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DI PADOVA



Dipartimento
di Fisica
e Astronomia
Galileo Galilei



Unveiling Imprints from Dark Symmetries:

Signatures of Axion Portal to Scalar Dark Matter

Tommaso Sassi

June 19 2025

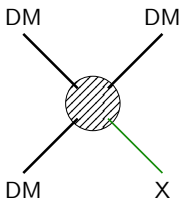


Breakthrough beyond the state of the art (in a nutshell)



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In literature



Fermionic DM

[Y. Cai and A. P. Spray: 1509.08481]

[F. D'Eramo and J. Thaler: 1003.5912]

Vector DM

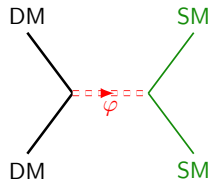
[F. D'Eramo *et al.*: 1210.7817]

[T. Hambye, M.H.G. Tytgat: 0907.1007]

[C.Arina *et al.*: 0912.4496v2]

Scalar DM

[G. Belanger *et al.*: 1202.2962]



Fermionic DM

[S. Gola *et al.*: 2106.00547 (N_R)]

[L. Fitzpatrick *et al.*: 2306.03128 (gluon)]

[G. Armando *et al.*: 2310.05827 (leptophilic)]

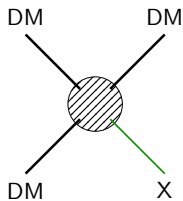
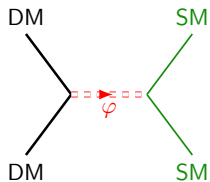
Vector DM

[K. Kaneta *et al.*: 1611.01466 (A'_μ)]

Breakthrough beyond the state of the art (in a nutshell)

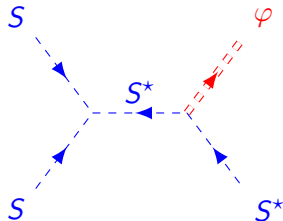


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NEW!: SCALAR DM

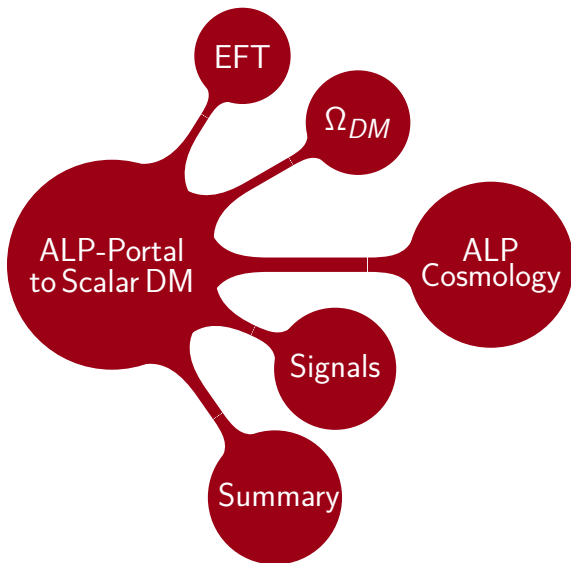
[F.D'Eramo, *TS*: arXiv:2502.19491]

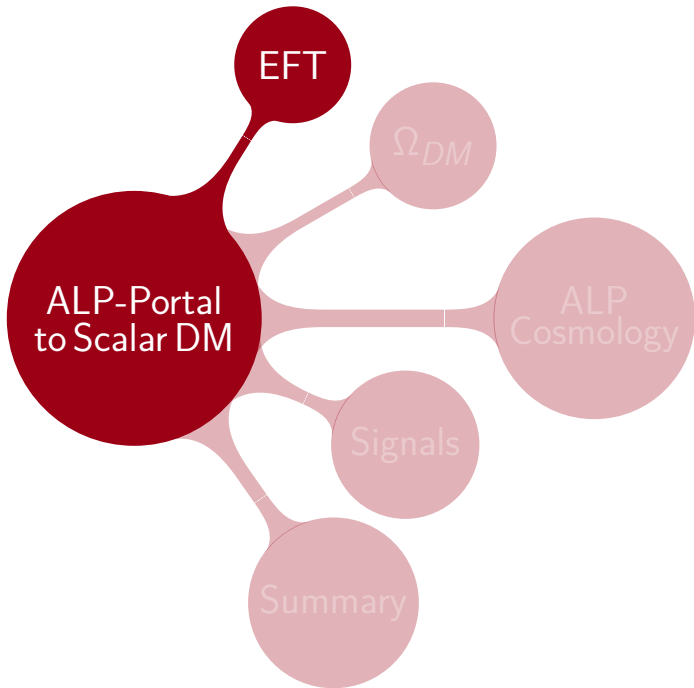


$\mathbb{Z}_3$ symmetry

Physical $c_S \frac{\partial_\mu \varphi}{2f_\varphi} S^\dagger i \overleftrightarrow{\partial}^\mu S$?

ALP-SM couplings (g, γ, ψ)





We start from the dimension 5 EFT ($\text{SM} + S, \varphi$) in the derivative basis ($\varphi \rightarrow \varphi + \text{const}$)

$V_{\text{mix}}(S, H)$ negligible [G.Arcadi *et al.*: 2403.15860]...

