SPIN CORRELATIONS:
TEVATRON vs. LHC

Greg Mahlon, Penn State Mont Alto

Mahlon and Parke, *Phys. Rev.* **D53**, 4886 (1996),

Parke and Shadmi, *Phys. Lett.* **B387**, 199 (1996),

Mahlon and Parke, *Phys. Rev.* **D81**, 074024 (2010).

See also

Jeżabek and Kühn, *Phys. Lett.* **B329**, 317 (1994),

Brandenburg, Si, and Uwer, *Phys. Lett.* **B539**, 235 (2002),

Bernreuther, Brandenburg, Si, and Uwer, *Nucl. Phys.* **B690**, 81 (2004),

and

Uwer, *Phys. Lett.* **B609**, 271 (2005).

3rd International Workshop on Top Quark Physics – June 3, 2010

Outline

I. Top pair production mechanism and spin structure

- A. Tevatron: mostly $q\bar{q} \to t\bar{t}$
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III. Strategies for observation of spin correlations

- A. Tevatron: joint $t, \bar{t}$ decay angular distribution
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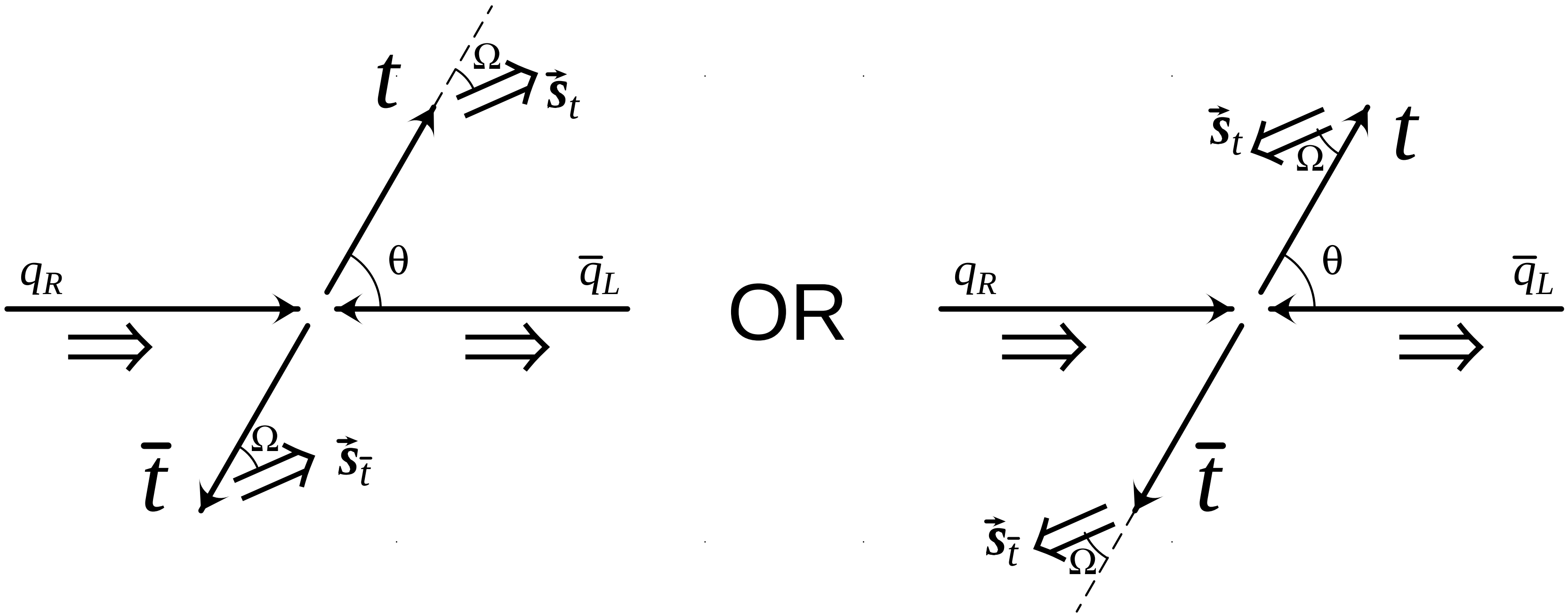
V. Summary and Conclusions

$q\bar{q} \to t\bar{t}$ Spin Structure

Incoming q, $\bar{q}$ must have opposite helicity to couple to s-channel gluon.

Off-diagonal basis: $\tan\Omega = (1-\beta^2)\tan\theta$

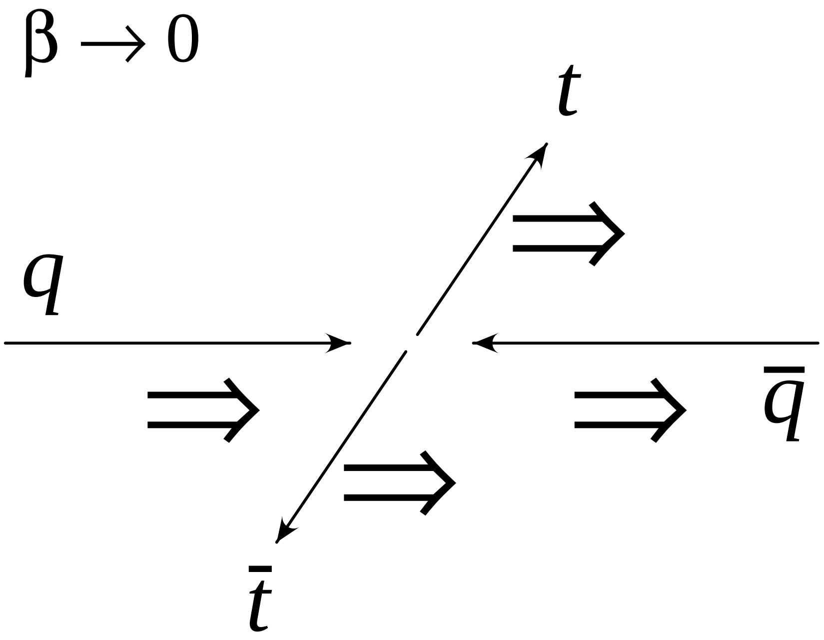


$$\left(1+\sqrt{1-\beta^2\sin^2\theta}\right)^2 : \left(1-\sqrt{1-\beta^2\sin^2\theta}\right)^2$$

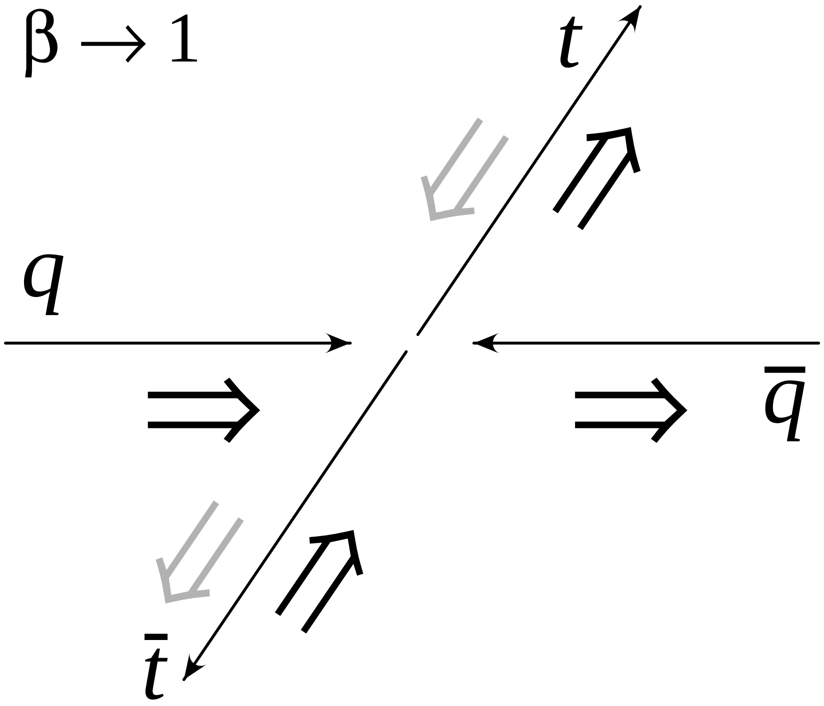
Reverse all spins for $q_L\bar{q}_R$ initial state.

