Bound-State Effects in Ttbar Production at Hadron Colliders

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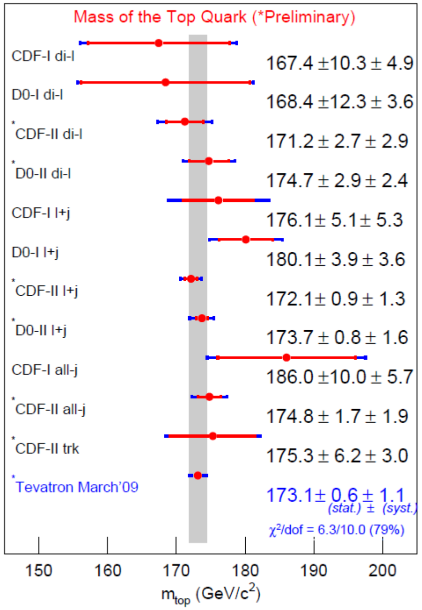
Outline :

- Introduction
- BS effects in ttbar inv.-mass dist.
- in differential cross-section
- Discussions
- Summary

TOP2010 2010.06.01 Bruges

1. Introduction (1)

Top-quark properties



- Mass measurement (CDF and D0 combined)

$$m_t = 173.1 \pm 0.6(\text{stat.}) \pm 1.1(\text{syst.})\ [\text{GeV}]$$

arXiv:0903.2503

- Decay-width (SM) : $\Gamma_t \simeq \frac{G_F m_t^3}{8\sqrt{2}\pi}|V_{tb}|^2 \sim 1.5\ [\text{GeV}]$

$$\Gamma_t \gg \Lambda_{\text{QCD}}$$

a unique property : top-quarks decay before hadronization, no t-hadron observed

spin information is preserved in decay products

- Cross-Section at the LHC

$$\sigma_{tt}(\text{LHC14TeV}) \sim 800\,\text{pb}$$
$$\sim 8\text{M/year}\,(\text{L} = 10\text{fb}^{-1})$$

LHC = top factory,

detail study can be possible

1. Introduction (2)

☐ Toponium = ttbar bound-state

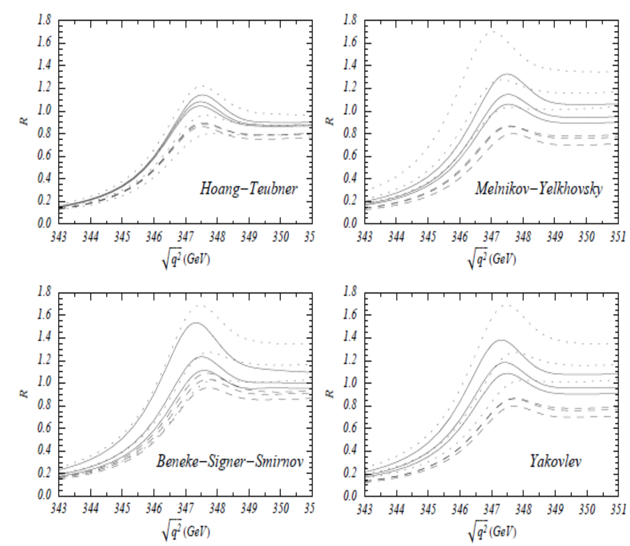
- The large width smears the resonance structure,

$$E_{1S} = \frac{m_t C_F^2 \alpha_s^2}{4} (\simeq 2\,[\mathrm{GeV}]) \simeq \Gamma_t$$

formation time of the toponium BS is $T = 1/E_{BS} \simeq \tau = 1/\Gamma_t$

But still a remnant of the resonances would remain.

hep-ph/0001286



- Threshold scan at e^+e^- colliders

dedicated theoretical studies in (P,v)NRQCD formalism up to NNLO

precise determinations on the top-quark mass, width and strong coupling constant are possible

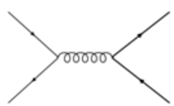
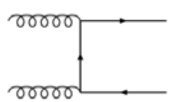
$$\delta m_t \leq 50\,[\mathrm{MeV}] \qquad \delta\Gamma_t/\Gamma_t \sim 20\,[\%]$$

1. Introduction (3)

Recent studies on bound-state effects in tt production at hadron colliders

1. **Hagiwara,Sumino,HY('08)** (approx. gluon radiation)
2. **Kiyo,Kuhn,Moch,Steinhauser,Uwer('08)** (exact up to NLO, resummation)

- Only a few work before them; Fadin,Khoze,Sjostrand('90),,
- Partonic subprocesses

			Tevatron	LHC
$q\bar{q} \to t\bar{t}$		Color: Octet |J|=1	85%	10%
$gg \to t\bar{t}$		Color: Singlet, Octet |J|=0,1,2,,,	15%	90%

- In contrast to e^+e^- case, at hadron colliders,

  Collision energy cannot be fixed → see the ttbar invariant-mass dist.

  ISR → large QCD correction

2. Bound-state effects at Hadron colliders

Coulomb corrections to all-orders

color-factor

$$\begin{cases} \text{singlet} & C^{(1)} = -C_F \\ \text{octet} & C^{(8)} = C_A/2 - C_F \end{cases}$$

- Coulomb singularity $\propto C^{(c)}\dfrac{\alpha_s}{\beta}$ $\quad \mathcal{O}(1)$ for $\beta \simeq \alpha_s$
- Summation of ladder diagrams = Sommerfeld factor Sommerfeld,Sakharov (QED)

$$S(z) = \frac{z}{1-\exp[z]}, \quad z = C^{(c)}\pi\alpha_s/\beta$$

$1 \qquad \alpha_s/\beta \qquad (\alpha_s/\beta)^2$

- Bound-state production : $\sigma_n \propto |\Psi_n(0)|^2\,\delta(E-E_n)$
- Green's function formalism (NRQCD) Fadin,Khoze('87)

$$\left[(E+i\Gamma_t) - \left\{-\frac{\nabla^2}{m_t} + V^{(c)}_{QCD}(r)\right\}\right] G^{(c)}(E,\vec{x}) = \delta^3(\vec{x})$$

Schrodinger's Eq.

finite width effects by complex energy

$$G(E,\vec{x}) = \sum_n \frac{\Psi_n(\vec{x})\Psi_n(0)^*}{E-E_n+i\Gamma_n/2} + \text{continuum}$$

Optical theorem

$\sigma_{\text{tot}}(s) \propto \text{Im}[G(E,\vec{r}=\vec{0})]$

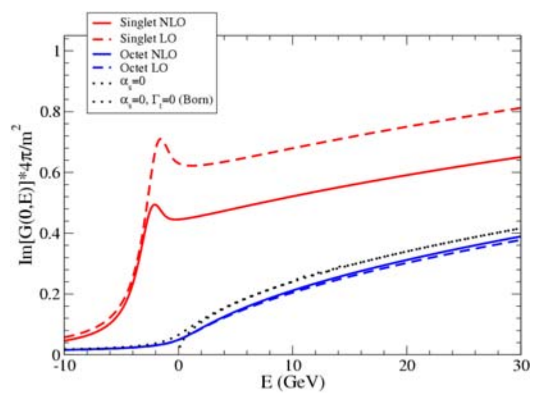
2.

