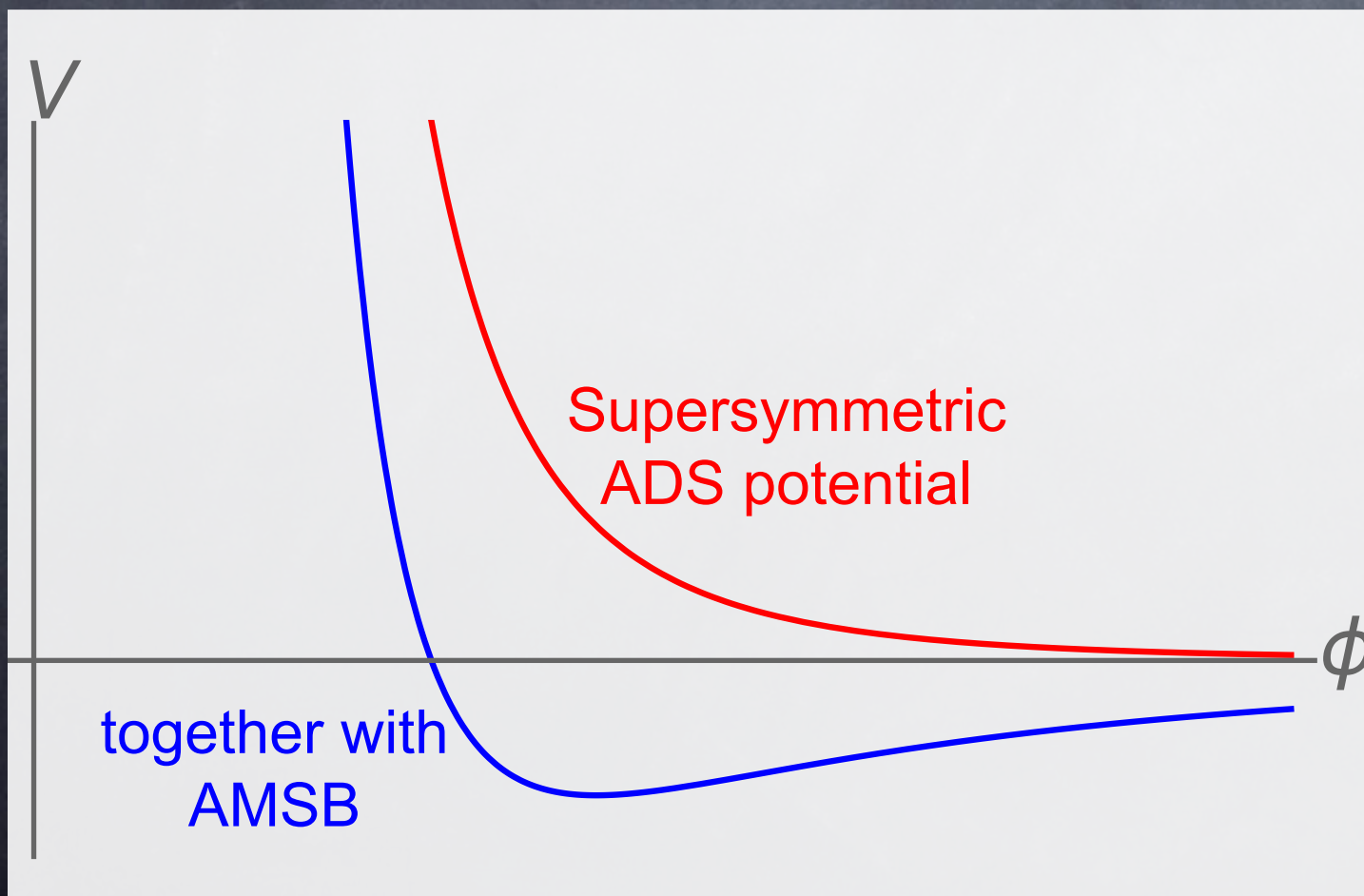
Phys.Rev.Lett. 126 (2021) 25, 251601

$$N_f < N_c$$

run-away superpotential for $M^{ij} = \tilde{Q}^i Q^j$

$$W = (N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{1/(N_c - N_f)} \quad M^{ij} = \delta^{ij} \phi^2$$

$$V = 2N_f \left| \frac{\Lambda^{3N_c - N_f}}{\phi^{N_c + N_f}} \right|^{\frac{1}{N_c - N_f}} - m(3N_c - N_f) \left(\frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}} \right)^{\frac{1}{N_c - N_f}} + c.c.$$



$$M_{ij} = \delta_{ij} \left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{N_c - N_f} \frac{\Lambda^{3N_c - N_f}}{m^{N_c - N_f}} \right)^{1/N_c}$$

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

χ SB! Proving Nambu
mesino loop $\rightarrow$ WZW term

$N_f=1$ special
no NGB, gapped

Deriving Chiral Lagrangian

$$Q = \begin{pmatrix} v\xi^T \\ 0 \end{pmatrix}, \quad \tilde{Q} = \begin{pmatrix} v\xi \\ 0 \end{pmatrix} \quad \xi \rightarrow g_L \xi h^T, \quad \xi \rightarrow h^* \xi g_R^T$$

$$U = \xi \xi \rightarrow g_L U g_R^T$$

$$|D_\mu Q|^2 + |D_\mu \tilde{Q}|^2 = \frac{v^2}{2} \text{Tr} \partial_\mu U^\dagger \partial^\mu U$$

$$f_\pi^2 = 2v^2 = 2 \left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{N_c - N_f} \frac{\Lambda^{3N_c - N_f}}{m^{N_c - N_f}} \right)^{1/N_c}$$

$$\Lambda^{3N_c - N_f} = M^{3N_c - N_f} e^{-8\pi^2 / g_h^2(M)}$$

$$= M^{3N_c - N_f} e^{-8\pi^2 / g_c^2(M)} Z^{-N_f} (g_c^2)^{-N_c}$$

$$= M^{3N_c - N_f} e^{-8\pi^2 N_c / g_{\text{tH}}^2(M)} Z^{-N_f} N_c^{N_c} (g_{\text{tH}}^2)^{-N_c}$$

