DRAKE 2: Dark matter Relic Abundance beyond Kinetic Equilibrium in 2-component dark sectors

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based on arXiv:2502.08725 (hep-ph)
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Outline

- Dark matter freeze-out beyond Kinetic Equilibrium: Why DRAKE?
- Production out-of-kinetic equilibrium: Details of the full Boltzmann Equation (fBE)
- fBE in two-component dark sector
- A look at how the DRAKE-2 works
- Benchmark Point results from DRAKE-2 for a 2-component coy dark matter model
- Summary

DM Freeze-out: Beyond Kinetic equilibrium?

Dark matter relic density measurement from the CMB is a well-measured quantity

$$\Omega_c h^2 = 0.1198 \pm 0.0012$$ PLANCK 2018

- Obtained from solving the Boltzmann equation

$$L[f_{DM}] = C[f_{DM}]$$

$$\partial_t f_{DM} - Hp\partial_p f_{DM} = C_{el}[f_{DM}] + C_{ann}[f_{DM}] + \ldots$$

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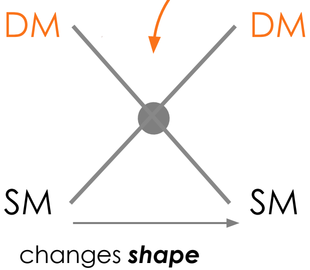

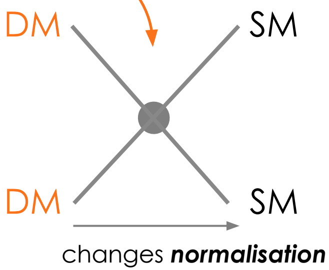

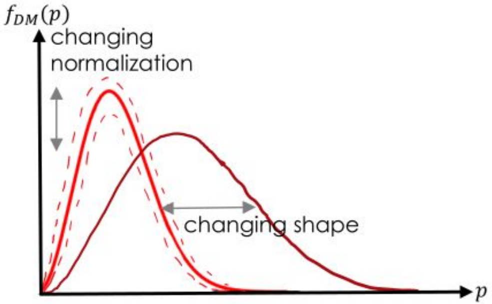

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Kinetic equilibrium $f_{DM}(T) \propto f_{eq}(T)$

Bernstein, Brown, Feinberg 1985

- Although typically a good assumption for $m_{DM} \gg m_{SM}$ …

$$\langle\sigma v\rangle_{scattering} \simeq \langle\sigma v\rangle_{annihilation},\ n_{SM}^{eq} \gg n_{DM}^{eq} \implies \Gamma_{scattering} \gg \Gamma_{annihilation} \simeq H_{FO}$$

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DRAKE

To solve the full Boltzmann equation (fBE) for the DM phase space distribution

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2. Sommerfeld enhanced annihilation
3. DM stabilised by say Z3 so crossing symmetry broken

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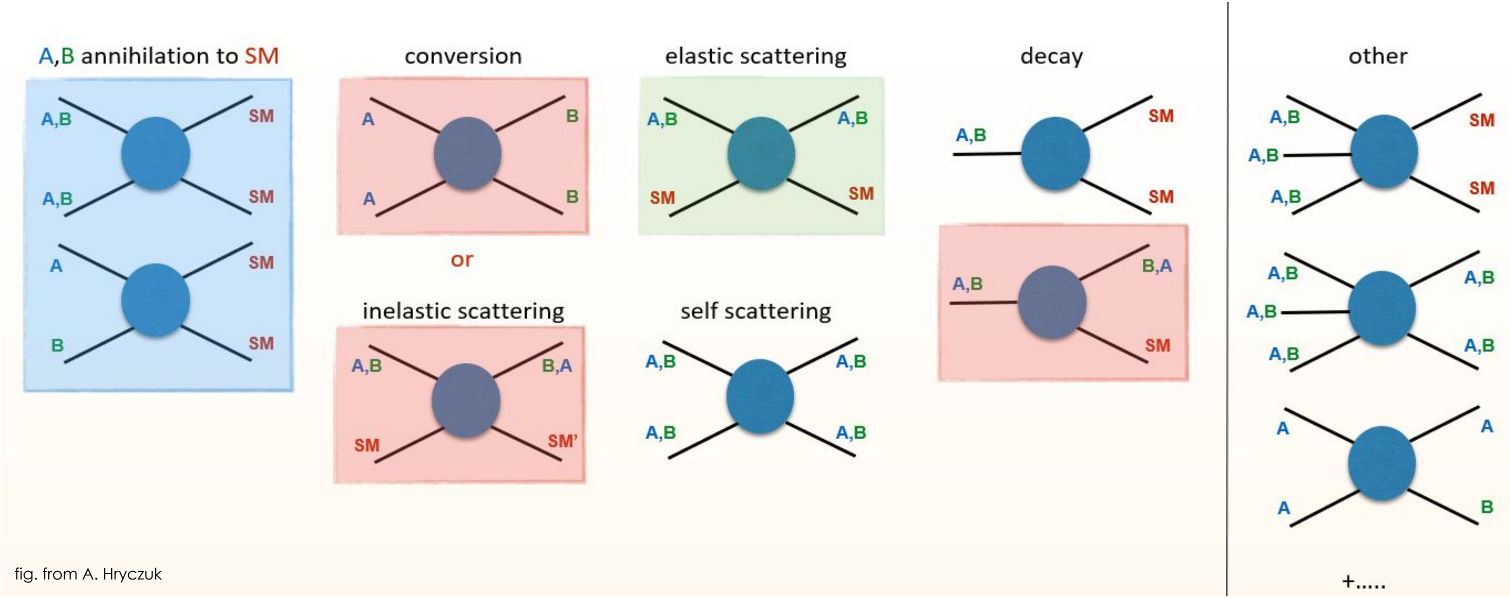
6. **Multicomponent dark sector**

Many more processes ⇒ Crossing symmetry between leading number changing process and leading elastic scattering process not guaranteed

Multicomponent Dark Sectors

- There can then exist many more processes (for change in number density as well as temperature)



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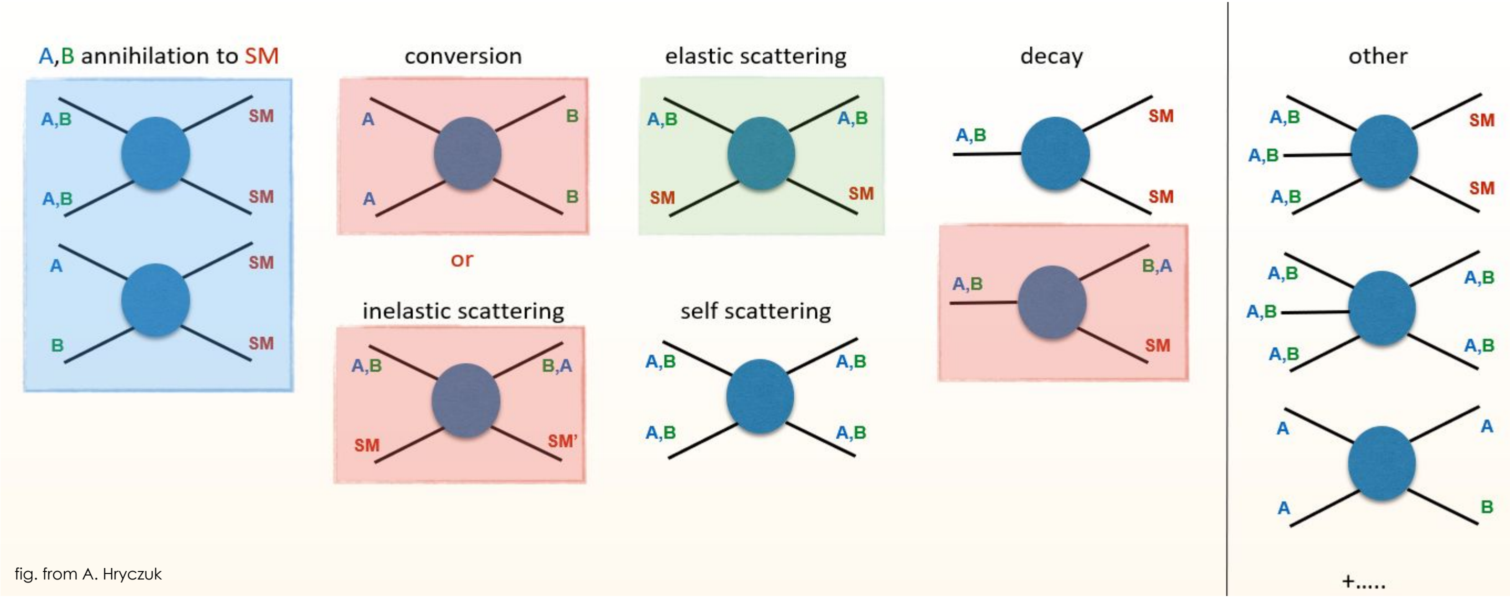

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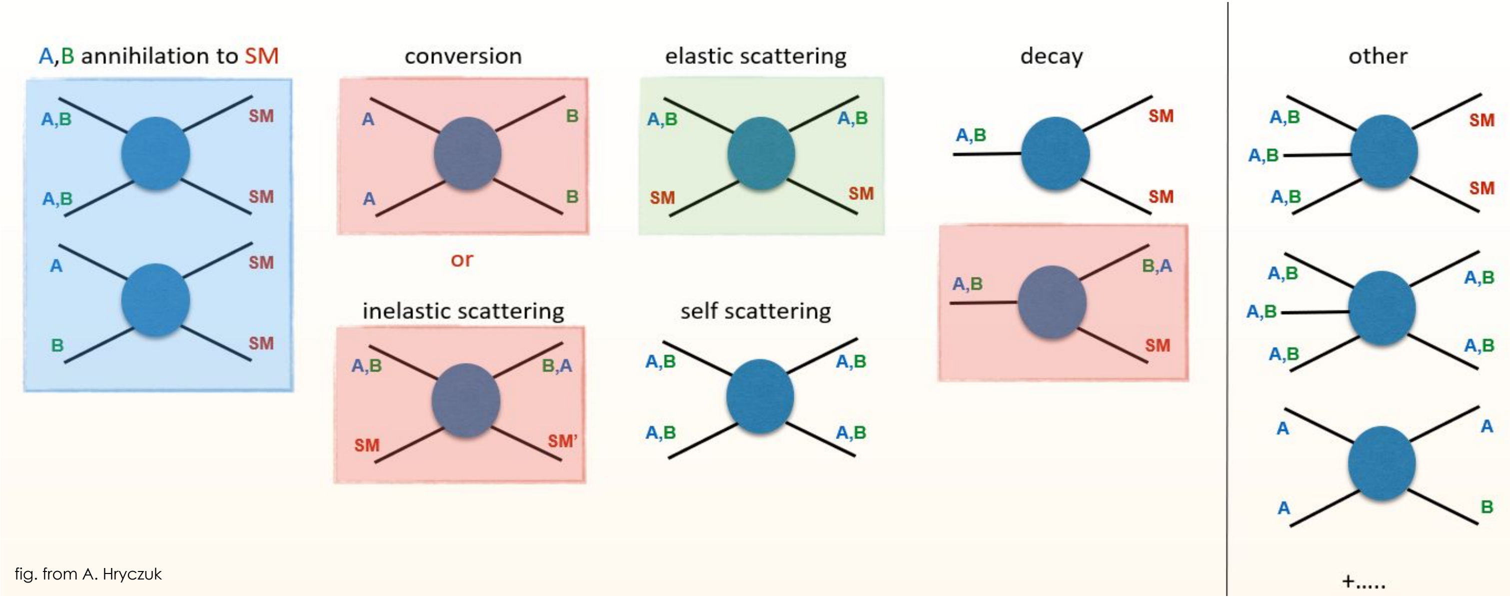

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full Boltzmann Equation (**fBE**): $E_{\chi_i}\left(\partial_t - Hp\partial_p\right) f_{\chi_i} = C_{\text{ann.}}[f_{\chi_i}] + C_{\text{el.}}[f_{\chi_i}] + C_{\chi_i,\chi_i \leftrightarrow \chi_j\chi_j}[f_{\chi_i}]$ + ...

2-component fBE: Technical details

$$E_{\chi_i}\left(\partial_t - Hp\partial_p\right) f_{\chi_i} = C_{\text{ann.}}[f_{\chi_i}] + C_{\text{el.}}[f_{\chi_i}] + C_{\chi_i,\chi_i \leftrightarrow \chi_j\chi_j}[f_{\chi_i}] \quad + \quad \ldots$$

$i \in$ A,B

$$C_{el}[f_{DM}] = \int d\Pi\, |M|^2_{DM,SM\to DM,SM}\left(f_{DM;A,B}(p_1) f_{eq}(p_3) - \boldsymbol{f_{DM;A,B}(p_2)} f_{eq}(p_4)\right)$$

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1. Discretization over momentum: map physical momentum p to q, *best suited to case at hand*

e.g., choice after chemical decoupling of DM, where the expansion alone changes the phase space density:

$$q \sim p/((g^s_{\text{eff}})^{1/3} T)$$

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   - Solving over **intervals in x=mDM/T**: choose momentum range each time, choosing max. and min. p to account for all physical processes relevant at those temperatures

- Conversions=> p-max>Sqrt(m2^2-m1^2)
- expansion alone=> p-max of relevance falls
- ...

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   - Introduced 2 switches to **choose this map from p to q**, possibly different for each particle ★

★ Change of variables from (p,T) to (q,x) introduces an extra derivative term which introduces numerical instabilities. Choices of map where q is a function of p/a are recommended in practice, where a is the scale factor

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   - Automatic check that choice of number of points of discretization and momentum range in fact cover the full (relevant) range distribution function

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2. Speed up the calculation of annihilation collision term:

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2. Speed up the calculation of **annihilation collision term**:
   - re-express ***integration over unknown distribution function as sum***
   - ***Pre-tabulate results*** of remaining integrals over the known equilibrium distribution functions, rewritten in terms of ***angular averaged cross sections***, given from model generation

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2. Speed up the calculation of annihilation collision term:
3. Speed up the calculation of **elastic scattering collision term**:
   - Typically **Fokker Planck Approximation** is used to reduce the collision term to a **DM distribution independent scattering γ** times a **differential operator** acting on **f** (DM distribution fn.)

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$$\ldots (2\pi^4)$$

The Fokker Planck approximation works well for:

1. Scattering particle with masses significantly smaller than DM mass (small reduced mass ⇒ small momentum transfer

&

2. DM temperatures close to the SM temperature (eg.: near kinetic decoupling)

&

3. Scattering amplitudes that aren't strongly dependent on momentum transfer (the dropped higher order terms are more relevant for an amplitude sensitive to said dropped quantity)

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$$C_{conv}[f_{DM}] = \int d\Pi \, |M|^2_{A,A \to B,B} \left(f_{DM,A}(p_1) f_{DM,A}(p_2) - f_{DM,B}(p_3) f_{DM,B}(p_4)\right)$$

$$d\Pi \equiv d\pi_{p_2} d\pi_3 d\pi_4 \delta^4\left(p_1 + p_2 - p_3 - p_4\right)\left(2\pi^4\right)$$

1. Discretization over momentum: map physical momentum p to q, best suited to case at hand.
2. Speed up the calculation of annihilation collision term.
3. Speed up the calculation of **elastic scattering collision term**:
   - Typically **Fokker Planck Approximation** is used to reduce the collision term to a **DM distribution independent scattering** γ times a **differential operator** acting on **f** (DM distribution fn.)
   - Full collision term can be re-expressed as:

$$\vec{C}^{\text{el.},\chi_i} = \text{EM}^{\chi_i} \cdot \vec{\text{f}}^{\chi_i}$$

2-component fBE: Technical details

$$E_{\chi_i}\left(\partial_t - Hp\partial_p\right) f_{\chi_i} = C_{\text{ann.}}[f_{\chi_i}] + C_{\text{el.}}[f_{\chi_i}] + C_{\chi_i,\chi_i \leftrightarrow \chi_j \chi_j}[f_{\chi_i}] \quad + \quad \dots$$

i ∈ A,B

$$C_{el}[f_{DM}] = \int d\Pi\, |M|^2_{DM,SM \to DM,SM} \left(f_{DM;A,B}(p_1) f_{eq}(p_3) - f_{DM;A,B}(p_2) f_{eq}(p_4)\right)$$

$$C_{ann}[f_{DM}] = \int d\Pi\, |M|^2_{DM,DM \to SM,SM} \left(f_{DM;A,B}(p_1) f_{DM;A,B}(p_2) - f_{eq}(p_3) f_{eq}(p_4)\right)$$

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pre-tabulated

- Generic process: *4-dim. Integration* done numerically*
- **One-Mandelstam variable dep. process: 2-dim. Integral done analytically** (Klasen et al 2022)

*to be implemented in future

2-component fBE: Technical details

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$i \in$ A,B

$$C_{el}[f_{DM}] = \int d\Pi\, |M|^2_{DM,SM\to DM,SM} \left(f_{DM;A,B}(p_1) f_{eq}(p_3) - f_{DM;A,B}(p_2) f_{eq}(p_4)\right)$$

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   - Typically **Fokker Planck Approximation** is used to reduce the collision term to a **DM distribution independent scattering γ** times a **differential operator** acting on **f** (DM distribution fn.)
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$$\vec{C}^{\text{el.},\chi_i} = \text{EM}^{\chi_i} \cdot \vec{\text{f}}^{\chi_i} \quad + \quad \text{Fokker Planck for subleading scatterings, } \textit{sorted automatically}$$

pre-tabulated

2-component fBE: Technical details

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1. Discretization over momentum: map physical momentum p to q, best suited to case at hand.
2. Speed up the calculation of annihilation collision term.
3. Speed up the calculation of elastic scattering collision term.
4. Efficient calculation of the **conversion term**:
   - None of the above ways to simplify can be used.

