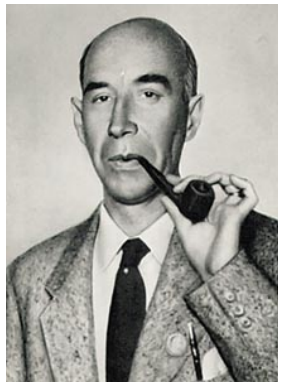
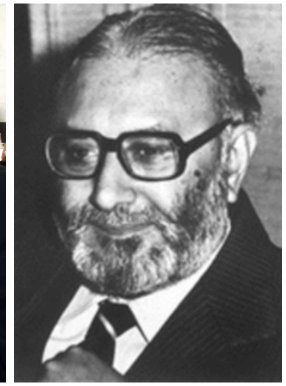
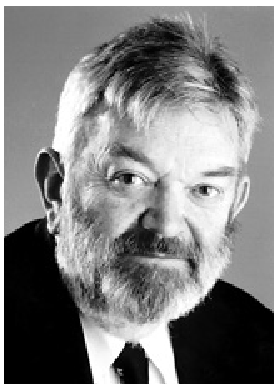
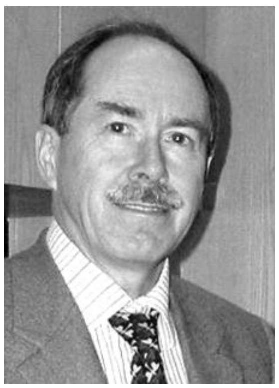
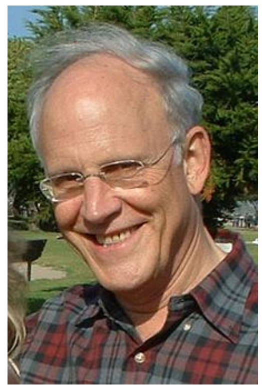
Experimental Foundations of the Standard Model II

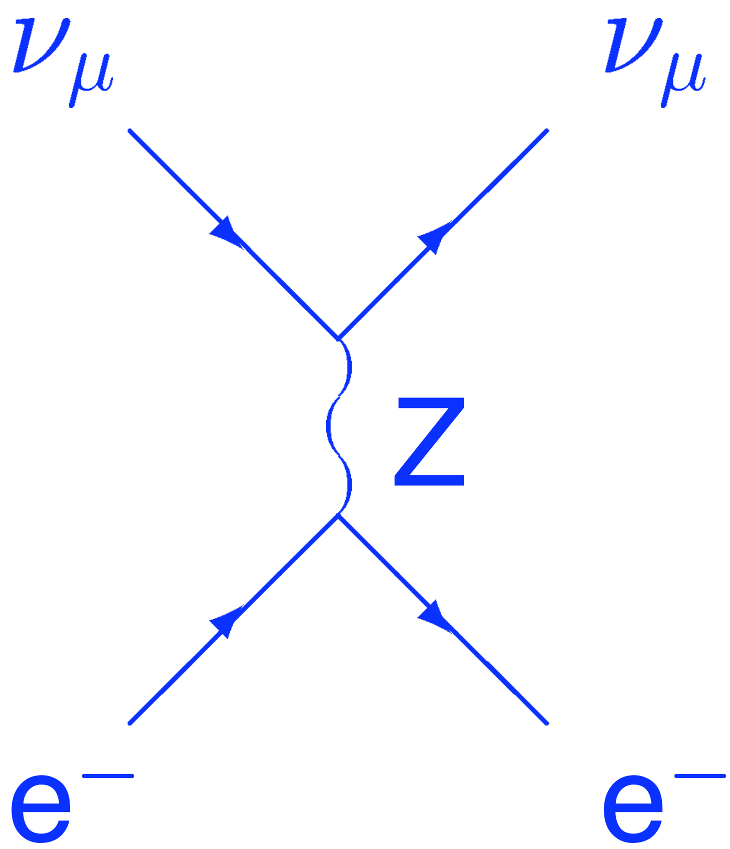
Postgraduate Course, UC Louvain, 2009

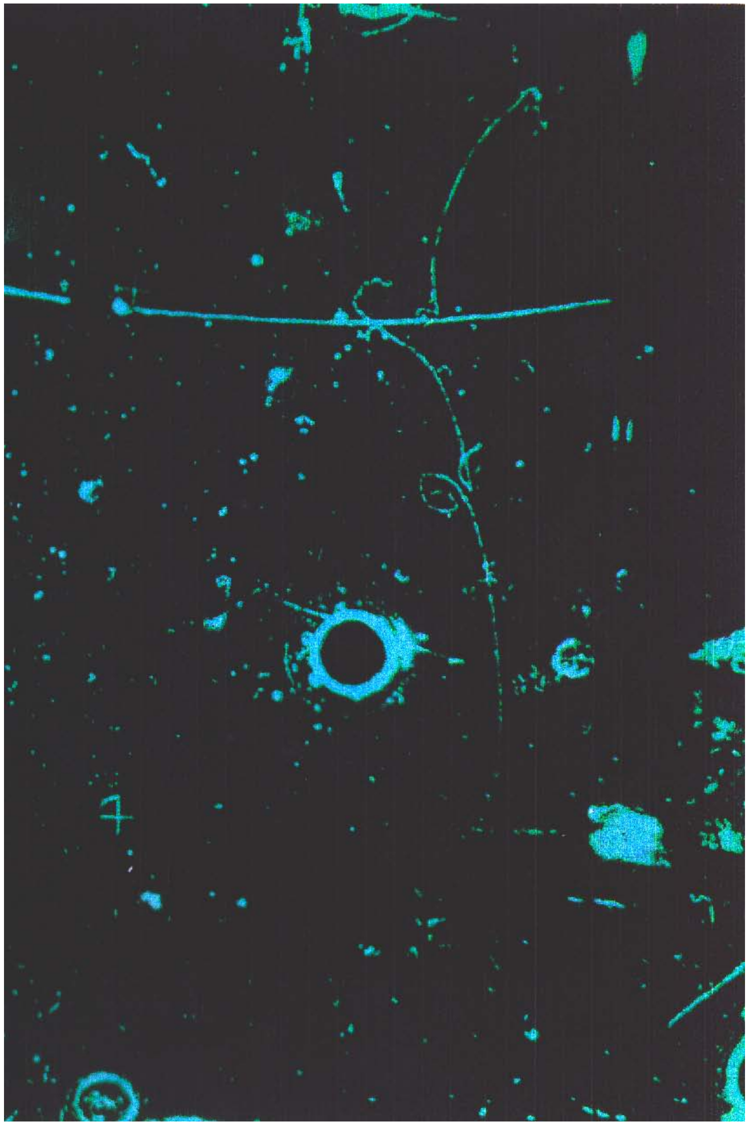
UNIVERSITÉ DE GENÈVE

Martin Pohl

The First Weak Neutral Current (1973)