QCD potential

- Perturbative QCD potential (NLO), since an IR cut-off by $r \lesssim \dfrac{1}{\Gamma_t}$

$$V^{(c)}_{\text{QCD}}(r) = C^{(c)}\frac{\alpha_s(\mu_B)}{r} \times \left[1 + \frac{\alpha_s}{\pi}v_1^{(c)}(r) + \cdots\right]$$

$$\begin{cases} \text{singlet} & C^{(1)} = -C_F \\ \text{octet} & C^{(8)} = C_A/2 - C_F \end{cases}$$



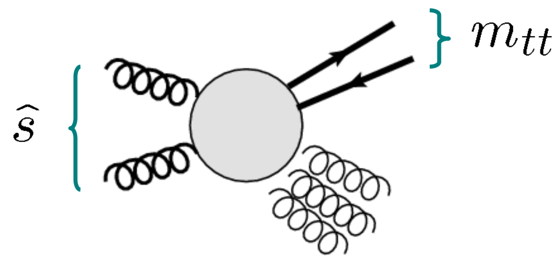
- Scales : $m_t \gg \mu_B > E_B \simeq \Gamma_t \gg \Lambda_{\text{QCD}}$
  - Bohr radius : $\mu_B \simeq m_t\alpha_s \simeq 20 - 30\text{GeV}$ typical momentum of Coulomb gluon
  - Binding energy : $E_B \simeq m_t\alpha_s^2 \simeq 2\text{GeV}$ If $\Gamma_t > E_B$, top-quark decays before bound-state formation

2.

- ttbar invariant-mass distribution (NLO)

Mangano,Nason,Ridolfi('92)

$$\frac{d\hat{\sigma}}{dm_{tt}}(\hat{s}, m_{tt}; m_t) \propto F(z = \frac{m_{tt}^2}{\hat{s}}, \rho = \frac{4m_t^2}{m_{tt}^2}) = F^{(1)}(z,\rho) + \frac{\alpha_s}{\pi} F^{(2)}(z,\rho) + \cdots$$



LO $\quad F^{(1)}(z,\rho) = C^{(1)}(\rho)\delta(1-z) \qquad C^{(1)}(\rho) \propto \alpha_s^2 \beta \qquad \beta = \sqrt{1-\rho}$

NLO $\quad F^{(2)}(z,\rho) = C^{(2,2)}(\rho)\left(\frac{\ln(1-z)}{1-z}\right)_+ + C^{(2,1)}(\rho)\frac{1}{(1-z)_+} + C^{(2,0)}(\rho)\delta(1-z) + \tilde{C}^{(2)}(z,\rho)$

$C^{(2,2)}(\rho) \propto C^{(1)}(\rho)$: soft-collinear gluon emission

$C^{(2,1)}(\rho)$: soft non-collinear gluon, depends on color

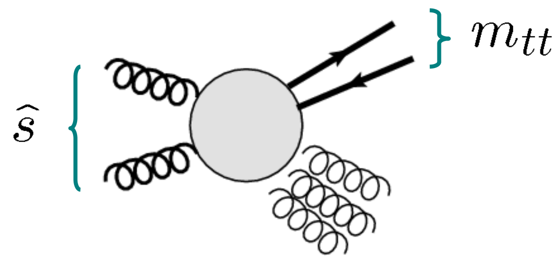
$C^{(2,0)}(\rho)$: includes Coulomb singularity, hard correction

$\tilde{C}^{(2)}(z,\rho)$: regular function of z

2.

ttbar invariant-mass distribution (Threshold region)

$$\frac{d\hat{\sigma}}{dm_{tt}}(\hat{s}, m_{tt}; m_t) \propto F(z = \frac{m_{tt}^2}{\hat{s}}, \rho = \frac{4m_t^2}{m_{tt}^2}) = F^{(1)}(z,\rho) + \frac{\alpha_s}{\pi}F^{(2)}(z,\rho) + \cdots$$



- Threshold limit : $\rho \to 1$ where relative motion of the ttbar is slow

  equivalent with Heavy Quarkonium production, except the Coulomb term,
  which is in the wave-function in the HQonium case.

- Factorized form ;

Kuhn,Mirkes('93),
Petrelli,Cacciari,Greco,Maltoni,Mangano('97)

$$F^{\mathrm{NLO}}(z,\rho) \sim C^{(0)}(\rho)\left(1 + \frac{\alpha_s}{\pi}\frac{\pi^2 C^{(c)}}{2\beta}\right)\left(1 + \frac{\alpha_s}{\pi}h\right)$$

$$\times\left[\delta(1-z) + \frac{\alpha_s}{\pi}\left\{c^{(2,2)}\left(\frac{\ln(1-z)}{1-z}\right)_+ + c^{(2,1)}\left(\frac{1}{1-z}\right)_+ + k\delta(1-z) + \tilde{c}^{(2)}(z)\right\}\right]$$

The formula :

ttbar invariant-mass distribution near threshold (BS effects+NLO)

$$\frac{d\sigma}{dm_{tt}}(s,m_{tt}^2) = \underline{\hat{\sigma}_{B,i}^{(c)}(m_{tt}^2)} \cdot \underline{K_i^{(c)}} \int_{\tau_0}^1 \frac{dz}{z} \underline{F_i^{(c)}(z)} \frac{d\mathcal{L}_i}{d\tau}(\tau_0/z)$$

- Bond-state cross-section :

  Green's fnc. with NLO potential

- Gluon Radiation (ISR) :