2-component fBE: Technical details

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1. Discretization over momentum: map physical momentum p to q, best suited to case at hand.
2. Speed up the calculation of annihilation collision term.
3. Speed up the calculation of elastic scattering collision term.
4. Efficient calculation of the **conversion term**:
   - Discretize the integrations over unknown **f** into sums
   - Large matrices obtained (2N)^3 for N-discretization
   - No remnant integration so pre-tabulation doesn't give any speed up–evaluated at each step
   - Significant reduction in time for one mandelstam variable only dependent process; the other cases be implemented in the code

DRAKE2: Code Overview

Feynrules model file → Model Generation → <model>.m, <model>.wl,

DRAKE2: Code Overview

Feynrules model file → Model Generation → <model>.m, <model>.wl,

1. List of all particles: 2-Dark Sector + rest in equilibrium with SM plamma
2. list of all processes: (co-)annihilations, conversion, elastic scatterings
3. squared amplitudes, cross sections of all processes
4. compiled functions for thermalised average of cross section: $\langle \sigma v \rangle$
5. Fokker Planck γ
6. decay width of particles

DRAKE2: Code Overview

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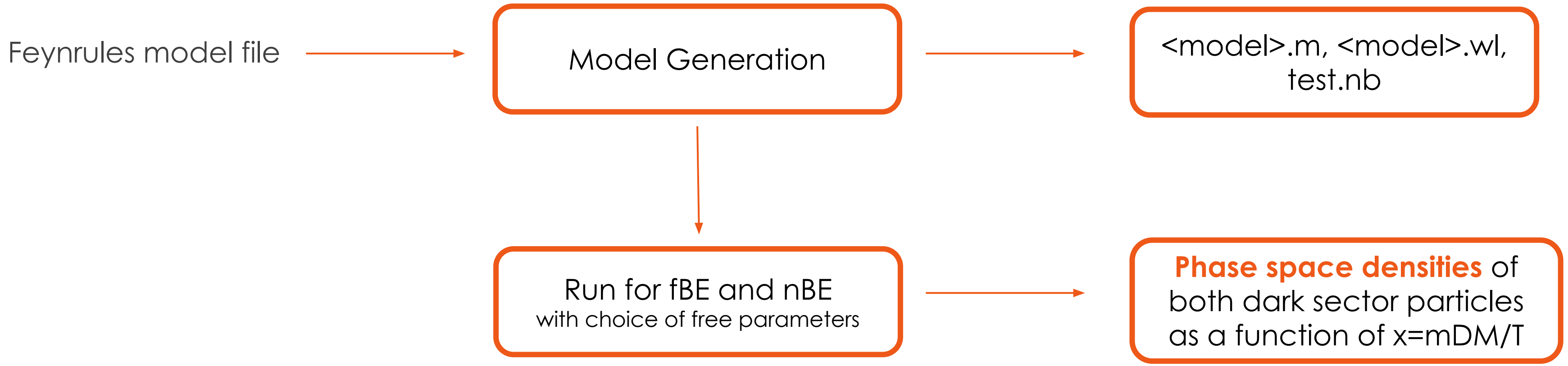
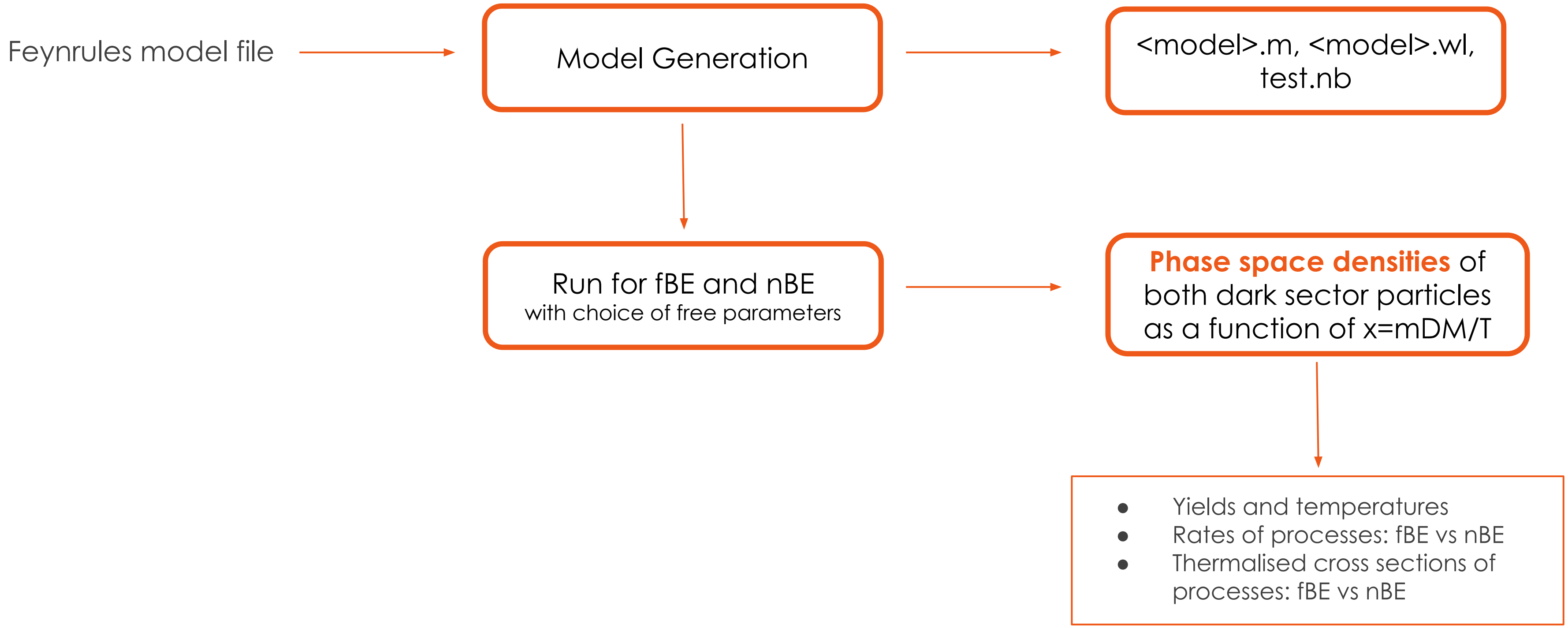
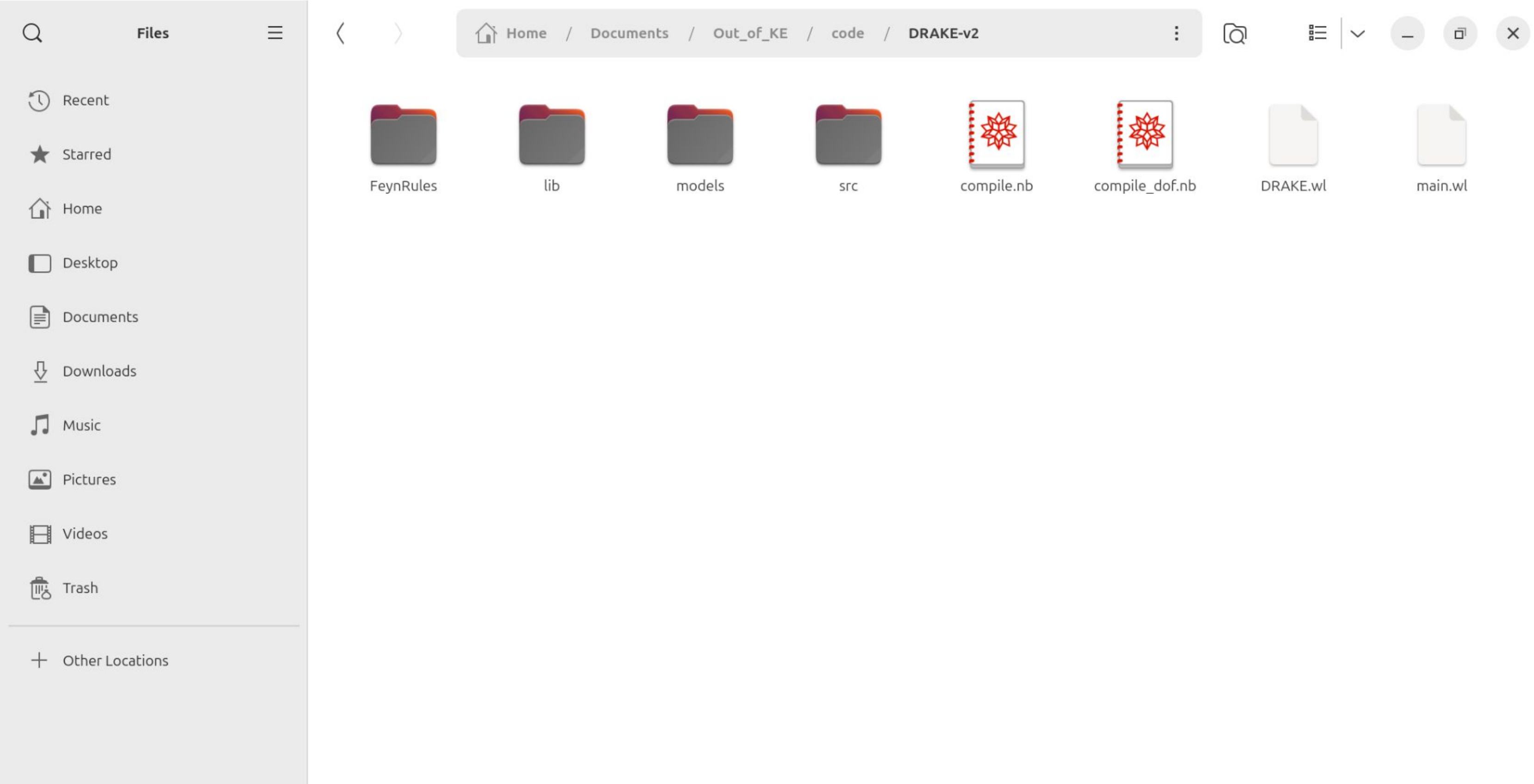


DRAKE2: Code Overview



DRAKE2: Code Overview



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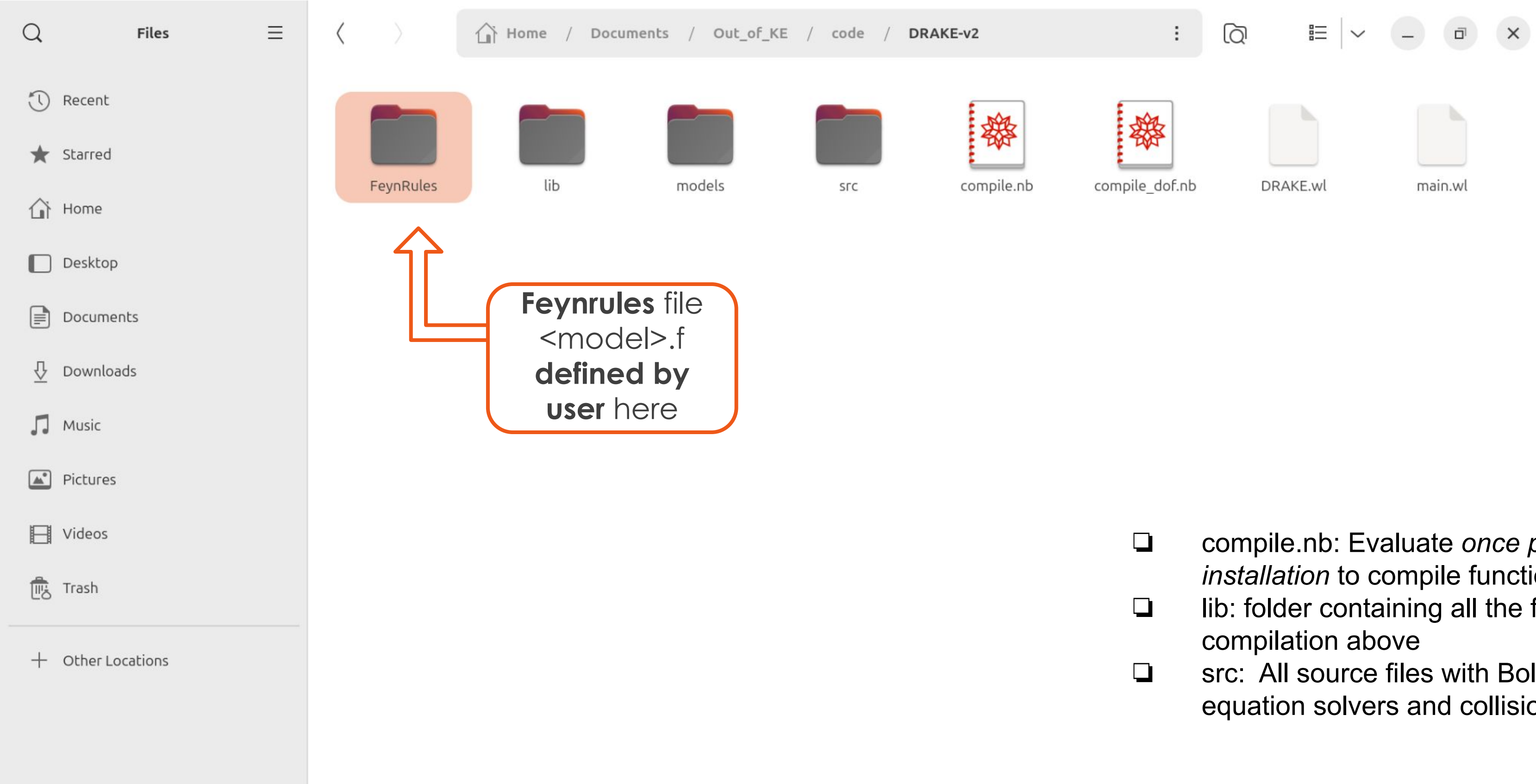
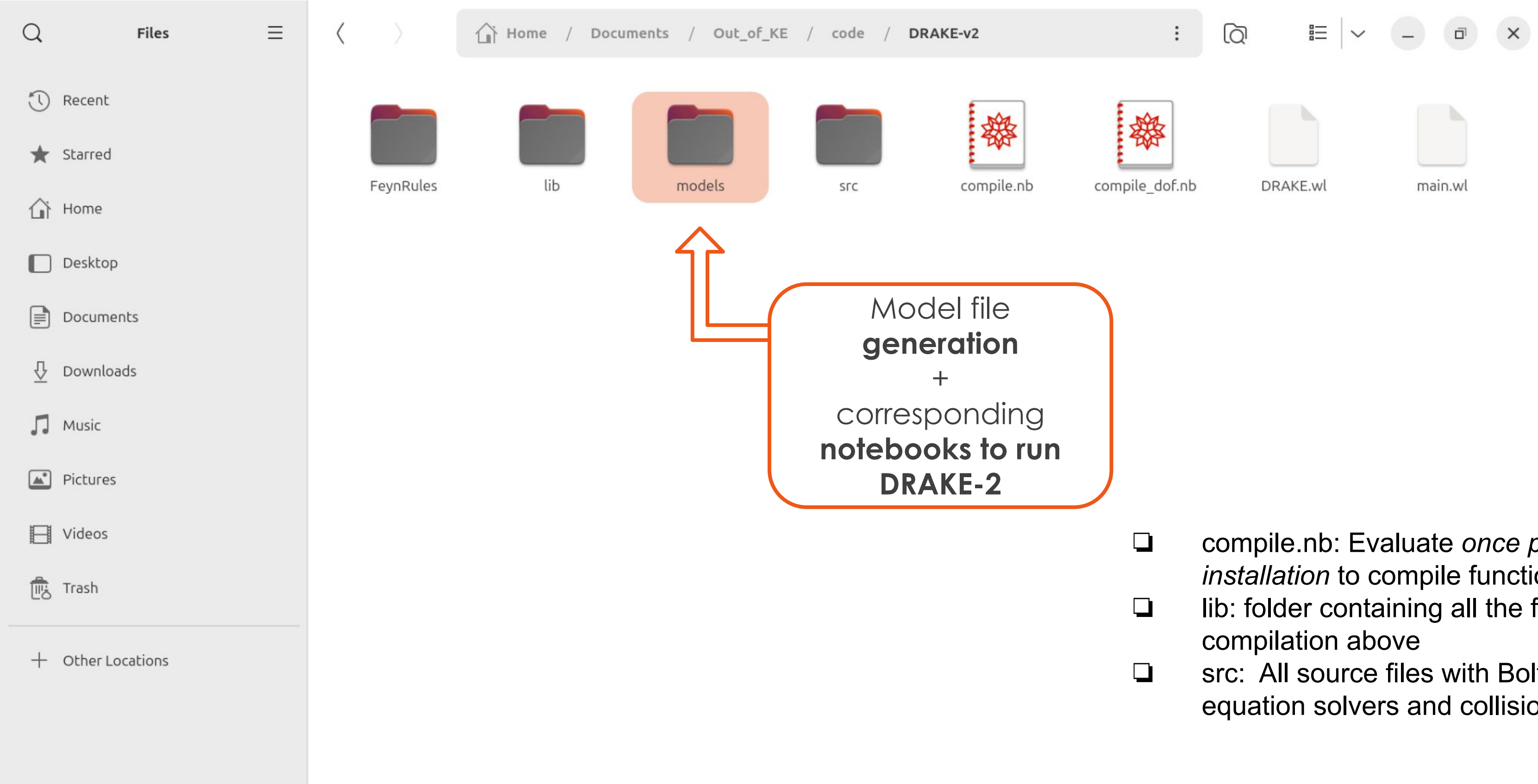


