A Material-Independent Upper Bound on the Absorption Rate of Solar Dark Photons

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Dark Photons

Possible

Simple

Mediator between
DM and SM

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The model:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{4}V_{\mu\nu}^2 - \frac{\kappa}{2}F_{\mu\nu}V^{\mu\nu} + \frac{m_V^2}{2}V_\mu V^\mu + eJ_{\text{em}}^\mu A_\mu$$

An, Pospelov, Pradler, arXiv:1304.3461

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Kinetic mixing

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Absorption of Solar Dark Photons

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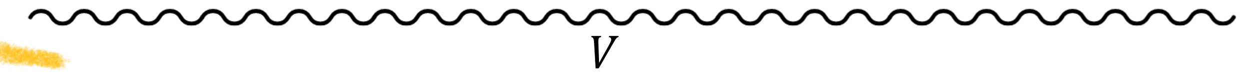
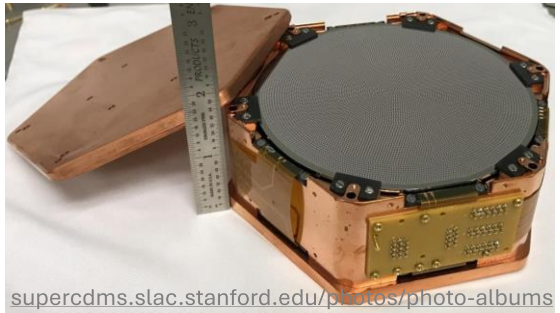
Production in the sun

Absorption in a detector





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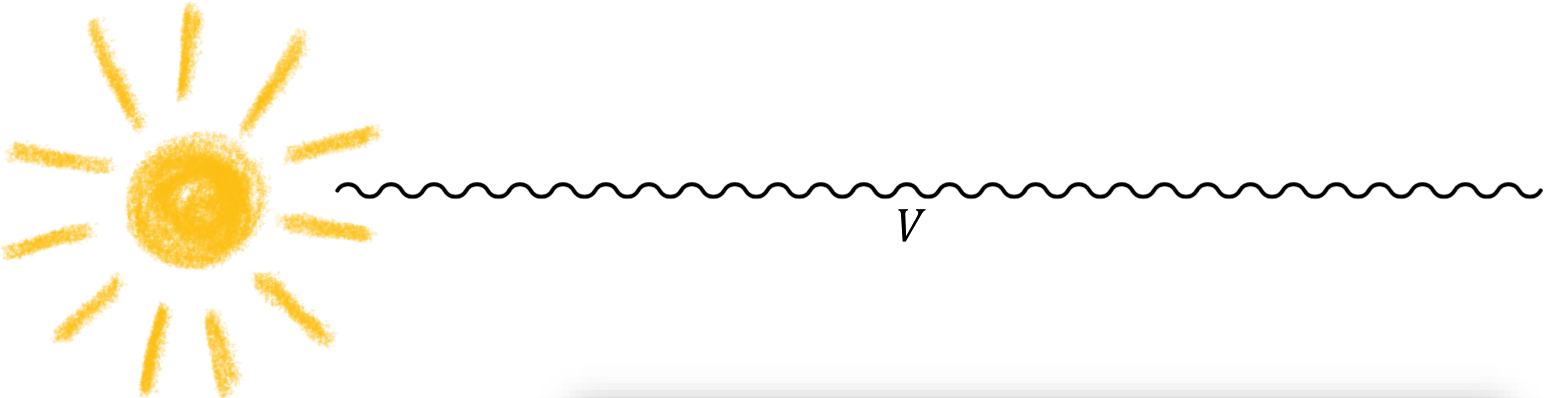

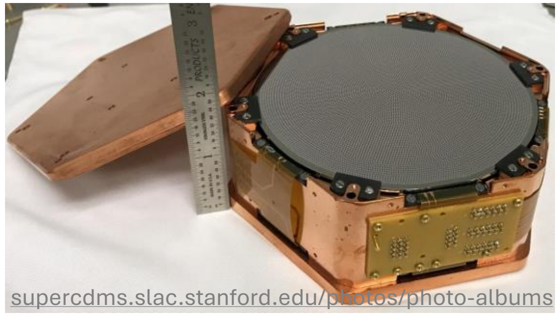

What is the best material for detecting dark photons?

