Correlators of conserved and nonconserved charges in QCD

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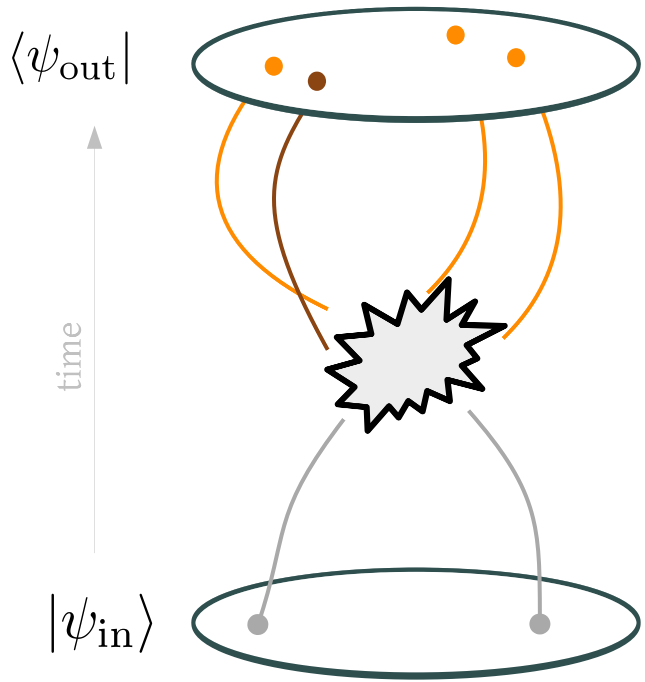
CERN

based on 2407.12082 w/ M. Son

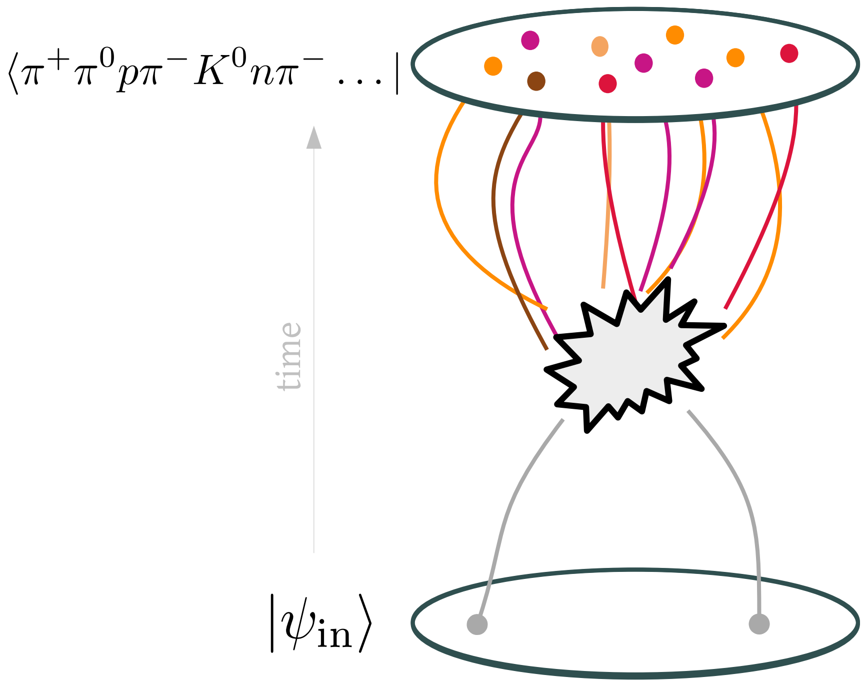
UCLouvain, 12th November 2024



Collider experiments transform an initial state, e.g. pp, into a final state.

(Almost) all we know is based on the different production rates of different states.

Fine for theories with a mass gap and suppressed multiparticle production.



Collider experiments transform an initial state, e.g. pp, into a final state.

(Almost) all we know is based on the different production rates of different states.

Fine for theories with a mass gap and suppressed multiparticle production.

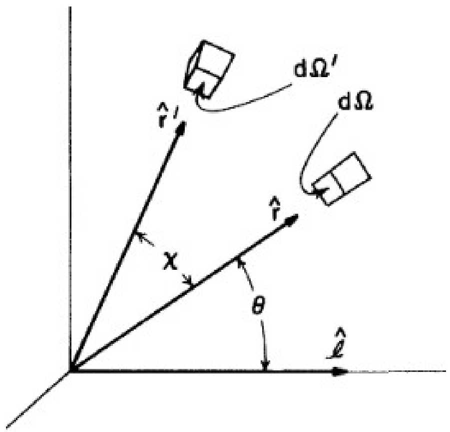
Not the case of real world at high energy!

Need to “coarse grain” your Hilbert space into jets... matching, merging...

Worse at high energies:
what does “diboson” means at $\sqrt{s}$=10TeV?

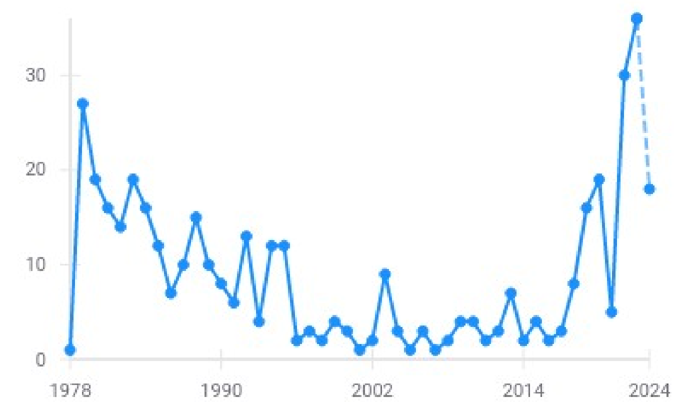
A different set of observables: correlators

Basham, Brown, Ellis, Love '78



An experimental measure is presented for a precise test of quantum chromodynamics. This measure involves the asymmetry in the energy-weighted opening angles of the jets of hadrons produced in the process $e^+e^- \to$ hadrons at energy W. It is special for several reasons: It is reliably calculable in asymptotically free perturbation theory; it has rapidly vanishing (order $1/W^2$) corrections due to nonperturbative confinement effects; and it is straightforward to determine experimentally.

$$\frac{d\langle E\rangle}{d\chi} = \sum_i \int d\Omega |\mathcal{A}|^2 E_i \delta(\cos\theta_i - \chi)$$



Jets from Quantum Chromodynamics

George Sterman and Steven Weinberg
Phys. Rev. Lett. **39**, 1436 – Published 5 December 1977

Discovery of Three-Jet Events and a Test of Quantum Chromodynamics at PETRA

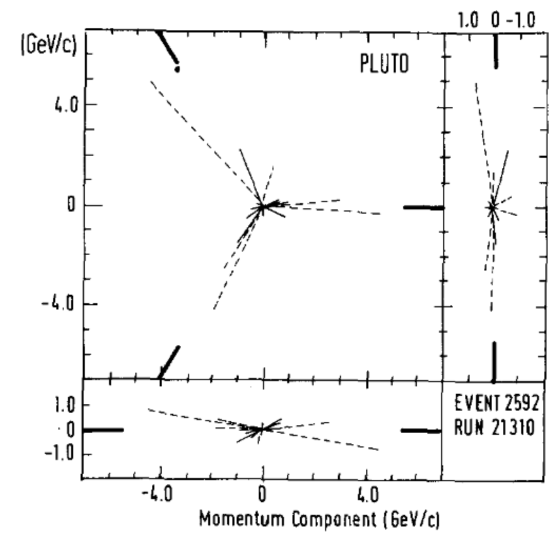
D. P. Barber *et al.*
Phys. Rev. Lett. **43**, 830 – Published 17 September 1979