  NLO (singular term, regular term), Resummation of threshold logs

  Kiyo,Kuhn,Moch,Steinhauser,Uwer('08)

- Hard-gluon correction (color-dependent) :

  from NLO HQonium production

  Petrelli,Cacciari,Greco,Maltoni,Mangano('98)

  Non-decoupling term Hagiwara,Sumino,HY('08)

  Confirmed by Czakon,Mitov('08)

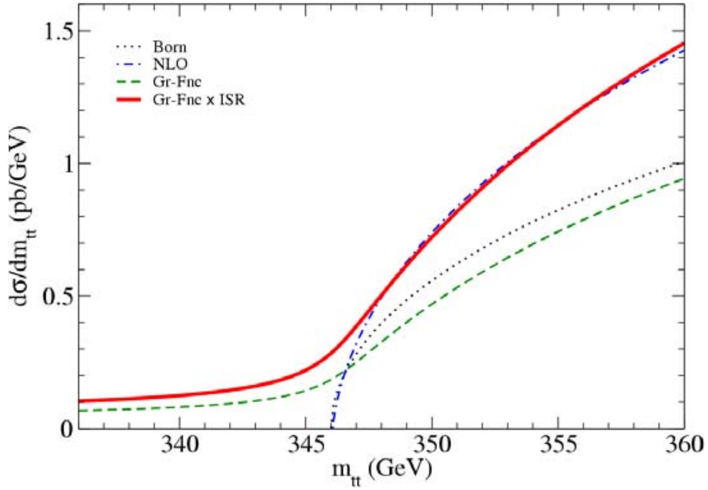
2.

m_t=173 GeV, Γ_t=1.5 GeV, CTEQ6M

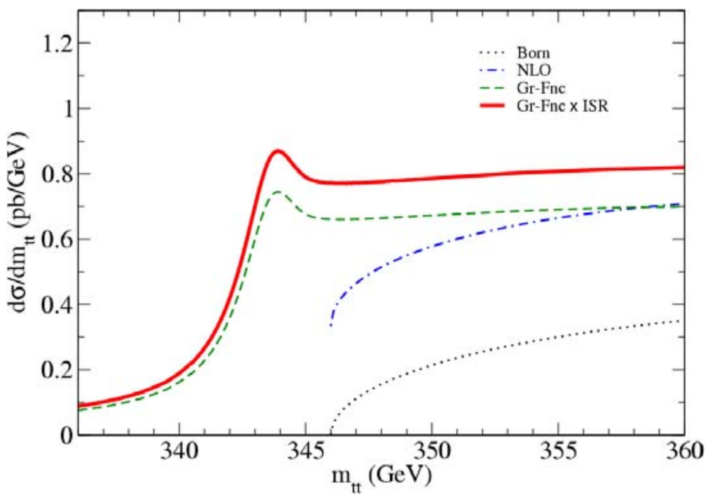
☐ Results

Black	: Born
Blue	: O(as) corr.
Green	: Gr-Fnc. without ISR
Red	: Gr-Fnc. with ISR

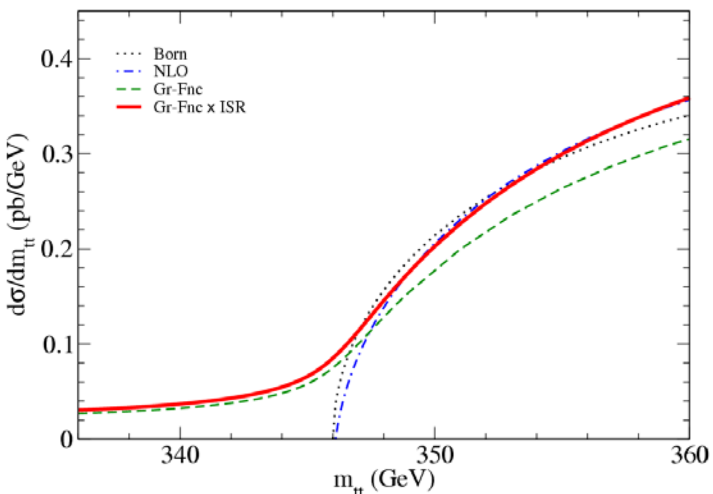
gg->tt, color-octet



gg->tt, color-singlet



qq->tt, color-octet



2.

Results

Kiyo,Kuhn,Moch,Steinhauser,Uwer('08)

- Effect of regular term :

$$(\mathcal{L}\otimes F)[gg \to {}^1S_0^{[1]}] = \begin{Bmatrix} 14.5 + (4.53 + 1.68)_{\mathcal{A}} \\ 14.0 + (5.66 + 1.58)_{\mathcal{A}} \\ 13.0 + (6.37 + 1.48)_{\mathcal{A}} \end{Bmatrix} \times 10^{-6}\,\mathrm{GeV}^{-2},$$

K(NLO)~1.4

$$(\mathcal{L}\otimes F)[gg \to {}^1S_0^{[8]}] = \begin{Bmatrix} 39.3 + (16.6 + 7.26)_{\mathcal{A}} \\ 37.4 + (18.8 + 6.52)_{\mathcal{A}} \\ 34.4 + (20.0 + 5.83)_{\mathcal{A}} \end{Bmatrix} \times 10^{-6}\,\mathrm{GeV}^{-2},$$

K(NLO)~1.6

$$(\mathcal{L}\otimes F)[q\bar{q} \to {}^3S_1^{[8]}] = \begin{Bmatrix} 16.7 + (3.50 + 2.91)_{\mathcal{A}} \\ 16.8 + (3.41 + 3.56)_{\mathcal{A}} \\ 16.4 + (3.28 + 3.97)_{\mathcal{A}} \end{Bmatrix} \times 10^{-6}\,\mathrm{GeV}^{-2}.$$

K(NLO)~1.4

- Effect of threshold resummation :

