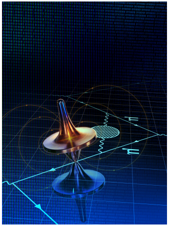
Probing the Standard Model to 0.37 ppm with the muon anomalous magnetic moment

Laurent Lellouch

CNRS & Aix-Marseille U.

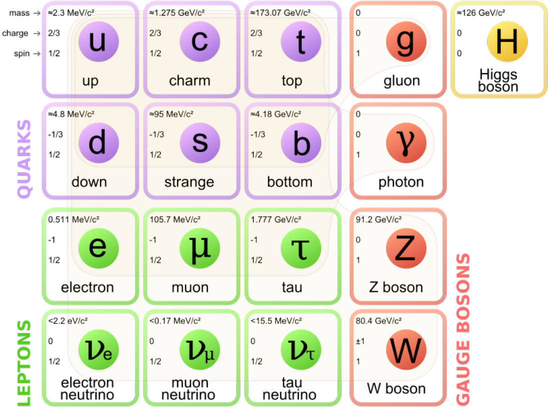


© Dani Zemba, Penn State

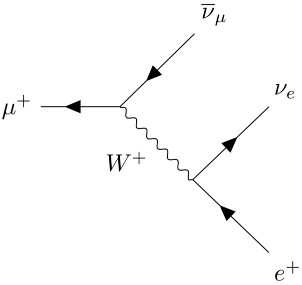
Budapest-Marseille-Wuppertal collaboration [BMW] & DMZ

2407.10913 ⟶ BMWc-DMZ '24 (or this work)
Nature 593 (2021) ⟶ BMWc '20
PRL 121 (2018) 022002 (Editors' Selection) ⟶ BMWc '17
Aoyama et al., Phys. Rep. 887 (2020) 1-166 ⟶ WP '20
Davier et al., EPJ C (2024) ⟶ DHLMZ '23

The muon in the Standard Model



(Wikimedia)



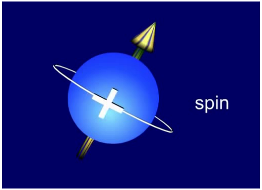
- muon (μ) $\sim$ electron (e): same couplings to gauge bosons, but not to the Brout-Englert-Higgs field

$\rightarrow$ $m_\mu \simeq 207 \times m_e \rightarrow \tau_\mu \simeq 2 \times 10^{-6}$ sec

Charged lepton magnetic moments

Muons are tiny magnets

A massive elementary particle w/ electric charge and spin behaves like a tiny magnet



(← Silver Swan)

Magnetic moment of the muon

$$\vec{\mu}_\mu = \pm g_\mu \frac{e}{2m_\mu} \vec{S}$$

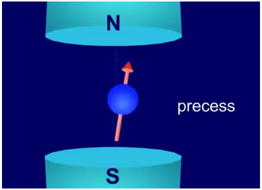
g_μ = Landé factor

In uniform magnetic field $\vec{B}$, $\vec{S}$ precesses w/ angular frequency

$$\omega_S = g_\mu \frac{e}{2m_\mu} |\vec{B}|$$

7×10^6 rotations per second for $|\vec{B}| = 1.45$ T

→ same principle as for MRI

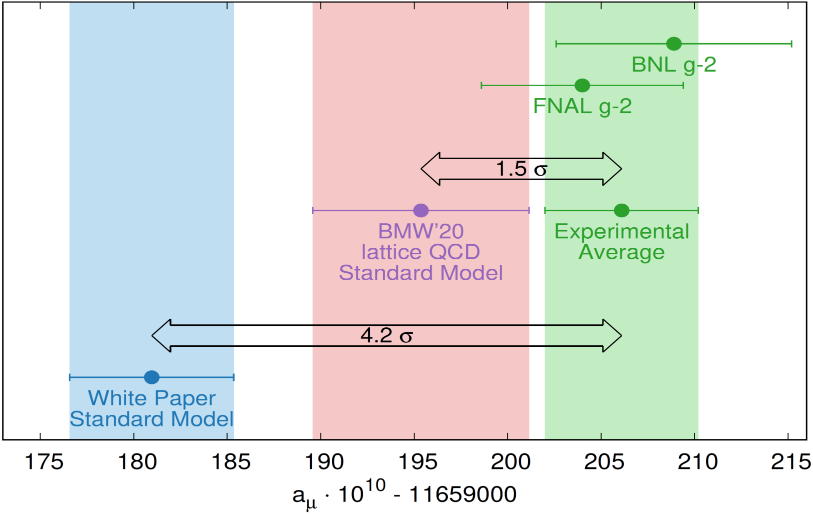


(Silver Swan)

Crucial point:

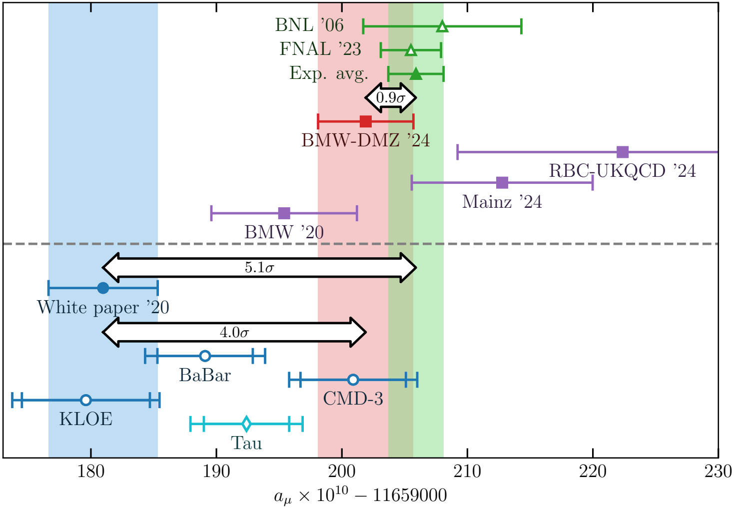
- g_μ can be **measured** & **calculated** very, very ... precisely
- measurement = SM prediction ?
  → Yes: another victory for the SM
  → No: we have uncovered new fundamental physics

Take home: muon magnetic moment 2021



New physics ?