Absorption of Solar Dark Photons

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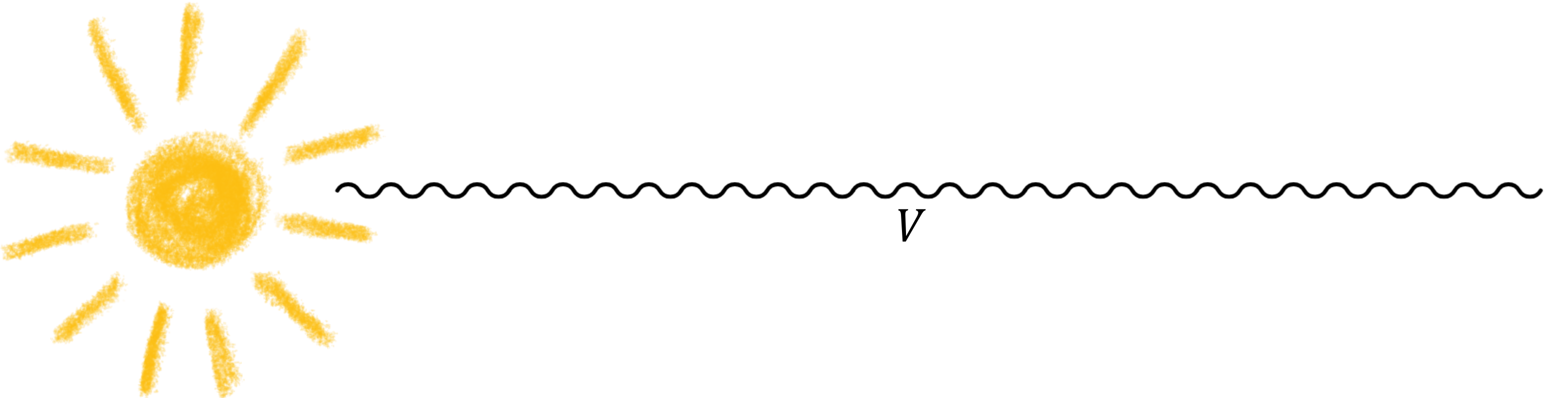

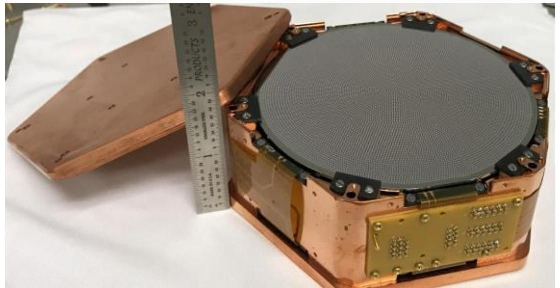

supercdms.slac.stanford.edu/photos/photo-albums

Main production mechanism:
Bremsstrahlung

Calculated by An et al., arxiv:1302.3884
and Redondo, arxiv:0801.1527

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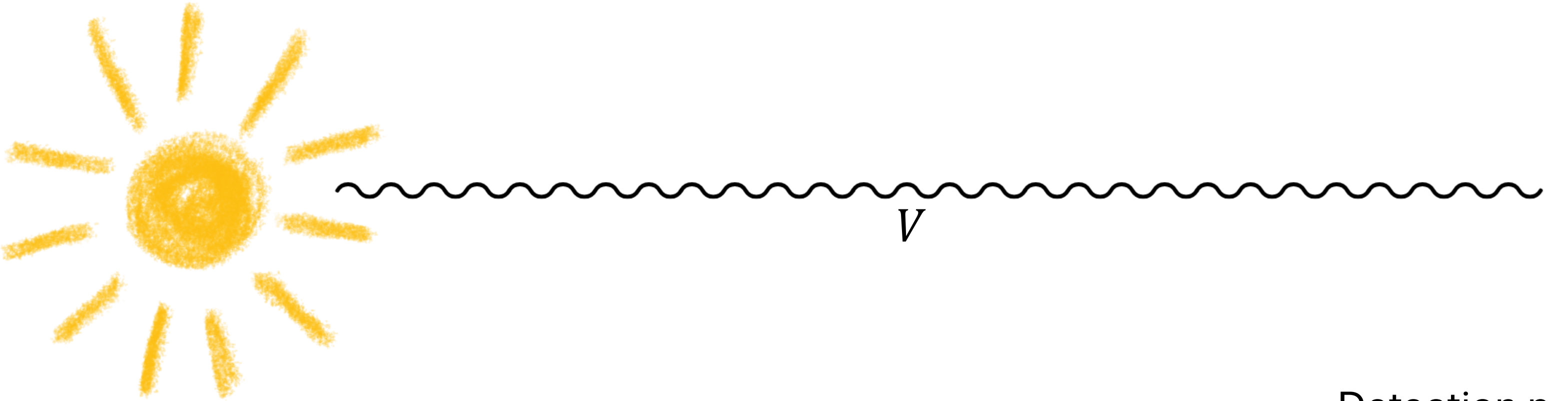