$$f_\pi^2 \propto N_c$$

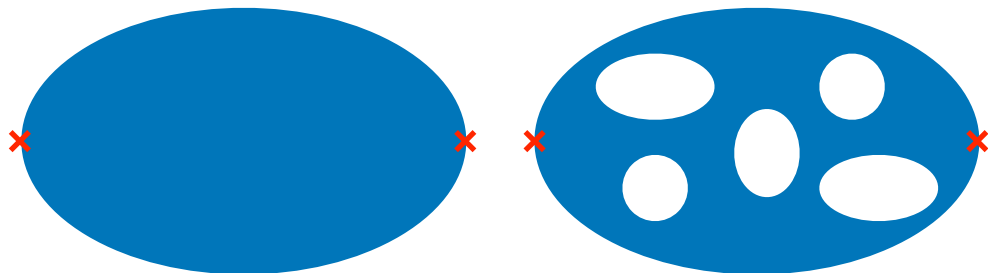
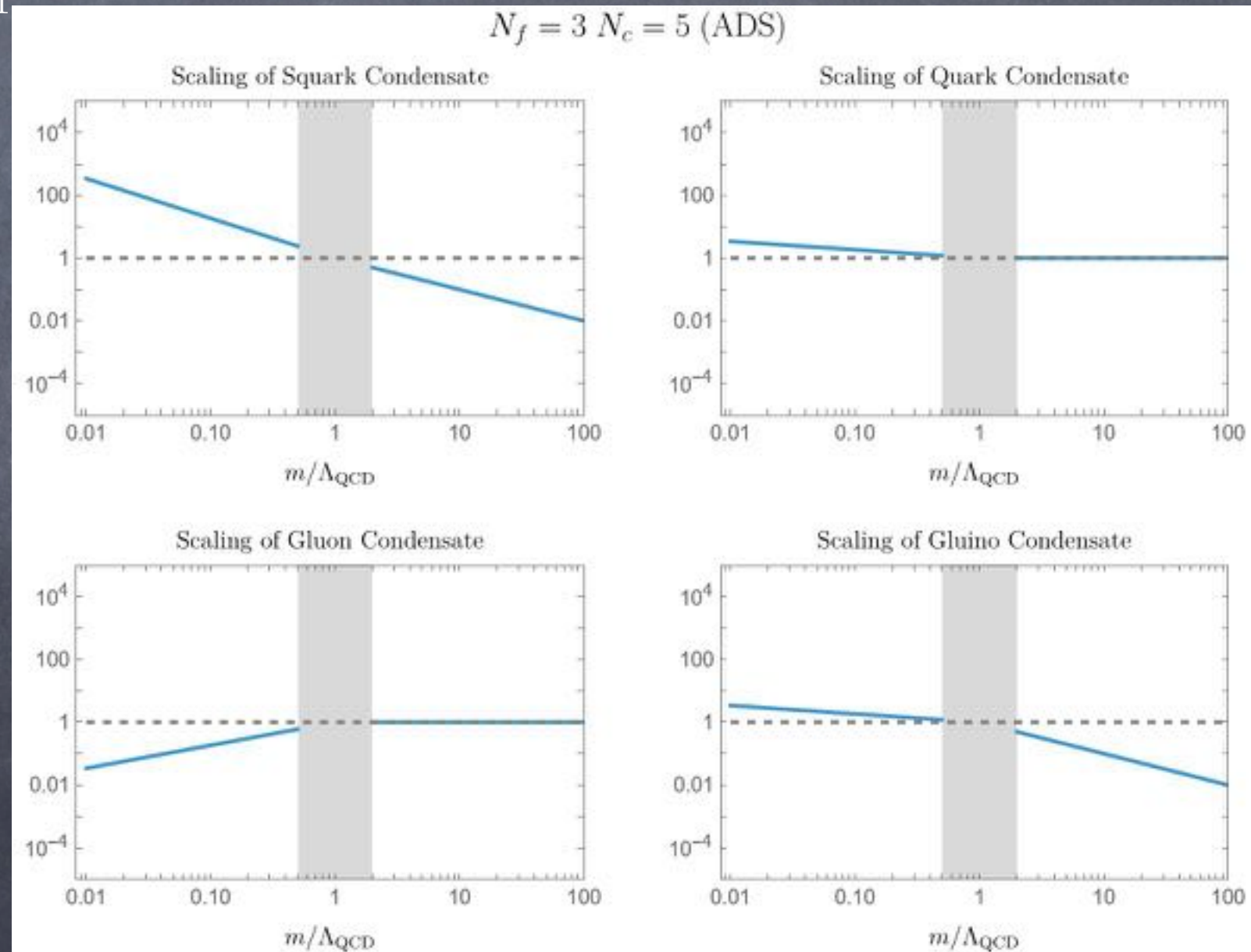
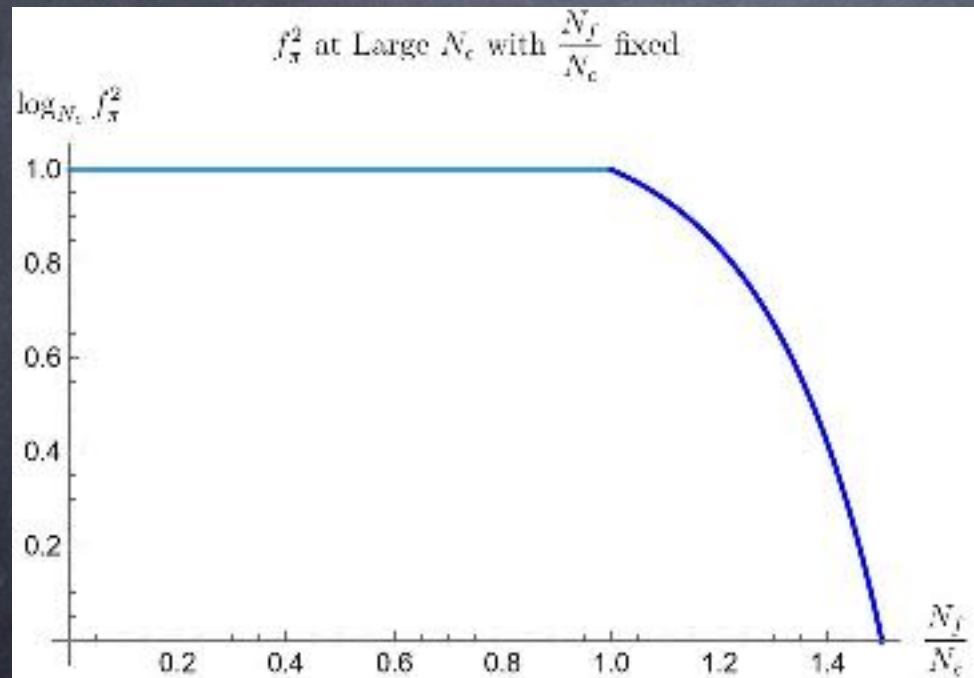
Martin & Wells didn't realize this