We start from the dimension 5 EFT (SM + S , φ) in the derivative basis ($\varphi \rightarrow \varphi + \text{const}$)

$V_{\text{mix}}(S, H)$ negligible [G.Arcadi *et al.*: 2403.15860]...

$$\mathcal{L}_{S\varphi}^{(\partial)} = c_S \frac{\partial_\mu \varphi}{2f_\varphi} S^\dagger i \overleftrightarrow{\partial}^\mu S + \mathcal{O}\left(\frac{1}{f_\varphi^2}\right),$$

$$\begin{aligned} \mathcal{L}_{\varphi SM}^{(\partial)} = & \frac{\varphi}{8\pi f_\varphi} c_V \alpha_V V_{\mu\nu} \widetilde{V}^{\mu\nu} + \frac{\partial_\mu \varphi}{2f_\varphi} \left(c_H H^\dagger i \overleftrightarrow{D}^\mu H \right) + \\ & + \frac{\partial_\mu \varphi}{2f_\varphi} \left(c_{\psi_L} \overline{\psi}_L \gamma^\mu \psi_L + \overline{\psi}_R \gamma^\mu \psi_R \right) + \mathcal{O}\left(\frac{1}{f_\varphi^2}\right). \end{aligned}$$

φ -dependent rotations remove derivative couplings

[Georgi, Kaplan, Randall: Phys. Lett. B, 169:73–78, 1986]

$$S \rightarrow \exp \left[i \mathcal{C}_S \frac{\varphi}{2f_\varphi} \right] S, \quad H \rightarrow \exp \left[i \mathcal{C}_H \frac{\varphi}{2f_\varphi} \right] H, \quad \psi \rightarrow \exp \left[i \mathcal{C}_\psi \frac{\varphi}{2f_\varphi} \right] \psi$$

and φ -SM interactions are redefined [M. Bauer *et al.*: 2012.12272]

$$\mathcal{L}_{\varphi SM}^{(N\partial)} = \frac{\varphi}{8\pi f_\varphi} \mathcal{C}'_V \alpha_V V_{\mu\nu} \tilde{V}^{\mu\nu} + \frac{\varphi}{2f_\varphi} \mathcal{C}'_\psi \overline{\psi}_L H \psi_R + \mathcal{O} \left(\frac{1}{f_\varphi^2} \right),$$

while $\mathcal{L}_{S\varphi} \dots$ it's gone ~~$\mathcal{L}_{S\varphi}$~~ !

But wait, where did the ALP-DM interaction go!?

The **stabilizing symmetry** acting on the DM is *crucial!* $\mathbb{Z}_3$

$$V_S^{(\partial)}(S) = m_S^2 S^\dagger S + \frac{1}{3!}(A S^3 + A^* S^{\dagger 3}) + \frac{\lambda_S}{4}(S^\dagger S)^2$$

The **stabilizing symmetry** acting on the DM is *crucial!* $\mathbb{Z}_3$

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$$A \leq 3\sqrt{\lambda_S m_S} \quad [\text{G. Belanger et al.: 1211.1014}]$$

$S \rightarrow \exp\left[i C_S \frac{\varphi}{2f_\varphi}\right] S$

$$V_S^{(N\partial)}(S) = V_S^{(\partial)}\left(e^{i C_S \frac{\varphi}{2f_\varphi}} S\right)$$

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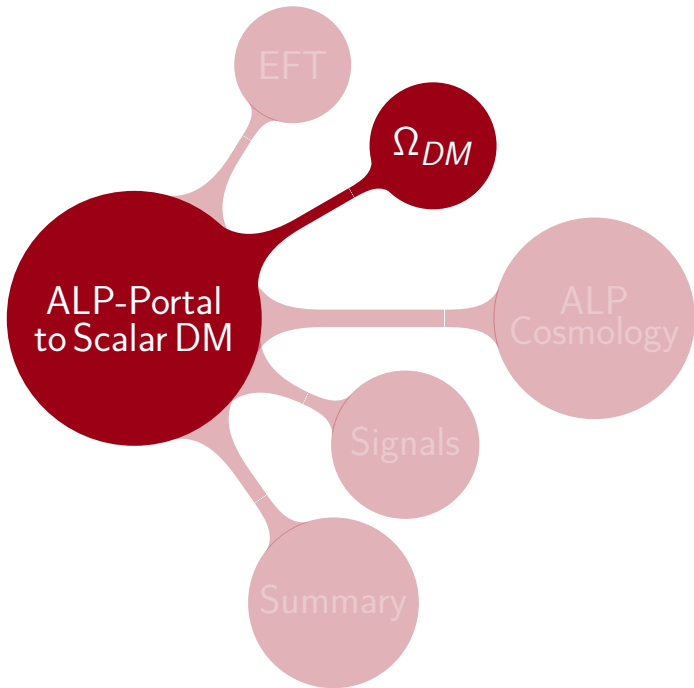
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$$V_S^{(N\partial)}(S) = V_S^{(\partial)}\left(e^{i C_S \frac{\varphi}{2 f_\varphi}} S\right)$$

$$\lambda_{S\varphi} \equiv i \frac{3 C_S A}{2 f_\varphi}$$

$$V_S^{(N\partial)}(S) \supset \frac{1}{3!} \left(\lambda_{S\varphi} S^3 + \lambda_{S\varphi}^* S^{\dagger 3} \right) \varphi$$

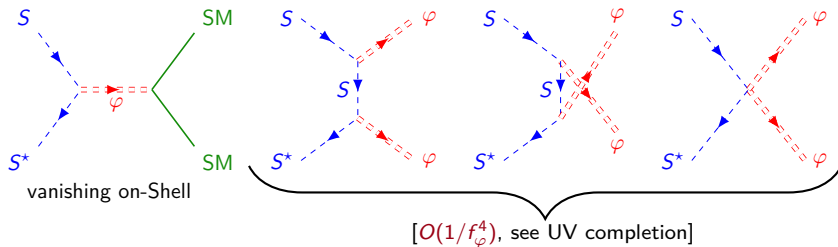


Dark Matter Freeze-Out

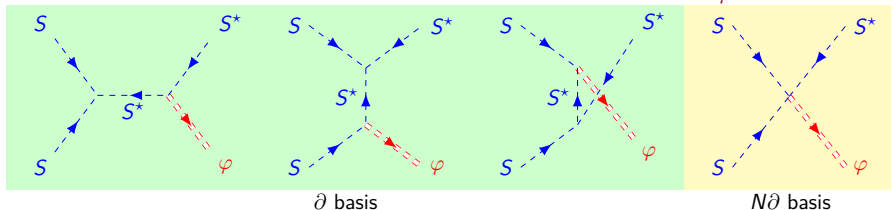


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Standard DM annihilations are absent...



