Status of $t\bar{t}$ cross section predictions

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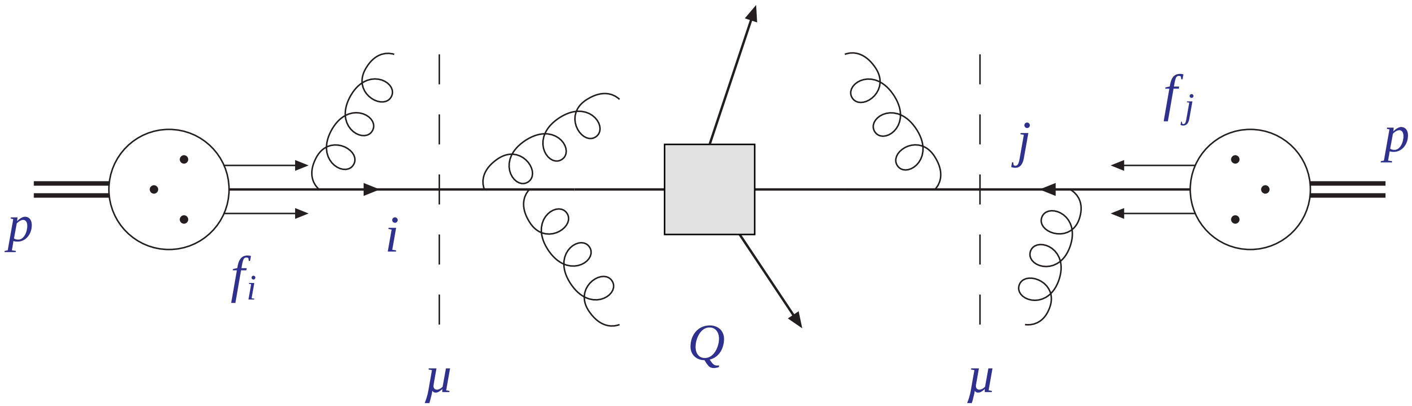
Perturbative QCD at colliders

- Hard hadron-hadron scattering
  - constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2)



- QCD factorization at scale μ
  - separate sensitivity to dynamics from different scales

$$\sigma_{pp\to t\bar{t}X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij\to t\bar{t}X}\left(\alpha_s(\mu^2), Q^2, \mu^2\right)$$

  - subprocess cross section $\hat{\sigma}_{ij\to t\bar{t}X}$ for parton types i, j

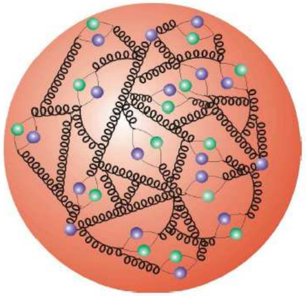
Hard scattering cross section

- Standard approach to uncertainties in theoretical predictions
  - variation of factorization scale μ: $\dfrac{d}{d \ln \mu^2} \sigma_{pp \to X} = \mathcal{O}(\alpha_s^{l+1})$

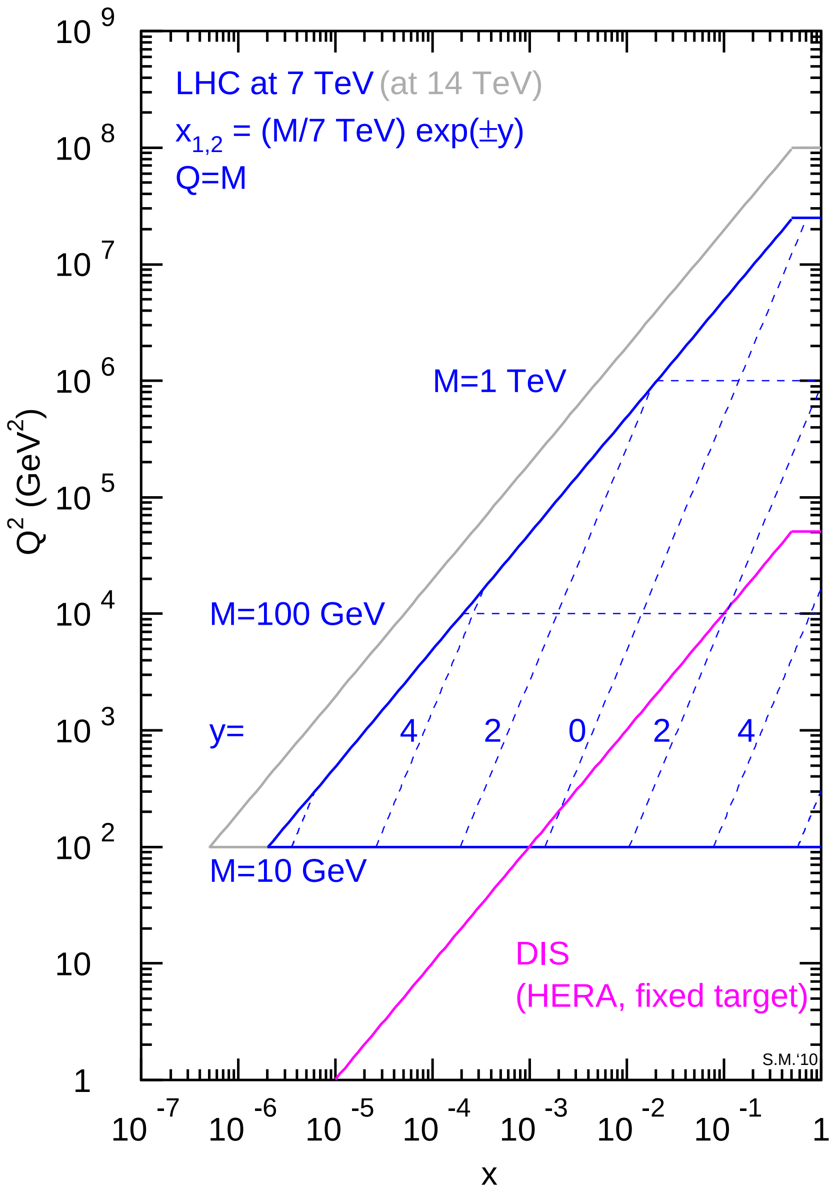
$$\sigma_{pp \to t\bar{t}X} = \sum_{ijk} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \to t\bar{t}} \left(\alpha_s(\mu^2), Q^2, \mu^2 \right)$$

- Parton cross section $\hat{\sigma}_{ij \to t\bar{t}}$ calculable pertubatively in powers of α_s
  - constituent partons from incoming protons interact at short distances of order $\mathcal{O}(1/Q)$

- Parton luminosity $f_i \otimes f_j$
  - proton: very complicated multi-particle bound state
  - colliders: wide-band beams of quarks and gluons



Parton luminosity at LHC

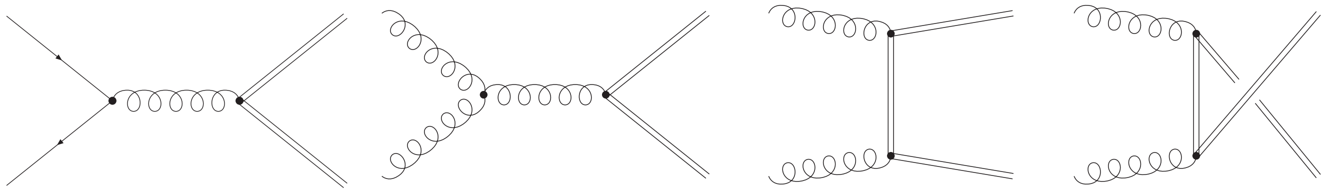


- LHC will run at 7 TeV until end of 2011 years, up to $1\,\mathrm{fb}^{-1}$
- $t\bar{t}$ cross section is $\simeq 1/4$ rate with respect to run at 14 TeV
  - larger PDF uncertainties
- Parton kinematics restricted to larger effective $\langle x\rangle = M/\sqrt{S}$
  - 100 GeV physics: small-x, sea partons
  - TeV scales: large-x
- Limited discovery reach
  - less phase space available for heavy mass states
  - use 7 TeV for accurate SM benchmarking

Top quark production

- Leading order Feynman diagrams

$$\mathrm{q} + \bar{\mathrm{q}} \longrightarrow \mathrm{Q} + \bar{\mathrm{Q}}$$
$$\mathrm{g} + \mathrm{g} \longrightarrow \mathrm{Q} + \bar{\mathrm{Q}}$$

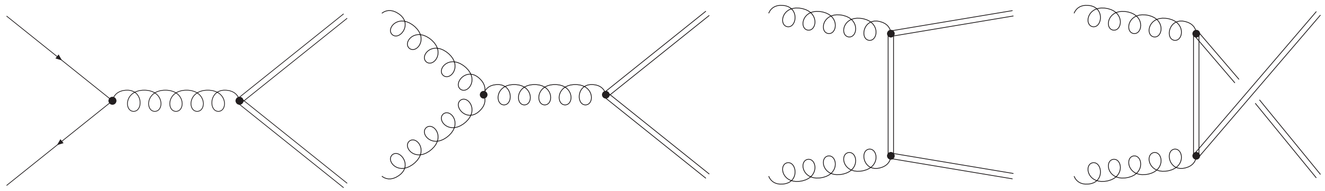


- NLO in QCD Nason, Dawson, Ellis ‘88; Beenakker, Smith, van Neerven ‘89; Mangano, Nason, Ridolfi ‘92; Bernreuther, Brandenburg, Si, Uwer ‘04; Mitov, Czakon ‘08; . . .
  - accurate to $\mathcal{O}(15\%)$ at LHC

Top quark production

- Leading order Feynman diagrams

$$\begin{aligned} \mathrm{q} + \bar{\mathrm{q}} &\longrightarrow \mathrm{Q} + \bar{\mathrm{Q}} \\ \mathrm{g} + \mathrm{g} &\longrightarrow \mathrm{Q} + \bar{\mathrm{Q}} \end{aligned}$$



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Challenge

- Improve theory precdictions and reduce theoretical uncertainty
  - hard scattering cross section $\hat{\sigma}_{ij \to t\bar{t}}$
  - parton luminosity $f_i \otimes f_j$

