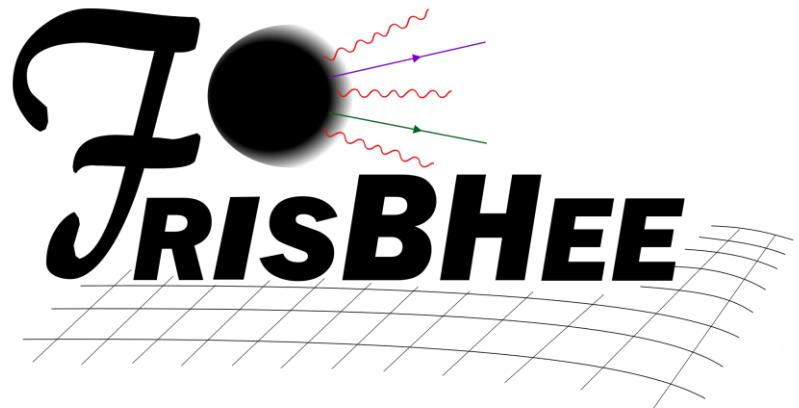




李政道研究所
TSUNG-DAO LEE INSTITUTE



:a code for evaporating black holes

DARKTOOLS 2025

Andrew Cheek,
based on [arXiv:2212.03878](https://arxiv.org/abs/2212.03878), [arXiv:2207.09462](https://arxiv.org/abs/2207.09462), [arXiv:2107.00013](https://arxiv.org/abs/2107.00013), [arXiv:2107.00016](https://arxiv.org/abs/2107.00016)
in collaboration with L. Heurtier, Y. F. Perez-Gonzalez, J. Turner

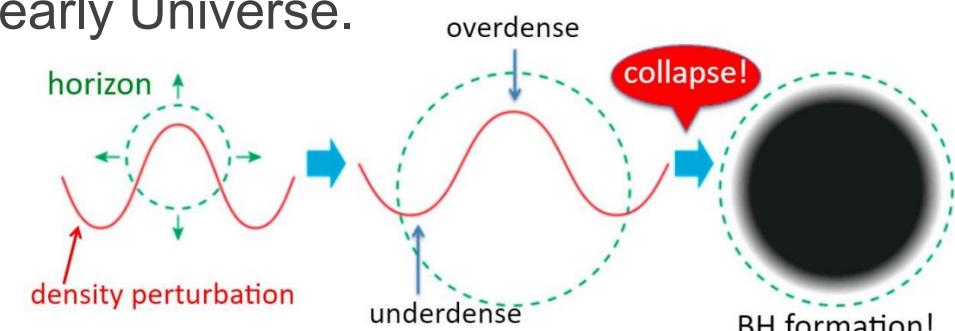
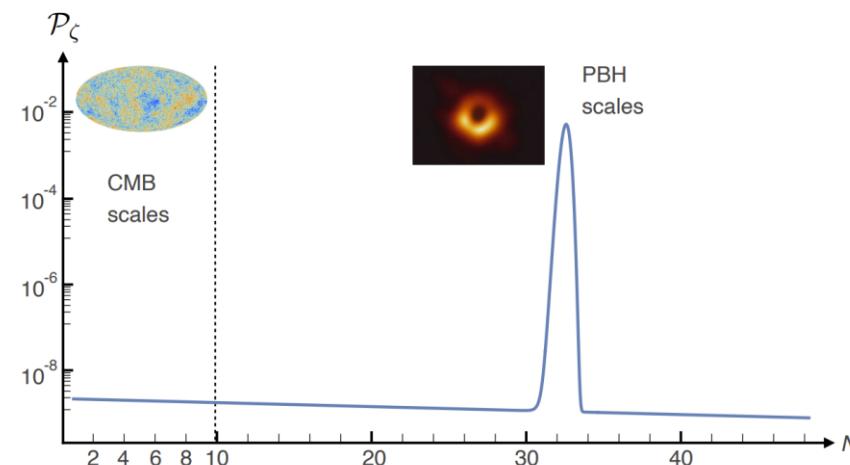
<https://github.com/yfperezg/frisbhee>

Primordial Black Holes

Hypothetical black holes formed before stellar formation.

Come from extremely dense matter fluctuations in the early Universe.

Not predicted from vanilla slow-roll inflation, but requires a big extrapolation



[picture borrowed from N. Kitajima]

No shortage of primordial black hole (PBH) production mechanisms:

Ultra slow-roll inflation, cosmological phase transitions, pre-heating... etc

Black holes evaporate

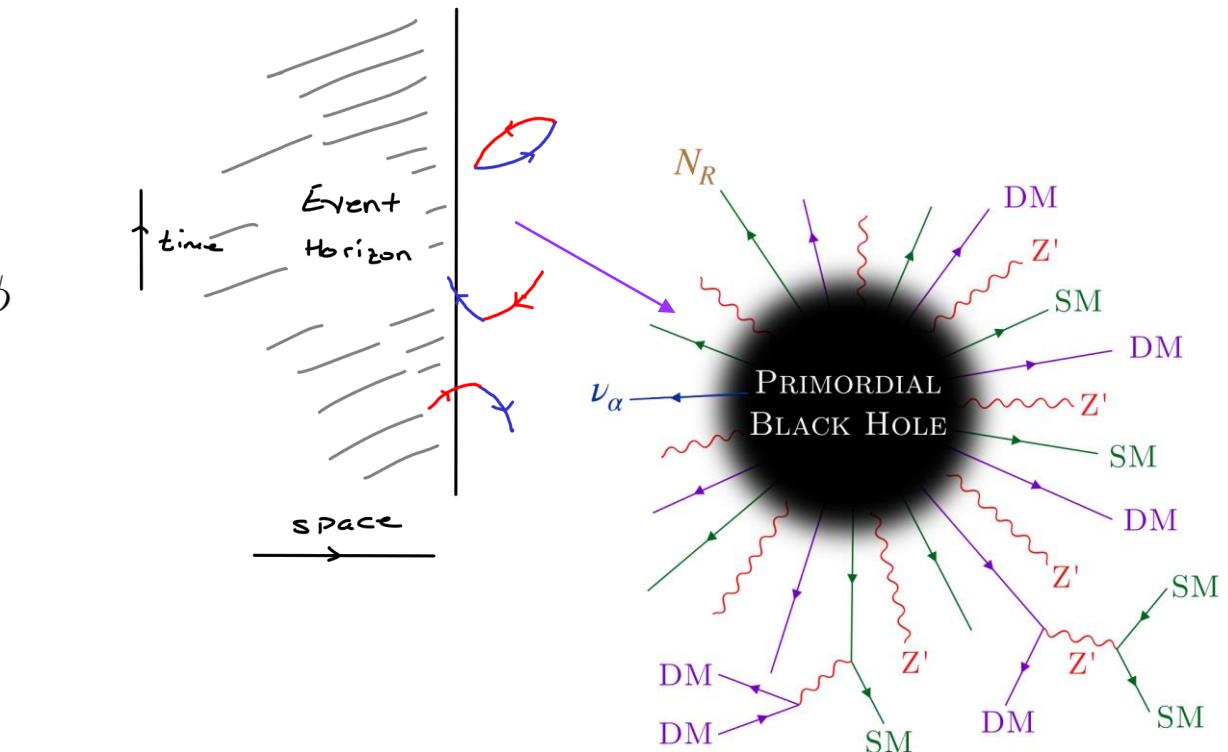
Hawking (1970s) showed that by solving the equation of motion for quantum fields in a curved spacetime

$$(\square + m^2) \phi \rightarrow \frac{1}{\sqrt{-g}} \partial_\mu (g^{\mu\nu} \sqrt{-g} \partial_\nu \phi) + m^2 \phi$$

You get particle emission from a black hole!

The production rate is insensitive to other particle interactions. So will happen for all particles, known or unknown.

One tool for calculating this is [BlackHawk](#).



