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Astrophysical neutrino point sources as a probe of new physics

arXiv:2304.08533

in collaboration with
Stefan Vogl

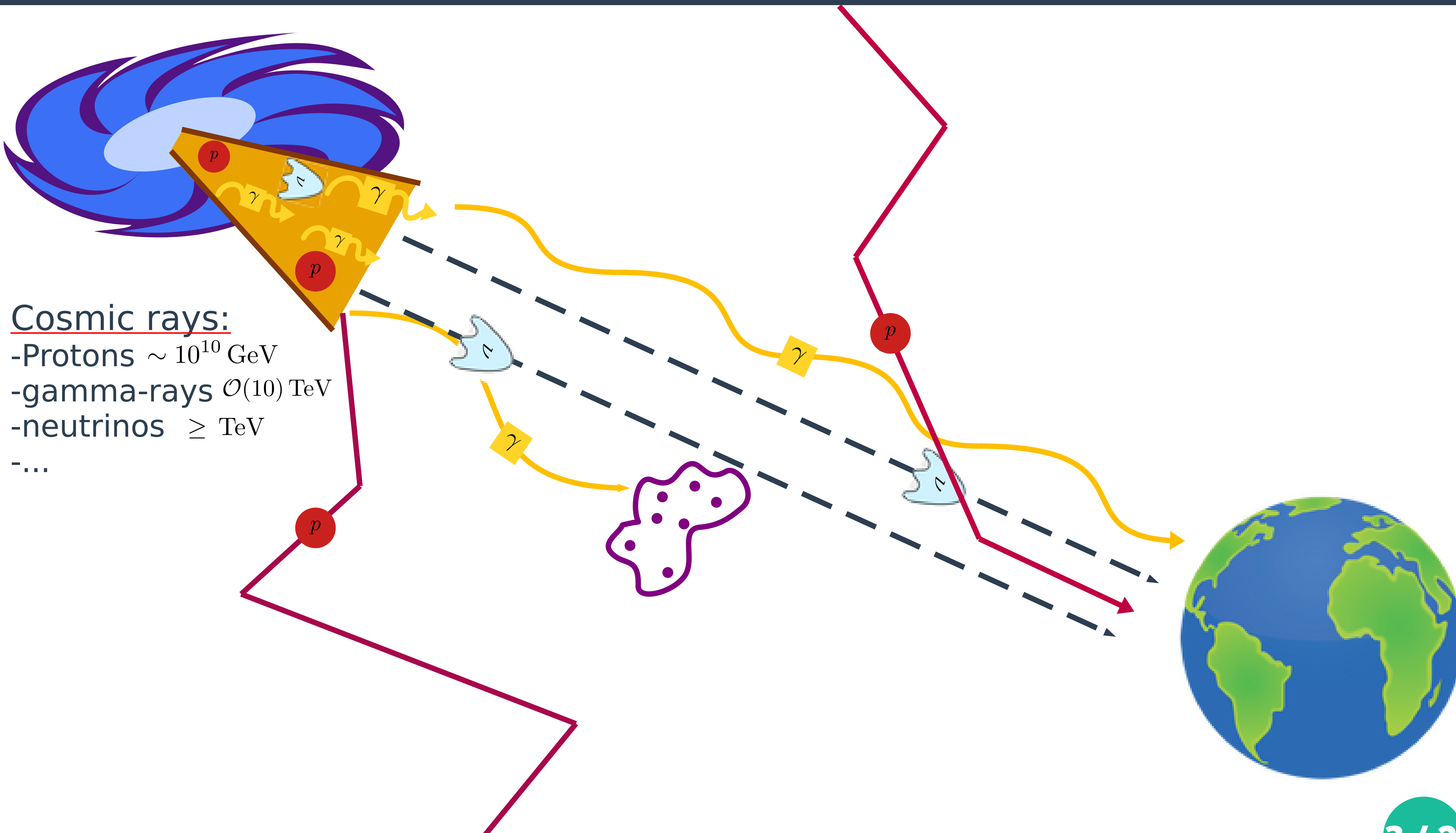
Christian Döring

Be.HEP-Meeting, KU Leuven – 21.06.24

Brief Overview

- **Introduction**
  - **Neutrinos and Cosmic Rays**
  - **Observations: TXS 0506+056 and NGC 1068**
- **New Physics and the Neutrino Signal**
  - Secret neutrino interaction and the cosmic neutrino background
- **Mean free path and flux**
  - Massfull case
  - Massless case
- **Results**
  - Universal Coupling
  - Tauphilic Coupling
  - Future Sources
- **Conclusion**

Cosmic Rays and Neutrinos



Cosmic rays:

- -Protons $\sim 10^{10}\,\mathrm{GeV}$
- -gamma-rays $\mathcal{O}(10)\,\mathrm{TeV}$
- -neutrinos $\geq \mathrm{TeV}$
- -...

Galactic Neutrino Signals

Blazar TXS 0506+056

2018

Neutrino emission from the direction of the blazar TXS 0506+056 prior to the IceCube-170922A alert

IceCube Collaboration*†

Multi-messenger observations of a flaring blazar coincident with high-energy neutrino IceCube-170922A

The IceCube, *Fermi*-LAT, MAGIC, *AGILE*, ASAS-SN, HAWC, H.E.S.S, *INTEGRAL*, Kanata, Kiso, Kapteyn, Liverpool telescope, Subaru, *Swift*/*NuSTAR*, VERITAS, and VLA/17B-403 teams*†

Facts:

Distance: $1.2\,\mathrm{Gpc}$

Flux: $\hat{\Phi}_0 = 1.2 \times 10^{-13} \frac{1}{\mathrm{TeV cm^2 s}}$

Spectral index: $\gamma = 2.0$

Energy: $40\text{-}4000\,\mathrm{TeV}$

arXiv:1807.08794
arXiv:1807.08816

Active Galaxy NGC 1068

2022

Evidence for neutrino emission from the nearby active galaxy NGC 1068

IceCube Collaboration*

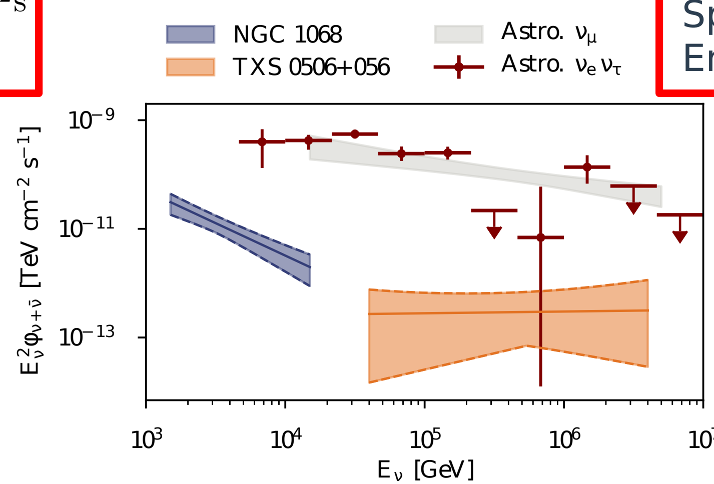
Facts:

Distance: $14.4\,\mathrm{Mpc}$

Flux: $\hat{\Phi}_0 = 5 \times 10^{-11} \frac{1}{\mathrm{TeV cm^2 s}}$

Spectral index: $\gamma = 3.2$

Energy: $1.5\text{-}15\,\mathrm{TeV}$

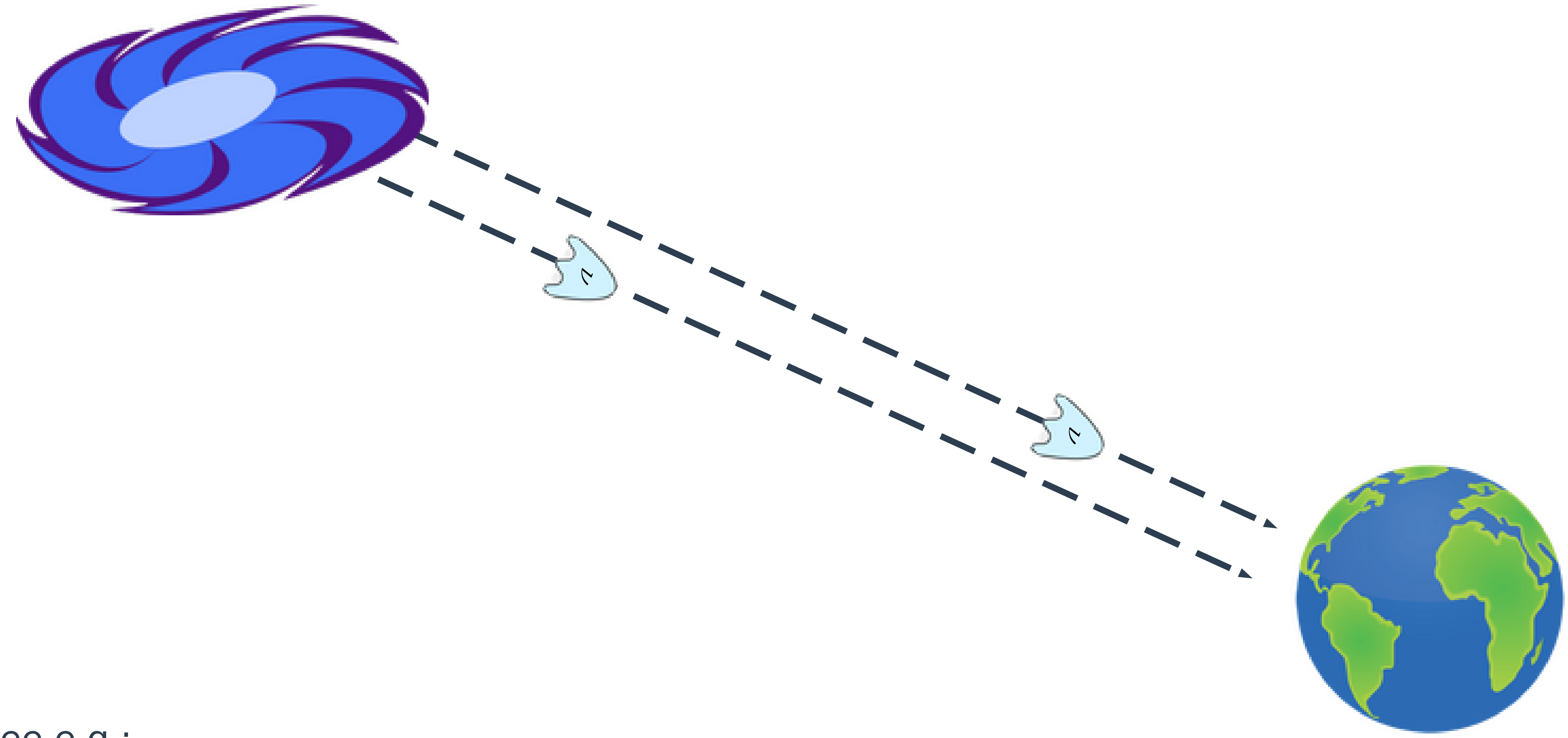


arXiv:2211.09972

What if…?

How to „stop“ Neutrinos

… there would be Sterile Neutrinos?



See e.g.:
arXiv:2211.16520v2
arXiv:2212.00737v2

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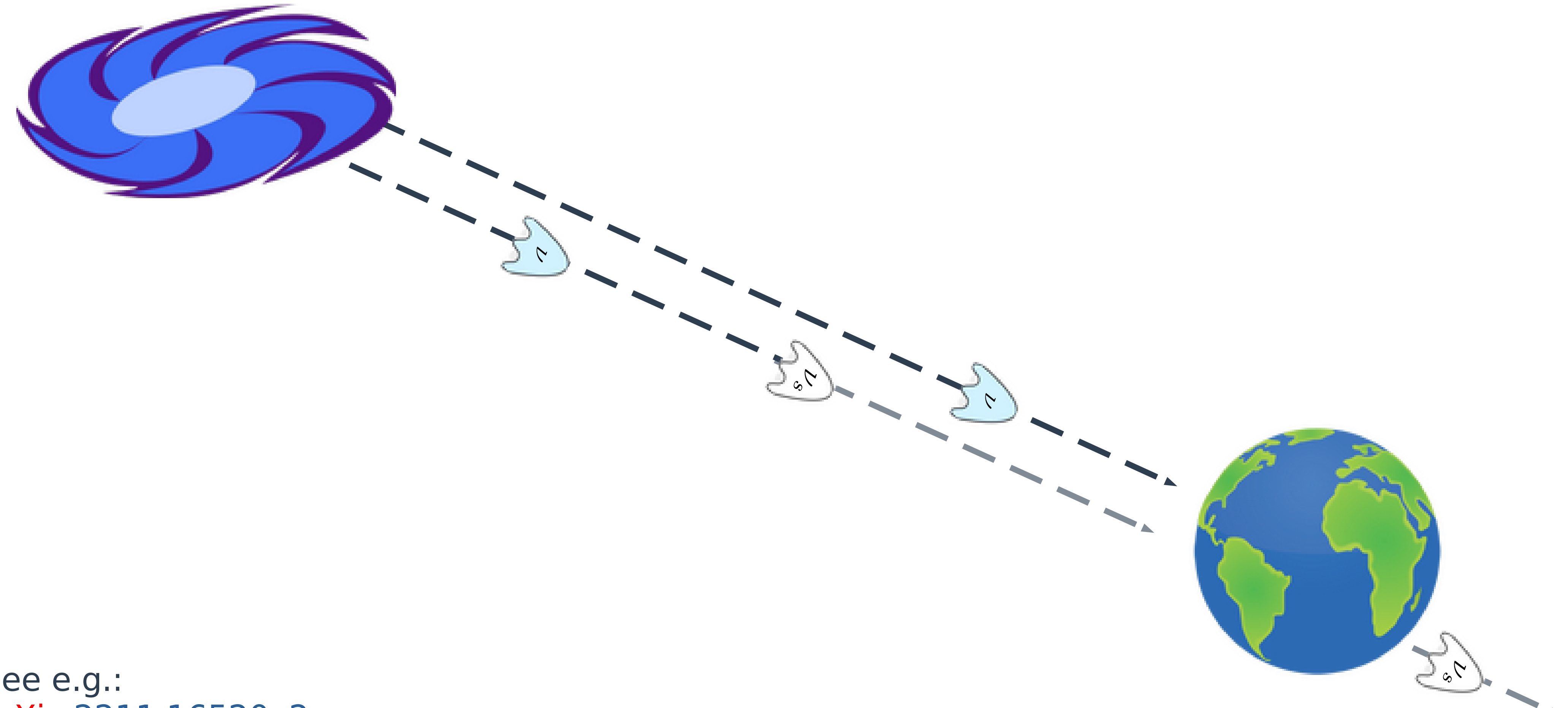