Physics Letters B
Volume 86, Issues 3–4, 8 October 1979, Pages 418-425

Evidence for gluon bremsstrahlung in e^+e^- annihilations at high energies

PLUTO Collaboration, Ch. Berger, H. Genzel, R. Grigull, W. Lackas, F. Raupach, A. Klovning, E. Lillestöl, E. Lillethun, J.A. Skard, H. Ackermann, G. Alexander[3], F. Barreiro, J. Bürger, L. Criegee, H.C. Dehne, R. Devenish[4], A. Eskreys[5], G. Flügge, G. Franke...K. Wacker[13]

The spectacular discovery of the gluon shifted the attention towards *jets*



Sveshnikov, Tkachov '95

Energy weights have an operatorial definition

$$\mathcal{O}_n = \lim_{r\to\infty} \int dt r^2 n_i T_{i0}(t, r\hat{n}) \quad \longrightarrow \quad \mathcal{O}_n \sim \int d^4k \delta(k^2)\delta^{(2)}(\Omega_{\vec{k}} - \Omega_{\vec{n}})\, k^0\, a^\dagger_k a_k$$

These act as "detectors" or "calorimeters": Extract the energy of particles along detector's direction.

$$\mathcal{O}_{\hat{n}_i}|\alpha\rangle = \sum_i E_i \delta(\hat{p}_i - \hat{n}_i)|\alpha\rangle$$

In particular, it implies that one can write these phase space reweightings as correlators:

$$\frac{d\langle E\rangle}{d\chi} = L_{\mu\nu} \int d^4x \langle 0|j^\mu(x)\mathcal{O}_{\hat{n}} j^\nu(0)|0\rangle$$

In hindsight, this was a breakthrough. As long as the operator is well defined, as is the case of the energy operator, this gives a perfectly robust definition of observables in a gauge theory,
avoiding the theoretical nuance of defining an S-matrix for a gauge theory.

Hoffman, Maldacena '08

Energy weights have an OPE

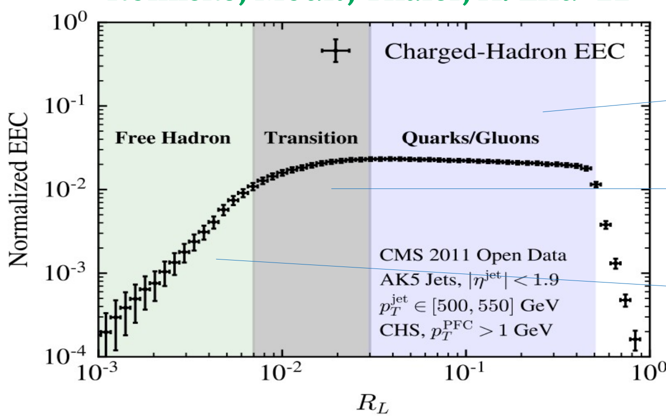
$$\mathcal{E}(n_1)\mathcal{E}(n_2) = \frac{1}{\theta_{12}^2}\int dt\, r^2\, \bar{\Psi}\gamma^+ D^+ D^+ \Psi + \ldots$$

Fixed by transf. under boosts & dilatations

"Measures" the energy squared

The scaling was measured using CMS open data:

Komiske, Moult, Thaler, X. Zhu '22



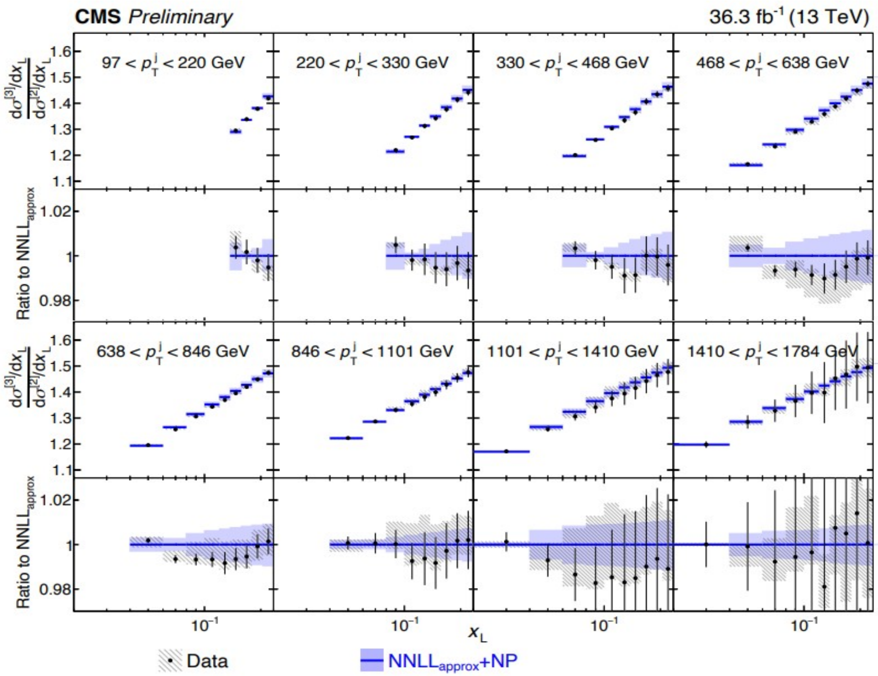
$$1 \gg \theta \gg \Lambda_{QCD}/E_{jet}$$

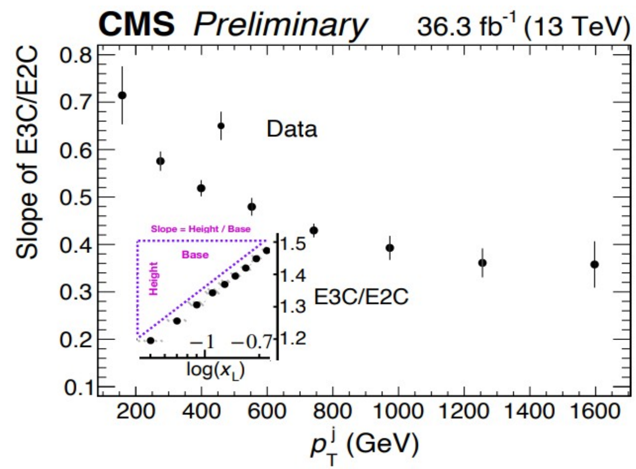
$$\theta \sim \Lambda_{QCD}/E_{jet}$$

$$\Lambda_{QCD}/E_{jet} \gg \theta$$

Strong coupling measurement inside jets

Chen, Gao, Li, Xu, Zhang, X. Zhu '23
CMS-PAS-SMP-22-015 '23
CERN-EP-2024-010 '24





$$\alpha_s(m_Z) = 0.1229^{+0.0040}_{-0.0050}$$

$$= 0.1229^{+0.0014(stat.)+0.0030(theo.)+0.0023(exp.)}_{-0.0012(stat.)-0.0033(theo.)-0.0036(exp.)}$$

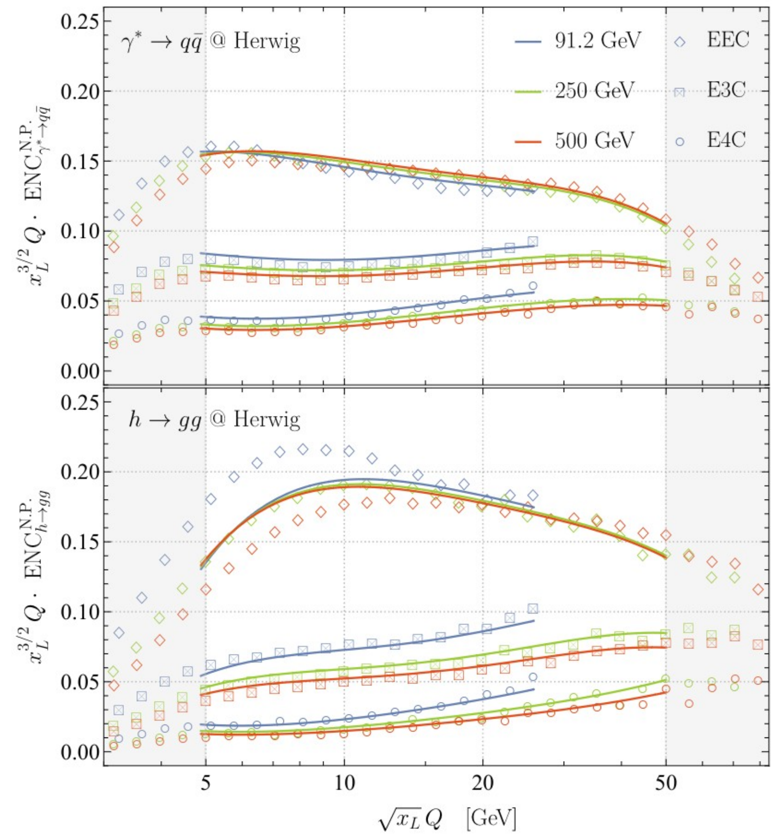
Best determination of α_s using jet substructure