[Image: L. Heurtier & Y. Perez-Gonzalez]

<https://blackhawk.hepforge.org/>

Evaporation means finite lifetime

By emitting radiation, the BH mass decreases, giving BHs a finite lifetime

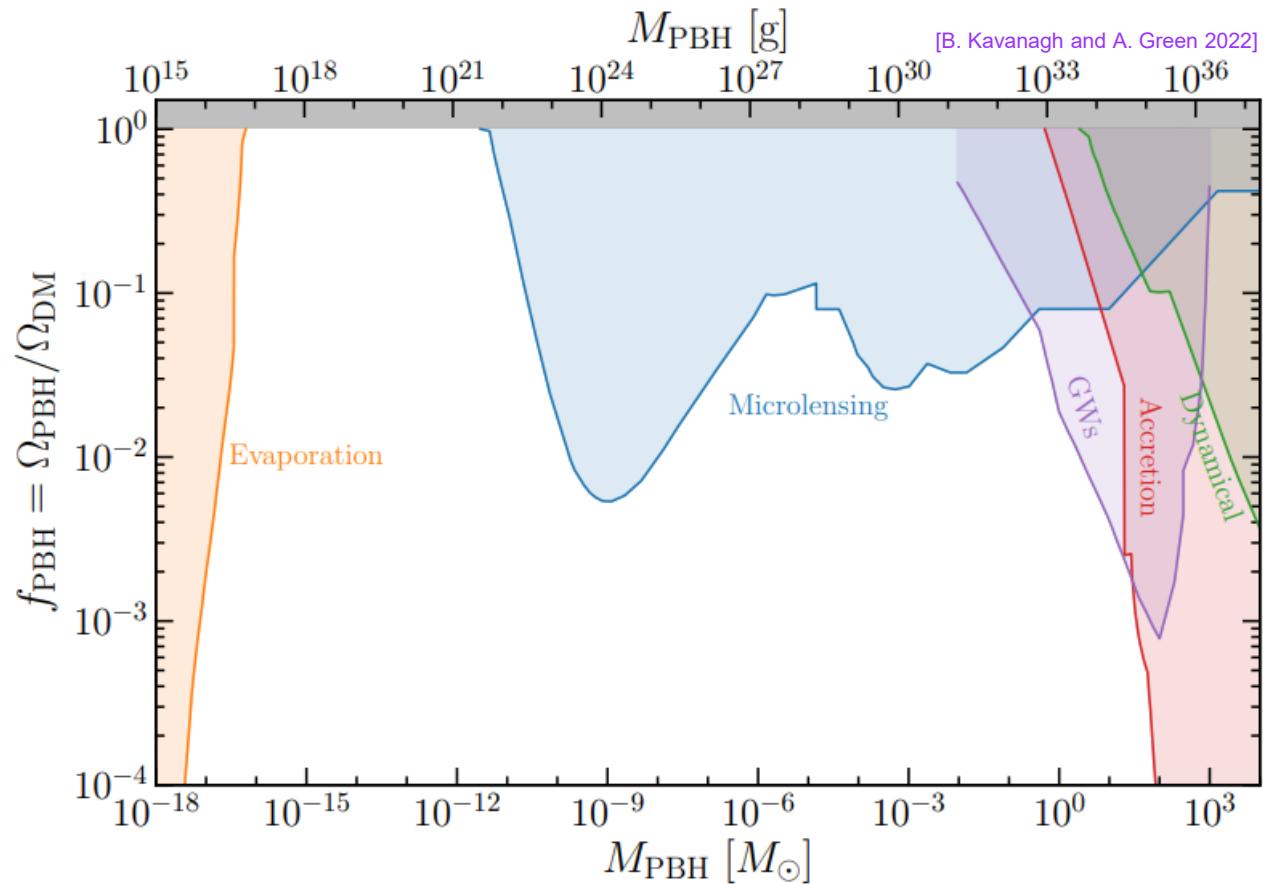
$$t_{\text{ev}} \sim (M_{\text{BH}}^{\text{in}})^3 / (3M_{\text{pl}}^4)$$

Heavier black holes live longer

Sometimes longer than t_{universe}

These black holes could contribute substantially to dark matter abundance

$$f_{\text{PBH}} = \frac{\Omega_{\text{PBH}}}{\Omega_{\text{DM}}}$$



Evaporated BHs parameter space

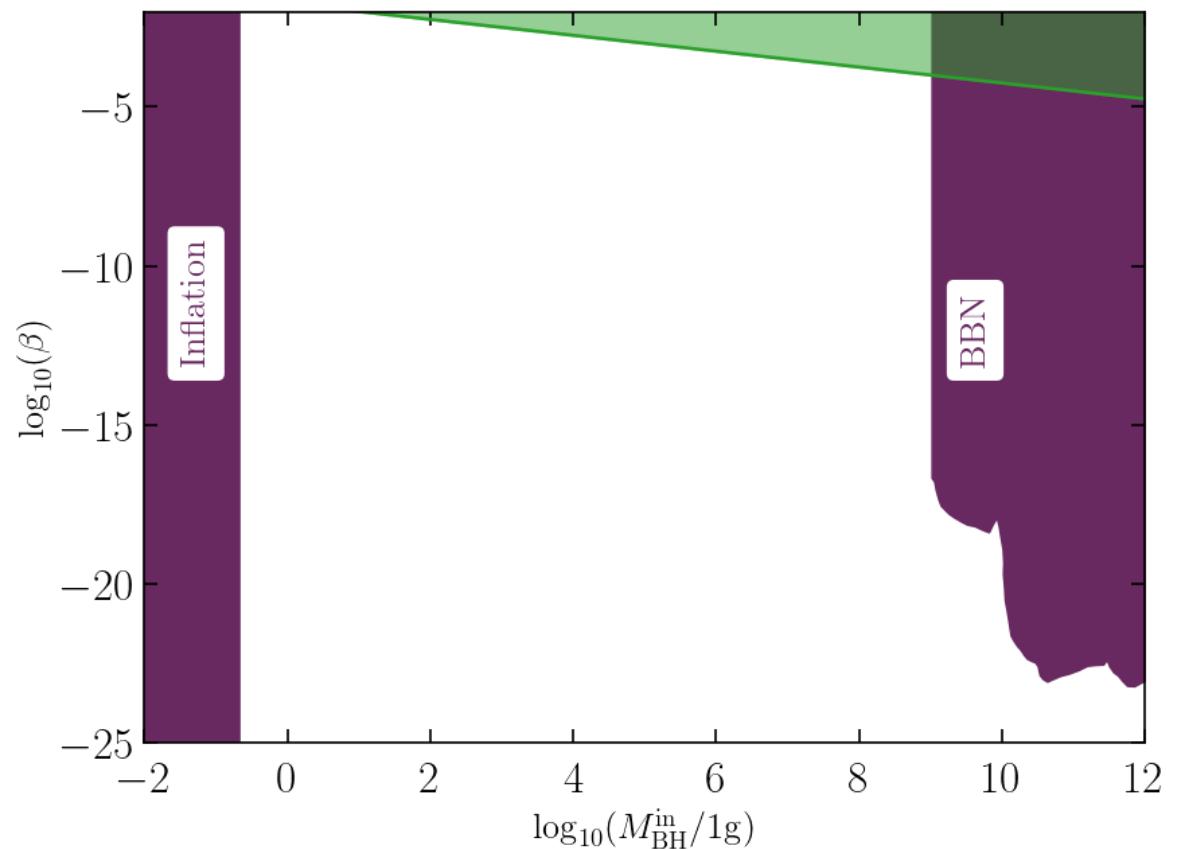
Big Bang Nucleosynthesis (BBN) and inflation provide constraints but lots of space in between!

$$\beta' \equiv \gamma^{1/2} \left(\frac{g_*(T_{\text{in}})}{106.75} \right)^{-1/4} \frac{\rho_{\text{PBH}}^{\text{in}}}{\rho^{\text{in}}}$$

In **green** GW constraints [\[G. Domenech et. al 2020\]](#)

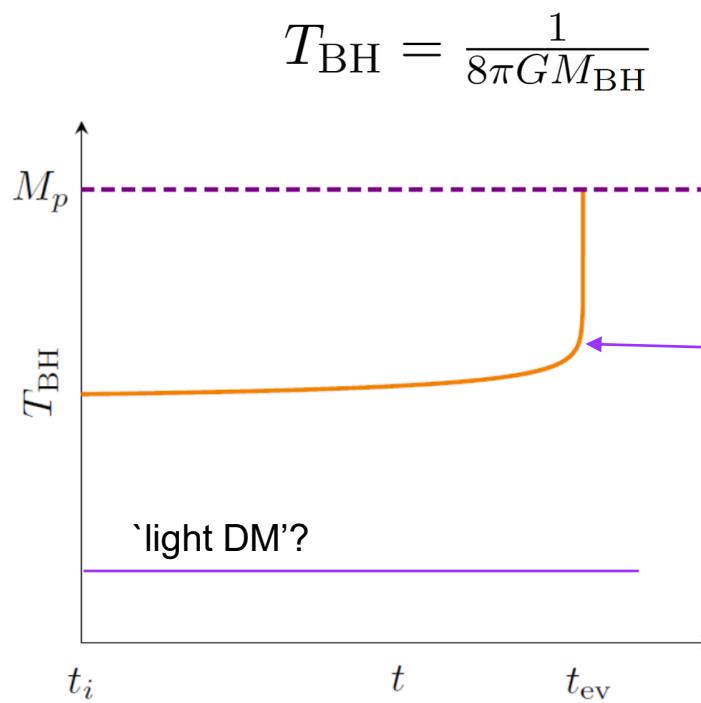
Evidence for evaporated PBHs would be interesting in of itself.

Implications for topics on the edge of our understanding, inflation, extra dimensions
GR at Planck scale, ...