$q\bar{q} \to t\bar{t}$ Spin Structure: Limits

Off-diagonal basis: $\tan\Omega = (1-\beta^2)\tan\theta$



Beamline basis



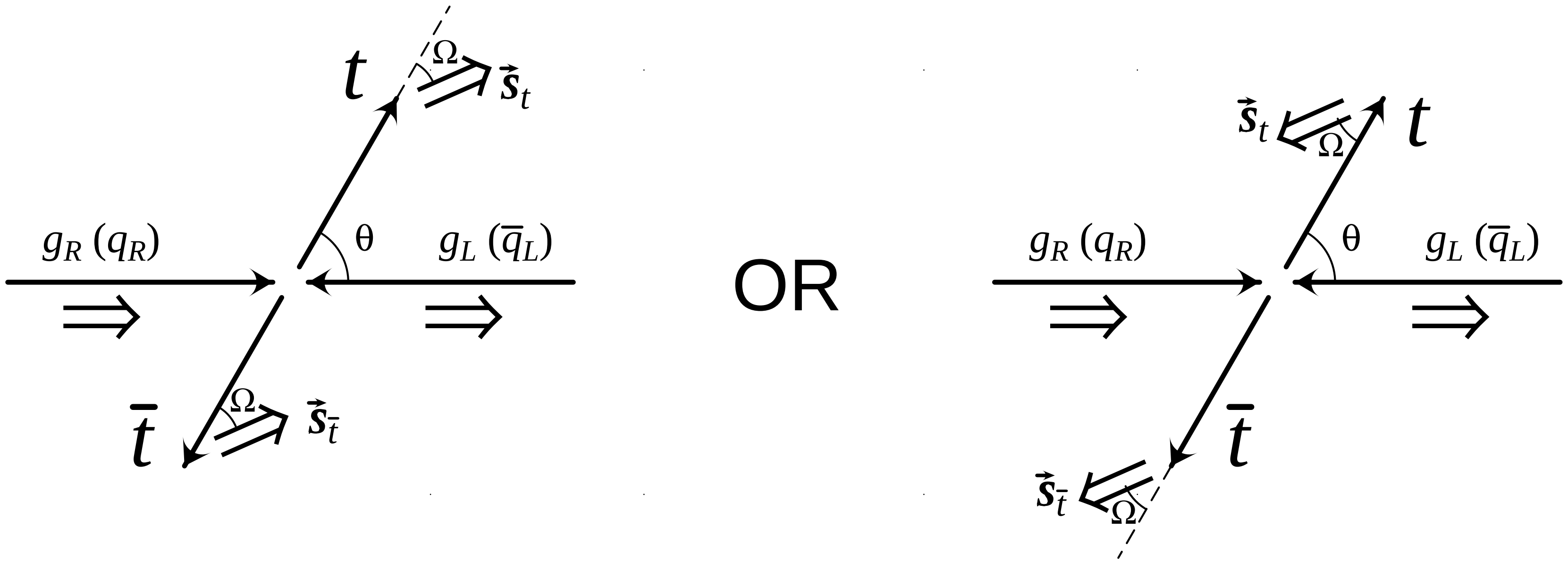
Helicity basis

Off-diagonal basis smoothly connects these two extremes.

$gg \to t\bar{t}$ Spin Structure

Opposite-helicity gluons dominate when $\beta\gamma\sin\theta > 1$ $\left(\beta^2 > \dfrac{1}{2-\cos^2\theta}\right)$: **same** correlations as $q\bar{q} \to t\bar{t}$.

$$\tan\Omega = (1-\beta^2)\tan\theta$$



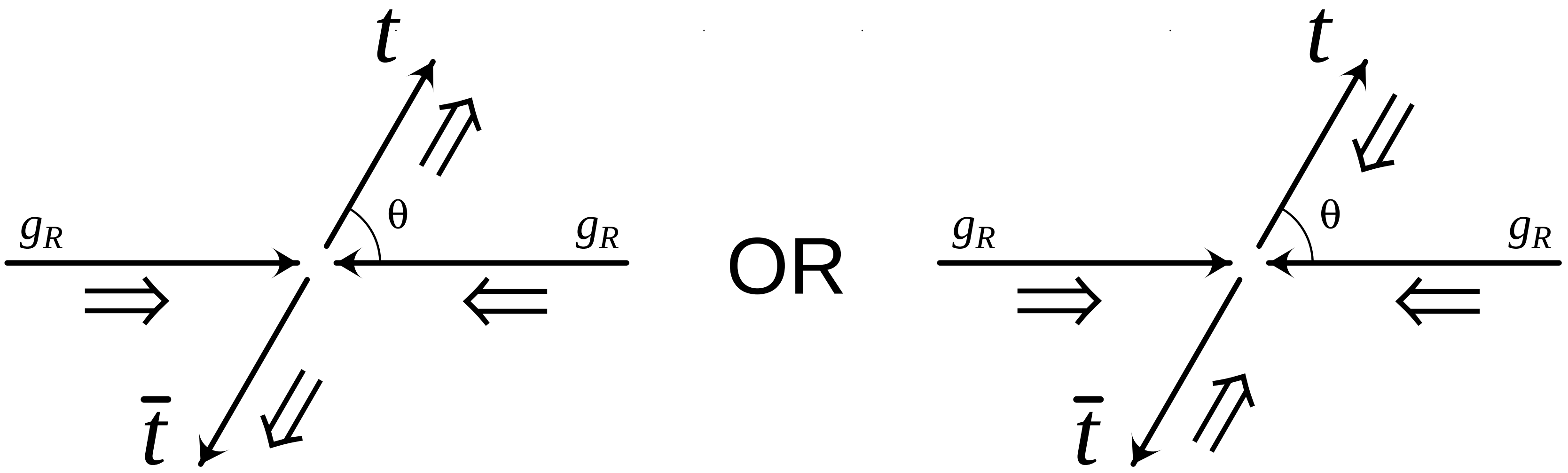
$$\left(1+\sqrt{1-\beta^2\sin^2\theta}\right)^2 : \left(1-\sqrt{1-\beta^2\sin^2\theta}\right)^2$$

Maximum correlation in off-diagonal basis.

$gg \to t\bar{t}$ Spin Structure

Like-helicity gluons dominate when $\beta\gamma \sin\theta < 1$:

maximum correlation in helicity basis (for all β) for these events.