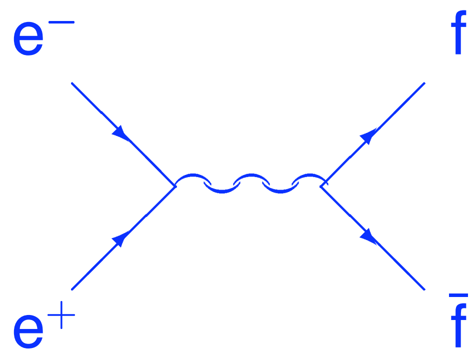
Heavy Liquid Bubble Chamber
Gargamelle CERN (1973)





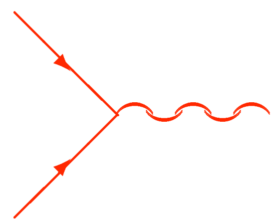
Standard Model Predictions

- Feynman graphs



- Propagators

- Vertices:
  coupling constant g: strength;
  operator Γ: transformation properties



$-\mathrm{i}g\Gamma$

'Small' Unification

- Three gauge couplings, but only two independent ones:

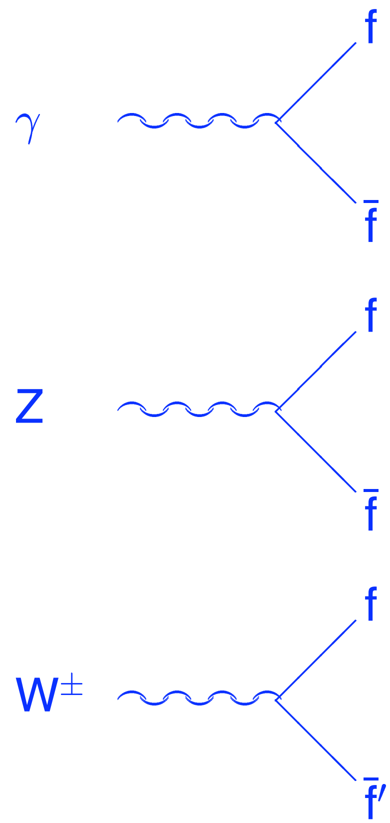
$$e = g \sin\theta_W = g' \cos\theta_W$$

Partial unification of electroweak forces.

- Higgs mechanism:

$$\sin^2\theta_W = \rho_0 \left(1 - \frac{M_W^2}{M_Z^2}\right)$$

Relation between couplings and gauge boson masses.



$$-ie\gamma^\mu$$

$$-\frac{ig'}{2\sin\theta_W}\gamma^\mu \left(g_V^f - g_A^f\gamma^5\right)$$

$$-\frac{ig}{2\sqrt{2}}\gamma^\mu \left(1 - \gamma^5\right)$$

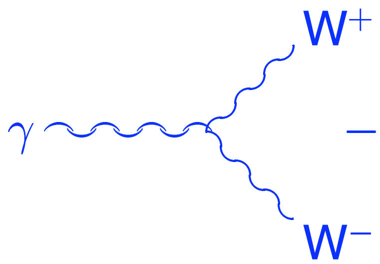
- Space-time structure of vertices given by electroweak multiplets:

$$g_V^f = \sqrt{\rho_0}\left(T_3^f - 2Q_f \sin^2\theta_W\right) \quad ; \quad g_A^f = \sqrt{\rho_0} T_3^f$$

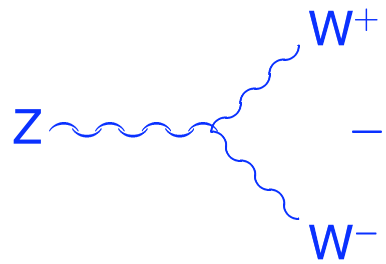
For Higgs doublets, $\sqrt{\rho_0} = 1$. Since $\sin^2\theta_W = 0.232 \simeq 1/4$: $g_V^l \ll g_A^l$ for leptons, but not for quarks.

Non-Abelian Gauge Theories

In a non-abelian gauge theory, gauge bosons couple to themselves. This fixes the coupling constants, without involving matter particles:

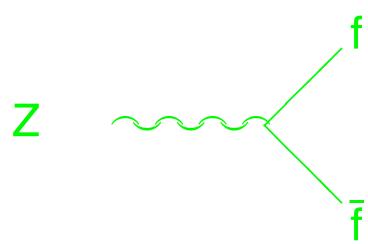


$-\mathrm{ie}\Gamma_\gamma$

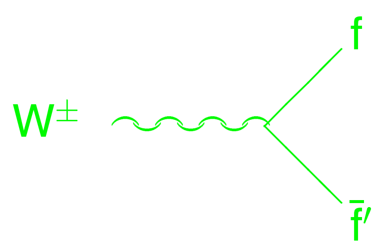


$-\mathrm{ie}\cot\theta_W\Gamma_Z$

- This automatically fixes all couplings to matter particles.
- Couplings are determined by the position in electroweak multiplets.
- Couplings are universal, independent of the generation.



$-\frac{\mathrm{ie}}{\sin\theta_W\cos\theta_W}\gamma_\mu\left(g_V^f - g_A^f\gamma^5\right)$

