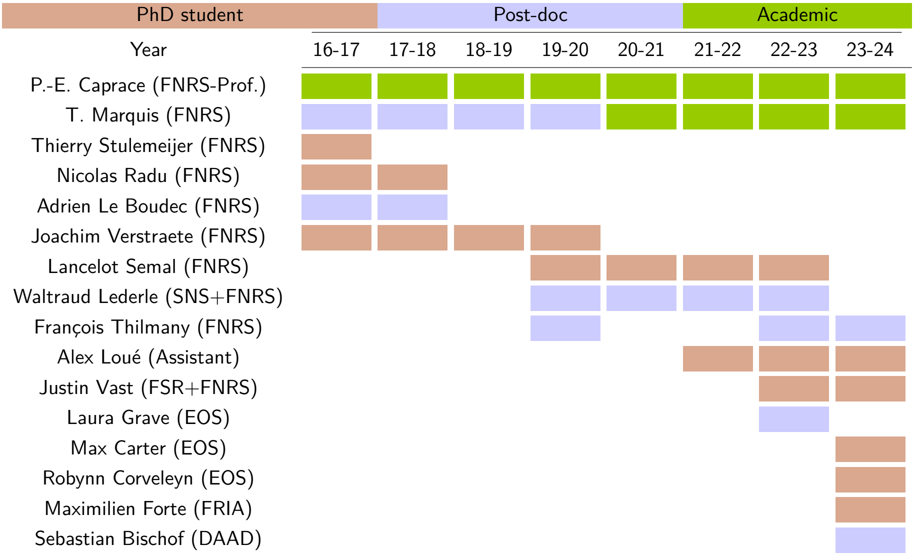
Geometric group theory at IRMP

2017–2022

Timothée Marquis

April 26, 2024

The evolution of the research team



Current members


P-E. Caprace


T. Marquis


François Thilmany


Sebastian Bischof


Alex Loué


Max Carter

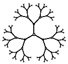

Robynn Corveleyn


Maximilien Forte


Justin Vast

Main research directions around geometric group theory

- Algebraic structure of (non-discrete) **locally compact groups**.



  - e.g. G closed subgroup of $\mathrm{Aut}(T_d)$
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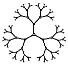

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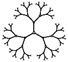

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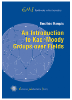

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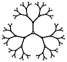

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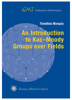

- **Kac–Moody groups**.
  - e.g. $G = \mathrm{SL}_n(\mathbb{K})$, $G = \mathrm{SL}_n(\mathbb{K}(\!(t)\!))$, non-linear groups.
  - $\mathrm{SL}_{n+1}(\mathbb{K}) = U^+ W U^+ T$ with $U^+ = \begin{pmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{pmatrix}$, $T = \begin{pmatrix} * & 0 & 0 \\ 0 & * & 0 \\ 0 & 0 & * \end{pmatrix}$, $W \cong \mathrm{Sym}(n)$.
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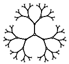

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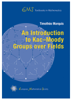

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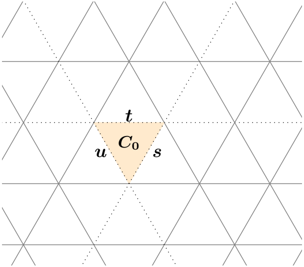
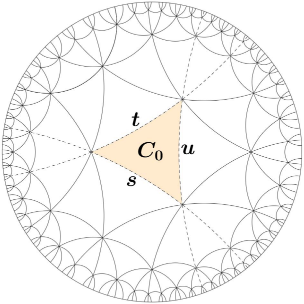
- KMS groups and connections to **high-dimensional expanders**
  - e.g. $G = U^+$ over $\mathbb{K}$ finite field.
  - Topic of EOS project with Tom De Medts (UGent).
  - **P.-E. Caprace**, M. Conder, M. Kaluba and S. Witzel, Hyperbolic generalized triangle groups, property (T) and finite simple quotients. **J. London Math. Soc.** 106 (2022), Nr. 4, pp. 3577–3637.

    P.-E. Caprace, M. Kassabov, Tame automorphism groups of polynomial rings with property (T) and infinitely many alternating group quotients. **Trans. Amer. Math. Soc.** 376 (2023), no. 11, pp. 7983–8021.

