Theoretical issues on the top mass reconstruction at hadron colliders

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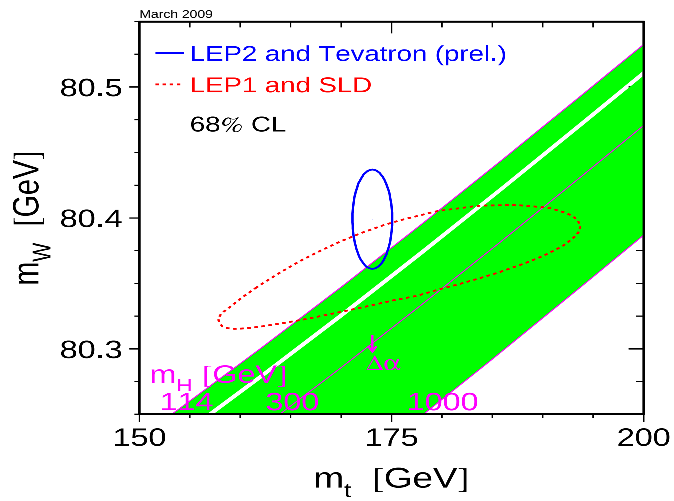
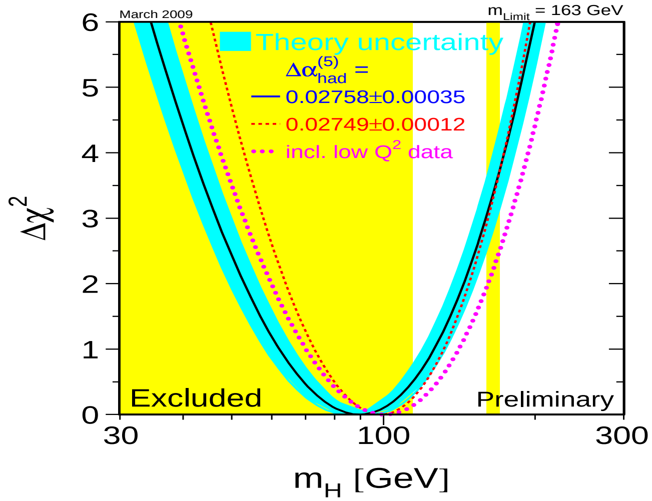
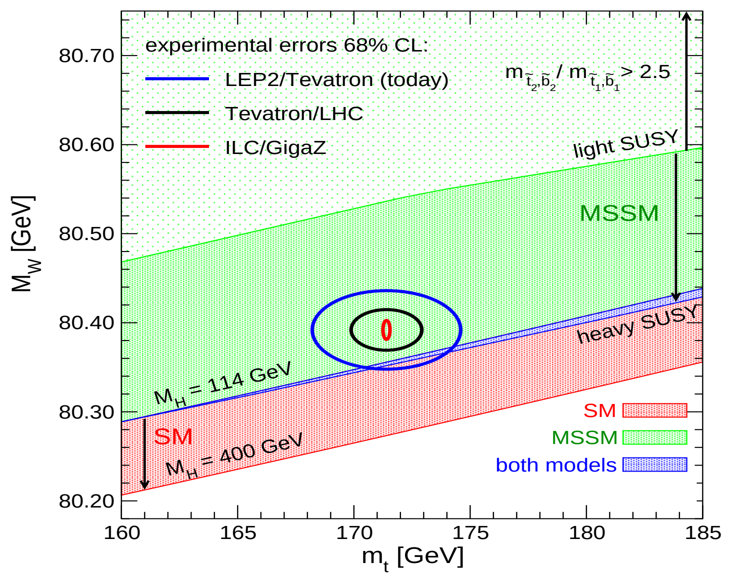
1. **Introduction**
2. **Top mass definitions and extraction**
3. **Systematic uncertainties due to bottom fragmentation in top decay and jet algorithms**
4. **Conclusions**

Based on work by Hoang, Stewart, Scimemi, Beneke, Langenfeld, Moch, Uwer, Mescia, Kharchilava, Drollinger, Seymour, Jain, Cerrito, Tevlin, Mitov, G.C. as well as Tevatron/LHC analyses

The top mass is fundamental in the SM as it constrains the Higgs mass





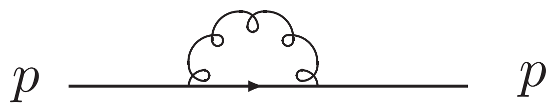


Heinemeyer, Hollik, Stöckinger, Weber, Weiglein

What top mass is measured? How should we interprete the theory error?

Top mass definitions

Subtraction of the UV divergences in the self energy $\Sigma(p)$



Renormalized propagator: $S^{-1}(p) = -i[\not{p} - m_t^0 + \Sigma^R(p, m_t^0, \mu)]$

Mass is solution of equation $\not{p} - m_t + \Sigma^R(p, m_t, \mu) = 0$

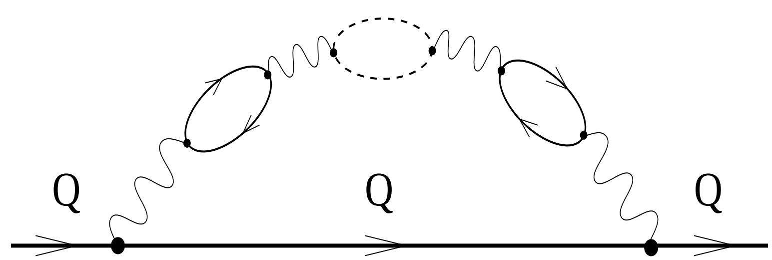
Pole mass:

$$\Sigma^R(p) = 0 \quad \text{and} \quad \frac{\partial \Sigma^R}{\partial \not{p}} = 0 \quad \text{for} \quad \not{p} = m$$

OK for electrons, but for quarks non-perturbative ambiguity: $\Delta m \sim \Lambda_{\rm QCD}$

Higher-order corrections lead to infrared renormalons:

$$\Sigma(m) \sim m \sum_n \alpha_S^{n+1} \, (2\beta_0)^n \, n!$$



$\overline{\rm MS}$ **mass $\bar{m}_t(\mu)$– dimensional regularization** $D = 4 - 2\epsilon$

$$\Sigma(p) = \frac{i\alpha_S C_F}{4\pi}\left\{\left[\frac{1}{\epsilon} - \gamma + \ln 4\pi + A(m_t^0, p, \mu)\right]\not{p} - \left[4\left(\frac{1}{\epsilon} - \gamma + \ln 4\pi\right) + B(m_t^0, p, \mu)\right] m_t^0\right\}$$

Counterterm to subtract $(1/\epsilon + \gamma_E - \ln 4\pi)$

Relation with the pole mass (coefficients c_i depending on $\ln[\mu^2/\bar{m}_t(\mu)^2]$)

$$m_t = \bar{m}_t(\mu)\left[1 + \alpha_S(\mu)c_1 + \alpha_S^2(\mu)c_2 + \dots\right]$$

Processes with off-shell top quarks; at threshold contributions $\sim (\alpha_S/v^2)^k$

Potential-subtracted mass: δm subtracts the IR divergences in the pole mass

$$m_{\rm PS}(\mu_F) = m_{\rm pole} - \delta m(\mu_F) \;\; ; \;\; \delta m(\mu_F) = \frac{1}{2}\int_{|q|<\mu_F}\frac{d^3q}{(2\pi)^3}\tilde{V}(q)$$

Short-distance masses in terms of an infrared scale R, e.g. μ_F, $\bar{m}(\mu)$, etc.

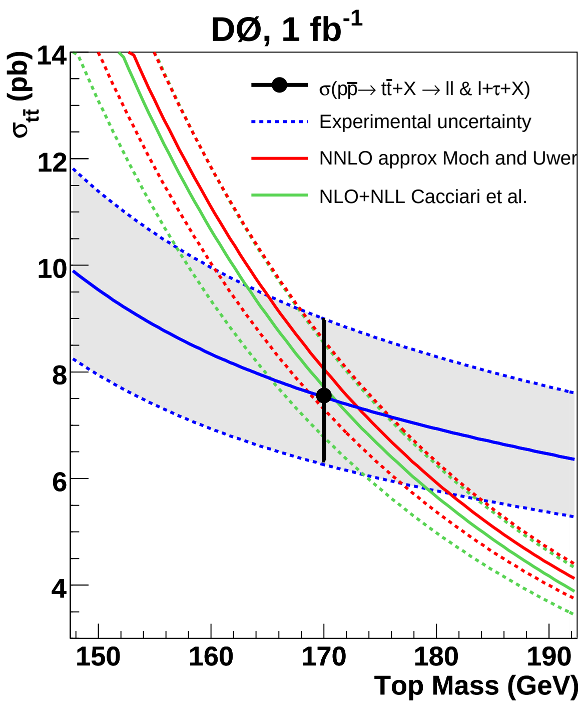
$$M_{\rm pole} = m(R,\mu) + \delta m(R,\mu) \;\; ; \;\; \delta m(R,\mu) = R\sum_{n=1}^{\infty}\sum_{k=0}^{n} a_{nk}\alpha_S(\mu)^n$$

$$\frac{dM_{\rm pole}}{d\ln\mu} = 0 \Rightarrow \frac{dm(R,\mu)}{d\ln\mu} = -R\gamma[\alpha_S(\mu)] \;\; \text{(Hoang)}$$

Top mass reconstruction from cross section (top production) or final-state observables (top decay)

$$\sigma^{\rm NLO}(pp \to t\bar{t}) = \sum_{a,b} \int dx_1 dx_2 f_a(x_1,\mu_F)^{\rm NLO} f_b(x_2,\mu_F)^{\rm NLO} \hat{\sigma}(ab \to t\bar{t}, \alpha_S(\mu_R), \mu_R, \mu_F)^{\rm NLO} + \mathcal{O}\left(\frac{\Lambda^2}{Q^2}\right)$$