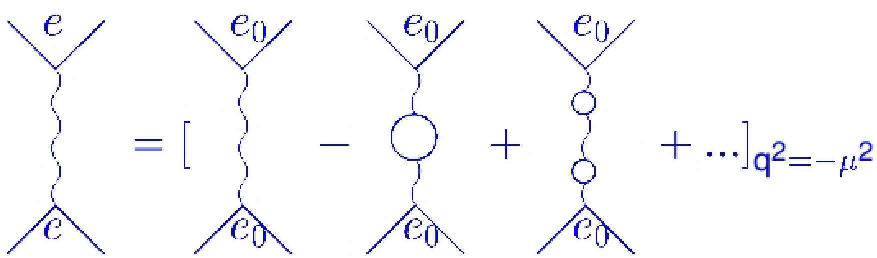


$-\frac{\mathrm{ie}}{\sqrt{2}\sin\theta_W}\gamma_\mu\left(1 - \gamma^5\right)$

Running Couplings

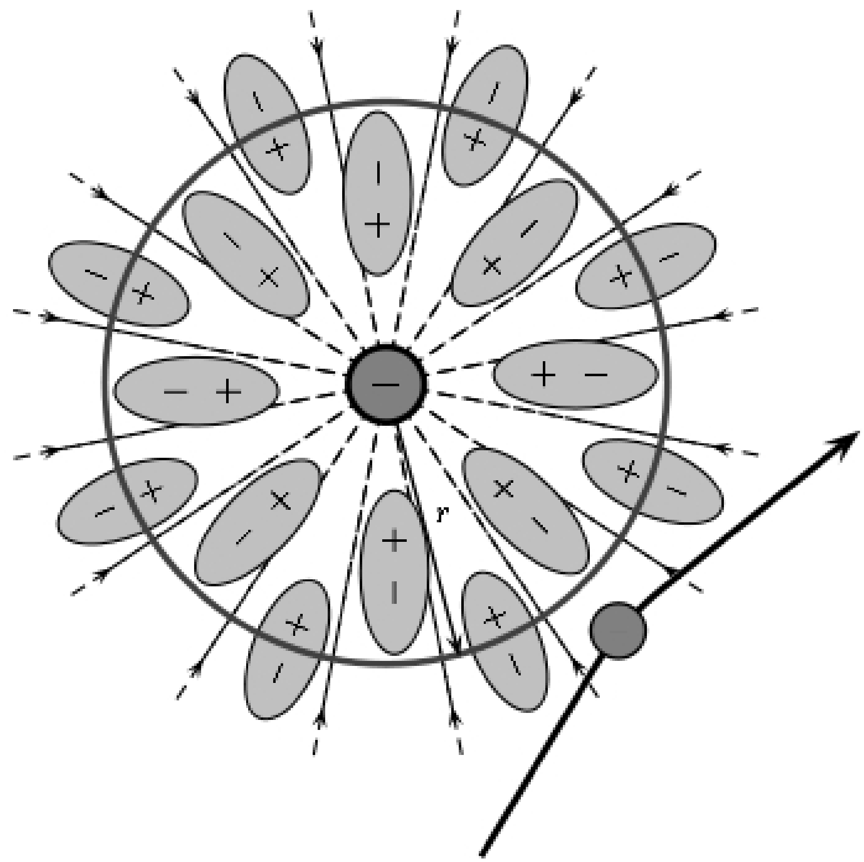
Higher order corrections for gauge boson propagators introduce a Q^2-dependence of the coupling constants.

Example for QED:



$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\log\frac{Q^2}{\mu^2}}$$

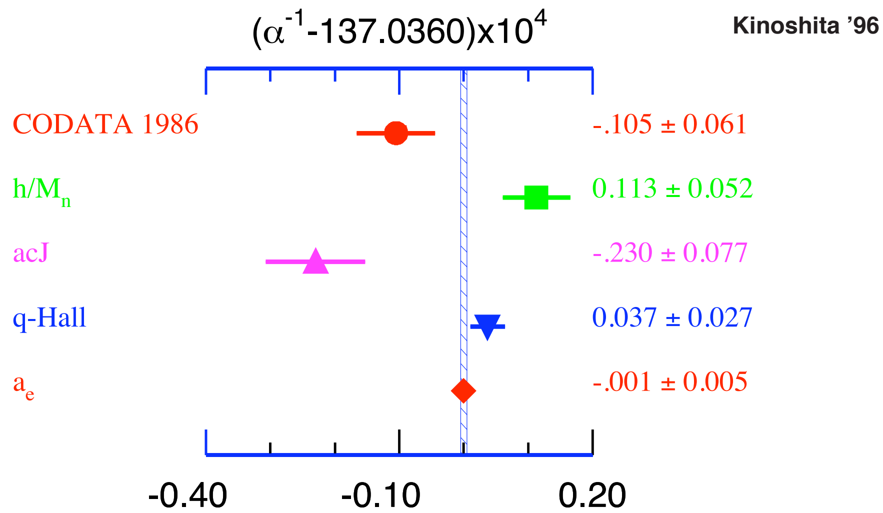
This can be understood in terms of vacuum polarisation screening the electric charge at large distance.



Fine Structure Constant

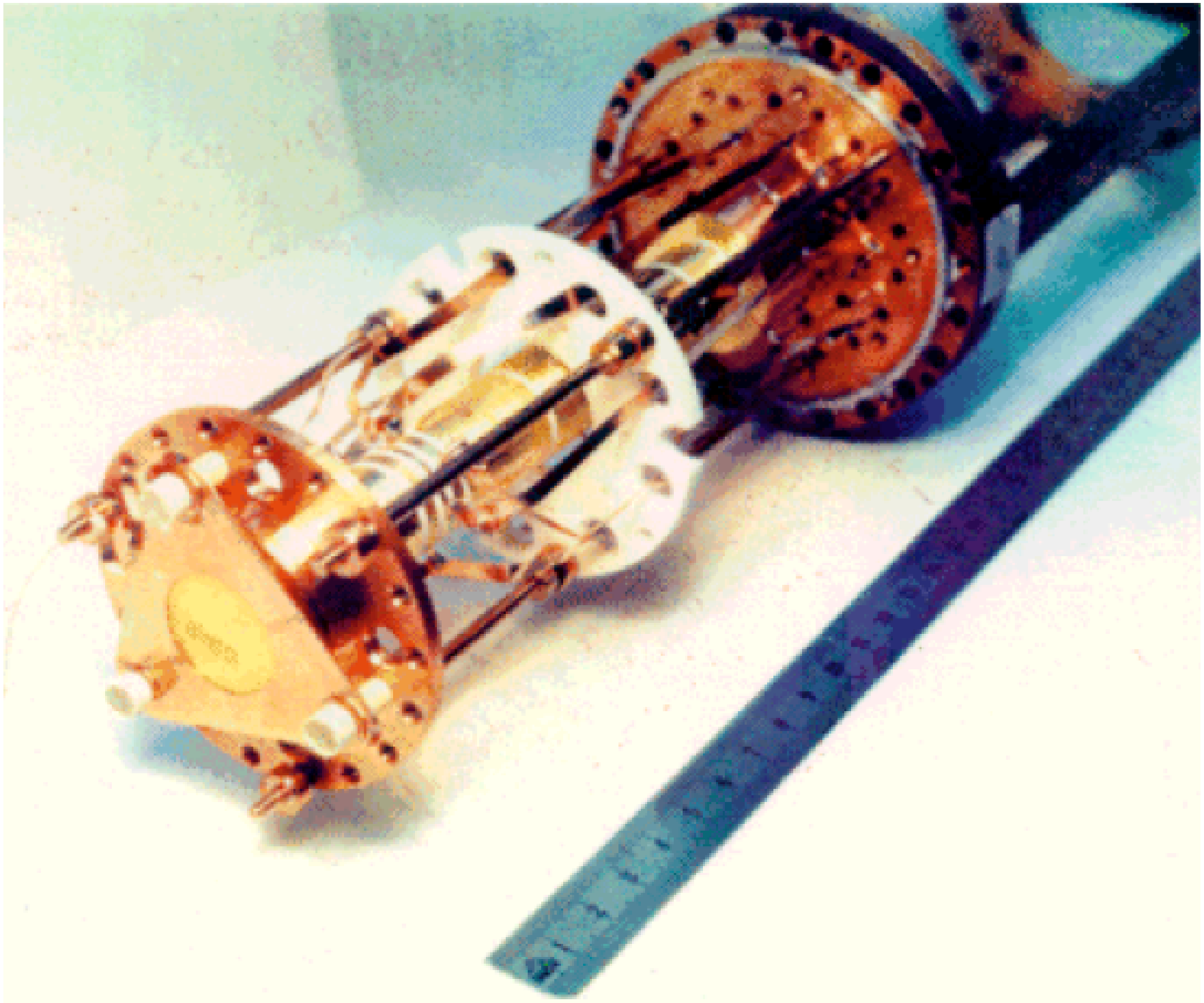
Methods to measure $\alpha(\mu^2 \simeq 0)$:

- Josephson effect
- Rydberg constant and de Broglie wave length of the neutron
- Quantum Hall effect
- Anomalous magnetic moment of the electron:

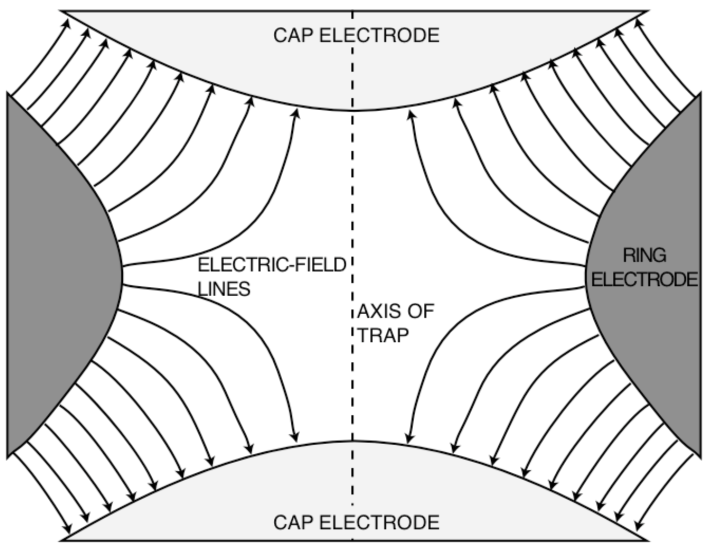


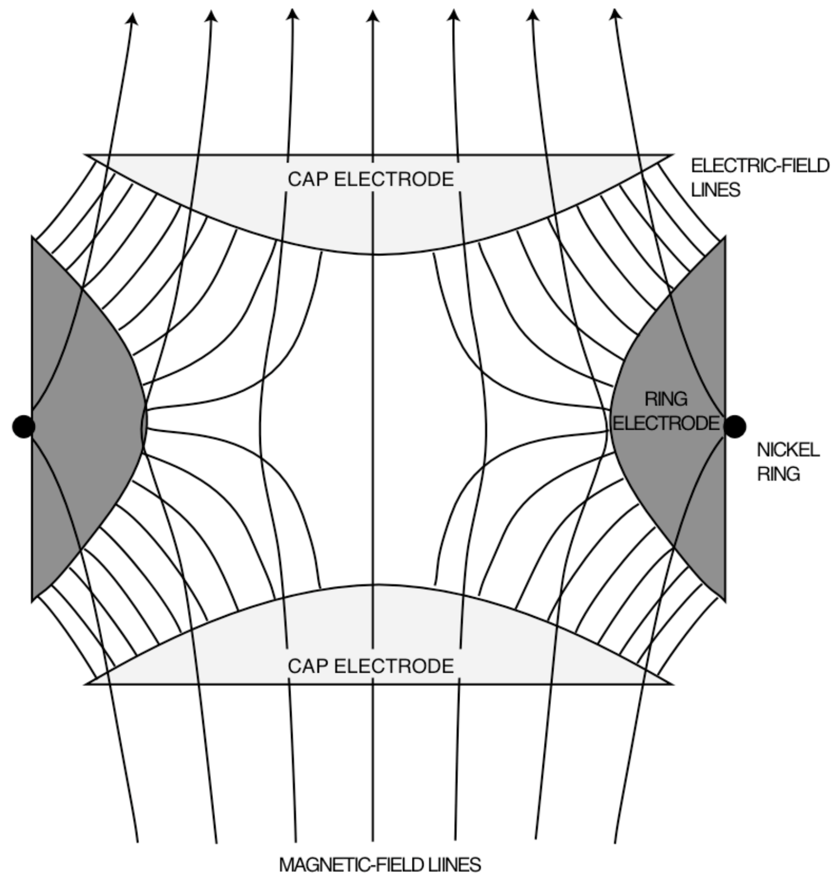
$$\begin{aligned} a_e &= \frac{g_e - 2}{2} \\ &= 0.5\left(\frac{\alpha}{\pi}\right) - 0.328478965\left(\frac{\alpha}{\pi}\right)^2 + 1.181241456\left(\frac{\alpha}{\pi}\right)^3 \\ &\quad -1.4092(384)\left(\frac{\alpha}{\pi}\right)^4 + 4.396(42) \times 10^{-12} - \ldots \end{aligned}$$

Anomalous Magnetic Moment: Penning Trap



Penning Trap





The electromagnetic configuration of a Penning trap consists of an axial magnetic field, which confines the electrons radially. A quadrupole filed creates a dip of the potential in axial direction. A nickel ring electrode slightly deforms the field such that a coupling between orbital energy and axial oscillation exists.