Handle on nonperturbative corrections:

Chen, Monni, Xu, X. Zhu '24

$$\lim_{n_1\to n_2} \mathcal{E}(n_1)\mathcal{E}(n_2) = \frac{1}{x_L}\vec{C}\cdot\vec{\mathbb{O}}^{[J=3]}_{\tau=2}(n_2) + \frac{\Lambda_{\rm QCD}}{x_L^{3/2}}\vec{D}\cdot\vec{\mathbb{O}}^{[J=2]}_{\tau=2}(n_2) + \cdots$$

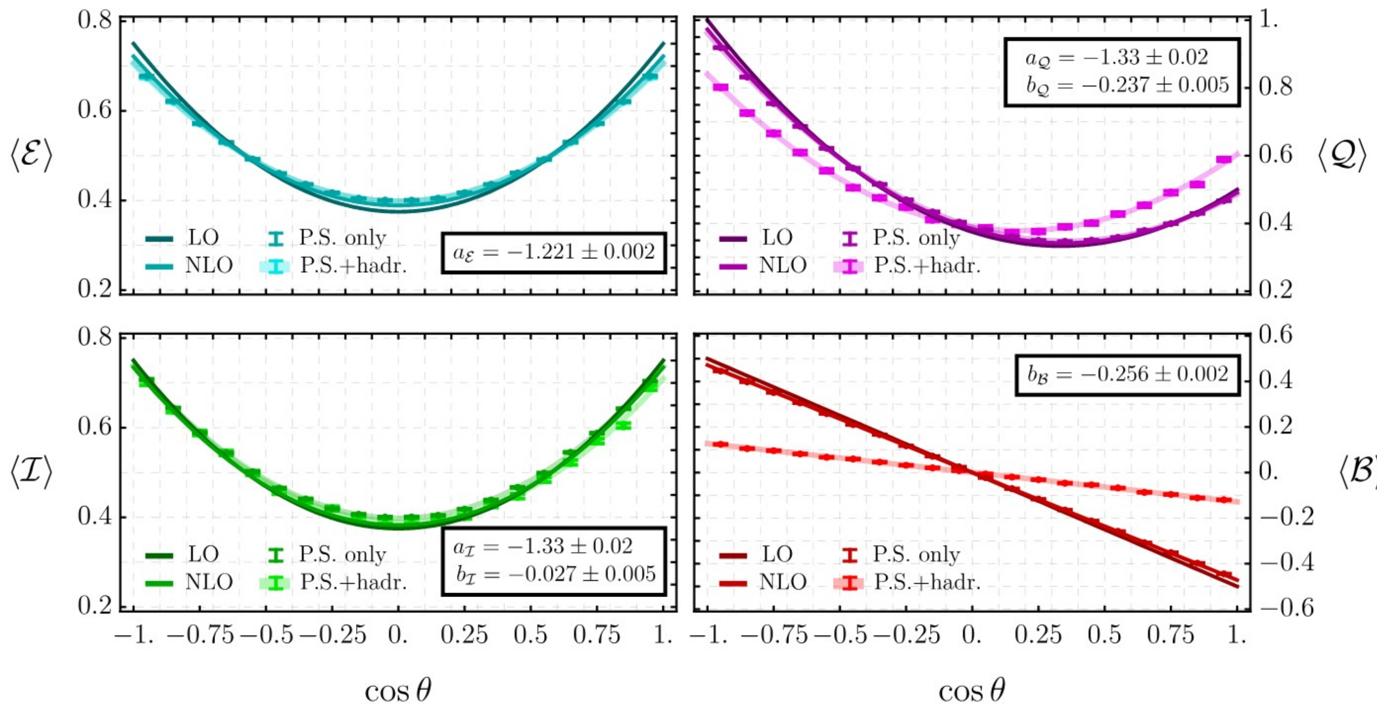
$$\mathrm{ENC}^{\mathrm{N.P.}}_{\Psi_q}(x_L,Q) \equiv \mathrm{ENC}_{\Psi_q}(x_L,Q) - \mathrm{ENC}^{\mathrm{P.T.}}_{\Psi_q}(x_L,Q)\,,$$



OPE structure predicts scaling of nonperturbative corrections, which can be matched across different scales

MR, M. Son ‘24

In the rest of the talk I tell you what are we seeing here:



For a global charge with an associated conserved current J^μ,

$$\mathcal{D}_n = \lim_{r\to\infty}\int_0^\infty dt r^2 n_i J^i(t, r\vec{n})$$

is a *detector* that measures the asymptotic flux of charge in the direction $\vec{n}$,

Acting on physical multiparticle states as

$$\mathcal{D}_n|\alpha\rangle = \sum_i q_i \delta^{(2)}(\Omega_n - \Omega_i)|\alpha\rangle$$

In the presence of an operator exciting the QCD vacuum, the one point correlator is given by

$$\langle\mathcal{D}_n\rangle = \frac{1}{\mathcal{N}}\int d^4x e^{ip\cdot x}\langle\mathcal{O}^\dagger(x)\mathcal{D}_n\mathcal{O}(0)\rangle$$

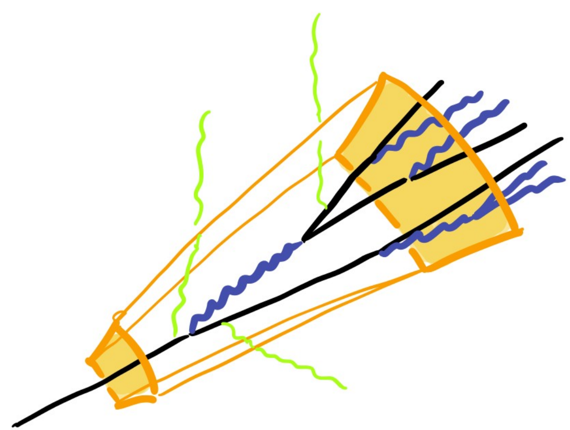
$$= \frac{1}{\mathcal{N}}\sum_{\alpha,i}\delta^{(d)}(p-p_\alpha)\,\omega_i\delta^{(2)}(\Omega_n-\Omega_i)|\langle\alpha|\mathcal{O}|0\rangle|^2$$

That J^μ is conserved has two important consequences:

i) IR finiteness against soft and collinear radiation.

Since the conserved charge is preserved under collinear radiation, the detector acts homogeneously on the set of collinear particles, measuring the total charge.

It is fully inclusive on the rest of the event, ensuring the cancellation of collinear IR divs.



That J^μ is conserved has two important consequences:

i) IR finiteness against soft and collinear radiation.

Soft radiation is instead annihilated by detectors of conserved charges. Using universality of soft interactions, a soft photon/gluon contributes as

$$\int d\Phi_2 \frac{p_q^\mu p_{\bar{q}}^\nu + p_q^\nu p_{\bar{q}}^\mu - q^2/2\eta^{\mu\nu}}{q^2} \sum_{i=q\bar{q}} w_i \delta^{(2)}(\Omega_i - \Omega_n)$$

which vanishes due to opposite fermion anti-fermion charges

That J^μ is conserved has two important consequences:

ii) Vanishing of anomalous dimension

$$U(1)$$

$$J^\mu = \pi^+ \overleftrightarrow{D}_\mu \pi^-$$

What you measure

$$J^\mu = \bar{q}\gamma^\mu q$$

What you compute

In general, operators get anomalous dimensions. Non-conserved quantities at hadron level are mapped to unknown operators at quark level.