Non-perturbative Condensates

$$M_{ij} = \delta_{ij} \left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{N_c - N_f} \frac{\Lambda^{3N_c - N_f}}{m^{N_c - N_f}} \right)^{1/N_c} \quad \langle \bar{q}_i q_j \rangle = 4m\delta_{ij} \left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{-N_f} \frac{\Lambda^{3N_c - N_f}}{m^{N_c - N_f}} \right)^{1/N_c}$$

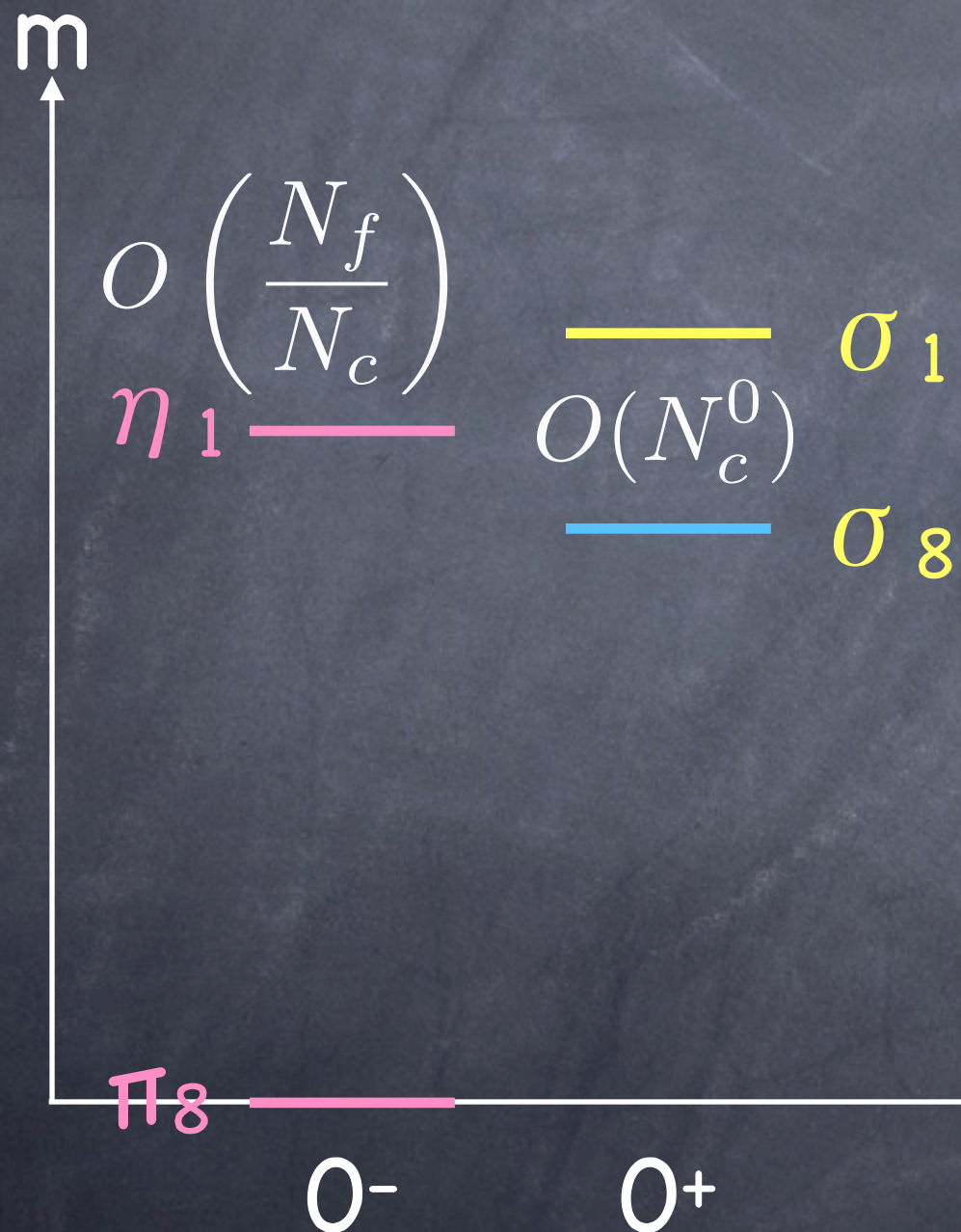
$$\langle \text{Tr} \lambda \lambda \rangle = 32\pi^2 \left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{-N_f} \Lambda^{3N_c - N_f} m^{N_f} \right)^{1/N_c} \quad \langle \text{Tr} G_{\mu\nu} G^{\mu\nu} \rangle = 32\pi^2 m \left(\left(\frac{N_c + N_f}{3N_c - N_f} \right)^{-N_c - N_f} \Lambda^{3N_c - N_f} m^{N_f} \right)^{1/N_c}$$

$$m_{\eta'}^2 = 2m^2 \frac{N_f(3N_c - N_f)^2}{(N_c - N_f)^2(N_c + N_f)} \propto N_c^{-1}$$