Penning Trap

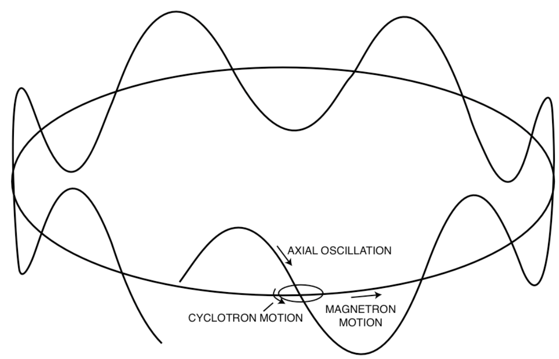
The force on an electron in such a field is:

$$\vec{F} = -e\vec{\nabla}V + e\left(\vec{v} \times \vec{B}\right) \;\; ; \;\; V = \frac{V_0}{2d^2}\left(z^2 - \frac{x^2}{2} - \frac{y^2}{2}\right) \;\; ; \;\; \vec{B} \simeq (0, 0B_z)$$

Consequently, with the cyclotron frequency $\omega_c = eB/m$, the equations of motion are:

$$\ddot{x} = \frac{eV_0}{2md^2}x + \omega_c\dot{y} \;\; ; \;\; \ddot{y} = \frac{eV_0}{2md^2}x - \omega_c\dot{x} \;\; ; \;\; \ddot{z} = -\frac{eV_0}{md^2}z$$

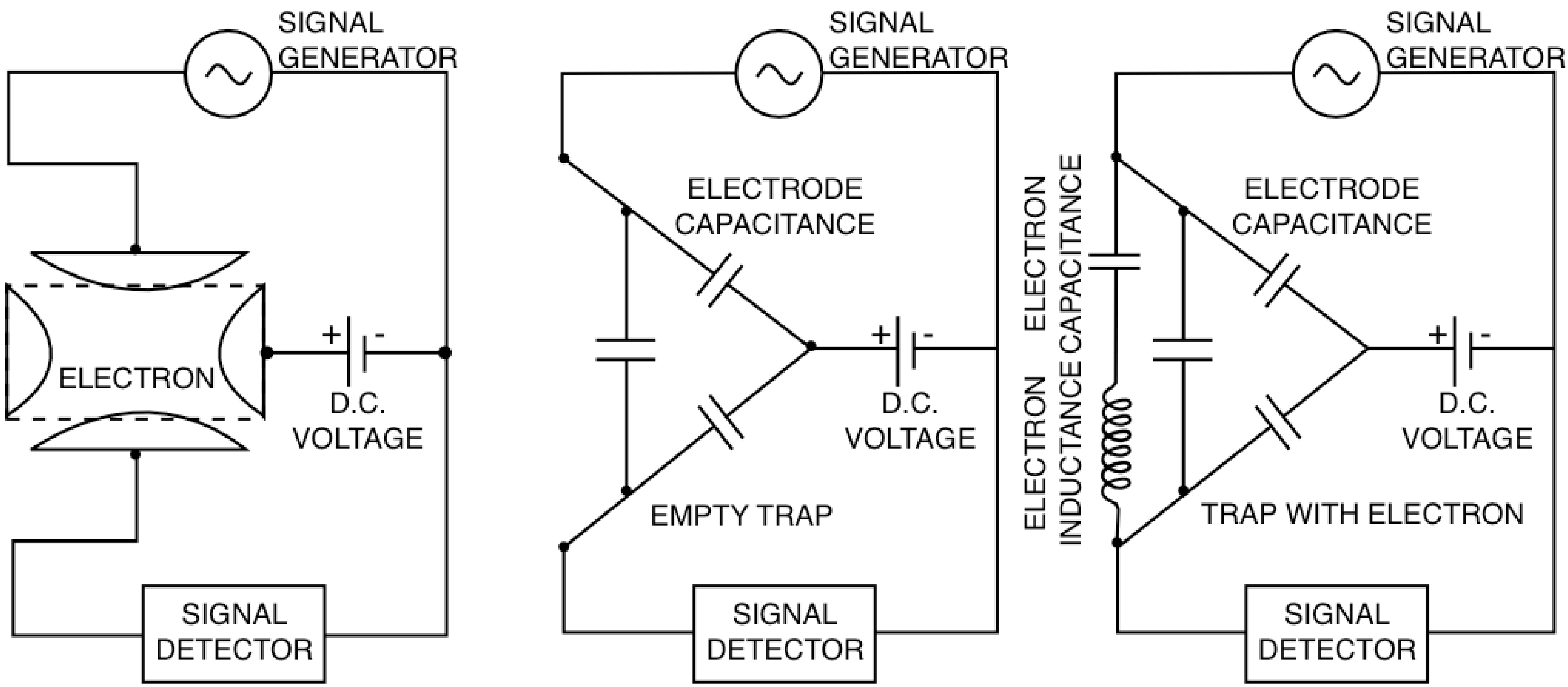
In the axial z direction, the movement is a harmonic oscillation with frequency $\omega_z = \sqrt{eV_0/md^2}$. It is decoupled from the movement in x and y, here it can be decomposed in a rapid cyclotron movement, with slightly modified frequency ω_c', and a slow magnetron drift with frequency ω_m':



$$\omega_c' = \frac{1}{2}\left(\omega_c + (\omega_c^2 - 2\omega_z^2)^{1/2}\right) \;\; ; \;\; \omega_m' = \frac{1}{2}\left(\omega_c - (\omega_c^2 - 2\omega_z^2)^{1/2}\right)$$