Feynrules file <model>.f **defined by user** here

- compile.nb: Evaluate *once per installation* to compile functions
- lib: folder containing all the files from the compilation above
- src: All source files with Boltzmann equation solvers and collision terms, etc.

DRAKE2: Code Overview



Model file **generation** + corresponding **notebooks to run DRAKE-2**

- ❑ compile.nb: Evaluate *once per installation* to compile functions
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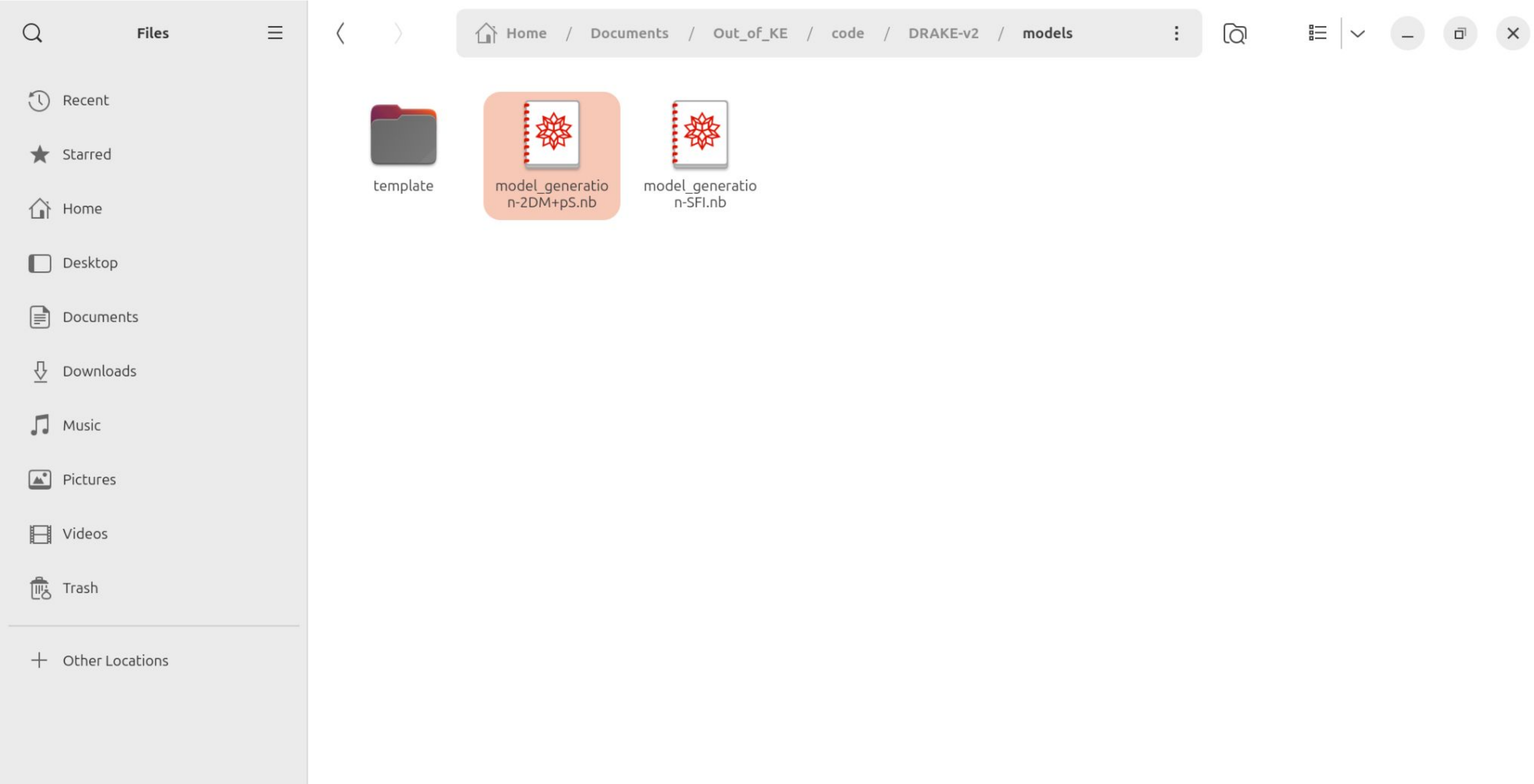
DRAKE2: Example with double coy DM

$$\mathcal{L} \supset -i\lambda_1 a\, \bar{\chi}_1 \gamma^5 \chi_1 - i\lambda_2 a\, \bar{\chi}_2 \gamma^5 \chi_2 - i\lambda_y \sum_{f \in SM} y_f\, a\, \bar{f} \gamma^5 f$$



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model_generation-2DM+pS.nb - Wolfram Mathematica 13.0

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Code Analysis | Analyze Notebook

```
(************************************************************************)

MODELNAME = "2DM+pS";

(************************************************************************)
```

Notebook creating a DRAKE model files based on a FeynRules .mod model file.

Prerequisites:
- installed FeynCalc, FeynArts

Input:
- model .mod file
- template files in the <DRAKE dir>/models/template (supplied with DRAKE)

User set variables:
- Model -- name of the model (also sets directory structure in DRAKE/models/)
- List of all particles *in given format*
- Model parameters

In[•]:=
```
(************************************************************************)
$verbose = True; (* True - print out all messages; False - don't print *)
$LoadAddOns = {"FeynArts"};
<< FeynCalc`
(************************************************************************)
```

FeynCalc 10.0.0 (stable version). For help, use the online documentation, visit the forum and have a look at the supplied examples.

DRAKE2: Example with double coy DM

$$\mathcal{L} \supset -i\lambda_1 a\, \bar{\chi}_1 \gamma^5 \chi_1 - i\lambda_2 a\, \bar{\chi}_2 \gamma^5 \chi_2 - i\lambda_y \sum y_f\, a\, \bar{f} \gamma^5 f$$

model_generation-2DM+pS.nb - Wolfram Mathematica 13.0

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Code Analysis Analyze Notebook

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SM assumed to be one fermion of a given mass for this example

model_generation-2DM+pS.nb * - Wolfram Mathematica 13.0

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Code Analysis   Analyze Notebook

```
In[◦]:= (*********************************************************************)
DRAKEdir = NotebookDirectory[] <> "../"; SetDirectory[DRAKEdir];
Get["./src/model_generation.wl"];
Model = MODELNAME;
ModFile = DRAKEdir <> "FeynRules/Models/" <> Model <> "/" <> Model <> "/" <> Model;
InitModel[ModFile]
(*********************************************************************)
```

S(0)	Mh	{}	h
S(1)	Ms	{}	s
F(0)	Mf	{}	f
F(1)	Mchi1	{}	Chi1
F(2)	Mchi2	{}	Chi2

```
(* User input:
  make sure it is in the same convention as the model file, i.e:
{FeynCalc symbol, mass parameter, spin, em.charge, own antiparticle, name, dark sector?} *)
DrakeParticles["All"] = {
    (* DS particles (for which f(p) is traced *)
    {F[1], Mchi1,  1/2, 0, False, "Chi1", 1},
    {F[2], Mchi2,  1/2, 0, False, "Chi2", 1},
    (* SM particles & equilibrium DS particles *)
    {S[0], Mh,    0, 0, True , "h", 0},
    {S[1], Ms,    0, 0, True , "s", 0},
    {F[0], Mf, 1/2, 0, False, "f", 0}
};

(* list of the parameters of the model: masses as in the list above + couplings etc. *)
ModelParameters = {Mchi1, Mchi2, Mf, Ms, llx1, llx2, lly};
(*********************************************************************)
```

DRAKE2: Example with double coy DM

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model_generation-2DM+pS.nb * - Wolfram Mathematica 13.0

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Code Analysis — Analyze Notebook

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Output

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model_generation-2DM+pS.nb * - Wolfram Mathematica 13.0

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Code Analysis Analyze Notebook

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Code Analysis — Analyze Notebook

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```

Output

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   (* SM particles & equilibrium DS particles *)
   {S[0], Mh,    0, 0, True , "h", 0},
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```

DRAKE2: Example with double coy DM

model_generation-2DM+pS.nb - Wolfram Mathematica 13.0

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Code Analysis   Analyze Notebook

Perform Calculation & Model Creation

Create list of processes (1-3 commented out)

M^2

σ, Γ,γ

Create & save the DRAKE model file

```
NotebookDirectory[];
Model = "2DM+pS+EL";
MODELNAME = Model;
```

```
In[•]:= SetDirectory[DRAKEdir <> "/models/"];
If[! DirectoryQ[Model], CreateDirectory[Model];
  CreateDirectory[Model <> "/Results"];
  CreateFile[Model <> "/Results/empty.m"]];
mfile = "./template/template.wl";
FileTemplateApply[FileTemplate[mfile], "./" <> Model <> "/" <> Model <> ".wl"];
sfile = "./template/s-template.wl";
FileTemplateApply[FileTemplate[sfile], "./" <> Model <> "/s-template.wl"];
CopyFile["./template/test-template.nb", "./" <> Model <> "/model_test-template.nb"]
CopyFile["./template/tests.nb", "./" <> Model <> "/tests.nb"]
SetDirectory[DRAKEdir <> "/models/" <> Model];

(* First argument is the name of the file,
2nd is its format: Choose ".m" if you want to inspect what's inside and ".mx" if you want fast loading later*)
CreateDrakeModelFile[Model, ".m" (* or .mx *)]
In[•]:=
```

DRAKE2: Example with double coy DM

model_generation-2DM+pS.nb - Wolfram Mathematica 13.0

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Code Analysis — Analyze Notebook

Perform Calculation & Model Creation

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M^2 »

σ, Γ,γ »

Create & save the DRAKE model file

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NotebookDirectory[];
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```

Rename before saving

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If[! DirectoryQ[Model], CreateDirectory[Model];
  CreateDirectory[Model <> "/Results"];
  CreateFile[Model <> "/Results/empty.m"]];
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DRAKE2: Example with double coy DM

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CopyFile["./template/tests.nb", "./" <> Model <> "/tests.nb"]
SetDirectory[DRAKEdir <> "/models/" <> Model];

(* First argument is the name of the file,
2nd is its format: Choose ".m" if you want to inspect what's inside and ".mx" if you want fast loading later*)
CreateDrakeModelFile[Model, ".m" (* or .mx *)]
```

Generate and save output

In[•]:=

DRAKE2: Example with double coy DM

model_generation-2DM+pS.nb - Wolfram Mathematica 13.0

File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

Code Analysis — Analyze Notebook

Perform Calculation & Model Creation

Create list of processes (1-3 commented out) »

M^2 »

σ, Γ,γ »

Create & save the DRAKE model file

```
NotebookDirectory[];
Model = "2DM+pS+EL";
MODELNAME = Model;
```

Rename before saving

```
In[∘]:= SetDirectory[DRAKEdir <> "/models/"];
If[! DirectoryQ[Model], CreateDirectory[Model];
  CreateDirectory[Model <> "/Results"];
  CreateFile[Model <> "/Results/empty.m"]];
mfile = "./template/template.wl";
FileTemplateApply[FileTemplate[mfile], "./" <> Model <> "/" <> Model <> ".wl"];
sfile = "./template/s-template.wl";
FileTemplateApply[FileTemplate[sfile], "./" <> Model <> "/s-template.wl"];
CopyFile["./template/test-template.nb", "./" <> Model <> "/model_test-template.nb"]
CopyFile["./template/tests.nb", "./" <> Model <> "/tests.nb"]
SetDirectory[DRAKEdir <> "/models/" <> Model];

(* First argument is the name of the file,
2nd is its format: Choose ".m" if you want to inspect what's inside and ".mx" if you want fast loading later*)
CreateDrakeModelFile[Model, ".m" (* or .mx *)]