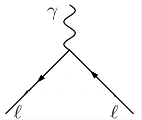
Take home: muon magnetic moment 2025



New physics ??

Early history: the electron

- $\textcolor{blue}{1928}$: Dirac's new theory predicts the existence of the positron and



$$g_e|_{\text{Dirac}} = 2$$

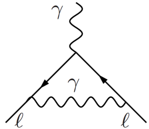
"That was really an unexpected bonus for me" (P.A.M. Dirac)

- 1934 : Kinsler & Houston confirm $g_e = 2$, w/ permil precision by studying spectrum of neon atom

- 1947 : Nafe, Nelson & Rabi, then Kusch & Foley measure hyperfine structure of hydrogen and deuterium, showing that $g_e > 2$ by 0.1%

  $\rightarrow$ there is a problem w/ Dirac!

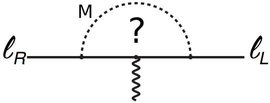
- 1947 : Schwinger understands very quickly that Dirac's theory neglects **quantum fluctuations** and manages to compute them to obtain the **"anomalous"** contribution



$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi} = 0.00116\ldots$$

$\rightarrow$ birth of QED and relativistic quantum field theory

Why are a_ℓ interesting?



$\ell_R \longrightarrow \ell_L$

$$\longrightarrow \quad \mathcal{L}_{\rm eff} = -\frac{Qe}{2}\frac{a_\ell}{2m_\ell}F^{\mu\nu}[\bar{\ell}_L\sigma_{\mu\nu}\ell_R] + \text{hc}$$

- a_ℓ loop-induced $\Rightarrow$ possibly sensitive to particles too heavy or too weakly coupled to be produced directly
- Flavor and CP conserving, chirality flipping $\Rightarrow$ sensitive to muon mass generation mechanism and complementary to EDMs, s & b decays, EWPO, direct LHC searches ...
- 1956 : Berestetskii notes that sensitivity of a_ℓ to contributions of heavy particles w/ $M \gg m_\ell$ typically goes like $\sim (m_\ell/M)^2$

  $\Rightarrow$ a_μ is $(m_\mu/m_e)^2 \sim 43,000$ times more sensitive to heavy particles than a_e

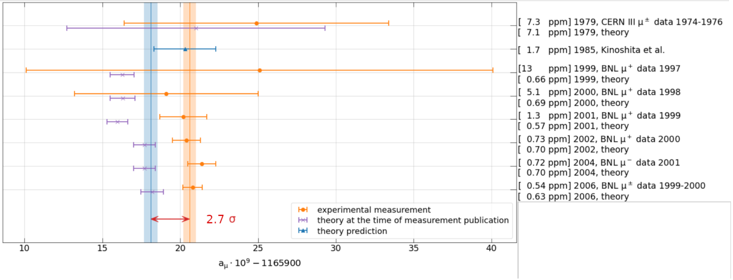
  $\Rightarrow$ a_μ is a good way to reveal possibly unknown, heavy particles

  $\rightarrow$ Today, we know that BSM models can give large contributions to a_μ
- 1960 : despite $\tau_\mu \sim 2\,\mu$s, Garwin et al manage to measure $g_\mu \simeq 2$

A brief history of a_μ

- $>$ 1960 : measurement of a_μ progressed in // with the development of the SM

[Adapted from G. Venanzioni. Colored error bands not relevant here. 2.7σ is between 2006 experiment and theory]



- 2006 : BNL final report [PRD 76, 2006]

$$a_\mu^{\text{exp}} = 11659208.0(6.3) \times 10^{-10}\ [0.54\,\text{ppm}], \qquad a_\mu^{\text{SM}} = 11659180.0(7.3) \times 10^{-10}\ [0.62\,\text{ppm}]$$

$$\Delta a_\mu^{\text{exp-SM}} = 26.1(9.4) \times 10^{-10}$$

$\longrightarrow$ 2.7σ discrepancy was **too small** to claim new physics, but **too large** to ignore ($\sim 2\times$ weak contribution!)

Muon: recent history and near future

To decide on possible presence of new fundamental physics:

Improve the measurement

Move BNL apparatus to Fermilab & significantly upgrade experiment

⇒ April 7, 2021: first results (run 1): 6% of planned data and 20% more precise than BNL [Abi et al, PRL 126 (2021)]

⇒ August 10, 2023: new results (runs 2/3): w/ runs 1-3, $\sim 6\times$ BNL statistics [Aguillard et al, PRL 131 (2023)]

⇒ 2025: soon final results (runs 4/5/6): w/ runs 1-3, $\sim 22\times$ BNL statistics

Improve the SM prediction

Important theoretical/experimental effort to improve SM prediction to comparable level of precision

⇒ White Paper from the muon $g-2$ Theory Initiative w/ reference SM prediction [Aoyama et al '20 = WP '20]

⇒ New measurements of $\sigma(e^+e^- \to \text{hadrons})$ to improve determination of QCD contribution that limits SM prediction precision

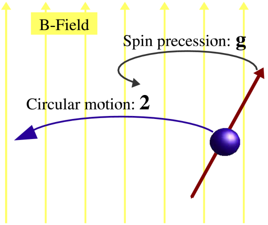
⇒ Onging *ab-initio* supercomputer calculations of all highly nonlinear QCD contributions

⇒ New White Paper in preparation

$$a_\mu^{\text{exp}} = a_\mu^{\text{SM}} \text{ ?}$$

Experimental measurement of a_μ

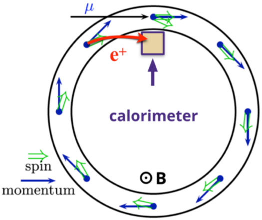
Measurement principle for a_μ



Precession determined by

$$\vec{\mu}_\mu = 2(1+a_\mu)\frac{Qe}{2m_\mu}\vec{S}$$

$$\vec{d}_\mu = \eta_\mu \frac{Qe}{2m_\mu}\vec{S}$$



$$\vec{\omega}_{a\eta} = \vec{\omega}_a + \vec{\omega}_\eta \simeq -\frac{Qe}{m_\mu}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right] - \eta_\mu \frac{Qe}{2m_\mu}\left[\vec{E}+\vec{\beta}\times\vec{B}\right]$$