D0 (PLB 679 (2009) 177): extraction of the pole mass from the cross section



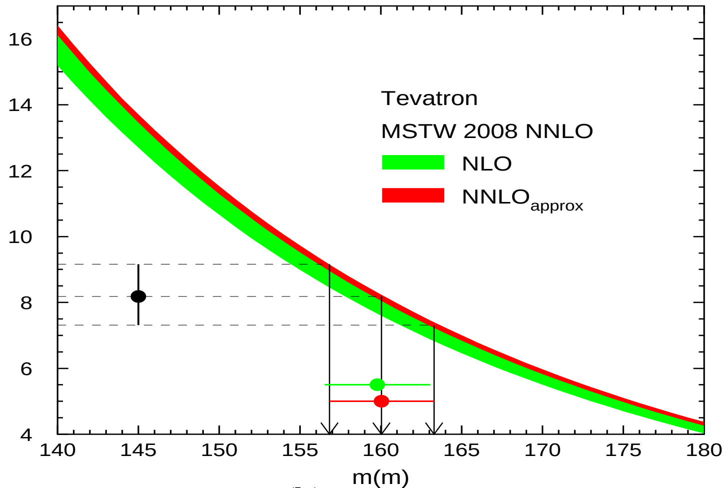
Moch and Uwer: also Sudakov logs $\alpha_S^4 \ln^k \beta$, $k \le 4$

Coulomb corrections $\sim 1/\beta, 1/\beta^2$ to NNLO in $q\bar{q}$ and gg: $M_t = 169.1^{+5.9}_{-5.2}$ GeV

Cacciari et al., NLO hard scattering + NLL threshold resummation: $M_t = 167.5^{+5.8}_{-5.6}$ GeV

P. Nadolsky et al. (NLO): $M_t = 165.5^{+6.1}_{-5.9}$ GeV

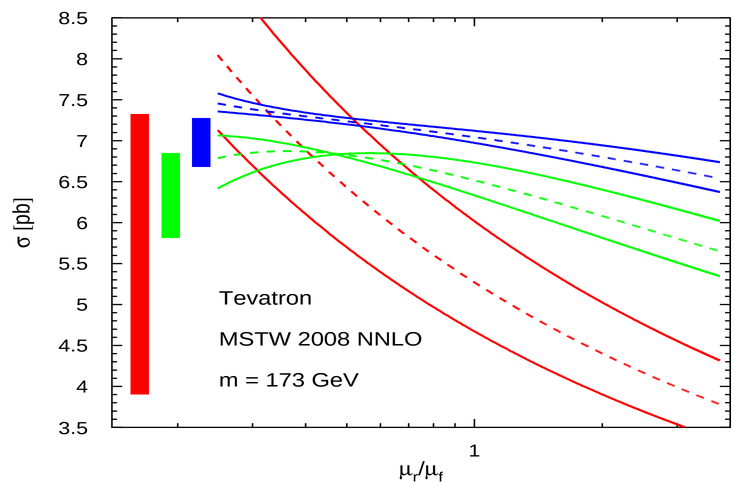
In terms of the $\overline{\mathrm{MS}}$ mass $\bar{m}_t(\bar{m}_t)$ mass (Langenfeld, Moch, Uwer)

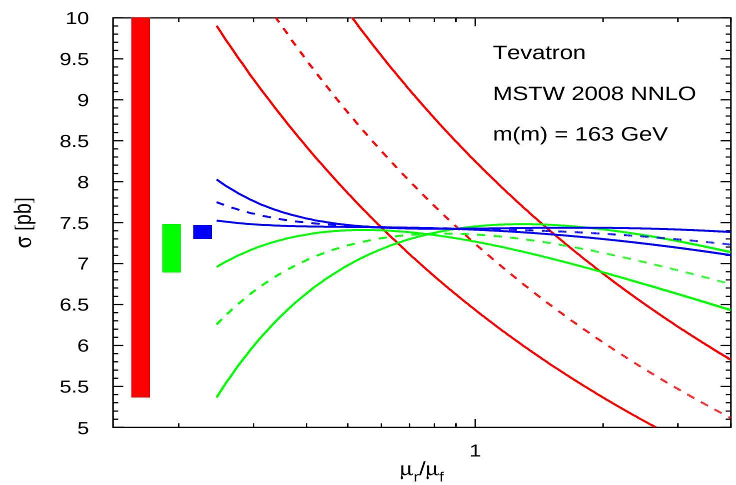


	$\bar{m}_t(\bar{m}_t)$	M_t
LO	159.2 GeV	159.2 GeV
NLO	159.8 GeV	165.8 GeV
NLO+"NNLO"	160.0 GeV	168.2 GeV

Scale dependence of σ [pb] in terms of pole and $\overline{\mathrm{MS}}$ mass, in the range $[m_t/2, 2m_t]$

Red: LO; Green: NLO; Blue: 'approximate NNLO'





The $\overline{\mathrm{MS}}$ mass yields a lower scale dependence with respect to the pole mass

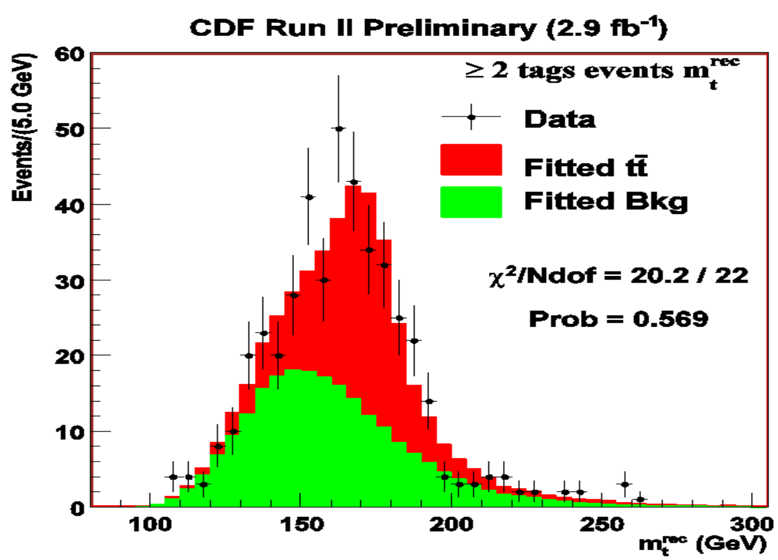
Top mass reconstruction at the Tevatron from final-state observables:

Matrix-element method: extract information from top events for a given theoretical prediction for $t\bar{t}$ production and decay:

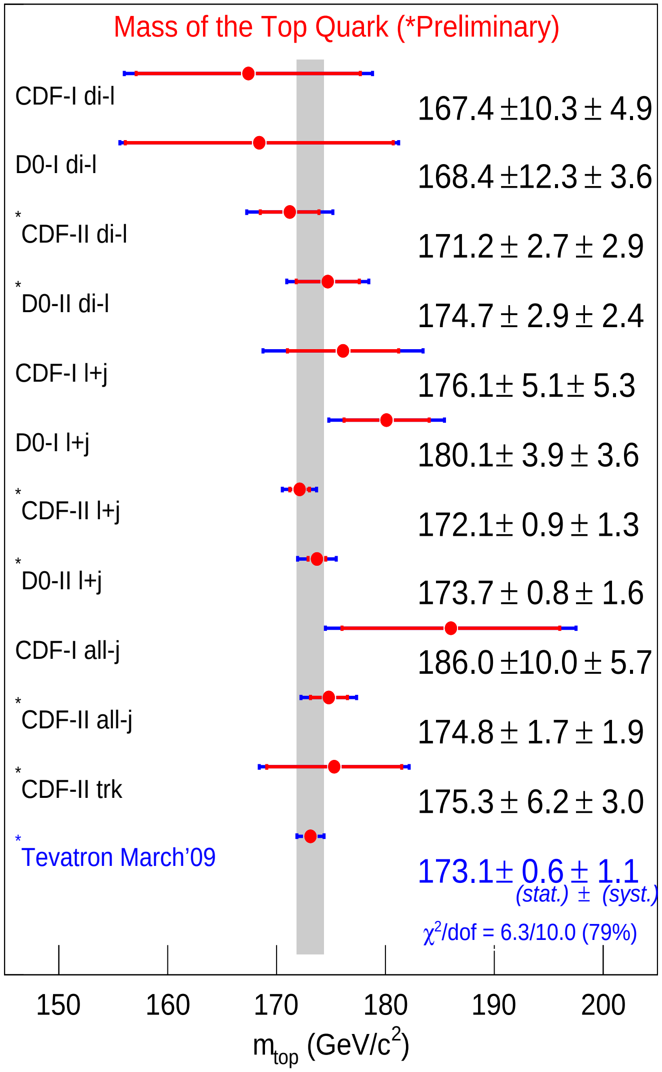
$$P(x|M_t) = \frac{1}{N}\int d\Phi |\mathcal{M}(ij \to t\bar{t} \to y)|^2 W(x,y,\mathrm{JES}) f_i f_j$$

W: probability of reconstructing the fermionic final state y given the measurement of x in the detector

Template measurement: find many variables correlated to the top mass, then form multivariate probability density function and fit for m_t (ex. all jets at CDF)



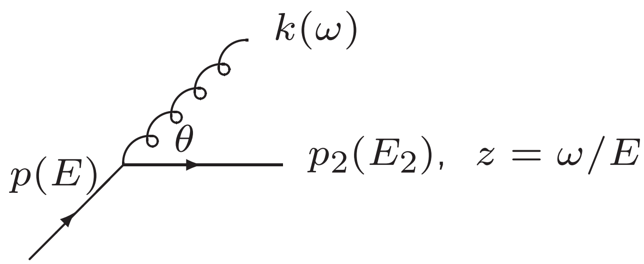
Top mass world average (Tevatron combination)