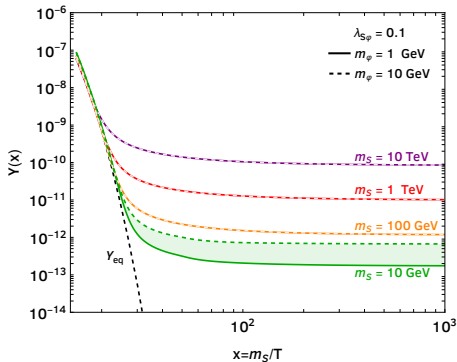
Only *semi-annihilations* are active at tree-level at $O(1/f_\phi^2)$!



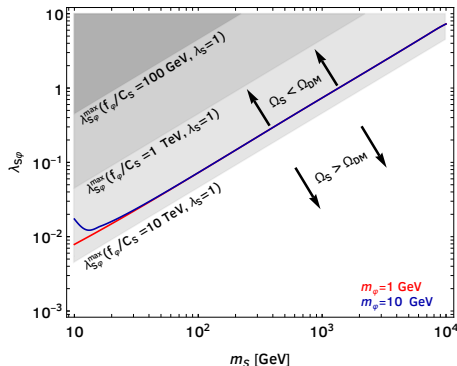
Relic density I: $N\partial$ Basis



$m_S \in [10, 10^4] \text{ GeV}$

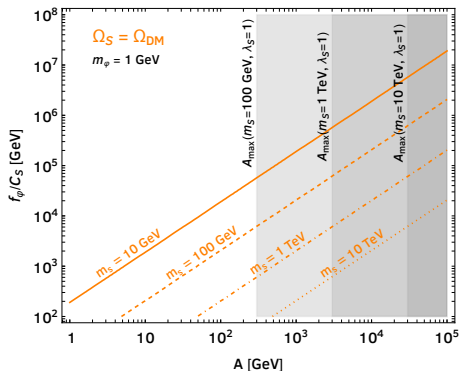
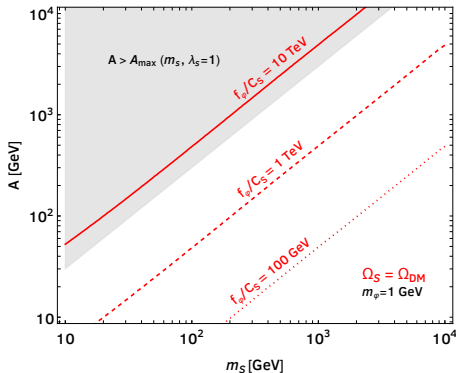


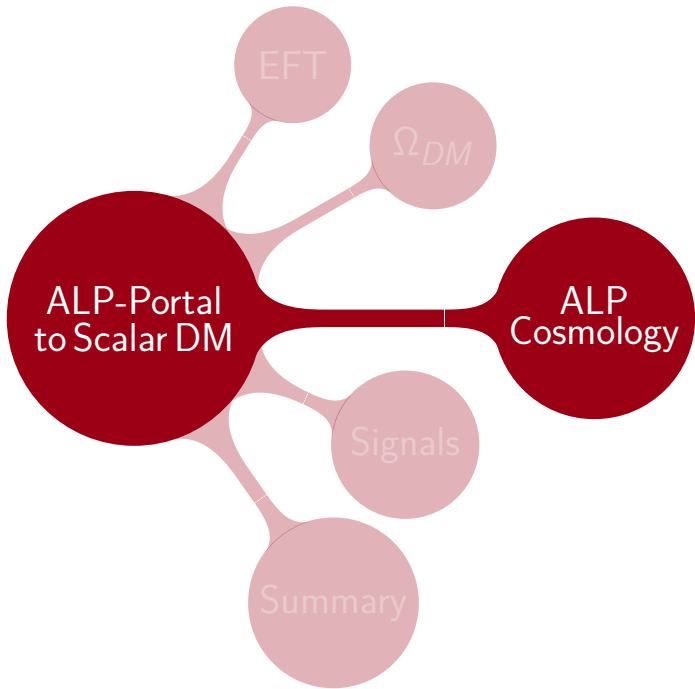
$m_\phi \in [1, 10] \text{ GeV}$



Unravel the three-dimensional parameters space (m_S , A , f_φ)

$$\lambda_{S\varphi} \equiv i \frac{3 C_S A}{2 f_\varphi} , \quad A \leq 3\sqrt{\lambda_S} m_S$$





- 1 Short lifetime $\tau_\varphi \ll 1 \text{ sec}$

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- Escape BBN constraints [M. Kawasaki *et al.*: 1709.01211]
- Escape CMB constraints [C. Balázs *et al.*: 2205.13549]
- $\Omega_\varphi = 0$ after BBN

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2 Thermal equilibrium at DM Freeze-Out

1 Short lifetime $\tau_\varphi \ll 1 \text{ sec}$

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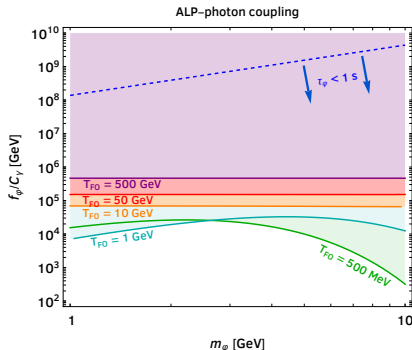
2 Thermal equilibrium at DM Freeze-Out

- $T_{DS} = T_\gamma$
- Sizeable ALP-SM couplings
- $\mathcal{C}_i$ hierarchy?
- $\left. \frac{\gamma_\alpha(T)}{H(T)s(T)} \right|_{T=T_{FO}} \gtrsim 1$

ALP coupling to SM gauge bosons

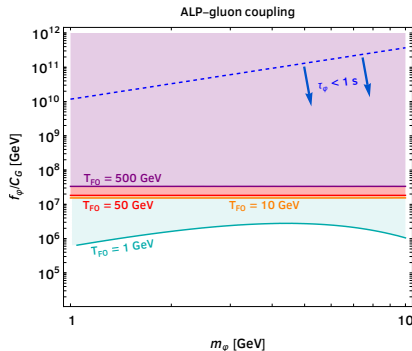


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No $\frac{C_\gamma}{C_S}$ hierarchy needed