- Experiment measures very precisely $\vec{B}$ with $|\vec{B}| \gg |\vec{E}|$ &

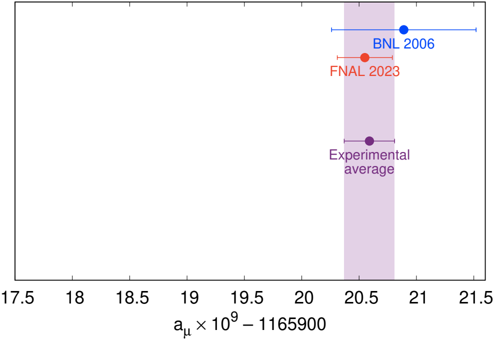
$$\Delta\omega \equiv \omega_S - \omega_C \simeq \sqrt{\omega_a^2+\omega_\eta^2} \simeq \omega_a$$

since $d_\mu = 0.1(9)\times 10^{-19} e\cdot\mathrm{cm}$ (Benett et al '09)

- Consider either magic $\gamma = 29.3$ (CERN/BNL/Fermilab) or $\vec{E}=0$ (J-PARC)

$$\to \Delta\omega \simeq a_\mu B \frac{e}{m_\mu}$$

a_μ: present experimental status



$$a_\mu = 11\,659\,205.9\,(2.2) \times 10^{-10} \qquad [0.19\,\text{ppm}]$$

Bathroom scale sensitive to weight even smaller than that of a single eyelash !!!

Based on $\sim$ 25% of Fermilab data $\rightarrow$ should get $\delta a_\mu \sim 0.10\,\text{ppm}$ in 2025

Reference standard model calculation of a_μ

[Aoyama et al '20 = WP '20]

At needed precision: all three interactions and all SM particles

$$
\begin{aligned}
a_\mu^{\rm SM} &= a_\mu^{\rm QED} + a_\mu^{\rm had} + a_\mu^{\rm EW} \\
&= O\left(\frac{\alpha}{2\pi}\right) + O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) + O\left(\left(\frac{\alpha}{16\pi \sin^2\theta_W}\right)\left(\frac{m_\mu}{M_W}\right)^2\right) \\
&= O\left(10^{-3}\right) + O\left(10^{-7}\right) + O\left(10^{-9}\right)
\end{aligned}
$$

QED contributions to a_ℓ

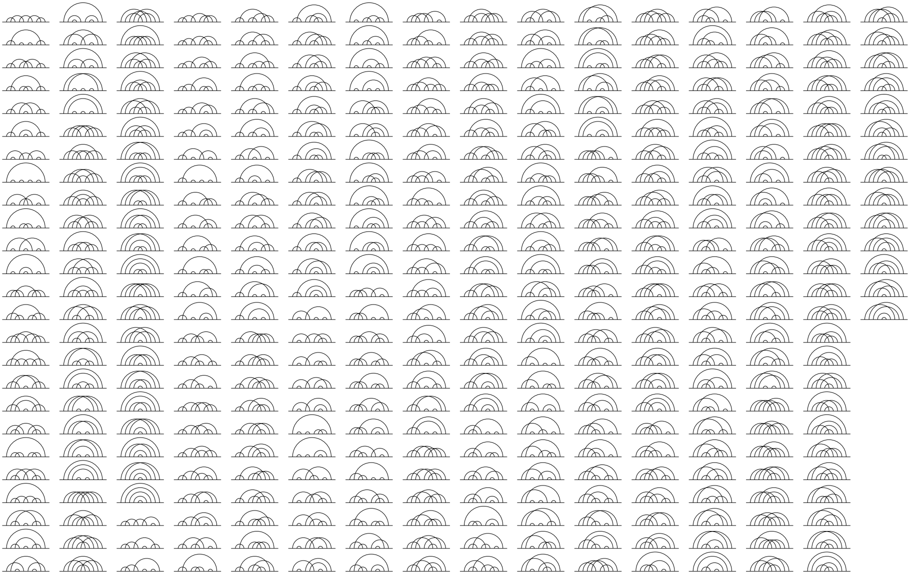
Loops with only photons and leptons: can expand in $\alpha = e^2/(4\pi) \ll 1$

$$a_\ell^{\rm QED} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5 + \cdots$$

$$C_\ell^{(2n)} = A_1^{(2n)} + A_2^{(2n)}(m_\ell/m_{\ell'}) + A_3^{(2n)}(m_\ell/m_{\ell'}, m_\ell/m_{\ell''})$$

- $A_1^{(2)}, A_1^{(4)}, A_1^{(6)}, A_2^{(4)}, A_2^{(6)}, A_3^{(6)}$ known analytically [Schwinger '48; Sommerfield '57, '58; Petermann '57; ...]
- $O((\alpha/\pi)^3)$: 72 diagrams [Laporta et al '91, '93, '95, '96; Kinoshita '95)
- $O((\alpha/\pi)^4; (\alpha/\pi)^5)$: 891;12,672 diagrams [Laporta '95; Aguilar et al '08; Aoyama et al '96-'19, Volkov '19-'24]
    - Automated generation of diagrams
    - Numerical evaluation of loop integrals
    - Calculations cross-checked

5-loop QED diagrams



[Aoyama et al '15]

QED contribution to a_μ

From Cs [Mueller et al '18] or Rb [Morel et al LKB'20] recoil measurements:

$\alpha^{-1}[\mathrm{Cs}] = 137.035\,999\,046(27)$ [0.2 ppb] $\qquad \alpha^{-1}[\mathrm{Rb}] = 137.035\,999\,206(11)$ [0.081 ppb]

Then:

				% of a_μ	order
$a_\mu^{\mathrm{QED}} \times 10^{10}$	$=$	11 614 097.3321	(23)	99.6133%	α
	$+$	41 321.7626	(7)	0.3544%	α^2
	$+$	3 014.1902	(33)	0.0259%	α^3
	$+$	38.1004	(17)	0.0003%	α^4
[needs negligible update]	$+$	0.5078	(6)	$4 \cdot 10^{-6}$	α^5
	$=$	11 658 471.8931	$(7)_{m_\tau}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_{\alpha}[104]$ [0.9 ppb]		

[Aoyama et al '12, '18, '19]