	Run I published					Run II preliminary					
	CDF			DØ		CDF				DØ	
	all-j	l+j	di-l	l+j	di-l	l+j	di-l	all-j	trk	l+j	di-l
$\int \mathcal{L}\, dt$	0.1	0.1	0.1	0.1	0.1	3.2	1.9	2.9	1.9	3.6	3.6
Result	186.00	176.10	167.40	180.10	168.40	172.14	171.15	174.80	175.30	173.75	174.66
iJES	0.00	0.00	0.00	0.00	0.00	0.74	0.00	1.64	0.00	0.47	0.00
aJES	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.91	1.32
bJES	0.60	0.60	0.80	0.71	0.71	0.38	0.40	0.21	0.00	0.07	0.26
cJES	3.00	2.70	2.60	2.00	2.00	0.32	1.73	0.49	0.60	0.00	0.00
dJES	0.30	0.70	0.60	0.00	0.00	0.08	0.09	0.08	0.00	0.84	1.46
rJES	4.00	3.35	2.65	2.53	1.12	0.40	1.90	0.21	0.10	0.00	0.00
lepPt	0.00	0.00	0.00	0.00	0.00	0.18	0.10	0.00	1.10	0.18	0.32
Signal	1.80	2.60	2.80	1.11	1.80	0.34	0.78	0.23	1.60	0.45	0.65
MC	0.80	0.10	0.60	0.00	0.00	0.51	0.90	0.31	0.60	0.58	1.00
UN/MI	0.00	0.00	0.00	1.30	1.30	0.00	0.00	0.00	0.00	0.00	0.00
BG	1.70	1.30	0.30	1.00	1.10	0.50	0.38	0.35	1.60	0.08	0.08
Fit	0.60	0.00	0.70	0.58	1.14	0.16	0.60	0.67	1.40	0.21	0.51
CR	0.00	0.00	0.00	0.00	0.00	0.41	0.40	0.41	0.40	0.40	0.40
MHI	0.00	0.00	0.00	0.00	0.00	0.09	0.20	0.17	0.70	0.05	0.00
Syst.	5.71	5.28	4.85	3.89	3.63	1.35	2.98	1.99	3.11	1.60	2.43
Stat.	10.00	5.10	10.30	3.60	12.30	0.94	2.67	1.70	6.20	0.83	2.92
Total	11.51	7.34	11.39	5.30	12.83	1.64	4.00	2.61	6.94	1.80	3.80

	Tevatron Combined
Result	173.12
iJES	0.48
aJES	0.33
bJES	0.23
cJES	0.19
dJES	0.30
rJES	0.13
lepPt	0.11
Signal	0.30
MC	0.49
UN/MI	0.03
BG	0.26
Fit	0.16
CR	0.41
MHI	0.07
Syst.	1.07
Stat.	0.65
Total	1.25

Top mass measurement driven by parton shower generators



$$dP = \frac{\alpha_S}{2\pi}\ P(z)dz\ \frac{dQ^2}{Q^2}\ \Delta_S(Q^2_{\max}, Q^2)$$

Q^2: **ordering variable**

$\Delta_S(Q^2_{\max}, Q^2)$ **: no radiation in** $[Q^2, Q^2_{\max}]$ **(soft/collinear virtual corrections)**

$$\Delta_S(Q^2_{\max}, Q^2) = \exp\left[-\frac{\alpha_S}{2\pi}\int_{Q^2}^{Q^2_{\max}}\frac{dQ'^2}{Q'^2}\int_{z_{\min}}^{z_{\max}} dz P(z)\right]$$

HERWIG : $Q^2 = E^2(1-\cos\theta) \simeq E^2\theta^2/2$ **(angular ordering);**

PYTHIA: $Q^2 = p^2$ **or** k_T^2

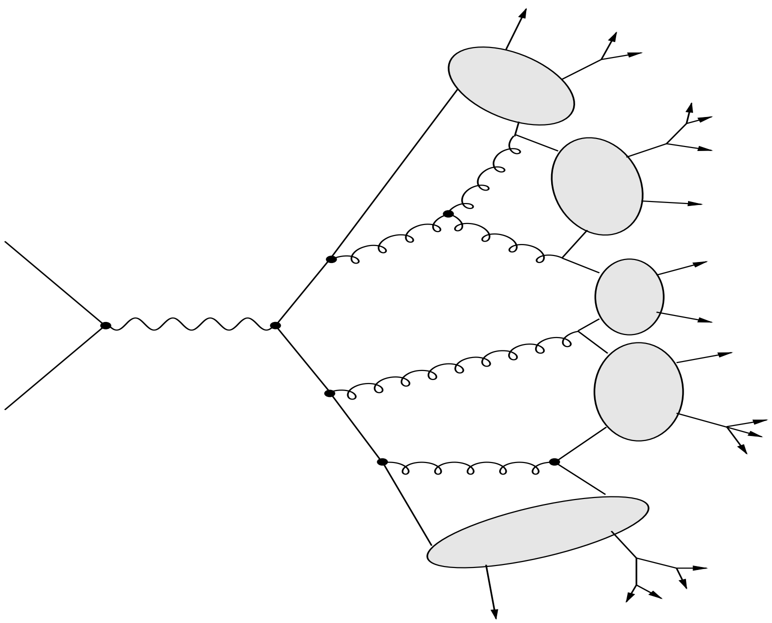
$\alpha_S(k_T^2)$ **with two-loop evolution in HERWIG and one-loop in PYTHIA**

Total cross section LO thanks to unitarity $(1 = R + V)$

Distributions equivalent to threshold LL resummation, + some NLLs

$\Lambda \to \Lambda_{\rm MC} = \Lambda \exp(4K\beta_0)$: NLL Sudakov form factor at large x (Catani, Marchesini and Webber)

Hadronization models



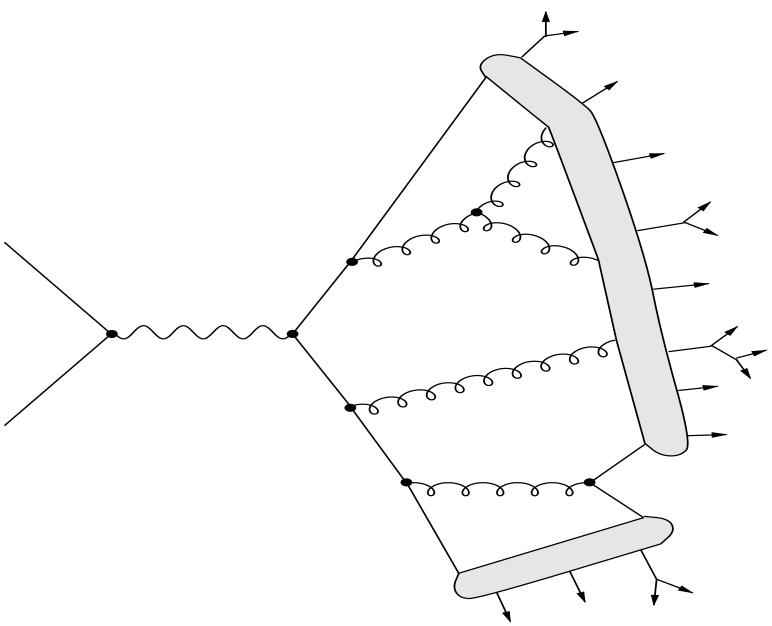
Cluster model (HERWIG)

Perturbative evolution ends at $Q^2 = Q_0^2$

Angular ordering $\Rightarrow$ colour preconfinement

Forced gluon splitting $(g \to q\bar{q})$

Colour-singlet clusters decay into the observed hadrons



String model (PYTHIA)

q and $\bar{q}$ move in opposite direction

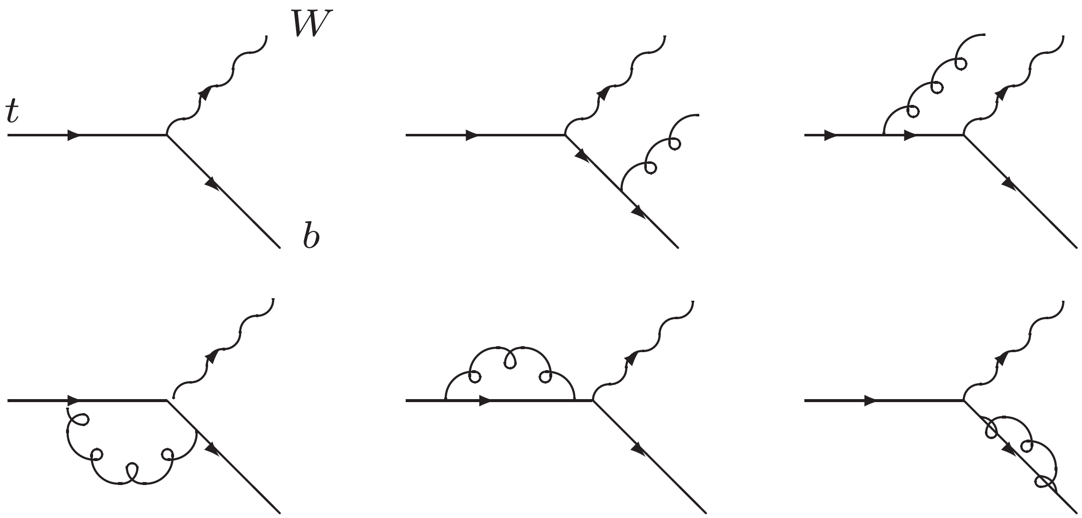
The colour field collapses into a string, with uniform energy density

$q\bar{q}$ pairs are produced

The string breaks into the observed hadrons

Figures from R.K. Ellis, W.J. Stirling and B.R. Webber, QCD and Collider Physics

Top decay at NLO (neglecting interference for $E_T \gg \Gamma_t$)



$t(q) \to b(p_b) W(p_W)\ (g(p_g))$

$$x_b = \frac{1}{1-m_W^2/m_t^2}\ \frac{2p_b \cdot p_t}{m_t^2}$$

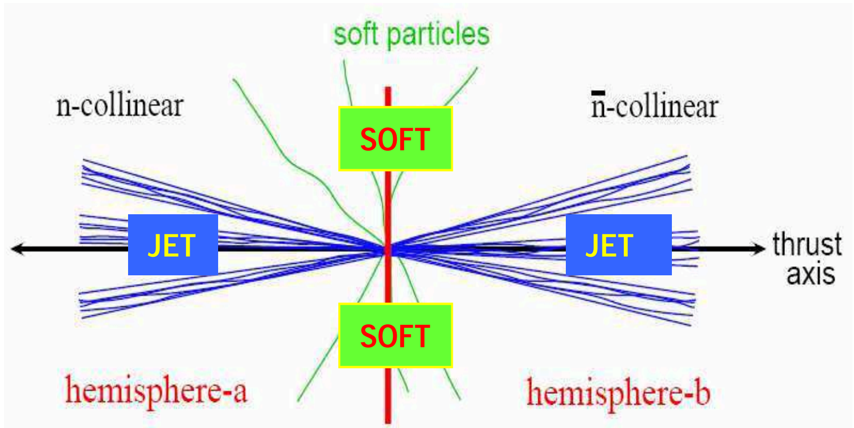
Total width up to NNLO; b spectrum at NLO+NLL threshold resummation available the PFF formalism and using the top pole mass

Parton showers matched to tree-level $\Gamma(t \to bWg)$ (hard/large-angle radiation)

The top mass in top decays in HERWIG/PYTHIA should be related to the pole mass (on-shell) and in fact the world average (relying on MC's) agrees with the pole mass extracted from the (N)NLO cross section

Open questions: width ambiguity? Missing higher orders in the top self-energy? Top colour connection? Hadronization corrections?