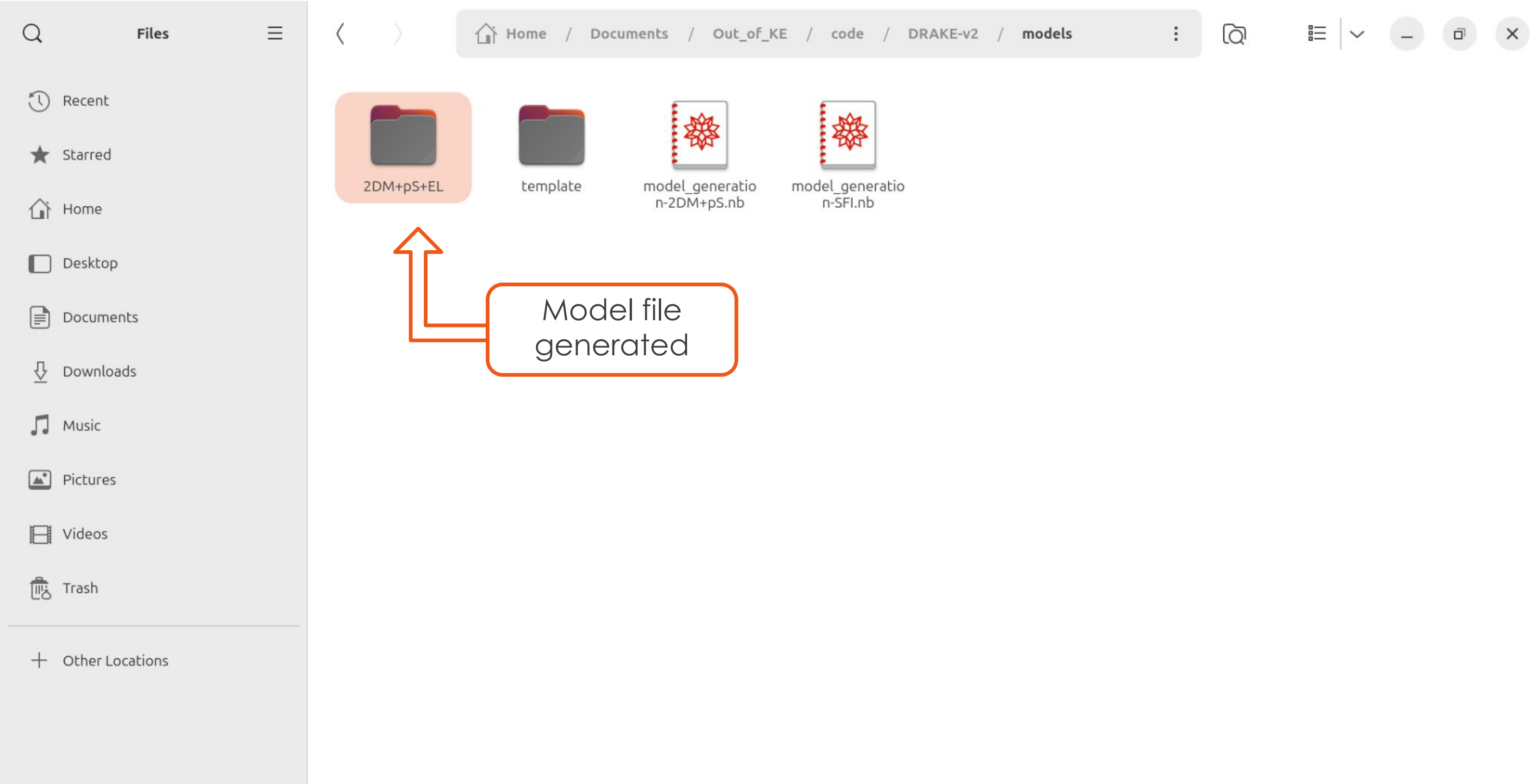
In[∘]:=
```

Output:

$$|\mathcal{M}|^2_{process}\,,\ \sigma_{proc.},\gamma_{FP},\ \langle\sigma v_{mol}\rangle_{proc.},\ \Gamma_i$$

Generate and save output

DRAKE2: Example with double coy DM



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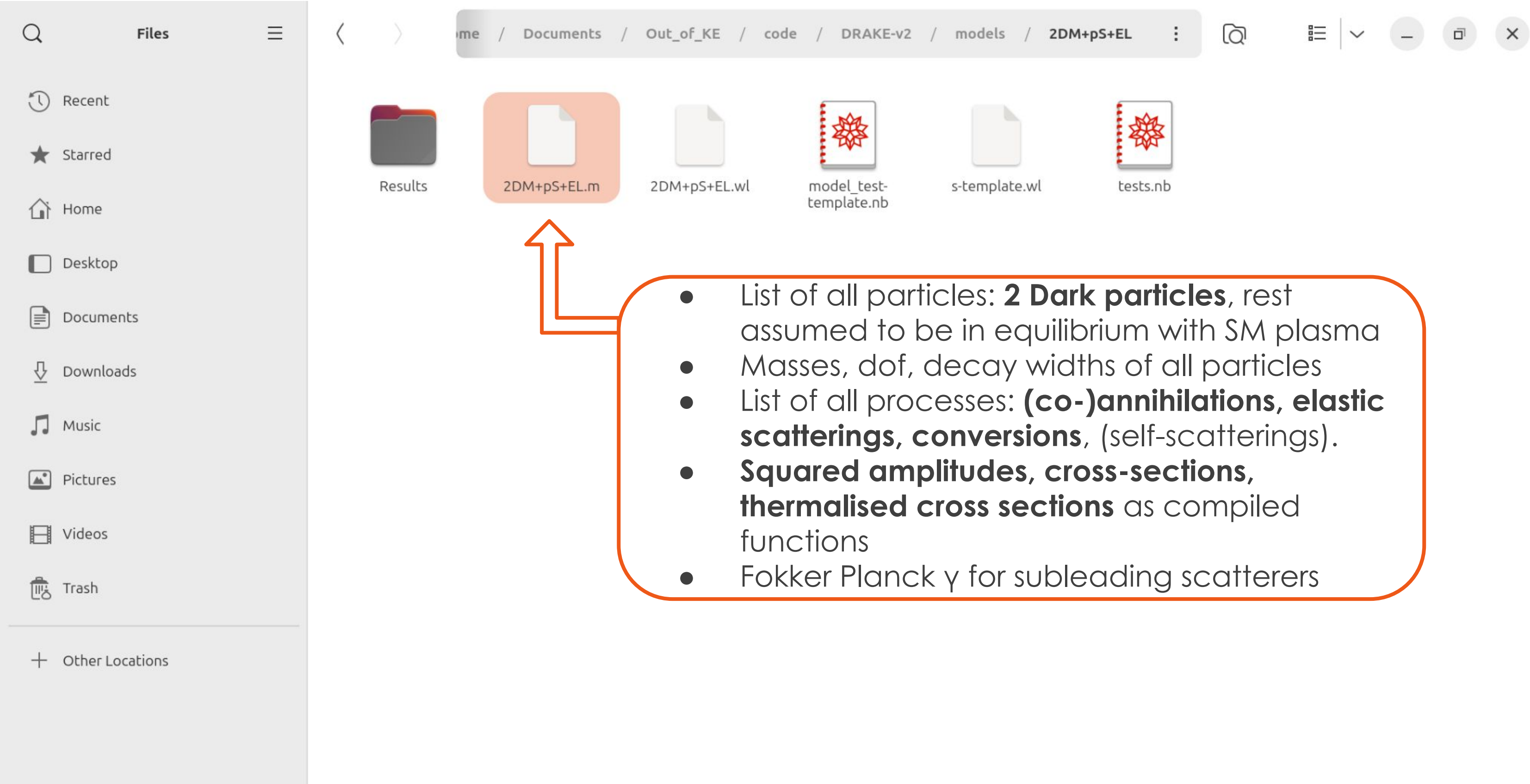
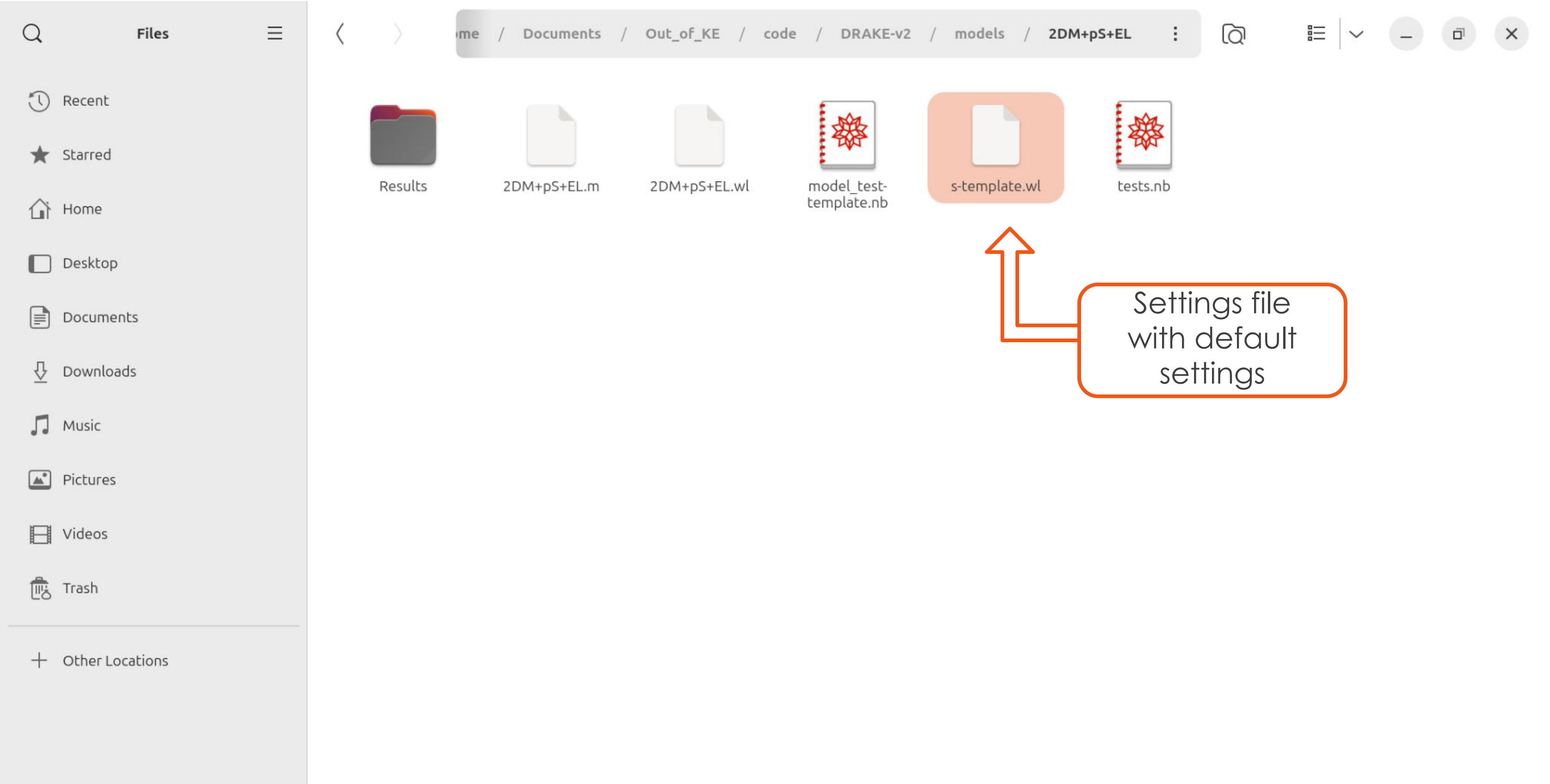
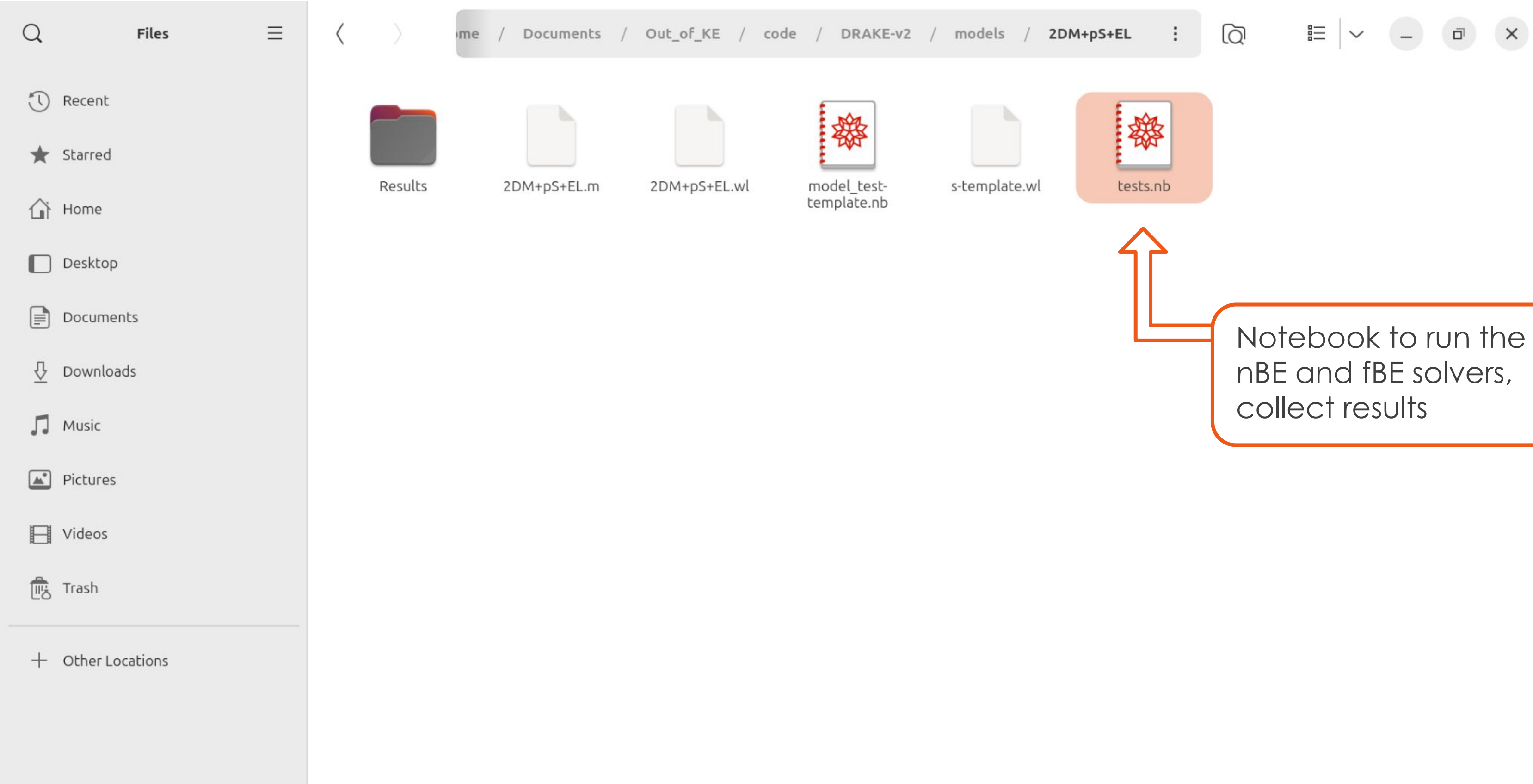


- List of all particles: **2 Dark particles**, rest assumed to be in equilibrium with SM plasma
- Masses, dof, decay widths of all particles
- List of all processes: **(co-)annihilations, elastic scatterings, conversions**, (self-scatterings).
- **Squared amplitudes, cross-sections, thermalised cross sections** as compiled functions
- Fokker Planck y for subleading scatterers

DRAKE2: Example with double coy DM



DRAKE2: Example with double coy DM

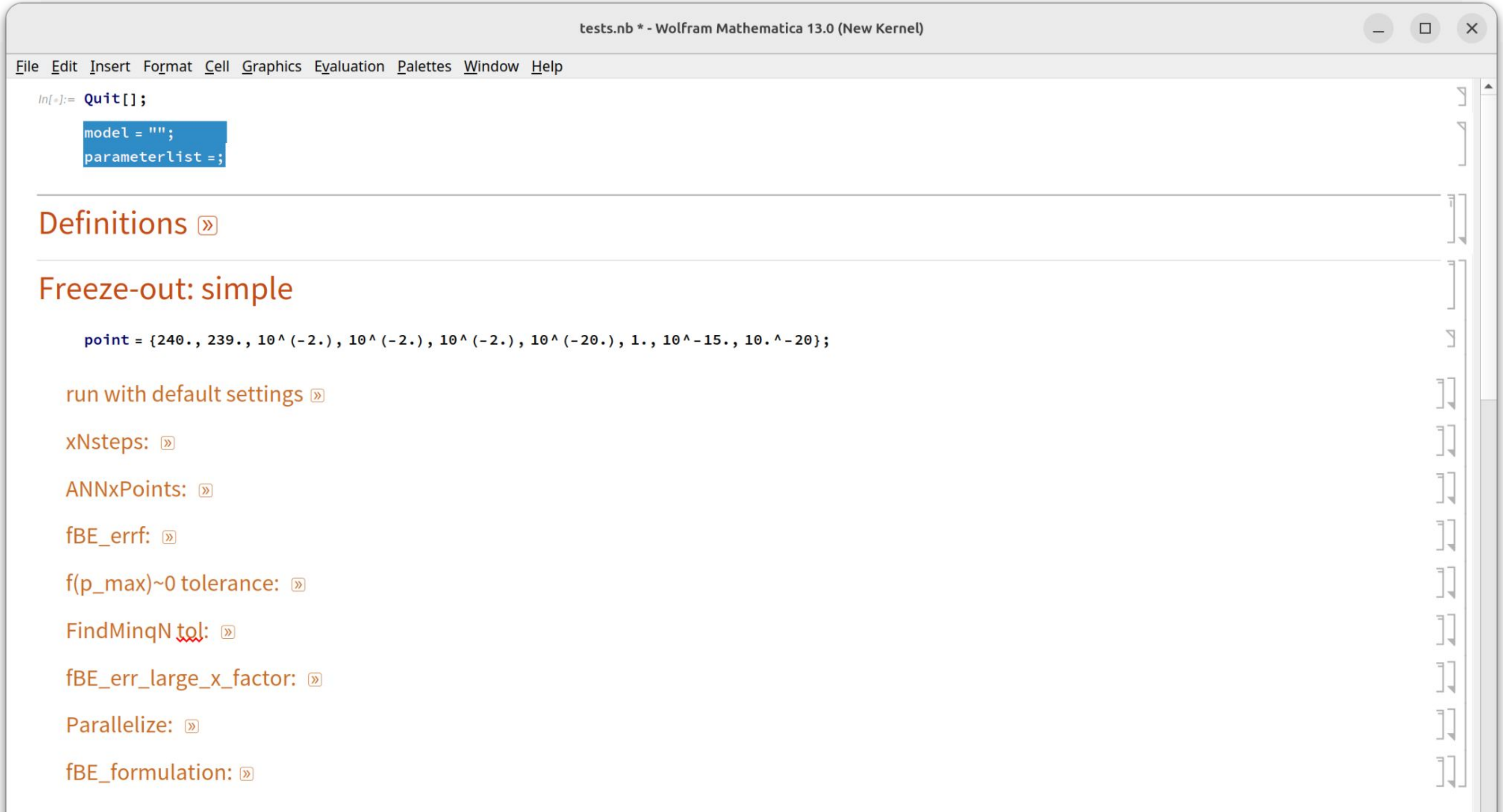


Notebook to run the nBE and fBE solvers, collect results

DRAKE2: Example with double coy DM

$$\mathcal{L} \supset -i\lambda_1 a\,\bar{\chi}_1\gamma^5\chi_1 - i\lambda_2 a\,\bar{\chi}_2\gamma^5\chi_2 - i\lambda_y \sum_{f\in SM} y_f\, a\,\bar{f}\gamma^5 f$$

SM assumed to be one fermion of a given mass for this example



DRAKE2: Example with double coy DM

$$\mathcal{L} \supset -i\lambda_1 a\, \bar{\chi}_1 \gamma^5 \chi_1 - i\lambda_2 a\, \bar{\chi}_2 \gamma^5 \chi_2 - i\lambda_y \sum_{f \in SM} y_f\, a\, \bar{f} \gamma^5 f$$

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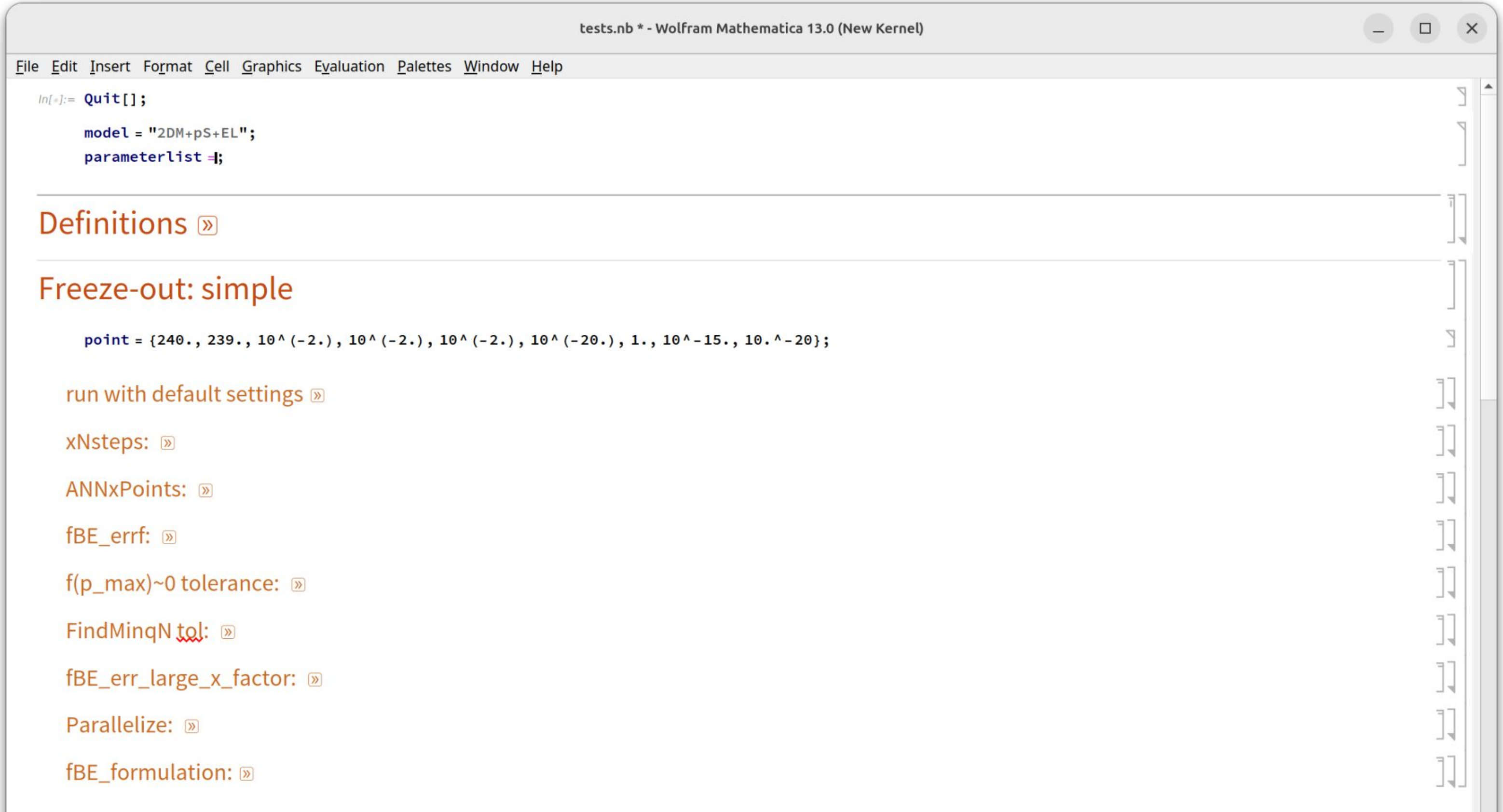

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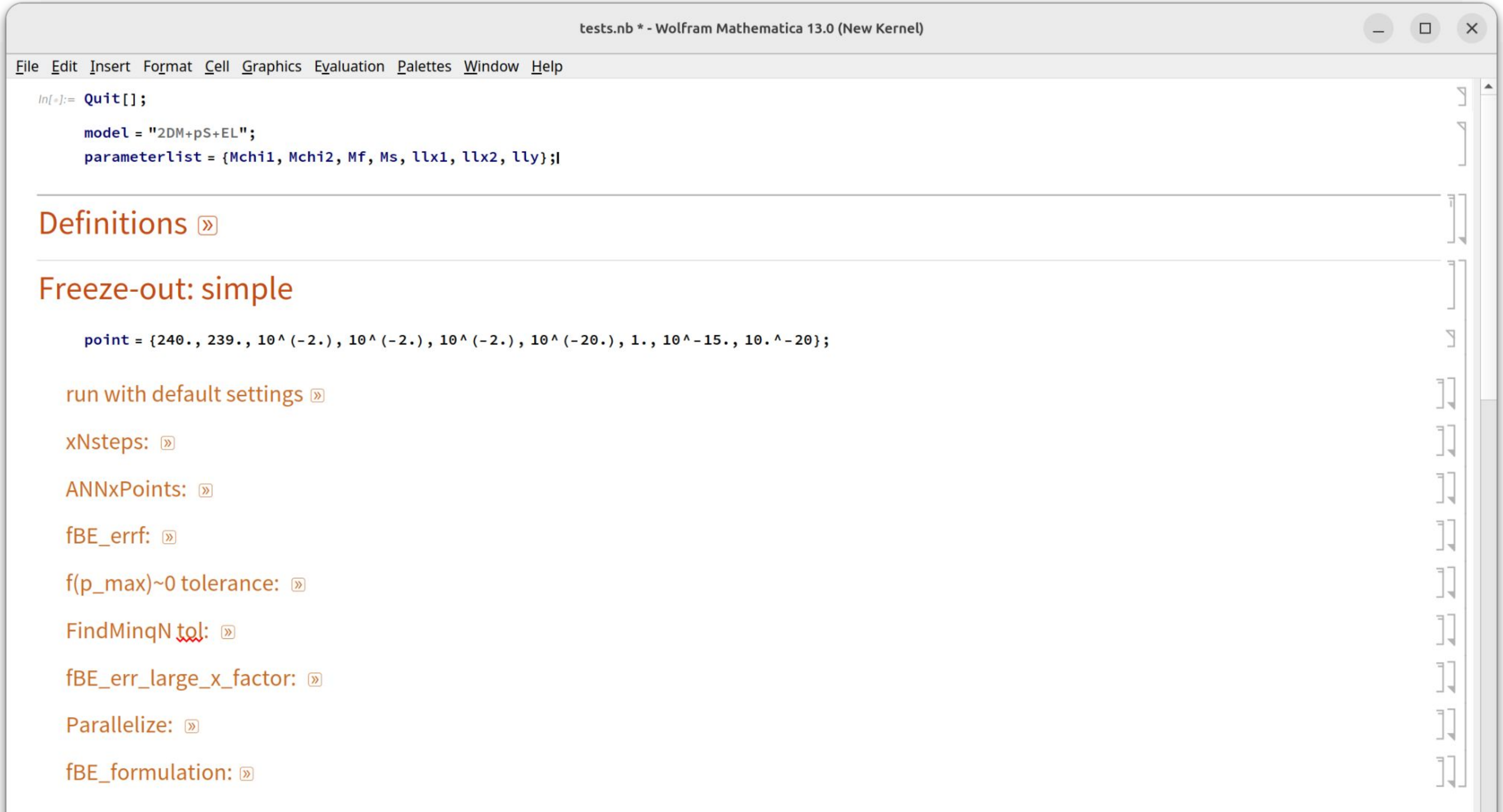

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SM assumed to be one fermion of a given mass for this example

tests.nb * - Wolfram Mathematica 13.0

File Edit Insert Format Cell Graphics Evaluation Palettes Window Help