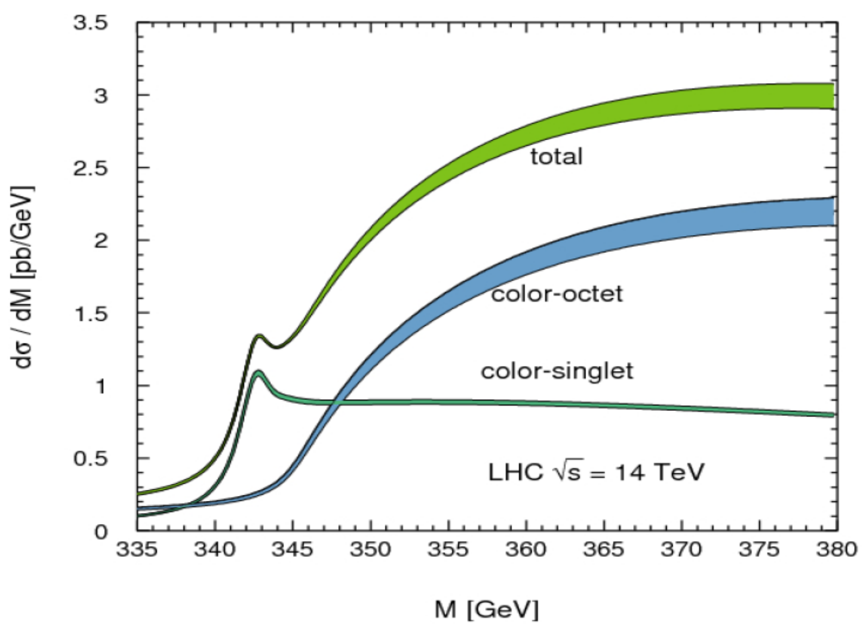
	NLO			resummed		
$gg \to {}^1S_0^{[1]}$	20.7	21.2	20.9	22.0	23.2	24.0
$gg \to {}^1S_0^{[8]}$	63.2	62.7	60.2	67.8	69.7	70.6
$q\bar{q} \to {}^3S_1^{[8]}$	23.1	23.8	23.6	23.8	24.0	23.6

Table 3: Comparison of the NLO and resummed result of the convolution $\mathcal{L}\otimes F$ (in $10^{-6}\,\mathrm{GeV}^{-2}$) for LHC at the reference point $M = 2m_t$. The three columns correspond to the scale choices $\mu_r = \mu_f = (m_t, 2m_t, 4m_t)$. The NLO results can also be found in Tab. 2.

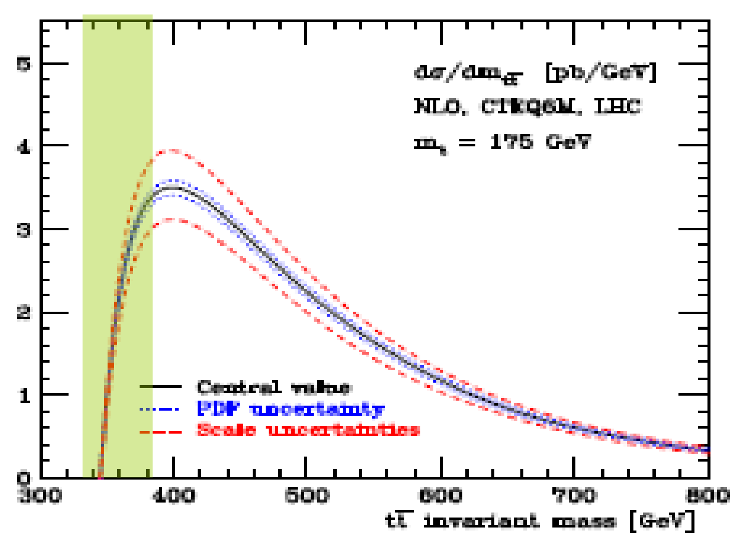
These contribute to enhance the normalization of the distribution.

2.

- In total at the LHC :
  - A broad resonance peak below threshold (observable in principal)
  - Deform the invariant-mass distribution near threshold
  - Enhancement in total cross-section only O(1%), several pb.



NLO inv. dist. using MCFM (Campbell,Ellis)
Figs. from Maltoni,Frederix('08)



2.

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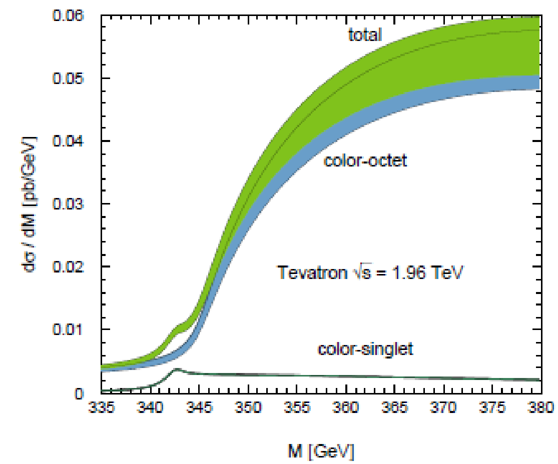
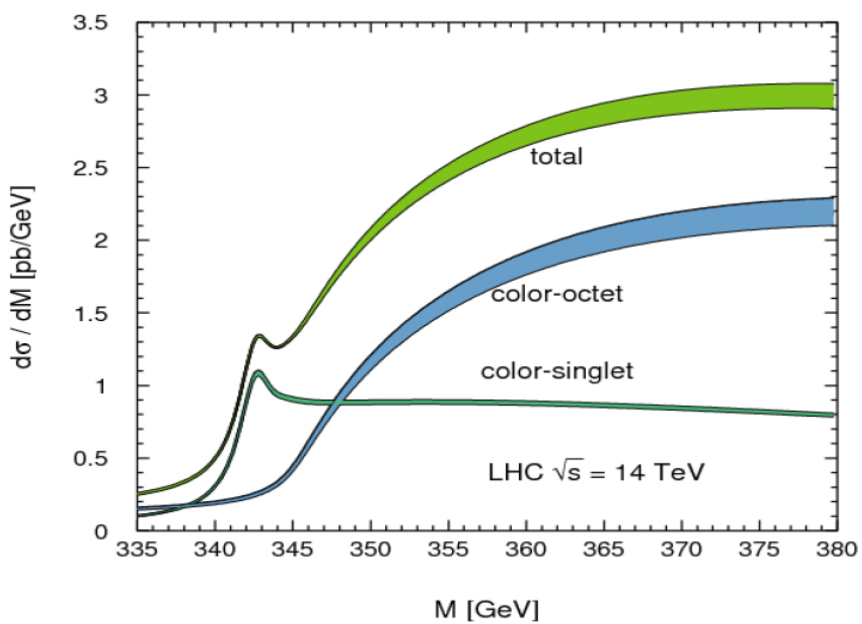
Tevatron