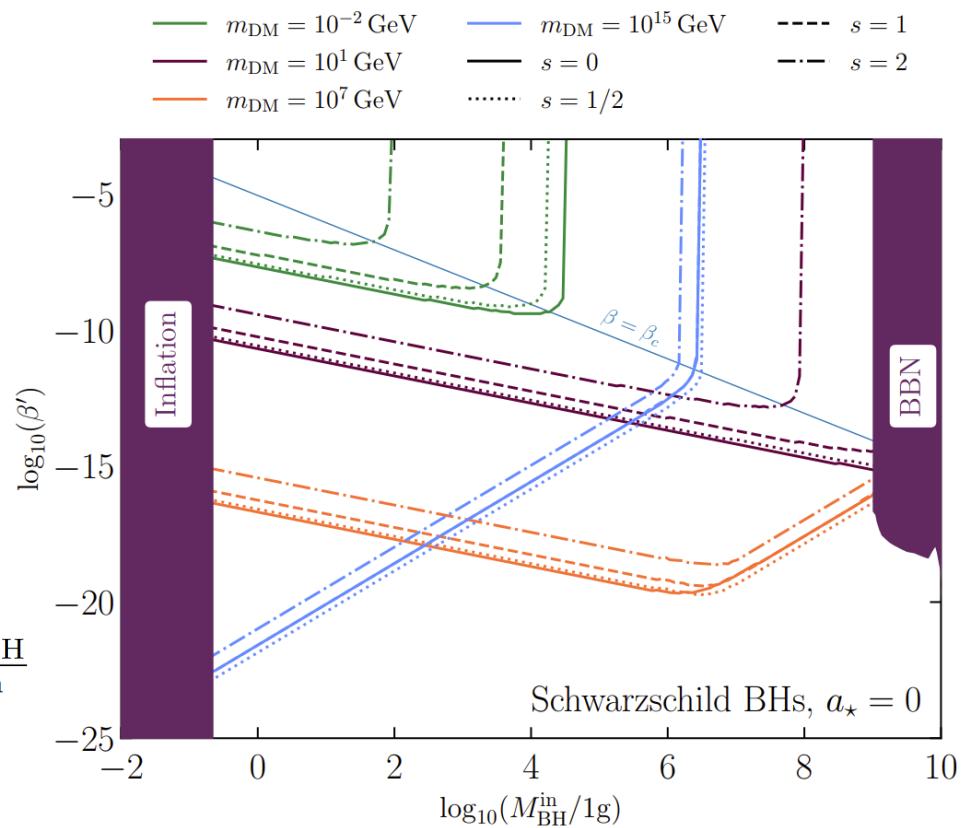
Hawking radiation and BSM

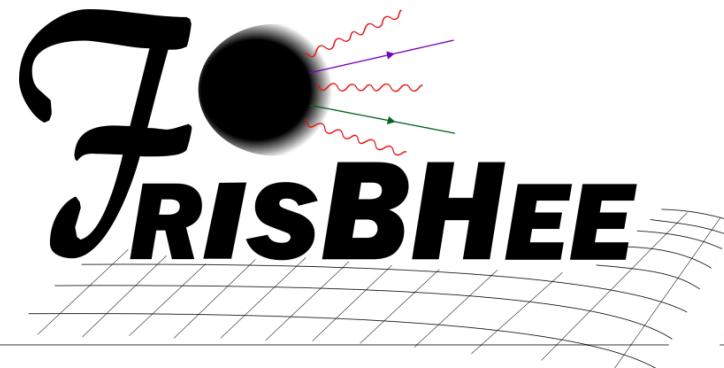
Black hole temperature increases as mass decreases



$$\beta' \equiv \gamma^{1/2} \left(\frac{g_*(T_{\text{in}})}{106.75} \right)^{-1/4} \frac{\rho_{\text{PBH}}^{\text{in}}}{\rho^{\text{in}}}$$

Very efficient production of dark matter!





We created

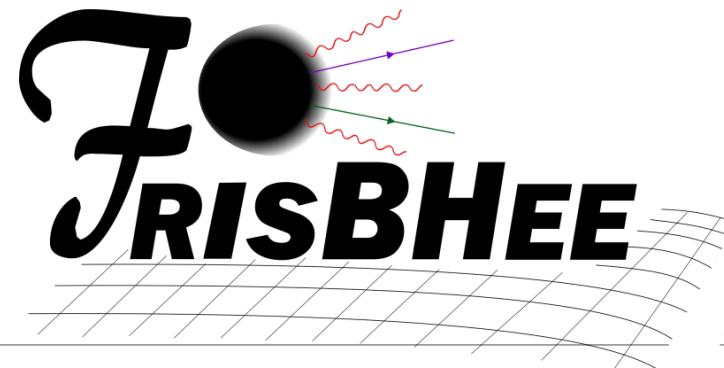
Our code **FRISBHEE**, FRiedmann Solver for Black Hole Evaporation in the Early universe, solves the system of coupled Friedmann and Boltzmann equations fully.

Publicly available on [GitHub](#) and ready to use!

$$\frac{dM_{\text{BH}}}{dt} = -\epsilon(M_{\text{BH}}, a_\star) \frac{M_p^4}{M_{\text{BH}}^2} \quad \frac{da_\star}{dt} = -a_\star [\gamma(M_{\text{BH}}, a_\star) - 2\epsilon(M_{\text{BH}}, a_\star)] \frac{M_p^4}{M_{\text{BH}}^3}$$

$$\frac{3H^2 M_p^2}{8\pi} = \rho_{\text{R}}^{\text{SM}} + \rho_{\text{DR}} + \rho_{\text{PBH}} \quad \dot{\rho}_{\text{PBH}} + 3H\rho_{\text{PBH}} = \frac{d \log M_{\text{BH}}}{dt} \rho_{\text{PBH}}$$

$$\dot{\rho}_{\text{DR}} + 4H\rho_{\text{DR}} = - \left. \frac{d \log M_{\text{BH}}}{dt} \right|_{\text{DR}} \rho_{\text{PBH}} \quad \dot{\rho}_{\text{R}}^{\text{SM}} + 4H\rho_{\text{R}}^{\text{SM}} = - \left. \frac{d \log M_{\text{BH}}}{dt} \right|_{\text{SM}} \rho_{\text{PBH}}$$



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Black hole dynamics

$$\frac{dM_{\text{BH}}}{dt} = -\epsilon(M_{\text{BH}}, a_*) \frac{M_p^4}{M_{\text{BH}}^2}$$

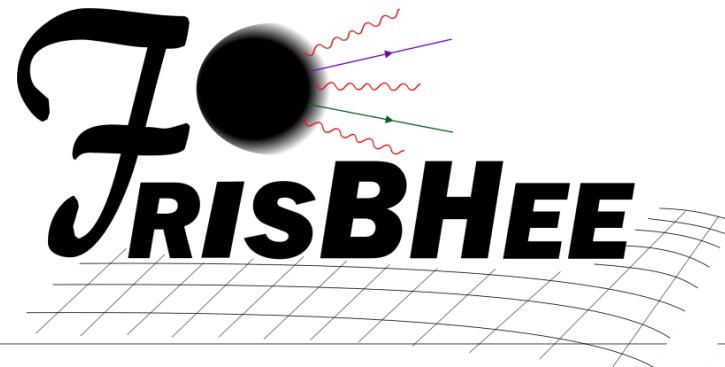
$$\frac{da_*}{dt} = -a_* [\gamma(M_{\text{BH}}, a_*) - 2\epsilon(M_{\text{BH}}, a_*)] \frac{M_p^4}{M_{\text{BH}}^3}$$

$$\frac{3H^2 M_p^2}{8\pi} = \rho_{\text{R}}^{\text{SM}} + \rho_{\text{DR}} + \rho_{\text{PBH}}$$

$$\dot{\rho}_{\text{PBH}} + 3H\rho_{\text{PBH}} = \frac{d \log M_{\text{BH}}}{dt} \rho_{\text{PBH}}$$

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$$\dot{\rho}_{\text{R}}^{\text{SM}} + 4H\rho_{\text{R}}^{\text{SM}} = - \left. \frac{d \log M_{\text{BH}}}{dt} \right|_{\text{SM}} \rho_{\text{PBH}}$$