[D. Cadamuro, J. Redondo: 1110.2895]



No $\frac{C_G}{C_S}$ hierarchy needed

[F. D'Eramo, F. Hajkarim, S. Yun:
2108.05371]

[K. Bouzoud, J. Ghiglieri: 2404.06113]

ALP coupling to fermions



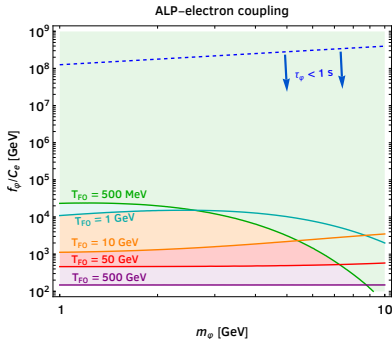
[A. Czarnecki *et al.*: 1110.2171]

[M. Bauer *et al.*: 1708.00443]

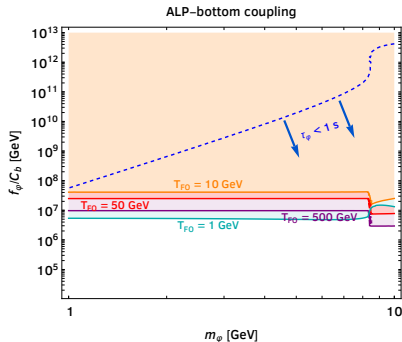
[J. L. Feng *et al.*: 9709411]

[A. Crivellin *et al.*: 1402.1173]

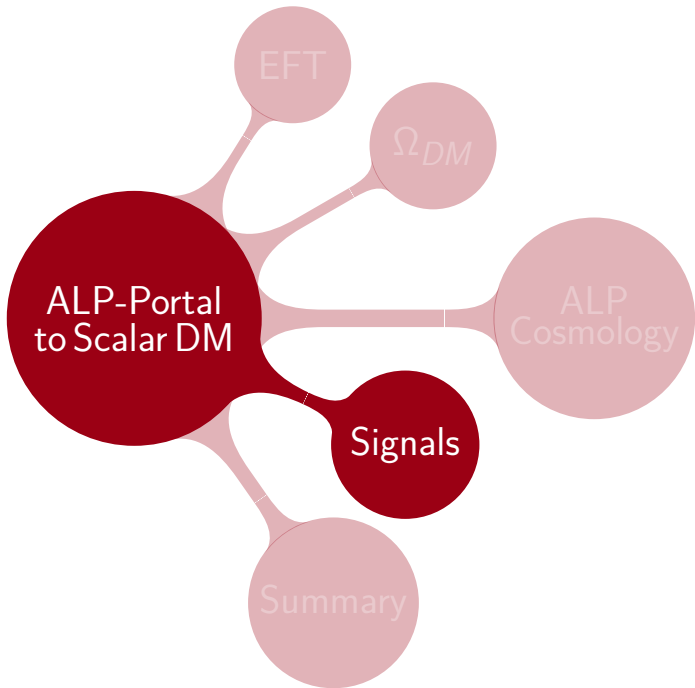
*Warning: QCD and threshold
effects in action*



Need for small $\frac{C_e}{C_S}$ hierarchy



No need for $\frac{C_b}{C_S}$ hierarchy

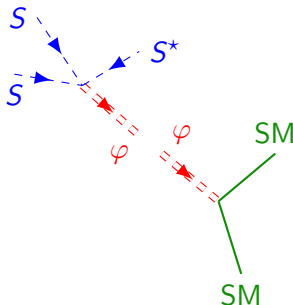


What to Expect



	Freeze-Out	Indirect	Direct	Collider
Fermion ($\mathbb{Z}_2$)		[M. Cirelli <i>et al.</i>: 2406.01705, T.R. Slatyer: 1710.05137]	[E. Del Nobile <i>et al.</i>: 1307.5955]	
Scalar ($\mathbb{Z}_3$)		followed by	[Berger <i>et al.</i>: 1912.05558]	[Toma: 2109.05911]

One-Step Cascade Semi-Annihilation



$$\frac{dN_X}{dx_S} = \frac{4}{3\zeta} \int_{x_\varphi^{\min}}^{x_\varphi^{\max}} \frac{dN_X}{dx_\varphi} \frac{dx_\varphi}{\sqrt{x_\varphi^2 - \epsilon_X^2}},$$
$$\zeta = \frac{4}{3} \frac{p_\varphi^{(GF)}}{m_S} \quad x_\varphi = \frac{E_X}{m_S}$$

Key features

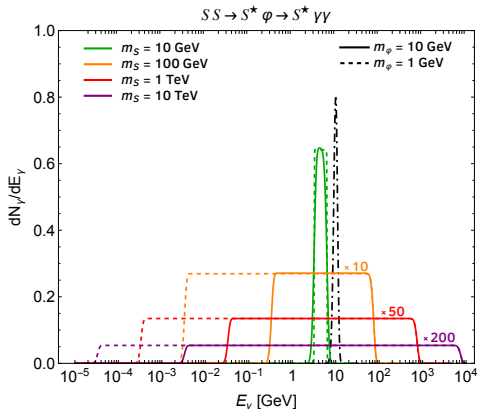
- $\varphi \rightarrow \text{SM} \overline{\text{SM}}$ shapes the spectrum
- Softer and broadened spectra for $\epsilon_\varphi \ll 1$
- Channel closes for $\epsilon_X \rightarrow 1$

Indirect Detection I: Box-Shaped Gamma Ray Spectrum



$$\frac{dN_\gamma}{dE_\gamma} = \frac{N_\gamma}{p_\phi} \Theta(E_\gamma - E_\gamma^-) \Theta(E_\gamma^+ - E_\gamma), \quad E_\gamma^\pm = (1 \pm \beta_L) E_\phi / 2$$

