Building spacetime from entanglement

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Be.HEP Leuven 2024

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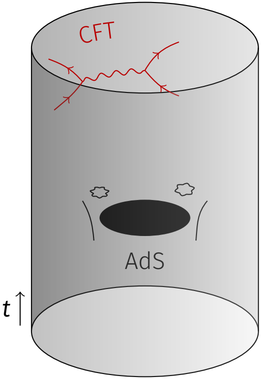
The AdS/CFT correspondence

Maldacena (1998):



Quantum gravity in $d+1$ dimensional negatively curved (Anti de Sitter) spacetime

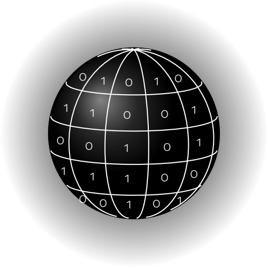
$\Leftrightarrow$

Conformal field theory on the d dimensional AdS boundary

Implies a one-to-one map between CFT and gravity observables

How is the AdS geometry encoded in the CFT state?

Entropy and area

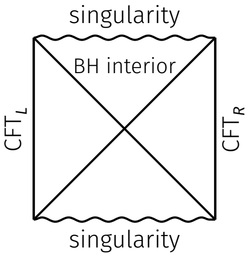


Bekenstein (1972), Hawking (1974):

$$S_{\mathrm{BH}} = \frac{\mathrm{Area}[\mathscr{E}_{\mathrm{horizon}}]}{4G_N}$$

In AdS/CFT:

$$S_{\mathrm{BH}} = -\mathrm{Tr}[\rho(T)\log\rho(T)] \quad \text{with} \quad T = T_{\mathrm{Hawking}}$$

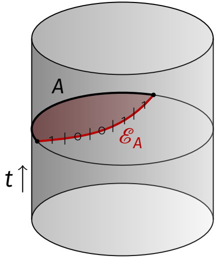


Maldacena (2001): Two-sided AdS black hole described by thermofield-double state

$$|\psi\rangle = \frac{1}{\sqrt{Z(\beta)}}\sum_n e^{-\beta E_n/2}|n\rangle_L|n\rangle_R$$

Black hole entropy due to entanglement between CFT_L and CFT_R

Entanglement entropy and area



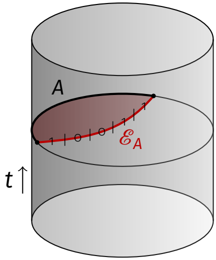
Ryu, Takayanagi (2006):

$$S_A = -\mathrm{Tr}[\rho_A \log \rho_A] = \min_{\mathcal{E}_A} \frac{\mathrm{Area}[\mathcal{E}_A]}{4G_N}$$

Applications:

- "entanglement builds geometry" (Swingle (2009), Van Raamsdonk (2010))
  - → Derivation of (linearized) Einstein's equations from first law of entanglement (Lashkari, McDermott, Van Raamsdonk (2013))
  - → Subregion-subregion duality (Czech, Karczmarek, Nogueira, Van Raamsdonk (2012))

Entanglement entropy and area



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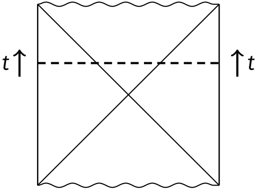
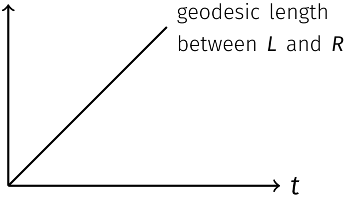
Applications:

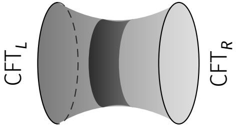
- "entanglement builds geometry" (Swingle (2009), Van Raamsdonk (2010))
- Resolution of information paradox for AdS black holes evaporating into a bath system (Penington; Almheiri, Engelhardt, Marolf, Maxfield (2019))
- Diagnostic of confinement/deconfinement transition (Kelbanov, Kutasov, Murugan (2007))
- ...

Issues with “entanglement builds geometry”

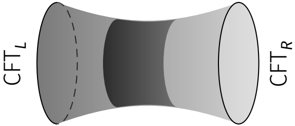
Hartman, Maldacena (2013), Susskind (2014): growth of wormhole length not captured by entanglement entropy







early times



late times

Issues with “entanglement builds geometry”

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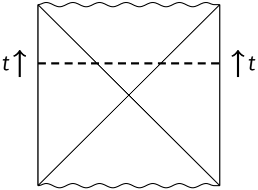

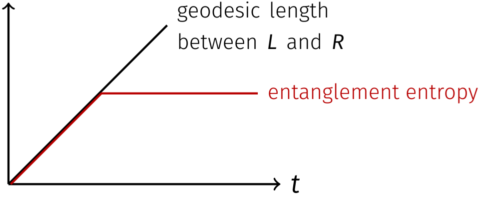

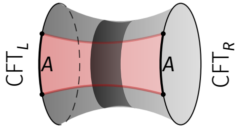

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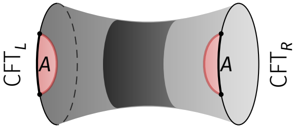

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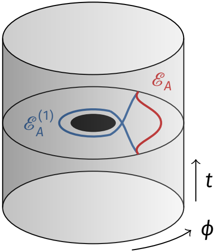

- Minimal surfaces $\mathcal{E}_A$ don’t probe all features of the geometry
- What about non-minimal surfaces (e.g. with non-zero winding number)?

$$S_A^{(w)} \stackrel{?}{=} \frac{\text{Area}[\mathcal{E}_A^{(w)}]}{4G_N}$$

→ Need to take into account entanglement not only between spatial DoF but also between different fields

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How does a holographic CFT look like?