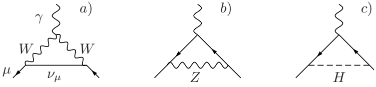
99.994% of a_μ are due to QED contributions!

$$a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{QED}} = 734.0(2.2) \times 10^{-10}$$

$$\stackrel{?}{=} a_\mu^{\mathrm{EW}} + a_\mu^{\mathrm{had}}$$

Electroweak contributions to a_μ: Z, W, H, etc. loops

1-loop

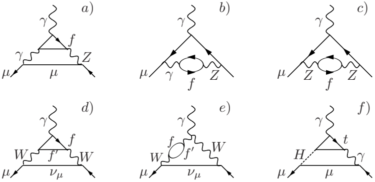


$$a_\mu^{\text{EW},(1)} = O\left(\frac{\sqrt{2}G_F m_\mu^2}{16\pi^2}\right)$$

$$= 19.479(1) \times 10^{-10}$$

(Gnendiger et al '15, Aoyama et al '20 and refs therein)

2-loop



$$a_\mu^{\text{EW},(2)} = O\left(\frac{\sqrt{2}G_F m_\mu^2}{16\pi^2}\frac{\alpha}{\pi}\right)$$

$$= -4.12(10) \times 10^{-10}$$

(Gnendiger et al '15 and refs therein)

$$a_\mu^{\text{EW}} = 15.36(10) \times 10^{-10}$$

Hadronic contributions to a_μ: quark and gluon loops

$$a_\mu^{\text{exp}} - a_\mu^{\text{QED}} - a_\mu^{\text{EW}} = 718.6(2.2) \times 10^{-10} \stackrel{?}{=} a_\mu^{\text{had}}$$

- Clearly right order of magnitude:

$$a_\mu^{\text{had}} = O\left(\left(\frac{\alpha}{\pi}\right)^2 \left(\frac{m_\mu}{M_\rho}\right)^2\right) = O\left(10^{-7}\right)$$

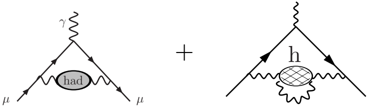
(already Gourdin & de Rafael '69 found $a_\mu^{\text{had}} = 650(50) \times 10^{-10}$)

- However, must be determined to subpercent accuracy & involves quarks and gluons at low energies
  - $\Rightarrow$ must be able to describe the highly nonlinear dynamics of the strong interaction in that regime
  - $\Rightarrow$ cannot rely on the perturbative methods used for QED and weak corrections
  - $\Rightarrow$ need methods that allow a fully nonperturbative calculation

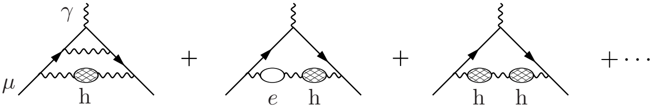
- Decompose:

$$a_\mu^{\text{had}} = a_\mu^{\text{LO-HVP}} + a_\mu^{\text{HO-HVP}} + a_\mu^{\text{HLbyL}} + O\left(\left(\frac{\alpha}{\pi}\right)^4\right)$$

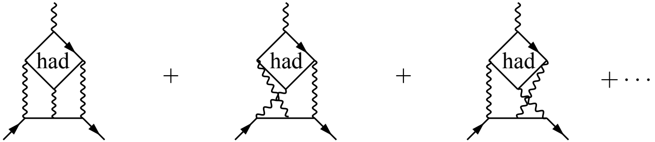
Hadronic contributions to a_μ: diagrams



$\rightarrow a_\mu^{\text{LO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^2\right)$



$\rightarrow a_\mu^{\text{NLO-HVP}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$



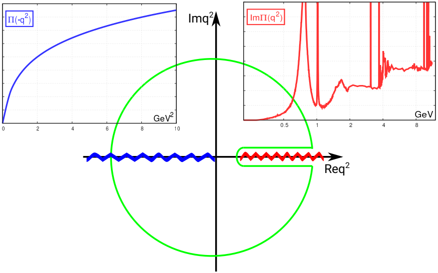
$\rightarrow a_\mu^{\text{HLbL}} = O\left(\left(\frac{\alpha}{\pi}\right)^3\right)$

Data-driven determination of HVP contribution

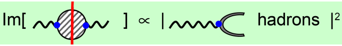
- $\Pi_{\mu\nu}(q) =$  $= (q_\mu q_\nu - g_{\mu\nu}q^2)\,\Pi(q^2)$

- $a_\mu^{\text{LO-HVP}} =$ weighted integral of $\hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$ for $q^2 = -Q^2$, $Q^2 = 0 \to \infty$

- $\hat{\Pi}(q^2)$ is real and analytic except for cut along real, positive q^2 axis



- **Analyticity**: can get $\hat{\Pi}(q^2)$ for $q^2 \le 0$ from $\mathrm{Im}\Pi(q^2)$ w/ $q^2 > 0$ via contour integral ([once subtracted] dispersion relation)

- **Unitarity** [Bouchiat et al '61]: 

$$\mathrm{Im}\Pi(s) = -\frac{R(s)}{12\pi}, \quad R(s) \equiv \frac{\sigma(e^+e^- \to \text{had})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Reference standard model prediction and comparison to experiment

[WP'20]

Reference SM result vs experiment

SM contribution	$a_\mu^{\text{contrib.}} \times 10^{10}$	Ref.
QED [5 loops]	11658471.8931 ± 0.0104	[Aoyama '19, WP '20]
EW [2 loops]	15.36 ± 0.10	[Gnendiger '15, WP '20]
HVP Tot. (R-ratio)	684.5 ± 4.0	[WP '20]
HLbL Tot.	9.2 ± 1.8	[WP '20]
SM [0.37 ppm]	11659181.0 ± 4.3	[WP '20]

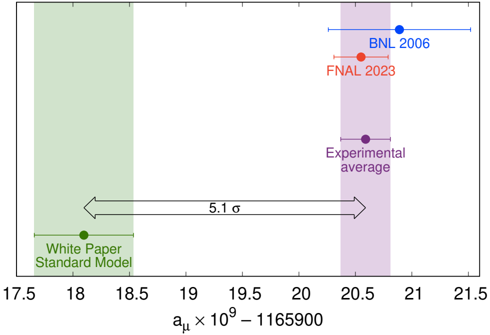
$$a_\mu|_{\text{exp.}} = 0.00116592059(19)$$
$$a_\mu|_{\text{ref.}} = 0.00116591810(43)$$
$$\text{diff.} = 0.00000000249(49)$$