Top jet mass from QCD to SCET/HQET framework $Q \gg m_t \gg \Gamma_t \gg \Lambda_{\rm QCD}$



Top-decay invariant masses: $M_t^2 = \left(\sum_{i\in a} p_i^\mu\right)^2,\ M_{\bar t}^2 = \left(\sum_{i\in b} p_i^\mu\right)^2$

Factorization theorem: (Hoang et al.)

$$\frac{d\sigma}{dM_t^2 dM_{\bar t}^2} \sim H_Q(Q,\mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \int d\ell^+ d\ell^- B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_+\left(\hat{s}_{\bar t} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S(\ell^+, \ell, \mu)$$

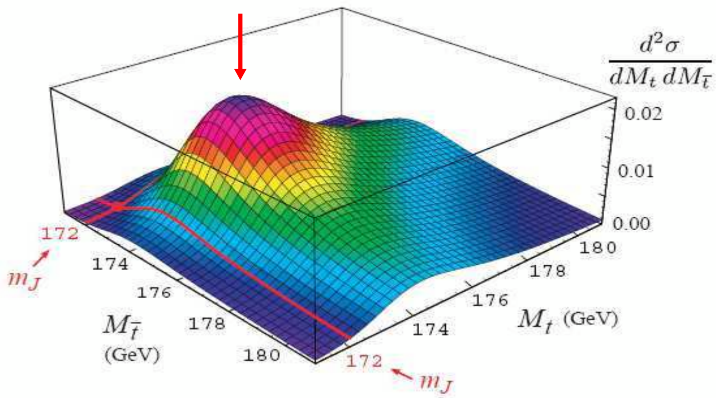
H_Q, H_m: hard scattering coefficient functions at scales Q and m_t

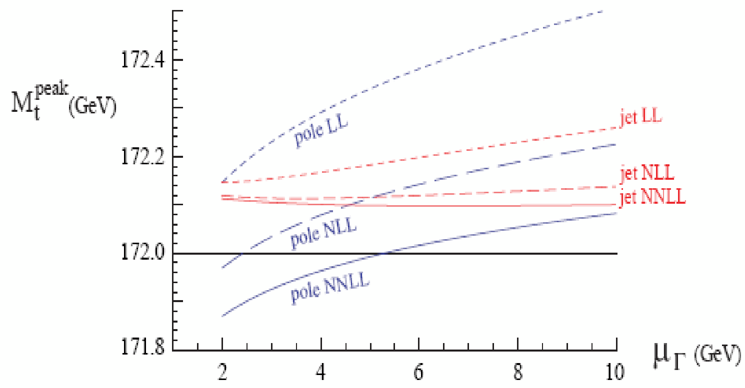
$B_\pm$: heavy-quark jet functions, describing top evolution into jets

$S(\ell^+, \ell^-, \mu)$: non-perturbative fragmentation function, depending on soft emissions, ruling dijet and mass distributions

Large logarithms $\ln(Q/m_t)$, $\ln(m_t/\Gamma_t)$, etc.

Double-differential distribution at NLL and mass peak position





Peak position is independent on the mass scheme:

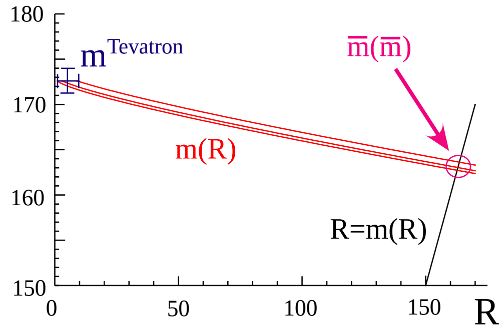
$$M_t^{\rm peak} = m + \Gamma_t(c_1\alpha_S + c_2\alpha_S^2) + \frac{c_3 Q\Lambda}{m}$$

Jet mass: short-distance (resonance) mass with $R \sim \Gamma_t$

$$\delta m_J \sim \left[\frac{d\ln \tilde{B}(y,\mu)}{dy}\right]_{y=-ie^{-\gamma_E}/R} \quad \Rightarrow \quad m_{\rm pole} = m_J(\mu) + e^{\gamma_E}\Gamma_t \frac{\alpha_S(\mu) C_F}{\pi}\left(\ln\frac{\mu}{\Gamma_t} + \frac{1}{2}\right) + \mathcal{O}(\alpha_S^2)$$

$\mu \simeq Q_0$: one can identify $m_J(\mu)$ with the Tevatron mass and evolve to $\bar{m}(\bar{m})$

How about $\delta m^{\rm MC}$?



Methods on the top mass reconstruction relying on b-quark fragmentation

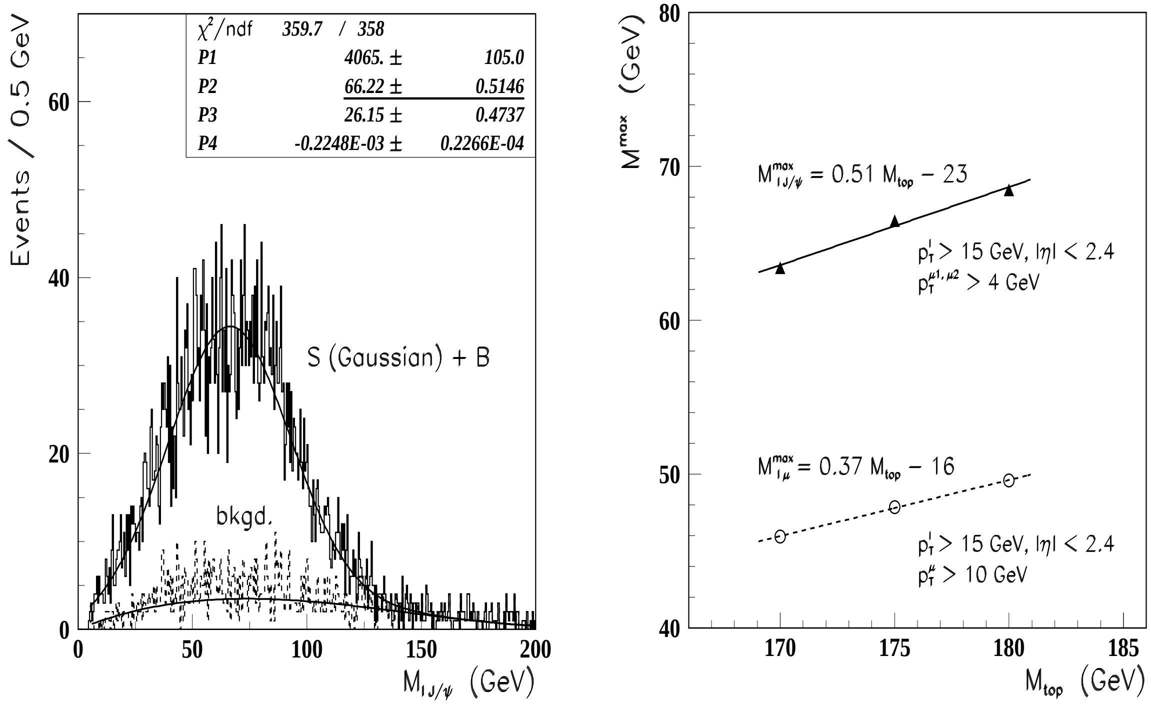
Final states with leptons and J/ψ, i.e. $W \to \ell\nu$ and $B \to J/\psi X$, $J/\psi \to \mu^+\mu^-$

A. Kharchilava, PLB 476 (2000) 73 **(PYTHIA + Peterson fragmentation function)**

$m_{\ell J/\psi} = 0.51\ m_t - 23$ GeV

$\Delta m_{\ell J/\psi} \simeq 0.5$ GeV $\Rightarrow \Delta m_t \simeq 1$ GeV

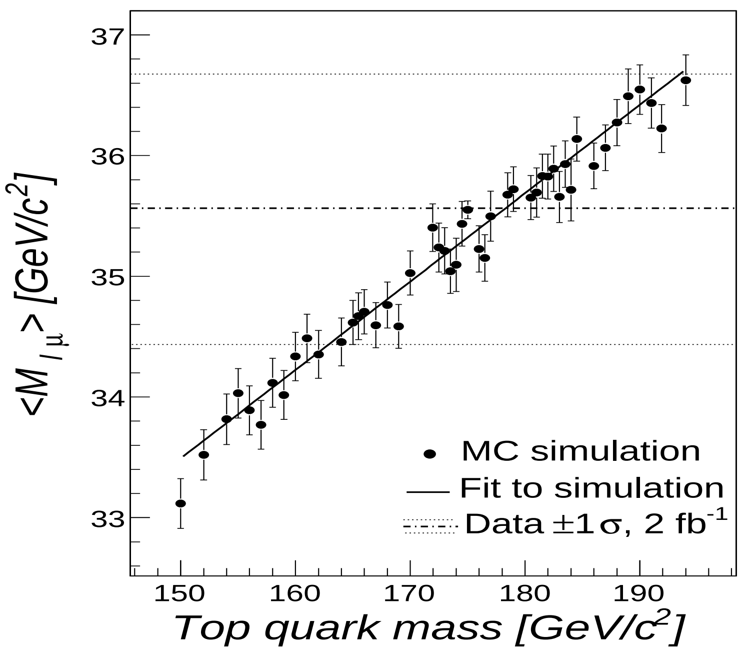
Δm_t (b-frag)$\simeq 0.6$ GeV **obtained varying** $\epsilon = (5.0 \pm 0.5) \times 10^{-3}$