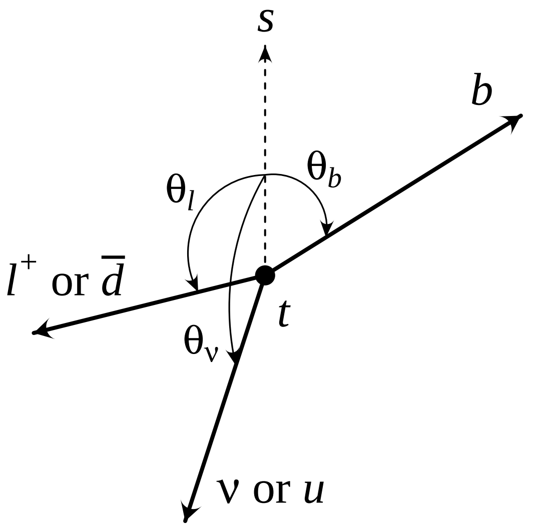
$$(1+\beta)^2 : (1-\beta)^2$$

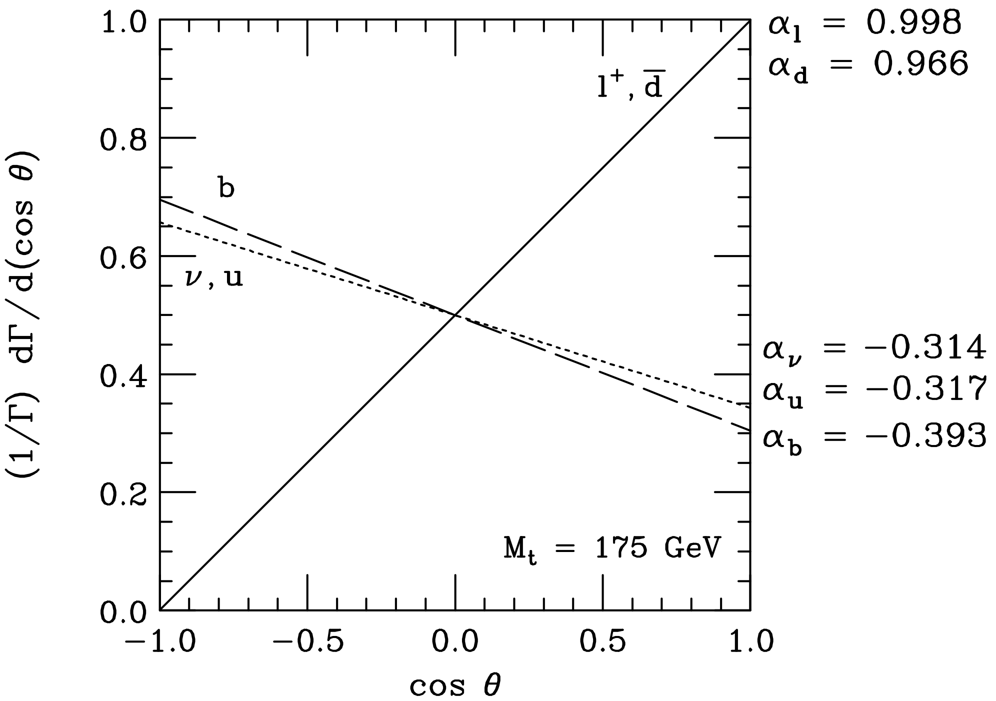
For $g_L g_L \to t\bar{t}$, flip the spins on both the gluons and the top quarks.

Polarized Top Decay

(LO: Reżabek and Kühn, *Phys. Lett.* **B329**, 317 (1994);
NLO: Brandenburg, Si, and Uwer, *Phys. Lett.* **B539**, 235 (2002).)

Define angles in top rest frame: $\frac{1}{\Gamma}\frac{d\Gamma}{d(\cos\theta_i)} = \frac{1+\alpha_i\cos\theta_i}{2}$





Note: Coefficients for b, u, and $\bar{d}$ are for partons; jets differ slightly at NLO.

Polarized Top Decay (cont.)

For a complete event,

$$p\bar{p} \to t\bar{t} \to \text{6-body state}$$

we have the distribution

$$\frac{1}{\sigma}\,\frac{d^2\sigma}{d(\cos\theta_i)d(\cos\bar{\theta}_{\bar{\imath}})} = \frac{1}{4}\left[1 + \frac{N_{\parallel}-N_{\times}}{N_{\parallel}+N_{\times}}\,\alpha_i\bar{\alpha}_{\bar{\imath}}\cos\theta_i\cos\bar{\theta}_{\bar{\imath}}\right].$$

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Dependence on production:

$N_{\parallel}$ is the number of like spin $t\bar{t}$ events

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- $N_{\|}$ is the number of like spin $t\bar{t}$ events
- $N_{\times}$ is the number of unlike spin $t\bar{t}$ events

Dependence on decay:

- θ_i, α_i: from t side of event (measure in t rest frame)
- $\bar{\theta}_{\bar{\imath}}$, $\bar{\alpha}_{\bar{\imath}}$: from $\bar{t}$ side of event (measure in $\bar{t}$ rest frame)

Polarized Top Decay (cont.)

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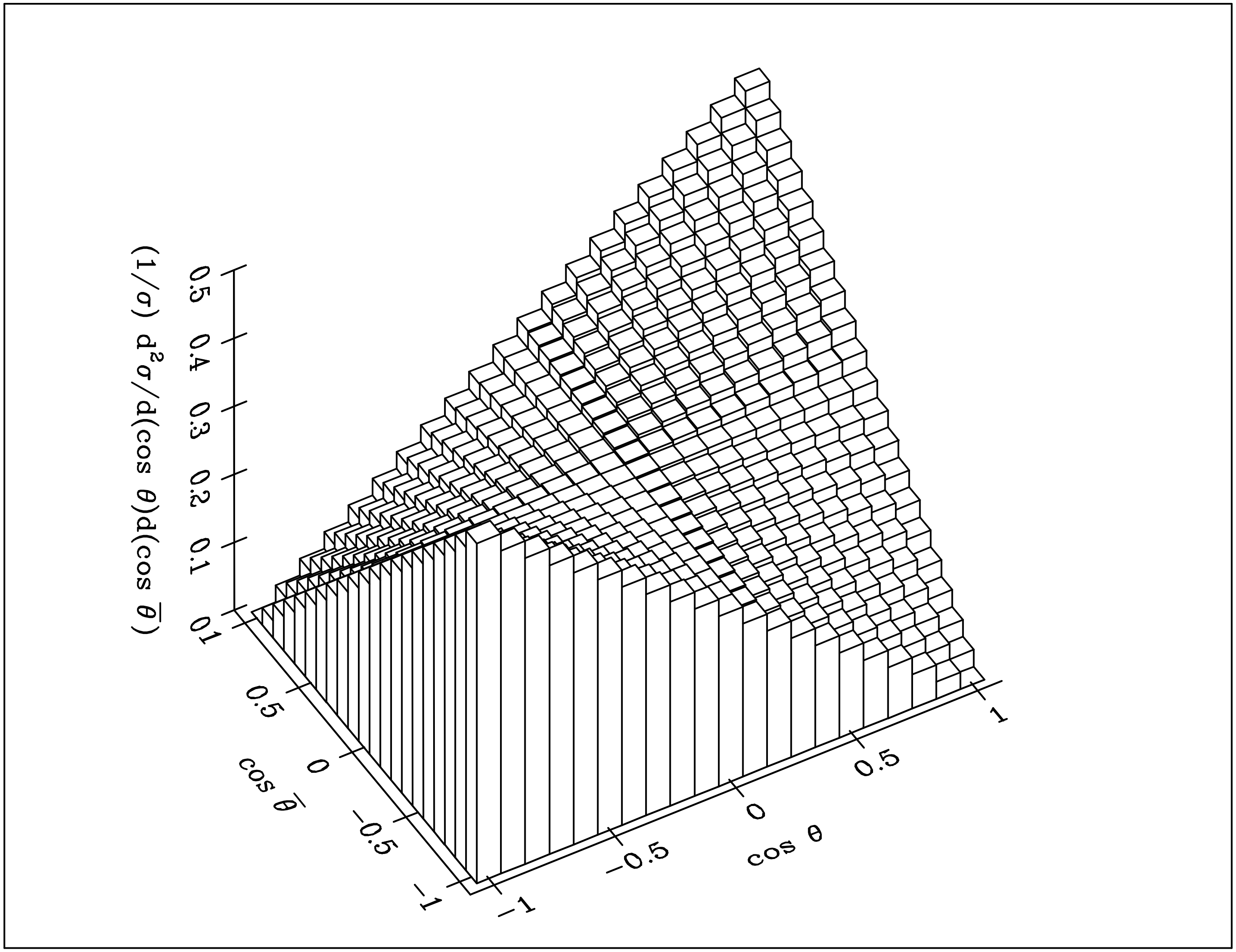
See Bernreuther, Brandenburg, Si, and Uwer, *Phys. Rev. Lett.* **87**, 242002 (2001) for NLO QCD corrections to this distribution.