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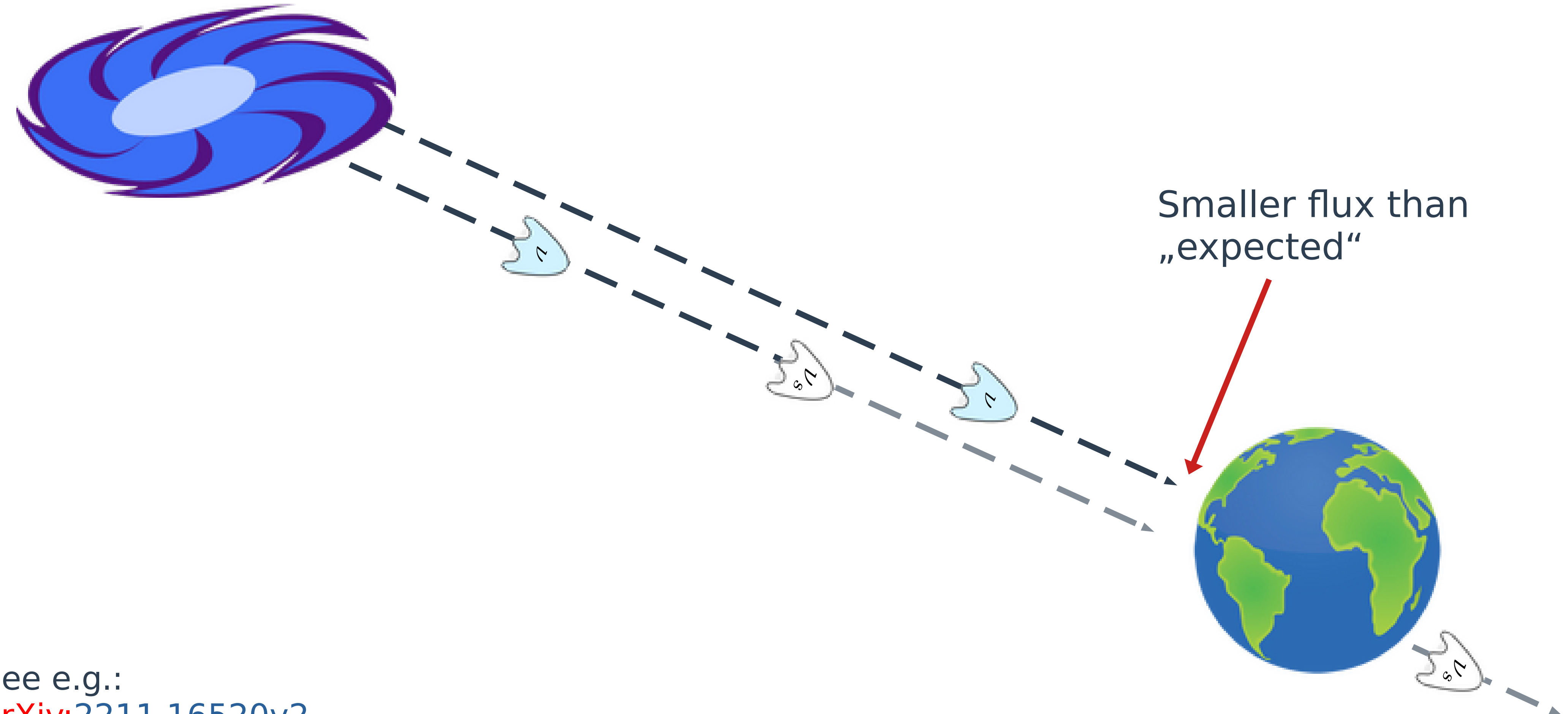

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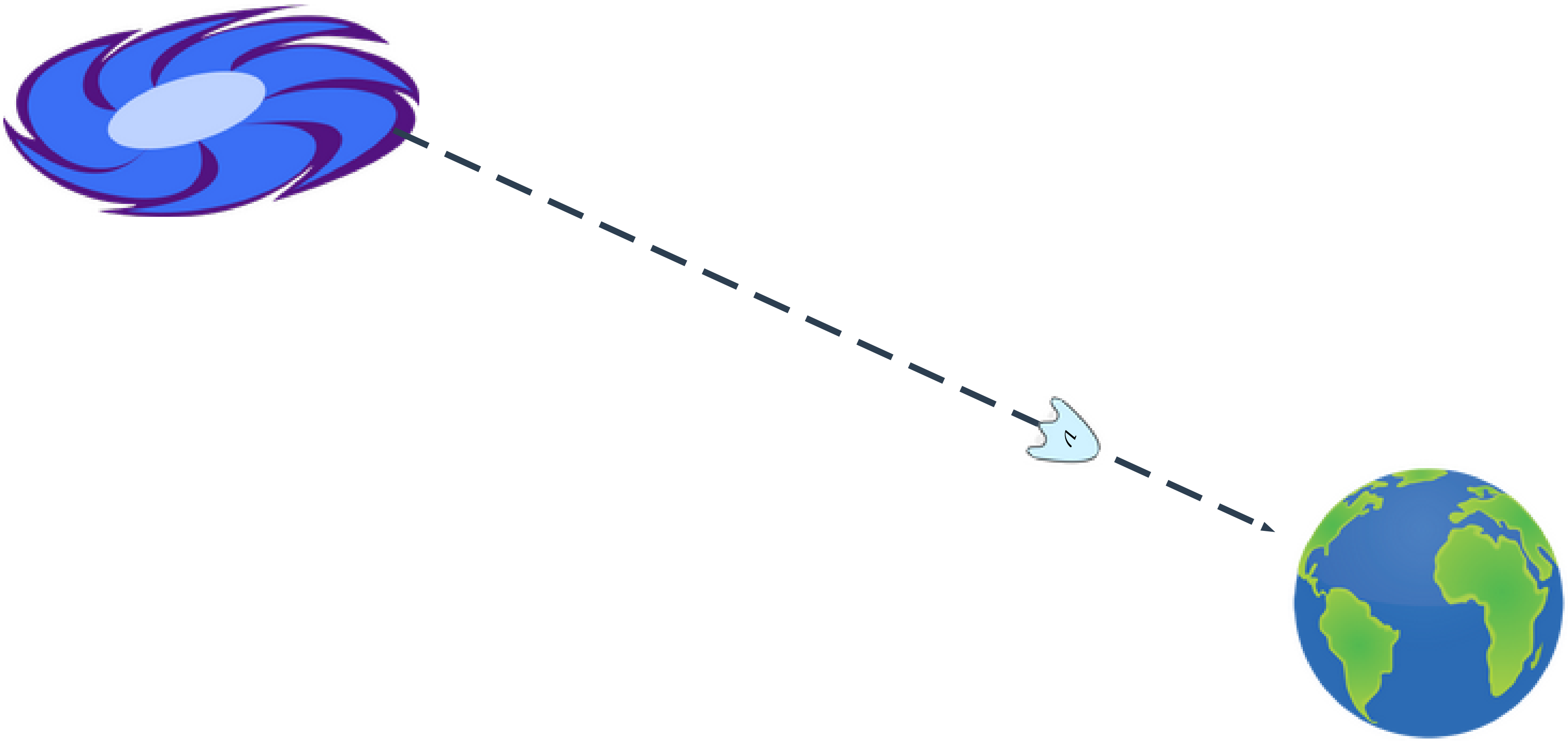
… there would DM-Neutrino interactions?



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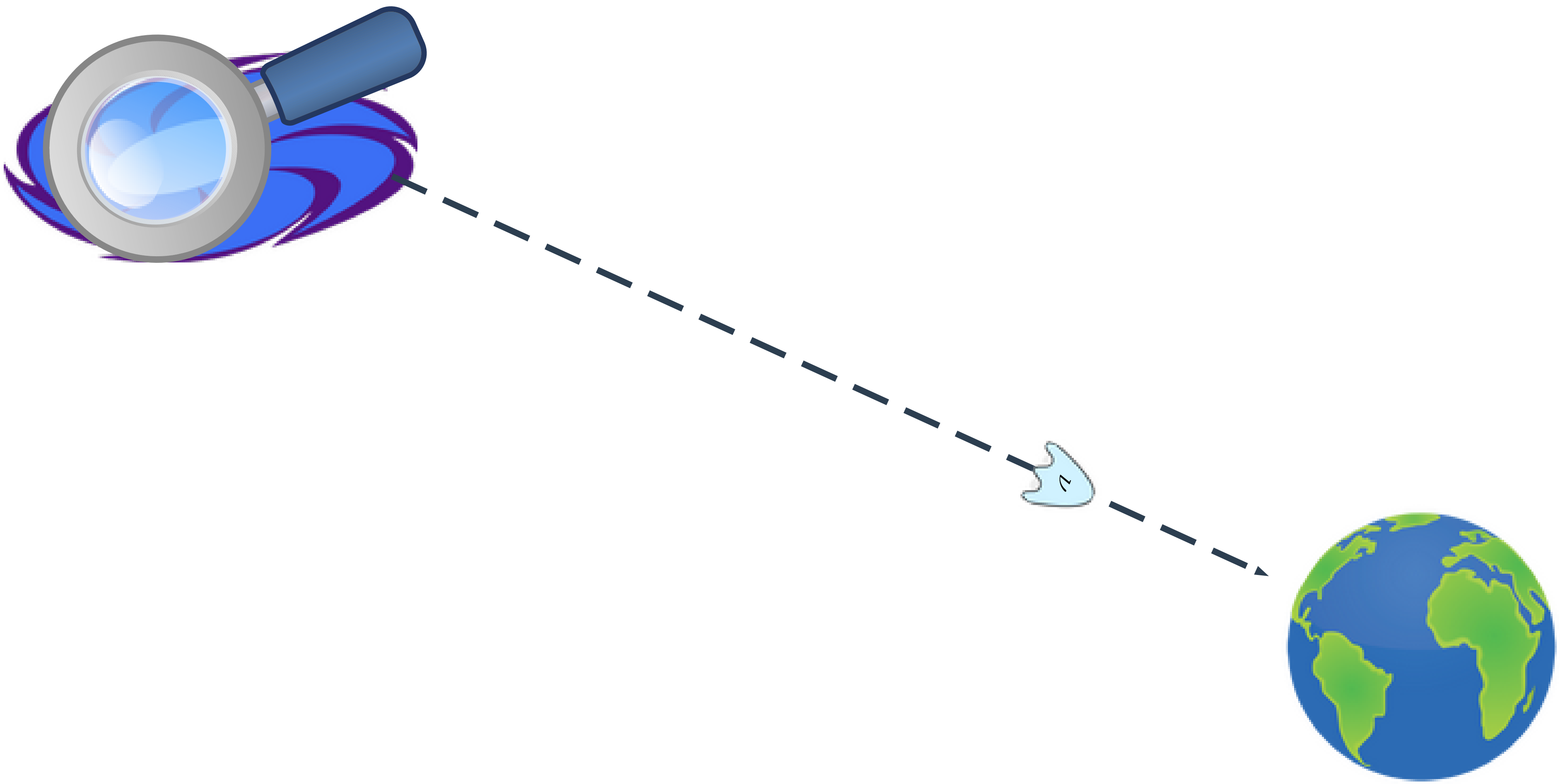