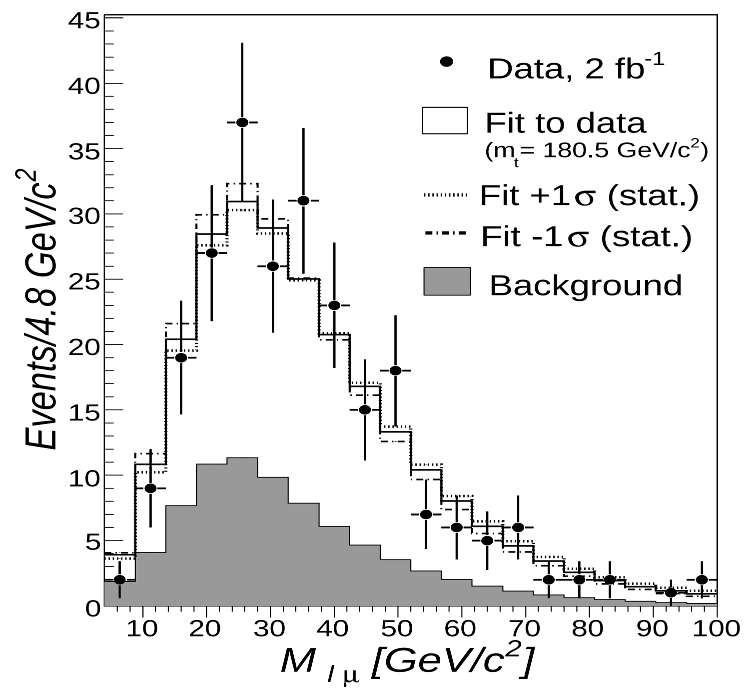


Soft-muon b-tagged events: $m_{\ell\mu}$, with μ from semileptonic B decays:

$m_t = 180.5 \pm 12.0 \pm 3.4 GeV$

Overall Monte Carlo uncertainty, including b-fragmentation, but not initial- and final-state radiation, $\Delta m_t \simeq 2.1$ GeV (L.Cerrito et al. [CDF] 2009)





Strong dependence on Monte Carlo modelling of top decays and $b \to B$ transition

Reconsidering b-fragmentation and tuning of cluster and string models

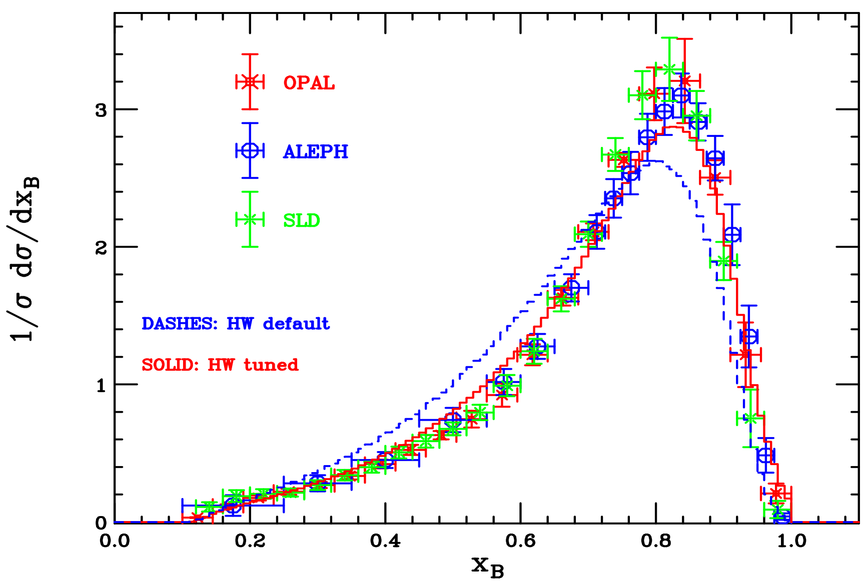
(G.C. and F. Mescia, EPJ C65 (2010) 171)

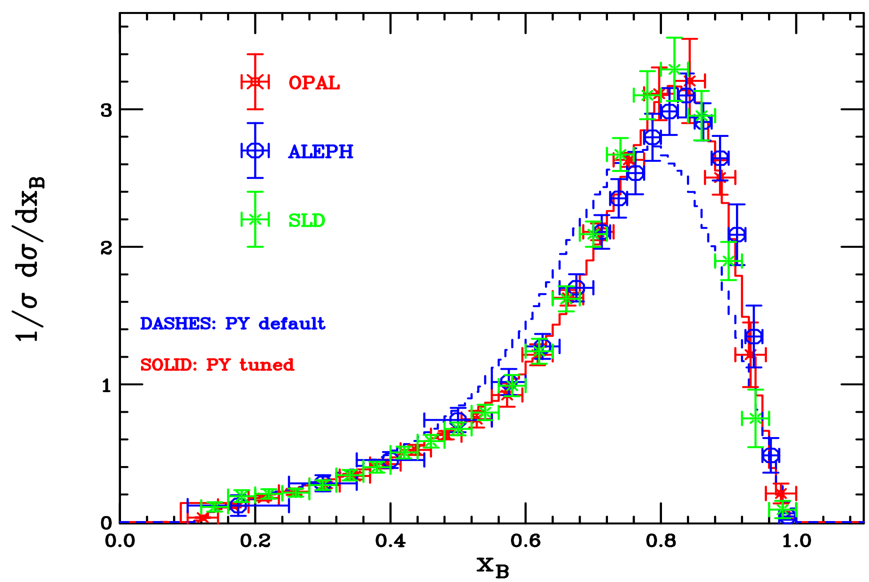
Monte Carlo tuning: $e^+e^- \to Z^0 \to b\bar{b} \to BX_{\bar{b}}$ $\quad x_B = 2E_B/m_Z$

(G. C. and V. Drollinger, NPB 730 (2005) 82)

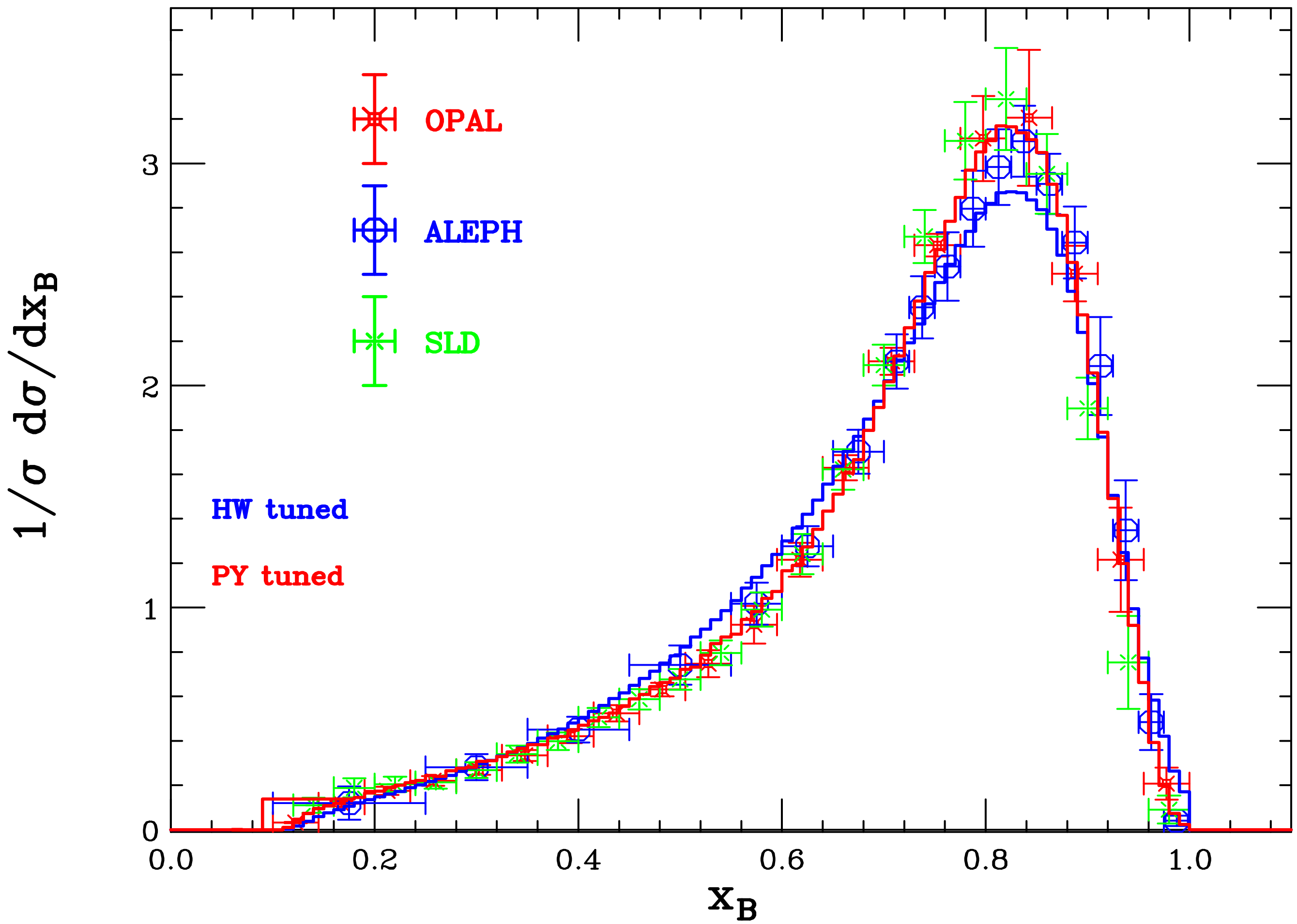
HERWIG	PYTHIA
CLSMR(1) = 0.4 (0.0)	
CLSMR(2) = 0.3 (0.0)	PARJ(41) = 0.85 (0.30)
DECWT = 0.7 (1.0)	PARJ(42) = 1.03 (0.58)
CLPOW = 2.1 (2.0)	PARJ(46) = 0.85 (1.00)
PSPLT(2) = 0.33 (1.00)	
χ^2/dof = 222.4/61 (739.4/61)	χ^2/dof = 45.7/61 (467.9/61)