Symmetry allows to map between scales.

Beyond one point:

As long as detectors measure conserved charges,

$$\langle \mathcal{D}_{n_1} \mathcal{D}_{n_2} \cdots \mathcal{D}_{n_m} \rangle$$

is collinear safe for sufficiently separated directions.

IR-safety under soft emissions puts restrictions on the operators, as they are forced to annihilate the soft sector.

This happens if one of the detectors measures the energy, or
a quark flavour highly suppressed during parton shower, like *b* or *c*.

$\langle \mathcal{Q}_{n_1} \mathcal{Q}_{n_2} \rangle$ ✗ $\langle \mathcal{E}_{n_1} \mathcal{Q}_{n_2} \rangle$ ✓ $\langle \mathcal{E}_{n_1} \mathcal{E}_{n_2} \mathcal{Q}_{n_3} \rangle$ ✓

$\langle \mathcal{I}_{n_1} \mathcal{B}_{n_2} \rangle$ ✗ $\langle b_{n_1} \mathcal{B}_{n_2} \rangle$ ✓

We will look at the one point correlator when the QCD vacuum is excited by a chiral current

$$J_h(x) = \varepsilon_h^{\mu} \bar{q} \gamma_{\mu} P_L q(x)$$

as it happens in the decay of an electroweak gauge boson. It leads to the density matrix

$$\langle \mathcal{D}_n \rangle_{hh'} = \frac{1}{\mathcal{N}} \int d^4 x e^{ip\cdot x} \langle J_{h'}^{\dagger}(x) \mathcal{D}_n J_h(0) \rangle$$

It represents the average charge measured at a given direction, in the presence of a vector decay with momentum p^{μ} and helicity $h, h' = +, -, 0$

The three point *JDJ* correlator

$$H_{\mathcal{D}}^{\mu\nu} = \int d^4x e^{ip\cdot x} \langle J^{\mu\dagger}(x) \mathcal{D}_n J^\nu(0) \rangle$$

can be decomposed as

$$\begin{aligned} H_{\mathcal{D}}^{\mu\nu} &= \left(-\eta^{\mu\nu} + p^\mu p^\nu / Q^2\right) H_{\mathcal{D}}^0 \\ &+ \left(\frac{Q^2 n_\mu n_\nu + p_\mu p_\nu}{Q n \cdot p} - \frac{n_\mu p_\nu + p_\mu n_\nu}{Q} \right) H_{\mathcal{D}}^n \\ &- \frac{i}{Q} \epsilon^{\mu\nu\alpha\beta} p_\alpha n_\beta H_{\mathcal{D}}^o \,, \end{aligned}$$

The three point *JDJ* correlator

$$H^{\mu\nu}_{\mathcal{D}} = \int d^4x e^{ip\cdot x}\langle J^{\mu\dagger}(x)\mathcal{D}_n J^\nu(0)\rangle$$

can be decomposed as

$$\begin{aligned} H^{\mu\nu}_{\mathcal{D}} &= \left(-\eta^{\mu\nu} + p^\mu p^\nu/Q^2\right) H^0_{\mathcal{D}} \\ &+ \left(\frac{Q^2 n_\mu n_\nu + p_\mu p_\nu}{Qn\cdot p} - \frac{n_\mu p_\nu + p_\mu n_\nu}{Q}\right) H^n_{\mathcal{D}} \\ &- \frac{i}{Q}\epsilon^{\mu\nu\alpha\beta} p_\alpha n_\beta H^o_{\mathcal{D}}\,, \end{aligned}$$

rest frame →

$$\begin{aligned} \langle \mathcal{D}_n \rangle_{hh'} = \frac{\langle \mathcal{D}\rangle}{4\pi}\Big[\delta_{hh'} + a_{\mathcal{D}}\Big((\vec{\epsilon}_h\cdot\vec{n})(\vec{\epsilon}^{\,*}_{h'}\cdot\vec{n}) - \frac{\delta_{hh'}}{3}\Big)\Big] \\ -\frac{i}{4\pi}\, b_{\mathcal{D}}\, \epsilon^{ijk}\epsilon^*_{h',i} n_j \epsilon_{h,k} \end{aligned}$$

Total charge, fixed by symm.

Parity-even, theory dependent

Parity-odd, theory dependent

$$\langle \mathcal{D}\rangle = \frac{H^0_{\mathcal{D}} + \frac{1}{3}H^n_{\mathcal{D}}}{\mathcal{N}}\,, \quad a_{\mathcal{D}} = \frac{H^n_{\mathcal{D}}}{H^0_{\mathcal{D}} + \frac{1}{3}H^n_{\mathcal{D}}}\,, \quad b_{\mathcal{D}} = \frac{H^o_{\mathcal{D}}}{\mathcal{N}}$$

Tree level:

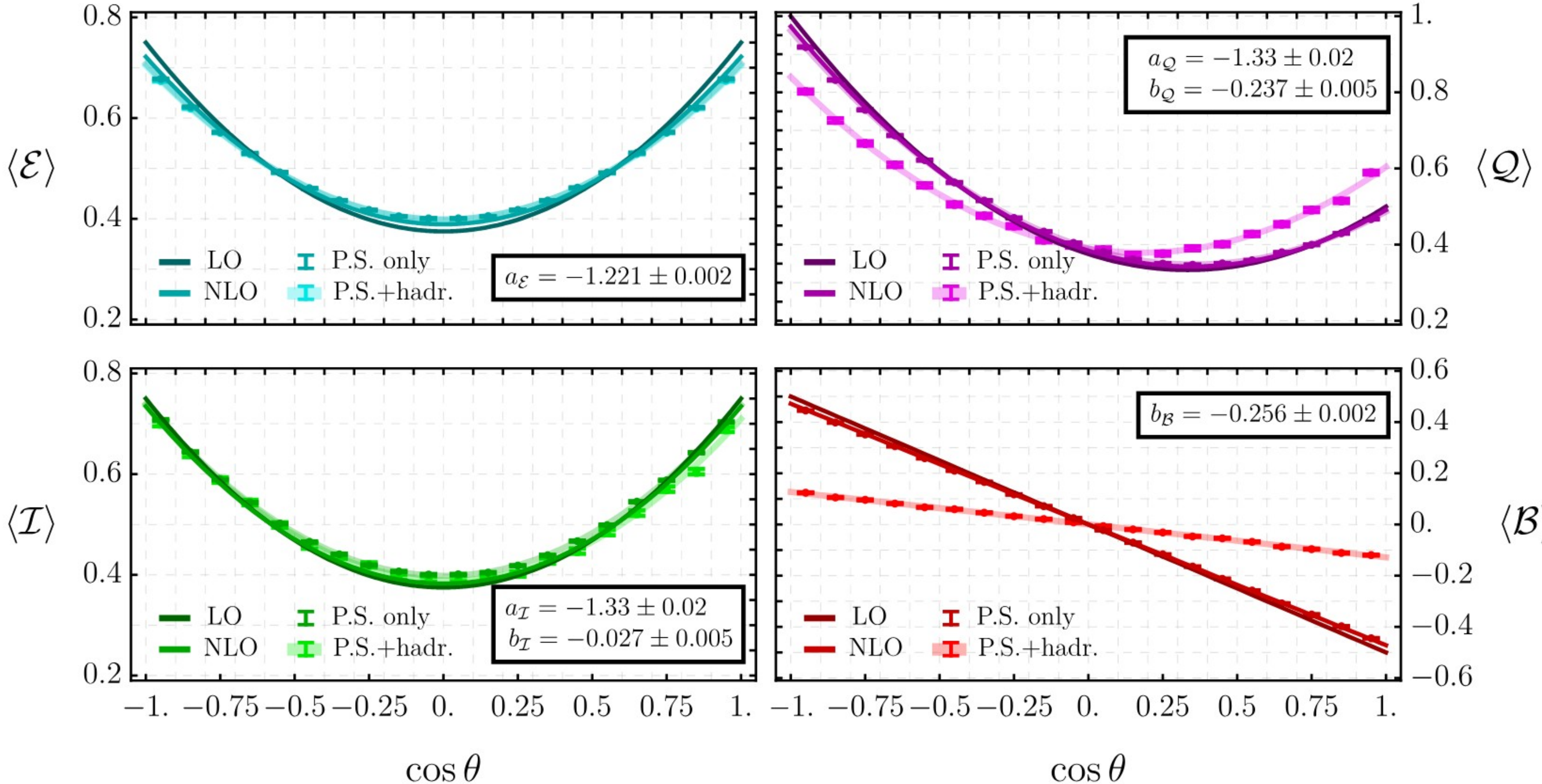
Values fixed by angular momentum, as only two-quark states contribute