Theoretical Shape of the Distribution



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Dilepton mode:

+ Maximum possible analyzing power.

– Low statistics.

– Two neutrinos: $\Rightarrow t, \bar{t}$ rest frames poorly determined.

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Dilepton mode:

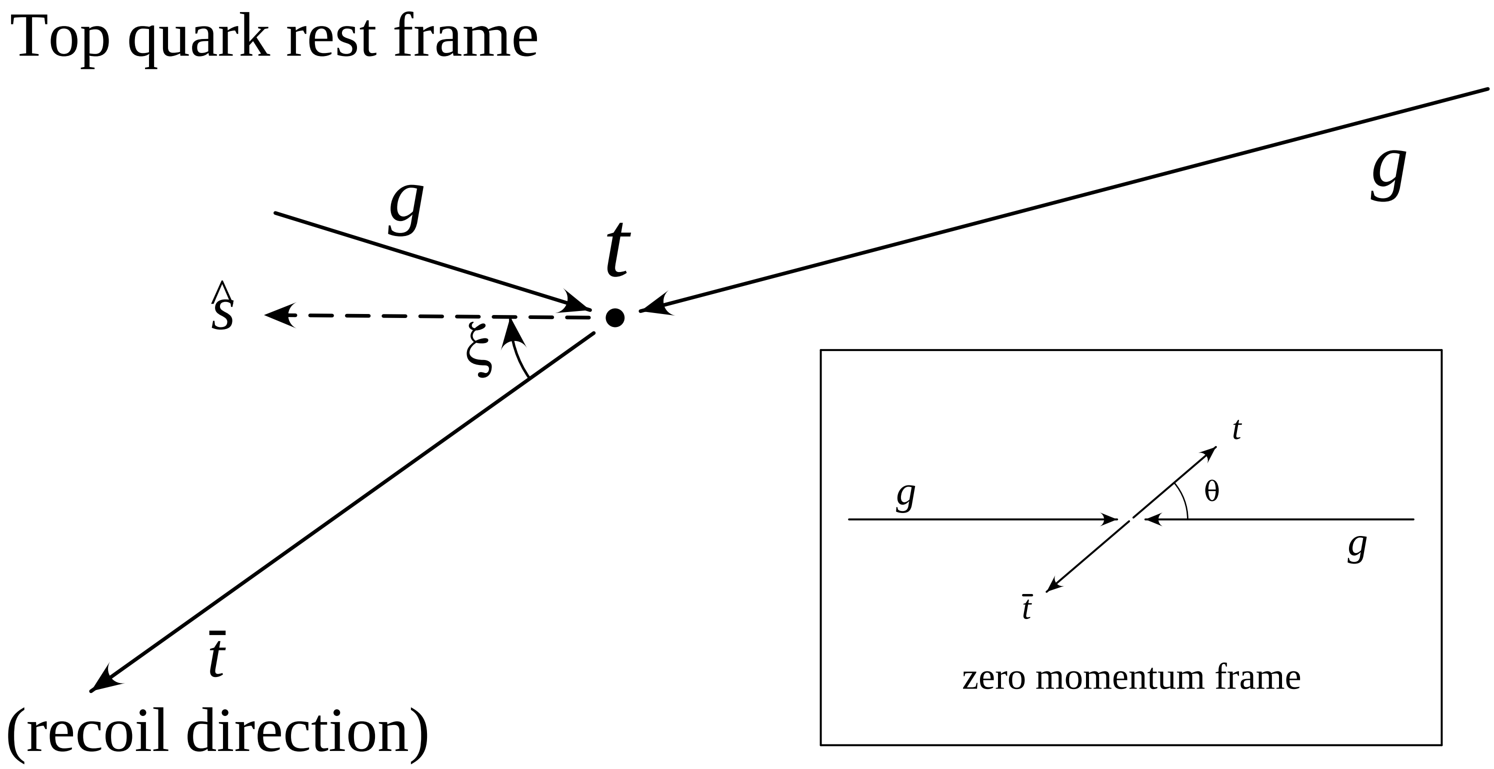
+ Maximum possible analyzing power.
– Low statistics.
– Two neutrinos: $\Rightarrow t, \bar{t}$ rest frames poorly determined.

Lepton + jets:

+ Higher statistics.
+ Only 1 neutrino: $t, \bar{t}$ rest frames better known.
– Loss of analyzing power since d-type quark selected probabilistically.

Spin Basis Optimization for $gg \to t\bar{t}$

Write amplitudes using arbitrary spin axis orientation.



Parke and Shadmi, *Phys. Lett.* **B387** (1996), 199.

Spin Basis Optimization for $gg \to t\bar{t}$ (continued)

Like-spin fraction is

$$f(\theta,\beta) = \frac{\gamma^{-2}(1+\beta^2\cos^2\xi) + \beta^2\sin^2\theta(\gamma^{-1}\sin\theta\cos\xi - \cos\theta\sin\xi)^2}{(1-\beta^4)+\beta^2\sin^2\theta(2-\beta^2\sin^2\theta)}.$$

Straightforward analytic exercise to find the extrema of this fraction (see Uwer, *Phys. Lett.* **B609**, 271 (2005) for numerical extremization).

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Extrema occur when ξ satisfies

$$\tan 2\xi_{\{\text{same, oppo}\}} = \frac{2\gamma^{-1}\sin^3\theta\cos\theta}{\sin^2\theta\cos^2\theta - \gamma^{-2}\sin^4\theta - \gamma^{-2}};$$

they are related as follows: $\xi_{\text{oppo}} = \xi_{\text{same}} + \pi/2$.