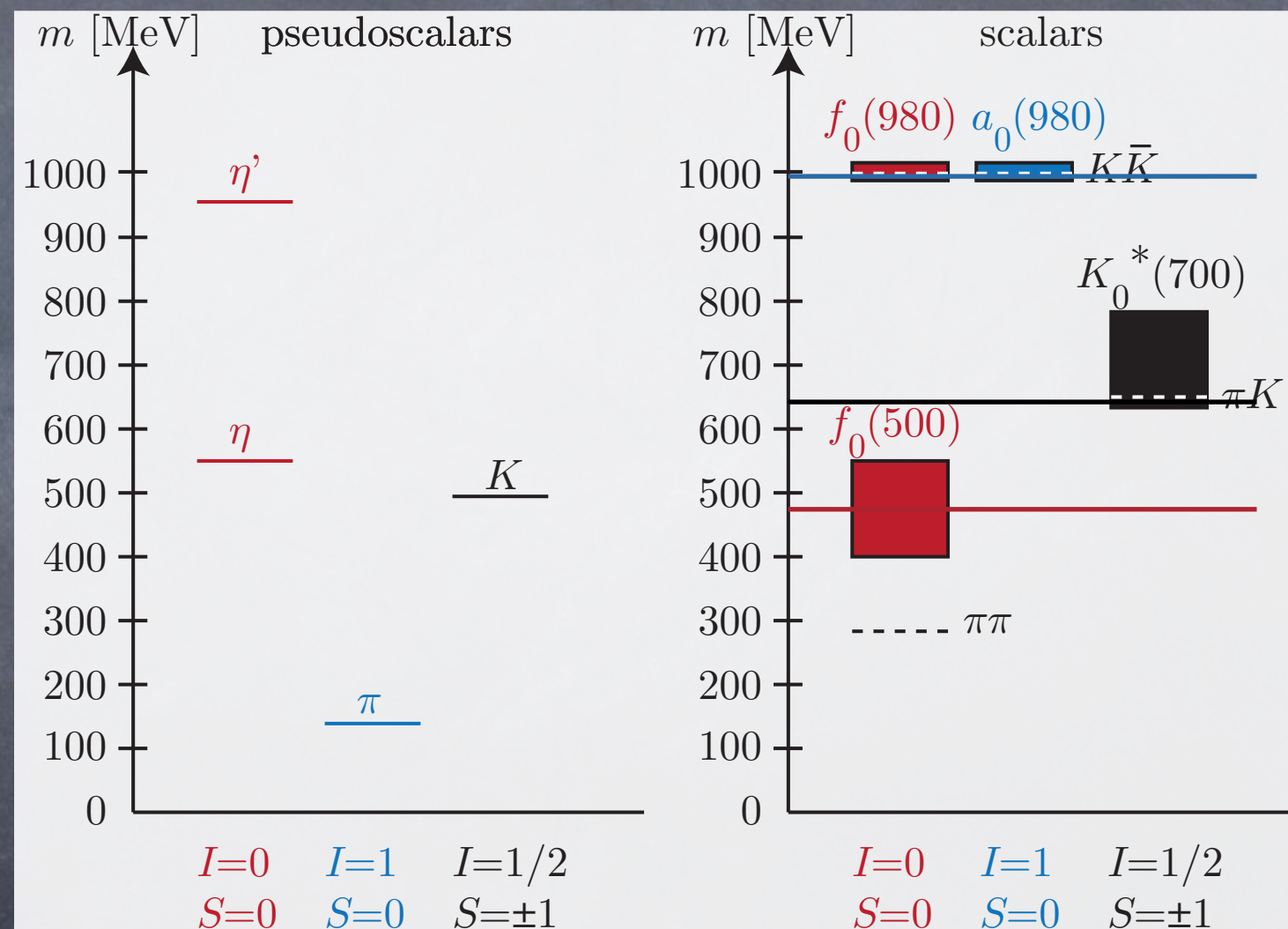


Back to near SUSY

Found $c' < 0$



$$m_\pi^2 = c(m_{q_i} + m_{q_j}) \quad m_\sigma^2 = m_8^2 + c'(m_{q_i} + m_{q_j})$$



$$c' = -2.2c$$

$$N_f = N_c + 1$$

“Confinement without χ SB”

$$W = \frac{\det M - \tilde{B} M B}{\Lambda^{2N_c - 1}}$$

$$W = \lambda \frac{\det M}{\Lambda^{N_f - 3}} - \kappa \tilde{B} M B$$

$$B_i = \epsilon_{ij_1 \dots j_{N_c}} Q^{j_1} \dots Q^{j_{N_c}}$$

$$V = N_f \lambda^2 \frac{|\phi|^{2N_f - 2}}{\Lambda^{2N_f - 6}} - \lambda(N_f - 3)m\phi^{N_f} + c.c.$$

$$\phi = \kappa^{-1} \Lambda \left(\frac{2N_f - 3N_c}{N_c} \frac{m}{\Lambda} \right)^{(N_f - N_c)/(2N_c - N_f)} \ll \Lambda$$

Supersymmetric
higher power potential

together with
AMSB

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

χ SB!

massless pions

massive baryons

$$m_B = \kappa f_\pi \approx 4\pi f_\pi$$

$$N_c + 2 \leq N_f < 3N_c/2$$

- “free magnetic phase”
- $SU(N_c)$ becomes strong, binds baryons, which break up into dual quarks

$$B^{i_1, \dots, i_{N_c}} = Q_{\alpha_1}^{i_1} \dots Q_{\alpha_{N_c}}^{i_{N_c}} \epsilon^{\alpha_1 \dots \alpha_{N_c}} = \epsilon^{i_1 \dots i_{N_c} i_{N_c+1} \dots i_{N_f}} b_{i_{N_c+1} \dots i_{N_f}}$$

$$b_{i_{N_c+1} \dots i_{N_f}} = q_{i_{N_c+1}}^{\beta_1} \dots q_{i_{N_f}}^{\beta_{N_f - N_c}} \epsilon_{\beta_1 \dots \beta_{N_f - N_c}}$$

- “magnetic” IR-free $SU(N_f - N_c)$ gauge theory

$$W = \frac{1}{\mu} M^{ij} q_i \tilde{q}_j \rightarrow \lambda \tilde{M}^{ij} q_i \tilde{q}_j$$

- λ dimensionless, only loop-level AMSB
- $N_f > 1.48 N_c$, magnetic quark has $m_q^2 < 0$
 - leads to $SU(N_f) \times SU(N_f - N_c)$ symmetry
 - can't match the anomalies w/o squarks
- χ SB is a local but consistent minimum