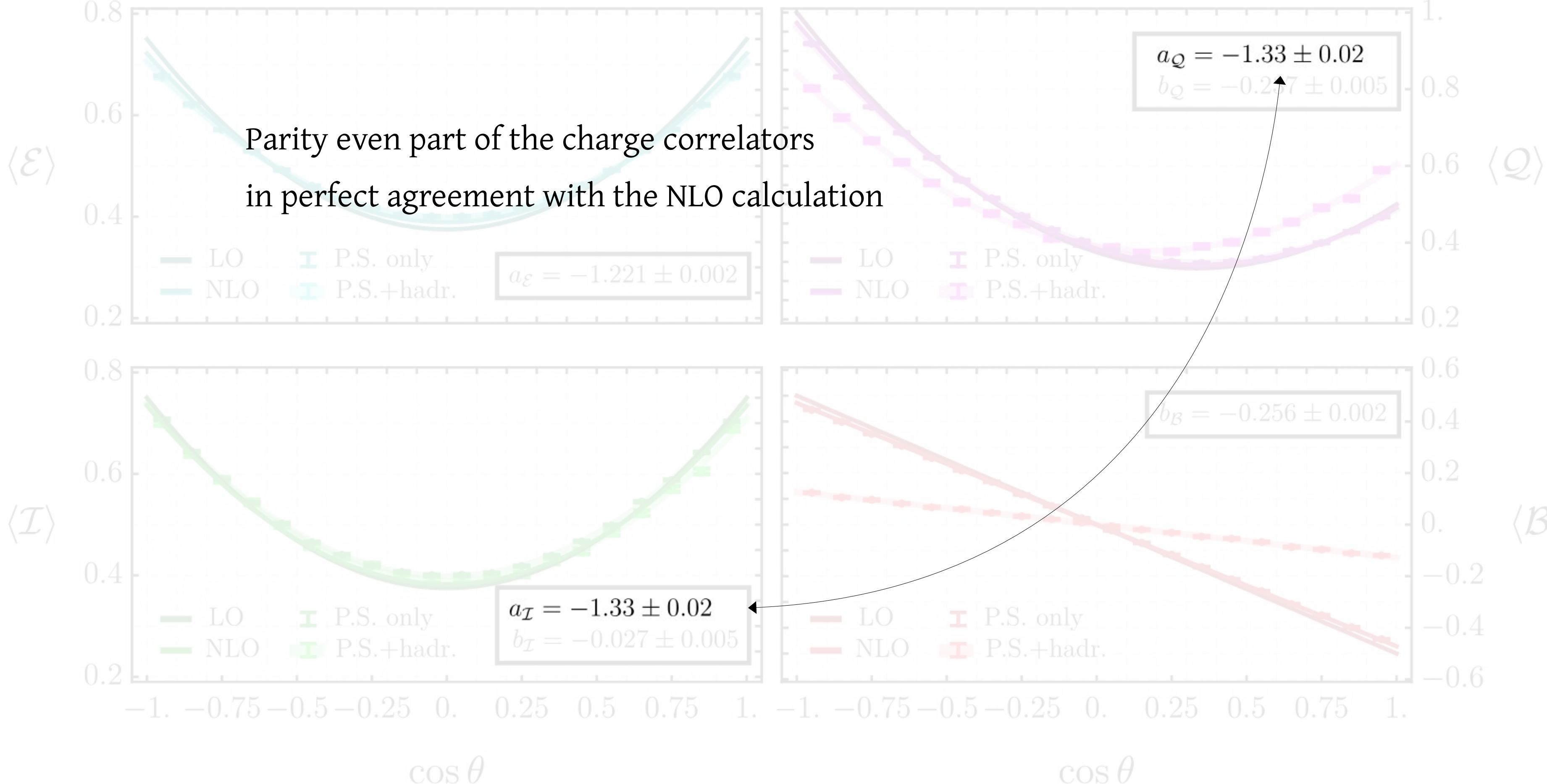
		$\langle \mathcal{D} \rangle$	$a_{\mathcal{D}}$	$b_{\mathcal{D}}$
energy	$\mathcal{E}$	m_W	$-3/2$	0
electric charge	$\mathcal{Q}$	1	$-3/2$	$-\frac{3}{2} \times \frac{1}{3}$
isospin	$\mathcal{I}$	1	$-3/2$	0
baryon number	$\mathcal{B}$	0	0	$-3 \times \frac{1}{3}$

One loop:

Cancellation of soft and collinear divergences

		$\langle \mathcal{D} \rangle$	$a_{\mathcal{D}}$	$b_{\mathcal{D}}$
energy	$\mathcal{E}$	m_W	$-3/2+\frac{9\alpha_S}{2\pi}$	0
electric charge	$\mathcal{Q}$	1	$-3/2+\frac{6\alpha_S}{2\pi}$	$-\frac{3}{2}\times\frac{1}{3}\times(1-\alpha_S/\pi)$
isospin	$\mathcal{I}$	1	$-3/2+\frac{6\alpha_S}{2\pi}$	0
baryon number	$\mathcal{B}$	0	0	$-3\times\frac{1}{3}\times(1-\alpha_S/\pi)$


$\langle\mathcal{E}\rangle$
$a_{\mathcal{E}} = -1.221 \pm 0.002$
LO
NLO
P.S. only
P.S.+hadr.
$\langle\mathcal{Q}\rangle$
$a_{\mathcal{Q}} = -1.33 \pm 0.02$
$b_{\mathcal{Q}} = -0.237 \pm 0.005$
$\langle\mathcal{I}\rangle$
$a_{\mathcal{I}} = -1.33 \pm 0.02$
$b_{\mathcal{I}} = -0.027 \pm 0.005$
$\langle\mathcal{B}\rangle$
$b_{\mathcal{B}} = -0.256 \pm 0.002$
$\cos\theta$
$\cos\theta$