Experimental uncertainties $<$ theory ones and 5.1σ disagreement (probability $\lesssim 1/3\,000\,000$)

$\Rightarrow$ usually signals new physics

Check most uncertain contribution (HVP) w/ fully independent methods

$\rightarrow$ *ab initio* calculations using **lattice quantum chromodynamics (QCD)**



Very brief introduction to lattice QCD ($+$ QED)

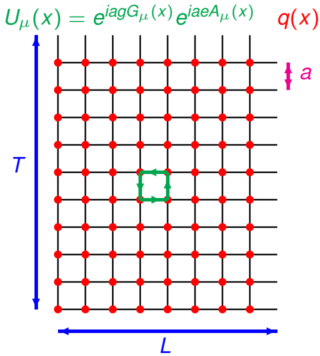
What is lattice QCD (LQCD) + QED?

To describe low-energy, strong (& electromagnetic) interaction phenomena w/ sub-% precision
$\rightarrow$ QCD + QED requires ≥ 132 numbers at every spacetime point
$\Rightarrow$ infinitely dense number of numbers in our continuous spacetime
$\Rightarrow$ must temporarily "simplify" the theory to calculate (regularization)
$\Rightarrow$ Lattice gauge theory $\longrightarrow$ mathematically sound definition of QCD (beyond PT) & QED:

- UV (& IR) cutoff $\rightarrow$ well defined functional integral in Euclidean spacetime:

$$\langle O\rangle = \int \mathcal{D}U\mathcal{D}A\mathcal{D}\bar{q}\mathcal{D}q\, e^{-S_G-\int \bar{q}D[M]q}\, O[U,A,q,\bar{q}]$$
$$= \int \mathcal{D}U\mathcal{D}A\, e^{-S_G}\, \det(D[M])\, O[U,A]_{\rm Wick}$$

- $\mathcal{D}U\mathcal{D}A\, e^{-S_G}\det(D[M]) \geq 0$ & finite # of dofs
$\rightarrow$ evaluate numerically using stochastic methods



L(QCD+QED) is really QCD+QED: must tune $m_q \rightarrow m_q^{\rm ph}$ & $\Lambda_{\rm QCD} \rightarrow \Lambda_{\rm QCD}^{\rm ph}$, $e \rightarrow e^{\rm ph}$, $a \rightarrow 0$ (after renormalization), $L, T \rightarrow \infty$ (and stats $\rightarrow \infty$)

HUGE conceptual and numerical ($10^{10} \rightarrow 10^{11}$ dofs) challenge

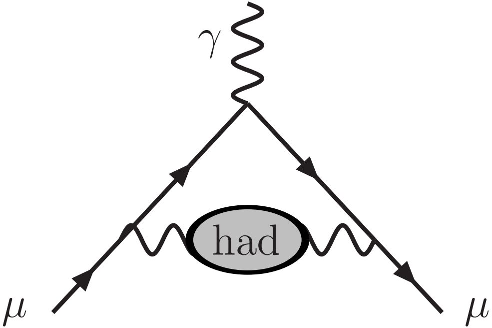
Our particle "accelerators"

Such computations require some of the world's most powerful supercomputers

- copyright Photothèque CNRS/Cyril Frésillon

- 1 year on HAWK supercomputer
  $O(10^5)$ years on laptop
- In Germany, those of the Forschungszentrum Jülich, the Leibniz Supercomputing Centre (Munich), and the High Performance Computing Center (Stuttgart); in France, of the Institute for Development and Resources in Intensive Scientific Computing (IDRIS) of the CNRS, the Centre Informatique National de l'Enseignement Supérieur (CINES) and the Very Large Computing Centre (TGCC) of the CEA by way of the French Large-scale Computing Infrastructure (GENCI); in Europe, those administered by EuroHPC.
- Soon in Europe: exaflop supercomputers ($\sim 10^{18}$ flop/s), i.e. $\sim 40\times$ faster

Lattice QCD calculation of $a_\mu^{\text{LO-HVP}}$



All quantities related to a_μ will be given in units of 10^{-10}

$a_\mu^{\text{LO-HVP}}$ from LQCD: introduction

Compute on $T \times L^3$ Euclidean-time lattice w/ spacing a [Bernecker et al '11]

$$C_L(t) = \frac{a^3}{3}\sum_{i=1}^{3}\sum_{\vec{x}}\langle J_i(x)J_i(0)\rangle$$

w/ $J_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c + \cdots$

Decompose ($C_L^{I=1} = \frac{9}{10}C_L^{ud}$)

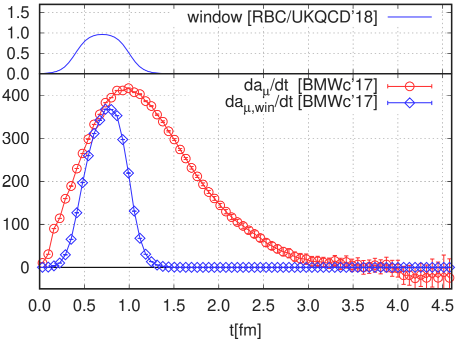
$$\begin{aligned} C_L(t) &= C_L^{ud}(t) + C_L^s(t) + C_L^c(t) + C_L^{\text{disc}}(t) \\ &= C_L^{I=1}(t) + C_L^{I=0}(t) \end{aligned}$$

Then get

$$a_{\mu,f}^{\text{LO-HVP}} = \lim_{\substack{a\to 0\\ L,T\to\infty}} \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{a}{m_\mu^2}\right) \sum_{t=0}^{T/2} K(tm_\mu)\,\mathrm{Re}C_L^f(t)$$

Define "windows" [RBC/UKQCD '18]

$K(\tau) \to W(\tau;\tau_i,\tau_f,\bar{\Delta})K(\tau)$



Challenges

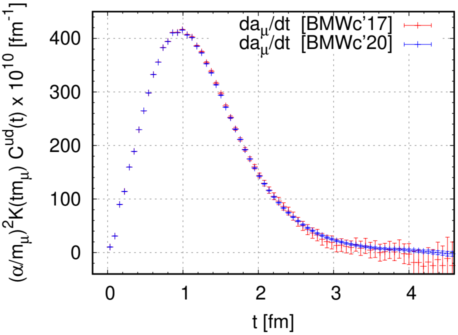
(a) Statistical uncertainties of light and disconnected contributions