Main research directions around geometric group theory

- **Coxeter groups**.
  - e.g. $W = \mathrm{Sym}(n)$, $W = \mathrm{Aut}(\text{tiling})$





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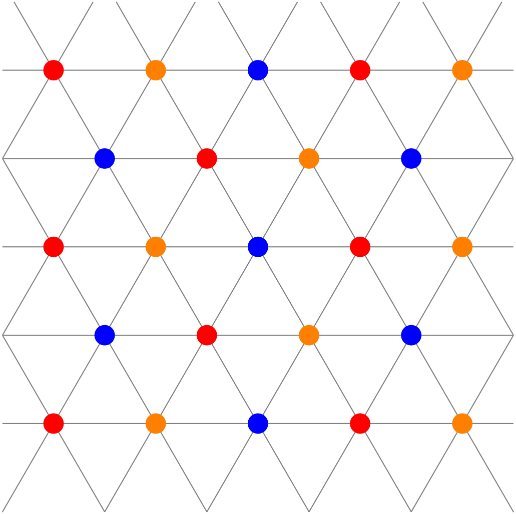
About geometric group theory

W = Weyl group of $\mathrm{SL}_3(\mathbb{K}(\!(t)\!)) = \mathrm{Aut}(X)$ where X is the colored tiling



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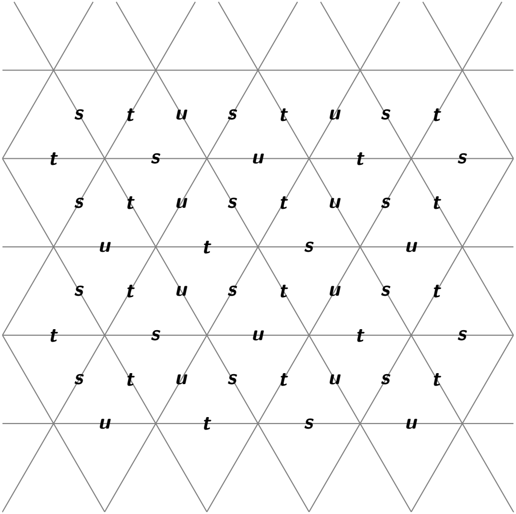

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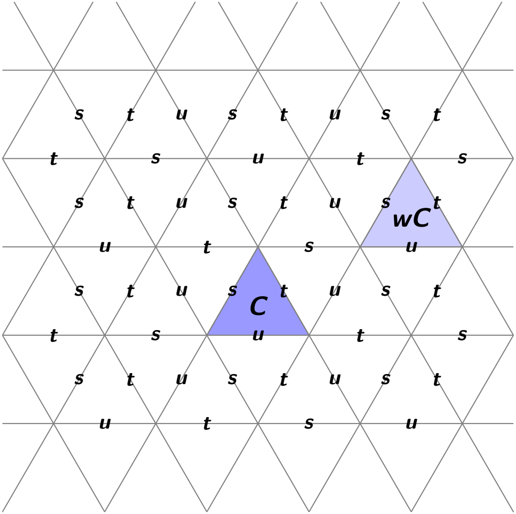

Triangles $\leftrightarrow W$: $wC \leftrightarrow w$

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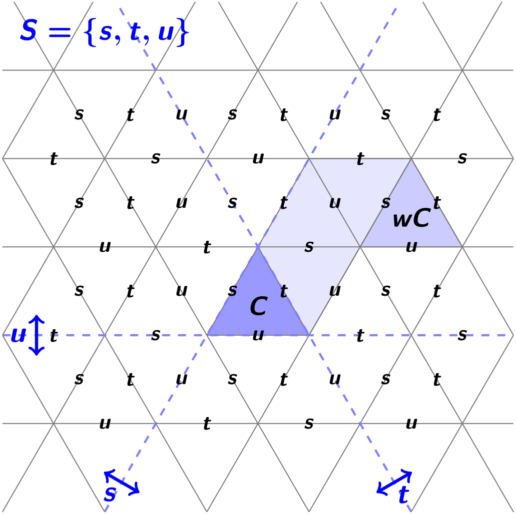

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ex: $w = tsus$

$\{\text{galleries } C \rightsquigarrow wC\} \leftrightarrow$
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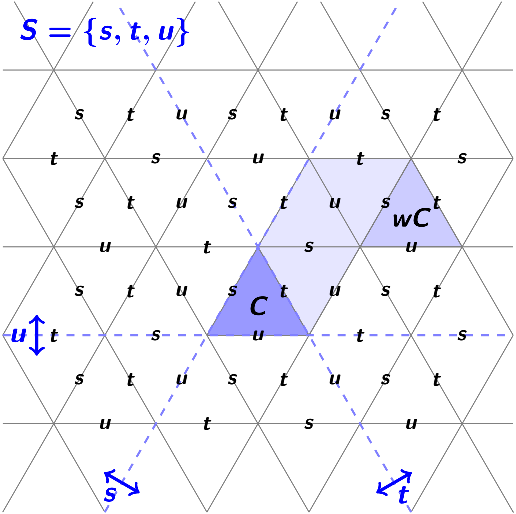

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Connectedness $\leftrightarrow$ $W = \langle s, t, u \rangle$

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$W = \langle s, t, u \rangle \Leftrightarrow$ every $w \in W$ can be represented by a word over the alphabet $S = \{s, t, u\}$ (e.g. $w = tsus = tusu$).