[A. Ibarra, S. L. Gehler, and M. Pato: 1205.0007]



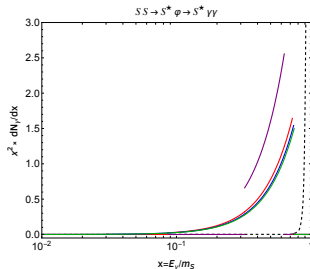
$$\epsilon_\phi \rightarrow 1$$

$\approx$ Direct decay
Dirac delta-like signal

$$\epsilon_\phi \ll 1$$

- Plateau
- Suppression $\propto m_S$

Indirect Detection II: γ -Ray Spectra from Semi-Annihilation



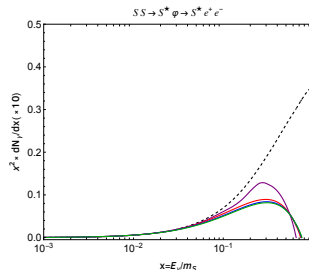
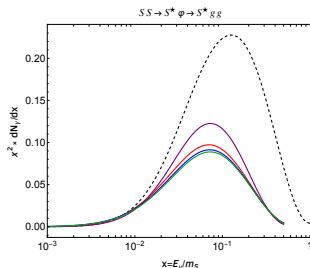
... $SS^* \rightarrow SM\overline{SM}$ via other portal

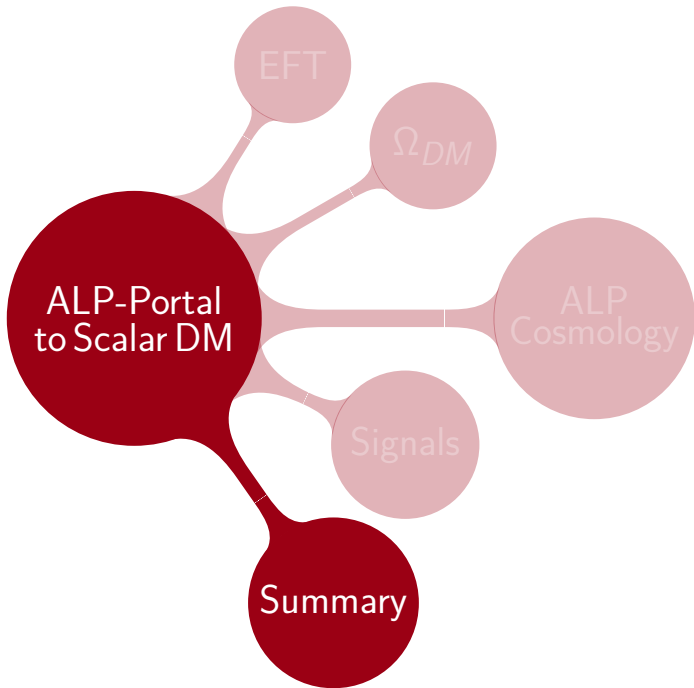
— $\epsilon_\varphi = 0.9$

— $\epsilon_\varphi = 0.5$

— $\epsilon_\varphi = 0.3$

— $\epsilon_\varphi = 0.1$

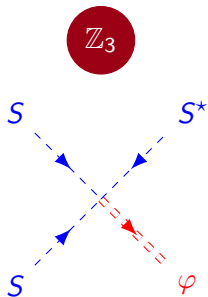




NEW! ALP-portal to *scalar* DM

[F.D'Eramo, TS: arXiv:2502.19491]

Governed by *semi-annihilations*



Imprints of symmetries on
phenomenology

- DM relic independent of $\mathcal{C}_{\varphi-SM}$
- Accessible parameters space
- Peculiar phenomenology
(suppressed DD, interesting ID)

Thank you!



SPARES

$$\mathcal{L}_{\varphi SM}^{(\partial)} = \frac{\varphi}{8\pi f_{\varphi}} \sum_V c_V \alpha_V V_{\mu\nu} \tilde{V}^{\mu\nu} + \frac{\partial_{\mu}\varphi}{2f_{\varphi}} \left(\mathcal{H}_{\varphi}^{\mu} + \mathcal{F}_{\varphi}^{\mu} \right) + \mathcal{O} \left(\frac{1}{f_{\varphi}^2} \right) ,$$

$$V = G, W, B ,$$

$$\mathcal{H}_{\varphi}^{\mu} = H^{\dagger} \overleftrightarrow{D}^{\mu} H ,$$

$$\begin{aligned} \mathcal{F}_{\varphi}^{\mu} = \sum_{j=1}^3 & \left[c_Q^j \bar{Q}_L^j \gamma^{\mu} Q_L^j + c_u^j \bar{u}_R^j \gamma^{\mu} u_R^j + c_d^j \bar{d}_R^j \gamma^{\mu} d_R^j + \right. \\ & \left. + c_E^j \bar{E}_L^j \gamma^{\mu} E_L^j + c_e^j \bar{e}_R^j \gamma^{\mu} e_R^j \right] \end{aligned}$$

$$\mathcal{L}_{\varphi SM}^{(N\partial)} = \frac{\varphi}{2f_{\varphi}} \mathcal{P}_{\varphi} + \frac{\varphi}{8\pi f_{\varphi}} \sum_V C'_V \alpha_V V_{\mu\nu} \tilde{V}^{\mu\nu} + \mathcal{O}\left(\frac{1}{f_{\varphi}^2}\right),$$

$$\begin{aligned} \mathcal{P}_{\varphi} = -i \sum_{j=1}^3 & \left[(C_u^j - C_Q^j - C_H) y_u^j \bar{Q}_L^j \tilde{H} u_R^j + \right. \\ & (C_d^j - C_Q^j + C_H) y_d^j \bar{Q}_L^j H d_R^j + \\ & \left. + (C_e^j - C_E^j + C_H) y_e^j \bar{E}_L^j H e_R^j \right] + \text{h.c.} . \end{aligned}$$

$$c'_G = c_G - \sum_{j=1}^3 \left(c_Q^j - \frac{1}{2} c_u^j - \frac{1}{2} c_d^j \right) ,$$

$$c'_W = c_W - \sum_{j=1}^3 \left(\frac{3}{2} c_Q^j + \frac{1}{2} c_E^j \right) ,$$

$$c'_B = c_B - \sum_{j=1}^3 \left(\frac{1}{6} c_Q^j + \frac{1}{2} c_E^j - \frac{4}{3} c_u^j - \frac{1}{3} c_d^j - c_e^j \right) .$$