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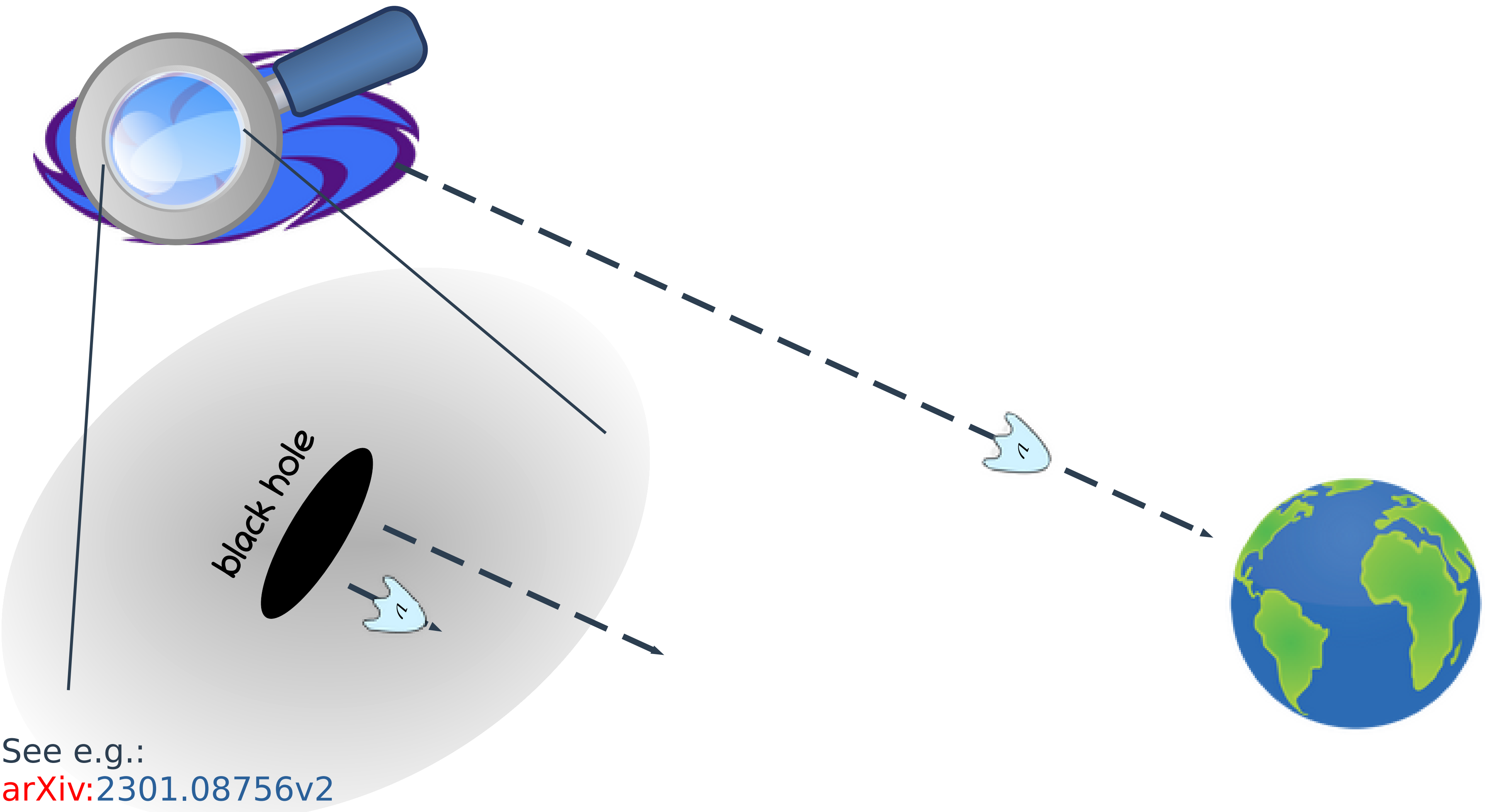

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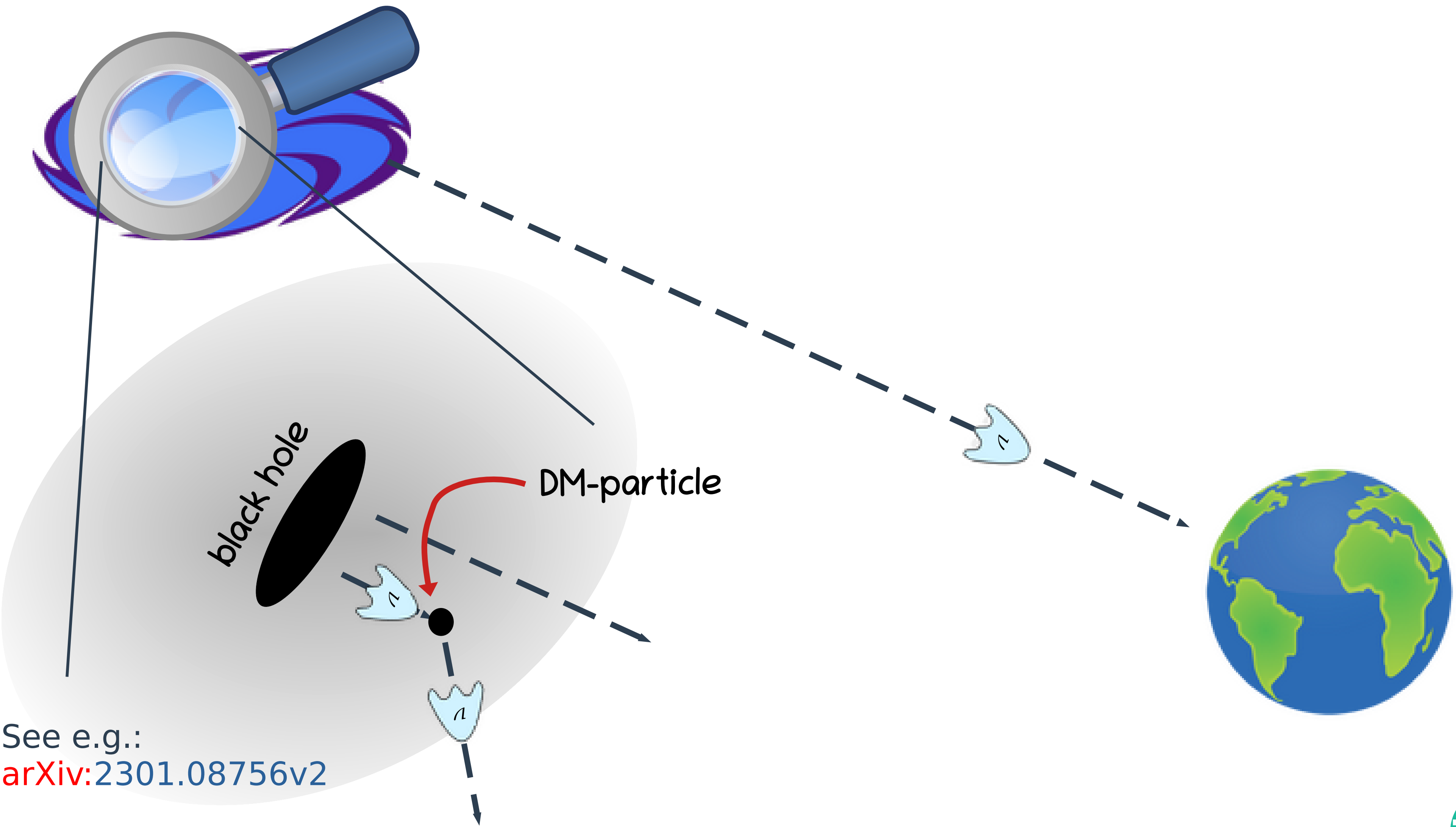

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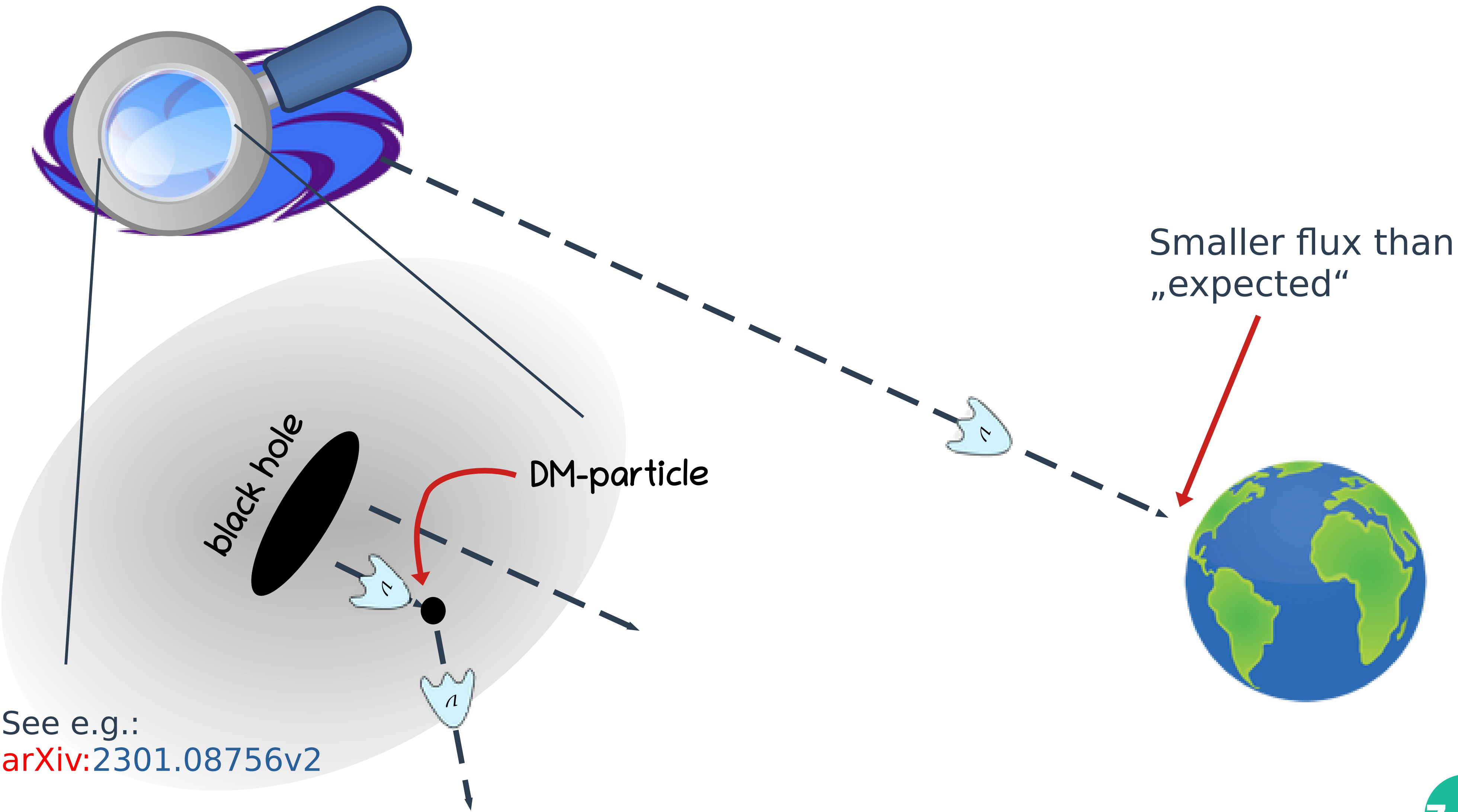

See e.g.:
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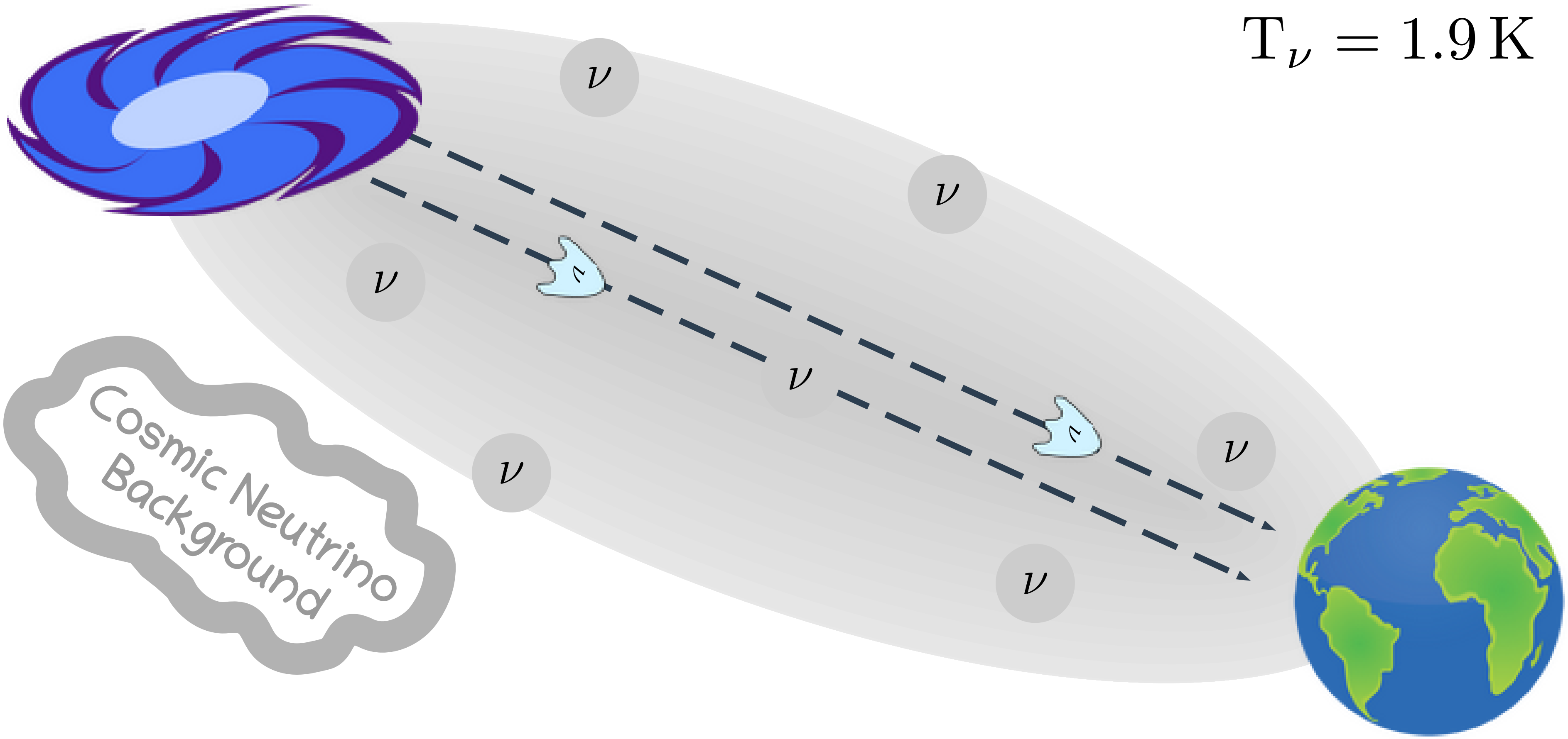
... there would be strong Neutrino-Neutrino interactions?