Simple examples of conformal field theories in 2d:

- free massless scalar

$$S_{\text{bos.}}[X] = \frac{1}{2\pi} \int d^2x (\partial_{x_+} X \partial_{x_-} X)$$

- free massless Majorana fermion

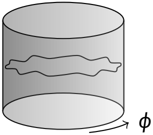
$$S_{\text{ferm.}}[\psi] = \frac{1}{8\pi} \int d^2x (\psi_+ \partial_{x_-} \psi_+ + \psi_- \partial_{x_+} \psi_-)$$

Example of holographic theory ("D1/D5" CFT, Strominger, Vafa (1996)): N indistinguishable copies of 4 scalars and fermions

$$S_{\text{hol.}} = \sum_{i=1}^{N} \sum_{a=1}^{4} (S_{\text{bos.}}[X_i^a] + S_{\text{ferm.}}[\psi_i^a]) + \text{interactions}$$

Properties:

- $N \sim 1/G_N$
- weak coupling: very large strings in the dual AdS space



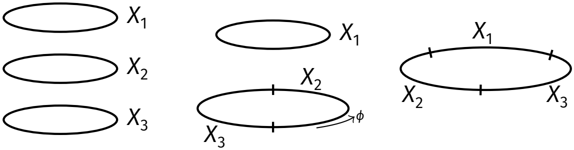
Boundary conditions and twisted sectors

- The holographic CFT admits states with non-trivial "twisted" boundary conditions

$$X_i(\phi + 2\pi) = X_{g(i)}(\phi) \quad \text{for} \quad g \in S_N$$

→ Strands of multiple fields joined together by the boundary conditions

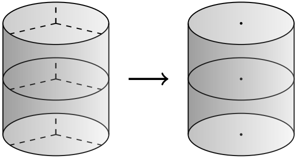
example: $N = 3$



- Long strands ~ winding strings in the AdS space (Eberhardt, Gaberdiel, Gopakumar (2018))

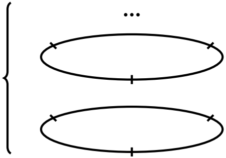
Example: naked singularity

Naked singularity: identification of $m \in \mathbb{N}$ wedges of empty AdS



dual to ground state of twisted sector with long strands of length m

N/m
strands



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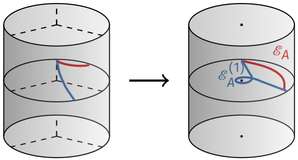

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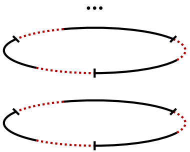
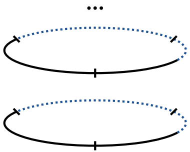


same subregion for all fields: no winding in AdS space



different subregions for different fields: non-zero winding

Balasubramanian, Chowdhury, Czech, de Boer (2014); Balasubramanian, Bernamonti, Craps, de Jonckheere, Galli (2016), Balasubramanian, Craps, de Jonckheere, Sárosi (2018), Erdmenger, MG (2019)

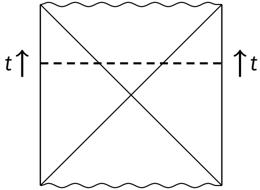
Example: black hole

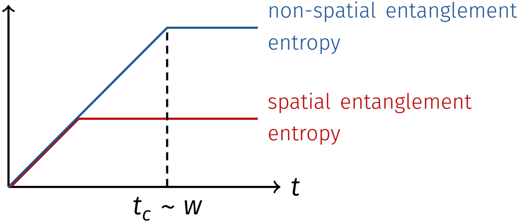
- CFT in thermal state with temperature set by Hawking temperature

→ mixture of all possible twisted sectors

- Entanglement only between spatial DoF dual to length of non-winding geodesic
- Winding geodesics also need entanglement between different fields
- Concretely: entanglement entropy dual to length of geodesic with winding number w (MG 2021):
    - choose bipartition where only long strands of length $m\mathbb{N}$ contribute
    - on these long strands, choose subregion covering w fields completely and another field partially

Two-sided black hole





- "Growth of wormhole" encoded in non-spatial entanglement
- Winding number limit $w < N \sim 1/G_N$: breakdown of entanglement/geometry connection as we approach a singularity?
- Motivation to study subleading orders in $G_N \sim 1/N$ expansion

$$S_A^{(w)} = \frac{\text{Length}[\mathscr{E}_A^{(w)}]}{4mG_N} + O(G_N^0)$$

- Small winding numbers: only small corrections (MG, D. He arXiv:24xx.xxxxx), confirming robustness of "entanglement builds geometry"

Summary

- AdS/CFT correspondence encodes the AdS geometry in the entanglement structure of the CFT state
- Entanglement between internal DoF is necessary to describe all features of the bulk geometry

Open questions:

- Entanglement between internal DoF in higher dimensions?
- String theoretic AdS/CFT constructions based on $\mathrm{AdS}_{d+1} \times$ internal space. Probe geometry of the internal space using entanglement?