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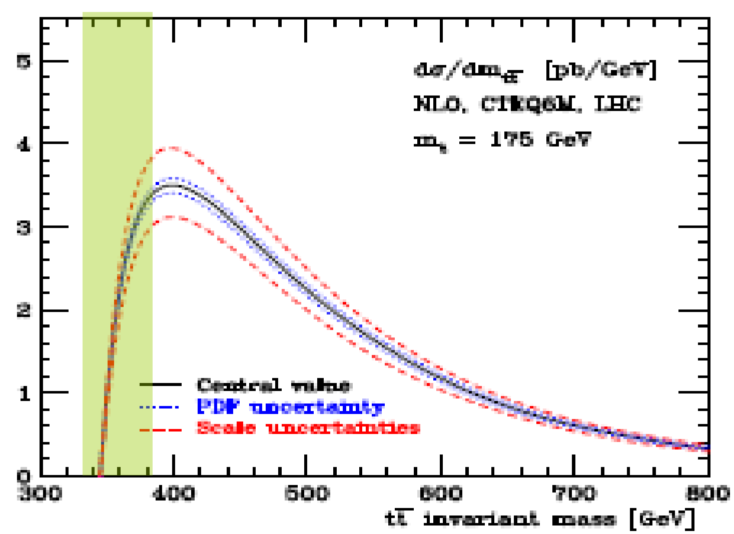


3. Differential Cross-section

☐ Beyond the invariant-mass distribution :

Sumino,HY in preparation

- Useful for simulation studies, event analysis,,,
- Coulomb correction affects the top-quark momentum distributions
- well-developed for e^+e^- collider case

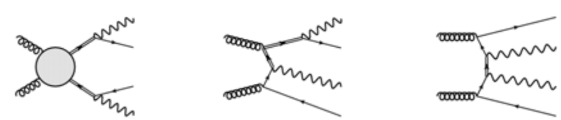
Jezabek,Kuhn,Teubner('92)
Sumino,Fujii,Hagiwara,Murayama,Ng('93)

- a quick description

Start from Matrix-Elements for gg/qq to bWbW process

Resonant diagrams and also non-resonant diagrams exist

$$\mathcal{M}^{(c)}(I \to bWbW) = \mathcal{M}^{(c)}_{t\bar{t}} + \mathcal{M}^{(c)}_{\mathrm{nr}}$$



A Correction factor to Matrix-Elements = Momentum-space Green's func.

$$\mathcal{M}^{(c)}_{t\bar{t}} \to \mathcal{M}^{(c)}_{t\bar{t}} \times \tilde{G}^{(c)}(E, \vec{p})$$

3.

◻ Event Generator (LO+Coulomb) :

Sumino,HY in preparation

- (almost) Full gg/qq→bWbW amplitudes, plus W-decays in MEs.
- Bound-state correction to the double-resonant amplitudes.
- Color-dependent K-factors to reproduce NLO m_{tt} dist. near threshold.

- Matrix-Elements : based on MadGraph/HELAS code.
  generated by "pp>(w+>mu+vm)(w->mu-vm~)bb~/azh"
  add color decomposition for gg.
- Green's Fnc. : pre-tabulated by solving Schrodinger Eq.
  in coordinate-space, then taking Fourier trans.
  NLO QCD potential
- Phase-Space Integral/Event Generation :
  BASES/SPRING, or put above into MadEvent
- Interface (LHE) to your favorite parton-shower, hadronization
  simulators (PYTHIA,HERWIG)

3.

Preliminary

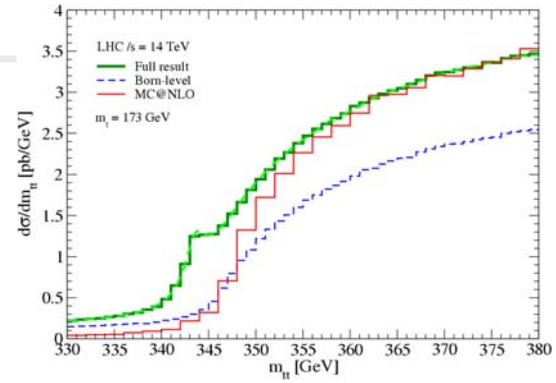
Some results (at partonic-level)

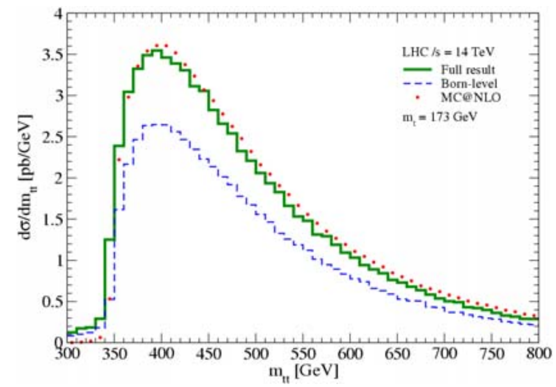
- m_{tt} distribution : check with previous results

  The only generator which generates threshold resonance

  Effectively, well reproduce MC@NLO at large m_{tt}

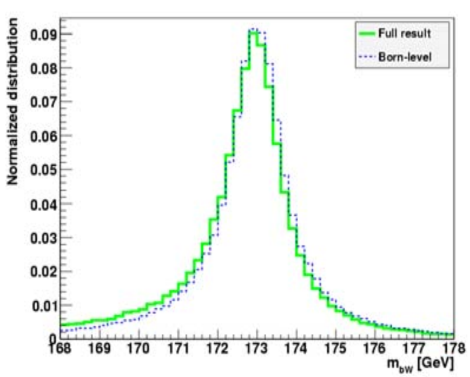
  Trick in scale choice $\mu = m_t \leftrightarrow \mu = \sqrt{m_t^2 + p_T^2}$

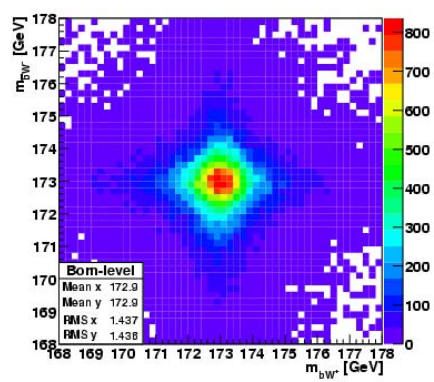


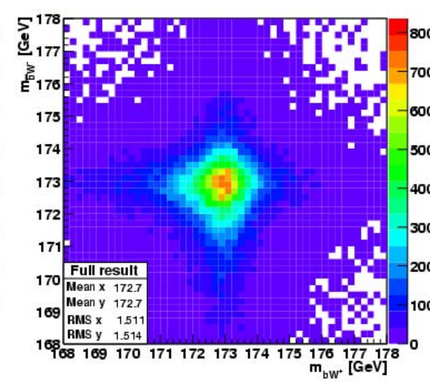


- Top-quark invariant-mass distribution, $m_{bW}=(p_b+p_W)^2$.

  limiting for the events with m_{tt}<370GeV (10% of total events) ⇒ $\delta m \simeq -200\,[\mathrm{MeV}]$







4. Discussions

Short-distance mass measurement

- Pole mass is known to be ambiguous due to its sensitivity to small momenta (IR-Renormalon), bad perturbative convergency.
- Use short-distance mass which don't have such ambiguity.