Lund/Bowler fragmentation function : $f_B(z) \propto \frac{1}{z^{1+brm_b^2}}(1-z)^a \exp(-bm_T^2/z)$

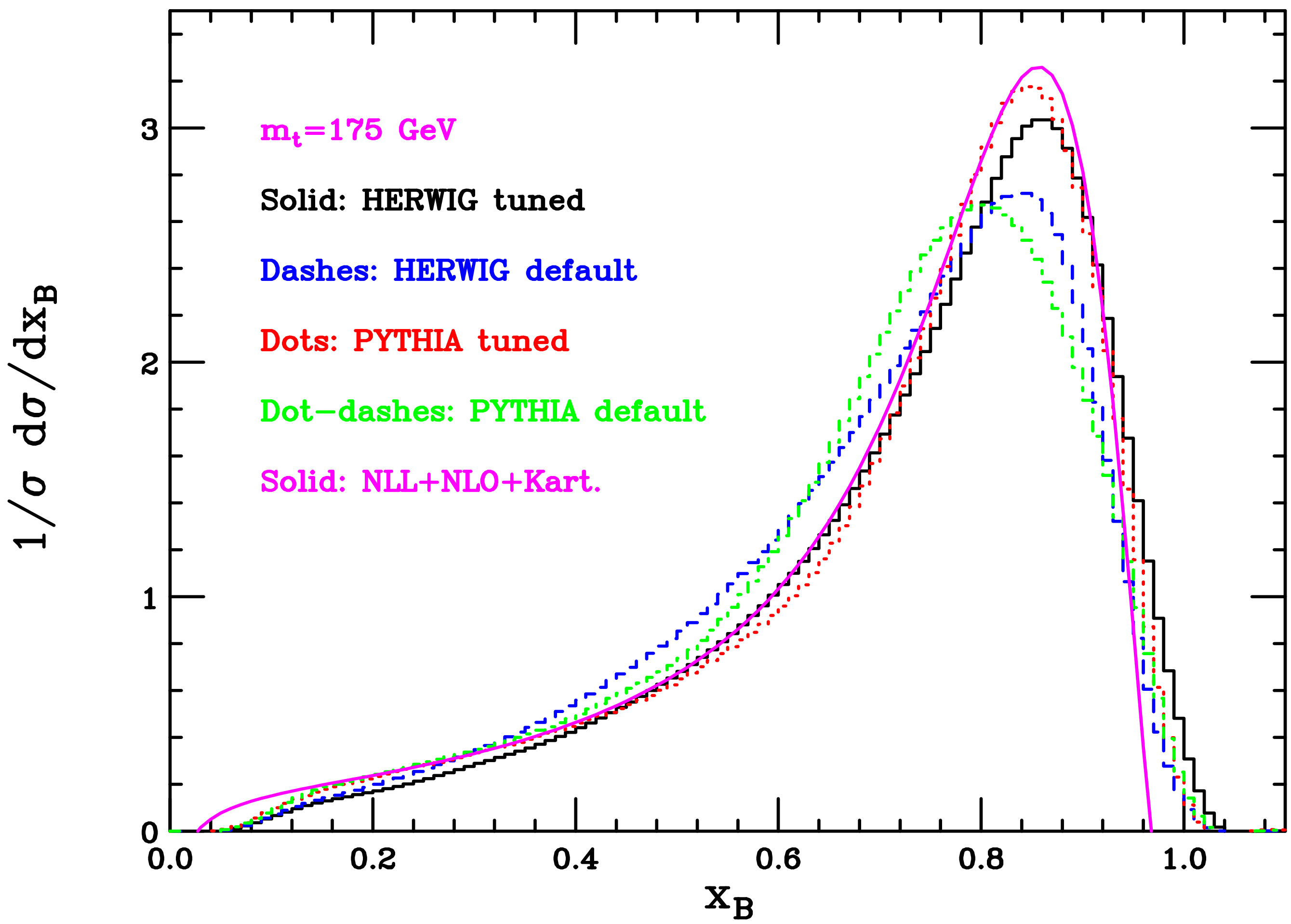




Comparing tuned HERWIG and PYTHIA



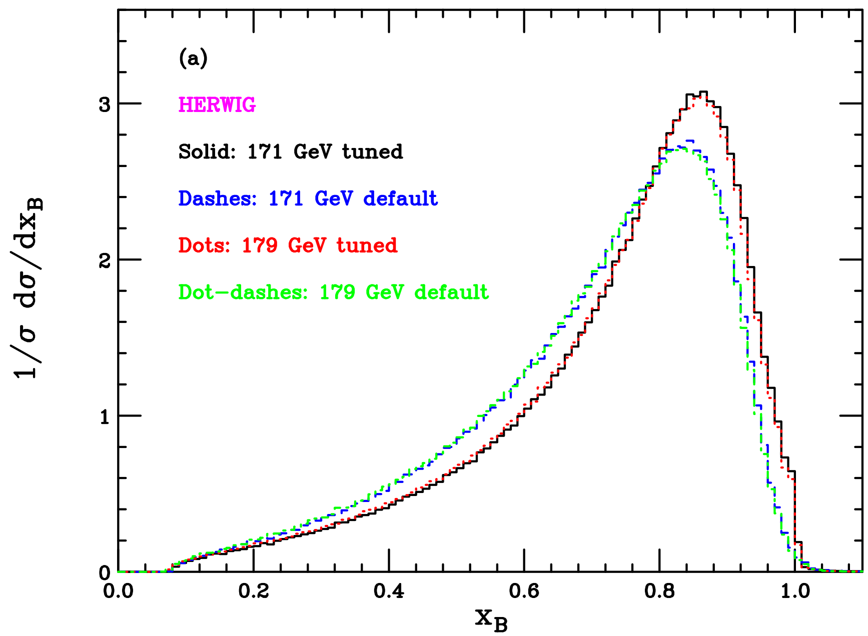
B-hadron spectrum in top decay : $x_B = \frac{1}{1-m_W^2/m_t^2}\frac{2p_B\cdot p_t}{m_t^2}$