Recent theory activities

- General structure of massive QCD amplitudes
  - relate massive to massless amplitudes in limit $m \to 0$ Mitov, S.M. '06
  - two-loop virtual corrections to $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$
    - small-mass limit $m^2 \ll s, t, u$ Czakon, Mitov, S.M. '07
    - complete IR singularities Ferroglia, Neubert, Pecjak, Yang '09
- Exact virtual amplitudes
  - one-loop squared terms (NLO $\times$ NLO)
    Anastasiou, Mert Aybat '08; Kniehl, Merebashvili, Körner, Rogal '08
  - two-loop virtual corrections for $q\bar{q} \to t\bar{t}$
    (analytic, n_f-terms) Bonciani, Ferroglia, Gehrmann, Maitre, Studerus '08;
    (analytic, two-loop planar) Bonciani, Ferroglia, Gehrmann, Studerus '09;
    (numerical result) Czakon '08
- Complete NLO corrections to $t\bar{t}$ in association with jets **$\longrightarrow$ talk by Worek**
  - $t\bar{t} + 1$ jet at NLO Dittmaier, Uwer, Weinzierl '07-'08; Melnikov, Schulze '10
  - $t\bar{t} + 2$ jets at NLO Bevilacqua, Czakon, Papadopoulos, Worek '10

Recent theory activities (cont'd)

- Threshold resummation
  - updates of cross section predictions based on resummation
    S.M., Uwer '08; Cacciari, Frixione, Mangano, Nason, Ridolfi '08; Kidonakis, Vogt '08;
    Beneke, Czakon, Falgari, Mitov, Schwinn '09;
    Ahrens, Ferroglia, Neubert, Pecjak, Yang '10
  - coulomb corrections
    Hagiwara, Sumino, Yokoya '08; Kiyo, Kühn, S.M., Steinhauser, Uwer '08
- Definition of mass parameter Hoang, Jain, Scimemi, Stewart '08
- Parton luminosity

  $\longrightarrow$ talk by Guffanti
  - precision PDFs for LHC (NNLO global analyses)
    Martin, Stirling, Thorne, Watt '08; Alekhin, Blümlein, Klein, S.M. '09;
    Jimenez-Delgado, Reya '09
  - correlation of cross section at NLO with gluon PDFs
    Nadolsky, Lai, Cao, Huston, Pumplin, Stump, Tung, Yuan '08
  - benchmarking of PDFs under way

NLO

- Cross section for heavy-quark hadro-production (scaling functions f_{ij})

$$\hat{\sigma}_{ij\to t\bar{t}} \quad = \quad \frac{{\alpha_s}^2}{m_t^2}\left\{f_{ij}^{(0)}(\rho) + (4\pi\alpha_s)\, f_{ij}^{(1)}(\rho, \mu_f/m_t, \mu_r/\mu_f)\right\}$$

- Perturbative expansion at NLO

NLO

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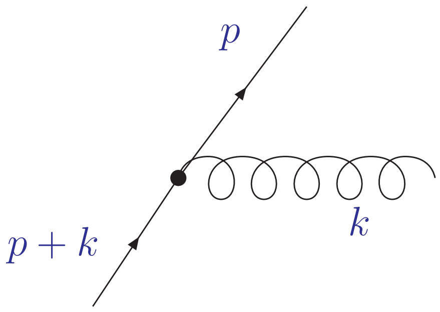
- Perturbative expansion at NLO

Strategy beyond NLO

- Use universal features of soft/collinear regions of phase space
  - double logarithms from singular regions in Feynman diagrams
  - propagator vanishes for: $E_g = 0$, soft θ_{qg} = 0 collinear

$$\frac{1}{(p+k)^2} \quad = \quad \frac{1}{2p\cdot k} \quad = \quad \frac{1}{2E_q E_g(1-\cos\theta_{qg})}$$



$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \quad \longrightarrow \quad \alpha_s \int dE_g\, d\sin\theta_{qg}\, \frac{1}{2E_q E_g(1-\cos\theta_{qg})}$$

$$\longrightarrow \quad \alpha_s \, \ln^2(\ldots)$$

Beyond NLO: all-order resummation

- Cross section for heavy-quark hadro-production (scaling functions f_{ij})

$$\hat{\sigma}_{ij\to t\bar{t}} \quad = \quad \frac{{\alpha_s}^2}{m_t^2}\{f_{ij}^{(0)}(\rho) + (4\pi\alpha_s)\, f_{ij}^{(1)}(\rho, \mu_f/m_t, \mu_r/\mu_f)$$
$$+ f_{ij}^{\text{resummed}}(\alpha_s, N, \mu_f/m_t, \mu_r/\mu_f) + \mathcal{O}(N^{-1}\ln^n N)\}$$

- All order resummation of large logarithms $\alpha_s^n \ln^{2n}(\beta) \longleftrightarrow \alpha_s^n\, \ln^{2n}(N)$
  - resummation in Mellin space (renormalization group equation)
  - long history Kidonakis, Sterman '97; Bonciani, Catani, Mangano, Nason '98; Kidonakis, Laenen, S.M., Vogt '01; . . .
- Upshot:
  - $f_{ij}^{\text{resummed}} \simeq \exp\left(\alpha_s \ln^2 N\right) + \mathcal{O}(N^{-1} \ln^n N)$

Beyond NLO: NNLO$_{\rm approx}$

- Cross section for heavy-quark hadro-production (scaling functions f_{ij})

$$\hat{\sigma}_{ij\to t\bar{t}} = \frac{{\alpha_s}^2}{m_t^2}\{f_{ij}^{(0)}(\rho) + (4\pi\alpha_s)\, f_{ij}^{(1)}(\rho, \mu_f/m_t, \mu_r/\mu_f) + (4\pi\alpha_s)^2 f_{ij}^{(2)}(\rho, \mu_f/m_t, \mu_r/\mu_f) + \mathcal{O}({\alpha_s}^3)\}$$

- General structure at NNLO
  - dependence on factorization and renormalization scale $L_M = \ln(\mu_f^2/m_t^2)$ and $L_R = \ln(\mu_r^2/\mu_f^2)$

$$f_{ij}^{(1)}(\rho, \mu_f/m_t, \mu_r/m_t) = f_{ij}^{(10)} + L_M f_{ij}^{(11)} + 2\beta_0 L_R f_{ij}^{(0)} ,$$

$$f_{ij}^{(2)}(\rho, \mu_f/m_t, \mu_r/m_t) = f_{ij}^{(20)} + L_M f_{ij}^{(21)} + L_M^2 f_{ij}^{(22)} + 3\beta_0 L_R f_{ij}^{(10)} + 3\beta_0 L_R L_M f_{ij}^{(11)} + 2\beta_1 L_R f_{ij}^{(0)} + 3\beta_0^2 L_R^2 f_{ij}^{(0)}$$

  - only unknown: $f_{ij}^{(20)}$ (but knowledge of threshold logarithms)
  - all other function known through renormalization group equations

Two-loop results

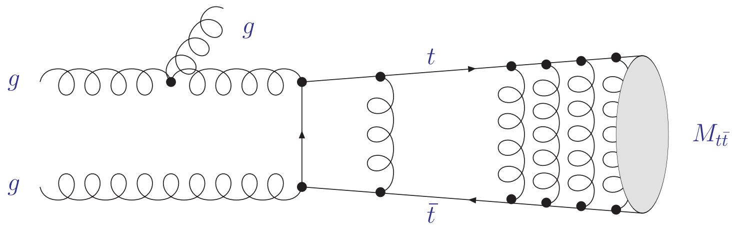
- NNLO cross section for heavy-quark hadro-production near threshold
  S.M, Uwer ‘08; Beneke, Czakon, Falgari, Mitov, Schwinn ‘09
  - e.g. gg-fusion for $n_f = 5$ light flavors at $\mu = m_t$

$$f_{gg}^{(10)} = \frac{f_{gg}^{(0)}}{(16\pi^2)} \left\{ 96 \ln^2 \beta - 9.5165 \ln \beta + 35.322 + 5.1698 \frac{1}{\beta} \right\}$$

$$f_{gg}^{(20)} = \frac{f_{gg}^{(0)}}{(16\pi^2)^2} \left\{ 4608 \ln^4 \beta - 1894.9 \ln^3 \beta + \left(-912.35 + 496.30 \frac{1}{\beta} \right) \ln^2 \beta \right.$$
$$\left. + \left(2456.7 + 321.14 \frac{1}{\beta} \right) \ln \beta + 68.547 \frac{1}{\beta^2} - 8.6226 \frac{1}{\beta} + C_{gg}^{(2)} \right\}$$