$$n_{tot} \approx 340\,\mathrm{cm}^{-3}$$

$$T_\nu = 1.9\,\mathrm{K}$$

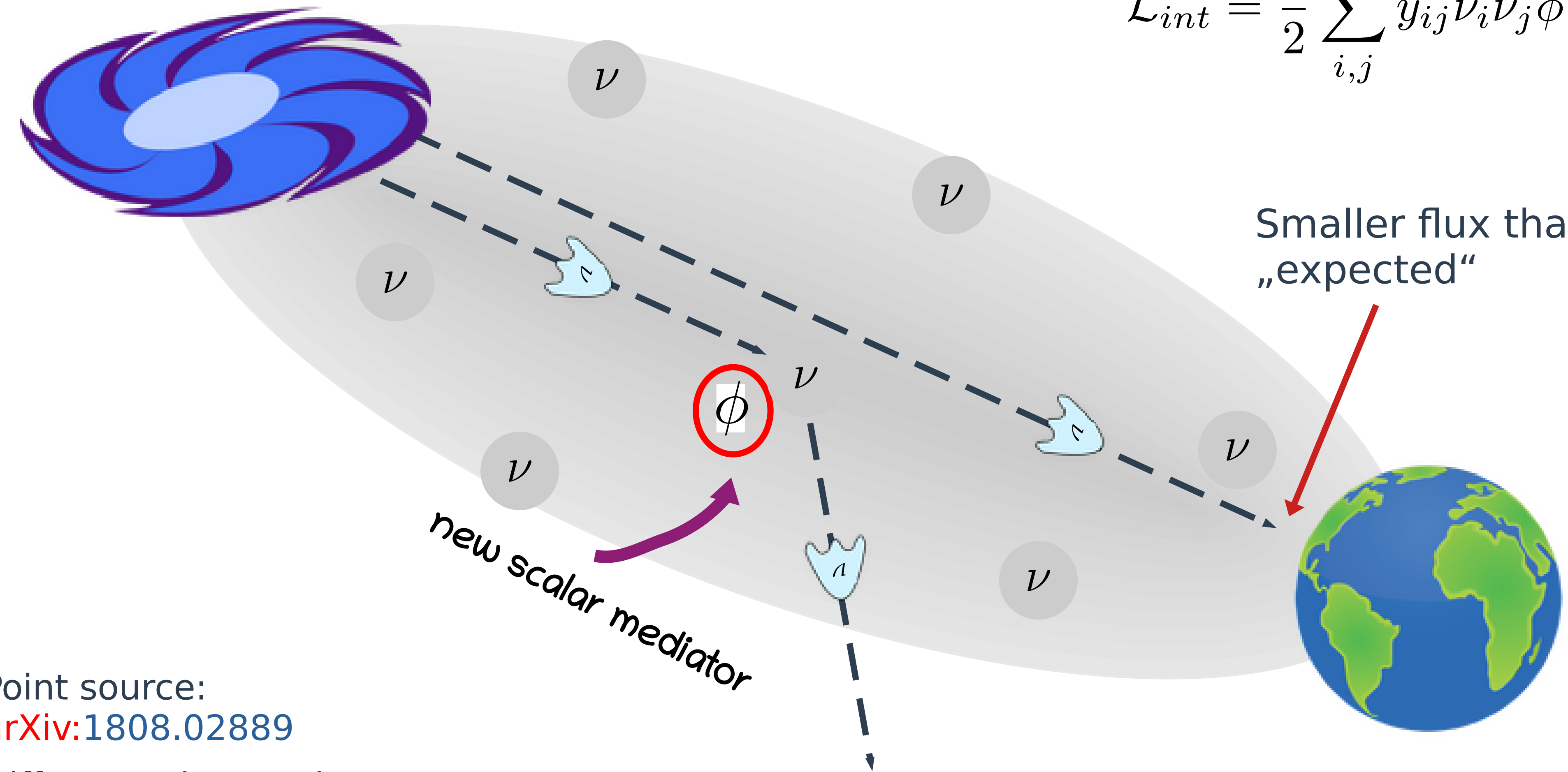


In the standard model the interaction
is negligibley small but...

Cosmic Neutrino Background as a Milk Glass

... in physics beyond the SM interaction can be sizeable!

$$\mathcal{L}_{int} = \frac{1}{2}\sum_{i,j} y_{ij}\bar{\nu}_i\nu_j\phi$$



Point source:
arXiv:1808.02889

Diffuse Background:
arXiv:2107.13568

How far can they come?
The mean free path

The Mean Free Path and the Reduced Flux

Flux

Spectral index

$$\Phi_0(E_\mathrm{a}) = \hat{\Phi}_0 \cdot \left(\frac{E_\mathrm{a}}{1\,\mathrm{TeV}}\right)^{-\gamma} \cdot \mathrm{T}(E_\mathrm{a})$$

Normalised flux

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Normalised flux

Source distance

Transmittance

$$\mathrm{T}(E_\mathrm{a}) = e^{-\frac{d}{\lambda_\mathrm{MFP}(E_\mathrm{a})}}$$

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Interaction Rate

$$\Gamma(E_\mathrm{a})$$

Mean Free Path

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Interaction Rate

Interaction rate

$$\Gamma_i(E_\mathrm{a}) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} f_i(\vec{p})\, v_{M\o l}(E_\mathrm{a}, \vec{p})\, \sigma(s(E_\mathrm{a}, \vec{p}))$$

Energy of
astrophysical neutrino

Interaction Rate

momentum
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Møllervelocity

Interaction Rate

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cross section

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Assumptions on the Neutrino Sector and the Mass of the Lightest Neutrino

Our Neutrino Sector (Assumptions):

-Flavor universal coupling
-Normal mass ordering
-Majorana fermion

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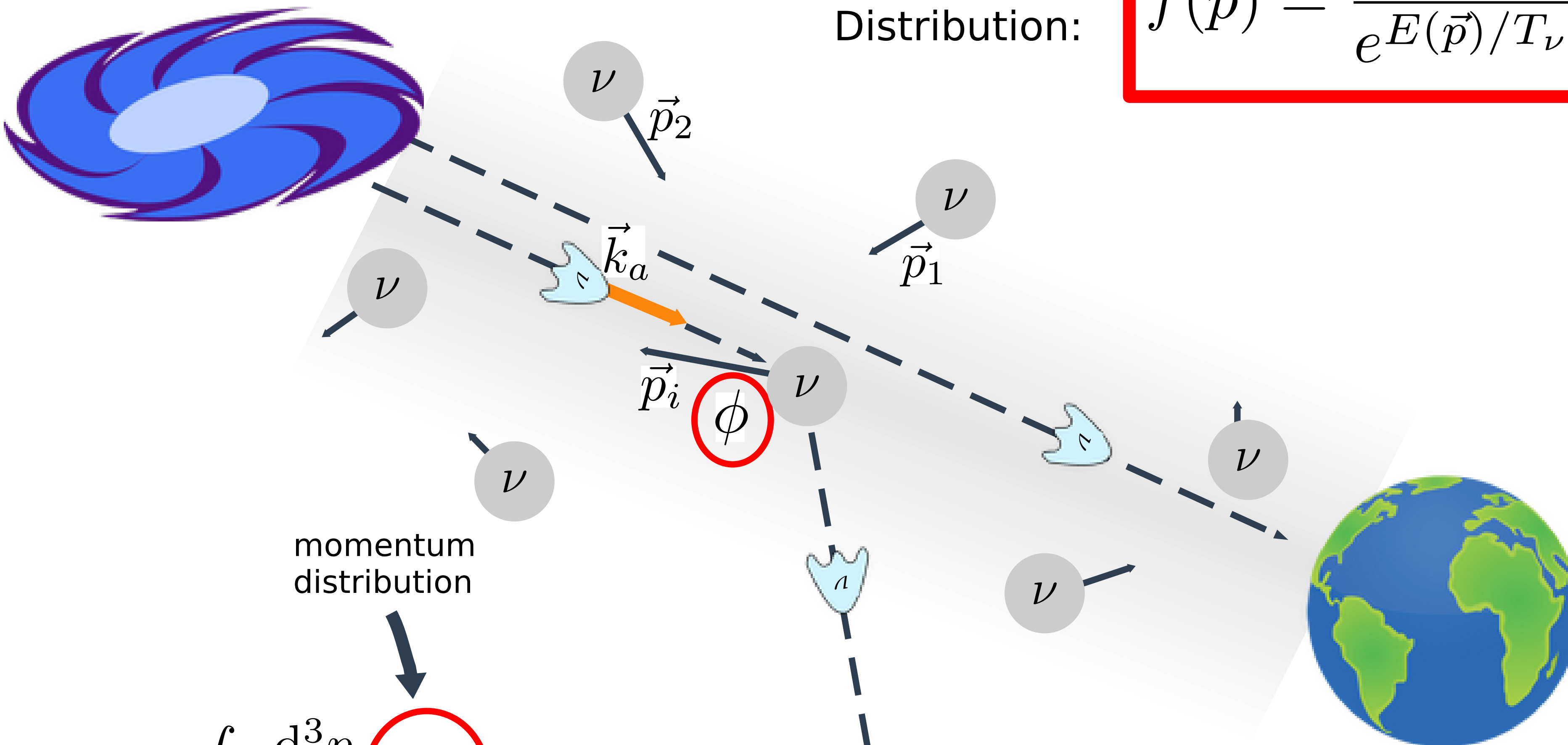
Interaction Rate for the Massless Neutrinos

Momentum-Distribution:

$$f(\vec{p}) = \frac{1}{e^{E(\vec{p})/T_\nu} + 1}$$



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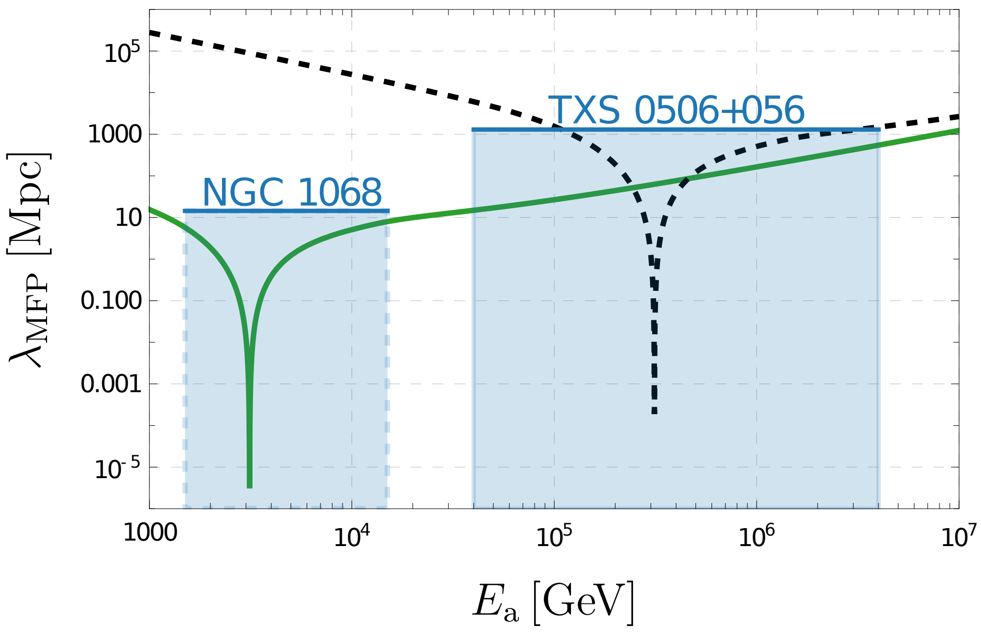
Mean Free Path