Counting Wilson Coefficients



We count the number of independent $\mathcal{C}$ to be:

$$\mathcal{C}_V \times 3 \quad V = G, W, B \quad +$$

$$\mathcal{C}_S \quad +$$

$$\{\mathcal{C}_u, \mathcal{C}_c, \mathcal{C}_t\} = \left\{ \frac{-\mathcal{C}_Q^1 + \mathcal{C}_u^1}{2}, \frac{-\mathcal{C}_Q^2 + \mathcal{C}_u^2}{2}, \frac{-\mathcal{C}_Q^3 + \mathcal{C}_u^3}{2} \right\} \quad +$$

$$\{\mathcal{C}_d, \mathcal{C}_s, \mathcal{C}_b\} = \left\{ \frac{-\mathcal{C}_Q^1 + \mathcal{C}_d^1}{2}, \frac{-\mathcal{C}_Q^2 + \mathcal{C}_d^2}{2}, \frac{-\mathcal{C}_Q^3 + \mathcal{C}_d^3}{2} \right\} \quad +$$

$$\{\mathcal{C}_e, \mathcal{C}_\mu, \mathcal{C}_\tau\} = \left\{ \frac{-\mathcal{C}_E^1 + \mathcal{C}_e^1}{2}, \frac{-\mathcal{C}_E^2 + \mathcal{C}_e^2}{2}, \frac{-\mathcal{C}_E^3 + \mathcal{C}_e^3}{2} \right\} \quad =$$

13

$$\frac{dY_S}{d \ln x} = - \left(1 - \frac{1}{3} \frac{d \ln g_{*s}}{d \ln x} \right) \frac{s(x) \langle \sigma_{SS \rightarrow S^* \varphi} v_{\text{Mø}} \rangle}{H(x)} \left[Y_S^2 - Y_S Y_S^{\text{eq}} \right]$$

$$\langle \sigma_{SS \rightarrow S^* \varphi} v_{\text{Mø}} \rangle = \frac{\int_{4m_S^2}^{\infty} ds \sqrt{s} (s - 4m_S^2) \sigma_{SS \rightarrow S^* \varphi}(s) K_1[\sqrt{s}/T]}{8m_S^4 T K_2[m_S/T]^2}$$

$$\simeq \frac{\lambda_{S\varphi}^2}{128\pi m_S^2} \sqrt{9 - 10 \left(\frac{m_\varphi}{m_S} \right)^2 + \left(\frac{m_\varphi}{m_S} \right)^4} + \mathcal{O} \left(\frac{T}{m_S} \right)$$

$$\frac{dY_\varphi}{d\ln x} = - \left(1 - \frac{1}{3} \frac{d\ln g_{*s}}{d\ln x} \right) \frac{\sum_\alpha \gamma_\alpha(x)}{H(x)s(x)} \left(1 - \frac{Y_\varphi}{Y_\varphi^{\text{eq}}} \right)$$

$$\gamma_{B_1 B_2 \rightarrow \varphi} = n_\varphi^{\text{eq}} \frac{K_1[m_\varphi/T]}{K_2[m_\varphi/T]} \Gamma_{\varphi \rightarrow B_1 B_2} \quad \text{Inverse Decays}$$

$$\begin{aligned} \gamma_{B_1 B_2 \rightarrow B_3 \varphi} &= n_{B_1}^{\text{eq}} n_{B_2}^{\text{eq}} \langle \sigma_{B_1 B_2 \rightarrow B_3 \varphi} v_{\text{Mø}} \rangle \\ &= n_{B_3}^{\text{eq}} n_\varphi^{\text{eq}} \langle \sigma_{B_3 \varphi \rightarrow B_1 B_2} v_{\text{Mø}} \rangle \end{aligned} \quad \text{Binary scatterings}$$

ALP coupled to SM gauge bosons: scattering rates



$$\gamma_{\text{scattering}}^{(\gamma)} = \frac{\alpha_{\text{em}}^3}{144\pi^2} \left(\frac{C_\gamma}{f_\varphi} \right)^2 T^3 n_Q(T) \left[\log \left(\frac{T^2}{m_\gamma^2(T)} \right) + 0.8194 \right]$$

$$\gamma_{\text{scattering}}^{(G)} = \frac{2\zeta(3)\mathcal{D}_G}{\pi^3} \left(\frac{C_G \alpha_s}{8\pi f_\varphi} \right)^2 T^6 F_G(T)$$

ALP coupled to SM gauge bosons: lifetime



$$\begin{aligned}\Gamma_{\varphi \rightarrow \gamma\gamma} &= \frac{C_\gamma^2 \alpha_{\text{em}}^2 m_\varphi^3}{256\pi^3 f_\varphi^2} \\ \downarrow \\ \tau_{\varphi \rightarrow \gamma\gamma} &= \Gamma_{\varphi \rightarrow \gamma\gamma}^{-1} \approx 0.5 \text{ s} \left(\frac{1}{C_\gamma} \right)^2 \left(\frac{0.01}{\alpha_{\text{em}}} \right)^2 \left(\frac{1 \text{ GeV}}{m_\varphi} \right)^3 \left(\frac{f_\varphi}{10^8 \text{ GeV}} \right)^2\end{aligned}$$

$$\begin{aligned}\Gamma_{\varphi \rightarrow gg} &= 8 \times \frac{C_G^2 \alpha_s^2 m_\varphi^3}{256\pi^3 f_\varphi^2} \\ \downarrow \\ \tau_{\varphi \rightarrow gg} &= \Gamma_{\varphi \rightarrow gg}^{-1} \approx 0.7 \text{ s} \left(\frac{1}{C_G} \right)^2 \left(\frac{0.3}{\alpha_s} \right)^2 \left(\frac{1 \text{ GeV}}{m_\varphi} \right)^3 \left(\frac{f_\varphi}{10^{10} \text{ GeV}} \right)^2\end{aligned}$$