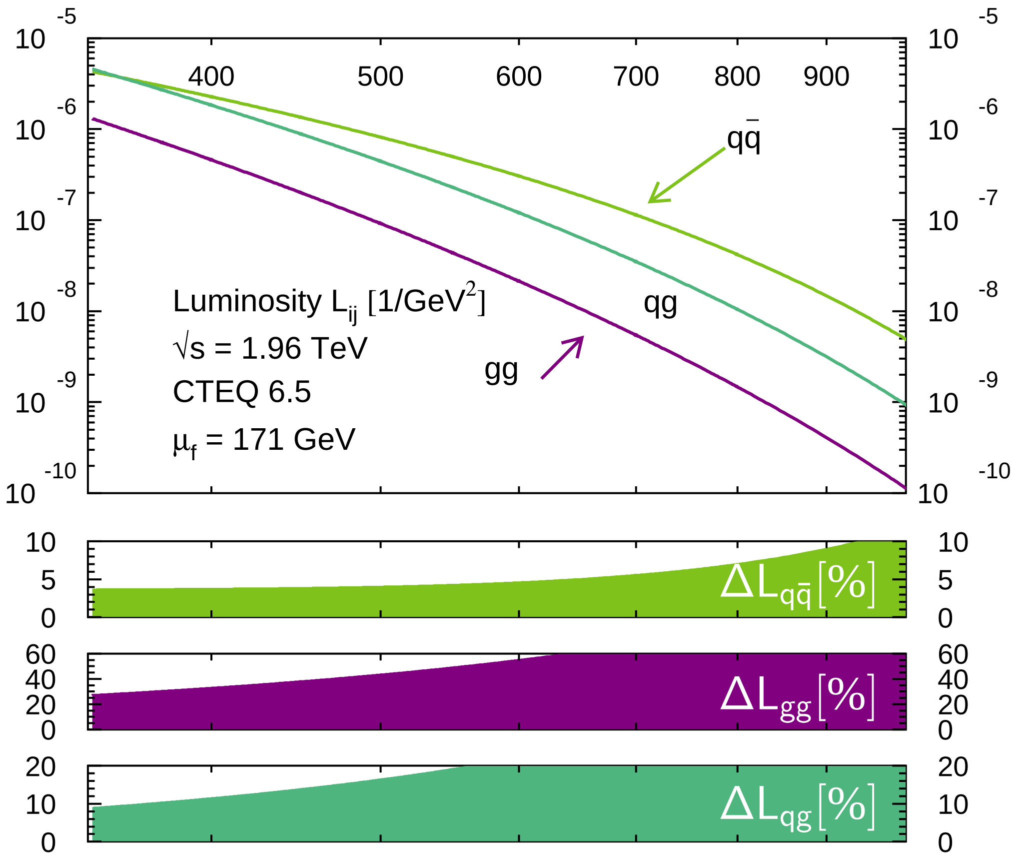
Who’s who

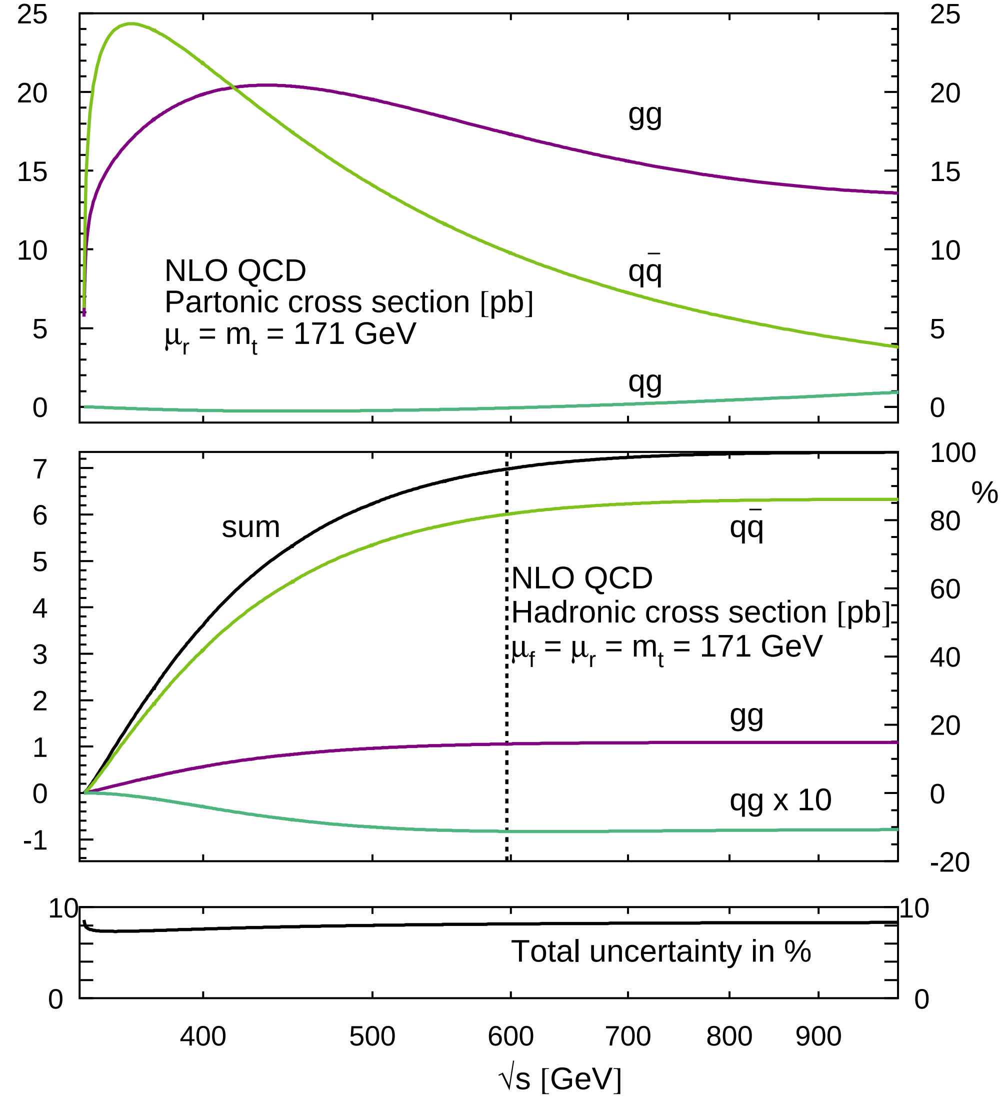
- Sudakov logarithms in $\ln \beta$ (generated from resummed cross sections)
- $1/\beta$-terms from Coulomb corrections
  resummation to all orders
  in non-relativistic QCD
- unkown constant $C_{gg}^{(2)}$



Total cross section at Tevatron

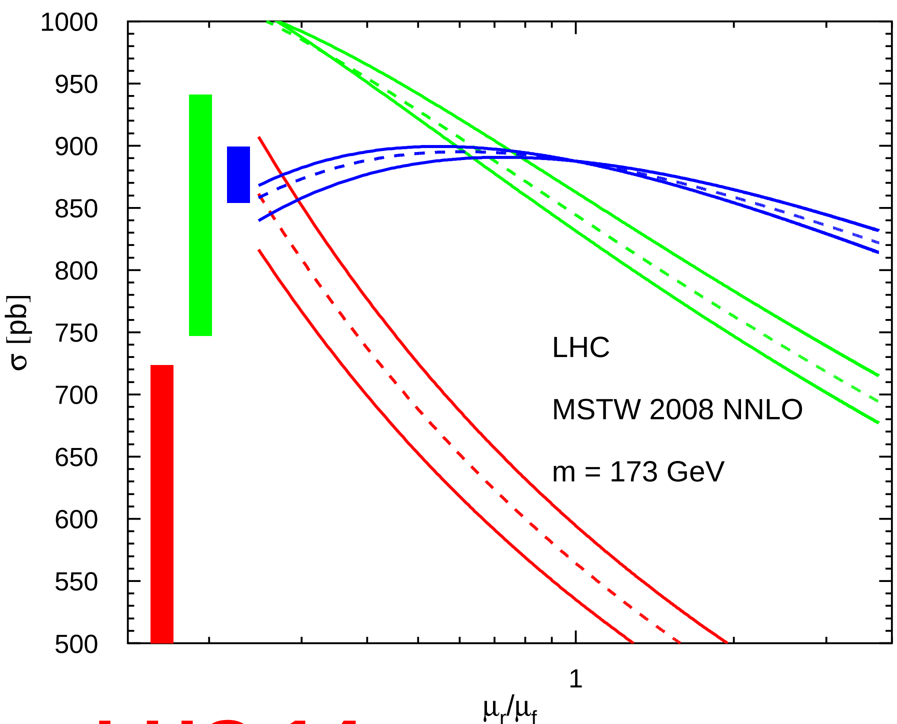
$$\sigma_{pp\to t\bar{t}} = \sum_{ij} f_i \otimes f_j \otimes \hat{\sigma}_{ij\to t\bar{t}}$$



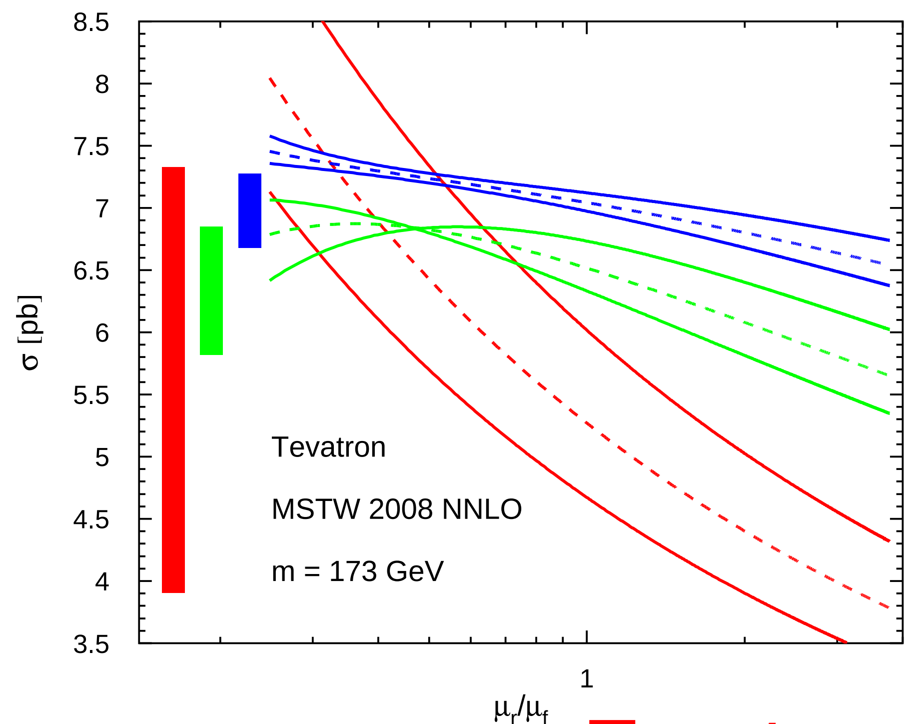


Scale dependence

- Theoretical uncertainty from variation of scales μ_R, μ_F
  - plot with PDF set MSTW 2008 (but largely independent on PDFs)
  - mass $m_t = 173$ GeV (from $m_t = 173.1 \pm 1.3$ GeV Tevatron winter '09)
  - stable predictions in range $\mu_R, \mu_F \in [m_t/2, 2m_t]$
    - $-3\% \le \Delta\sigma \le +1\%$ at LHC
    - $-5\% \le \Delta\sigma \le +3\%$ at Tevatron



LHC 14



Tevatron

The total cross section

- Theory prediction at NNLO$_{\rm approx}$ accuracy
  - pole mass $m_t = 173$ GeV
  - theoretical uncertainity from variation of $\mu_R, \mu_F \in [m_t/2, 2m_t]$
  - NNLO PDF set and α_s, e.g. MSTW 2008

$$\sigma_{\rm LHC} \quad = \quad 157.0 \text{ pb} \quad {}^{+2.1}_{-6.5} \text{ pb (scale)} \quad {}^{+4.4}_{-4.4} \text{ pb (MSTW2008)}$$

$$\sigma_{\rm TeV} \quad = \quad 6.93 \,\text{pb} \quad {}^{+0.15}_{-0.32} \,\text{pb (scale)} \quad {}^{+0.14}_{-0.14} \,\text{pb (MSTW2008)}$$