```
In[•]:= Quit[];

model = "2DM+pS+EL";
parameterlist = {Mchi1, Mchi2, Mf, Ms, llx1, llx2, lly};
```

Definitions »

Freeze-out: simple

```
point = {300., 320., 1., 50., .1, .04, 0.3};
```

run with default settings

```
In[73]:= RunPoint[point]

In[•]:= Manipulate[
  {messages[[i]],
   fplot[i],
   fplot2[i] (*,
   {analyze[i]}*)}
  , {i, 1, Min[Length[tf], Length[messages]], 1}]
```

xNsteps: »

ANNxPoints: »

fBE_errf: »

f(p_max)~0 tolerance: »

With defaults settings, the code:

- **Checks if Freeze-out/Freeze-in** from rates — sets x-range, initial conditions
- Solves **nBE** and **fBE**
- Generates results plots

DRAKE2: Example with double coy DM (eg timings)

SM assumed to be one fermion of a given mass for this example

```
RunPoint[point]

Loading DRAKE.wl: time=0.07472

----------------- Model: 2DM+pS+EL ----------------

{}

Determining [xmin,xmax] based on the nBE annihilation rates...

{xmin,xmax}={10., 10000.}

Determining initial condition based on xmin...

Initialization done...time=0.518692

Present day cross sections (1,1), (2,2), (1,1)->(2,2) & (2,2)->(1,1):{1.18105×10^-27, 1.65805×10^-28, 0., 6.4109×10^-30}

Present day cross-sections {1,1}->{s,s},{2,2}->{s,s}:{3.18943×10^-29, 7.18896×10^-31}

-------------------------------

Rates <sv> done... time=27.3647

nBESolver done... time=1.60038

Oh2nBE={2.29053, 12.8577}

γ done... time=0.299427

(# of calculated x points = 40  for total # of channels: 9 )

xFO={42.8157, 34.069}  xKD={41.3322, 41.1656}  x12eq=19.5996  xConv={16.185, 17.5029}

fBE solved on: [xmin,xmax]={10., 484.2447616840665}

-------------------------------

Running on {10., 62.6939}  with qN=40  {pmax1,pmax2} = {689.23, 675.028}  {Q1,Q2}={3, 3}

A matrices done... time=6.63702

E matrices done... time=38.716

step of fBESolver done... 173.107

-------------------------------
```

M1=300. GeV, M2=320. GeV, mSM=1. GeV, mpS=50. GeV, llx1=0.1, llx2=0.04, lly=0.3

DRAKE2: Example with double coy DM (eg timings)

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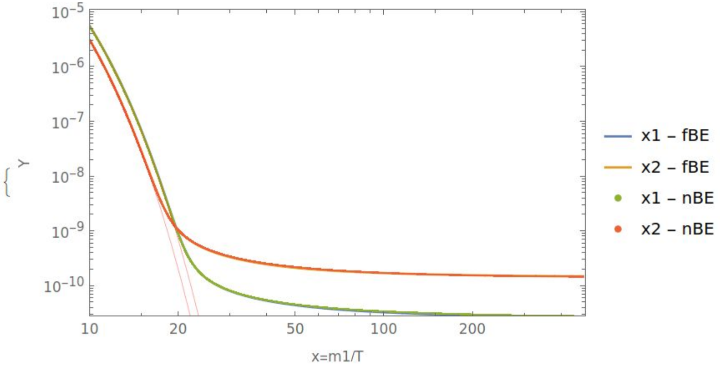
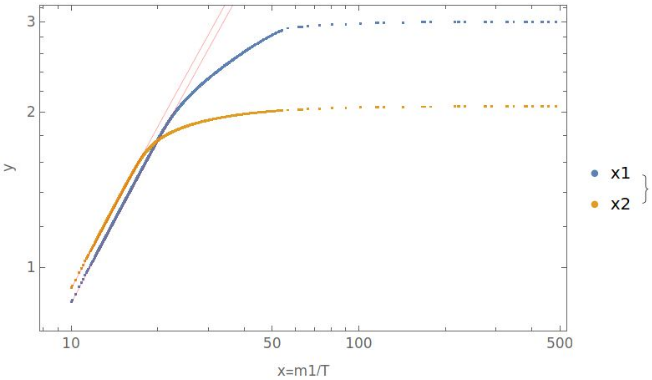
-------------------------------
```

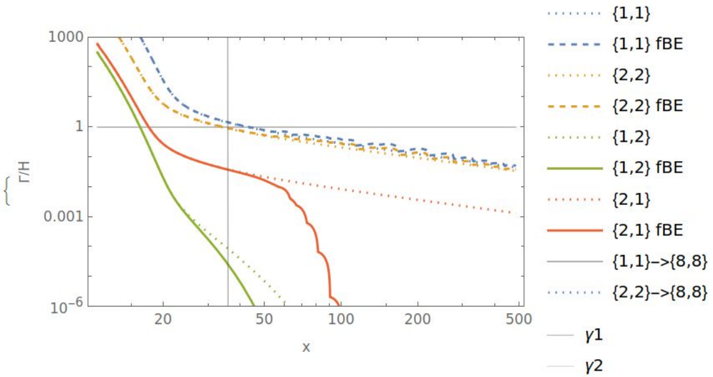
M1=300. GeV, M2=320. GeV, mSM=1. GeV, mpS=50. GeV, llx1=0.1, llx2=0.04, lly=0.3