  Threshold mass (PS mass, 1S mass, Kinetic mass,,,), MSbar mass, Jet-mass,,,
- Example: 1S-mass ; *defined as a half of the 1S toponium mass* Hoang,Teubner ('99)

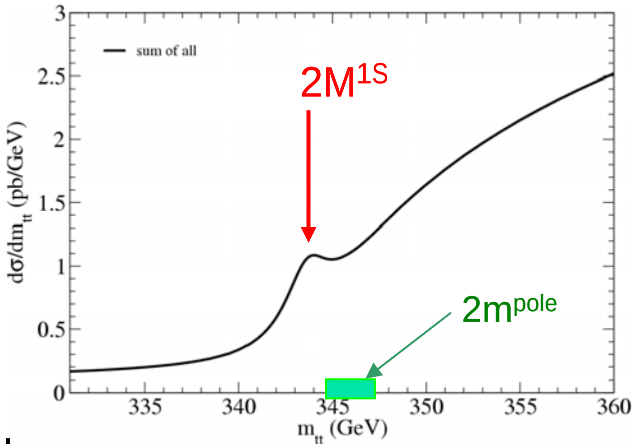
◆ Suppose, you measure the peak in m_{tt} dist. at the LHC

SD mass could be obtained very cleanly.

Peak consists of color-singlet resonance

→ Free from low momentum color-connection

Theoretically good perturbative convergence



Relation to the pole mass or the other SD mass is well known;

$$M^{1S}(\mu) = m_t^{\text{pole}} - \frac{m_t^{\text{pole}} C_F^2 \alpha_s^2}{8}\left[1 + \frac{\alpha_s}{\pi} d_1 + \left(\frac{\alpha_s}{\pi}\right)^2 d_2 + \cdots\right]$$

4. Discussions

Threshold events are worthwhile!

- But not easy in real life ;

  Small fraction of the threshold events

  Errors and combinatorials disturb the m_{tt} measurement

  → Large-m_{tt} events contribute as significant BG

- Challenging task at the LHC ;

  How to pick-up the threshold events in good accuracy?

  How much are the errors in measuring m_{tt} for each decay channel?

  How to treat additional jets from initial- and final- state radiation?

- Statistics may not be a problem (not a physics for first few years.)

4. Discussions

- Which decay-channel is better to see threshold events?

Key is the Jet Energy Scale, especially for B-jets.

- all-hadron : 6 jets
- lepton+jets : 4 jets (2-fold sol.)
- di-lepton : 2 jets (8-fold sol.)

- (Reference) Systematics errors in top-quark mass measurement

for ATLAS Borjanovic etal('05)

Table 14. Summary of the systematics errors in the top mass measurement, in the lepton plus jets channel, in the all jets channel and in the dilepton channel

Source of error in GeV	Lepton+jets inclusive sample	Lepton+jets large clusters sample	Dilepton	All jets high pT sample
Energy scale				
Light jet energy scale	0.2	–	–	0.8
b-jet energy scale	0.7	–	0.6	0.7
Mass scale calibration	–	0.9	–	–
UE estimate	–	1.3	–	–
Physics				
Background	0.1	0.1	0.2	0.4
b-quark fragmentation	0.1	0.3	0.7	0.3
Initial state radiation	0.1	0.1	0.1	0.4
Final state radiation	0.5	0.1	0.6	2.8
PDF	–	–	1.2	–

- all-hadron : $\delta m_{t,(\mathrm{sys.})} \sim 3\,[\mathrm{GeV}]$
- lepton+jets : $\delta m_{t,(\mathrm{sys.})} \sim 1.5\,[\mathrm{GeV}]$
- di-lepton : $\delta m_{t,(\mathrm{sys.})} \sim 1.5\,[\mathrm{GeV}]$

$$\delta m_{tt,(\mathrm{sys.})} \sim \sqrt{2}\delta m_{t,(\mathrm{sys.})}?\ \delta m_{t,(\mathrm{sys.})}?$$

4. Discussions

☐ Selecting threshold events; di-lepton example

- Mahlon,Parke('10) :

  (for the purpose of spin correlation study)

  Solve the system with the on-shell conditions for 4 particles (t,tbar,W^+,W^-).

  Take the average of the (at most) 8-fold solutions.

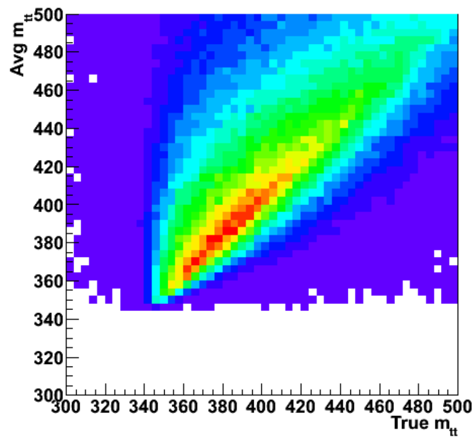
  Events with m_{tt}(Avg)<400GeV contain less events with m_{tt}(True)>400 GeV.

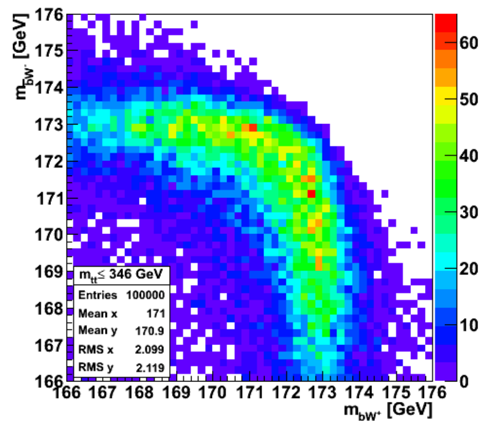
- However, for the threshold events, (both of) top-quarks on-shell condition cannot be used to reconstruct momenta.