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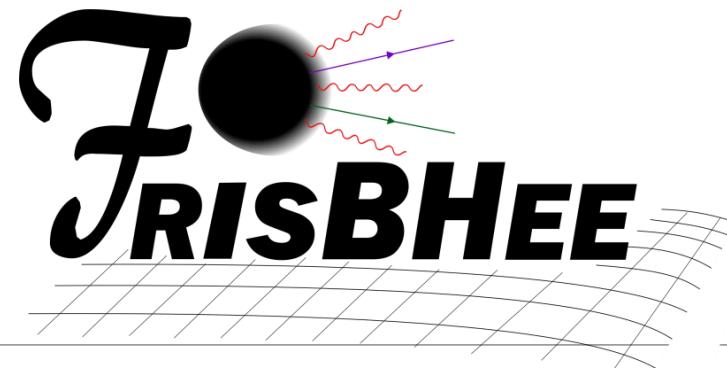
$$\frac{da_\star}{dt} = -a_\star [\gamma(M_{\text{BH}}, a_\star) - 2\epsilon(M_{\text{BH}}, a_\star)] \frac{M_p^4}{M_{\text{BH}}^3}$$

$$\frac{3H^2 M_p^2}{8\pi} = \rho_{\text{R}}^{\text{SM}} + \rho_{\text{DR}} + \rho_{\text{PBH}}$$

$$\dot{\rho}_{\text{DR}} + 4H\rho_{\text{DR}} = - \left. \frac{d \log M_{\text{BH}}}{dt} \right|_{\text{DR}} \rho_{\text{PBH}}$$

$$\dot{\rho}_{\text{PBH}} + 3H\rho_{\text{PBH}} = \frac{d \log M_{\text{BH}}}{dt} \rho_{\text{PBH}}$$

$$\dot{\rho}_{\text{R}}^{\text{SM}} + 4H\rho_{\text{R}}^{\text{SM}} = - \left. \frac{d \log M_{\text{BH}}}{dt} \right|_{\text{SM}} \rho_{\text{PBH}}$$



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Boltzmann

$$\dot{\rho}_{\text{R}}^{\text{SM}} + 4H\rho_{\text{R}}^{\text{SM}} = - \left. \frac{d \log M_{\text{BH}}}{dt} \right|_{\text{SM}} \rho_{\text{PBH}}$$

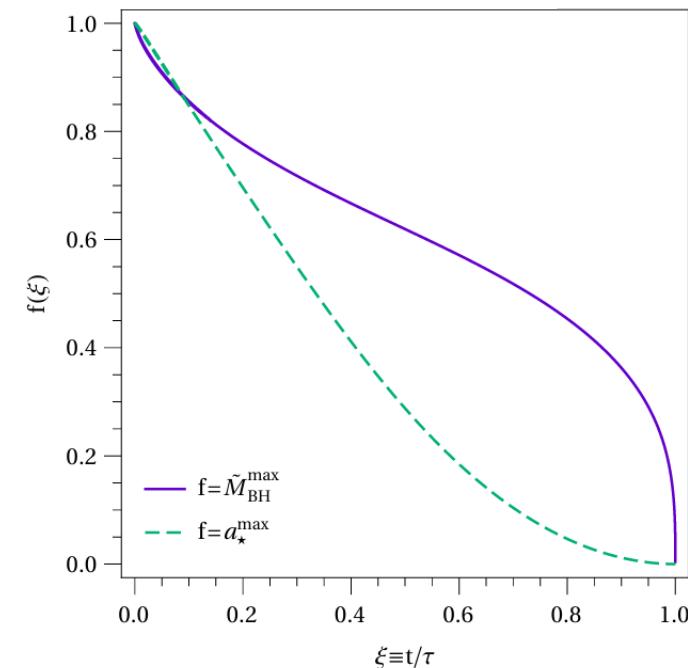
Successes of FRISBHEE

- Tracking the spin evolution proved to be an important **use case** for FRISBHEE.

$$\frac{dM_{\text{BH}}}{dt} = -\epsilon(M_{\text{BH}}, a_{\star}) \frac{M_p^4}{M_{\text{BH}}^2}$$

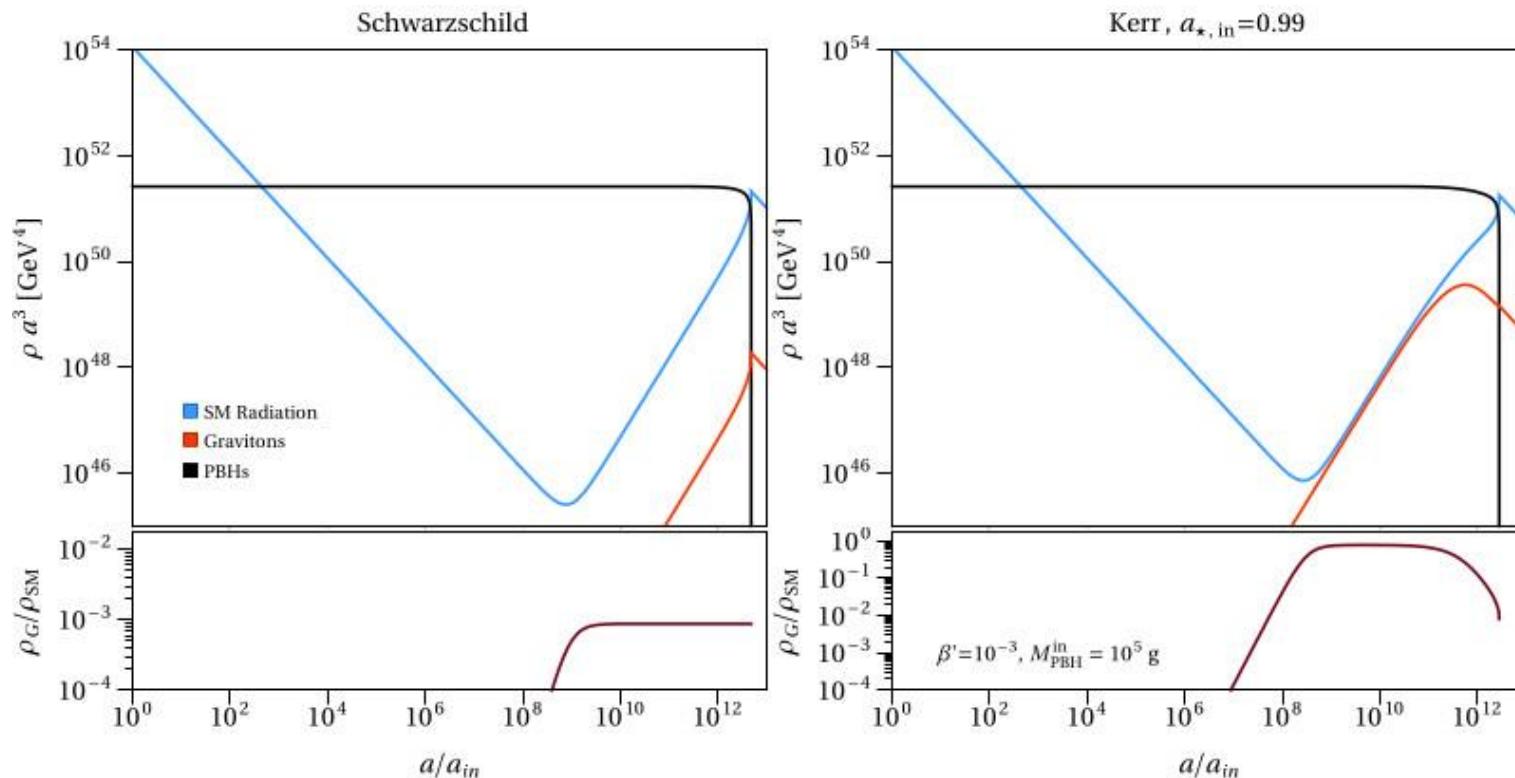
$$\frac{da_{\star}}{dt} = -a_{\star} [\gamma(M_{\text{BH}}, a_{\star}) - 2\epsilon(M_{\text{BH}}, a_{\star})] \frac{M_p^4}{M_{\text{BH}}^3}$$

- Been known for half a century that Kerr black holes shed angular momentum more quickly than their mass. [\[D. Page 1976\]](#)
- For maximally spinning black holes, only around 40% of mass has been lost when 90% of the spin has gone.



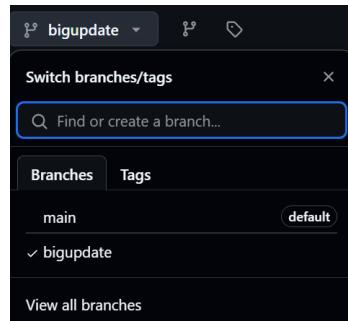
Successes of FRISBHEE

This results in an entropy injection after $a_* \sim 0$ diluting high spin radiation



Using FRISBHEE (dark radiation)