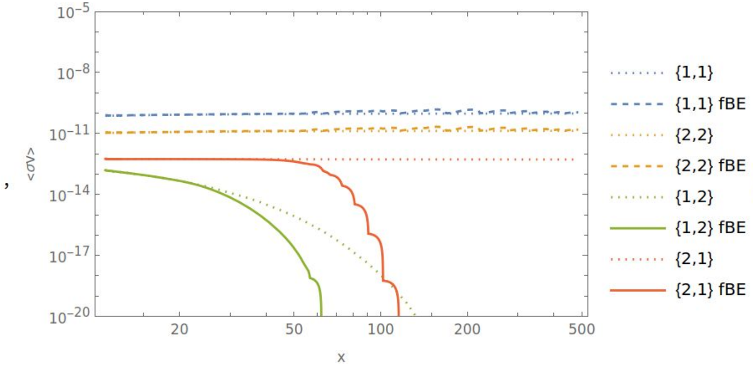
```
Running on {431.551, 484.245}   with qN=40   {pmax1,pmax2} = {14.604, 14.3031}   {Q1,Q2}={3, 3}
A matrices done... time=6.66525
E matrices done... time=41.5823
step of fBESolver done... 0.659584
Oh2_1=2.29053   Oh2_1=2.27927   nBE/fBE=1.00494
Oh2_2=12.8577   Oh2_2=13.2265   nBE/fBE=0.972118
```





M1=300. GeV,
M2=320. GeV,
mSM=1. GeV,
mpS=50. GeV,
llx1=0.1, llx2=0.04,
lly=0.3





Total time=647s

```
Running on {431.551, 484.245}   with qN=40   {pmax1,pmax2} = {14.604, 14.3031}   {Q1,Q2}={3, 3}
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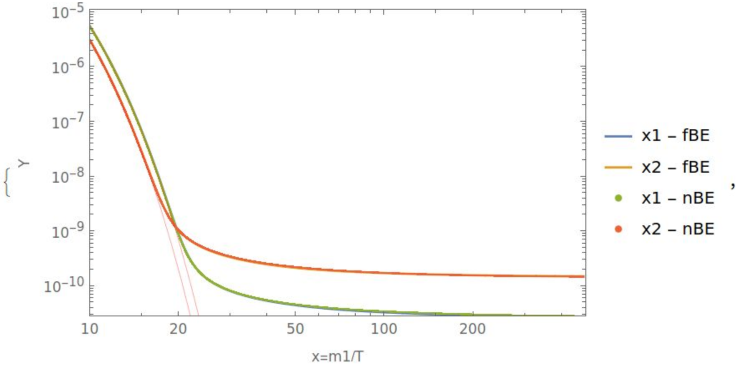
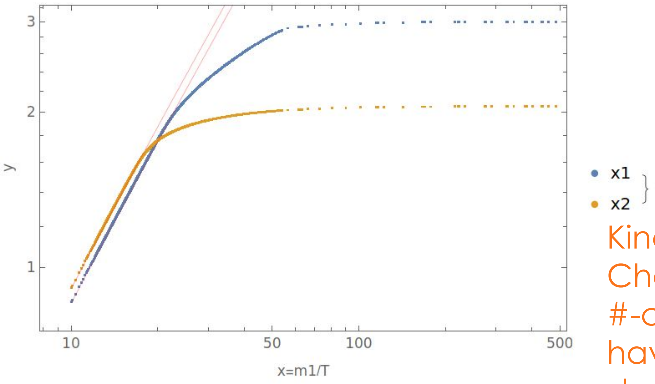
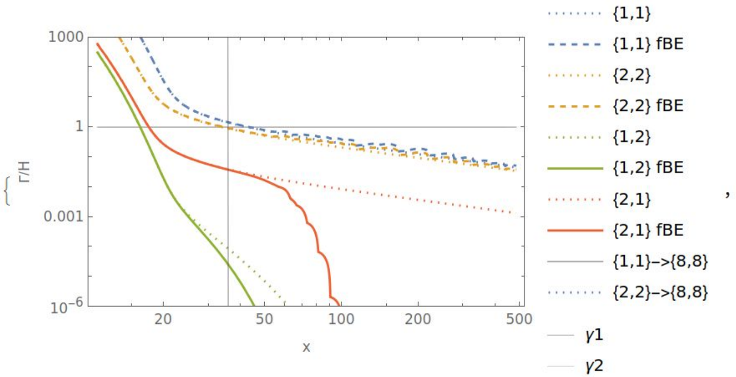
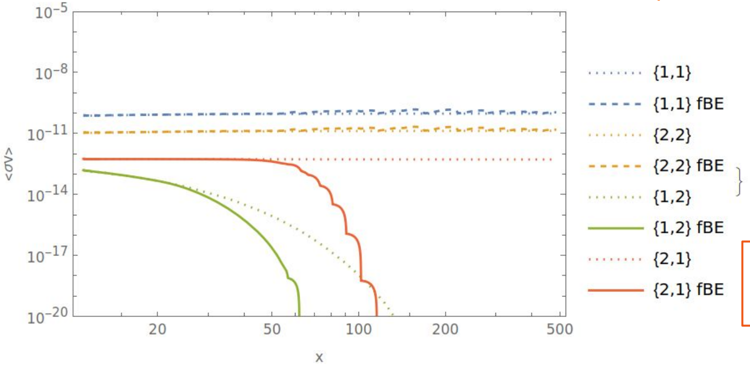
```





Kinetic dec. coincides with Chemical dec. but #-changing processes do not have large momentum dependence so fBE/nBE~=1



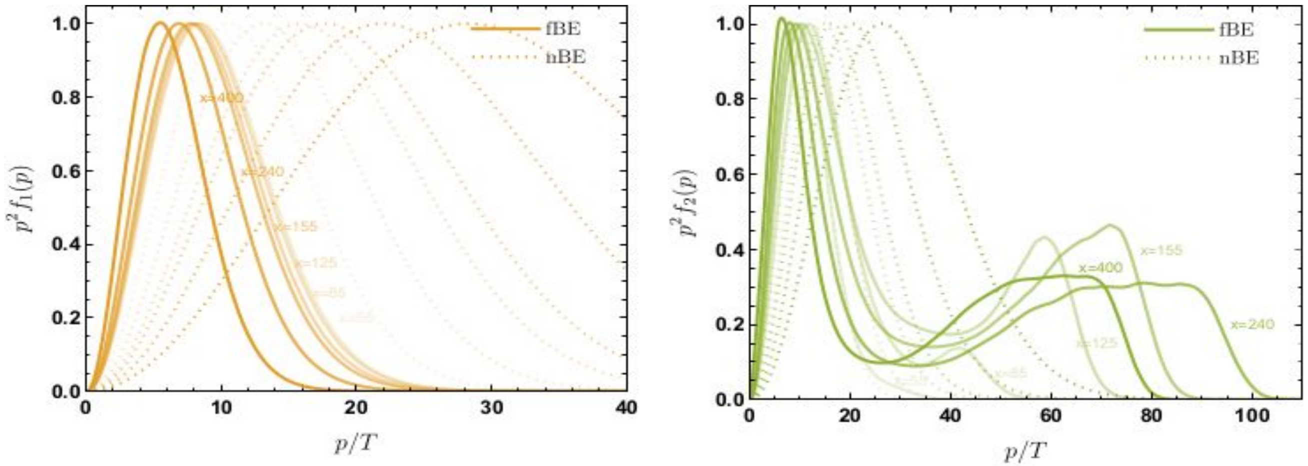


Total time=647s

BP: Double Coy Dark Matter

$$\mathcal{L} \supset -i\lambda_1 a\, \bar{\chi}_1\gamma^5\chi_1 - i\lambda_2 a\, \bar{\chi}_2\gamma^5\chi_2 - i\lambda_y \sum_{f\in SM} y_f\, a\, \bar{f}\gamma^5 f$$

$$\mathrm{M}_{\chi_1} = 44\,\mathrm{GeV},\ \mathrm{M}_{\chi_2} = 38\,\mathrm{GeV}$$
$$\mathrm{M}_s = 80\,\mathrm{GeV}$$
$$\lambda_{\chi_1} = 0.023,\ \lambda_{\chi_2} = 0.39,\ \lambda_y = 0.3$$



SC, A. Hryczuk '25

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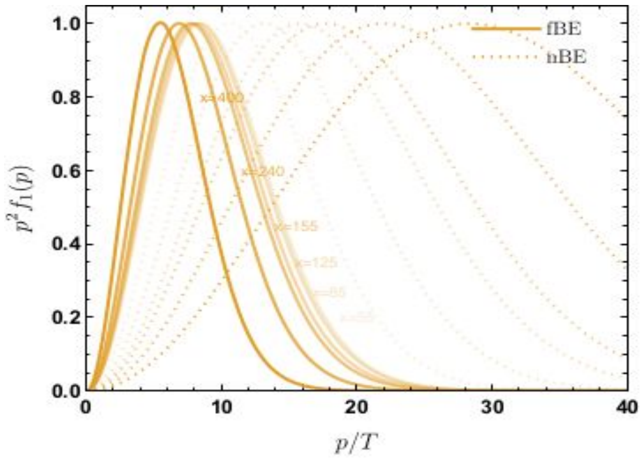
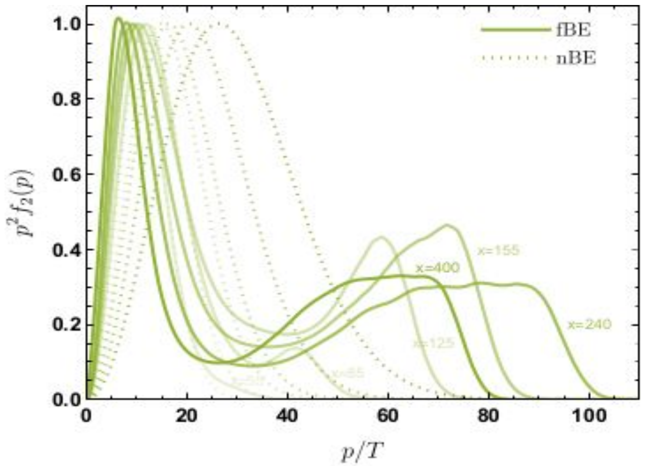
- Resonant annihilation of χ_2 prefers momentum of χ_2 ≃12 GeV
- Conversions $\chi_1 \rightarrow \chi_2$ with momentum ≥ 22 GeV





SC, A. Hryczuk '25

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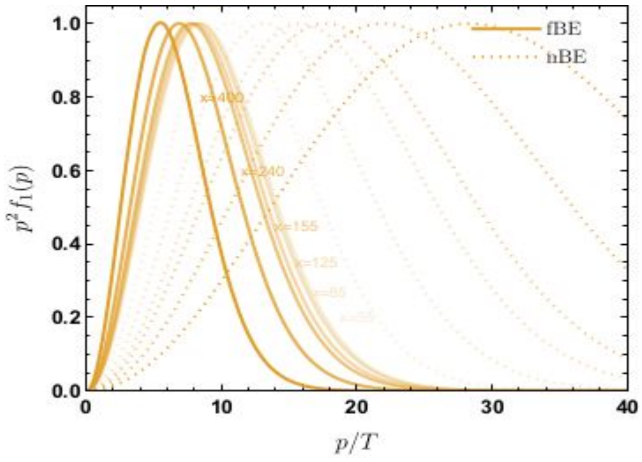
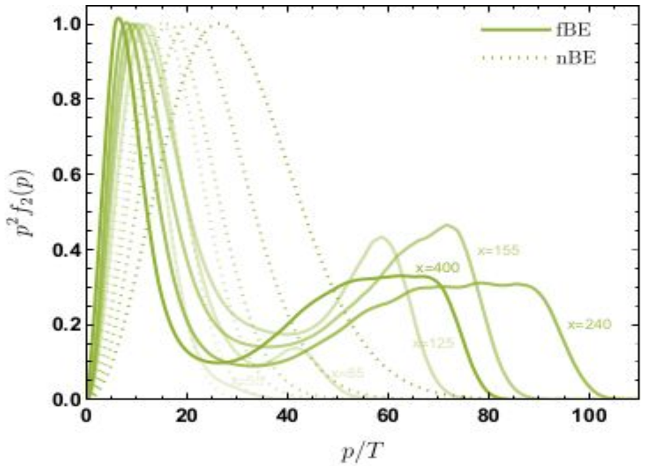
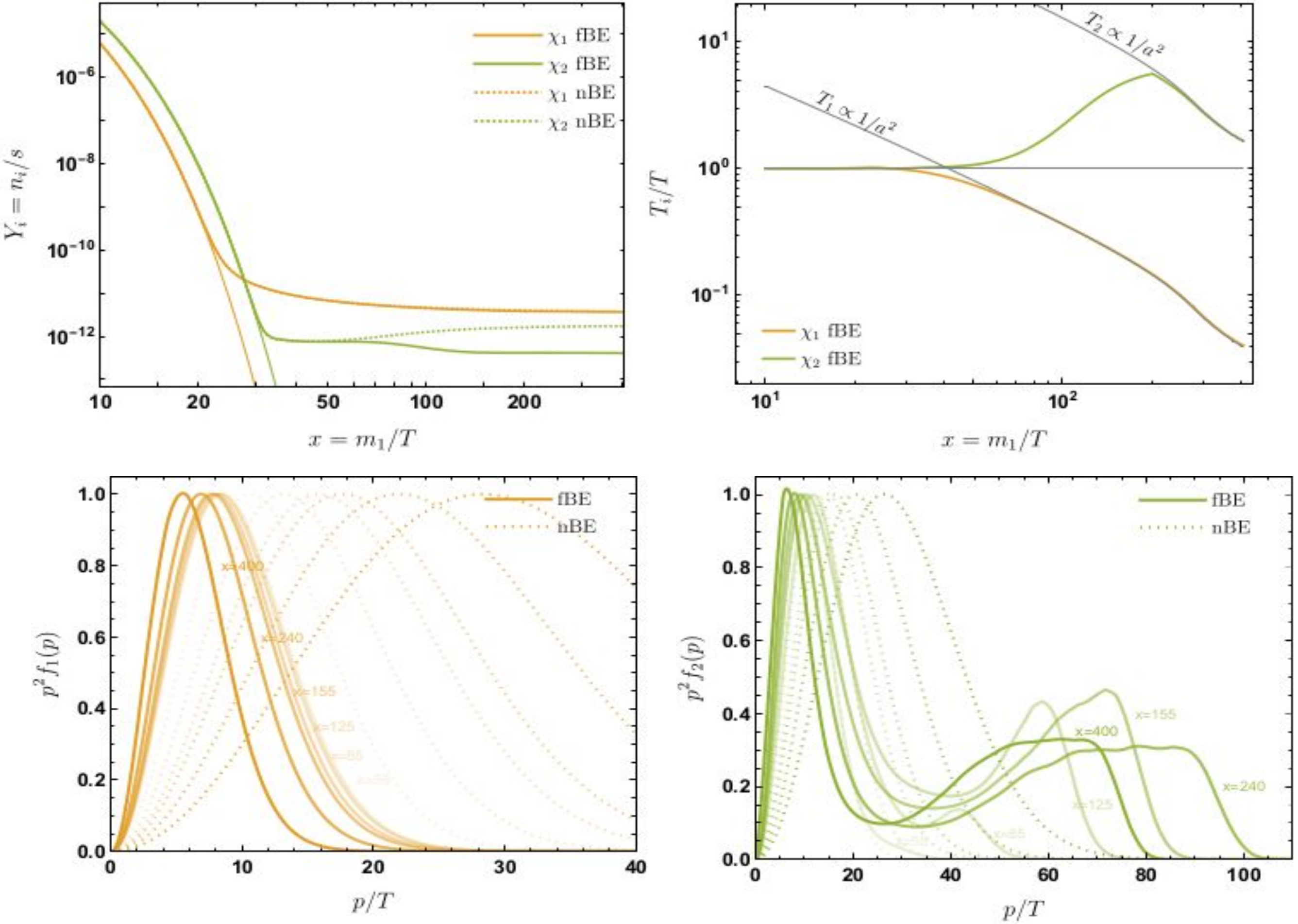