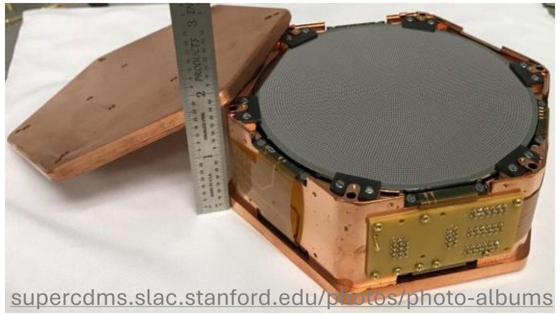

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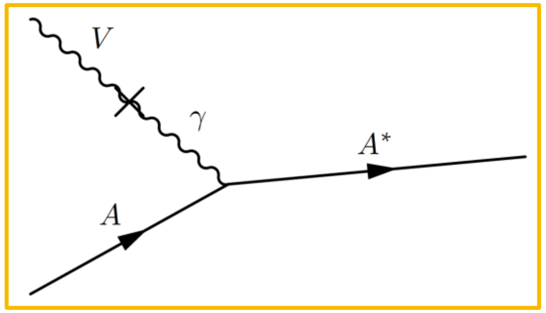

Detection Rates

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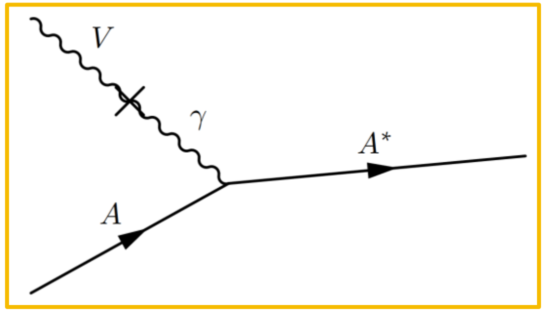

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Detection rate:

$$n = \int_{w_{min}}^{w_{max}} \frac{\omega d\omega}{|\boldsymbol{q}|}\left(\frac{d\Phi_L}{d\omega}\Gamma_L + \frac{d\Phi_T}{d\omega}\Gamma_T\right)$$

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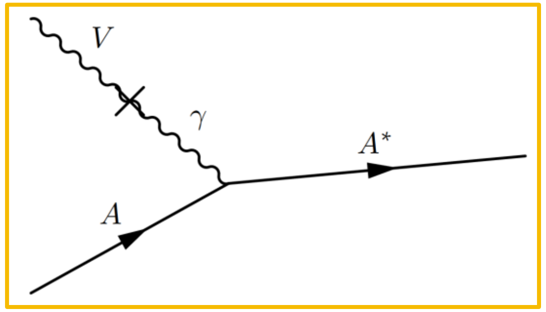

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$$n_L = \int_{\omega_{min}}^{\omega_{max}} \frac{d\omega}{\omega} \mathrm{Im} \left(\frac{-1}{\varepsilon_r} \right) \kappa^2 m_V^2 \frac{\omega}{|\boldsymbol{q}|} \frac{d\Phi_L}{d\omega}$$

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$$n_T = \int_{\omega_{min}}^{\omega_{max}} \frac{d\omega}{\omega} \mathrm{Im} \left(\frac{-1}{(\varepsilon_r - 1)\omega^2 + m_V^2} \right) \kappa^2 m_V^4 \frac{\omega}{|\boldsymbol{q}|} \frac{d\Phi_T}{d\omega}$$

What is the Best Material for Detecting Dark Photons?