Mean free path: $\lambda_{\mathrm{MFP}} = 1/\sum_i \Gamma_i(E_{\mathrm{a}})$

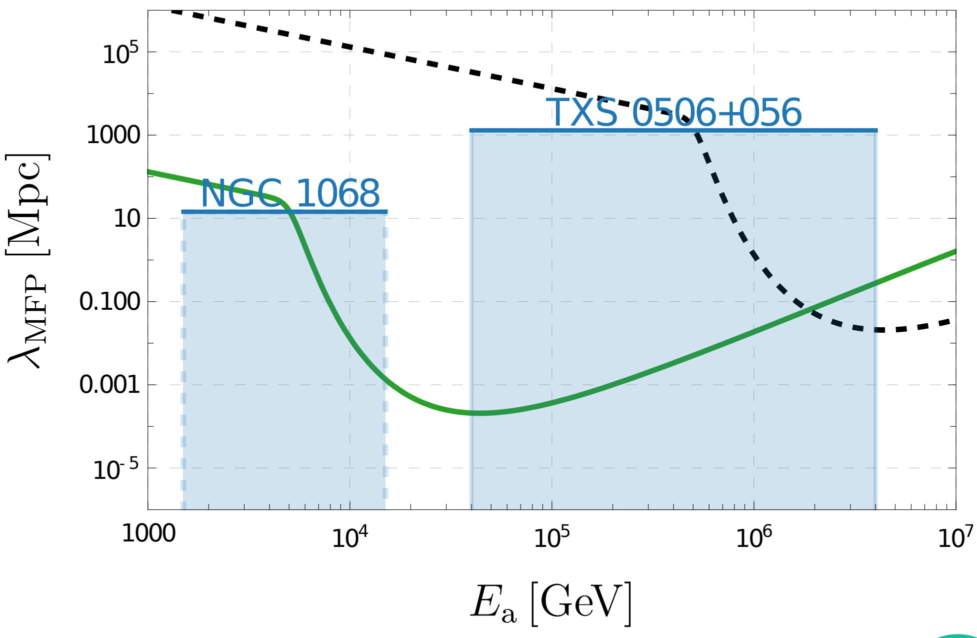
y=0.05
$\mathrm{m}_\phi \in \{0.25, 2.5\}\,\mathrm{MeV}$

Illustrative example:
single neutrino species

Non-relativistic today $\mathrm{m}_i = 0.01\,\mathrm{eV}$



Relativistic today



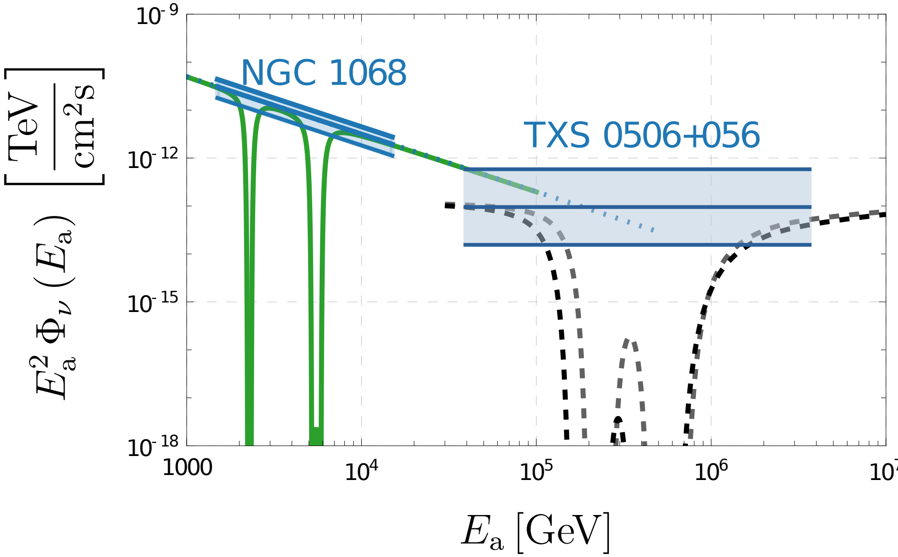
Flux

$$\Phi_0(E) = \hat{\Phi}_0 \cdot \left(\frac{E}{1\,\mathrm{TeV}}\right)^{-\gamma} \cdot \mathrm{T}(E)$$

Transmittance

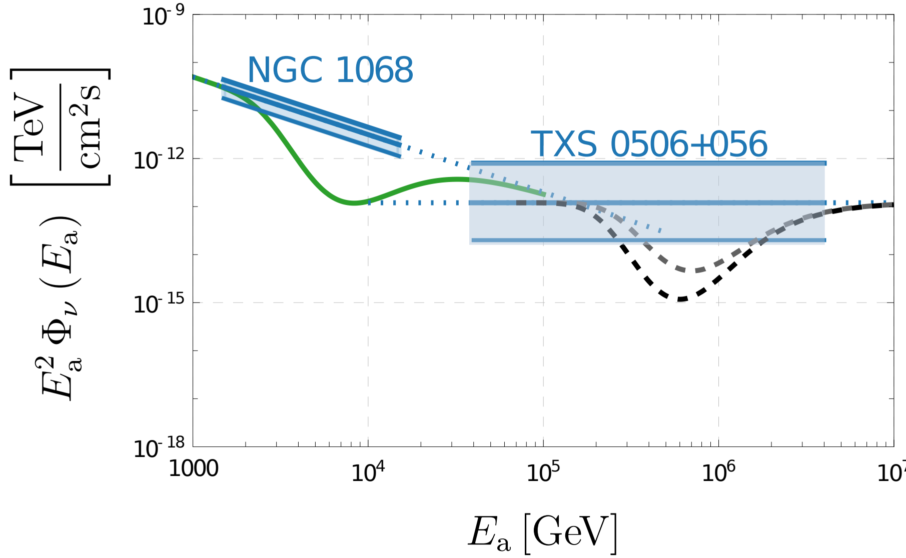
$$\mathrm{T(E)} = e^{-\frac{d}{\lambda_{\mathrm{MFP}}(E)}}$$

Non-relativistic today



$y \in \{0.03, 0.05\}$
$m_\phi \in \{0.5, 5\}$ MeV

Relativistic today



$y \in \{2.5 \cdot 10^{-4}, 2.5 \cdot 10^{-4}\}$
$m_\phi \in \{0.1, 1\}$ MeV

Flux

Transmittance

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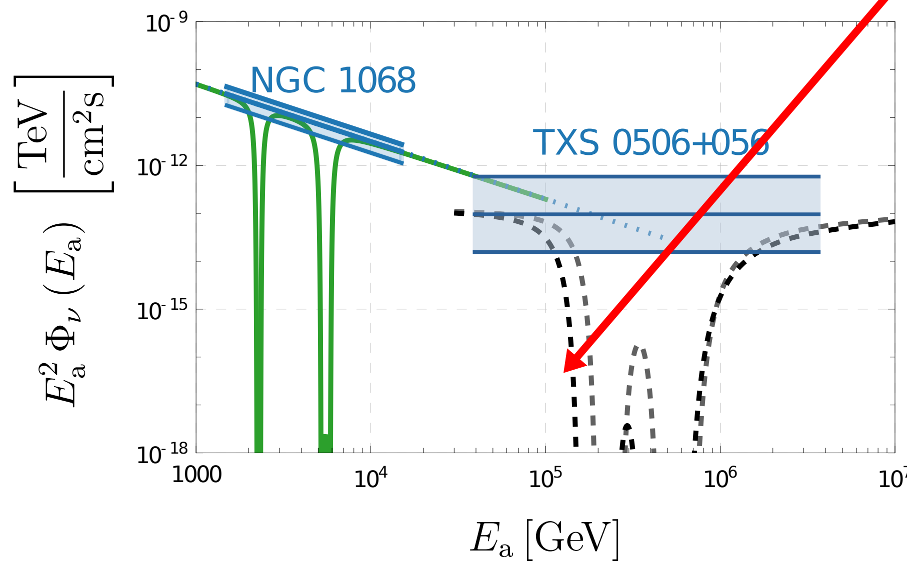
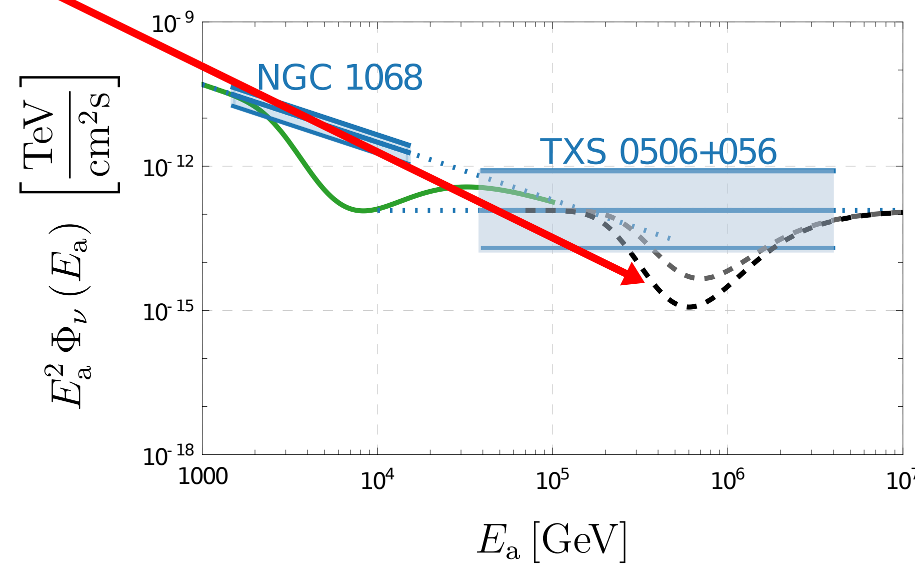
Redshift broadening

Non-relativistic today



$y \in \{0.03, 0.05\}$
$m_\phi \in \{0.5, 5\}$ MeV

Relativistic today



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Estimating the amount of absorbed neutrinos

Problem: We don‘t know the original amount of neutrinos emitted by the source…

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Estimate:

$$\frac{n}{n_0} = \frac{\int_{E_{\min}}^{E_{\max}} \mathrm{d}E \, A_{\mathrm{eff}}(E) \, \Phi(E)}{\int_{E_{\min}}^{E_{\max}} \mathrm{d}E \, A_{\mathrm{eff}}(E) \, \Phi_0(E)} \geq q$$

with absorption (milky)

measured number (transparent)

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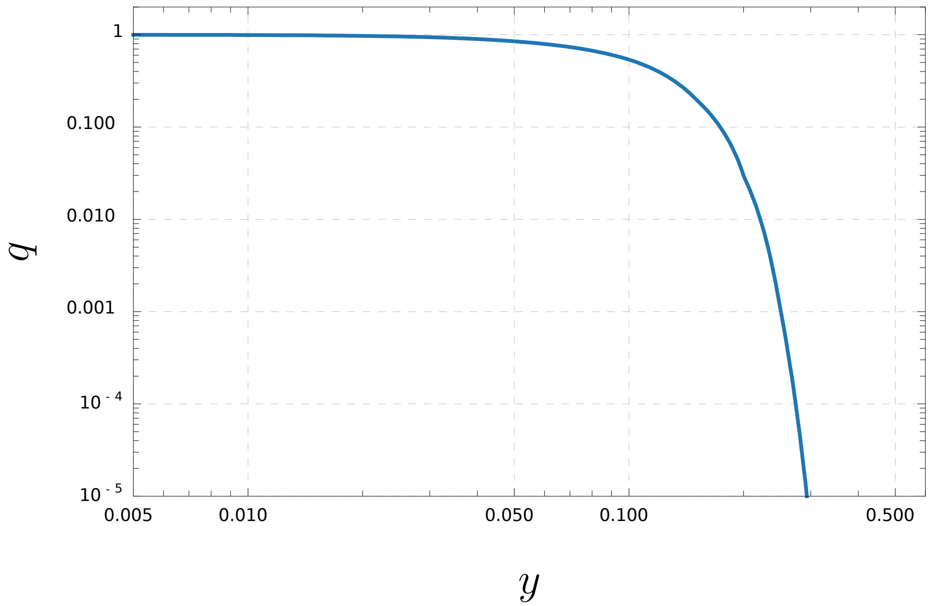

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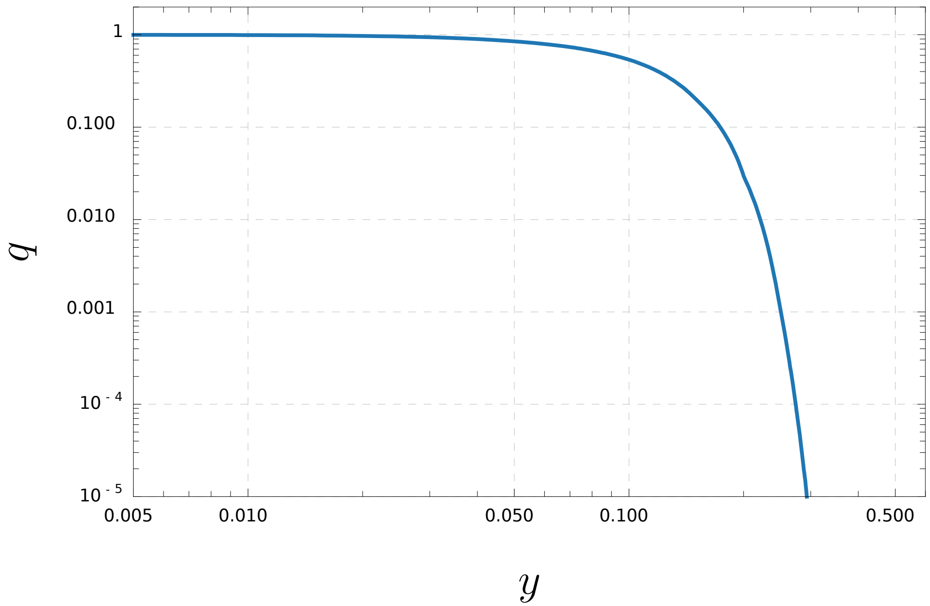