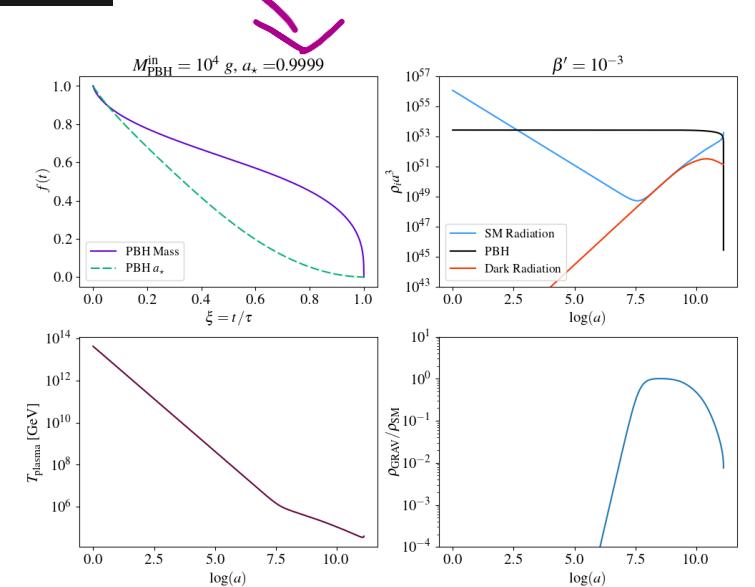
- How do you generate these plots yourself?



```
'Tools/frisbhee$ python -m dneff.ex_mono.py'
```

```
(base) andrew@Andrew:~/Documents/Tools/frisbhee$ python -m dneff.ex_mono.py
Warning : Page time passed, proceed with caution
Page time = 3.784958E-01 * t_ev, PBH mass at Page time = 6.783476E-01 * Min
M_PBH = 10^4 g, a_*=0.9999: DNeff = 0.02170739906303125
```

```
dneff/ex_mono.py
43  #-----#
44  # Main Parameters #
45  #-----#
46
47  Mi = 4    # Log10@ Initial BH mass in g
48  asi = 0.9999 # Initial a* value, a* = 0. -> Schwarzschild
49  bi = -3   # Log10@beta^prime
50  sDR = 2.0 # Spin of Dark Radiation
51
52  Dic_sDR = {0.:'scl', 0.5:'fer', 1.:'vec', 2.:'gra'}
```



Results from PBH domination

Previous studies overestimated the graviton contribution to the relativistic degrees of freedom at recombination.

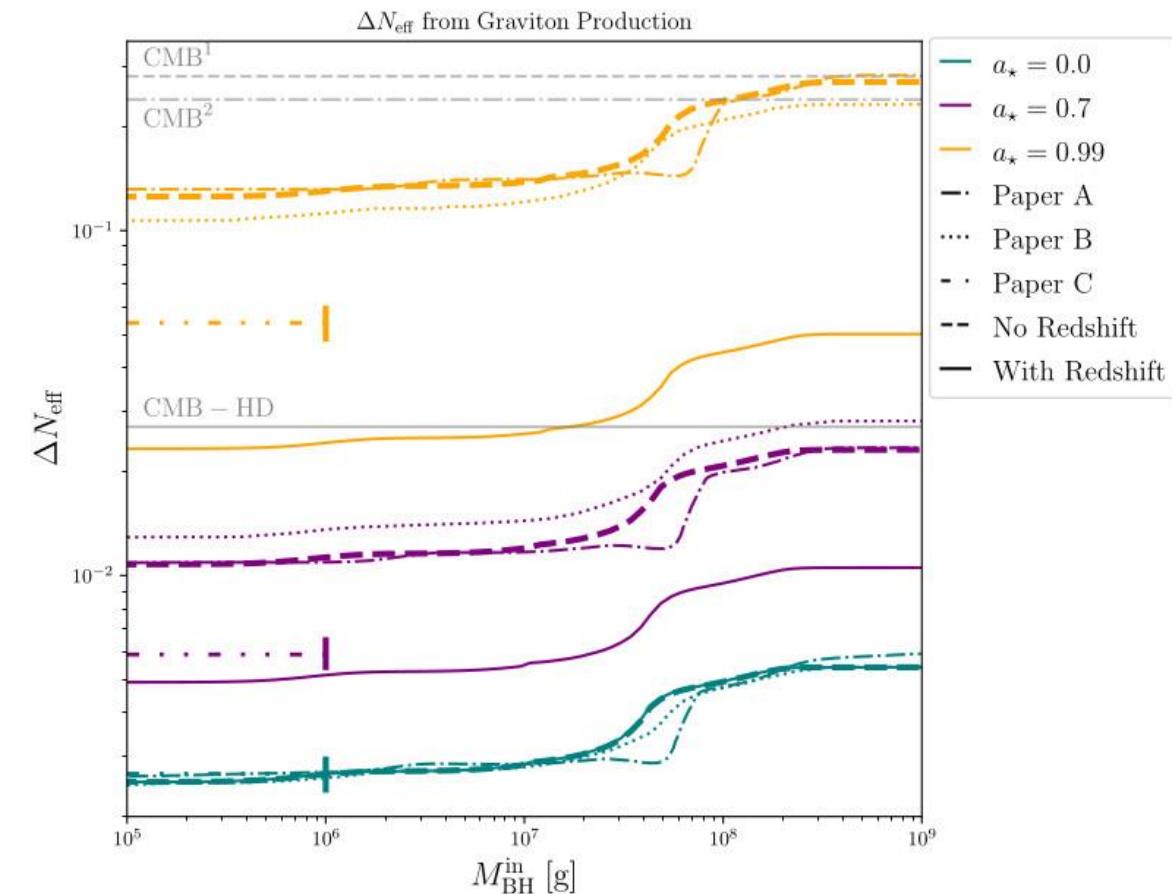
Paper A = Hooper et. al. [arXiv:2004.00618](https://arxiv.org/abs/2004.00618)

Paper B = Arbey et. al. [arXiv:2104.04051](https://arxiv.org/abs/2104.04051)

Paper C = Masina [arXiv:2103.13825](https://arxiv.org/abs/2103.13825)

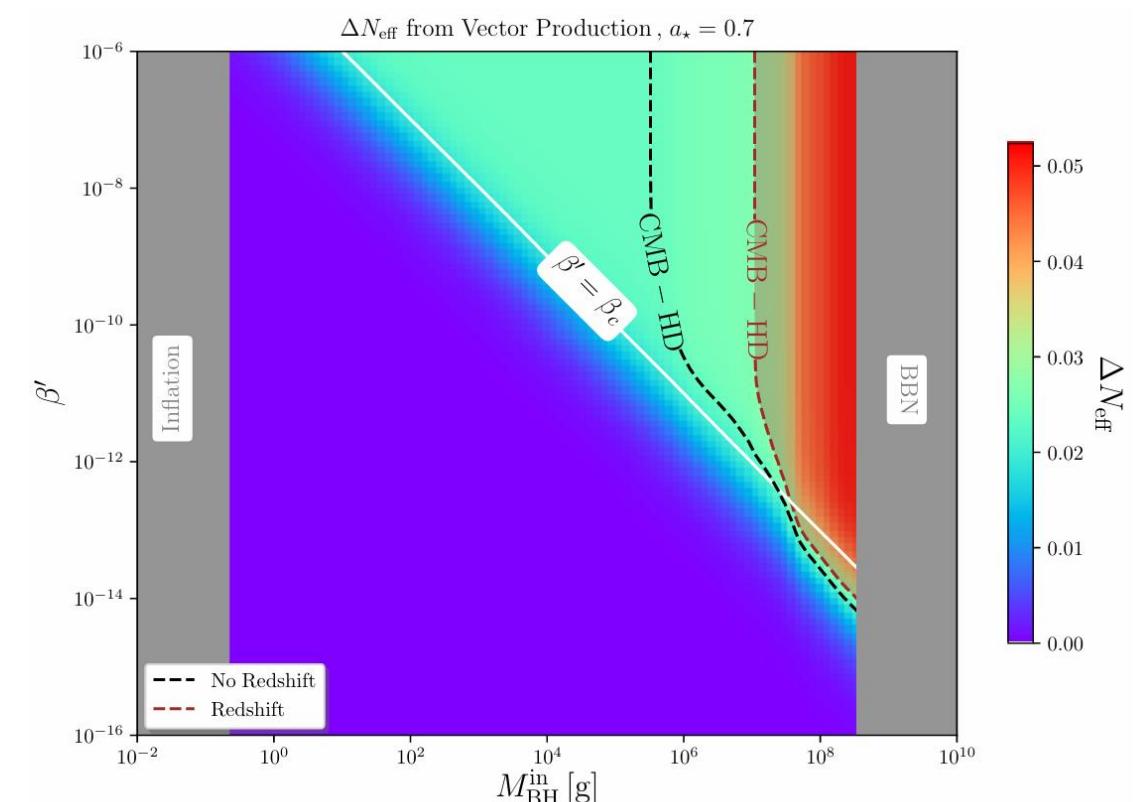
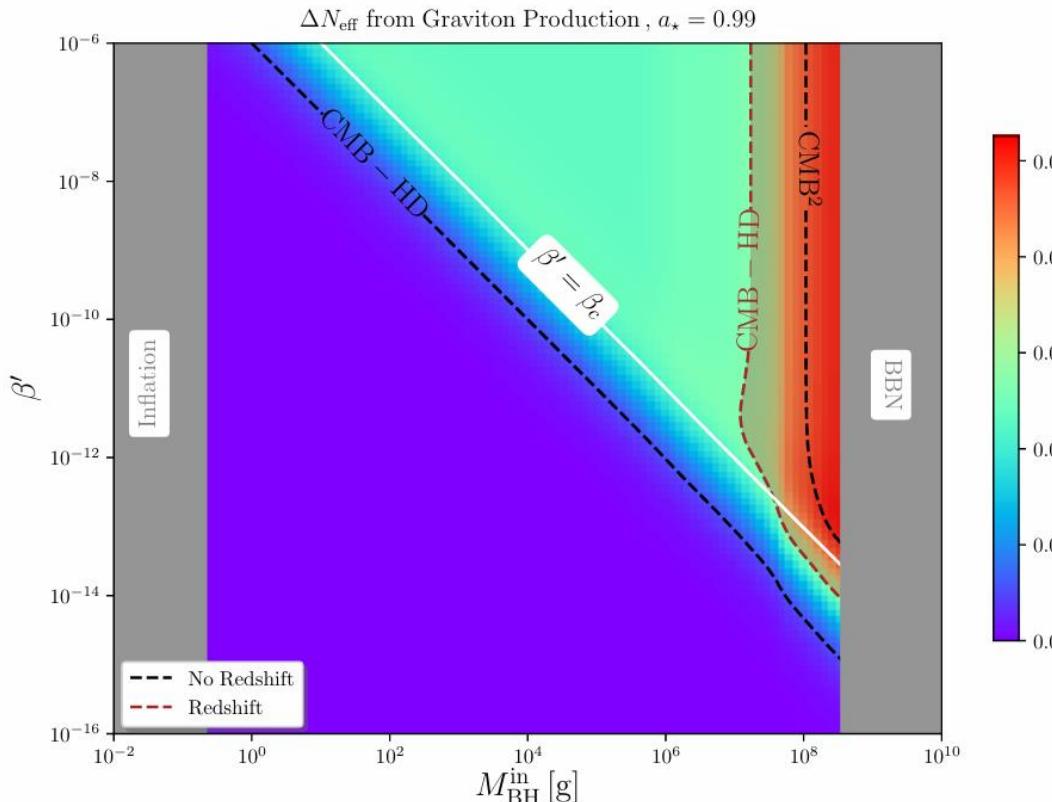
The prospects for future CMB probes become less optimistic.