Our Strategy to Find an Upper Limit on n_L and n_T

Kramers-Kronig relation:

$$\int_0^\infty \frac{d\omega}{\omega} \mathrm{Im}\,\chi(\omega, \boldsymbol{q}) = \frac{\pi}{2}\chi(0, \boldsymbol{q})$$

This holds if

1. $\lim_{\omega\to\infty} \chi(\omega, \boldsymbol{q}) = 0$
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e.g. Energy Loss Function

$$\int_{\omega_{min}}^{\omega_{max}} \frac{d\omega}{\omega} \mathrm{Im} \left(-\frac{1}{\varepsilon_r(\omega, q)} \right) \leq \frac{\pi}{2} \left(1 - \frac{1}{\varepsilon_r(0, q)} \right)$$

Analytical Results

$$\boxed{\int_{\omega_{min}}^{\omega_{max}} \frac{d\omega}{\omega} \mathrm{Im}\,\chi(\omega, \boldsymbol{q}) \leq \frac{\pi}{2}\chi(0, \boldsymbol{q})}$$

Longitudinal part:

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$\Downarrow$

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For $m_V^2 \ll \omega_p^2$ or $m_V^2 \geq 2\omega_p^2$:

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Putting everything together

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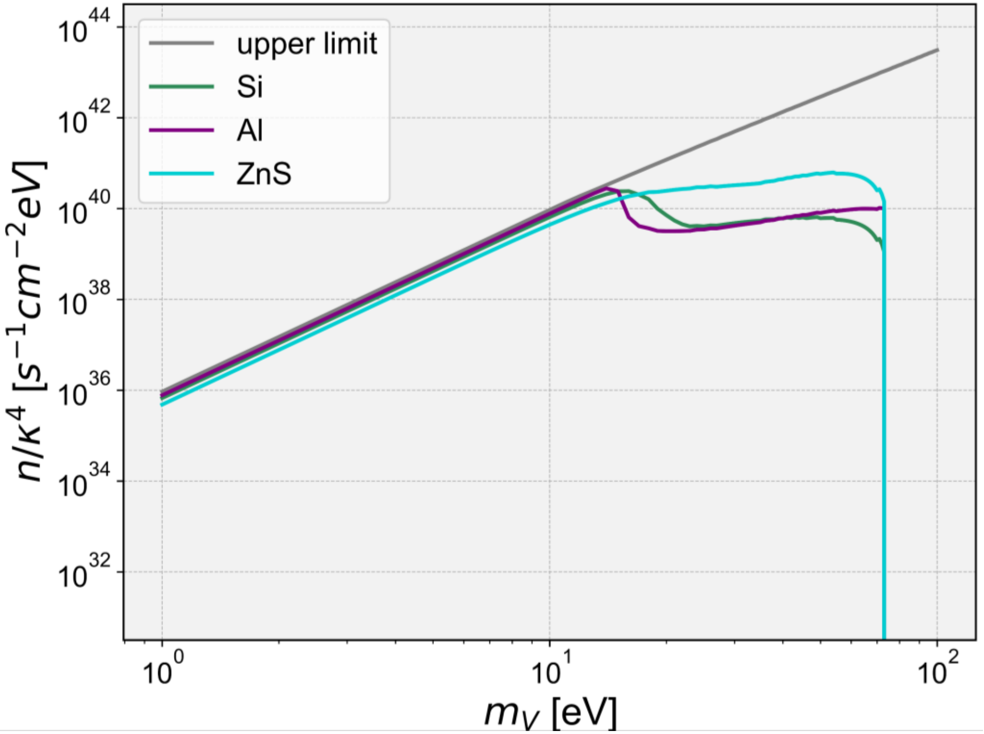