  (It is still OK to use them for the selection of threshold events)

  → How to reconstruct m_{tt},in di-lepton channel?

using parton-level momenta





events for $m_{tt}<2m_t$

5. Summary

- At the LHC, gluon-fusion process dominates, and the ttbar pair can be color-singlet.
- The bound-state effects are calculated for the m_{tt} distribution at Hadron Colliders up to NLO (Green's func., gluon radiation, hard-correction).
- Large correction in m_{tt} dist. near threshold, and there appears a broad resonance below threshold.
- Differential cross-sections are also calculated including BS effects and decays, to enable event generation for the threshold region
- Short-distance mass measurement using threshold events sounds interesting which decay-channel? systematics errors? how to extract threshold events?
- Width dependence from the shape of the peak? → needs very high resolution.
- (**AD**) We will be pleased to provide our generator code and generated events.

Back-up slides

3.

Some details on diff. cross-sections

- Top-quark momentum distribution

$$\frac{d\sigma}{d^3\vec{p}} \simeq |\tilde{G}(E,\vec{p})|^2 \qquad \text{color-singlet : } \delta p > 0 \qquad \text{color-octet : } \delta p < 0$$

- Phase-space suppression due to the running top-width

$$\left(\int d\Phi_{bW}\right)^2 |\mathcal{M}_{tt}|^2 \propto \frac{\sqrt{s_t}\Gamma_t(s_t)}{m_t\Gamma_t}\frac{\sqrt{s_{\bar{t}}}\Gamma_t(s_{\bar{t}})}{m_t\Gamma_t}$$

It is known to cancel with t-bbar (tbar-b) Coulomb int.

Jezabek,Kuhn,Teubner('92),Modritsch,Kummer('94)

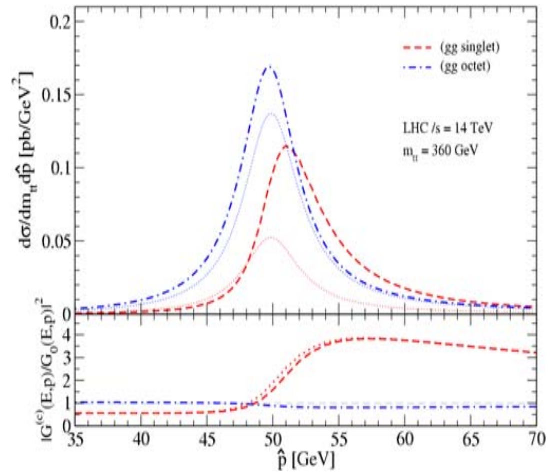
→ multiply a factor which cancel this suppression, plus a counter term in non-res amp.

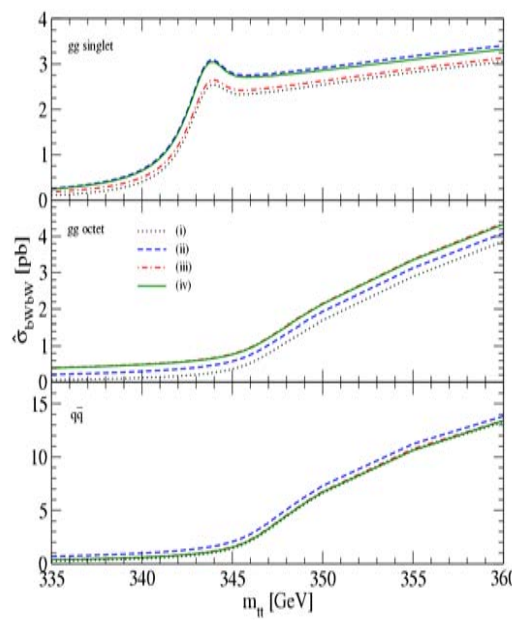
- Non-resonant diagrams : $\mathcal{M}^{(c)}(I \to bWbW) = \mathcal{M}^{(c)}_{t\bar{t}} + \mathcal{M}^{(c)}_{\mathrm{nr}}$

  Both are gauge-variant, but the sum is invariant.

  Note, FWS violates the gauge invariance, but small.

  Numerically, small scheme dependence

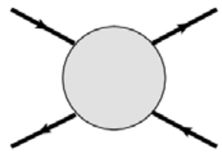




3.

Green's function at high-energy

$$G = \langle f | \frac{1}{m_{tt} - H + i\Gamma_t} | i \rangle$$



with the total energy of the tt system $m_{tt} = 2m_t + E$

and the Hamiltonian of the tt system

$$H = 2\sqrt{\vec{p}^2 + m_t^2} + V(\vec{r})$$
$$\simeq 2m_t + \frac{\vec{p}^2}{m_t} - \frac{\vec{p}^4}{8m_t^3} + \cdots + V(\vec{r})$$

higher-order term non-negligible

Solve the on-shell relation with free Hamiltonian $m_{tt} = H_0$

then we obtain $E + \frac{E^2}{4m_t} = \frac{\vec{p}^2}{m_t}$

Thus, define $G' = \langle f | \frac{1}{E' - \frac{\vec{p}^2}{m_t} + i\Gamma_t} | i \rangle$ with $E' = E + \frac{E^2}{4m_t}$

It has the same form with non-rela. Green fnc. which we know well, and respect relativistic on-shell pole at high-energy.

4. Discussions

Short-distance mass measurement

- What we have from (e.g. bjj) invariant-mass reconstruction at Tevatron is supposed to be "pole mass", but it is NOT TRUE!!

"pole mass = position of the pole in the propagator"

It can be different from the reconstructed invariant-mass, due to the color-connection at hard-process, parton-shower and hadronization.

- Pole mass is known to be ambiguous due to its sensitivity to small momenta (IR-Renormalon).

bad pertrubative convergency, ambiguous by $O(\Lambda_{QCD})$

- Solution: define "Short-Distance mass" which doesn't include such ambiguity.