$$\Delta N_{\text{eff}} \equiv \left\{ \frac{8}{7} \left(\frac{4}{11} \right)^{-\frac{4}{3}} + N_{\text{eff}}^{\text{SM}} \right\} \frac{\rho_{\text{DR}}}{\rho_{\text{R}}^{\text{SM}}}$$



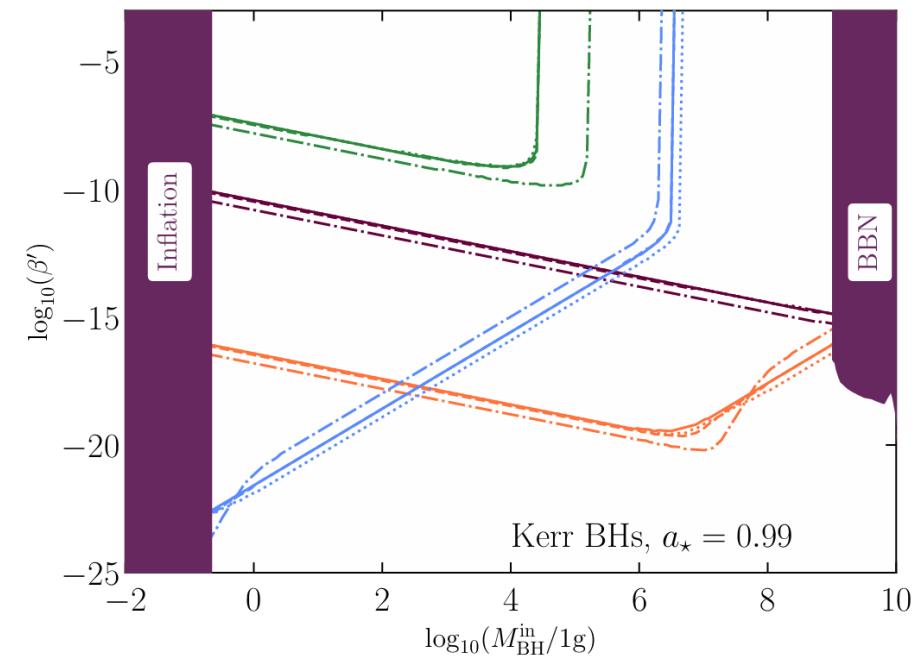
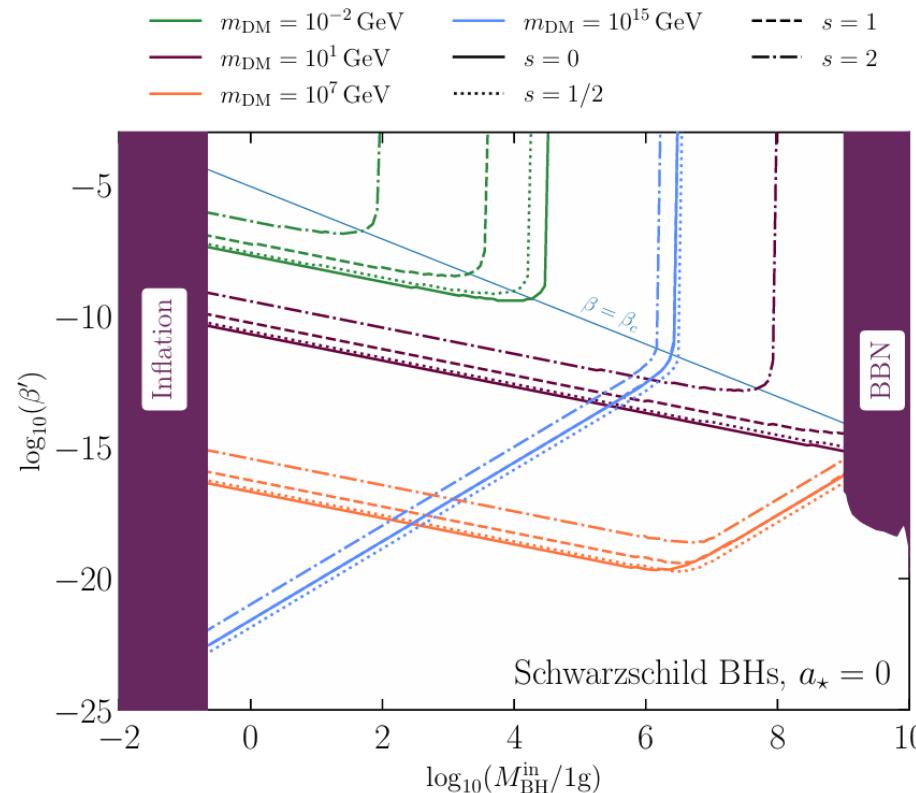
Scan results for graviton and vector

- With FRISBHEE we can perform full scans even when black holes don't dominate.



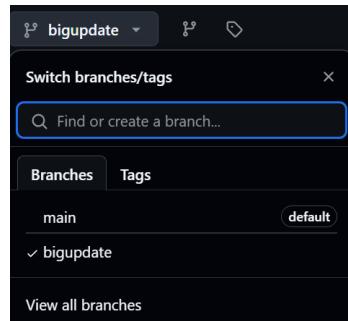
Dark matter relic abundance

- Making precise calculation of Ω_{DM} for different particles and BH spins



Using FRISBHEE (dark matter)

- How to get the dark matter abundance yourself?

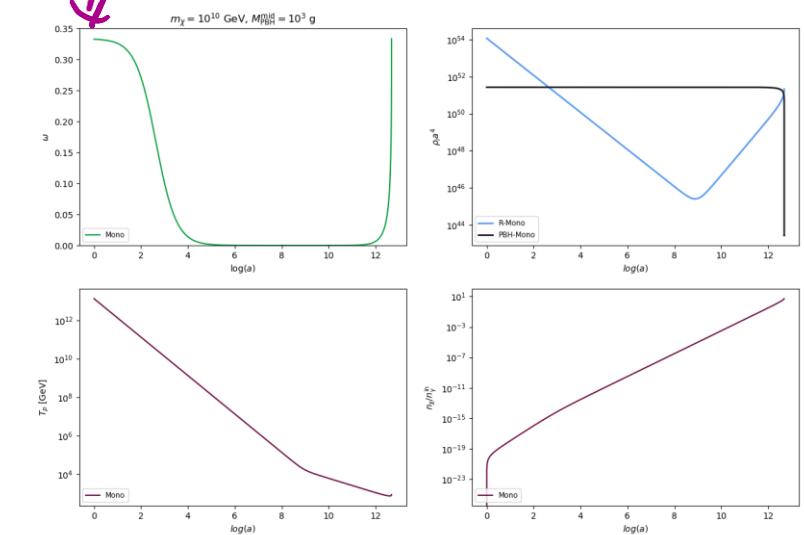


/frisbhee\$ python -m dm.ex_mono.py



```
Warning : Page time passed, proceed with caution
Page time = 5.103435E-01 * t_ev, PBH mass at Page time = 7.881932E-01 * Min
Oh^2 = 2.579915E+00
```