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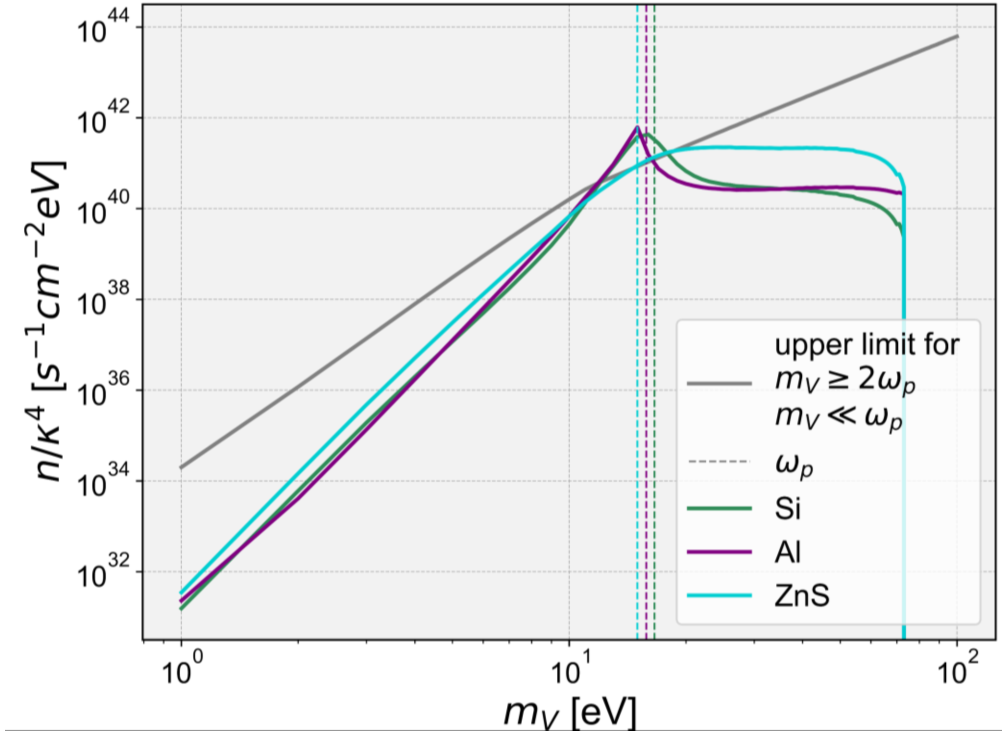

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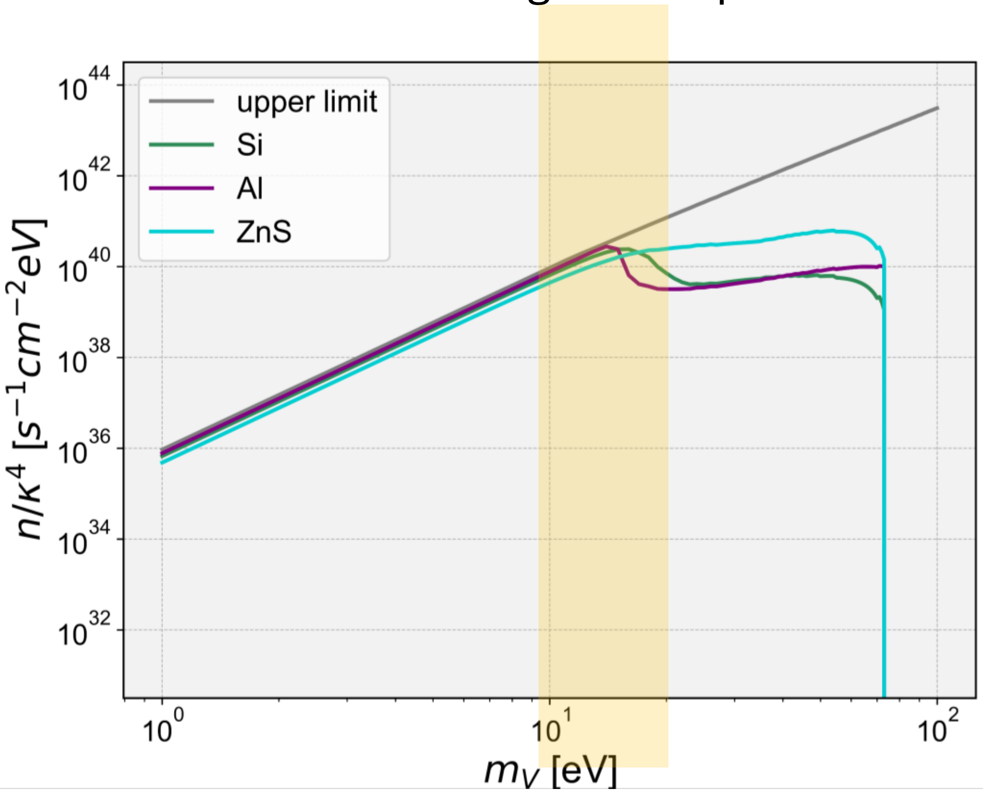