Parity even part of the charge correlators

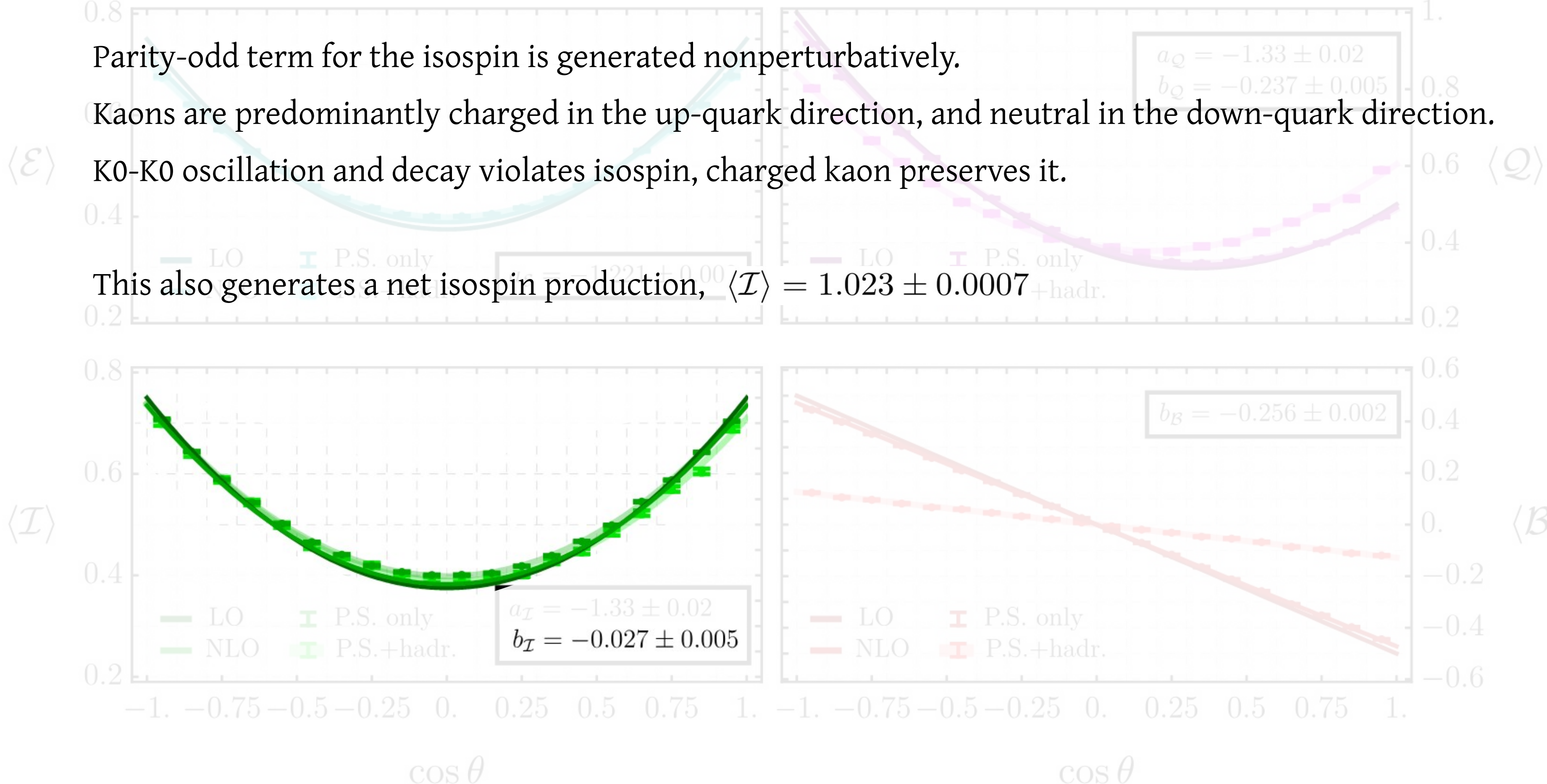
in perfect agreement with the NLO calculation

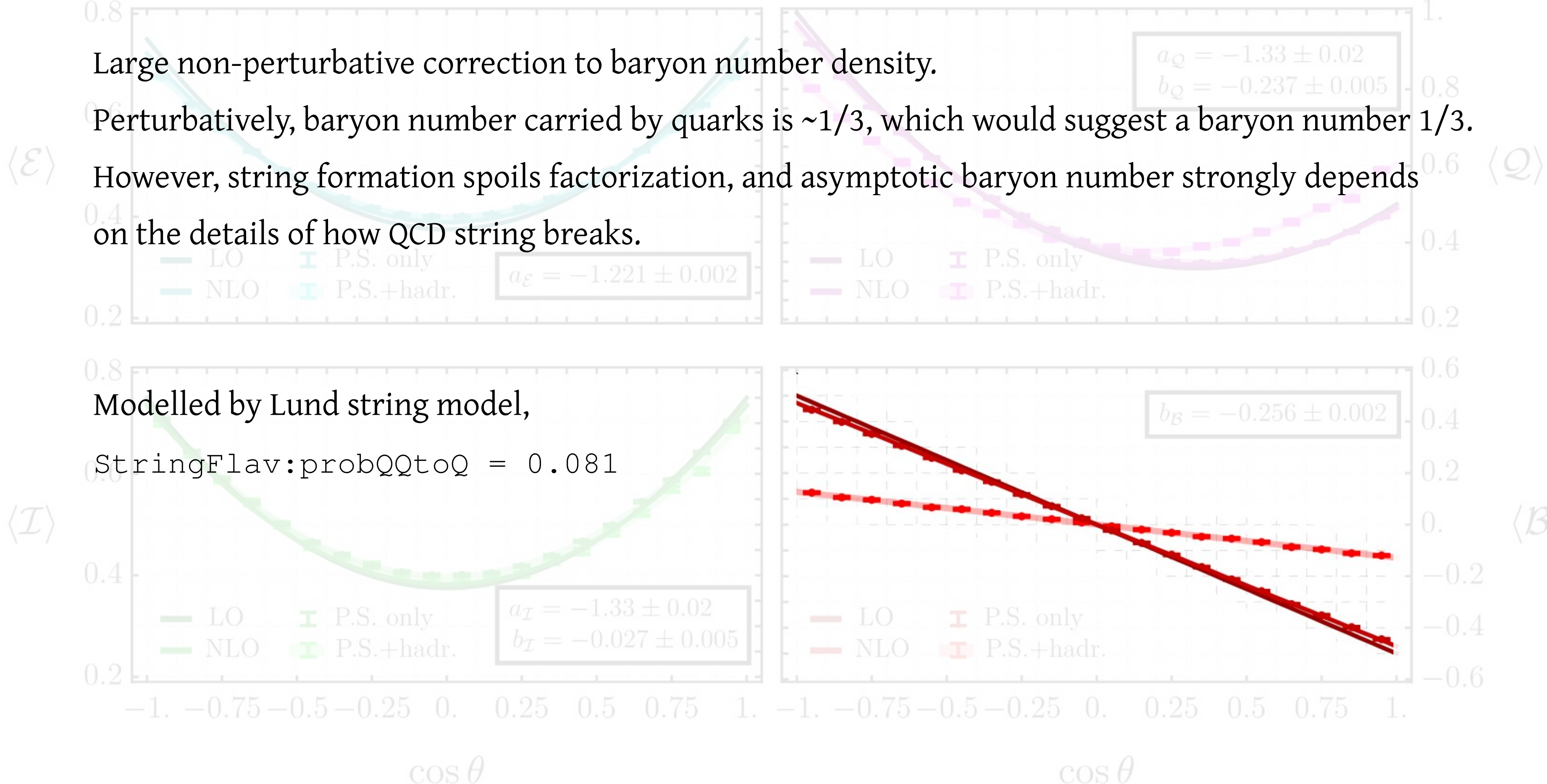
Parity-odd term for the isospin is generated nonperturbatively.

Kaons are predominantly charged in the up-quark direction, and neutral in the down-quark direction.

K0-K0 oscillation and decay violates isospin, charged kaon preserves it.

This also generates a net isospin production, $\langle \mathcal{I} \rangle = 1.023 \pm 0.0007$