SC, A. Hryczuk '25

BP: Double Coy Dark Matter



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SC, A. Hryczuk '25

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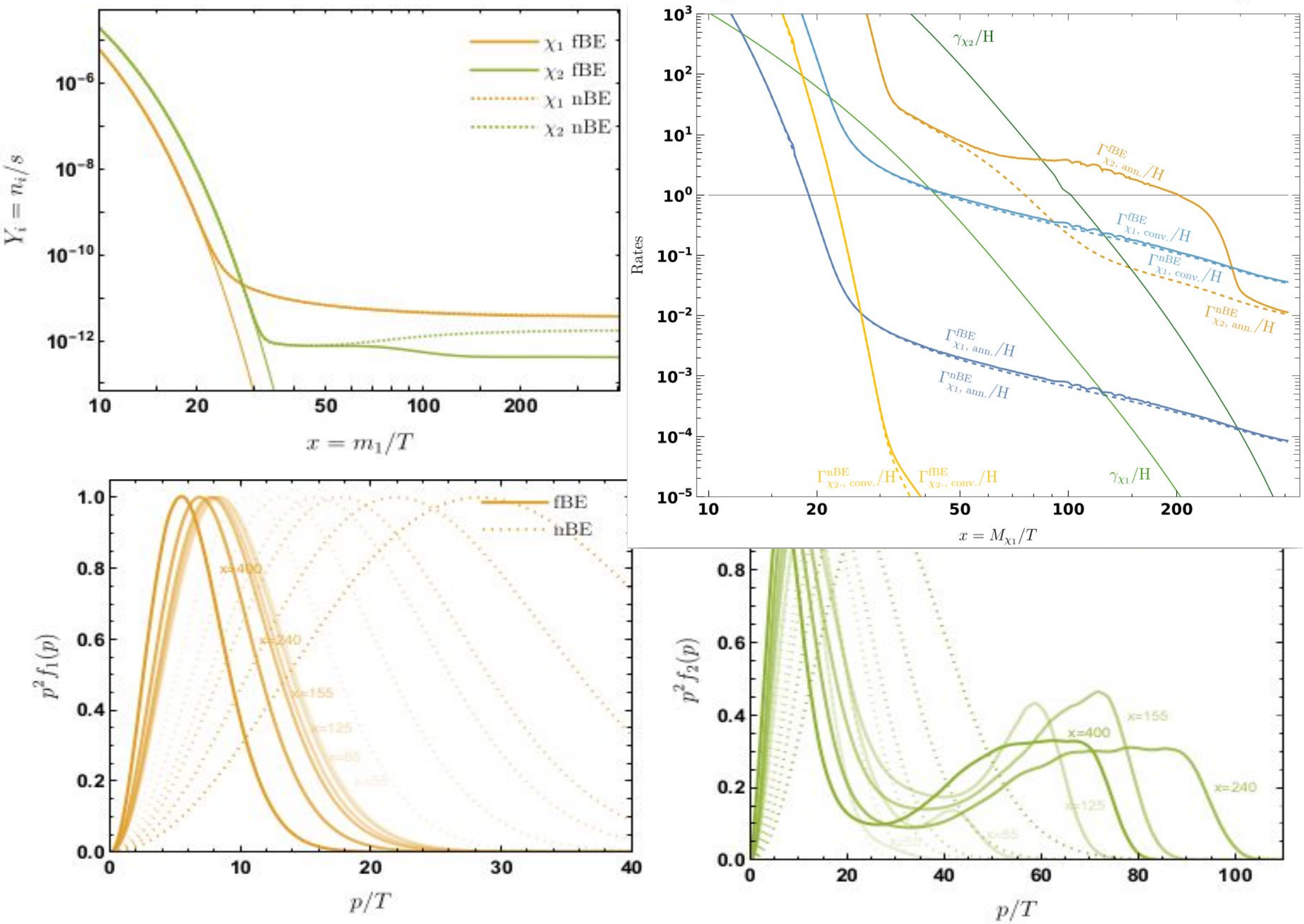

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SC, A. Hryczuk '25

Summary

- The production of DM beyond kinetic equilibrium requires for a solver of the full Boltzmann equation—DRAKE is a publicly available package capable of this

- In a multicomponent dark sector, it is more difficult to apriori ensure that kinetic equilibrium is maintained during freeze-out requiring a solution of the full Boltzmann equation (fBE) at the phase-space level for a precise determination of the relic abundance.
- We develop the extend the existing package to **DRAKE-2**, now adding
  - **Conversions, Decays & Elastic Scatterings without approximations**
  - **consistent model generation.**
  - **Freeze-out, Freeze-in (with dark freeze-out)**
  - Tested with the double coy DM with results published in arxiv: 2502.08725 [hep-ph],
  - Showing that departure from kinetic equilibrium can alter the predictions for the total DM abundance by more than 100% (while being -20% to 50% in most of the interesting parameter space)
  - Currently possible to tailor the code to given model
  - Ongoing study of interesting* physics cases, adding processes as required by the scenario
  - Future release enabled to solve two-component full Boltzmann equation for a generic BDM model planned after the end of this summer

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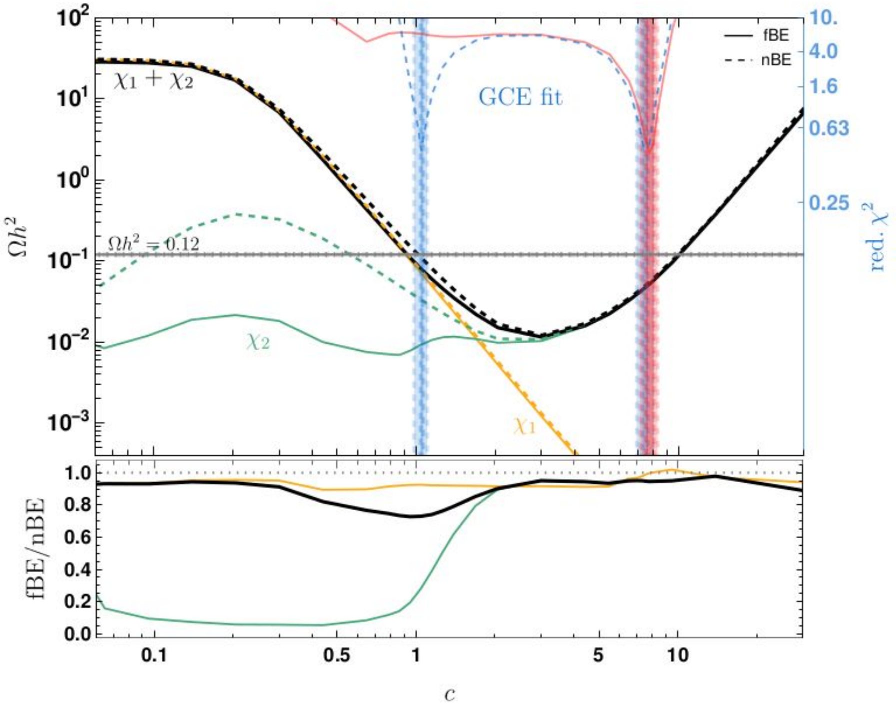
Thank you!

Back-up slides

Results: Doubled Coy Dark Matter

- Changing conversion strength ‘c’ keeping annihilation strength constant

$$\lambda_y \to \lambda_y/c,\;\; \lambda_{\chi_1} \to \lambda_{\chi_1} c,\;\; \lambda_{\chi_2} \to \lambda_{\chi_2} c \quad \Longrightarrow \quad \begin{array}{c} \sigma_{\chi_i,\chi_i \leftrightarrow \mathrm{SM,SM}} \propto \lambda_y^2 \lambda_{\chi_i}^2 \propto \text{constant} \\ \sigma_{\chi_1,\chi_1 \leftrightarrow \chi_2\chi_2} \propto \lambda_{\chi_1}^2 \lambda_{\chi_2}^2 \propto c^4 \end{array}$$

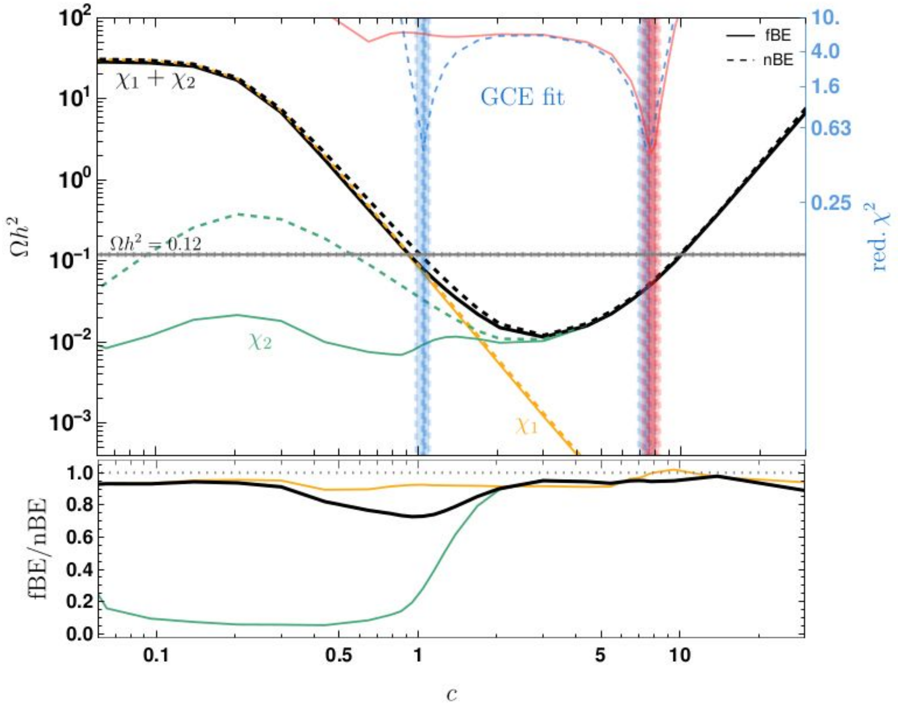


$$\mathrm{M}_{\chi_1} = 44\,\mathrm{GeV},\; \mathrm{M}_{\chi_2} = 38\,\mathrm{GeV}$$
$$\mathrm{M}_s = 80\,\mathrm{GeV}$$
$$\lambda_{\chi_1} = 0.023,\; \lambda_{\chi_2} = 0.39,\; \lambda_y = 0.3$$

Results: Doubled Coy Dark Matter

- Changing conversion strength 'c' keeping annihilation strength constant

$$\lambda_y \to \lambda_y/c, \;\; \lambda_{\chi_1} \to \lambda_{\chi_1} c, \;\; \lambda_{\chi_2} \to \lambda_{\chi_2} c \;\; \Longrightarrow \;\; \begin{aligned} \sigma_{\chi_i,\chi_i \leftrightarrow \mathrm{SM,SM}} &\propto \lambda_y^2 \lambda_{\chi_i}^2 \propto \text{constant} \\ \sigma_{\chi_1,\chi_1 \leftrightarrow \chi_2\chi_2} &\propto \lambda_{\chi_1}^2 \lambda_{\chi_2}^2 \propto c^4 \end{aligned}$$



Modification of the abundance of *subdominant component* completely changes the preferred region for the GCE fit

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The Fokker Planck approximation

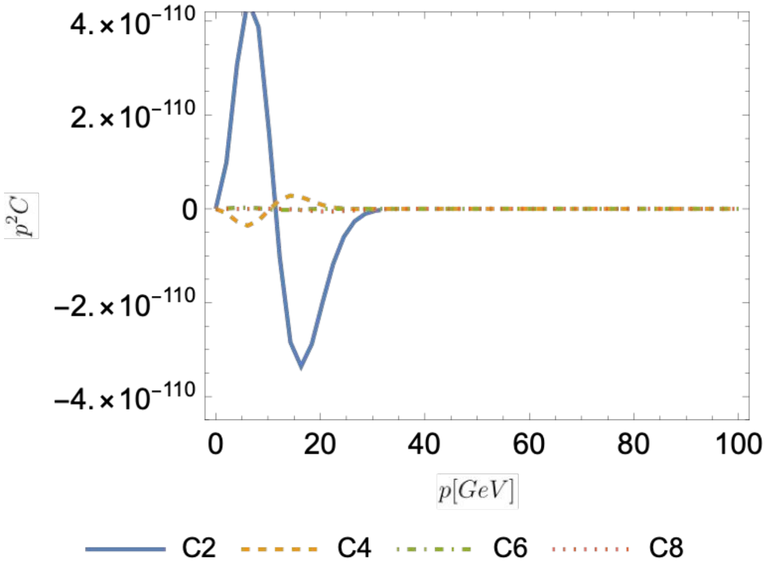
$$C_{el}[f_{DM}] = C_2 + C_4 + C_6 + C_8 + \cdots$$

$$C_{FP} = \frac{1}{2E_1}\gamma(f_{eq})\,\widehat{FP}(p_1).\boldsymbol{f_{DM}(p_1)}$$

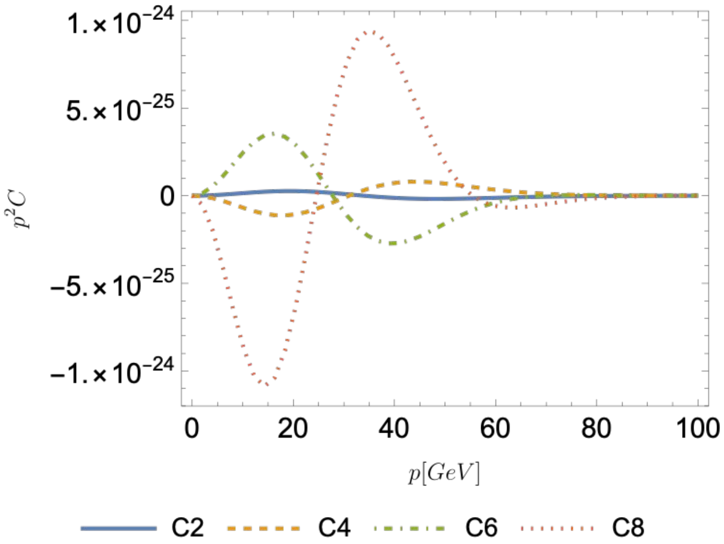
Has all the nice features:
- ✓ no integration on f_{DM}
- ✓ number conserving
- ✓ 0 on equilibrium distribution

$$x \equiv \frac{m_{DM}}{T} = \frac{m_\chi}{T}$$

$\frac{\Delta\vec{p}}{p} <$, $\frac{p_1}{E_1} < 1$



$m_\chi = 100$ GeV, $m = 1$ GeV, $x = 250$



$m_\chi = 100$ GeV, $m = 100$ GeV, $x = 25$

When does the Fokker Planck approx. work?

- Arrived at by *dropping higher order terms* in $\Delta\vec{p}/\vec{p}$ and p_1/E_1.
- Very good "approximation" (O(1%)) while the conditions of the expansion hold true.