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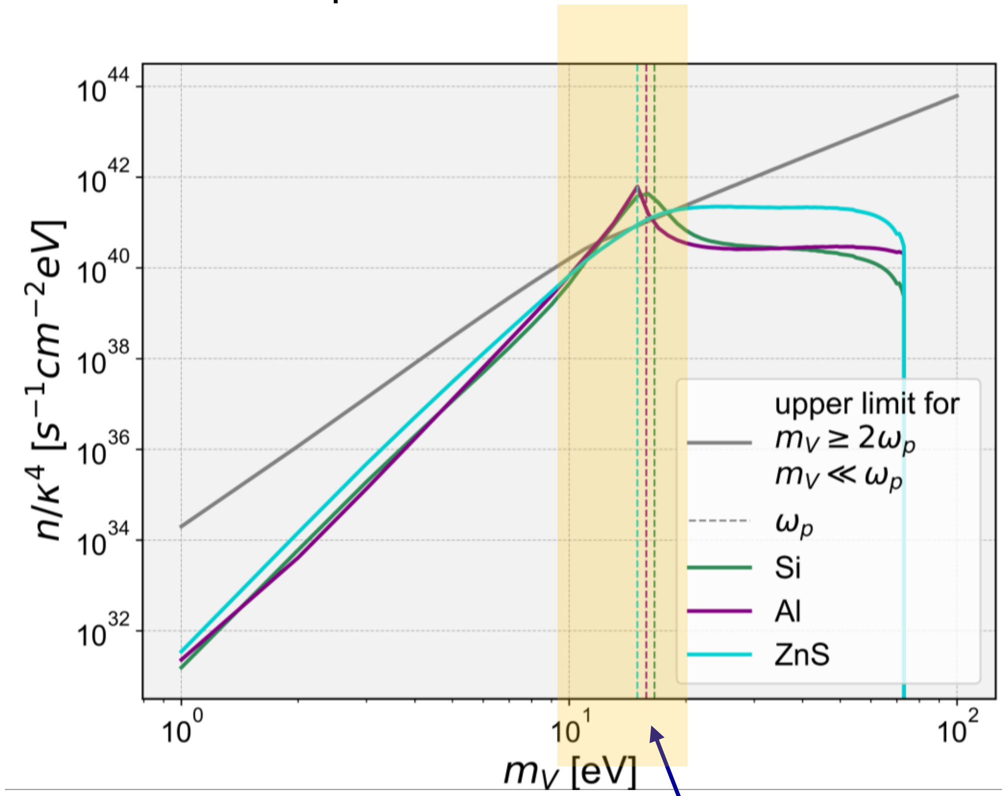