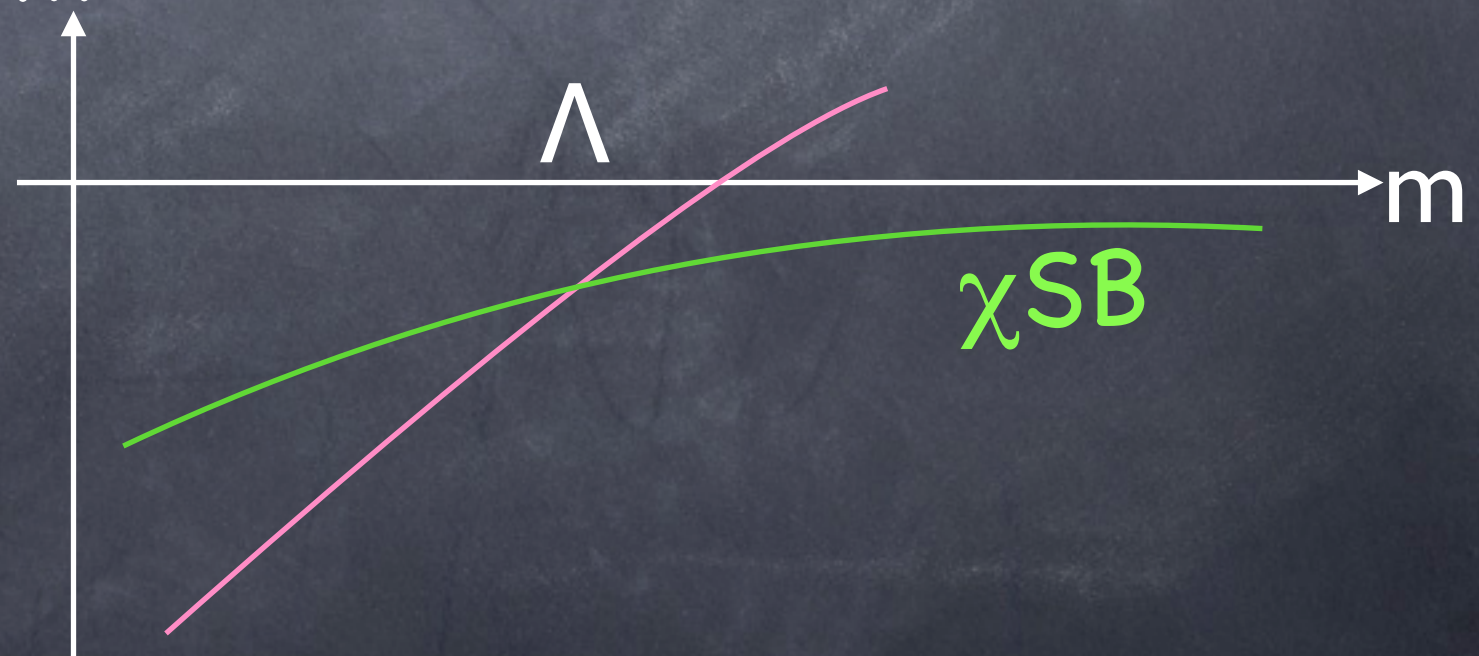
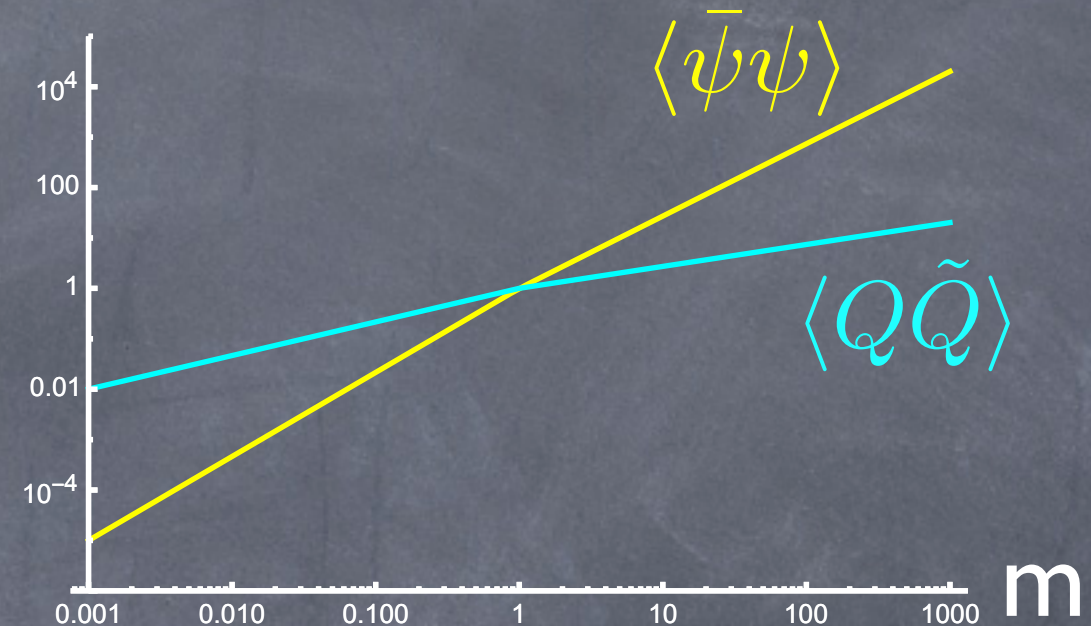
$$N_f > 1.48 N_c$$