Q: How to know when the FP approximation works?

$$|M|^2 \longrightarrow \underbrace{t^{n_1}}_{\propto \text{ transfer momentum}} \underbrace{(s-(m_{DM}+m_{SM})^2)}_{\propto \text{ relative velocity}}{}^{n_2} \underbrace{(u-(m_{DM}-m_{SM})^2)}_{\propto \text{ velocities}}{}^{n_3}$$

With an efficiently implemented fully numerical[1] solver for the Boltzmann equation into DRAKE, we find that The Fokker Planck approximation works well for:

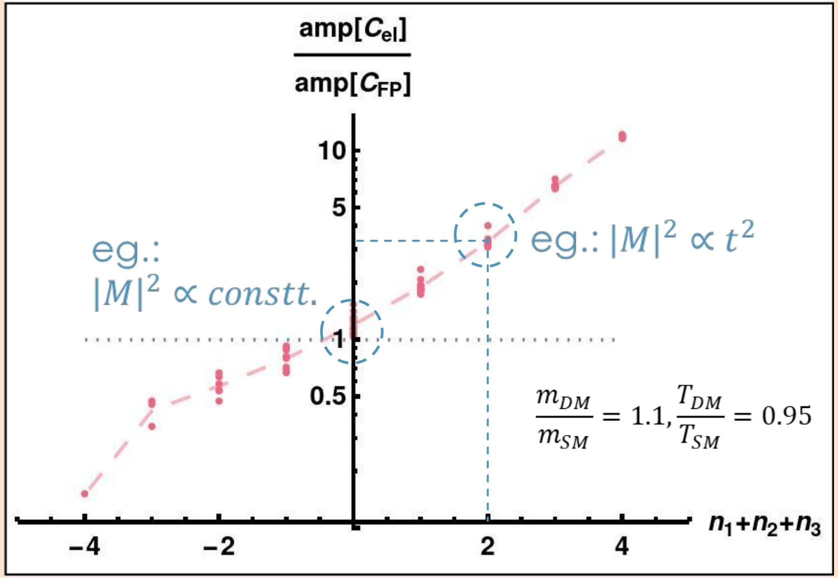
1. Scattering particle with masses significantly smaller than DM mass (small reduced mass ⇒ small momentum transfer

   &

2. DM temperatures close to the SM temperature (eg.: near kinetic decoupling)

   &

3. Scattering amplitudes that aren't strongly dependent on momentum transfer (the dropped higher order terms are more relevant for an amplitude sensitive to said dropped quantity)



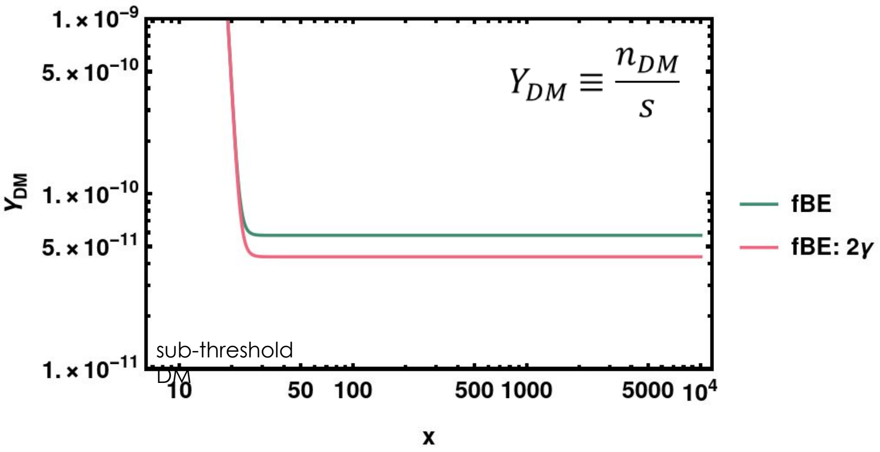
[1] Ala-Mattinen, Kainulainen '19
Hryczuk, Laletin '20
Aboubrahim, Klasen, Wiggering '23
Beauchesne, Chiang '24;

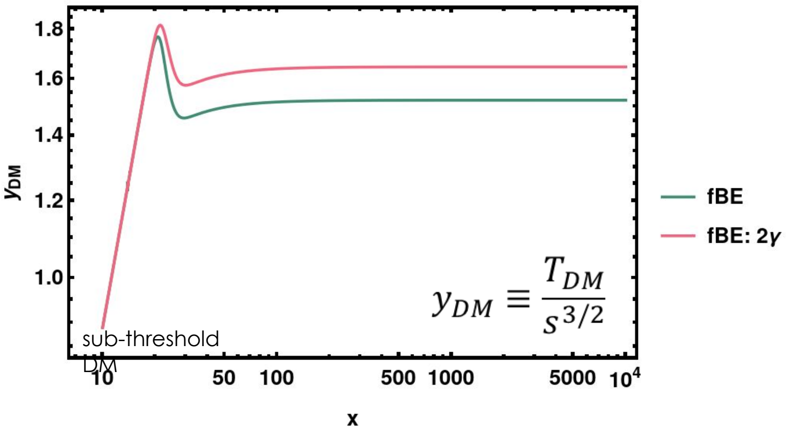
Improvement on Fokker Planck: Relic density

$$\partial_t f_{DM} - Hp\partial_p f_{DM} = C_{el}[f_{DM}] + C_{ann}[f_{DM}]$$

$$C_{el}[f_{DM}] \simeq C_{FP} = \frac{1}{2E_1}\gamma(f_{eq})\,\widehat{FP}(p_1).\boldsymbol{f_{DM}(p_1)}$$

An overall factor 2 at the level of collision operator ⟹ 25% change in DM relic density

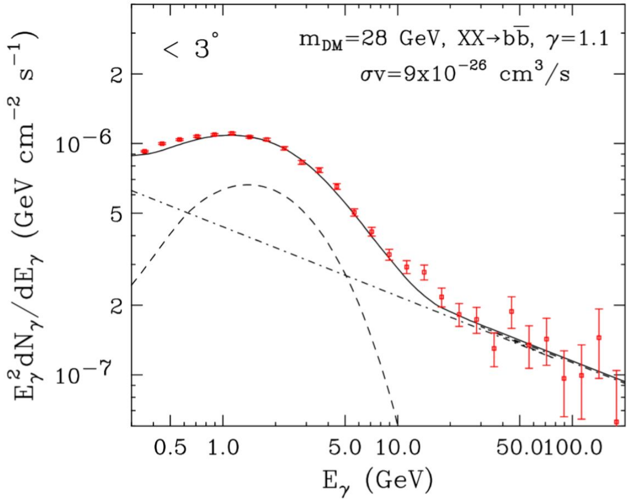




Galactic Centre Excess: Coy Dark Matter

- Fermi-LAT observes an excess in the spatially extended γ-rays from the Galactic Centre with a spectrum that peaks at a few GeV. Leading explanations:
  - DM annihilation
  - Millisecond Pulsar (MSP)

Fit to Galactic Centre Excess (GCE) from DM annihilation:



Goodenough, Hooper arXiv:0910.2998

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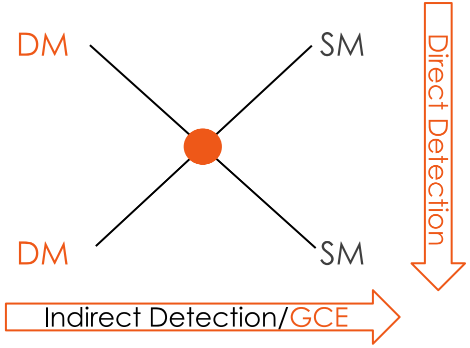

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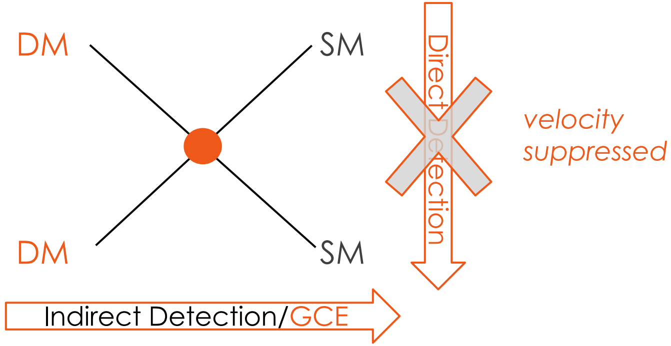

- Coy DM: fermionic DM with pseudoscalar mediator and coupling with SM proportional to Yukawa couplings of the SM fermions (Minimal Flavor Violation)

$$\mathcal{L} \supset -i\lambda_\chi s\,\bar{\chi}\gamma^5\chi - i\lambda_y \sum_{f\in\mathcal{SM}} y_f s\,\bar{f}\gamma^5 f$$

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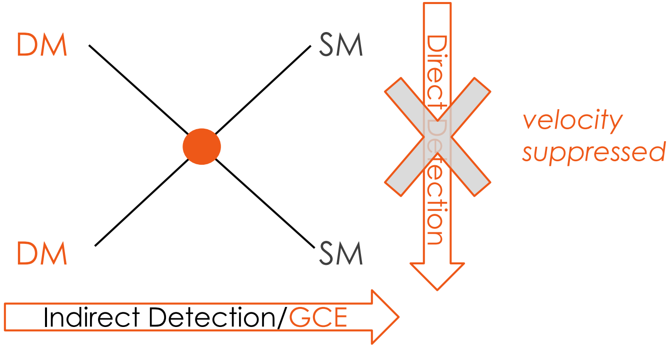
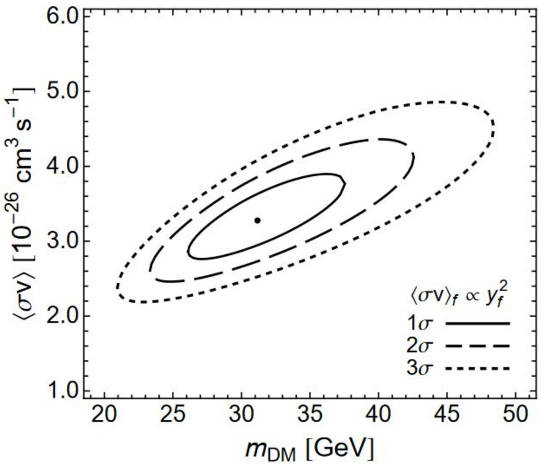


Fit to Galactic Centre Excess (GCE) from Coy DM annihilation:



Boehm et al 2014

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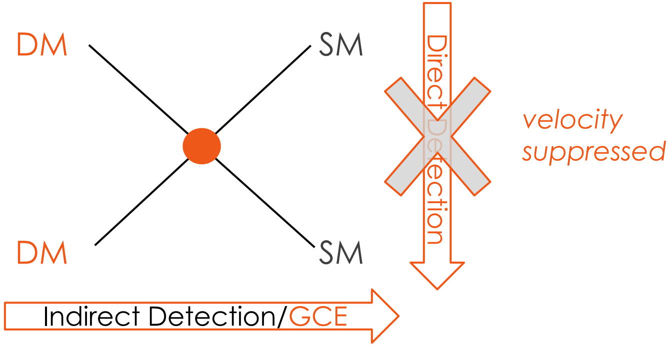
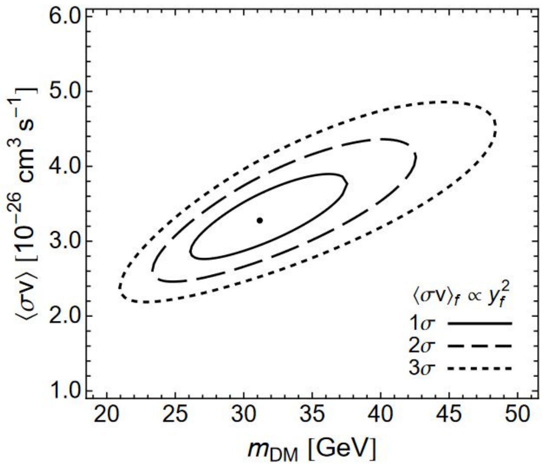


Fit to Galactic Centre Excess (GCE) from Coy DM annihilation:



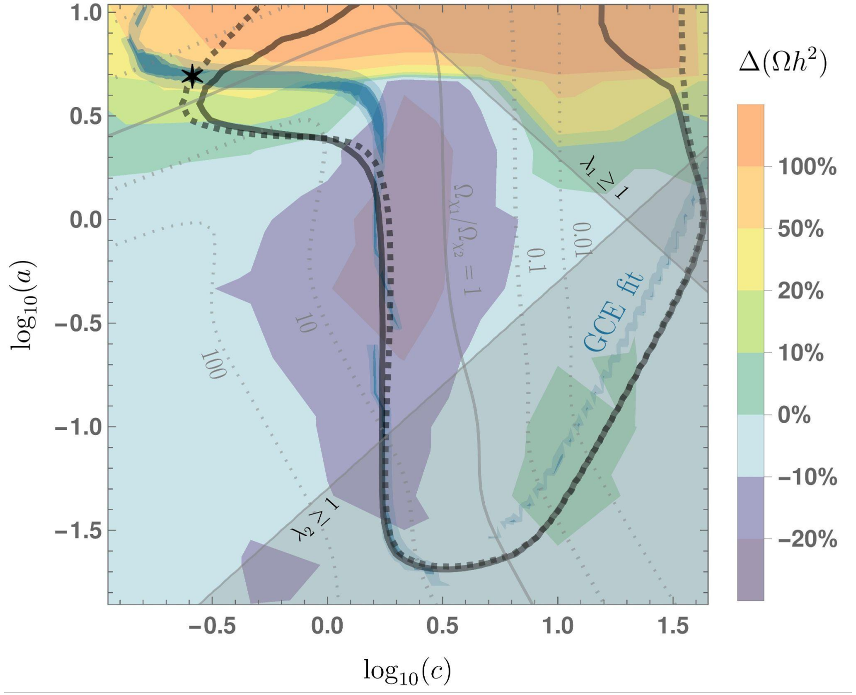
Boehm et al 2014

- Minimally extended coy DM: Two fermions (χ_1, χ_2) with pseudoscalar mediator (s) and coupling with SM proportional to Yukawa couplings of the SM fermions (Minimal Flavor Violation)

$$\mathcal{L} \supset -i\lambda_{\chi_1} s\,\bar{\chi}_1\gamma^5\chi_1 - i\lambda_{\chi_2} s\,\bar{\chi}_2\gamma^5\chi_2 - i\lambda_y \sum_{f\in SM} y_f s\,\bar{f}\gamma^5 f$$

Results: Doubled Coy Dark Matter

$$\lambda_y \to \lambda_y/c,\ \ \lambda_{\chi_1} \to \lambda_{\chi_1} c\, a\,,\ \ \lambda_{\chi_2} \to \lambda_{\chi_2}\, c/a \quad \Longrightarrow \quad \begin{aligned} \sigma_{\chi_1,\chi_1\leftrightarrow \mathrm{SM,SM}} &\propto \lambda_y^2\lambda_{\chi_1}^2 \propto a^2 \\ \sigma_{\chi_2,\chi_2\leftrightarrow \mathrm{SM,SM}} &\propto \lambda_y^2\lambda_{\chi_2}^2 \propto 1/a^2 \\ \sigma_{\chi_1,\chi_1\leftrightarrow \chi_2\chi_2} &\propto \lambda_{\chi_1}^2\lambda_{\chi_2}^2 \propto c^4 \end{aligned}$$



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$$\frac{(\Omega h^2)_{fBE} - (\Omega h^2)_{nBE}}{(\Omega h^2)_{nBE}} \times 100$$ is valued -20% to 100%

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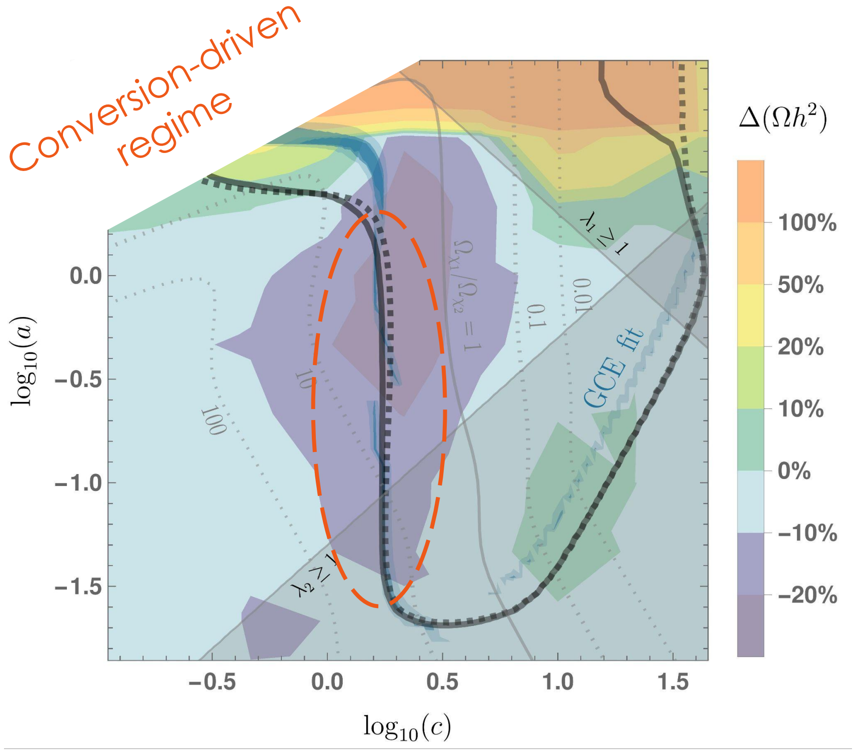

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