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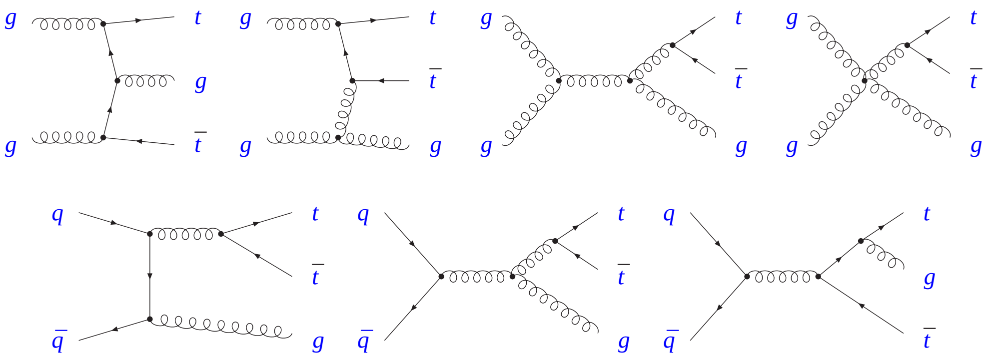
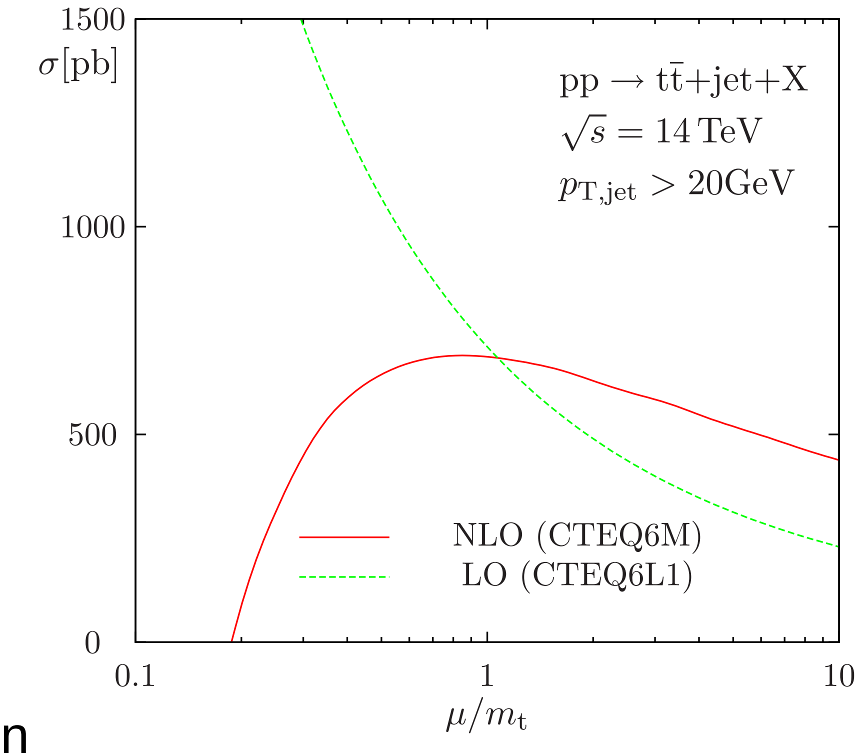
Quality control

- Check of systematics: variation of $C_{gg}^{(2)}$
  - recall $f_{gg}^{(20)} \simeq f_{gg}^{(0)} \{4608 \ln^4 \beta + \dots + C_{gg}^{(2)}\}$
- Estimate systematic uncertainty of total cross section $\Delta\sigma \sim \mathcal{O}(2\%)$
  - $\Delta\sigma \sim \mathcal{O}(3-4)\,{\rm pb}$ at LHC 7 TeV ($\mathcal{O}(10-15)\,{\rm pb}$ at 14 TeV)
  - $\Delta\sigma \sim \mathcal{O}(0.15-0.2)\,{\rm pb}$ at Tevatron

$t\bar{t}+$ jet production (I)

- LHC: large rates for production of $t\bar{t}$-pairs with additional jets
- NLO corrections to $t\bar{t}$+jet production are part of NNLO corrections for inclusive $t\bar{t}$ production





- Check of systematics: hard radiation
  - at scale $\mu_R = \mu_F = m_t$ corrections are almost zero
  - threshold resummation captures dominant contributions

$t\bar{t}+$ jet production (II)

Tevatron

$p_{\mathrm{T,jet,cut}}$ [GeV]	$\sigma_{\mathrm{t\bar{t}jet}}[\mathrm{pb}]$ LO	$\sigma_{\mathrm{t\bar{t}jet}}[\mathrm{pb}]$ NLO
20	$1.583(2)^{+0.96}_{-0.55}$	$1.791(1)^{+0.16}_{-0.31}$
30	$0.984(1)^{+0.60}_{-0.34}$	$1.1194(8)^{+0.11}_{-0.20}$
40	$0.6632(8)^{+0.41}_{-0.23}$	$0.7504(5)^{+0.072}_{-0.14}$
50	$0.4670(6)^{+0.29}_{-0.17}$	$0.5244(4)^{+0.049}_{-0.096}$

LHC 14

$p_{\mathrm{T,jet,cut}}$ [GeV]	$\sigma_{\mathrm{t\bar{t}jet}}[\mathrm{pb}]$ LO	$\sigma_{\mathrm{t\bar{t}jet}}[\mathrm{pb}]$ NLO
20	$710.8(8)^{+358}_{-221}$	$692(3)3^{-40}_{-62}$
50	$326.6(4)^{+168}_{-103}$	$376.2(6)^{+17}_{-48}$
100	$146.7(2)^{+77}_{-47}$	$175.0(2)^{+10}_{-24}$
200	$46.67(6)^{+26}_{-15}$	$52.81(8)^{+0.8}_{-6.7}$

- Cross section $\sigma_{\mathrm{t\bar{t}jet}}$ for different values of $p_{\mathrm{T,jet,cut}}$ for $\mu = \mu_R = \mu_F = \{m_{\mathrm{t}}/2, m_{\mathrm{t}}, 2m_{\mathrm{t}}\}$ with PDF sets CTEQ6L1, CTEQ6M
  Dittmaier, Uwer, Weinzierl '07-'08

Concordance approach

Cacciari, Czakon, Mangano, Mitov, S.M., Nason, Uwer [to appear]

- Philosophy
  - genuine NLO approach improved with threshold logarithms only
  - technically: truncation of $f_{ij}^{(2)} = f_{ij}^{(20)} + L_M f_{ij}^{(21)} + L_M^2 f_{ij}^{(22)} + \dots$ to logarithmic accuracy in $\ln\beta$
- Conservative estimate of theoretical uncertainty
  - scale variation of $\mu_R, \mu_F \in [m_t/2, 2m_t]$ with $1/2 \leq \mu_R/\mu_F \leq 2$
  - e.g. pole mass $m_t = 173$ GeV and NNLO PDF set MSTW08

$$\sigma_{\rm LHC} = 155.4\ {\rm pb}\ {}^{+1.3}_{-6.3}\ {\rm pb\ (scale)}\ {}^{+4.4}_{-4.4}\ {\rm pb\ (MSTW2008)}$$

$$\sigma_{\rm TeV} = 6.92\,{\rm pb}\ {}^{+0.01}_{-0.41}\,{\rm pb\ (scale)}\ {}^{+0.14}_{-0.14}\,{\rm pb\ (MSTW2008)}$$

preliminary

HATHOR

C++ package

Aliev, Lacker, Langenfeld, S.M. Uwer [to appear]

- Cross section evaluation done entirely in Hathor class

```
unsigned int scheme = Hathor::LO | Hathor::NLO | Hathor::NNLO;
double mt = 171., muf=171., mur=171.;
double val,err,chi2a,up,down;

Lhapdf pdf("MSTW2008nnlo68cl");
Hathor XS(pdf)
```

⟵ **PDF choice**

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Lhapdf pdf("MSTW2008nnlo68cl");
Hathor XS(pdf)

XS.setPrecision(Hathor::MEDIUM);
XS.getXsection(mt,mur,muf);
XS.getResult(0,val,err,chi2a);
```

$\longleftarrow$ **PDF choice** (for `Lhapdf pdf("MSTW2008nnlo68cl");`)

$\longrightarrow \sigma = 164.3^{+4.6}_{-9.2}\,\mathrm{pb}$ **(scale unc.)** (for `XS.getXsection(mt,mur,muf);`)

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Hathor XS(pdf)

XS.setPrecision(Hathor::MEDIUM);
XS.getXsection(mt,mur,muf);
XS.getResult(0,val,err,chi2a);

XS.setScheme(scheme | Hathor::PDF_SCAN);
XS.setPrecision(Hathor::LOW);
XS.getXsection(mt,mur,muf);
XS.getPdfErr(up,down);
```

$\longleftarrow$ **PDF choice** (for `Lhapdf pdf("MSTW2008nnlo68cl");`)