(b) Finite *V* (and *T*) corrections on $I = 1$ contribution

(c) Continuum limits

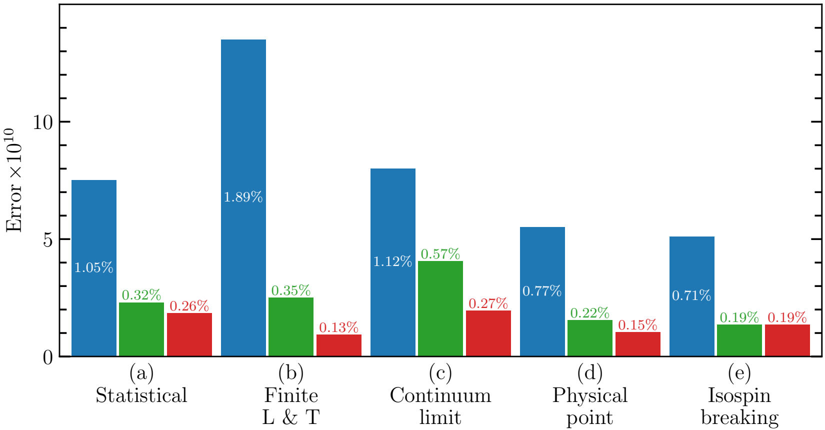
(e) Tuning of physical point $\leftrightarrow$ very precise determination QCD parameters: scale and m_u, m_d, m_s, m_c masses

(f) For subpercent accuracy, must include small effects from electromagnetism (QED) and due to fact that masses of *u* and *d* quarks are not quite equal (SIB)



Uncertainty reduction

2017 → 2020 → 2024



⇒ uncertainty reduced by:

- 2017 → 2020: ÷3.4 or 19. → 5.5
- 2020 → 2024: ÷1.7 or 5.5 → 3.3

Strategy for improvement

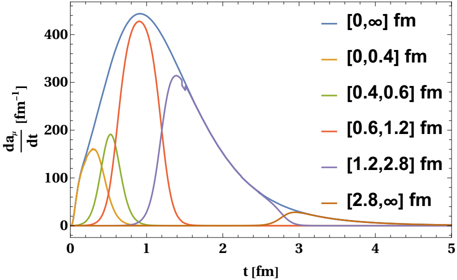
- New simulations on finer ("Monster") lattice: $a = 0.064\,\mathrm{fm}$ $[96^3 \times 144] \longrightarrow a = 0.048\,\mathrm{fm}$ $[128^3 \times 192]$
  - $\rightarrow$ 80% nearer continuum limit (in a^2)
  - $\rightarrow$ reduces $a \rightarrow 0$ error

- Break up analysis into optimized set of windows: $0-0.4$, $0.4-0.6$, $0.6-1.2$, $1.2-2.8\,\mathrm{fm}$

  Continuum extrapolate $I = 0$ instead of disconnected
  - $\rightarrow$ better control over $a \rightarrow 0$ limit
  - $\rightarrow$ overall reduction of uncertainties

- Data-driven evaluation of tail: $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ (proposed and used w/ $1\,\mathrm{fm} \rightarrow \infty$ [RBC/UKQCD '18])
  - $\rightarrow$ reduces FV correction $18.5(2.5) \rightarrow 9.3(9)$, i.e. cv $\div 2$ & err $\div 3$
  - $\rightarrow$ reduces long-distance (LD) noise
  - $\rightarrow$ reduces $a \rightarrow 0$ error some

- Calculation was fully blinded

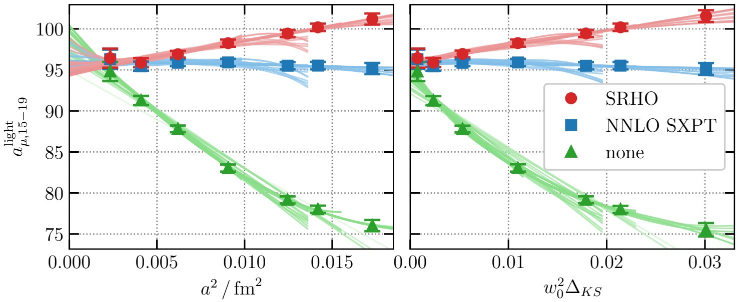


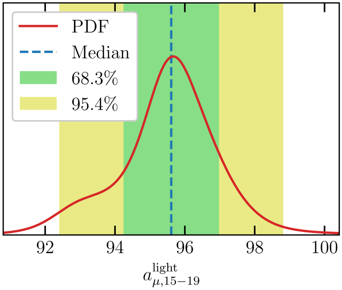
[plot made w/ KNT '18 data set]

July 12, 2024: unblinding

Preprint uploaded to arXiv on July 15, 2024

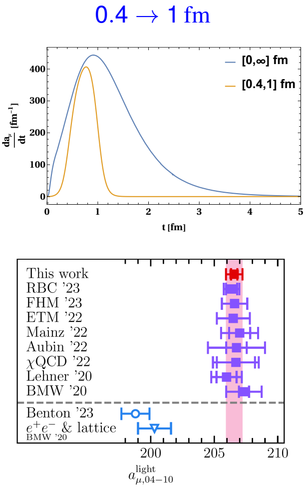
1.5−1.9 fm window [Aubin et al '22]