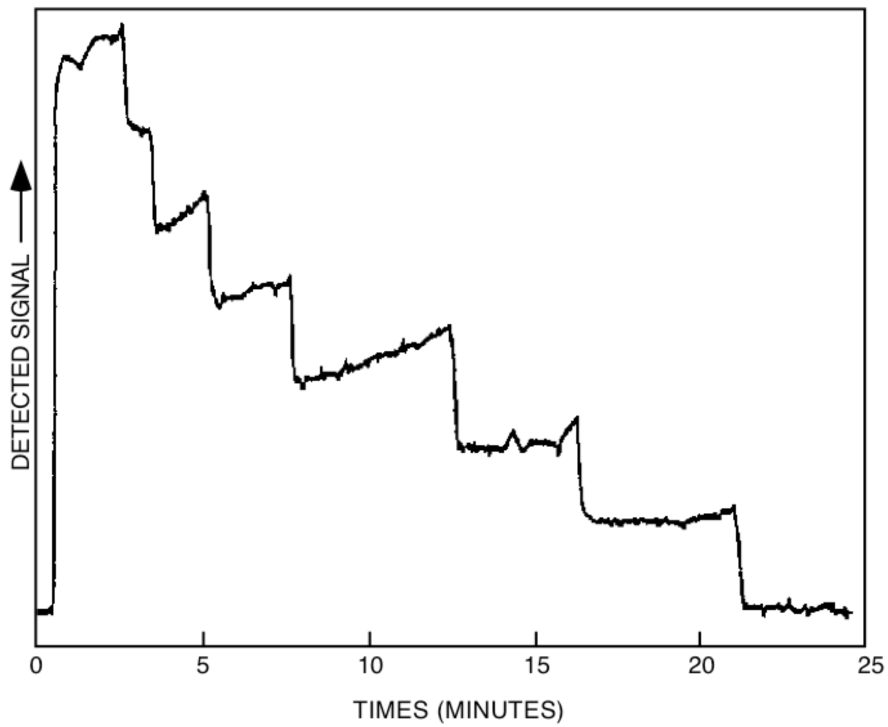
For electrons with $\omega_c = 150$GHz, typical conditions are $\omega_c' = 30$GHz and $\omega_m' = 3$kHz. The interaction with the electron spin modifies these frequencies.

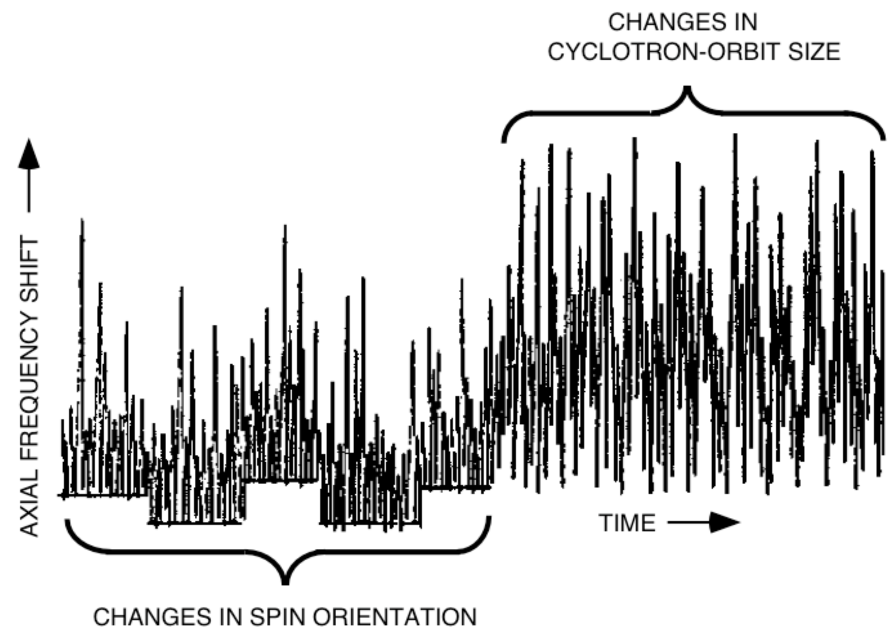
Penning Trap



Electrical equivalent of an empty and filled *penning trap*.

Multiple electron signals in a Penning Trap





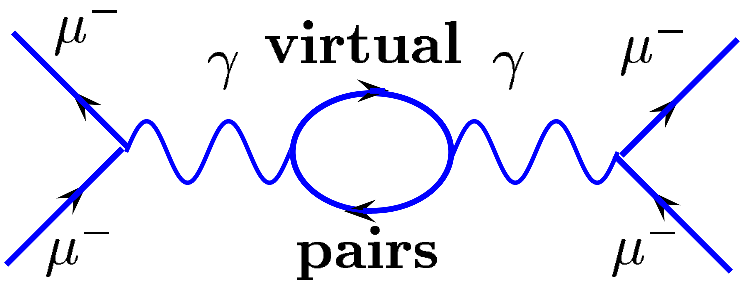
Signal as a function of time, starting with seven electrons in the trap, which are lost one by one.

Frequency shift as a function of time. The minimal shifts correspond to the two orientations of the electron spin, with respect to the magnetic axis.

Current accuracy: $\boldsymbol{\sigma_\alpha = 0.004}$ppm

Running α

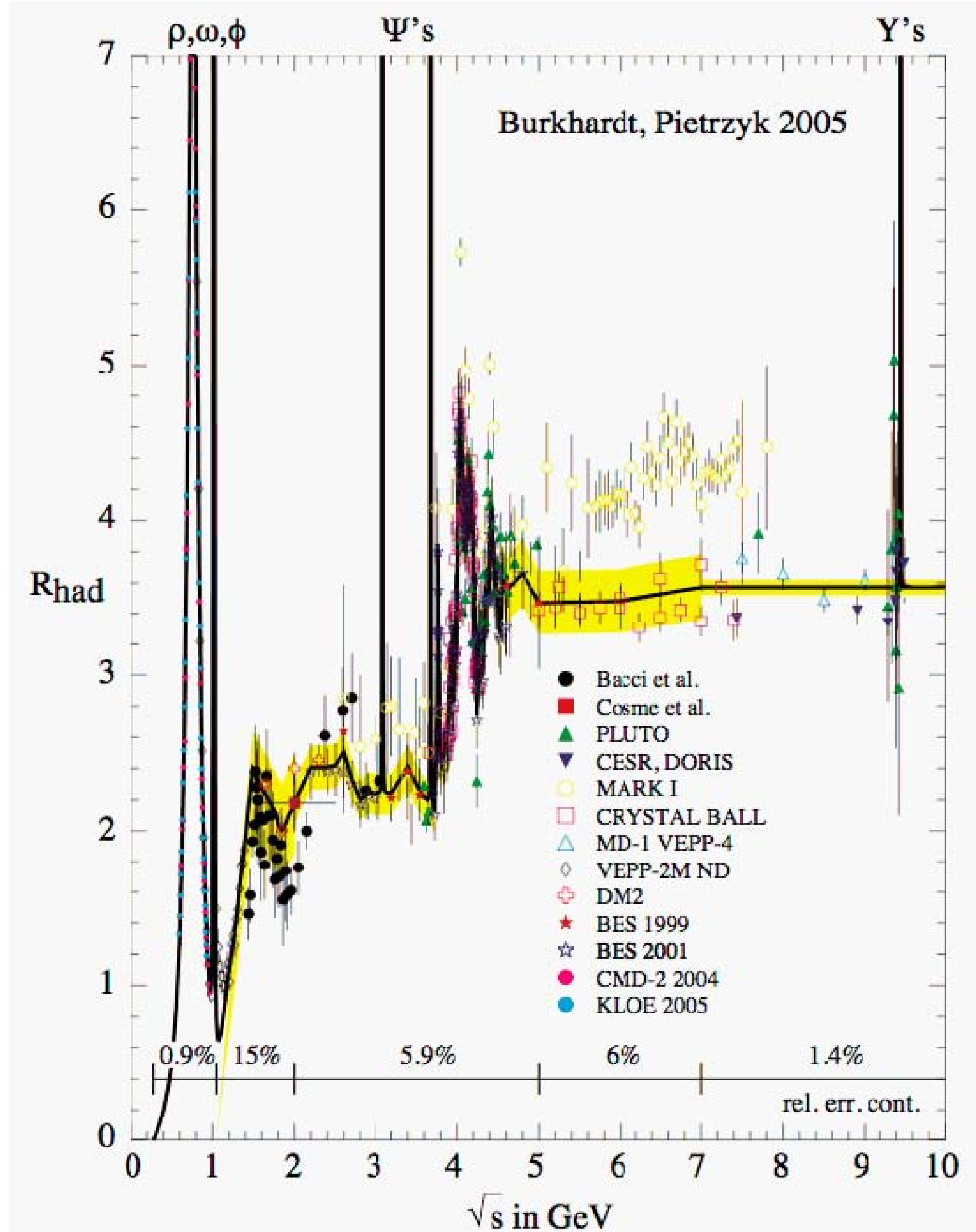
Fantastic accuracy is lost on the low walk from $\alpha(q^2 \simeq 0) \simeq 1/137$ to $\alpha(M_Z^2) \simeq 1/128$:



$$\gamma^* \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-, u\bar{u}, d\bar{d}, \cdots \rightarrow \gamma^*$$

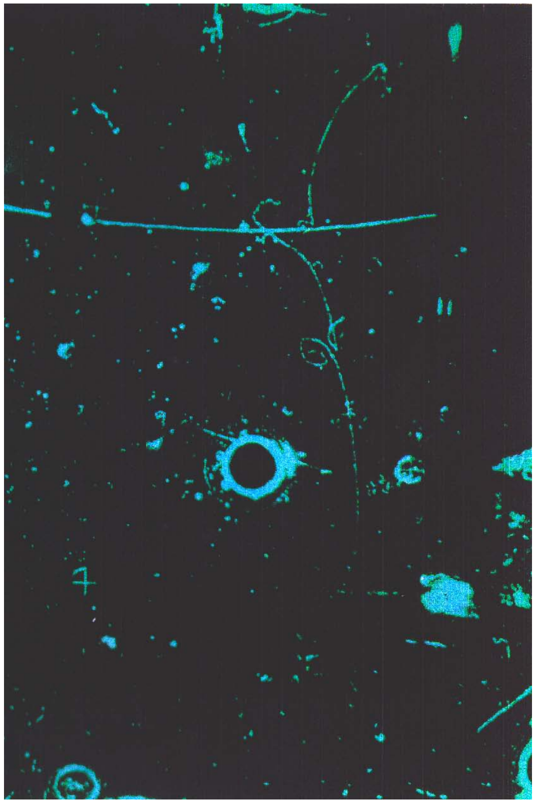
Loss is due to hadronic vacuum polarization, calculation must be replaced by measurements of $e^+e^- \rightarrow$ hadrons (and a bit of current algebra).

$$\alpha^{-1}(M_Z^2) = 128.940 \pm 0.48$$

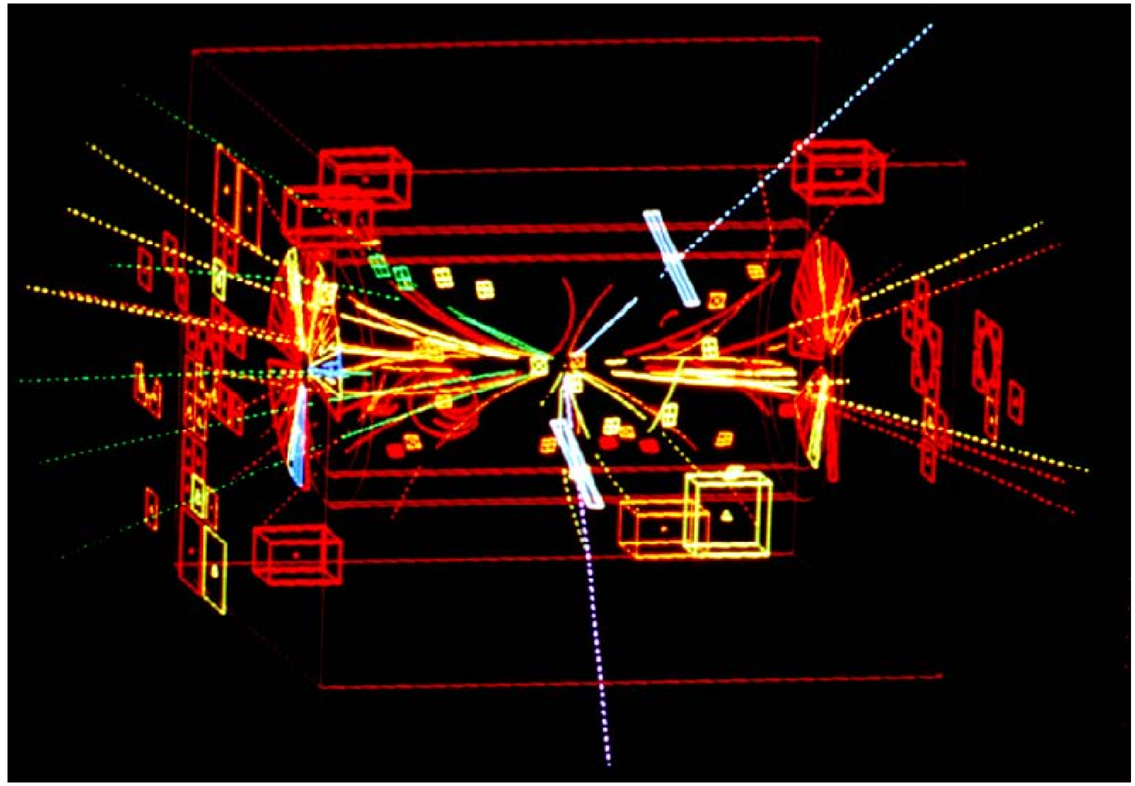


Discovery of the Z Boson

Gargamelle 1973



UA1 1973



VALUE (GeV)	EVTS	DOCUMENT ID		TECN	COMMENT
$91.74 \pm 0.28 \pm 0.93$	156	[9] ALITTI	92B	UA2	$E_{cm}^{p\bar{p}}$= 630 GeV
$90.9 \pm 0.3 \pm 0.2$	188	[10] ABE	89C	CDF	$E_{cm}^{p\bar{p}}$= 1.8 TeV
91.14 ± 0.12	480	[11] ABRAMS	89B	MRK2	E_{cm}^{ee}= 89 – 93 GeV
$93.1 \pm 1.0 \pm 3.0$	24	[12] ALBAJAR	89	UA1	$E_{cm}^{p\bar{p}}$= 546,630 GeV