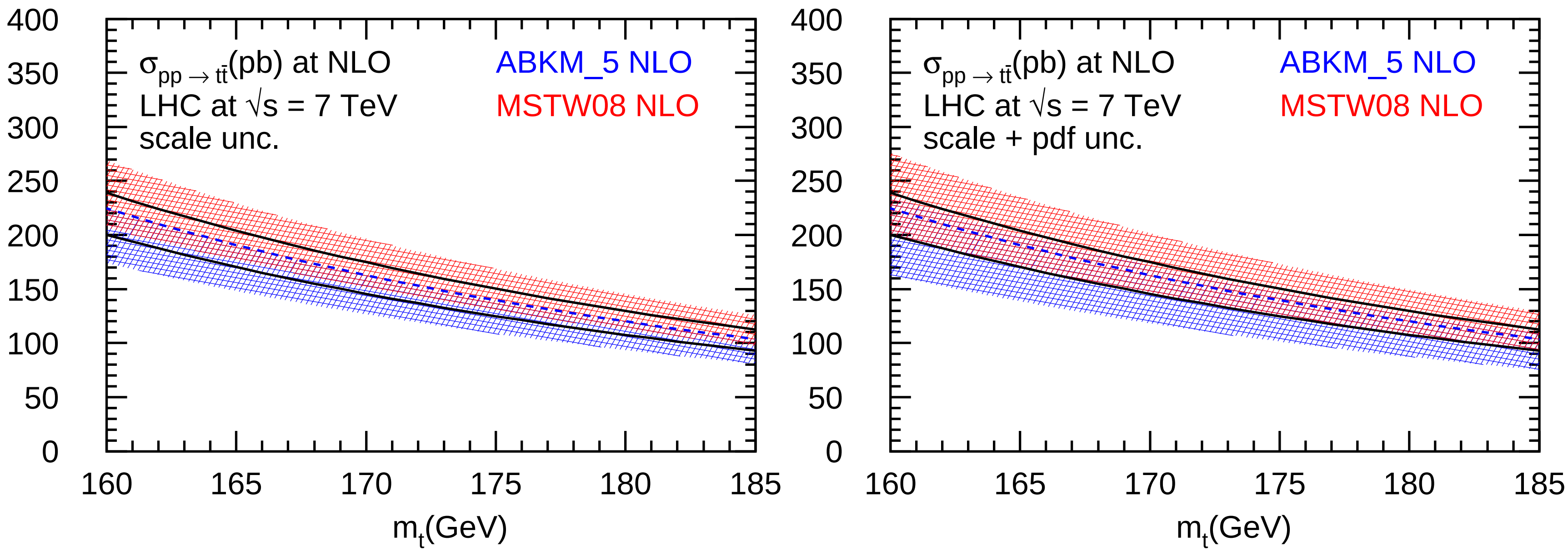
$\longrightarrow \sigma = 164.3^{+4.6}_{-9.2}\,\mathrm{pb}$ **(scale unc.)** (for `XS.getXsection(mt,mur,muf);`)

$\longleftarrow$ **with PDF error scan** (for `XS.setScheme(scheme | Hathor::PDF_SCAN);`)

$\} \longrightarrow \sigma = 164.3^{+4.6}_{-9.2}\,\mathrm{pb}$ **(sc.)** $^{+4.4}_{-4.4}\,\mathrm{pb}$ **(PDF unc.)** (for `XS.getXsection(mt,mur,muf);` and `XS.getPdfErr(up,down);`)

Dependence on parton distributions

- Comparision of NLO and NNLO$_{\text{approx}}$
- Theoretical uncertainty from scale variation: $\mu_R, \mu_F \in [m_t/2, 2m_t]$
  - uncertainties: scale (left) and scale + PDF (right)
- Sizeable difference between the ABKM/MSTW sets at NNLO well outside the PDF uncertainty
  - due to value of α_s and shape of gluon PDFs at $\langle x \rangle = 2m_t/\sqrt{s}$
  - can only be settled with first LHC data



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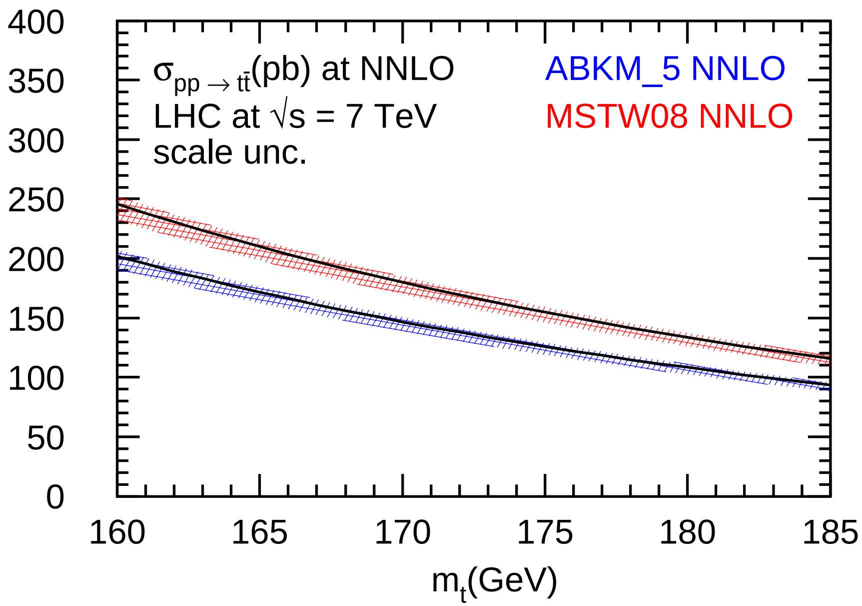
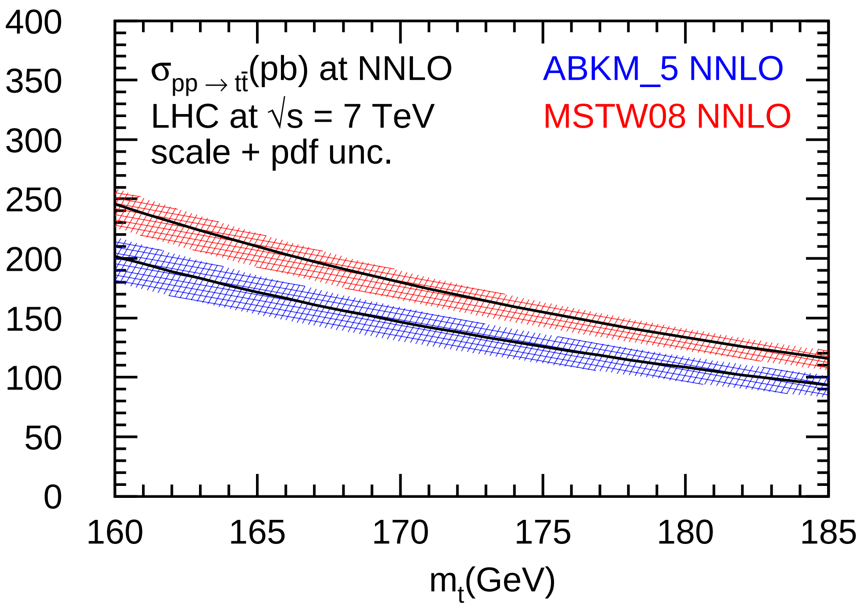
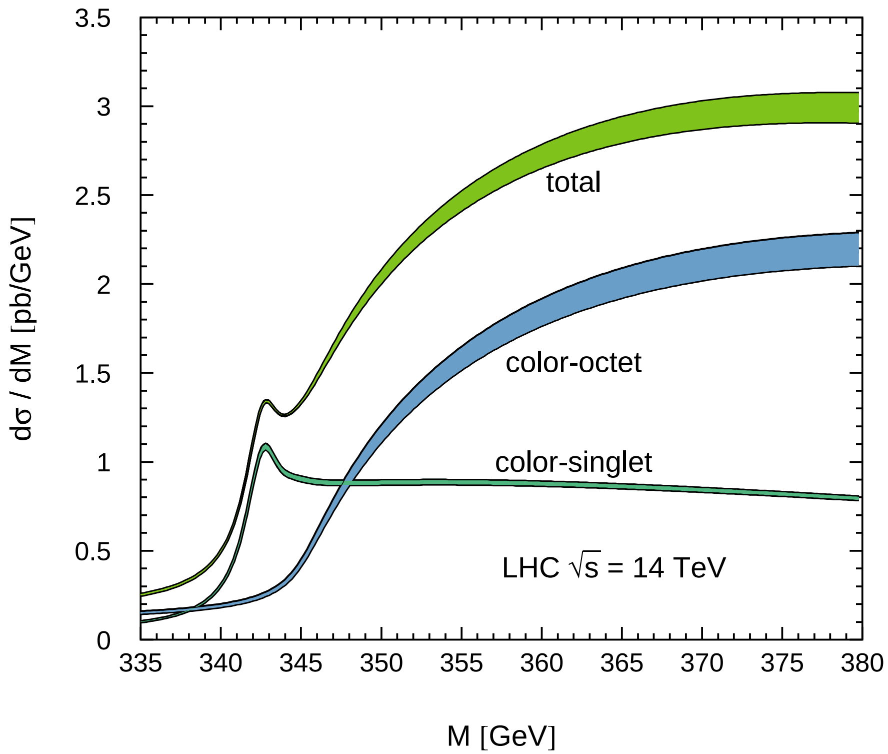
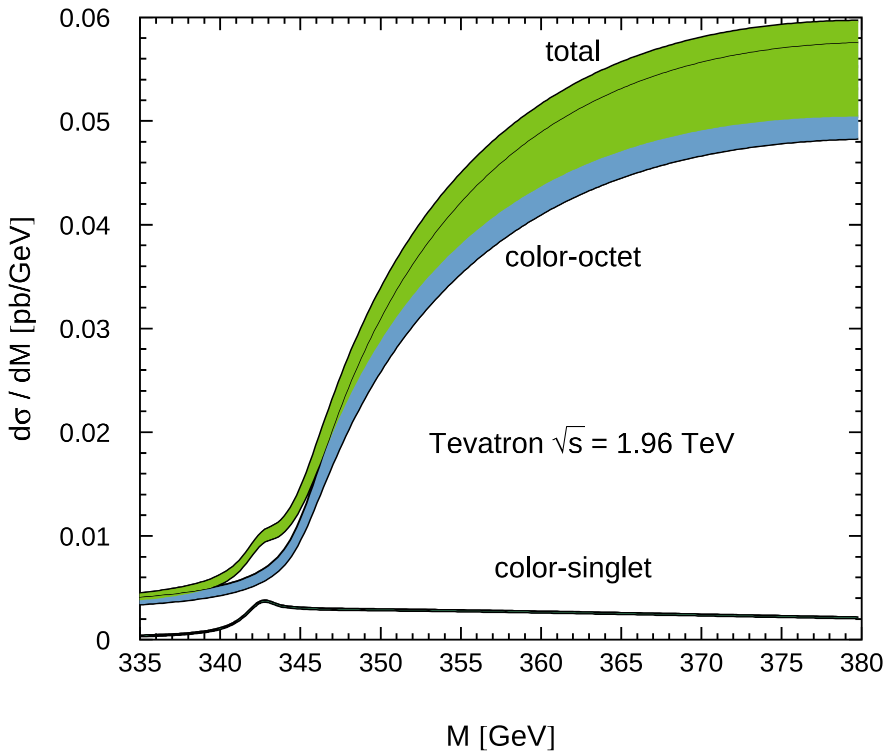