Word problem

Is there an algorithm deciding whether two S-words represent the same group element?

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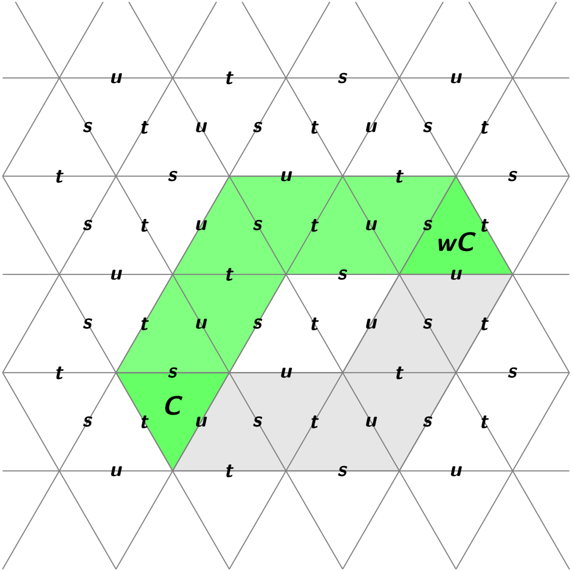

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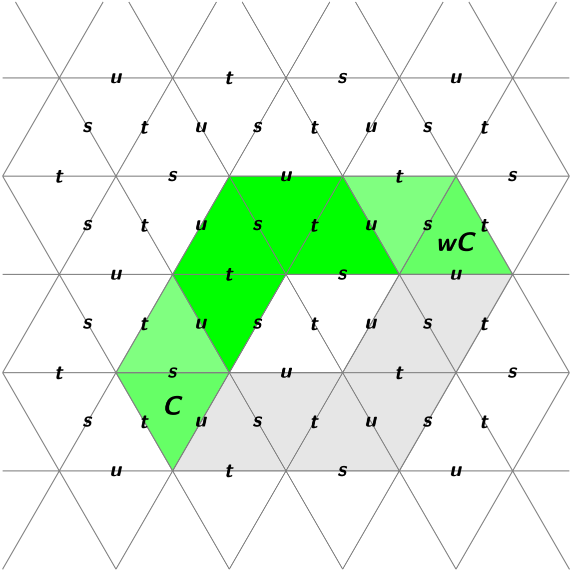

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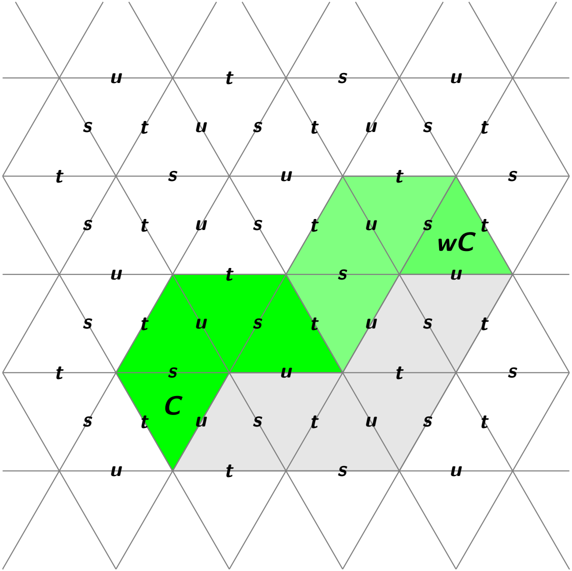

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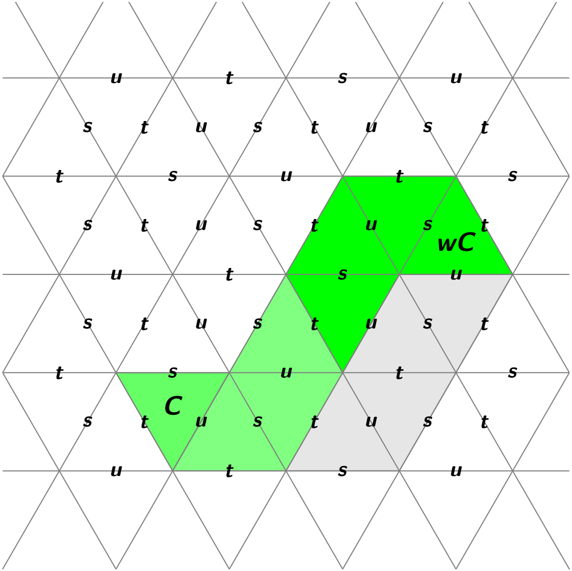

$$\begin{aligned} w &= sutstus \\ &= sustsus \\ &= usutsus \\ \\ &= ustutsu \end{aligned}$$