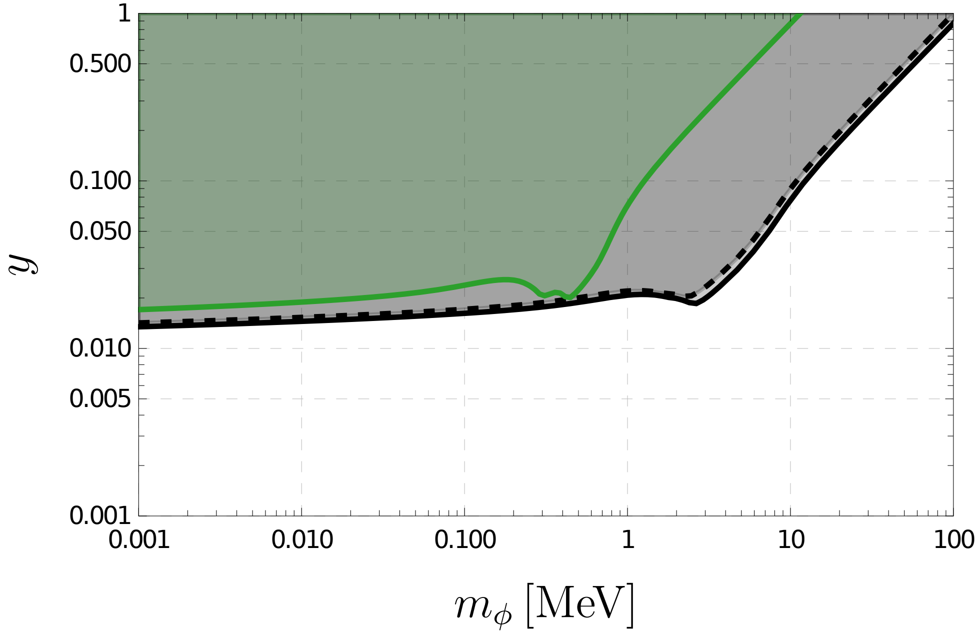
More dedicated analysis, see arXiv:2307.02361

Results

Results: Estimated Limits

Color Code:
NGC 1068
TXS 0506+056

Non-relativistic today



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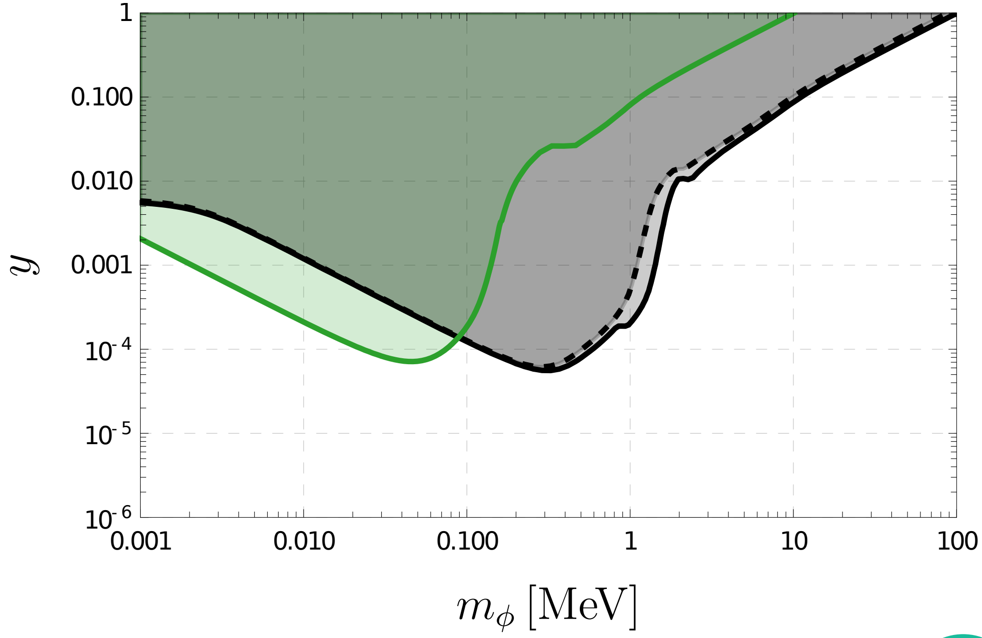

Results: Estimated Limits

Color Code:
NGC 1068
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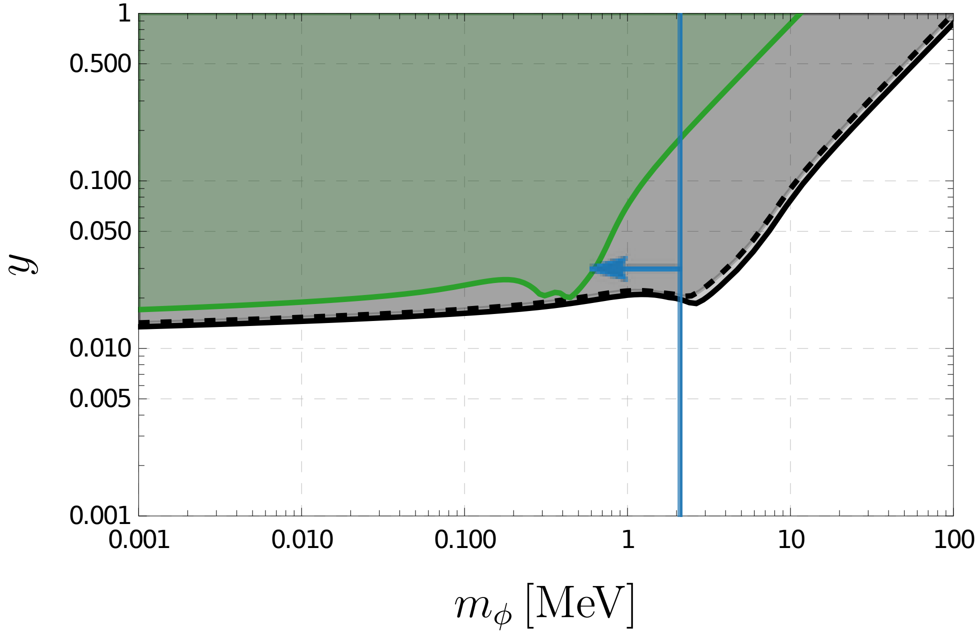
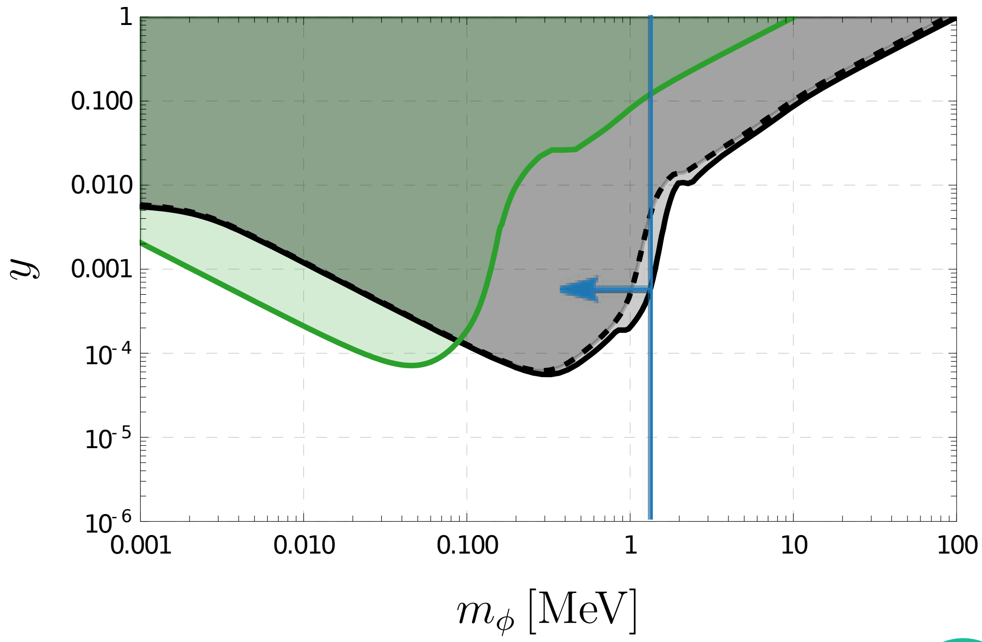
- Blue Line: BBN constrain (N_{eff})

Non-relativistic today



Relativistic today



Results: Estimated Limits

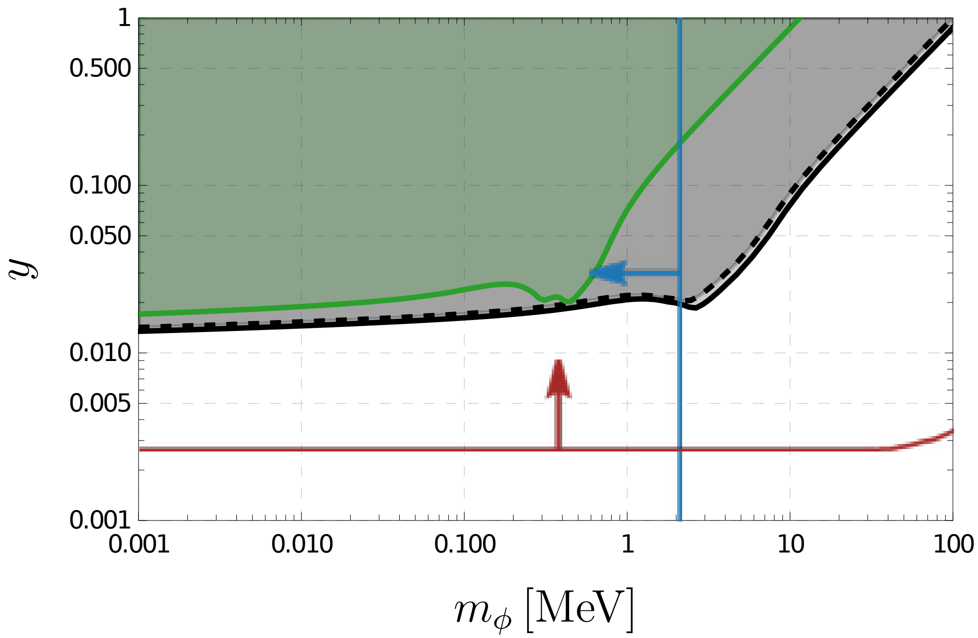
Color Code:

NGC 1068

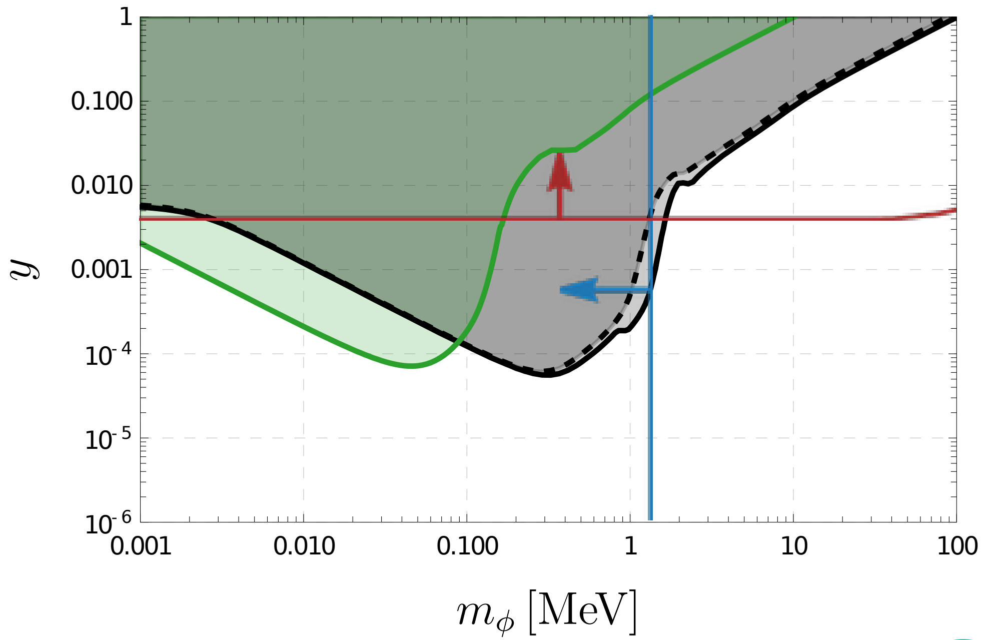
TXS 0506+056

- **Blue Line**: BBN constrain ($\mathrm{N_{eff}}$)
- **Brown Line**: Lab K^- decay (flavor dependent)