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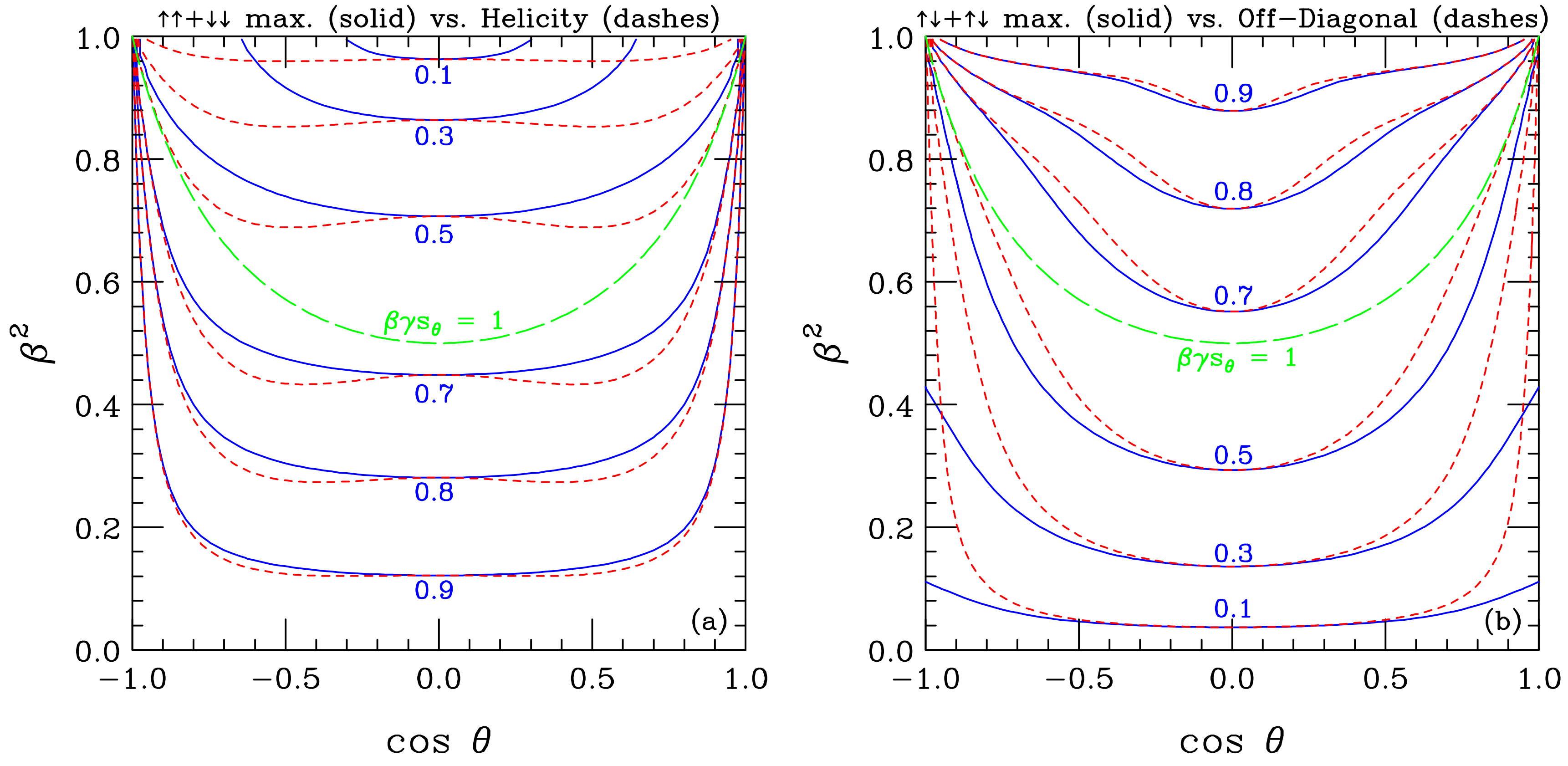
Learn the following:

- Best spin basis maximizes $\uparrow\uparrow + \downarrow\downarrow$ fraction when $\beta\gamma\sin\theta < 1$ (like-helicity gluons dominate this region).
- Best spin basis maximizes $\uparrow\downarrow + \downarrow\uparrow$ fraction when $\beta\gamma\sin\theta > 1$ (opposite-helicity gluons dominate this region).

Spin Basis Optimization for $gg \to t\bar{t}$ (continued)



- If $\beta\gamma\sin\theta < 1$, spin basis that maximizes $\uparrow\uparrow + \downarrow\downarrow$ fraction is very close to the helicity basis (like-helicity gluons dominate this region).
- If $\beta\gamma\sin\theta > 1$, spin basis that maximizes $\uparrow\downarrow + \downarrow\uparrow$ fraction is very close to the off-diagonal basis (opposite-helicity gluons dominate this region).

Separating Gluons

- Plenty of $t\bar{t}$ production at LHC energies (about 10^6 $t\bar{t}$ pairs per fb^{-1} at full beam energy; even at reduced energy get 10^4 to 10^5 pairs per fb^{-1}).
- Can implement significant cuts before statistical uncertainties rival systematic uncertainties.

⇒ Focus on $\gamma\beta\sin\theta < 1$ region:

- ⋆ Like-helicity gluons to like-helicity $t\bar{t}$ pairs dominates these events.
- ⋆ Spin correlations not masked by large boosts.
  - ⇒ In principle, could employ a Tevatron-style analysis.
- ⋆ Another option exists, however.

Comparison of Correlated and Uncorrelated Tops

"Uncorrelated" = spherical top quark decays (in their rest frames)
+ normal W decays.

Examine the ratio:

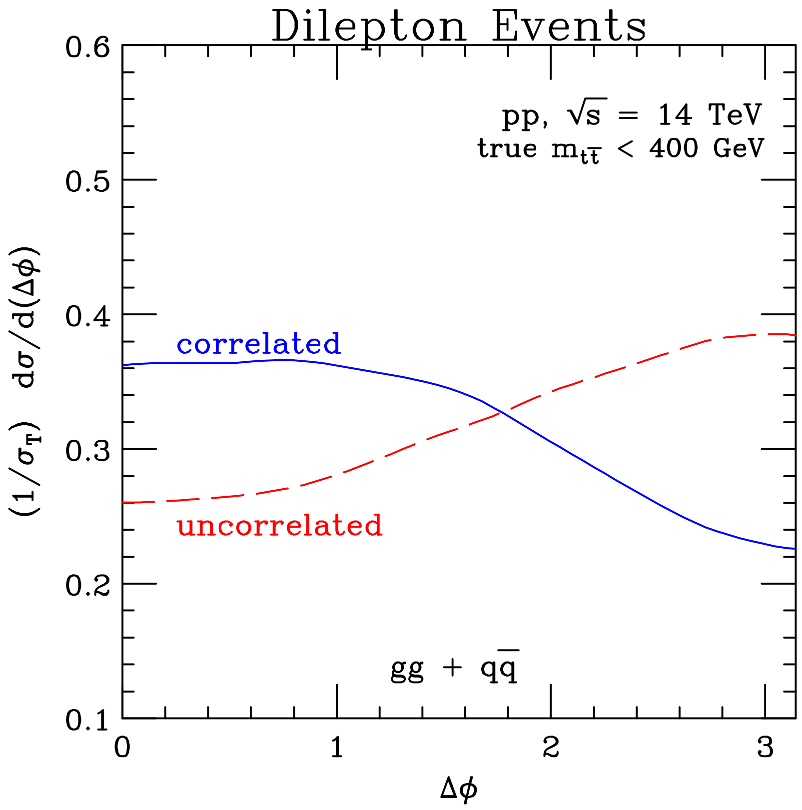
$$\begin{aligned}\mathcal{S} &\equiv \frac{(|\mathcal{A}|^2_{RR} + |\mathcal{A}|^2_{LL})_{\rm corr}}{(|\mathcal{A}|^2_{RR} + |\mathcal{A}|^2_{LL})_{\rm uncorr}} \\ &= \left[\frac{1-\beta^2}{1+\beta^2}\right]\left[\frac{(1+\beta^2)+(1-\beta^2)c_{\bar{e}\mu} - 2\beta^2 c_{t\bar{e}}c_{\bar{t}\mu}}{(1-\beta c_{t\bar{e}})(1-\beta c_{\bar{t}\mu})}\right] \\ &\sim 1 + c_{\bar{e}\mu} \quad \text{when } \beta \to 0.\end{aligned}$$

⇒ Examine $\Delta\phi$, $\Delta\eta$, and ΔR for the two charged leptons.