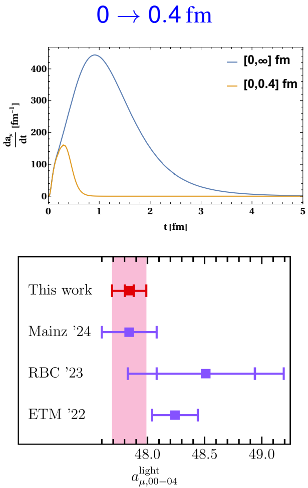


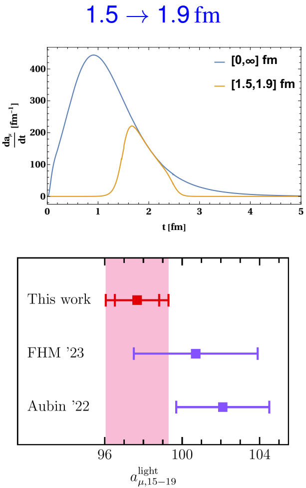


Median	95.61	
Total error	1.60	1.68 %
Statistical error	1.14	1.19 %
Systematic error	1.13	1.18 %
Pseudoscalar fit range	0.03	0.03 %
Physical value of M_{ss}	0.01	0.01 %
w_0 scale setting	0.67	0.70 %
Taste breaking correction	0.40	0.42 %
Lattice spacing cuts	0.11	0.12 %
Order of fit polynomials	0.21	0.22 %
Continuum parameter (Δ_{KS} or a^2)	0.34	0.36 %

Benchmarking of lattice calculation: windows





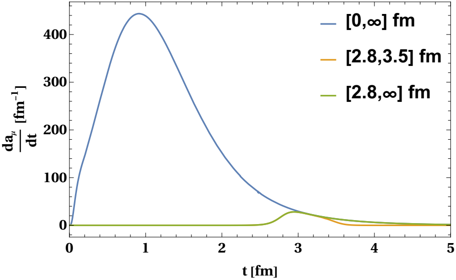


light $=$ *ud* contribution to long-distance window ($1 \to \infty$ fm):

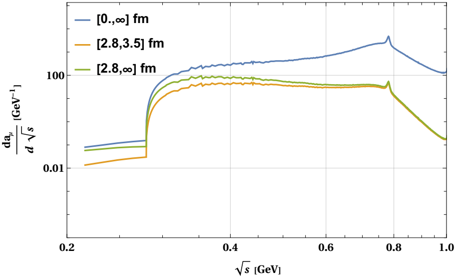
411.4[4.9] [RBC/UKQCD '24] ; 410.7[5.9] [Mainz '24, BMW world]

401.2[4.3] [RBC/UKQCD '24]

Tail contribution from $\sigma(e^+e^- \to \text{hadrons})$



$\updownarrow$

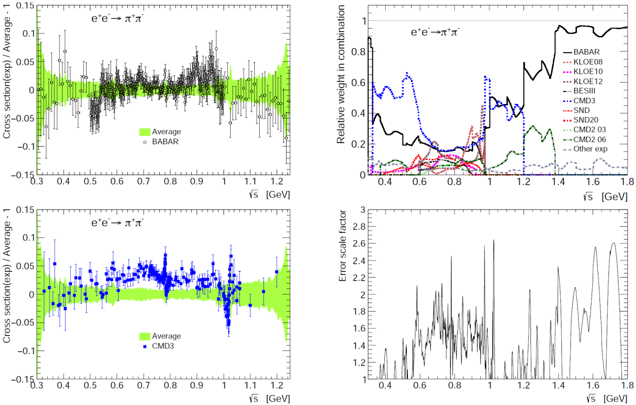


- Lattice computation for $t \leq 2.8\,\text{fm}$: $> 95\%$ of final result for $a_\mu^{\text{LO-HVP}}$
- Tail $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ computed using $e^+e^- \to$ hadrons for $t > 2.8\,\text{fm}$: $\lesssim 5\%$ to final result for $a_\mu^{\text{LO-HVP}}$
- Tail dominated by cross section below ρ peak: $\sim 75\%$ for $\sqrt{s} \leq 0.63\,\text{GeV}$
- Partial tail $a_{\mu,28\text{-}35}^{\text{LO-HVP}}$ ($2.8\,\text{fm} < t \leq 3.5\,\text{fm}$) for comparison with lattice dominated by cross section below ρ peak: $\sim 70\%$ for $\sqrt{s} \leq 0.63\,\text{GeV}$
- Region well controlled by theory (χPT, analyticity, unitarity, ...) and other experimental constraints (e.g. $\langle r_\pi^2 \rangle$)

[plots made w/ KNT '18 data set]

$\sigma(e^+e^- \to \text{hadrons})$ for the tail

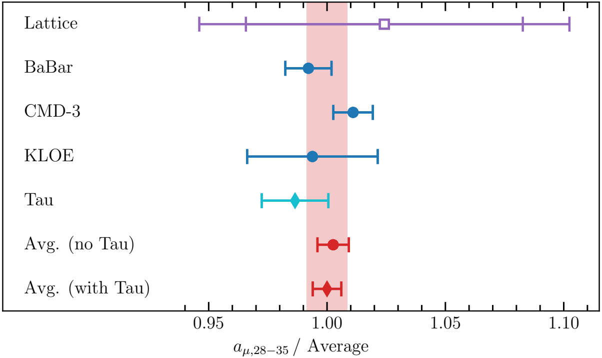
Tail $a_{\mu,28\text{-}\infty}^{\text{LO-HVP}}$ dominated cross section below ρ peak: $\sim$ 75% for $\sqrt{s} \leq 0.63\,\text{GeV}$



All measurements agree to within 1.4σ for $\sqrt{s} \lesssim 0.55\,\text{GeV}$

$\Rightarrow$ tensions that plague $a_\mu^{\text{LO-HVP}}$ & $a_{\mu,\text{win}}^{\text{LO-HVP}}$ not present here