Thermalization receives three main contributions:

$$\gamma_{\text{Inv-D}}^{(\psi)} = n_{\varphi}^{\text{eq}} \frac{K_1[m_{\varphi}/T]}{K_2[m_{\varphi}/T]} \Gamma_{\varphi \rightarrow \bar{\psi}\psi} \quad \text{Inverse Decays}$$

$$\gamma_{\text{Pair}}^{(\psi)} = (n_{\psi}^{\text{eq}})^2 \langle \sigma_{\psi\bar{\psi} \rightarrow V\varphi} v_{\text{Mø}} \rangle \quad \text{Fermion Pair Annihilation}$$

$$\gamma_{\text{Compton}}^{(\psi)} = 2 \times n_{\psi}^{\text{eq}} n_V^{\text{eq}} \langle \sigma_{\psi V \rightarrow \psi\varphi} v_{\text{Mø}} \rangle \quad \text{Inverse Compton}$$

Accounting for m_φ effects:

$$\sigma_{\psi\bar{\psi}\rightarrow V\varphi}(s) = \frac{\mathcal{C}_\psi^2 g_V^2}{N_{\text{avg}} \pi f_\varphi^2} \frac{m_\psi^2}{s(s - 4m_\psi^2)(s - m_\varphi^2)} \times$$

$$\times \left[(s^2 - 4m_\varphi^2 m_\psi^2 + m_\varphi^4) \tanh^{-1} \left(\sqrt{1 - \frac{4m_\psi^2}{s}} \right) - s m_\varphi^2 \sqrt{1 - \frac{4m_\psi^2}{s}} \right]$$

$$\sigma_{\psi V \rightarrow \psi \varphi}(s) = \frac{\mathcal{C}_\psi^2 g_V^2}{N_{\text{avg}} \pi f_\varphi^2} \frac{\sqrt{\lambda(s, m_\psi, m_\varphi)} m_\psi^2}{s^2 (s - m_\psi^2)^3} \times$$

$$\times \left[4s^2 \frac{\lambda(s, m_\psi, m_\varphi) + m_\varphi^4}{\sqrt{\lambda(s, m_\psi, m_\varphi)}} \coth^{-1} \left(\frac{s + m_\psi^2 - m_\varphi^2}{\sqrt{\lambda(s, m_\psi, m_\varphi)}} \right) + \right.$$

$$\left. + m_\varphi^2 (7s^2 + 2s m_\psi^2 - m_\psi^4) - (s - m_\psi^2)^2 (3s - m_\psi^2) \right]$$

$$\Gamma_{\varphi \rightarrow e^+ e^-} = \frac{C_e^2 m_\varphi m_e^2}{8\pi f_\varphi^2} \sqrt{1 - \frac{4m_e^2}{m_\varphi^2}}$$

$$\tau_{\varphi \rightarrow gg} = \Gamma_{\varphi \rightarrow gg}^{-1} \approx 0.7 \text{ s} \left(\frac{1}{C_G} \right)^2 \left(\frac{0.3}{\alpha_G} \right)^2 \left(\frac{1 \text{ GeV}}{m_\varphi} \right)^3 \left(\frac{f_\varphi}{10^{10} \text{ GeV}} \right)^2$$

$$\Gamma_\varphi = \begin{cases} \Gamma_{\varphi \rightarrow b\bar{b}} = 3 \times \frac{C_b^2 m_\varphi m_b^2}{8\pi f_\varphi^2} \sqrt{1 - \frac{4m_b^2}{m_\varphi^2}} & , \quad m_\varphi > 2m_b \\ \begin{cases} \Gamma_{\varphi \rightarrow \psi' \bar{\psi}'} = N_c^{\psi'} \times \frac{C_{\psi'}^2 m_\varphi m_{\psi'}^2}{8\pi f_\varphi^2} \sqrt{1 - \frac{4m_{\psi'}^2}{m_\varphi^2}} \\ \Gamma_{\varphi \rightarrow VV} = D_V \frac{C_V^2 \alpha_V^2 m_\varphi^3}{256\pi^3 f_\varphi^2} \end{cases} & , \quad m_\varphi \leq 2m_b \end{cases}$$

$$C_{\psi'} \simeq -C_b N_c^b \frac{Y_b^2}{8\pi^2} \ln\left(\frac{f_\varphi}{m_\varphi}\right), \quad C_\gamma = 2C_\psi N_c^\psi Q_\psi^2 \left(1 - \frac{\arcsin^2(x_\psi)}{x_\psi^2}\right), \quad C_G = C_b \left(1 - \frac{\arcsin^2(x_\psi)}{x_\psi^2}\right)$$

$$\tau_\varphi = \begin{cases} \Gamma_{\varphi \rightarrow b\bar{b}}^{-1} \approx 1.4 \text{ s} \left(\frac{1}{C_b}\right)^2 \left(\frac{10 \text{ GeV}}{m_\varphi}\right) \left(\frac{f_\varphi}{5 \times 10^{12} \text{ GeV}}\right)^2, & m_\varphi > 2m_b \\ \Gamma_{\varphi \rightarrow g\bar{g}}^{-1} \approx 0.6 \text{ s} \left(\frac{1}{C_b}\right)^2 \left(\frac{2 \text{ GeV}}{m_\varphi}\right)^7 \left(\frac{f_\varphi}{5 \times 10^8 \text{ GeV}}\right)^2, & m_\varphi \leq 2m_b \end{cases}$$