Resummation of Coulomb corrections

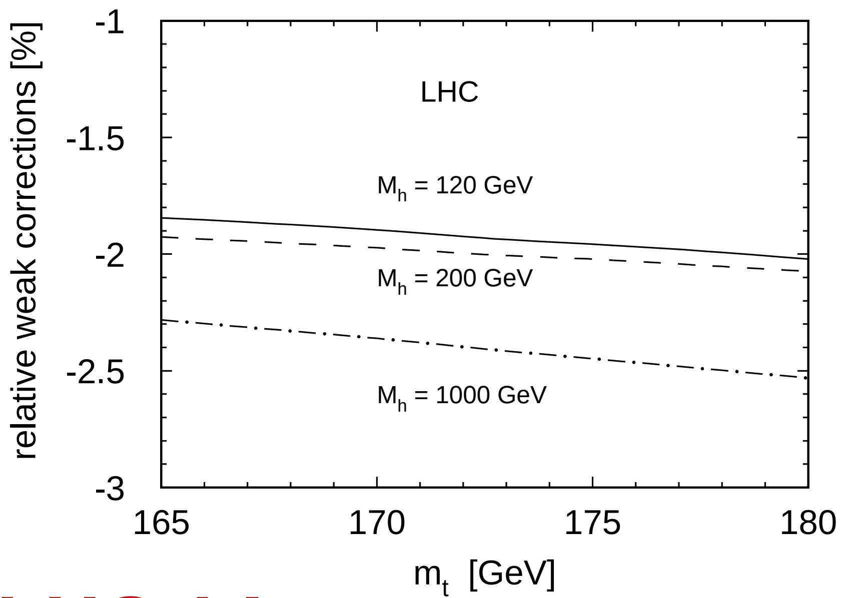
- Invariant mass distristribution $d\sigma/dM_{t\bar{t}}$
  - at LHC $gg \to t\bar{t}\left({}^1S_0^{[1]}\right)$ dominates; driven by large gluon luminosity
  - at Tevatron with small bound state effects; $q\bar{q}$-channel large with only color-octet configurations only



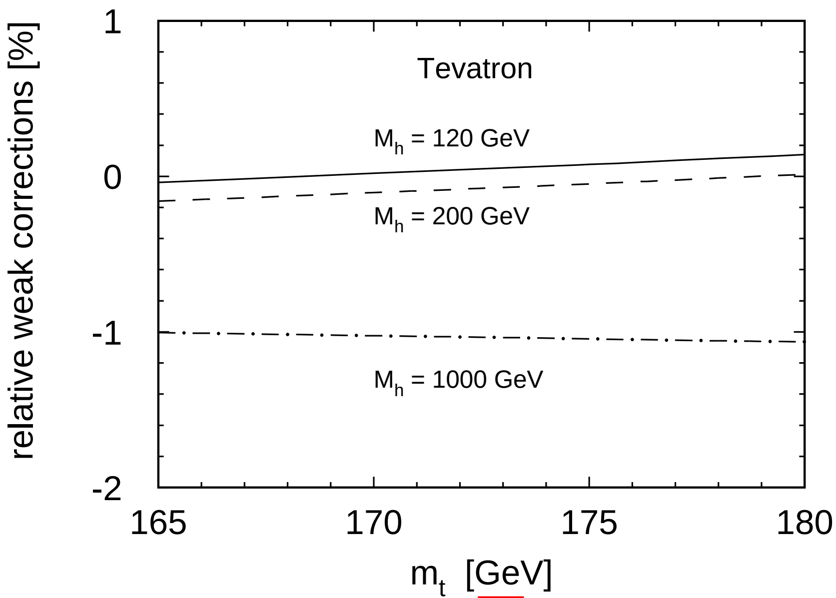


Electroweak corrections

- Electroweak corrections (ratio of $\sigma_{\rm EW}/\sigma_{\rm LO}$) Beenakker, Denner, Hollik, Mertig, Sack, Wackeroth '94; Bernreuther, Fücker '05; Kühn, Uwer, Scharf '06
- Effect depends on Higgs mass
(choices $m_H = 120$GeV, $m_H = 200$GeV, $m_H = 1000$GeV)



LHC 14



Tevatron

- Tevatron: vanishing contribution for light Higgs
- LHC: $\mathcal{O}(2\%)$ with respect to $\sigma_{\rm LO}$
negative contribution to total cross section $\Delta\sigma_{\rm EW} \simeq \mathcal{O}(10-15)$ pb

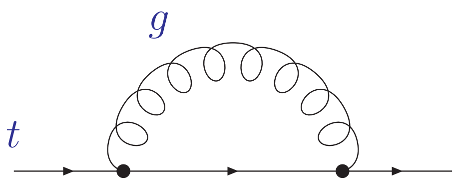
Mass dependence of cross section

Pole mass scheme

→ talk by Corcella

- Based on (unphysical) concept of top-quark being a free parton

$$\not{p} - m_t - \Sigma(p, m_t)\Big|_{p^2=m_t^2}$$



  - heavy-quark self-energy $\Sigma(p, m_t)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{\mathrm{QCD}})$

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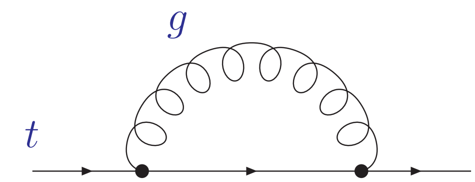
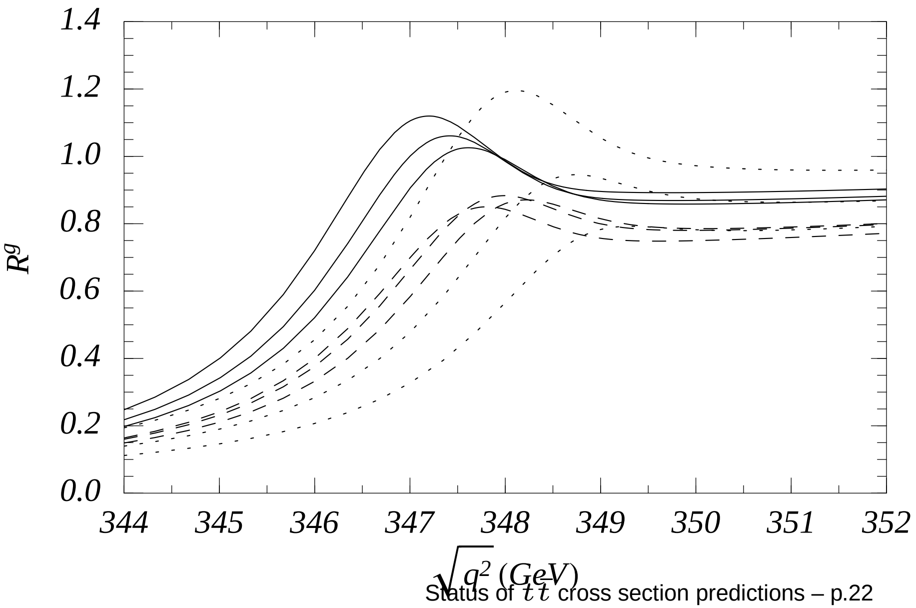


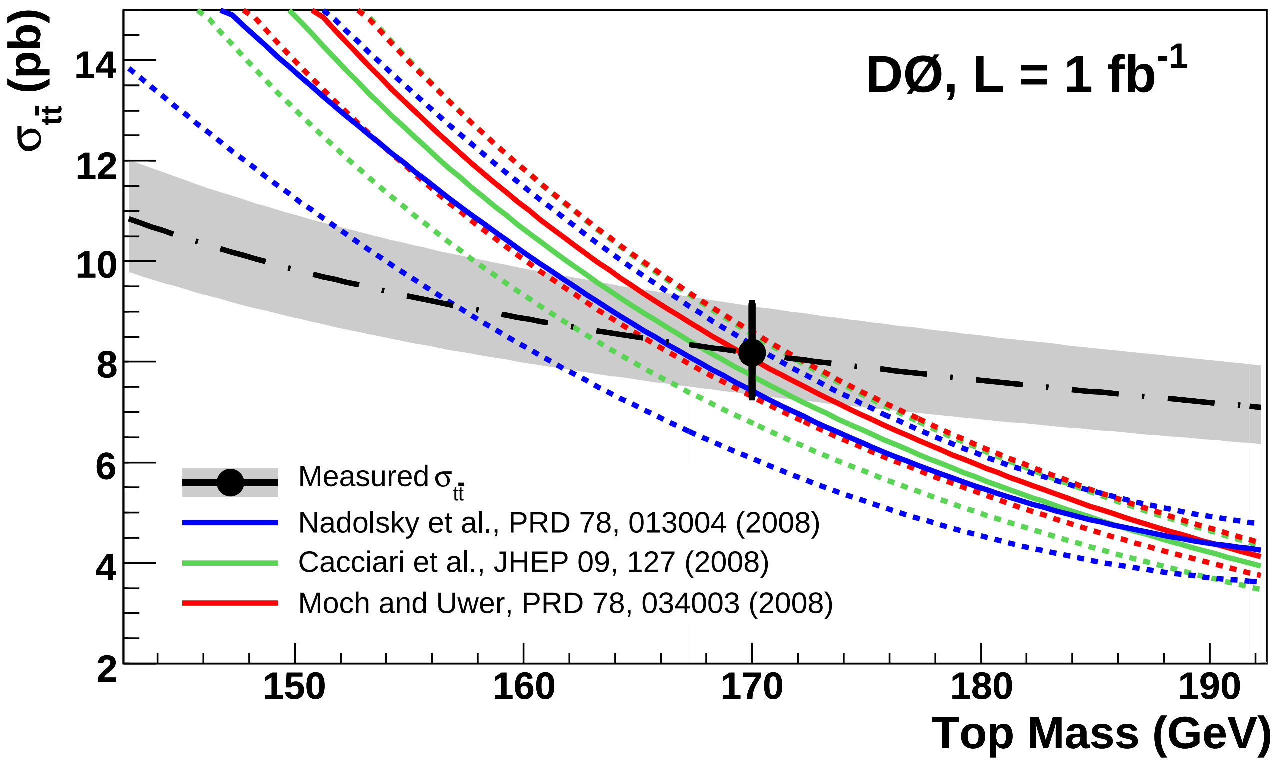
  - heavy-quark self-energy $\Sigma(p, m_t)$ receives contributions from regions of all loop momenta – also from momenta of $\mathcal{O}(\Lambda_{\rm QCD})$