Threshold mass (PS mass, 1S mass, Kinetic mass,,,), MSbar mass, Jet-mass,,,

4. Discussions

☐ Another SD-mass : $\overline{\text{MS}}$ mass

$$m_t^{\text{pole}} = \bar{m}(\bar{m}) + \bar{m}(\bar{m})\left[\frac{\alpha_s}{\pi}d_1 + \left(\frac{\alpha_s}{\pi}\right)^2 d_2 + \cdots\right] \qquad d_1 = \frac{4}{3},,,$$

known up to 3-loop

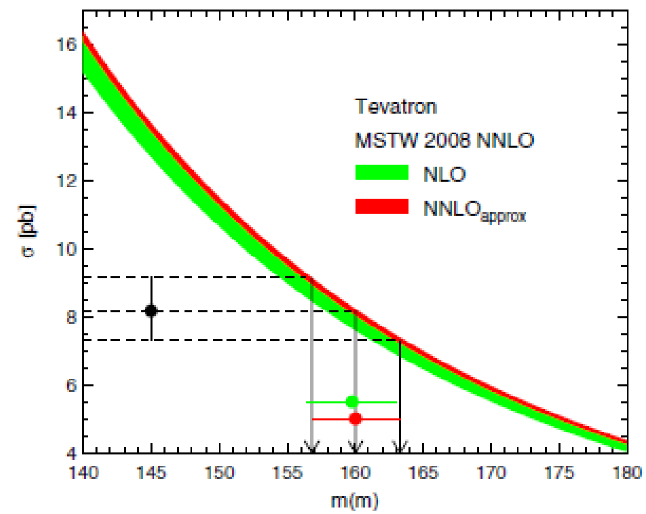
difference between the two scheme is large, $\delta m \sim 10\,[\text{GeV}]$

- Extracted from the total cross-section at the Tevatron

Langenfeld,Moch,Uwer ('09)

PHYSICAL REVIEW D 80, 054009 (2009)

$$\bar{m}(\bar{m}) = 160.0^{+3.3}_{-3.2}\,[\text{GeV}] \quad \text{NNLO}$$
$$159.8^{+3.3}_{-3.3} \quad \text{NLO}$$
$$159.2^{+3.5}_{-3.4} \quad \text{LO}$$



(b)

- The corresponding value in pole mass scheme (NNLO)

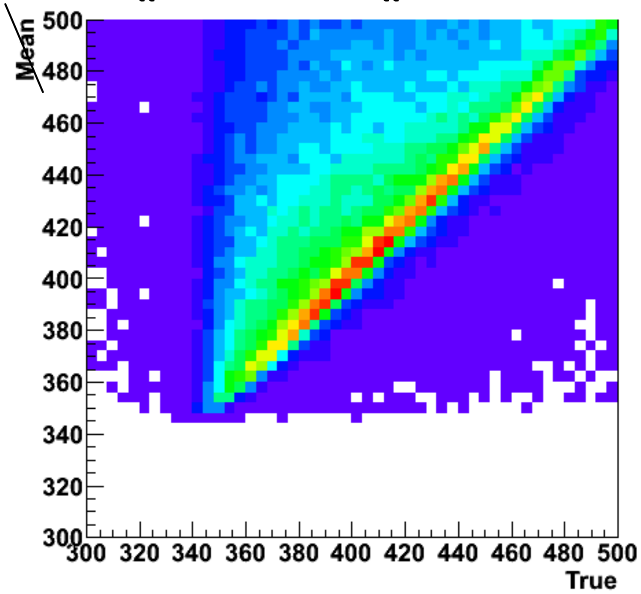
$$m_t^{\text{pole}} = 168.9^{+3.5}_{-3.3}\,[\text{GeV}]$$

is consistent with current world average.

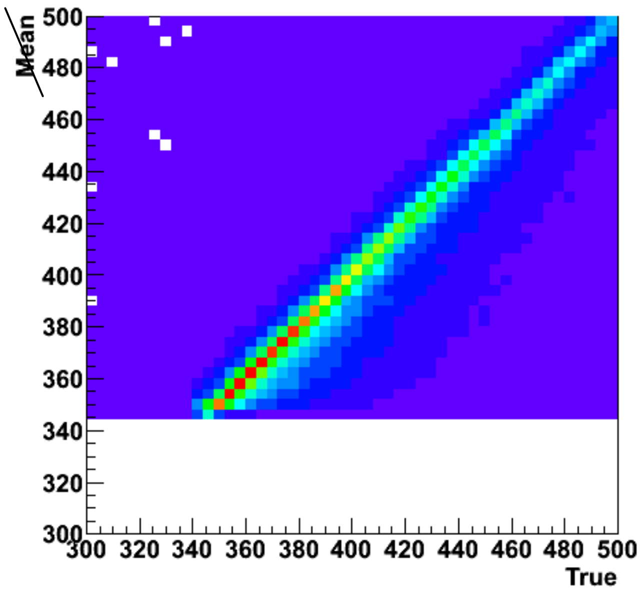
4. Discussions

- 8-fold solution in dilepton channel

m_{tt}(Max) vs. m_{tt}(True)



m_{tt}(Min) vs. m_{tt}(True)

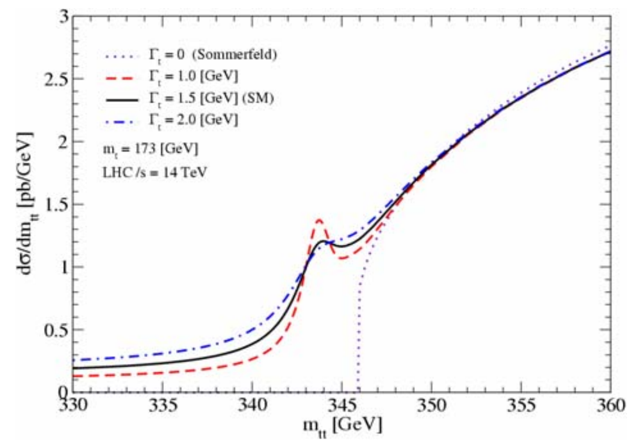


4. Discussions

Width dependence

- Examine by varying t-b-W coupling to preserve unitarity relation

$$\Gamma_t \propto |g_{tbW}|^2 \frac{m_t^3}{m_W^2}$$



- The narrower the peak observed, the smaller the decay-width
- If $\Gamma_t > E_{1S} \simeq \frac{m_t C_F^2 \alpha_s^2}{4}$, no time to form BS, no peak
- Needs very high resolution, otherwise smeared-out.