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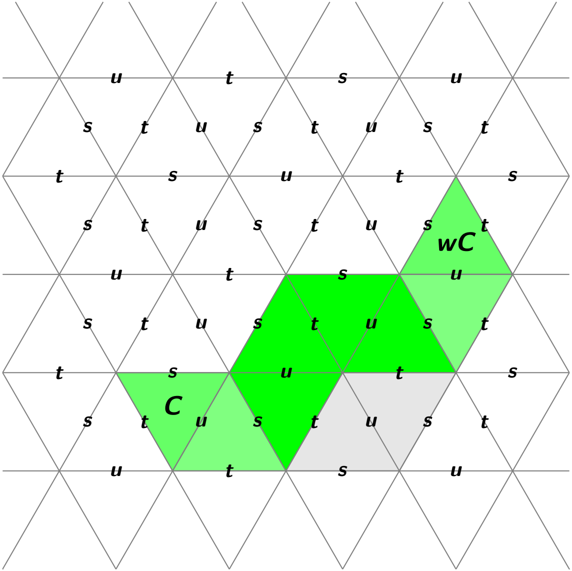

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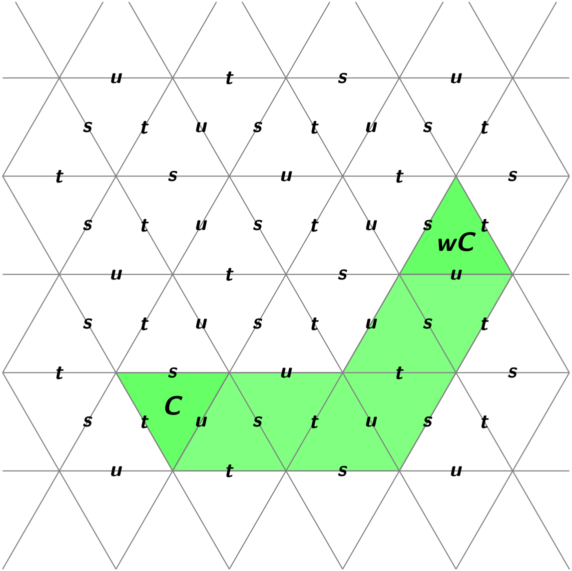

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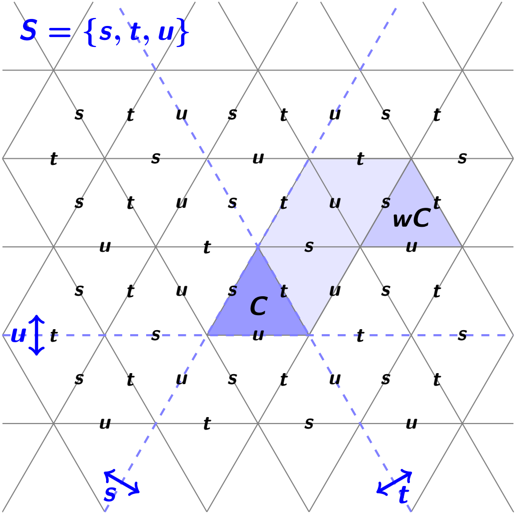

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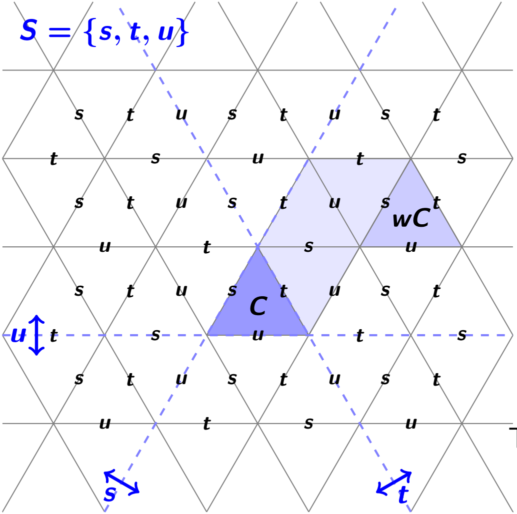

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Simple connectedness $\leftrightarrow$
Tits' solution to the word problem

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Some key points on impact and visibility

- EOS funding on "High-dimensional expanders and KMS groups" (total budget: €1,577,000).
- Summer school on high-dimensional expanders in May 2023 ($\sim$ 80 participants).
- P.-E. Caprace has been a speaker in the Algebra Section of the International Congress of Mathematicians in 2022.
- P.-E. Caprace was elected member of the Académie Royale des Sciences in 2023.