Non-relativistic today



Relativistic today

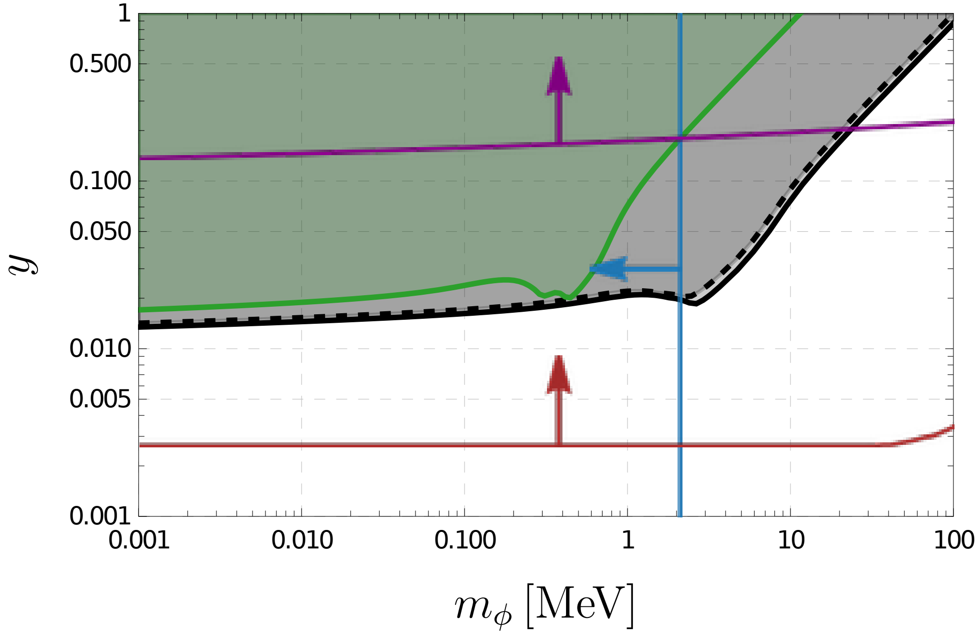


Results: Estimated Limits

Color Code:
NGC 1068
TXS 0506+056

- Blue Line: BBN constrain (N_{eff})
- Brown Line: Lab K^{-} decay (flavor dependent)
- Pink Line: Lab constrain Z-decay

Non-relativistic today



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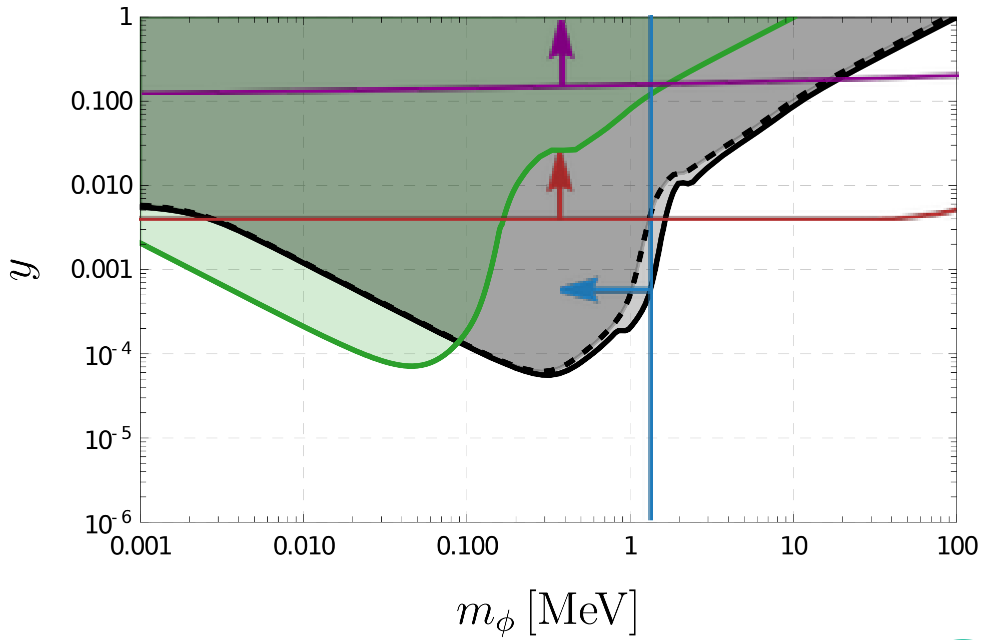
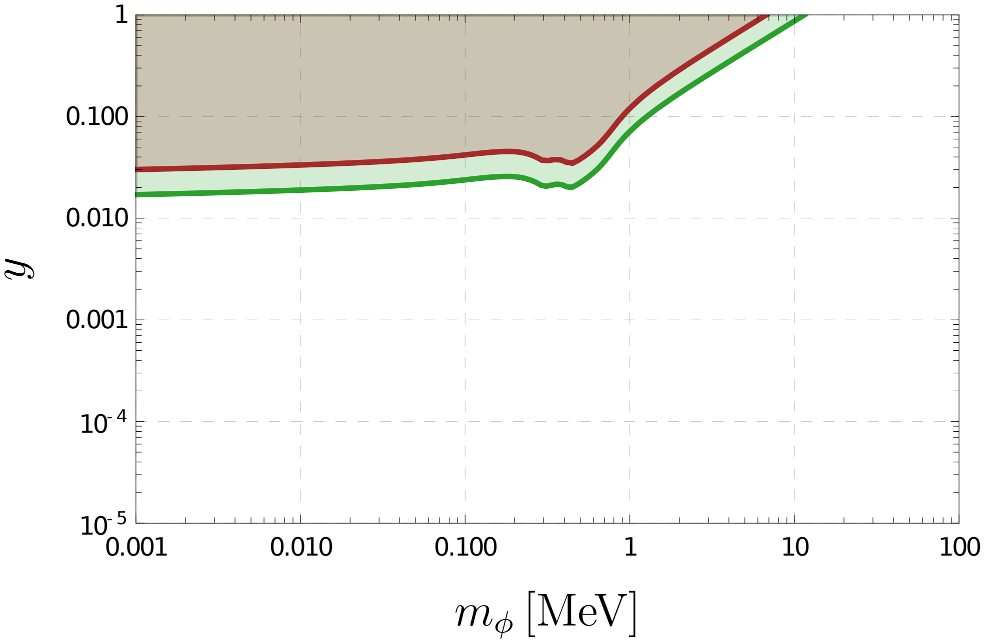


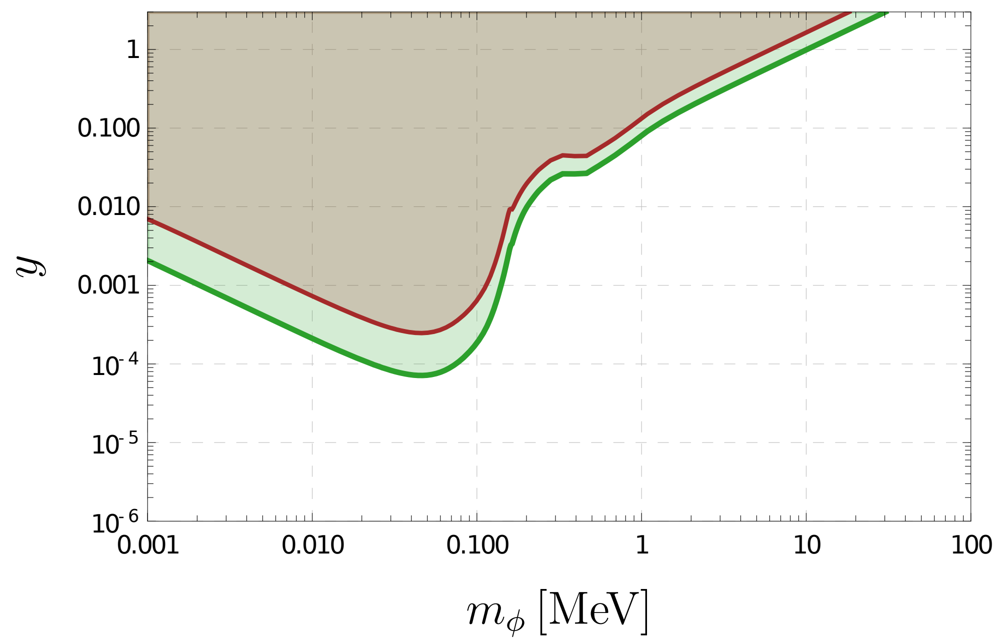
Tauphilic interactions

Example: NGC 1068
Flavor Universal Coupling
Coupling only to Tau-Neutrinos

Non-relativistic today



Relativistic today

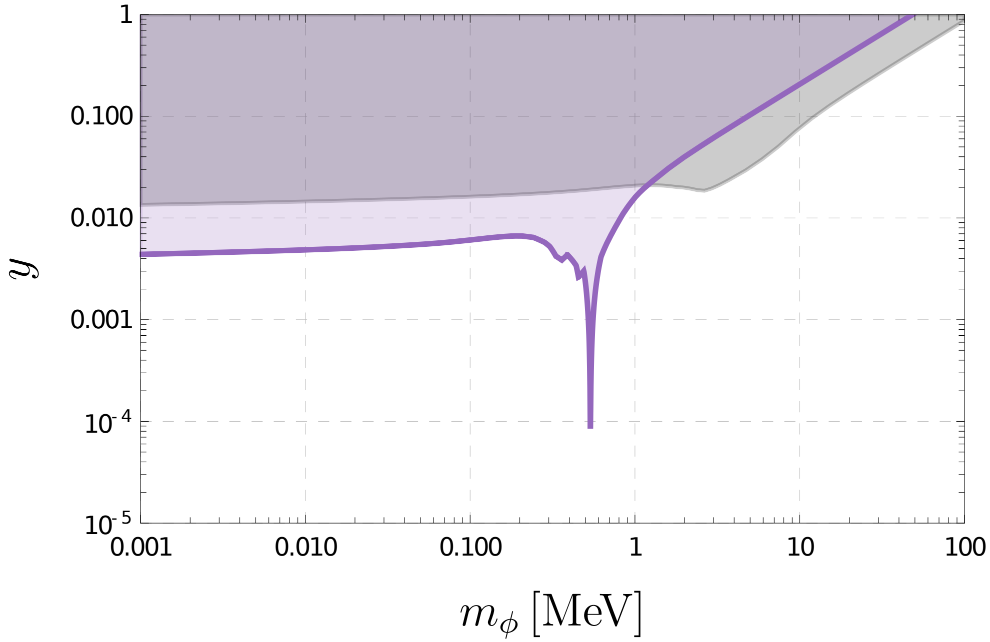


Future Sources – Outlook to PKS 1424+240

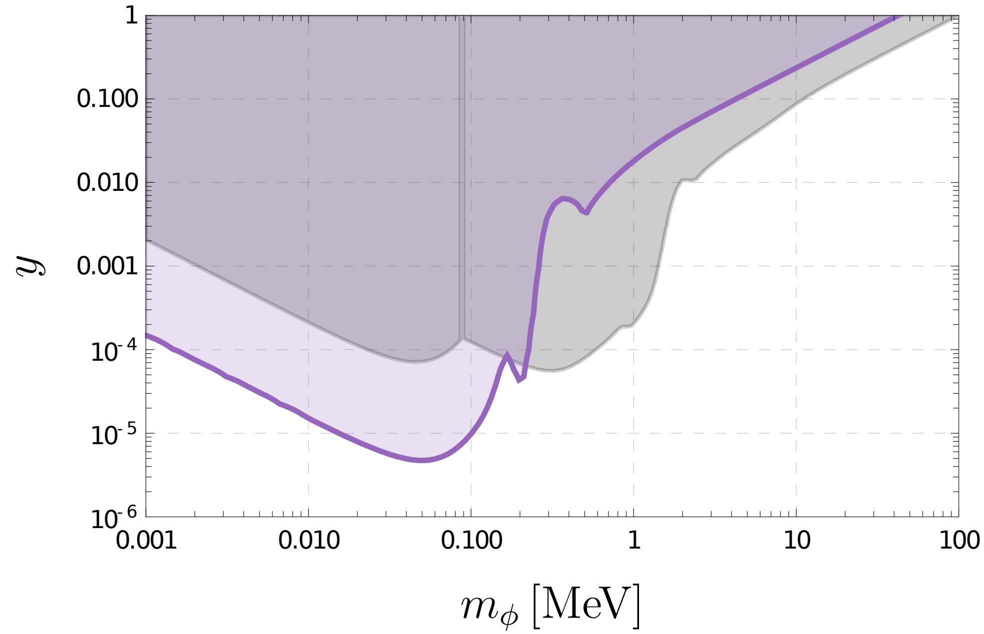
Color Code:
PKS 1424+240
Combined estimated limits:
NGC 1068 and TXS 0506+056

Facts:
Distance: $1.8\,\mathrm{Gpc}$
Flux: $\hat{\Phi}_0 \approx \hat{\Phi}_{0,\mathrm{NGC\ 1068}}$
Spectral index: $\gamma = 3.5$
Energy: $[E_{\mathrm{min}}, E_{\mathrm{max}}]_{\mathrm{NGC\ 1068}}$

Non-relativistic today



Relativistic today



Conclusion

- Neutrinos from astrophysical point sources have been measured by IceCube and are great messengers for astro- and particlephysics
- New physics (e.g. a scalar) can lead to interactions with the CNuB and thus turn the Universe opaque for Neutrinos
- Using the two observed sources TXS 0506+056 and NGC 1068 we put new estimated constraints on light scalar masses and neutrino coupling
- Two cases: lightest neutrino relativistic vs non-relativistic today
- Only estimate: the original neutrino emission at the source is not known
- More sources and higher energetic neutrinos could improve the constraints as well as a better understanding of the original neutrino luminosity of these sources

THANK YOU !