- Find almost no difference in $(\Delta\eta)_{\rm corr}$ and $(\Delta\eta)_{\rm uncorr}$.
- Modest difference in $(\Delta R)_{\rm corr}$ and $(\Delta R)_{\rm uncorr}$.
- Significant difference in $(\Delta\phi)_{\rm corr}$ and $(\Delta\phi)_{\rm uncorr}$!

Di-lepton azimuthal opening angle ($\Delta\phi$ distribution)

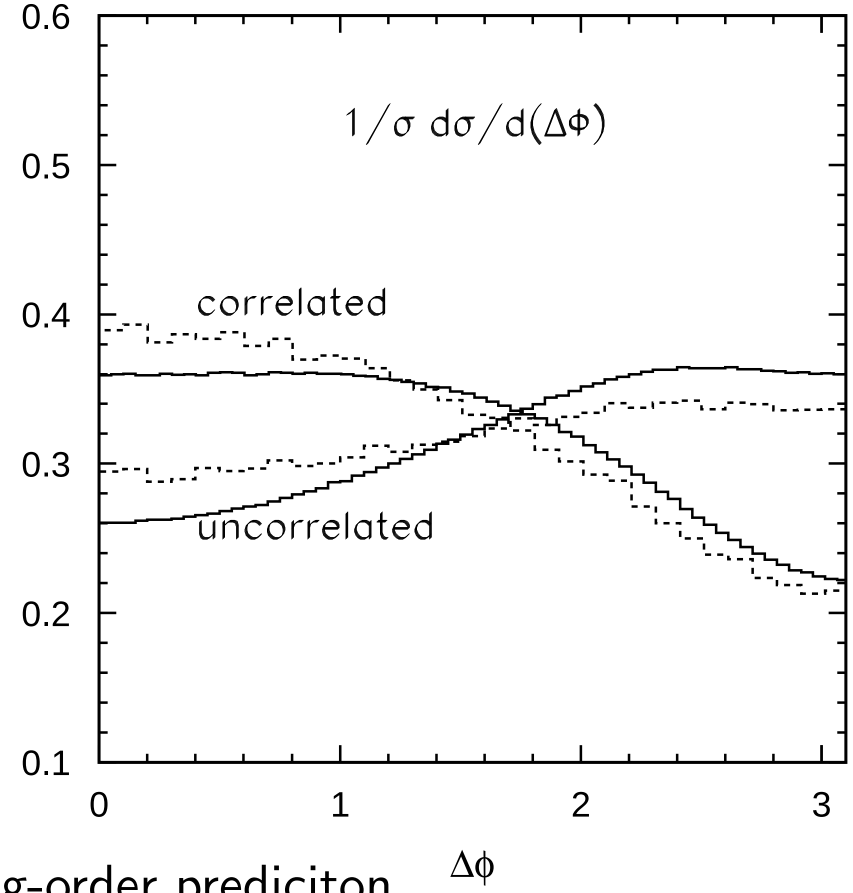
Restrict $\beta\gamma\sin\theta$ to small values (like-helicity gluons) by limiting $m_{t\bar{t}}$.



Advantage: $\Delta\phi$ invariant under longitudinal boosts: measure in lab frame!

Disadvantage: Two ν's make $m_{t\bar{t}}$ unobservable.

$\Delta\phi$ distribution at NLO



Solid lines: leading-order prediciton.

Dashed lines: next-to-leading order in strong and weak gauge couplings.

Figure from Bernreuther and Si, arXiv:1003.3926v1 [hep-ph]

Di-lepton Event Reconstruction

8 unknowns: $(E_\nu, \nu_x, \nu_y, \nu_z)$ and $(E_{\bar{\nu}}, \bar{\nu}_x, \bar{\nu}_y, \bar{\nu}_z)$.

6 linear constraint equations:

- 2 from missing E_T.
- 2 top mass constraints.
- 2 W mass constraints.

2 quadratic constraint equations:

- ν and $\bar{\nu}$ mass constraints.

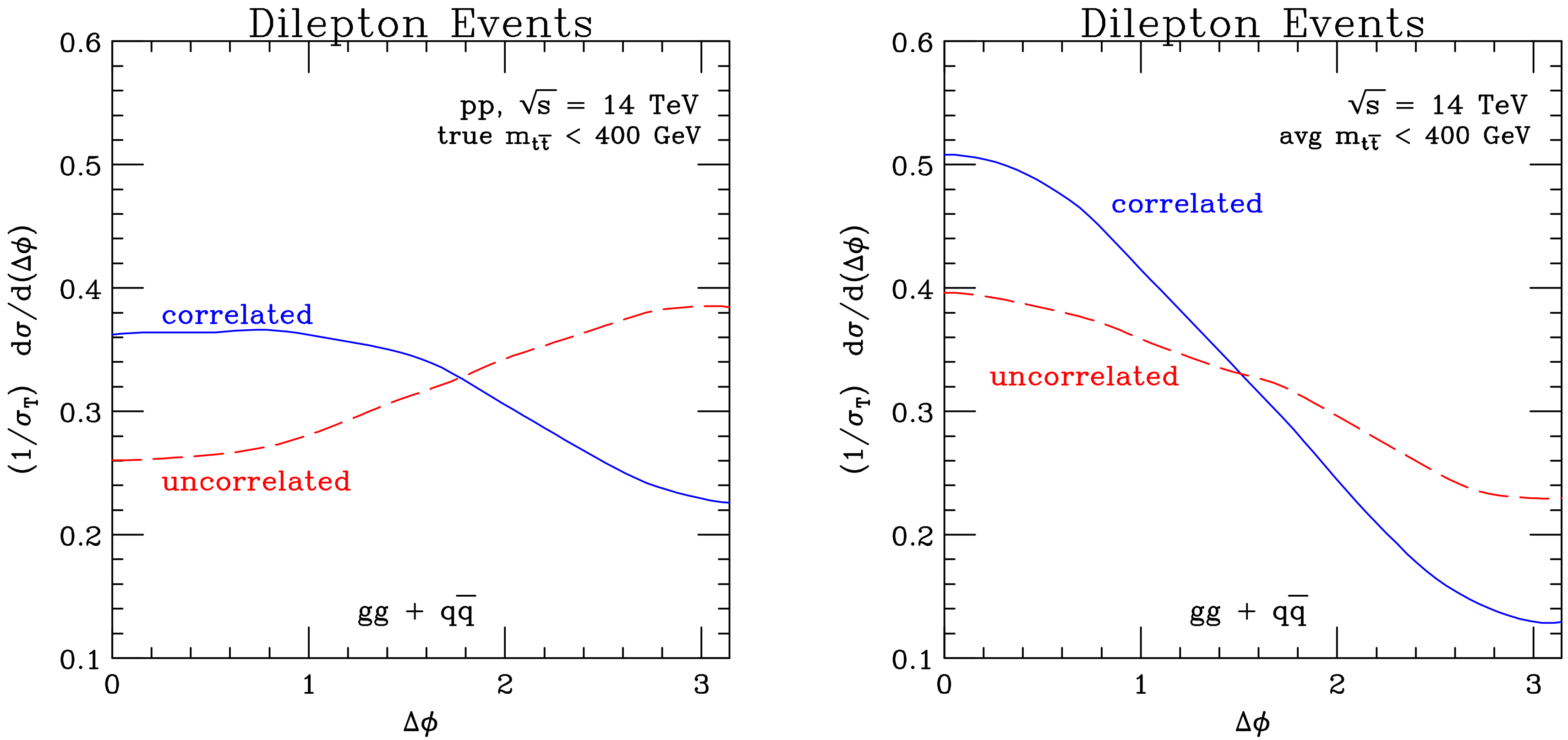
Di-lepton Event Reconstruction

Strategy:

- Pick a b-jet/lepton pairing.
- Solve 6×8 system for 2-parameter family of solutions.
  ⇒ Insert into quadratic constraints.
- Combine 2 quadratic equations with 2 unknowns into single quartic equation.
  - ⋆ Use complex root-finder to extract 4 solutions to quartic equation.
  - ⋆ Discard physically unacceptable solutions.
- Repeat process for other possible b-jet/lepton pairing.
  ⇒ Up to **8** viable solutions result (8 possible values of $m_{t\bar{t}}$).

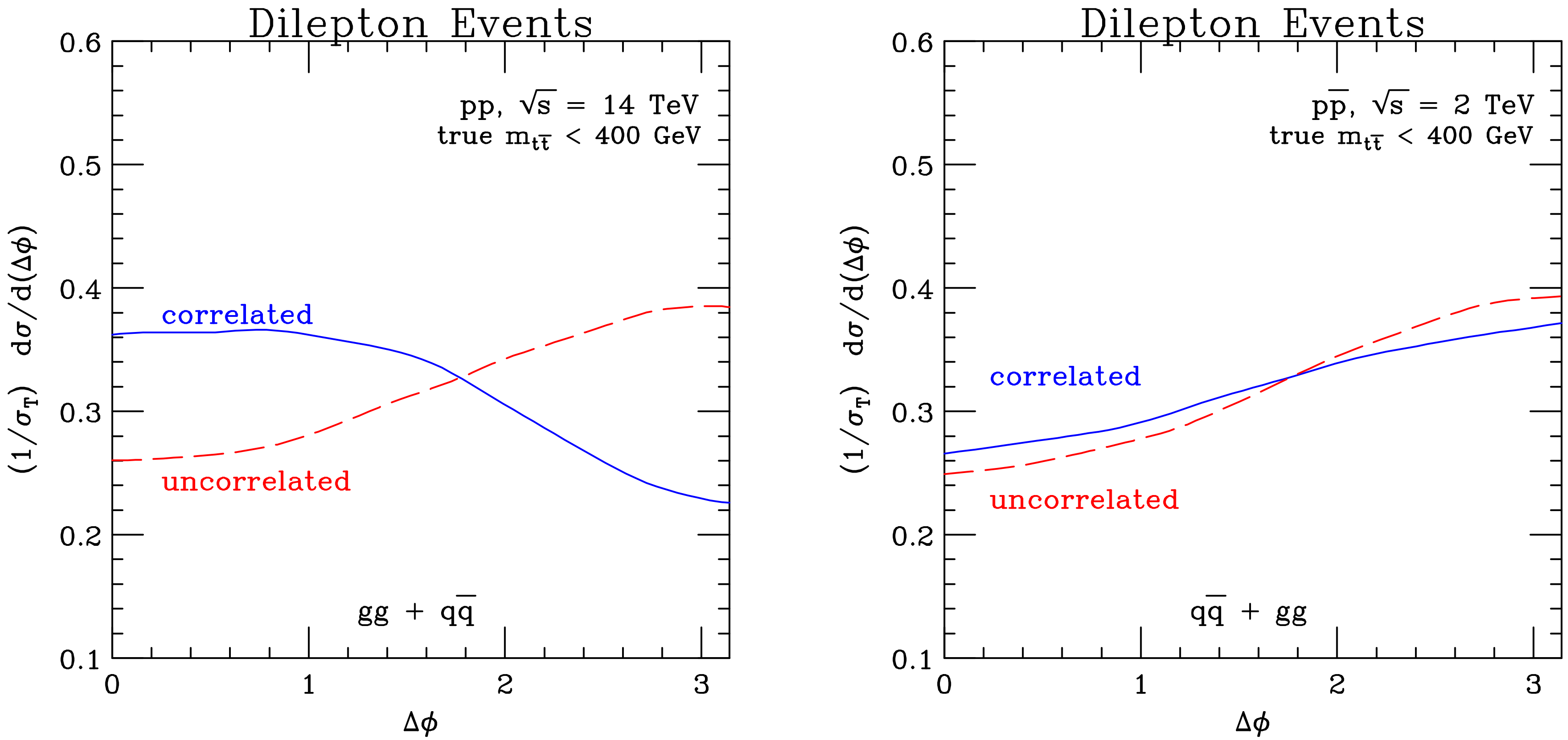
Simplest option: form the (naïve) unweighted average of these values for use in place of (unknown) true value of $m_{t\bar{t}}$.

Di-lepton azimuthal opening angle ($\Delta\phi$ distribution)



- Use of $\langle m_{t\bar{t}} \rangle$ introduces systematic depletion of events near $\Delta\phi = \pi$.
- Significant discriminating power remains, however!
  - ⋆ Need to understand systematics, NLO, very well.
  - ⋆ Does a better substitute for $m_{t\bar{t}}$ exist?

Di-lepton azimuthal opening angle ($\Delta\phi$ distribution)



- $\Delta\phi$ distribution shows almost no effect for the Tevatron!
  - ⋆ Consequence of $q\bar{q}$ vs. gg dominance in initial state.

Observation of significant $\Delta\phi$ correlations at LHC = evidence for gg production.

Correlations using lepton+jets mode

Advantage: single neutrino is over-constrained.

$\Rightarrow$ Can do a good job of locating the ZMF/calculating $m_{t\bar{t}}$.

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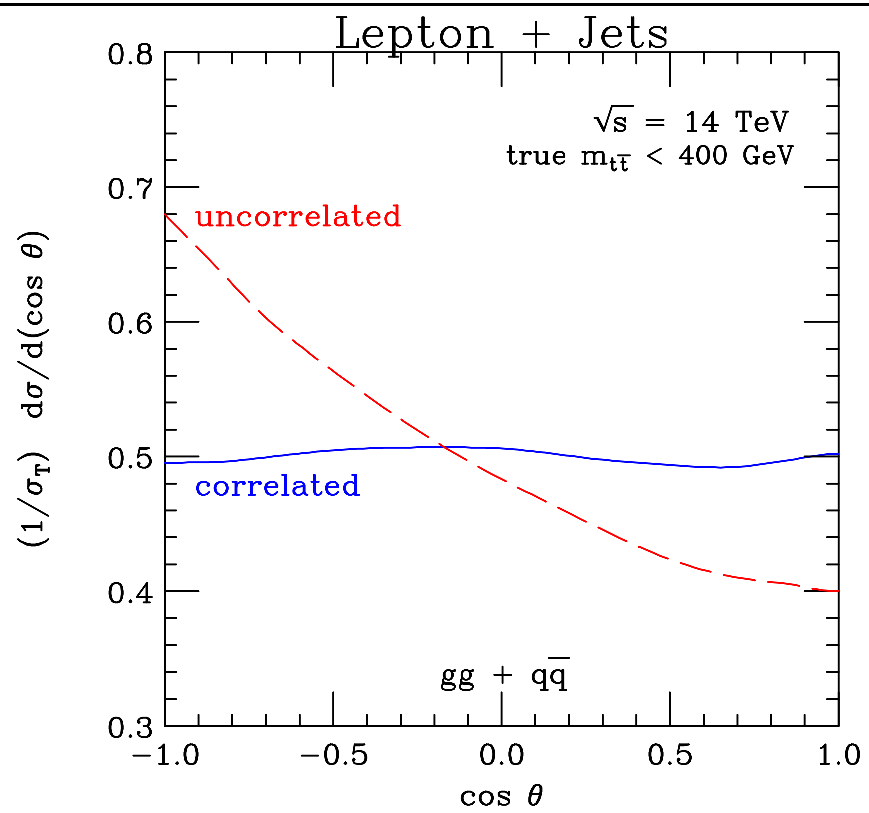
$\Rightarrow$ Use best informed guess:

- Choose jet which smallest spatial separation from b jet in W rest frame [Mahlon and Parke, *Phys. Rev.* **D53**, 4886 (1996)].
- Equivalent to using jet with lowest energy in t rest frame [Ježabek, *Nucl. Phys. Proc. Suppl.* **37B**, 197 (1994)].
- Somewhat reduced correlation ($\alpha \sim 0.5$), but still potentially useful.

Correlations using lepton+jets mode: $\cos\theta_{\bar{e}d}$ in ZMF



Half of area between the curves = 0.07 (compare to 0.11 for $\Delta\phi$ distributions)

- Reduced sensitivity vs. $\Delta\phi$ offset by higher statistics?
- Different systematics, so worth investigating further!

The future: what can be done with $\mathcal{O}(10^6)$ $t\bar{t}$ pairs?

Precision measurements/more tests of the Standard Model

- Probe top, gluon couplings.
  - ⋆ Three diagrams for $gg \to t\bar{t}$ are related in a specific way due to gauge-invariance of QCD.
  - ⋆ All three contribute to the predicted spin correlations.

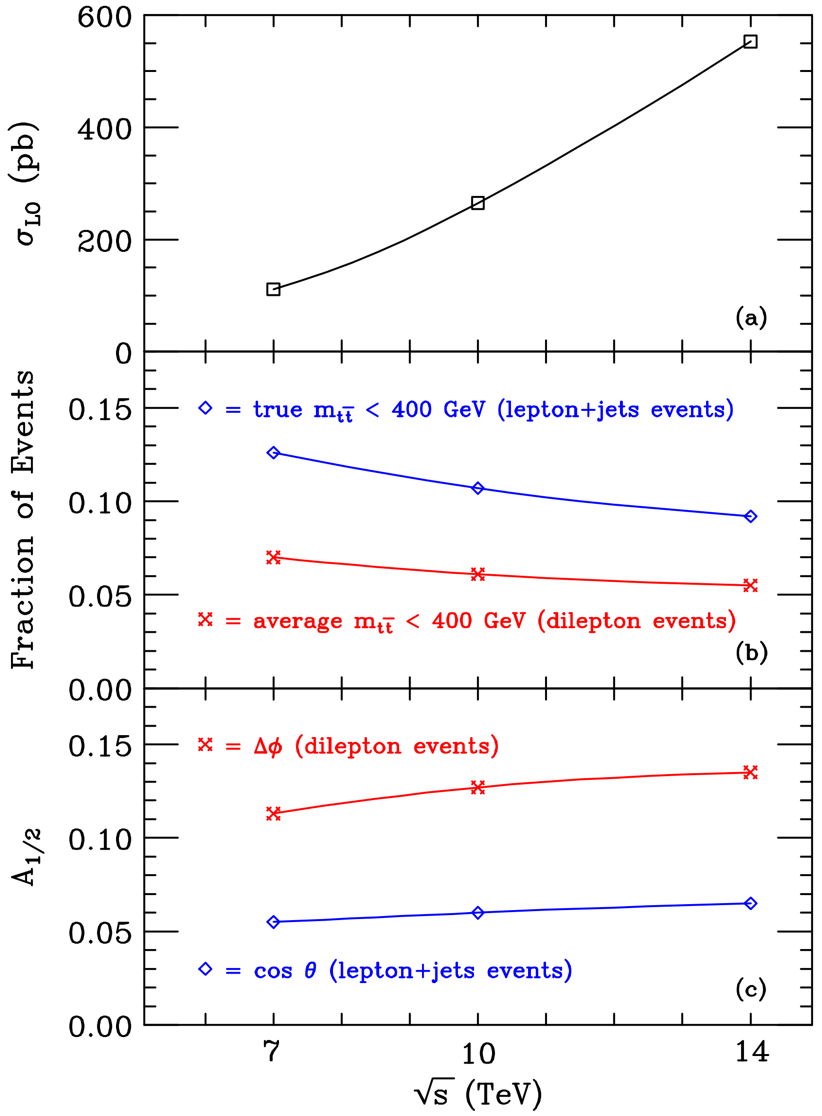
    ⇒ Test of spin correlations = test of QCD gauge invariance.
- Probe tbW vertex.
  - ⋆ Direct test of $V-A$ structure for quarks.
- Look for non-S.M. couplings, particles.

  K.K. gluons?

  "Extra" Higgses?

  "Extra" Z bosons?

Machine energy dependence



Summary and Conclusions

- The Tevatron and LHC probe two different aspects of spin correlations in top quark pair production and decay.

  - Tevatron production is mostly $q\bar{q} \to t\bar{t}$

    1. Spin correlations are largest in the off-diagonal spin basis (opposite spin top pairs dominate)

    2. Try to extract joint decay angular distribution in t,$\bar{t}$ rest frames.

    3. Measurement hampered by low statistics.

Summary and Conclusions

- LHC production is mostly $gg \to t\bar{t}$

  1. Rich spin structure in $gg \to t\bar{t}$

  2. Opposite-helicity gluons: maximize correlations with off-diagonal basis (same as $q\bar{q} \to t\bar{t}$).

  3. Like-helicity gluons: maximize correlations with helicity basis (like-helicity top pairs dominate).

  4. Restrict $m_{t\bar{t}}$ to enhance contributions from like- or opposite-helicity gluons.

     → Like helicity: $\gamma\beta\sin\theta < 1$

     → Opposite helicity: $\gamma\beta\sin\theta > 1$

  5. Promising variables for observing correlations:

     → Di-lepton mode: $\Delta\phi_{\bar{e}\mu}$

     "Easy" lab-frame measurement!

     → Lepton+jets mode: $\cos\theta_{\bar{e}d}$

Summary and Conclusions

The story of top pair spin correlations has only just begun.

- Tevatron on verge of unambiguously seeing the correlations for the first time (but statistics-limited).
- The LHC (even running at 7 TeV) ought to have sufficient statistics to firmly establish a spin-correlation effect.
    - ⋆ Observation of spin correlations $\Rightarrow$ upper limit on top quark lifetime.
    - ⋆ Observation of correlation effect in $\Delta\phi$ distribution provides evidence of top pair production via gluon fusion.

With millions of $t\bar{t}$ events on the horizon, precision (%-level) measurements of correlation parameters will be possible.