A Possible UV Completion I



The UV theory contains extra fields and a global U_φ symmetry

Field	Φ	S	Ψ_L	Ψ_R
Type	Complex scalar	Complex scalar	Weyl	Weyl
U_φ charge	-3	1	ξ_Ψ	$\xi_\Psi - 3$

$$\mathcal{L}_{\text{UV}} \supset - \left(y_\Psi \Phi^\dagger \bar{\Psi}_L \Psi_R + \text{h.c.} \right) + \cancel{\lambda_{\Phi H}}^{\ll 1} \Phi^\dagger \Phi H^\dagger H + \cancel{\lambda_{\Phi S}}^{\ll 1} \Phi^\dagger \Phi S^\dagger S + \left(\kappa_\star \Phi S^3 + \text{h.c.} \right)$$

A Possible UV Completion II



After Φ taking a VEV $\Phi \rightarrow \left(\frac{v_\Phi + \rho}{\sqrt{2}} \right) \exp \left[i \frac{\varphi}{v_\Phi} \right] \dots$

$$\mathcal{L}_\varphi = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{v_\Phi}{\sqrt{2}} \left[\kappa_\star \exp \left[i \frac{\varphi}{v_\Phi} \right] S^3 + y_\psi \exp \left[-i \frac{\varphi}{v_\Phi} \right] \bar{\Psi}_L \Psi_R + \text{h.c.} \right]$$

and integrating out heavy modes ...

$$\mathcal{L}_\varphi^{N\partial} \supset \frac{\varphi}{8\pi v_\Phi} \left[\alpha_G G_{\mu\nu} \tilde{G}^{\mu\nu} + 2Y_\psi^2 \alpha_B B_{\mu\nu} \tilde{B}^{\mu\nu} \right] + \frac{v_\Phi}{\sqrt{2}} \left[\kappa_\star \exp \left[i \frac{\varphi}{v_\Phi} \right] S^3 + \text{h.c.} \right]$$

$\mathbb{Z}_3$

arises!

Matching UV completion onto our EFT



The UV theory matches onto our EFT as ($\mathcal{C}_G = 1$):

$$f_\varphi = v_\Phi, \quad \mathcal{C}_B = 2Y_\Psi^2, \quad A = 3\sqrt{2} \kappa_\star v_\Phi$$

Higher order potential terms are naturally suppressed:

$$V_S(S) \supset \frac{\lambda_5}{4!} \frac{1}{v_\Phi} S^4 S^\dagger, \quad \lambda_5 = \frac{6\sqrt{2} \kappa_\star \lambda_\Phi S}{\lambda_\Phi} \ll 1$$

φ -dependent rotations $S \rightarrow e^{[i\varphi/(3f_\varphi)]} S$ recover ∂ -basis Lagrangian up to dim-6 interaction terms

$$\delta\mathcal{L}_{\text{INT}}^{(6)} = \frac{\mathcal{C}'_S{}^2}{4f_\varphi^2} |\partial_\mu \varphi|^2 S^\dagger S \quad \mathcal{C}_S = \mathcal{C}'_S = \frac{2}{3}$$

ALP particles are produced in $SS \rightarrow S^* \varphi$ processes with

$$E_{\varphi}^{(\text{GF})} = \frac{3}{4} m_S \left(1 + \frac{\epsilon_{\varphi}^2}{3} \right) \quad p_{\varphi}^{(\text{GF})} = m_S \sqrt{\lambda \left(1, \frac{1}{2}, \frac{\epsilon_{\varphi}}{2} \right)} \quad \epsilon_{\varphi} = \frac{m_{\varphi}}{m_S}$$

Subsequent visible $\varphi \rightarrow XX$ decay products are **isotropic and monochromatic**

$$\frac{dN_X}{dE'_X d \cos \theta'} = \delta \left(E'_X - \frac{m_{\varphi}}{2} \right) \quad \int dE_X d \cos \theta \frac{dN_X}{dE_X d \cos \theta} = 2$$

Kinematic variables in the GF are related to those in the ALP rest frame

$$E_X^{(\text{GF})} = \gamma_L (E'_X + \beta_L p'_X \cos \theta') .$$

Differential energy injection spectra in the GF are computed as

$$\frac{dN_X}{dE_X} = \int d \cos \theta' dE'_X \frac{dN_X}{dE'_X d \cos \theta'} \delta \left(E_X - E_X^{(\text{GF})} \right) .$$

Introduce dimensionless variables

$$\epsilon_X \equiv \frac{2m_X}{m_\varphi}, \quad x_\varphi \equiv \frac{2E'_X}{m_\varphi}, \quad x_S \equiv \frac{E_X}{m_S}$$

and apply this change of variable to the differential spectrum

$$\frac{dN_X}{dx_S} = 2 \int d \cos \theta' dx_\varphi \frac{dN_X}{dx_\varphi d \cos \theta'} \times$$

Master Formula $\times \delta \left(2x_S - \frac{3}{4} \left(x_\varphi \left(1 + \frac{\epsilon_\varphi^2}{3} \right) + \zeta \sqrt{x_\varphi^2 - \epsilon_X^2} \cos \theta' \right) \right)$

Upon isotropy assumption and performing the $\int_{-1}^1 d \cos \theta$ integral

$$\frac{dN_X}{dx_S} = \frac{4}{3\zeta} \int_{x_\varphi^{\min}}^{x_\varphi^{\max}} \frac{dx_\varphi}{\sqrt{x_\varphi^2 - \epsilon_X^2}} \frac{dN_X}{dx_\varphi} ,$$

$$x_\varphi^{\min} = \frac{8x_S (3 + \epsilon_\varphi^2) - 3\zeta \sqrt{64x_S^2 - \epsilon_X^2} \left[(3 + \epsilon_\varphi^2) - 9\zeta^2 \right]}{(3 + \epsilon_\varphi^2) - 9\zeta^2} ,$$

$$x_\varphi^{\max} = \text{Min} \left\{ 1, \frac{8x_S (3 + \epsilon_\varphi^2) + 3\zeta \sqrt{64x_S^2 - \epsilon_X^2} \left[(3 + \epsilon_\varphi^2) - 9\zeta^2 \right]}{(3 + \epsilon_\varphi^2) - 9\zeta^2} \right\} .$$