Highest detection rates for $m_V^2 \approx \omega_p^2$

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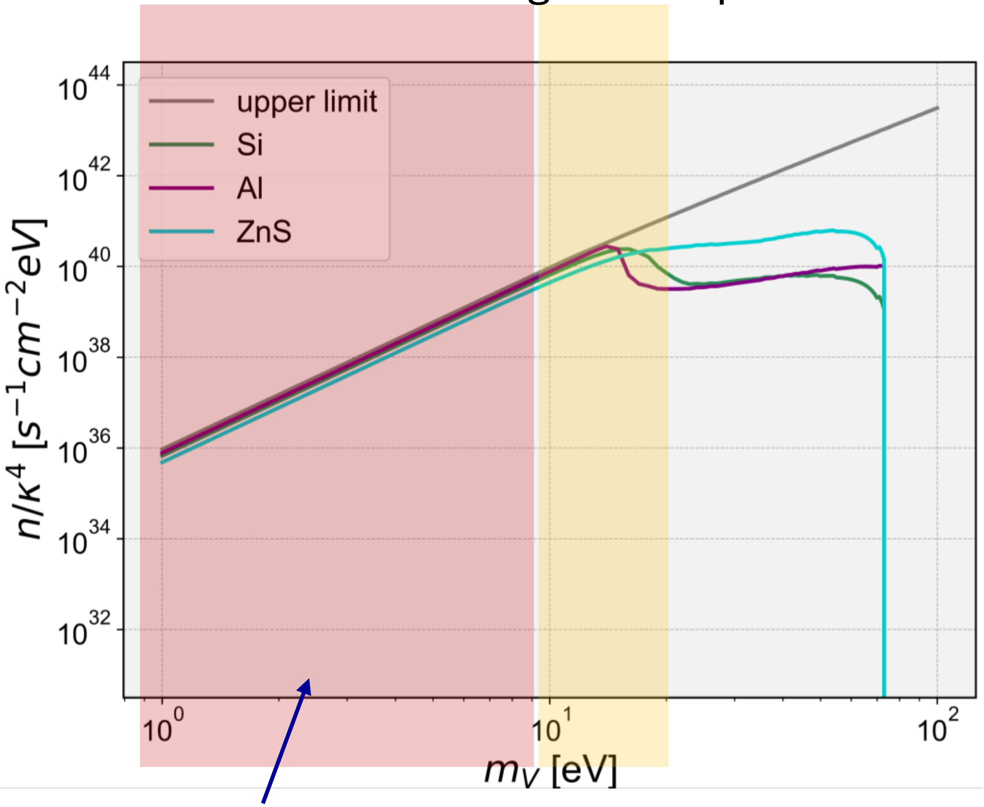

For $m_V^2 \ll \omega_p^2$ longitudinal part dominates

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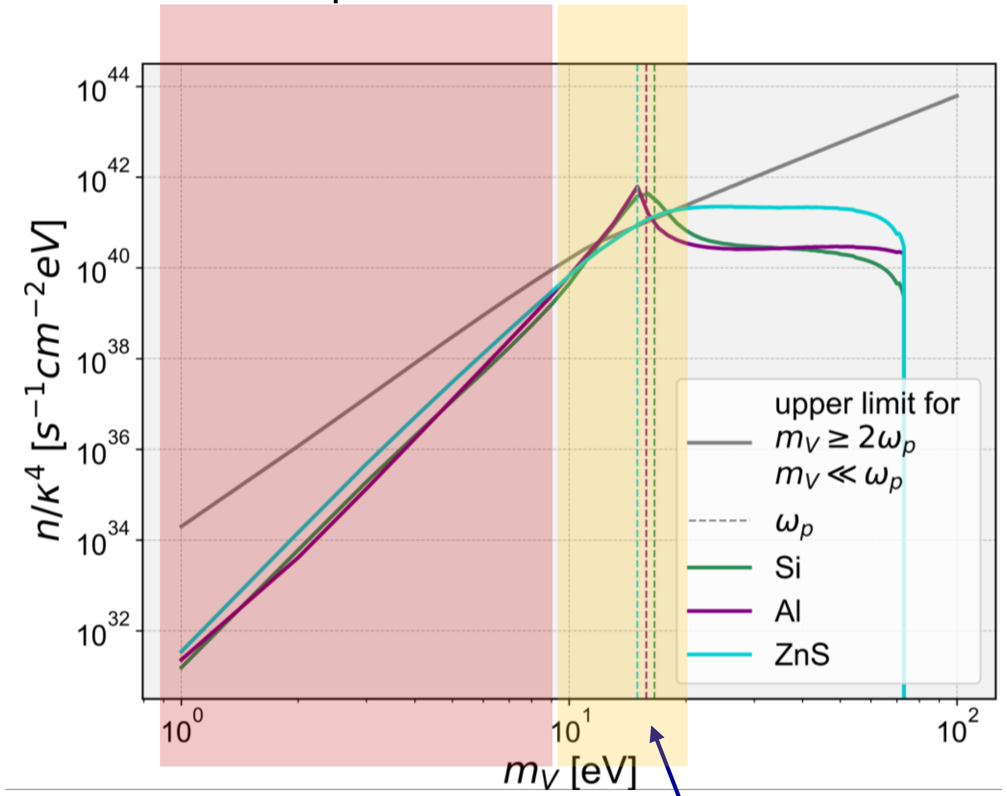

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Thank you for your attention.