BACKUP

Crosssection

Flavor universal neutrino scattering cross section

$$\sigma_{\nu\nu}(s) = \frac{y^4}{32\pi((m_\phi^2 - s)^2 + m_\phi^2\Gamma_\phi^2)s^2}\left(\frac{s(5m_\phi^6 - 9m_\phi^4 s + 6s^3)}{m_\phi^2 + s} + \frac{2(5m_\phi^8 - 9m_\phi^6 s + 4m_\phi^2 s^3)\log(\frac{m_\phi^2}{m_\phi^2+s})}{2m_\phi^2 + s}\right)$$

ϕ-pair production $\mathrm{E_{CM}} \geq m_\phi$

$$\sigma_{\phi\phi}(s) = \frac{y^4}{64\pi s^2}\left(\frac{s^2 - 4m_\phi^2 s + 6m_\phi^4}{s - 2m_\phi^2}\log\left[\left(\frac{(s(s-4m_\phi^2))^{1/2} + s - 2m_\phi^2}{(s(s-4m_\phi^2))^{1/2} - s + 2m_\phi^2}\right)^2\right] - 6(s(s-4m_\phi^2))^{1/2}\right)$$

See also arXiv:2107.13568

Massless Neutrino

Rate approximations in different limit cases:

Heavy mediator mass:

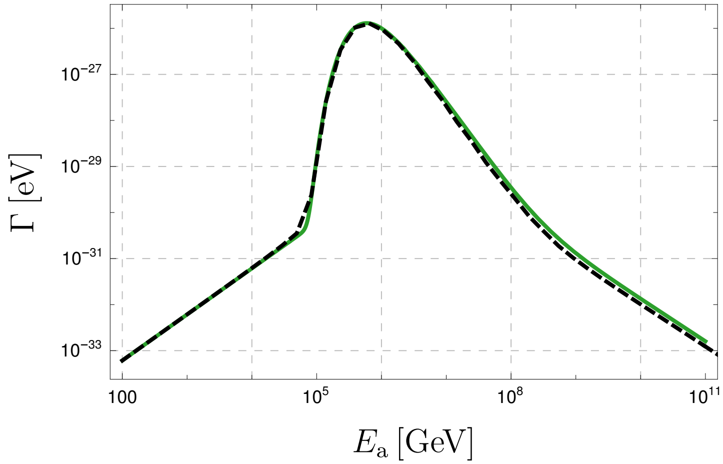
$$\Gamma_{\rm heavy} \approx \frac{7\pi^3 y^4}{2592\,\zeta(3)} \frac{E_{\rm a} T_\nu}{m_\phi^4} n_{\nu_1}$$

Small mediator mass:

$$\Gamma_{\rm light} \approx \frac{\pi y^4}{192\,\zeta(3)} \frac{1}{E_{\rm a}\, T_\nu} n_{\nu_1}$$

Resonance:

$$\Gamma_{\rm NWA} \approx \frac{y^4}{384\zeta(3)} \frac{m_\phi^3}{E_\nu^2\, T_\nu^2\, \Gamma_\phi} \log[1 + e^{-\frac{m_\phi^2}{4E_\nu T_\nu}}] n_{\nu_1}$$



Dashed-Black: Analytical Approximation
Green: Numerical Result

Redshift broadening

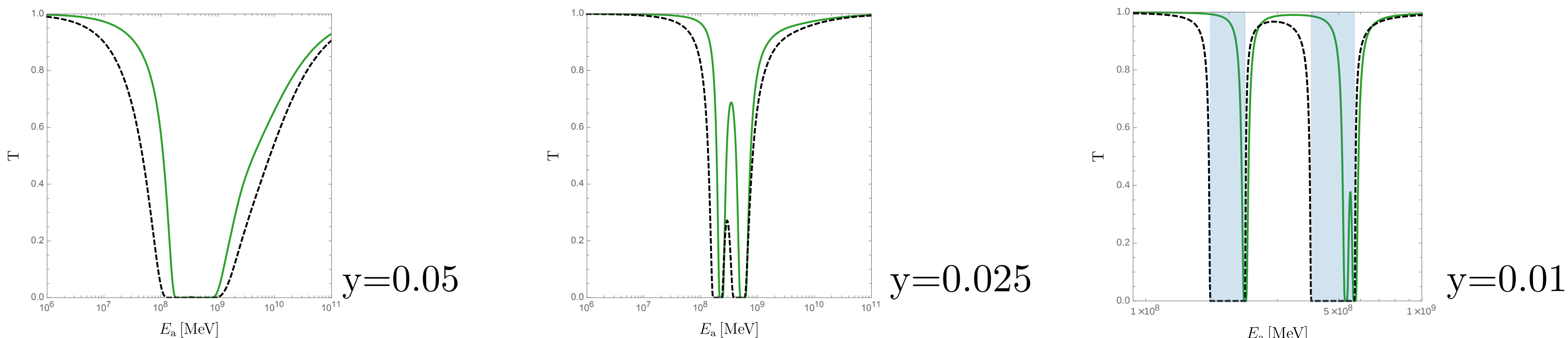
In **expanding Universe**: Flux evolves according to transport equation

$$\frac{\partial \Phi(t, E_\mathrm{a})}{\partial t} = \frac{\partial}{\partial E_\mathrm{a}}[H(t) E_\mathrm{a} \Phi(t, E_\mathrm{a})] - \Phi(t, E_\mathrm{a}) \Gamma(E_\mathrm{a}, t)$$

Which becomes $\frac{\partial Z(z, E_\mathrm{a})}{\partial z} = \frac{Z(z, E_\mathrm{a}) \Gamma(E_\mathrm{a}, z)}{H(z)(1+z)}$ with $Z(z, E_\mathrm{a}) := (1+z) \Phi(z, E_\mathrm{a}[1+z])$

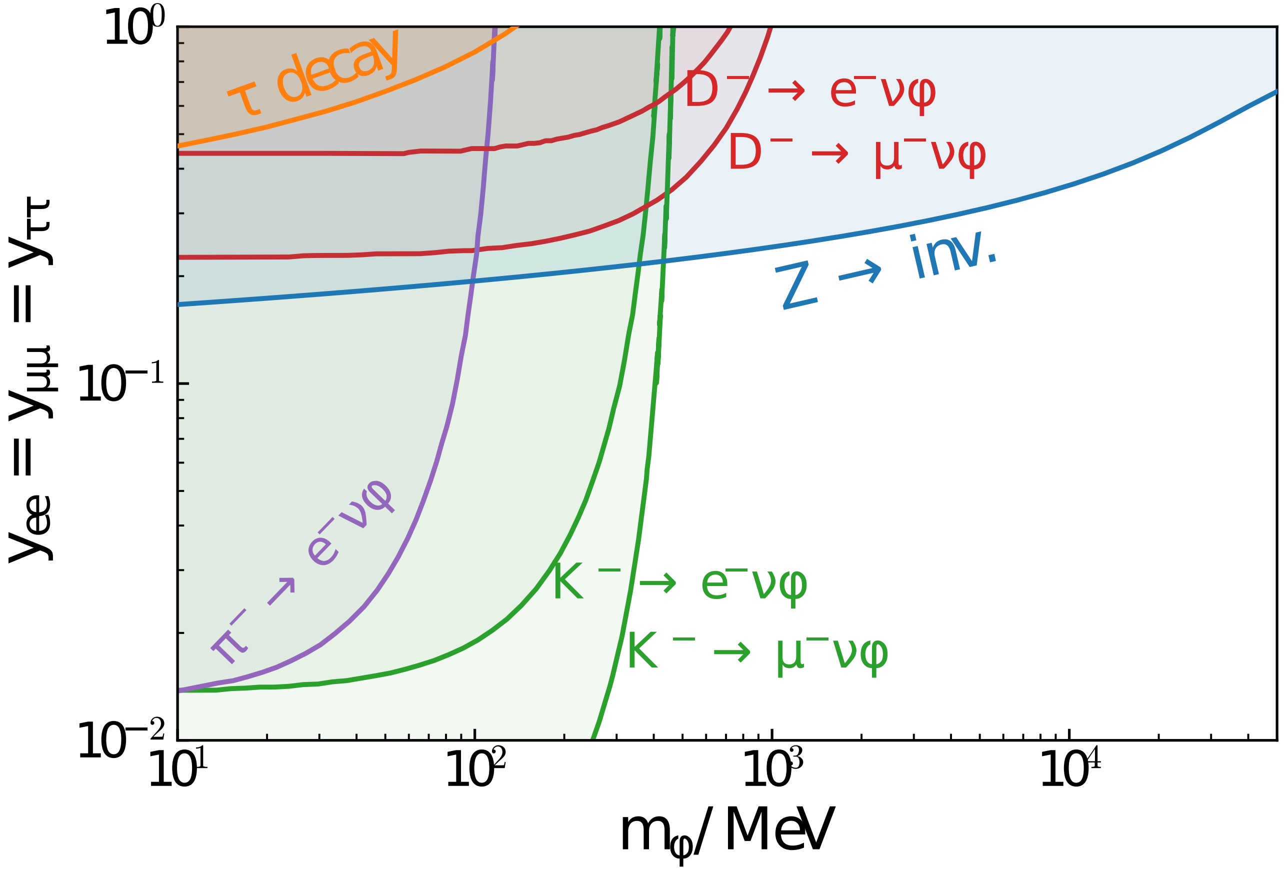
The redshift dependent rate is: $\Gamma_i(E_\mathrm{a}, z) = \int \frac{\mathrm{d}^3 p}{(2\pi)^3} (1+z)^3 f_i(\vec{p}(1+z))\, v_{M\o l} \sigma_{\nu\nu}(s(E_\mathrm{a}(1+z), \vec{p}(1+z)))$

Transmittance: $T = \frac{Z(0, E_\nu)}{Z(z, E_\nu)} = \mathrm{Exp}\left[-\int_0^z \frac{1}{H(z')(1+z')} \Gamma(E_\nu, z')\, \mathrm{d}z' \right]$



See also arXiv:2107.13568

Labconstraints



See:
arXiv:1802.00009
arXiv:2003.05339

Scalar mediated Nu-Nu Interactions Details on: Tauphilic Coupling vs Flavor-Universal Coupling

Relation between oberserved flux and source flux:

$$\phi_{obs} = \sum_i \exp\left(-\tau_i\right) \frac{\phi_{source}}{3}$$

Case I: Flavor-Universal Coupling:

$$\phi_{obs} = \exp(-d \sum_j \sigma_u n_j) \phi_{source}$$

Case II: Flavor-Spezific Coupling:

$$\phi_{obs} = \sum_i \exp\left(-d \sum_j \sigma_{ij} n_j\right) \frac{\phi_{source}}{3}$$

Attention: depend also on neutrino-mass m_j !!!

Comparison:

$$\exp\left(d \sum_j \sigma_u n_j\right) = \frac{1}{3} \sum_i \exp\left(-d |U_{\tau i}|^2 \sum_j |U_{j,\tau}|^2 \sigma_\tau n_j\right) .$$

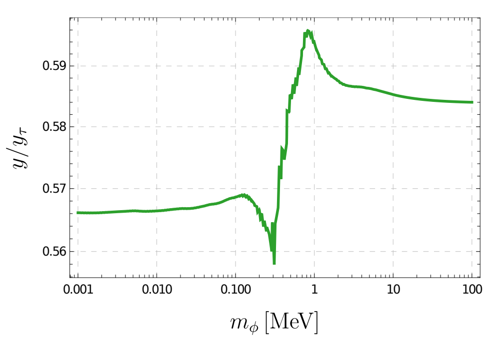
Approximation: small couplings

$$\sum_j \sigma_u n_j = \frac{1}{3} \sum_j |U_{\tau j}|^2 \sigma_\tau n_j$$

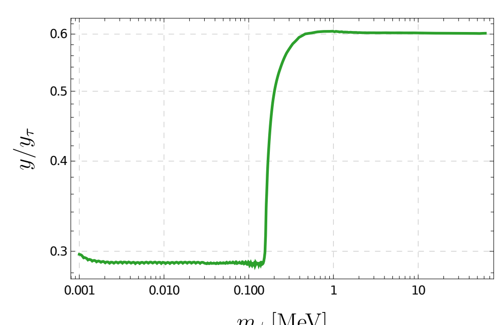
Relation of couplings:

Tails: $\sigma \propto y^4$ ➔ $\frac{y_u}{y_\tau} \approx \left(\frac{|U_{\tau j}|^2}{3}\right)^{1/4} \approx 0.58$

Resonance: $\sigma \propto y^2$ ➔ $\frac{y_u}{y_\tau} \approx \sqrt{\frac{|U_{\tau j}|^2}{3}} \approx 0.28$



Non-relativistic today



Relativistic today

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