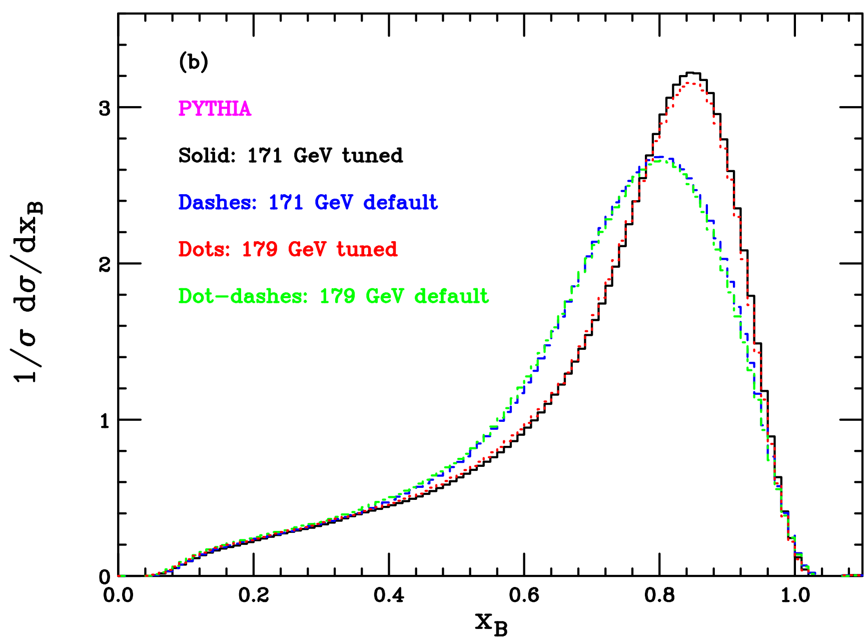


Also shown is the NLO+NLL prediction, using the Kartvelishvili model

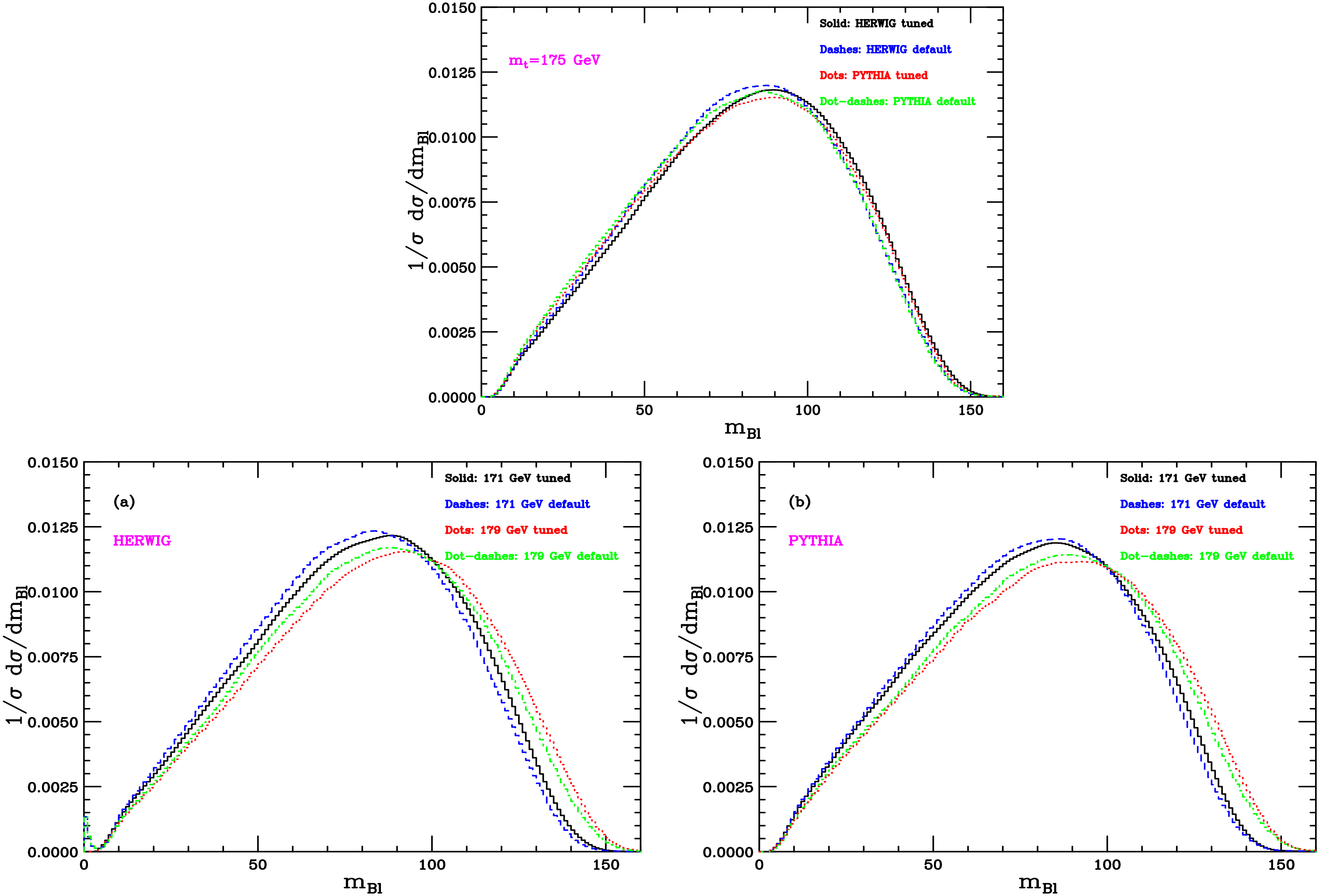
$D_{\rm np}(x_B,\gamma) = (1+\gamma)(2+\gamma)x_B(1-x_B)^\gamma$, $\gamma = 17.178 \pm 0.303$ (LEP/SLD)

x_B exhibits very mild dependence on top mass in both HERWIG and PYTHIA:





Investigating other observables: invariant mass $m_{B\ell}$



Mellin moments - full $m_{B\ell}$ spectrum - tuned HERWIG and PYTHIA

HERWIG:

m_t (GeV)	$\langle m_{B\ell} \rangle$ (GeV)	$\langle m^2_{B\ell} \rangle$ (GeV^2)	$\langle m^3_{B\ell} \rangle$ (GeV^3)	$\langle m^4_{B\ell} \rangle$ (GeV^4)
171	78.39	7.01×10^3	6.82×10^5	7.02×10^8
173	79.52	7.22×10^3	7.12×10^5	7.43×10^8
175	80.82	7.45×10^3	7.46×10^5	7.91×10^8
177	82.02	7.67×10^3	7.79×10^5	8.37×10^8
179	83.21	7.89×10^3	8.13×10^5	8.86×10^8

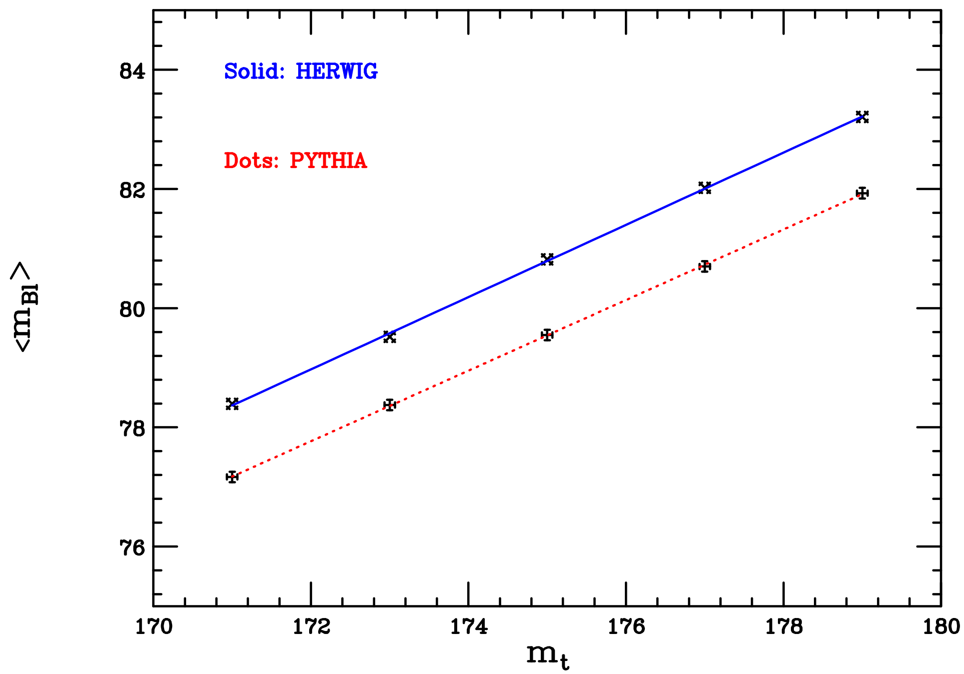
PYTHIA:

m_t (GeV)	$\langle m_{B\ell} \rangle$ (GeV)	$\langle m^2_{B\ell} \rangle$ (GeV^2)	$\langle m^3_{B\ell} \rangle$ (GeV^3)	$\langle m^4_{B\ell} \rangle$ (GeV^4)
171	77.17	6.85×10^3	6.62×10^5	6.81×10^8
173	78.37	7.06×10^3	6.94×10^5	7.23×10^8
175	79.55	7.27×10^3	7.25×10^5	7.67×10^8
177	80.70	7.48×10^3	7.56×10^5	8.12×10^8
179	81.93	7.71×10^3	7.91×10^5	8.61×10^8

Linear fits to extract m_t from $m_{B\ell}$

HERWIG: $\langle m_{B\ell}\rangle_{\rm H} \simeq -25.31~{\rm GeV} + 0.61~m_t$; $\delta = 0.043$ GeV

PYTHIA: $\langle m_{B\ell}\rangle_{\rm P} \simeq -24.11~{\rm GeV} + 0.59~m_t$; $\delta = 0.022$ GeV



$\Delta\langle m_{B\ell}\rangle \simeq 1.2$ GeV $\Rightarrow \Delta m_t \simeq 2$ GeV **(full range)**

Truncated Mellin moments 50 GeV $< m_{B\ell} <$ 120 GeV

HERWIG:

m_t (GeV)	$\langle m_{B\ell} \rangle$ (GeV)	$\langle m_{B\ell}^2 \rangle$ (GeV^2)	$\langle m_{B\ell}^3 \rangle$ (GeV^3)	$\langle m_{B\ell}^4 \rangle$ (GeV^4)
171	84.64	7.52×10^3	6.97×10^5	6.70×10^8
173	85.01	7.59×10^3	7.06×10^5	6.81×10^8
175	85.43	7.66×10^3	7.17×10^5	6.94×10^8
177	85.78	7.72×10^3	7.25×10^5	7.04×10^8
179	86.09	7.78×10^3	7.32×10^5	7.13×10^8

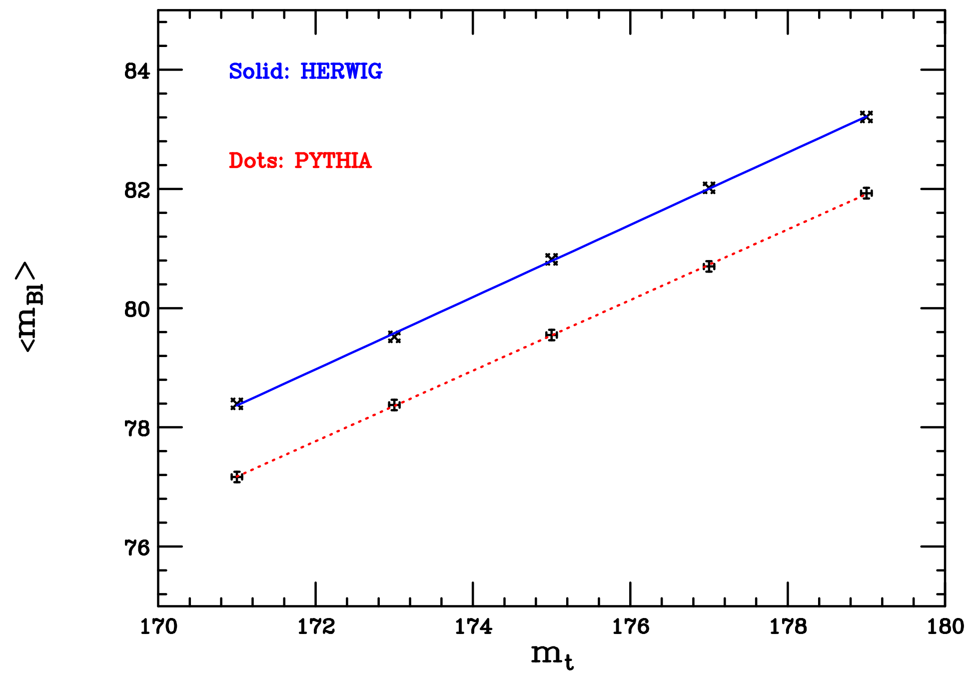
PYTHIA:

m_t (GeV)	$\langle m_{B\ell} \rangle$ (GeV)	$\langle m_{B\ell}^2 \rangle$ (GeV^2)	$\langle m_{B\ell}^3 \rangle$ (GeV^3)	$\langle m_{B\ell}^4 \rangle$ (GeV^4)
171	84.42	7.49×10^3	6.93×10^5	6.65×10^8
173	84.79	7.55×10^3	7.02×10^5	6.77×10^8
175	85.13	7.61×10^3	7.10×10^5	6.87×10^8
177	85.45	7.67×10^3	7.18×10^5	6.97×10^8
179	85.77	7.73×10^3	7.26×10^5	7.06×10^8