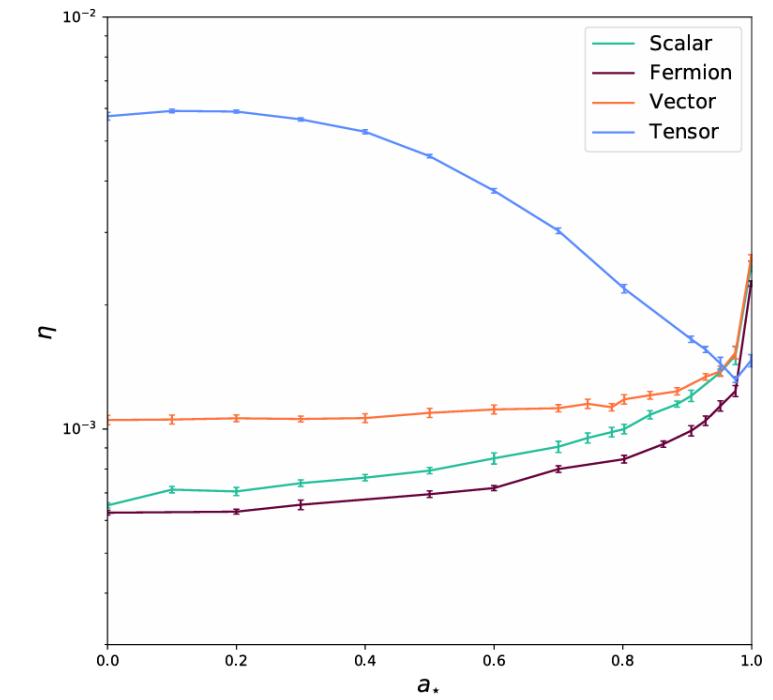
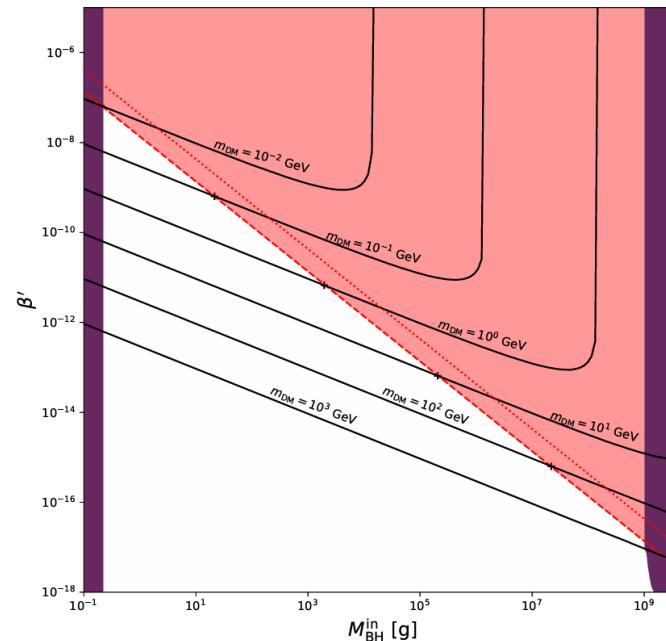
```
37 #-----#
38 #          Main Parameters          #
39 #-----#
40
41 Mi   = 5. # Peak mass in g at formation --> Tak
42 asi  = 0. # PBH initial rotation a_star factor
43 bi   = -3. # Initial PBH fraction
44 mDM = 1. # Log10 @ Dark Matter mass
45 sDM  = 2. # Dark Mater spin
```



Warm dark matter constraints

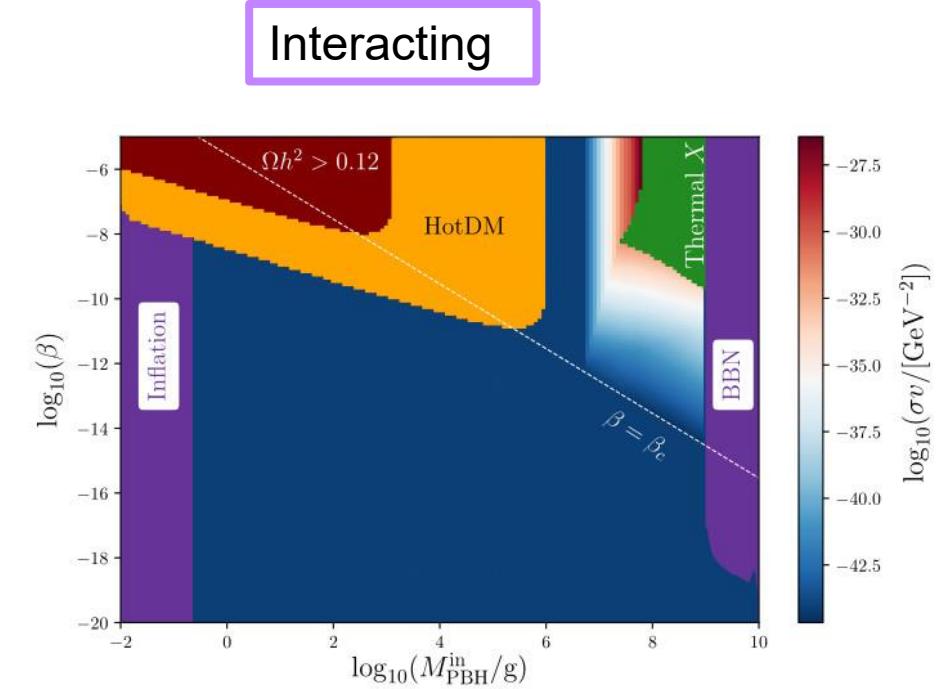
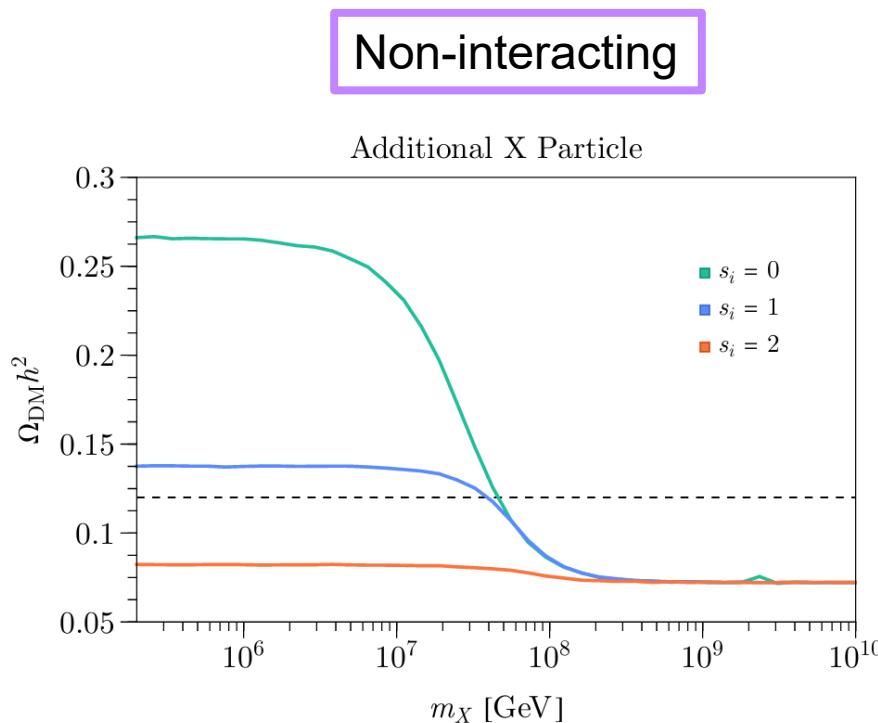
- We can use FRISBHEE to generate the output for structure formation codes like CLASS
- The main constraint we use is from Lyman- α forest
- Define constraint parameter η