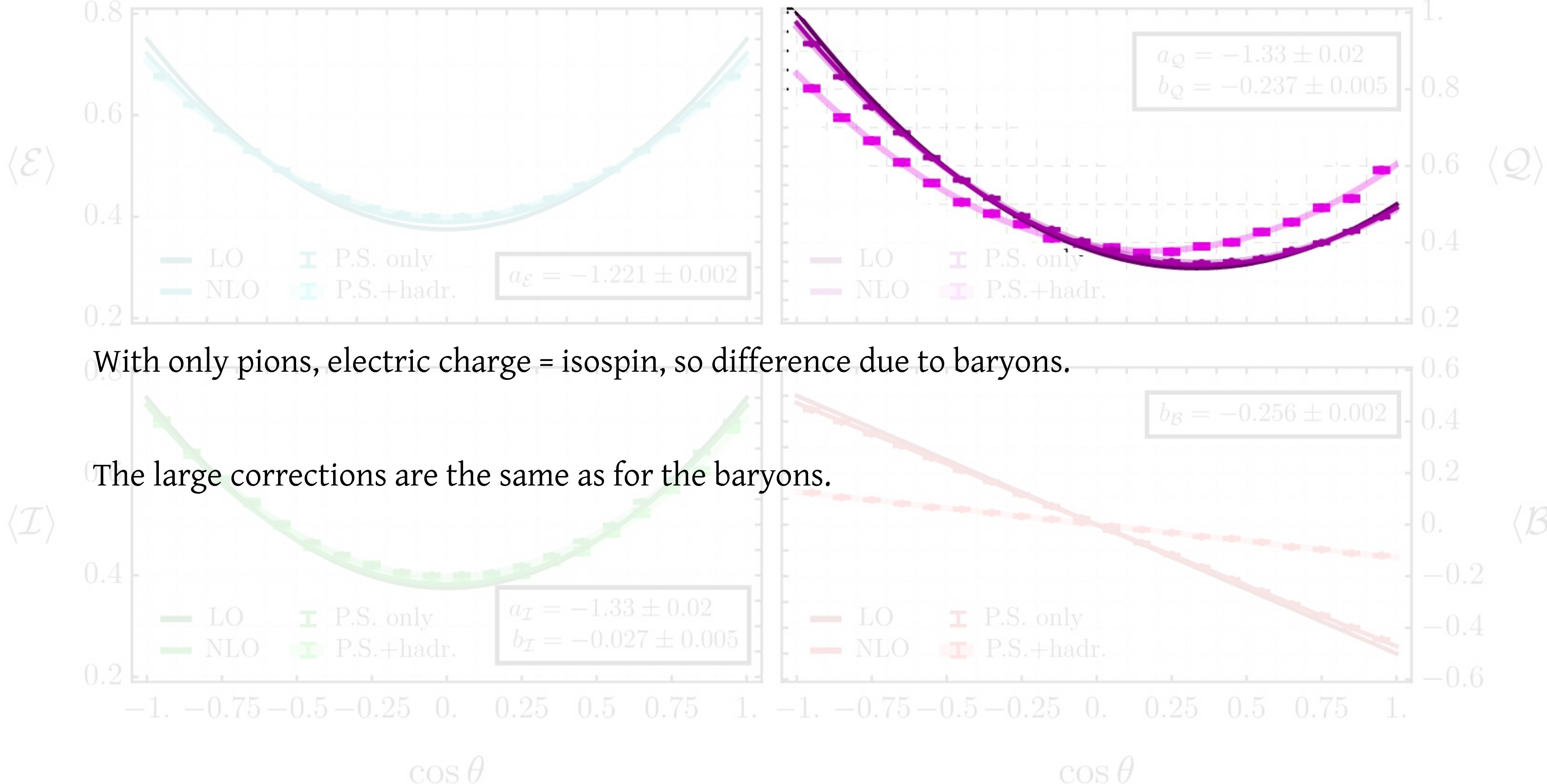
Large non-perturbative correction to baryon number density.

Perturbatively, baryon number carried by quarks is ~1/3, which would suggest a baryon number 1/3.

However, string formation spoils factorization, and asymptotic baryon number strongly depends on the details of how QCD string breaks.

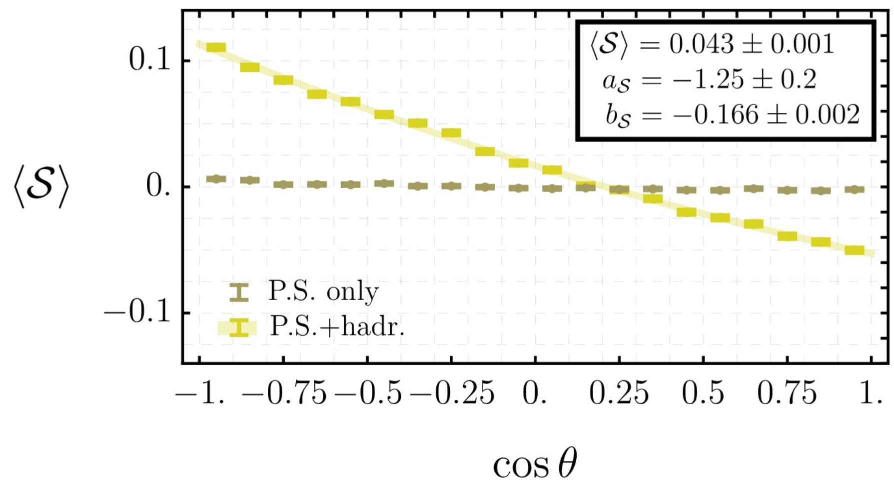
Modelled by Lund string model,

```
StringFlav:probQQtoQ = 0.081
```



With only pions, electric charge = isospin, so difference due to baryons.

The large corrections are the same as for the baryons.



Strangeness density is generated nonperturbatively.

Mechanism is the one described by isospin, as production of kaons is responsible for both effects.

The strangeness generated is indeed equal to the generated isospin.

The constriants due to unitarity can be seen from evaluating the spatial components of the correlator at the center of mass,

$$H_{\mathcal{D}}^{\mu\nu} = \int d^4x e^{ip\cdot x}\langle J^{\mu\dagger}(x)\mathcal{D}_n J^{\nu}(0)\rangle$$

$$\sum_{\alpha, i\in\alpha} \delta^{(4)}(p_\alpha - p)E_i\delta^{(2)}(\Omega_i - \Omega_n)\langle 0|J^{i\dagger}|\alpha\rangle\langle\alpha|J^j|0\rangle$$

$$\begin{aligned} H_{\mathcal{D}}^{\mu\nu} &= \left(-\eta^{\mu\nu} + p^\mu p^\nu/Q^2\right) H_{\mathcal{D}}^0 \\ &+ \left(\frac{Q^2 n_\mu n_\nu + p_\mu p_\nu}{Q n\cdot p} - \frac{n_\mu p_\nu + p_\mu n_\nu}{Q}\right) H_{\mathcal{D}}^n \\ &- \frac{i}{Q}\epsilon^{\mu\nu\alpha\beta} p_\alpha n_\beta H_{\mathcal{D}}^o\,, \end{aligned}$$

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$$\sum_{\alpha, i\in\alpha} \delta^{(4)}(p_\alpha - p) E_i \delta^{(2)}(\Omega_i - \Omega_n) \langle 0 | J^{i\dagger} | \alpha \rangle \langle \alpha | J^{i} | 0 \rangle > 0$$

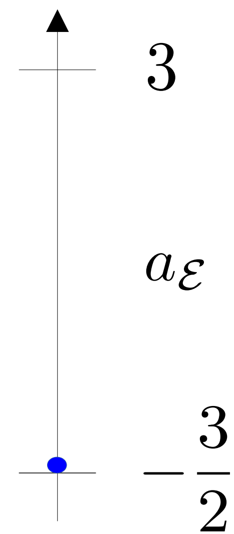
$$H_{\mathcal{D}}^{\mu\nu} = \left(-\eta^{\mu\nu} + p^\mu p^\nu / Q^2\right) H_{\mathcal{D}}^0 + \left(\frac{Q^2 n_\mu n_\nu + p_\mu p_\nu}{Q n\cdot p} - \frac{n_\mu p_\nu + p_\mu n_\nu}{Q}\right) H_{\mathcal{D}}^n - \frac{i}{Q}\epsilon^{\mu\nu\alpha\beta} p_\alpha n_\beta H_{\mathcal{D}}^o\,,$$

which implies that

$$H_{\mathcal{E}}^0 \geq 0 \qquad H_{\mathcal{E}}^0 + H_{\mathcal{E}}^n \geq 0 \quad \Rightarrow \quad -\frac{3}{2} < a_{\mathcal{E}} = \frac{H_{\mathcal{E}}^n}{H_{\mathcal{E}}^0 + \frac{1}{3} H_{\mathcal{E}}^n} < 3$$