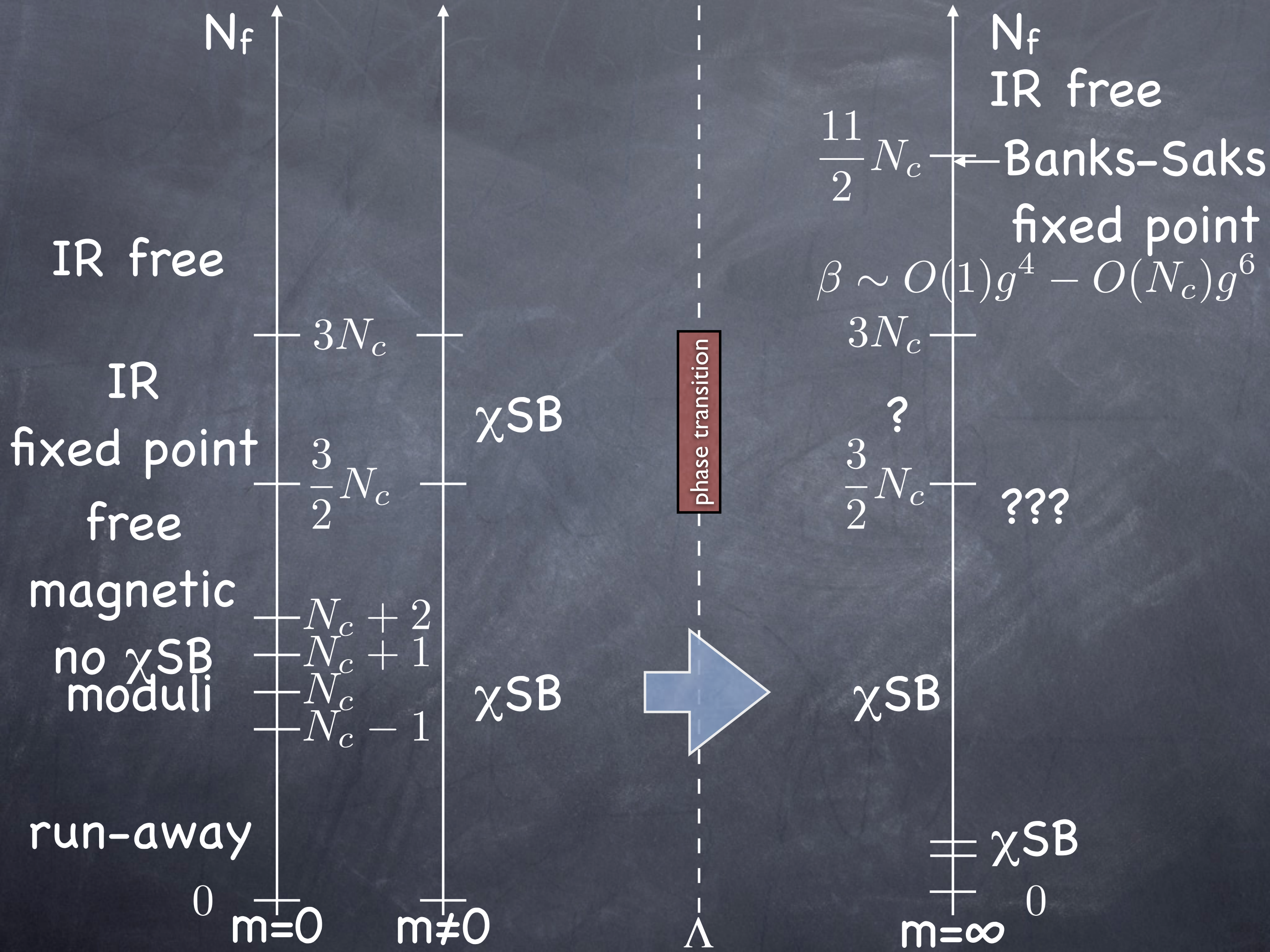
+Csáki, Gomes, Noether, Varier

χ_{SB}

baryon
condensate

χ_{SB} is a local minimum
but well-defined
and still useful
no similar issues
with SO , Sp



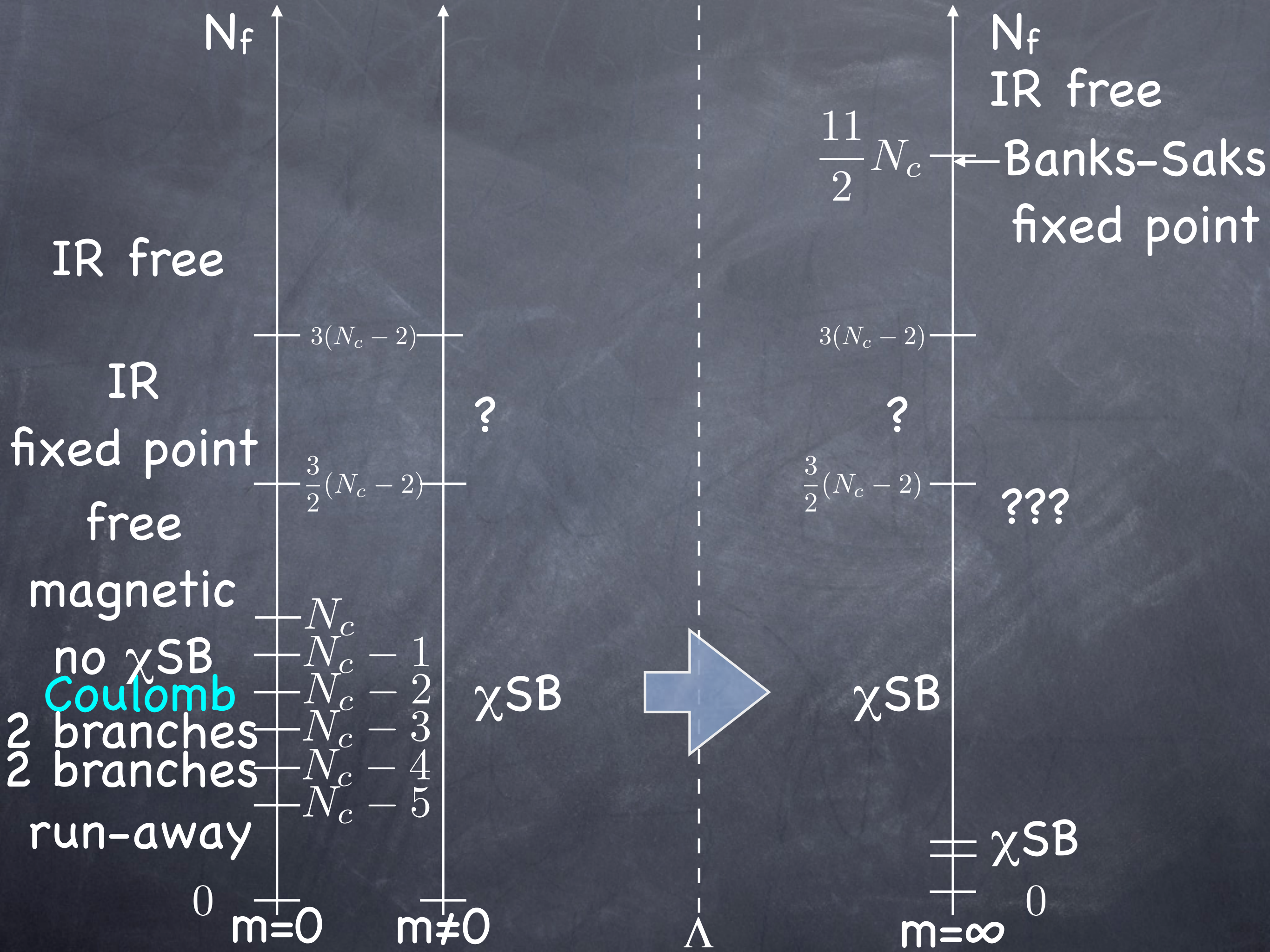


confinement and χ SB

Csáki, Gomes, HM, Telem, 2106.10288, 2107.02813

confinement vs screening

- We've derived χ SB in $SU(N_c)$ QCD
 - it has no confinement
 - massless quarks in the fundamental rep can screen any color charges
 - Wilson loop is perimeter law $\langle \text{Tr} e^{i \oint A dx} \rangle \propto e^{-L}$
- $SO(N_c)$ QCD with quarks in vector rep
 - cannot screen Z_2 center (e.g. spinor rep)
 - rigorous definition of confinement $\langle \text{Tr} e^{i \oint A dx} \rangle \propto e^{-A}$
 - can we see an interplay with χ SB?
- Complete classification of t-confining theories



$$N_f = N_c - 2$$

- for $M^{ij} = Q^i Q^j \neq 0$ with rank $M = N_f$, $SO(N_c)$ is broken to $SO(2)$