$$\beta' \leq \eta \left(\frac{M_p}{M_{\text{BH}}^{\text{in}}} \right)$$



Dark sectors

- We have created the framework that allows for more complicated dark sectors



Interacting DM

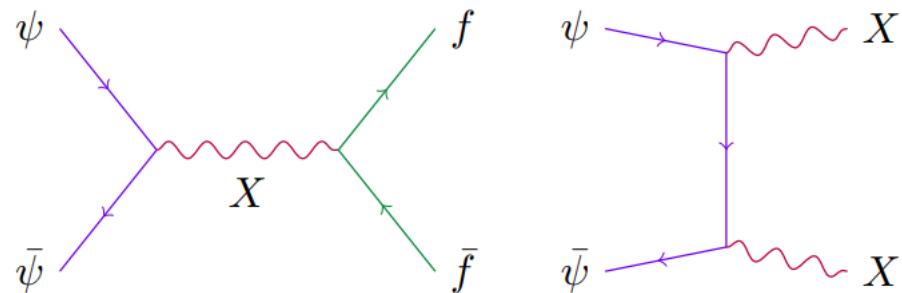
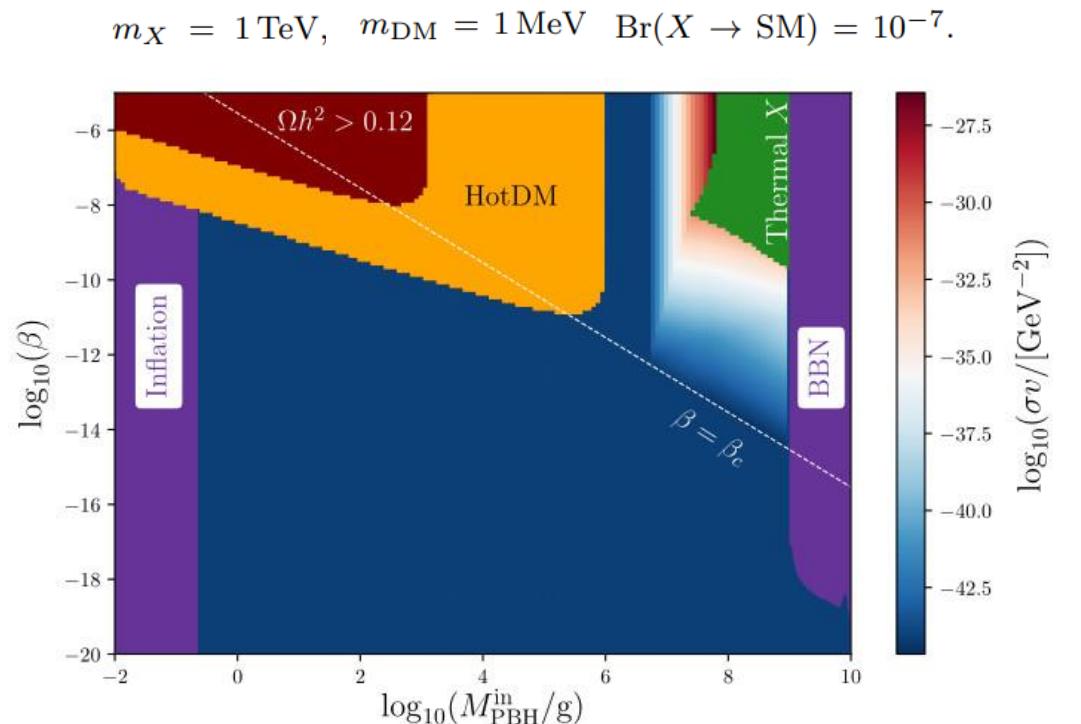


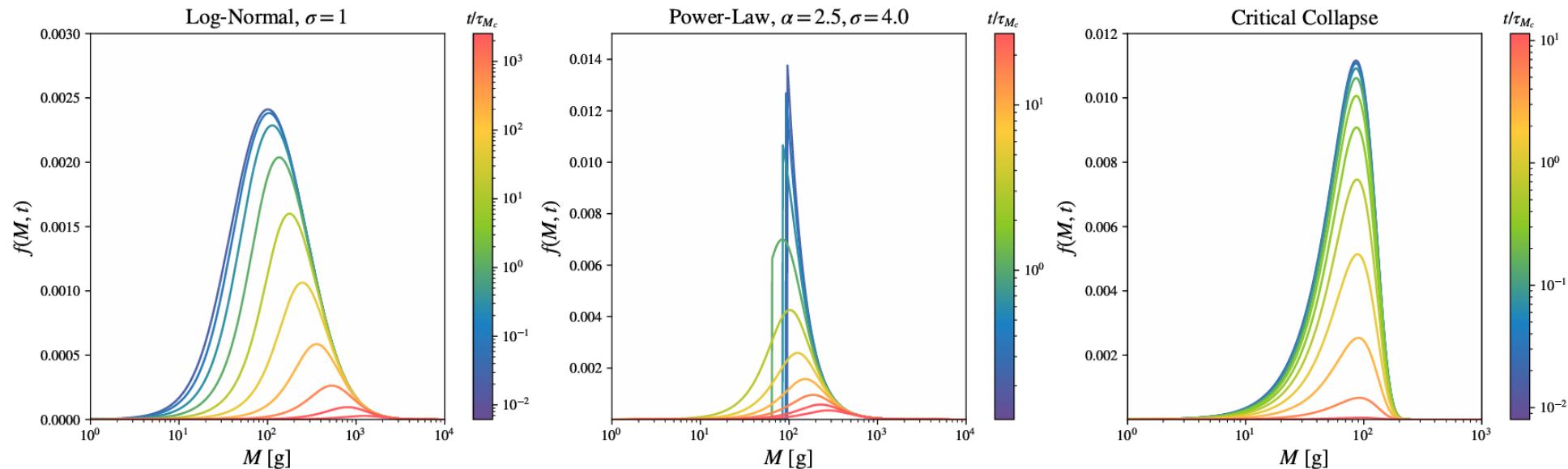
Fig. 1. Example of processes leading the production of thermalized DM particles.

$$\begin{aligned} \mathcal{L} = & \mathcal{L}_{\text{SM}} + \bar{\psi}(i\cancel{\partial} - m_{\text{DM}})\psi + \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \frac{1}{2}M_X^2X_\mu X^\mu \\ & - g_D X_\mu \bar{\psi} \gamma^\mu \psi - g_V X_\mu \bar{f} \gamma^\mu f, \end{aligned} \quad (1)$$



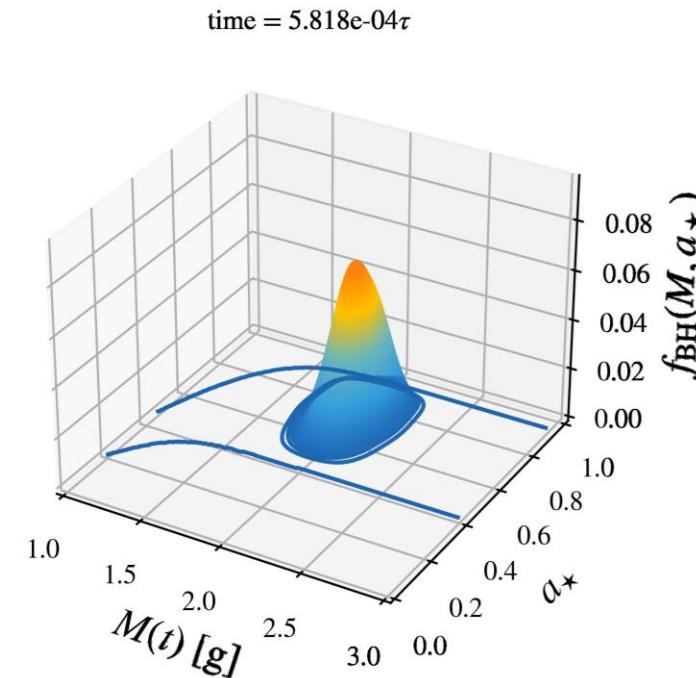
PBH distributions

- All results above have shown monochromatic distributions in mass and spin.
- Many production mechanisms lead to broader distributions.
- FRISBHEE can deal with distributions



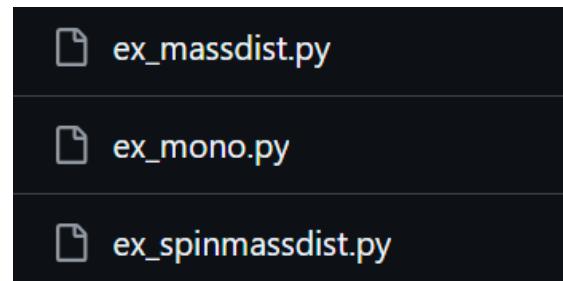
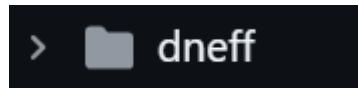
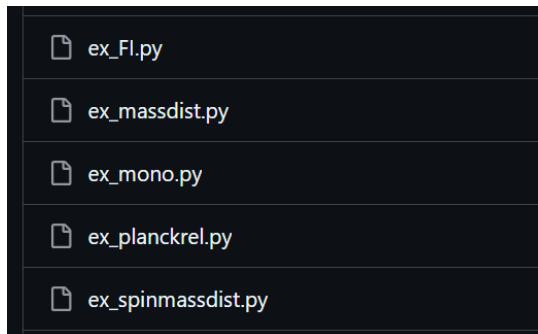
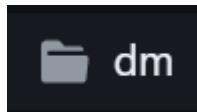
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FRISBHEE now

- There is an updated branch of the code with many examples, what I've presented here and others!
- The “big update” branch will soon become “main”



- We are working on other more updates on the physics side.
- Implementing evaporation variation at late stages of evaporation.

Summary

- FRISBHEE is code that solves the coupled equations for evaporating black holes in the early universe. It is public and available for all.
- We show that the accurate tracking of the coupled Friedmann-Boltzmann equations is vital.
- CMB probes of the additional relativistic degrees of freedom are less constraining than we previously thought.
- Dark matter will be produced by PBH evaporation so there will be an interplay with interacting models.
- Our code can handle distributions of black holes.