Data-driven partial-tail comparison with lattice

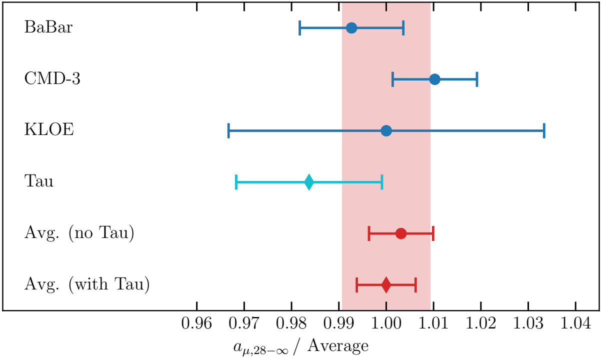


- Window from $2.8 \to 3.5$ fm
- All data-driven result agree very well
- Weighted average taken w/ and w/out τ: $\chi^2/\text{dof} = 1.1$ for both
- Final number: average w/ τ, PDG factor, and systematic = full difference τ/no-τ added linearly

$$a_{\mu,28\text{-}35}^{\text{LO-HVP}} = 18.12(11)(5)[16]$$

- Excellent agreement w/ lattice, but uncertainty reduced by factor ~ 15

Data-driven tail

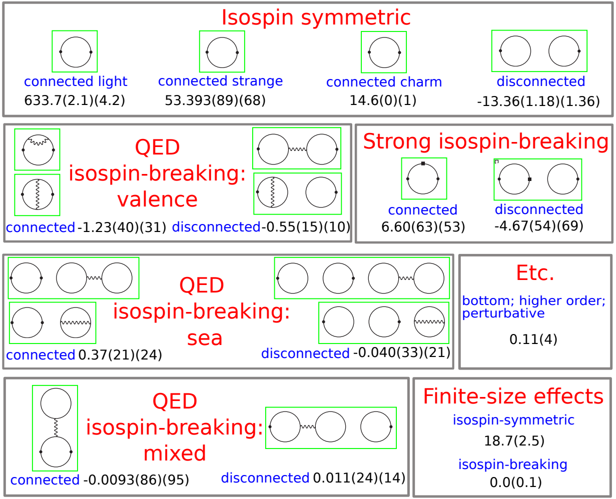


- Window from $2.8 \to \infty$ fm
- All data-driven result agree very well
- Weighted average taken w/ and w/out τ: $\chi^2/\text{dof} = 1.0$ and 0.8
- Final number: average w/ τ, and systematic = full difference τ/no-τ added linearly

$$a_{\mu,28\text{-}\infty}^{\text{LO-HVP}} = 27.59(17)(9)[26]$$

- Only $\lesssim 5\%$ of final result for a_μ

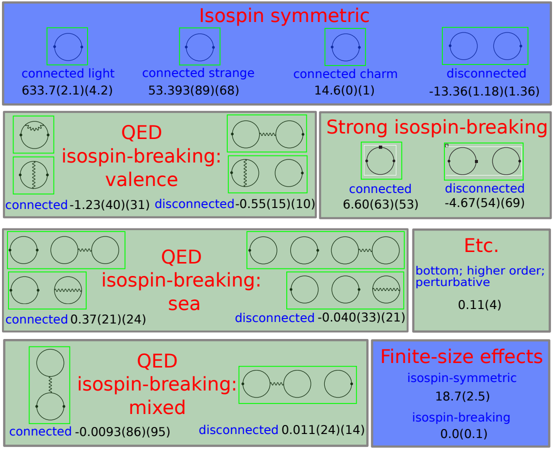
Summary of contributions to $a_\mu^{\text{LO-HVP}}$: 2020



$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

Improved in new work

Some checks in new work



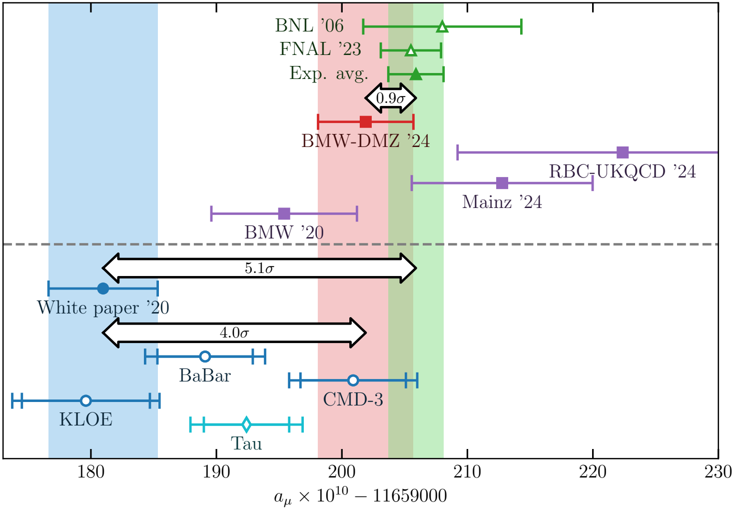
$$10^{10}\times a_\mu^{\text{LO-HVP}} = 707.5(2.3)_{\text{stat}}(5.0)_{\text{sys}}[5.5]_{\text{tot}}$$

↓ 2024

$$10^{10}\times a_\mu^{\text{LO-HVP}} = 714.1(2.2)_{\text{stat}}(2.5)_{\text{sys}}[3.3]_{\text{tot}}$$

Corresponds to a 4.6 per mil total uncertainty

BMW-DMZ '24 vs $g-2$ measurement



Indicates Standard Model confirmed to 0.37 ppm !

Conclusions

- New calculation of $a_\mu^{\text{LO-HVP}}$ to 0.46%
- Fully blinded analysis
- Lattice calculation of $0 \to 2.8\,\text{fm}$ window $> 95\%$ of total
- Data-driven evaluation of $2.8 \to \infty\,\text{fm}$ window $\leq 5\%$ of total
- Error reduction:
  - $\sim 37\%$ from lattice improvements
  - additional $\sim 30\%$ from data-driven tail
- Checks on QED and SIB corrections
- Our result indicates that SM confirmed to 0.37 ppm
- Lattice calculation agrees w/ others in windows: $0 \to 0.4\,\text{fm}$, $0.4 \to 1.0\,\text{fm}$ & $1.5 \to 1.9\,\text{fm}$
- Even newer lattice calculations [RBC/UKQCD '24, Mainz '24] are $\sim 1.5\sigma$ larger
- Lattice calculations of long-distance contribution $1 \to \infty\,\text{fm}$ of u and d quarks [RBC/UKQCD '24, Mainz '24, FHM '24] important step to further confirm agreement w/ data-driven tail

Conclusions

- Eagerly await
  - Fermilab $\sim$ 0.1 ppm measurement of a_μ in 2025
  - J-PARC entirely new method for a_μ measurement
  - Lattice results for complete $a_\mu^{\text{LO-HVP}}$ by FHM expected soon
  - New BaBar $e^+e^- \to$ hadrons analysis by early 2025
  - New KLOE analysis
  - New BES III, BELLE-II, CMD-3, SND-2 data and analysis
  - MUonE @ CERN for spacelike HVP