- Pole mass measurements are strongly order-dependent
  - e.g. threshold scan of cross section in e^+e^- collision
    Beneke, Signer, Smirnov '99;
    Hoang, Teubner '99;
    Melnikov, Yelkhovsky '98;
    Penin, Pivovarov '99;
    Yakovlev '99
  - LO (dotted), NLO (dashed), NNLO (solid)



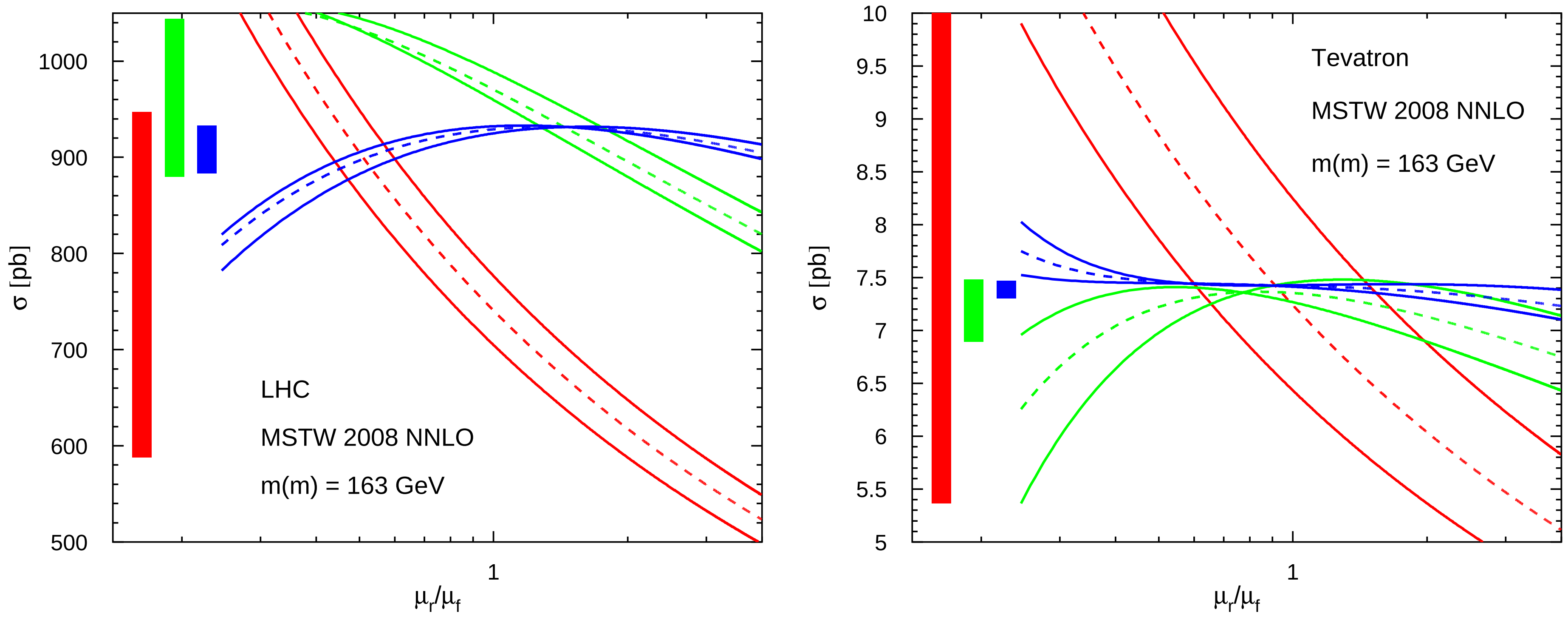
Tevatron analyses

- Total cross section and different channels of Tevatron analyses (theory uncertainty band from scale variation)
- Determination of m_t from total cross section (slope $d\sigma/dm_t$)
  - e.g. DZero '09: NLO $m_t = 165.5^{+6.1}_{-5.9}$; NNLO $m_t = 169.1^{+5.9}_{-5.2}$; ...



The running top-quark mass

- $\overline{MS}$ mass definition $m(\mu_R)$ realizes running mass (scale dependence)
  - short distance mass probes at scale of hard scattering
  - conversion between pole mass and $\overline{MS}$ mass definition in perturbation theory: $m_t = m(\mu_R)\left(1 + a_s(\mu_R)d^{(1)} + a_s(\mu_R)^2 d^{(2)}\right)$
- Scale dependence greatly reduced



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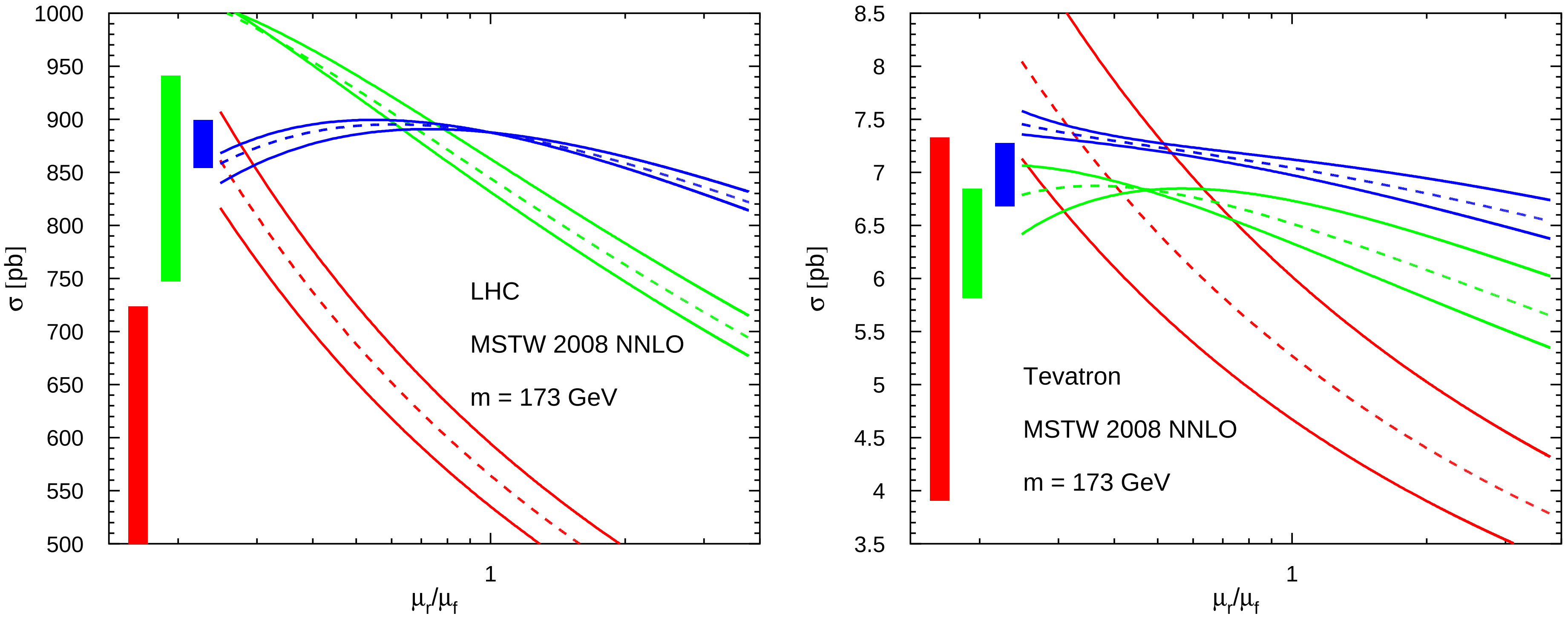
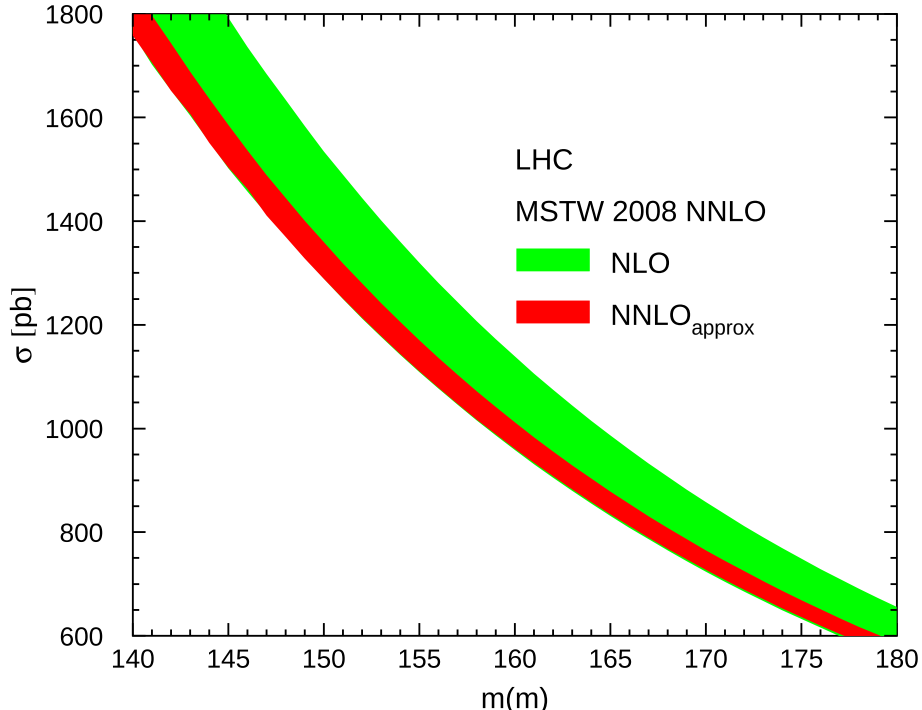
- Pole mass scheme for comparison