(

The bound is saturated by two-particle states



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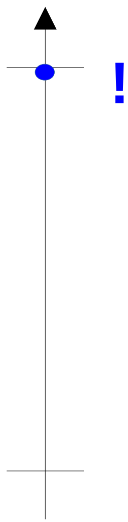
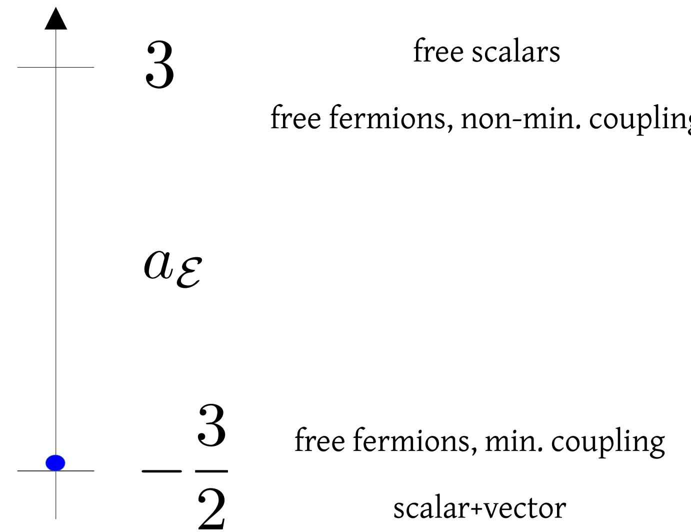


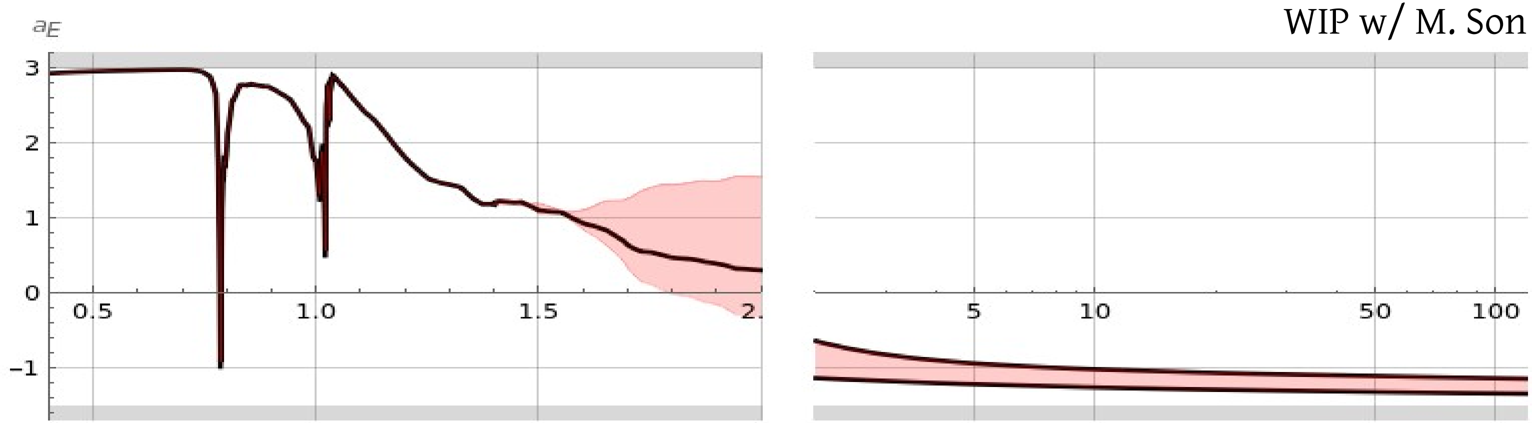
Low energy



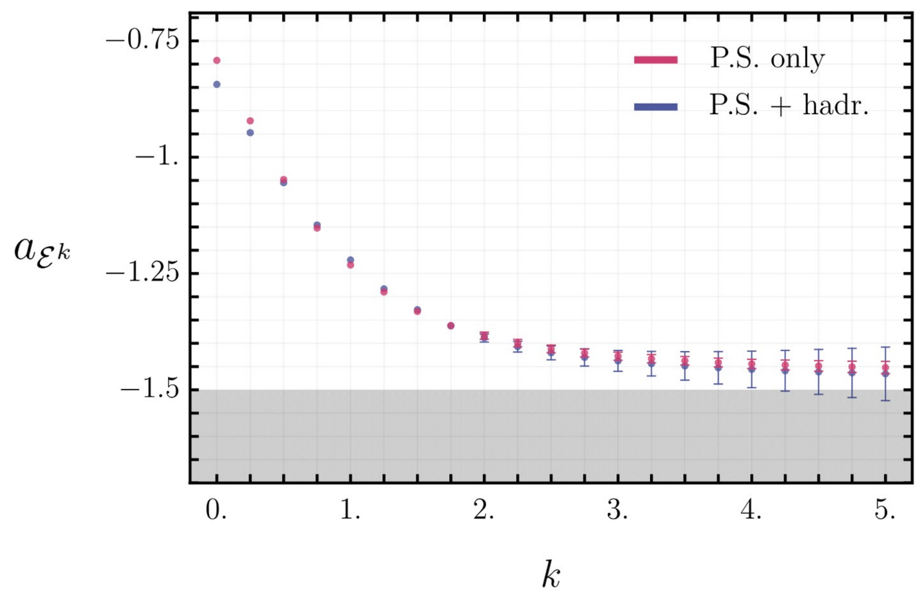
High energy

QCD flows from one boundary to the other!

WIP w/ M. Son



All applies to operators other than energy, as long as they are positive.



At low k, radiative corrections become more important

At k=0, the observable is the particle number density

At high k, radiation is highly suppressed and only two particle states contribute

Densities at different *k* constrained with each other.

The scalar functions can be schematically written as

$$H(k) = \int d\Phi \rho(\Phi)(\mathcal{E}(\Phi))^k$$

$$\rho(\Phi) \geq 0$$
$$\mathcal{E}(\Phi) \geq 0$$
$$\mathcal{E}(\Phi) \leq 1$$

So the derivatives in *k* obey

$$(-1)^N \frac{d^N H(k)}{dk^N} > 0$$

i.e. it is a completely monotonic function in *k*.

It can be shown that evaluating the function at *k, k+a, k+2a,...*leads to the *H(k)*'s to obey the Hausdorff moment problem.

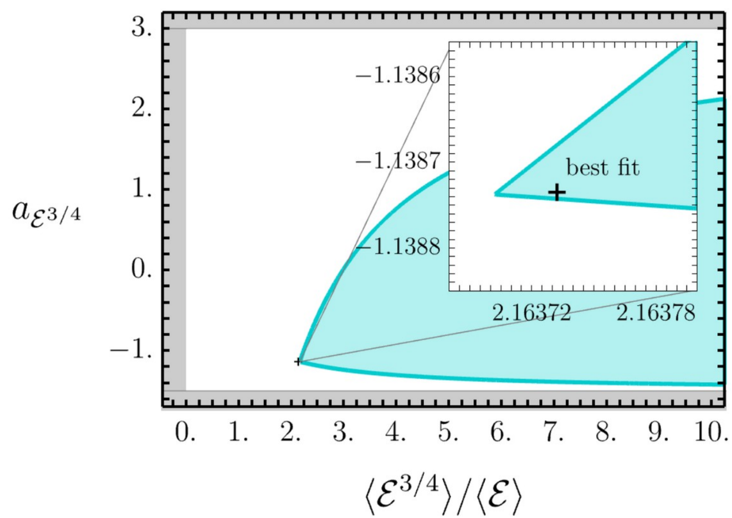
$$H^0 \succ 0\,, \quad H^1 \succ 0\,,$$
$$H^0 - H^1 \succ 0\,, \quad H^1 - H^2 \succ 0$$

with $(H^p)_{ij} = H(k + (i + j + p - 2)a)$

Densities at different *k* constrained with each other.

Higher *k* densities are very efficiently constrained from lower *k* ones.

Lower *k* densities have however one sided bounds. Data is close to the kink.



Correlator-based observables offer a different perspective on collider data, with strong theoretical roots that constrain and simplify them.

One point (and some higher point) correlators of conserved charges are perturbatively IR safe.

Thank you!