Linear fits to extract m_t for 50 GeV $< m_{B\ell} <$ 120 GeV

HERWIG : $\langle m_{B\ell} \rangle_{\rm H} \simeq 53.33 \text{ GeV} + 0.18\ m_t$; $\delta = 0.034$ GeV

PYTHIA : $\langle m_{B\ell} \rangle_{\rm P} \simeq 55.83 \text{ GeV} + 0.17\ m_t$; $\delta = 0.020$ GeV



$\Delta\langle m_{B\ell}\rangle \simeq 0.3$ GeV $\Rightarrow \Delta m_t \simeq 1.5$ GeV

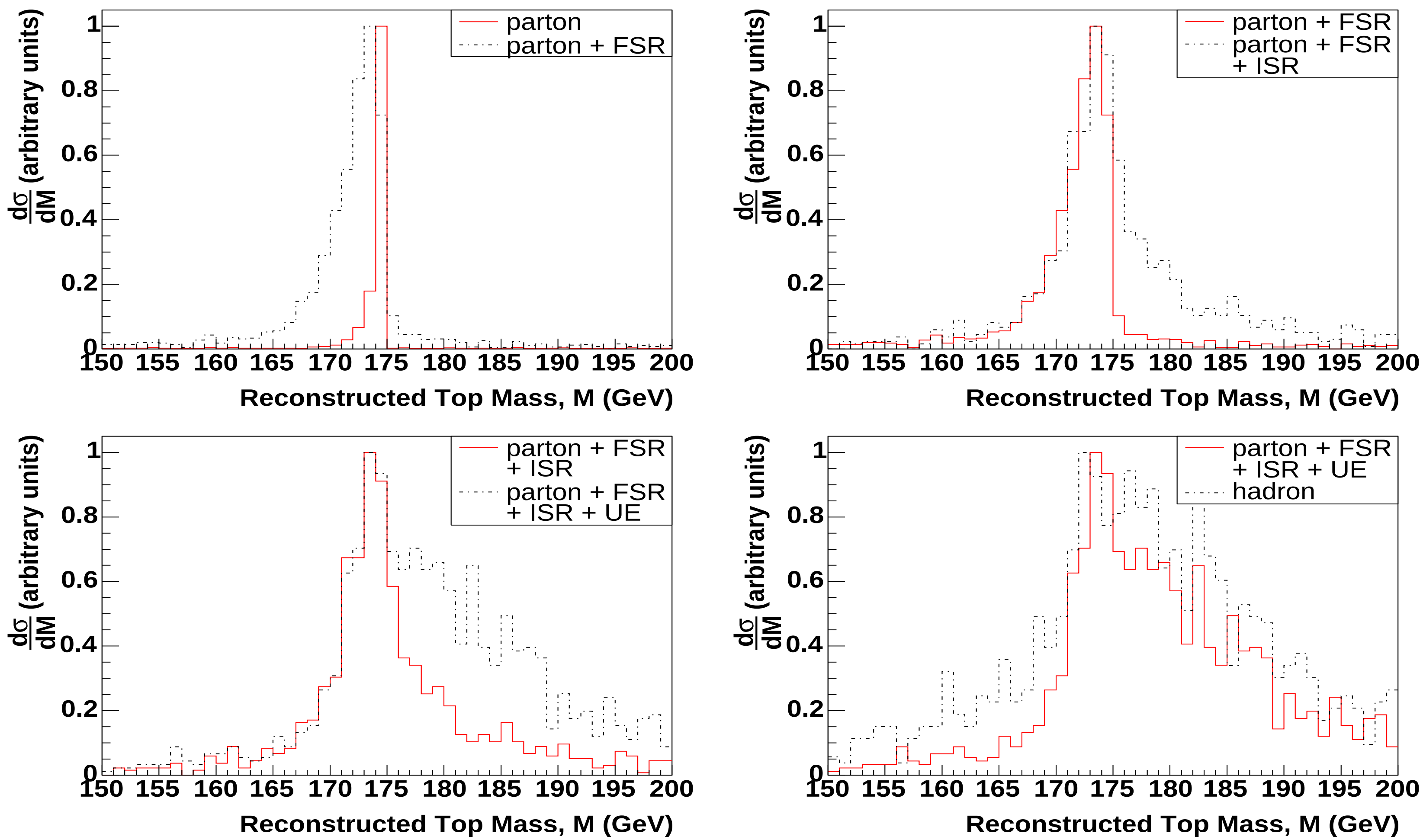
In progress: using the Professor code to improve the overall fit

Systematic errors on the top mass reconstruction (M.H. Seymour and C. Tevlin)

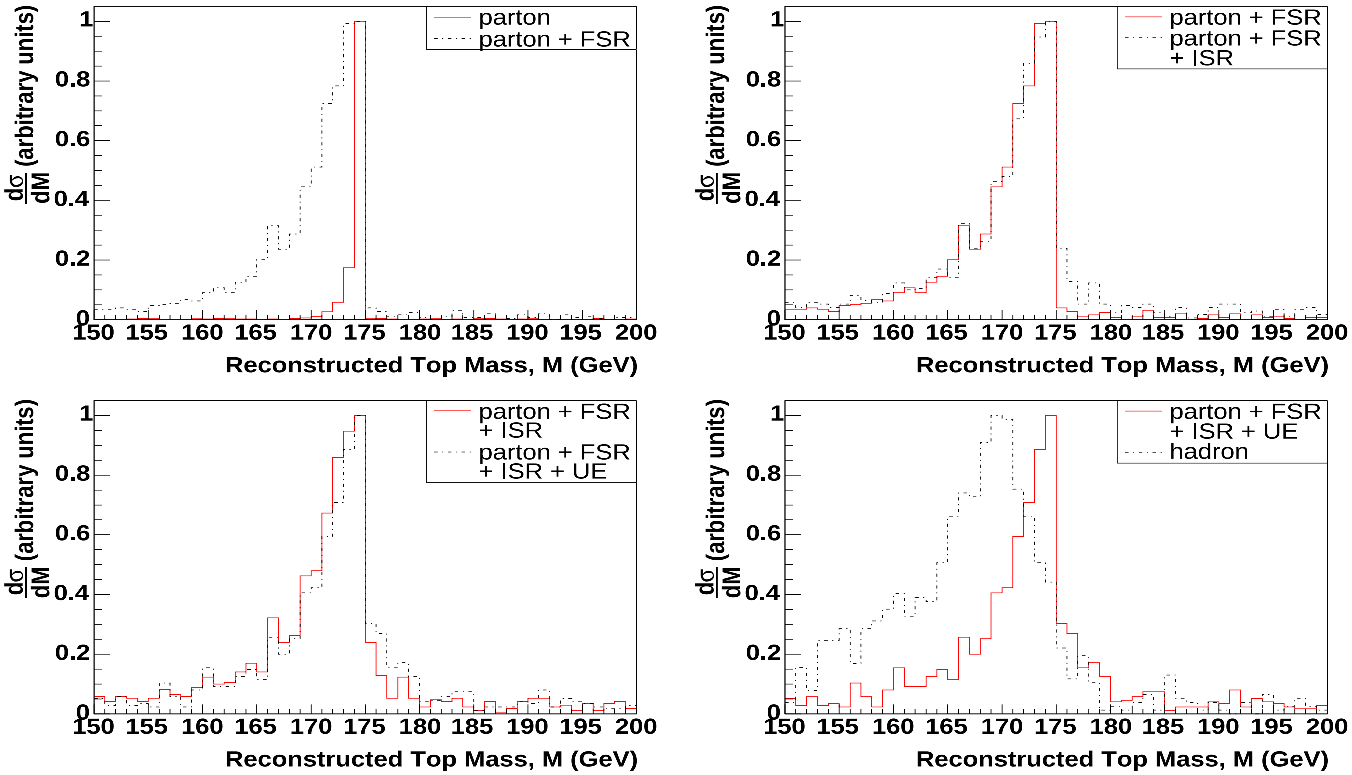
'Lepton + jets': top quark reconstruction as $W + b-$jet combination

Investigating FSR, ISR, underlying event and hadronization

k_T clustering algorithm (KtJet package)



Cone algorithm (PxCone, mid-point algorithm, infrared safe)



k_T algorithm mostly affected by ISR and UE; cone algorithm by FSR and hadronization

Useful employing both algorithms

Conclusions and outlook

The top-quark mass is a fundamental parameter of the Standard Model, but it is necessary to define the mass scheme

Several theoretical definitions appropriate for different energy regimes

Recent work shows stability of $\overline{\rm MS}$ and top jet masses

Extraction of the pole mass at NLO (plus some NNLO or NLL threshold resummation) from the cross section measurement at D0

Tevatron/LHC analyses strongly relying on parton shower generators

Devoted study on the Monte Carlo uncertainty due to b-fragmentation (HERWIG vs. PYTHIA)

Impact $\Delta m_t \simeq 1.5 - 2$ GeV when extracting m_t from $m_{B\ell}$

Studies on the sensitivity of the top mass reconstruction on the jet algorithms (cone and k_T)

Ongoing work towards extraction of the top mass consistently with theoretical definitions and fair determination of the Monte Carlo uncertainties

HERWIG ++: improved fragmentation model $(\chi^2/\mathrm{dof} \simeq \mathcal{O}(1))$