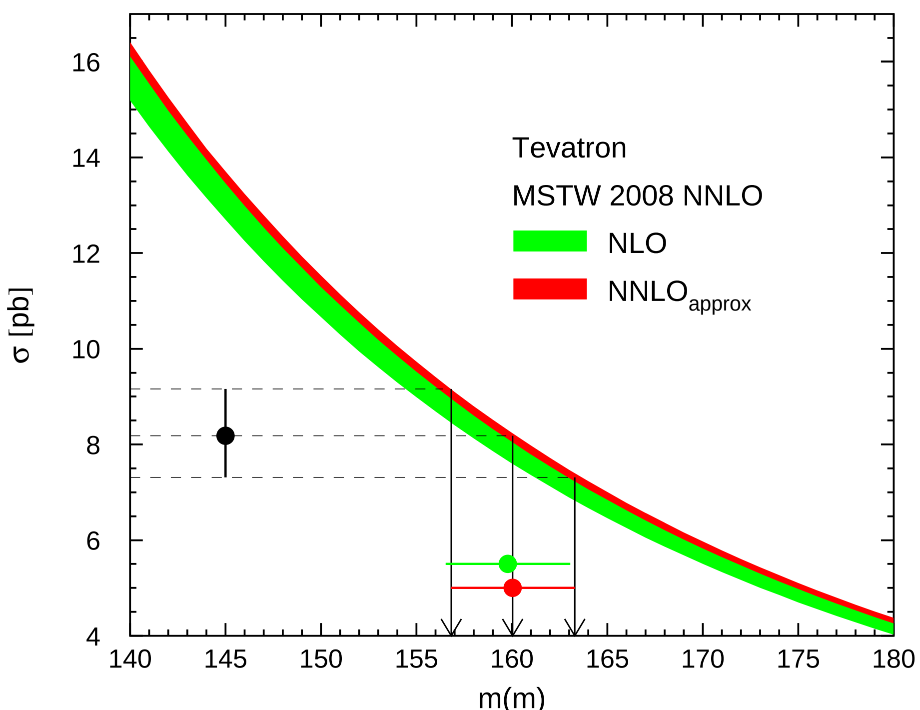


Mass dependence in $\overline{MS}$ mass scheme

- Total top-quark cross section as function of $\overline{m}$
  - theoretical uncertainity (band) due to variation of $\mu_R \in [\overline{m}/2, 2\overline{m}]$ for fixed set $\mu_F \in \overline{m}/2, \overline{m}, 2\overline{m}$



LHC 14

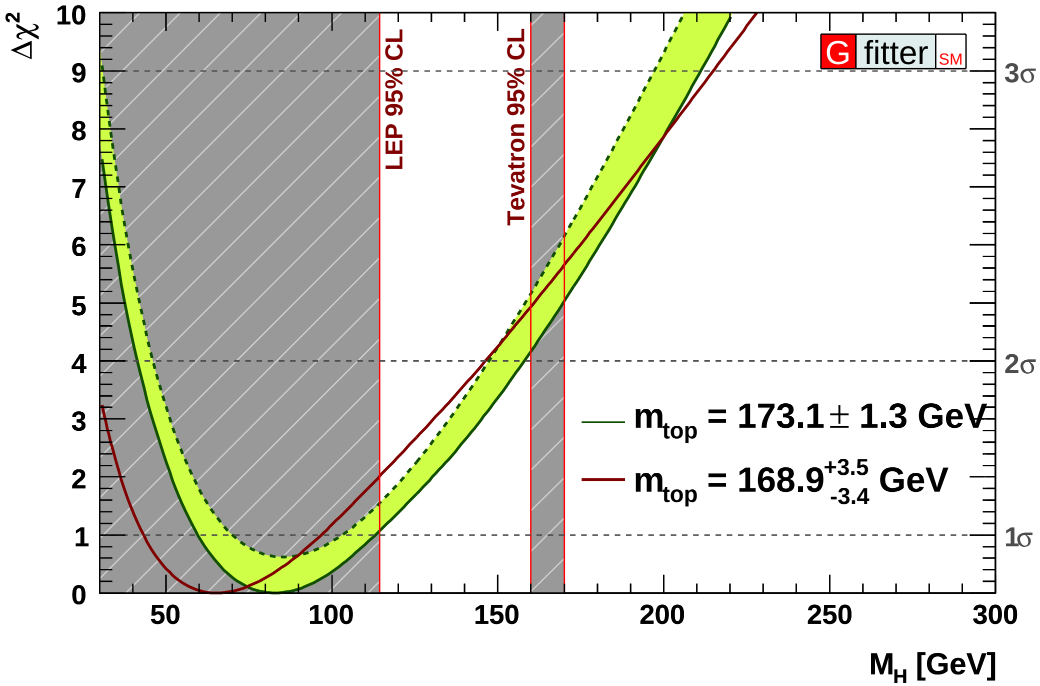


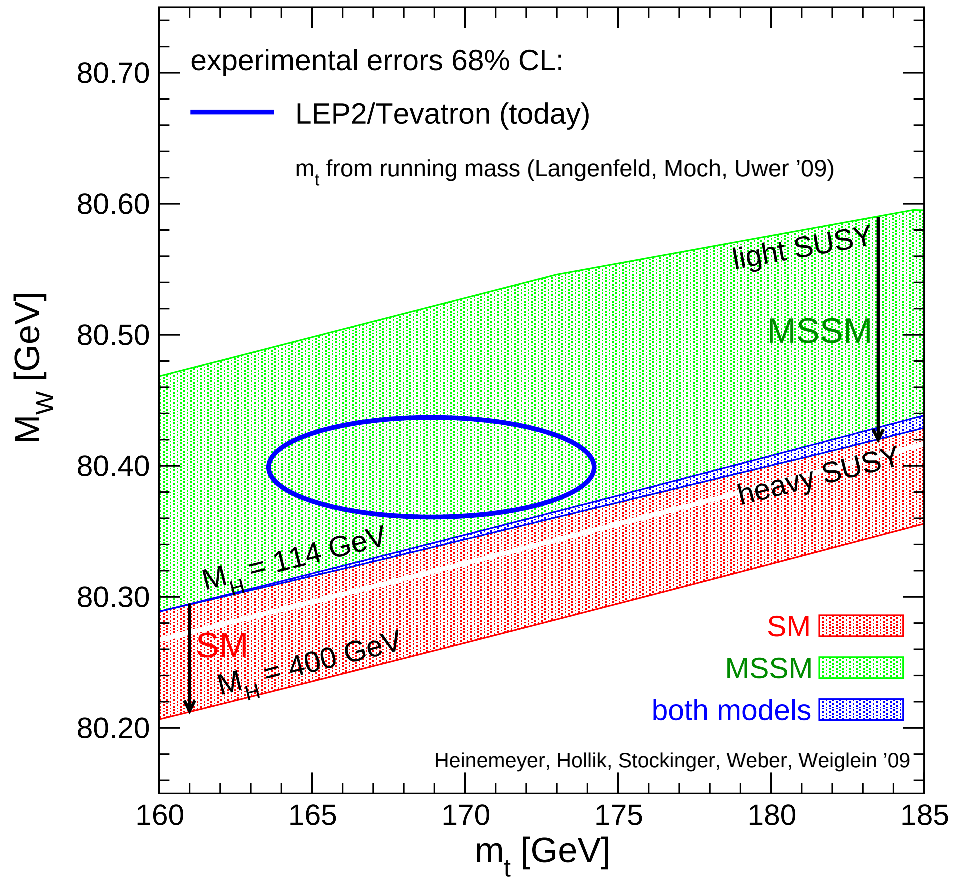
Tevatron

- First direct determination of $m(m)$ Langenfeld, S.M., Uwer '09
  - $\overline{MS}$ mass $m(m) = 160.0^{+3.3}_{-3.2}\text{GeV}$
  - conversion to pole mass $m_t = 168.9^{+3.5}_{-3.4}\text{GeV}$

Implications for indirect Higgs searches

- Electroweak precision data constrains M_H





- pole mass $m_t = 168.9^{+3.5}_{-3.4}\mathrm{GeV}$
  (lighter top-quark masses disfavor SM Higgs sector)

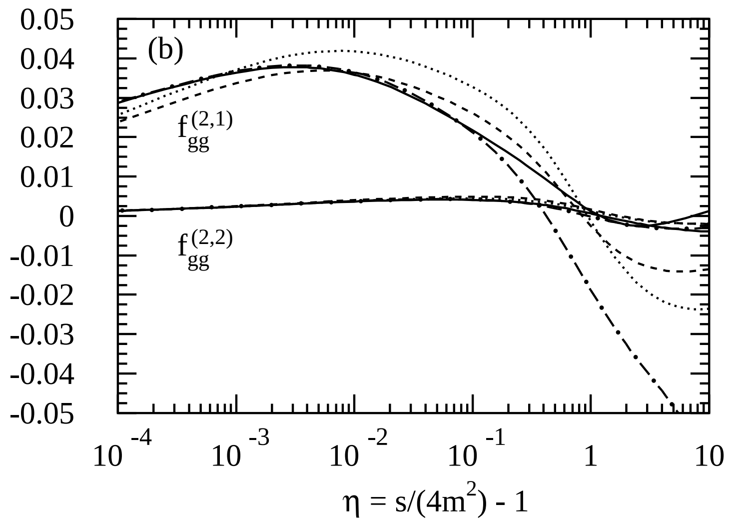
Summary

- Top quark theory
  - much recent progress for Tevatron and LHC phenomenology
  - improved understanding of theory and application of new concepts
- Total cross section
  - NNLO$_{\mathrm{approx}}$ prediction with exact scale dependence $\mu_R \neq \mu_F$ ($\ln(\mu_R/m)$, $\ln(\mu_F/m)$-terms)
  - cross check on systematics with NLO correction to $t\bar{t}+$ jet
  - electroweak corrections
  - bound state effects for $t\bar{t}$-system
- $\overline{MS}$ mass definition
  - running top-quark mass of $m(\mu = m) = 160.0^{+3.3}_{-3.2}$ GeV
  - greatly reduced scale dependence
  - much improved convergence of perturbation theory

Extra slides

Other recent developments

- Total cross section from invariant mass distristribution $d\sigma/dM_{t\bar{t}}$
  Ahrens, Ferroglia, Neubert, Pecjak, Yang '10
  - resummation of threshold logarithms in $\ln(1 - M_{t\bar{t}}^2/s)/(1 - M_{t\bar{t}}^2/s)$
- Cross section numbers significantly lower
  - $\sigma_{\rm LHC} = 146\ {\rm pb}\ \ {}^{+7}_{-7}\ {\rm pb\ (scale)}\ \ {}^{+8}_{-8}\ {\rm pb\ (MSTW2008)}$
  - $\sigma_{\rm TeV} = 6.48\,{\rm pb}\ \ {}^{+0.17}_{-0.21}\,{\rm pb\ (scale)}\ \ {}^{+0.32}_{-0.25}\,{\rm pb\ (MSTW2008)}$
- Open issues:
  - impact of scale choices μ_s, μ_h, μ_f
  - compatibility at high energy (BFKL asymptotics at small-x)



Kidonakis, Laenen, S.M., Vogt '01