- Coulomb branch $u = \det M$
- two singularities $V \approx - \left(\frac{\lambda^2}{16\pi^2} \right)^4 m^4$
- $u = \det M = 0$

- dyons: $q_i^\pm$ $W = \frac{1}{\mu} M^{ij} q_i^+ q_j^-$

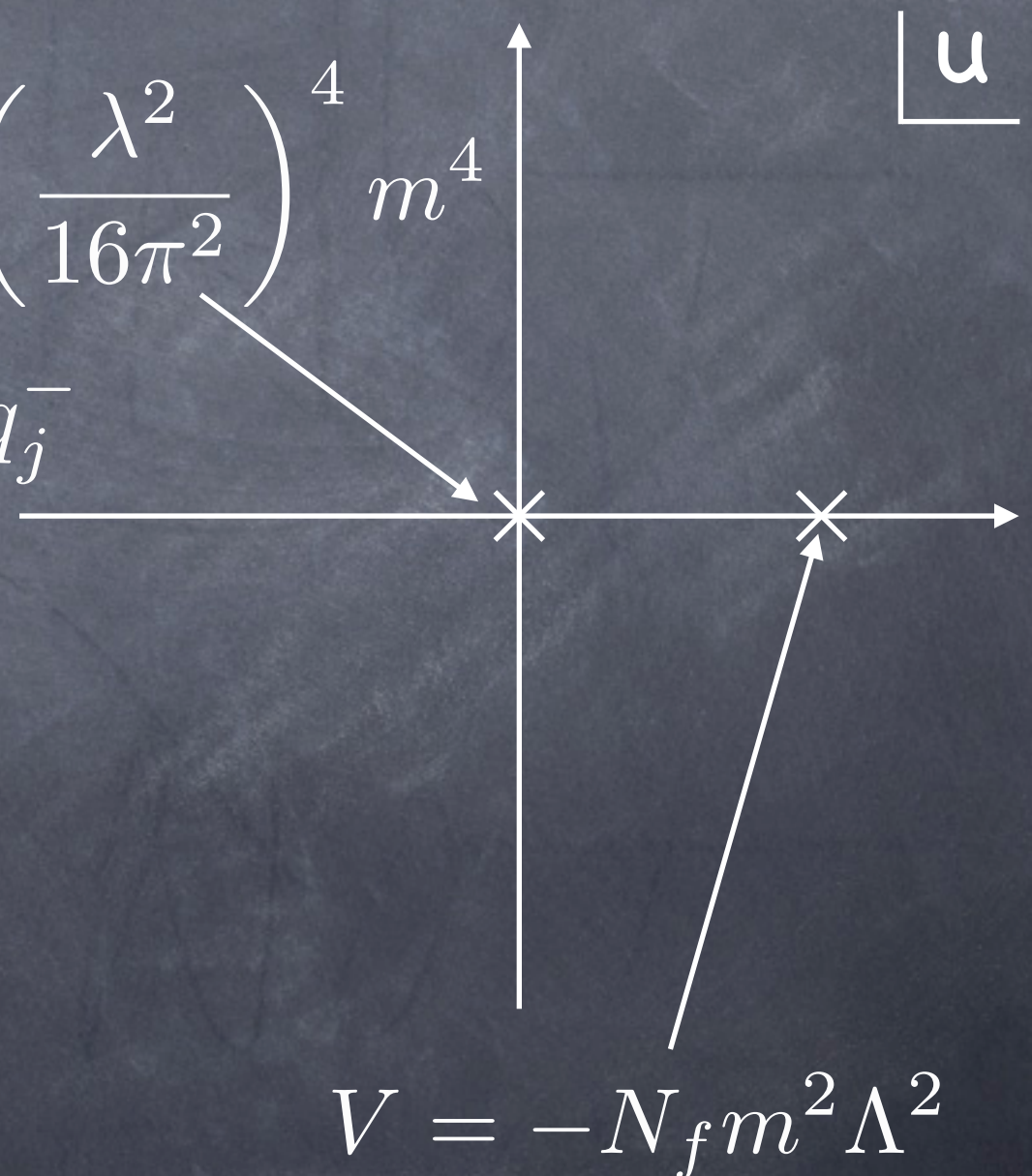
- $u = \det M = \Lambda^{2N_f}$

- monopoles:

$$W = (u - \Lambda^{2N_f}) E^+ E^-$$

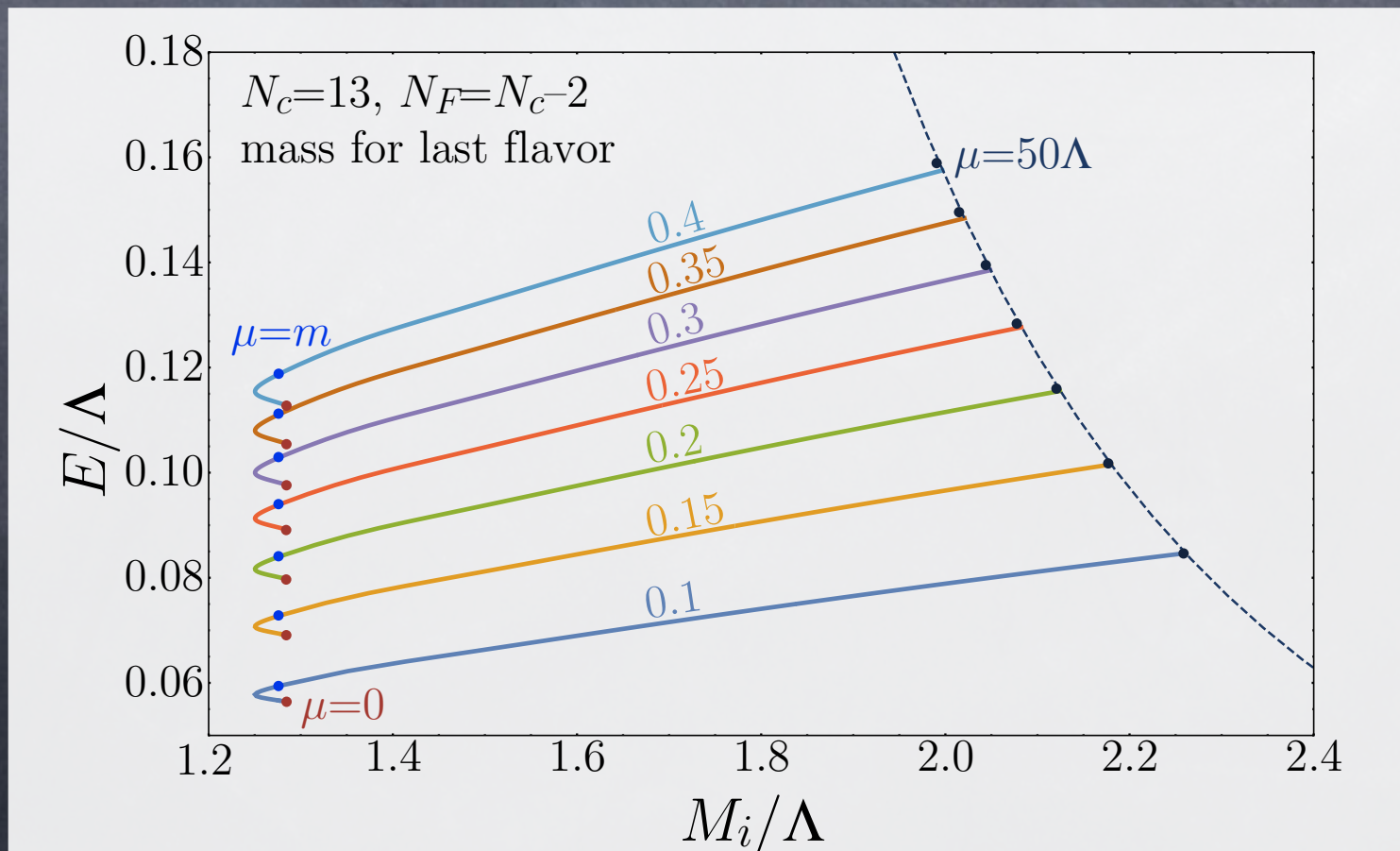
$$|E^\pm| = (m\Lambda)^{1/2}$$

- both monopoles and meson condense!



$$N_f < N_c - 2$$

- add mass m_q to some of the quarks
- can show monopole VEVs persist $m_q \rightarrow \infty$
- demonstration of confinement and chiral symmetry breaking for all $N_f \leq N_c - 2$



Chiral gauge theories

Nature is chiral

- Standard model is a chiral gauge theory
- but chiral gauge theories are difficult to regularize
- no lattice simulations yet!
- tumbling hypothesis (Dimopoulos, Raby, Stuart)
 - gauge theory breaks itself in the Most Attractive Channel (MAC) until it becomes vector-like
- $SU(N_c)$ with $A + (N_c-4) F^*$, global $SU(N_c-4) \times U(1)$
 - $MAC \langle A^{\alpha\beta} F_{i\beta}^* \rangle = \Lambda^3 \delta_i^\alpha : SU(N_c) \rightarrow SU(4) \times SU(N_c-4)$
 - $A = (6, 1) + (4, (N_c-4)^*)$, $F^* = (4^*, (N_c-4))$
 - $SU(4)$ vector-like, $SU(N_f)$ unbroken
 - massless composite fermions $A^{\alpha\beta} F_{\alpha}^* \{i, F_{\beta}^* j\}$
 - never proven

UV theory		SUSY + AMSB		Tumbling	
N_F	G_{global}	H_{global}	$m_f = 0$	H_{global}	$m_f = 0$
SU(N_C) + $A(\square) + (N_C - 4 + N_F)\tilde{F}(\bar{\square}) + N_FF(\square)$, $N_C = \text{Odd}$ [15, 20]					
0	SU($N_C - 4$) $\times$ U(1)	Sp($N_C - 5$) $\times$ U(1)	$\square + 1$	SU($N_C - 4$) $\times$ U(1)	$\square\square$
1	SU($N_C - 3$) $\times$ U(1) ²	Sp($N_C - 3$) $\times$ U(1)	$\square$	SU($N_C - 4$) $\times$ U(1)	$\square\square$
2	SU($N_C - 2$) $\times$ SU(2) $\times$ U(1) ²	Sp($N_C - 3$) $\times$ U(1) ²	$\square$	SU($N_C - 4$) $\times$ SU(2) $\times$ U(1)	($\square\square$, 1)
SU(N_C) + $A(\square) + (N_C - 4 + N_F)\tilde{F}(\bar{\square}) + N_FF(\square)$, $N_C = \text{Even}$ [15, 20]					
0	SU($N_C - 4$) $\times$ U(1)	Sp($N_C - 4$)	none	SU($N_C - 4$) $\times$ U(1)	$\square\square$
1	SU($N_C - 3$) $\times$ U(1) ²	Sp($N_C - 4$) $\times$ U(1)	none	SU($N_C - 4$) $\times$ U(1)	$\square\square$
2	SU($N_C - 2$) $\times$ SU(2) $\times$ U(1) ²	Sp($N_C - 4$) $\times$ SU(2) $\times$ U(1)	none	SU($N_C - 4$) $\times$ SU(2) $\times$ U(1)	($\square\square$, 1)
SU(N_C) + $S(\square\square) + (N_C + 4)\tilde{F}(\bar{\square})$ [18]					
0	SU($N_C + 4$) $\times$ U(1)	SO($N_C + 4$)	none	SU($N_C + 4$) $\times$ U(1)	$\square$
SO(10) + $N_F\psi(\mathbf{16})$ [19]					
1	none	none	none	none	none
2	SU(2)	SU(2)	none	SO(2)	none
3	SU(3)	SO(3)	none	SO(3)	none
$E_6 + N_F\psi(\mathbf{27})$ [This Work]					
1	none	none	none	none	none
2	SU(2)	none	none	SO(2)	none
3	SU(3)	SU(3)	$\square\square\square$	SO(3)	none
		U(1) $\times$ U(1)	$S_{iii}(i = 1, 2, 3)$		
		none	none		

New BSM model building tools

- SUSY + AMSB appears useful to gain insight into strong gauge dynamics
- new possibilities:
 - massless composite fermions
 - light composite scalars
 - new symmetry breaking patterns
 - hidden jets must be re-studied
- also many (in principle) testable predictions on dynamics
 - look forward to future developments on lattice

Conclusions



Conclusions

- SUSY + AMSB help us understand quantum dynamics



Conclusions

- SUSY + AMSB help us understand quantum dynamics
- obtain results consistent with Yukawa, Nambu
- no obvious signs of phase transition as SUSY breaking is increased
- applied to systems not understood yet, results very different from past conjectures
- can be applied to 2+1D, 1+1D systems (not yet)
- interested in strongly-coupled dark sector