[9] Enters fit through W/Z mass ratio given in the W Particle Listings. The ALITTI 1992B systematic error (±0.93) has two contributions: one (±0.92) cancels in m_W/m_Z and one (±0.12) is noncancelling. These were added in quadrature.

[10] First error of ABE 1989 is combination of statistical and systematic contributions; second is mass scale uncertainty.

[11] ABRAMS 1989B uncertainty includes 35 MeV due to the absolute energy measurement.

[12] ALBAJAR 1989 result is from a total sample of 33 $Z \rightarrow e^+ e^-$ events.

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CERN Large Elecron Positron Collider LEP

- e^+e^- collider, 27 km circumference
- 8 interaction points, 4 equipped with detectors: Aleph, Delphi, L3, Opal
- Maximum Center-of-mass energy $\sqrt{s} > 200$ GeV
- Running from 1989 to 2000

Precise Mass Measurement by Resonance

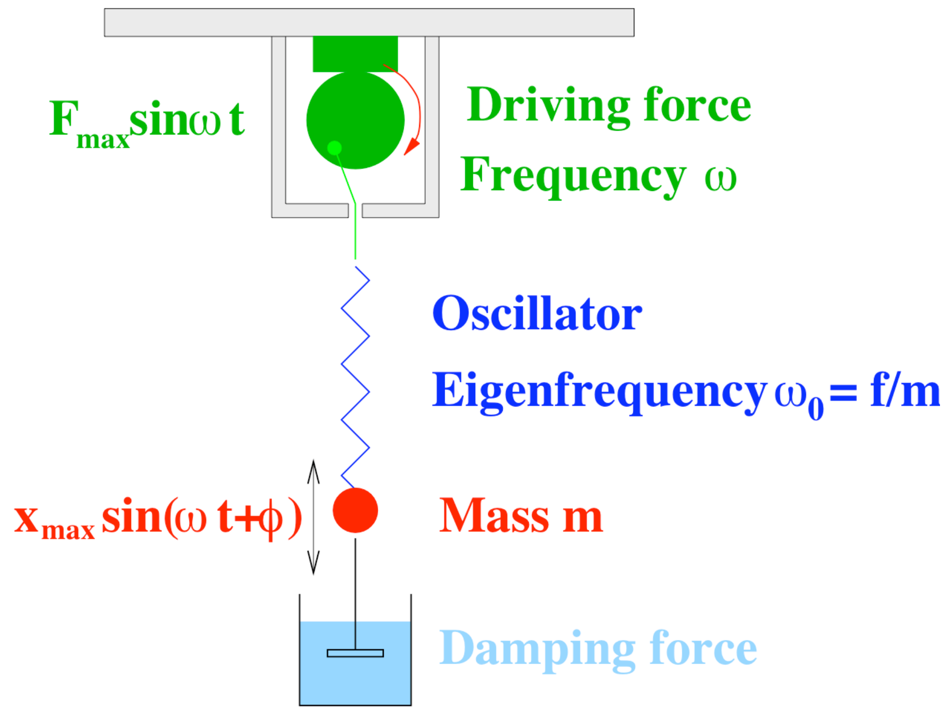
$$-kx - b\frac{dx}{dt} + F_0 \cos \omega t = m\frac{d^2x}{dt^2}$$

Solution:

$$x(t) = \frac{F_0 \sin\left(\omega t - \phi\right)}{\sqrt{m^2(\omega^2 - \omega_0^2)^2 + b^2\omega^2}}$$

with $\omega_0^2 = k/m$.

$$\begin{aligned} \omega &\leftrightarrow \text{total energy } E/\hbar \\ k &\leftrightarrow \text{coupling constant} \\ b &\leftrightarrow \text{width} = \tau^{-1} \\ m &\leftrightarrow \text{mass} \end{aligned}$$

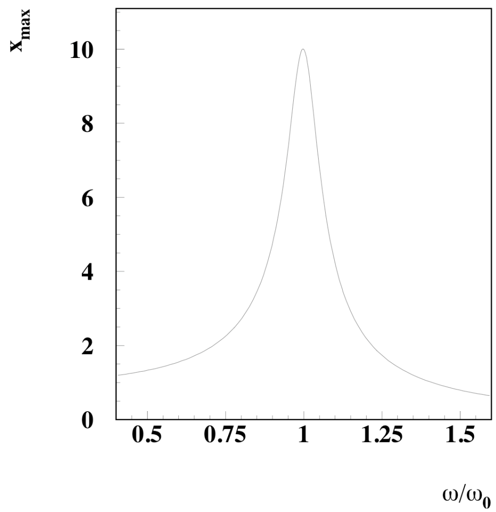


$x_{max}^2 \leftrightarrow$ cross section

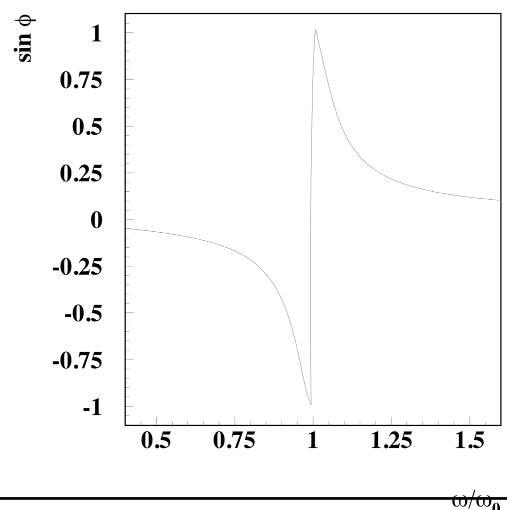
$\sin\phi \leftrightarrow$ angular asymmetry

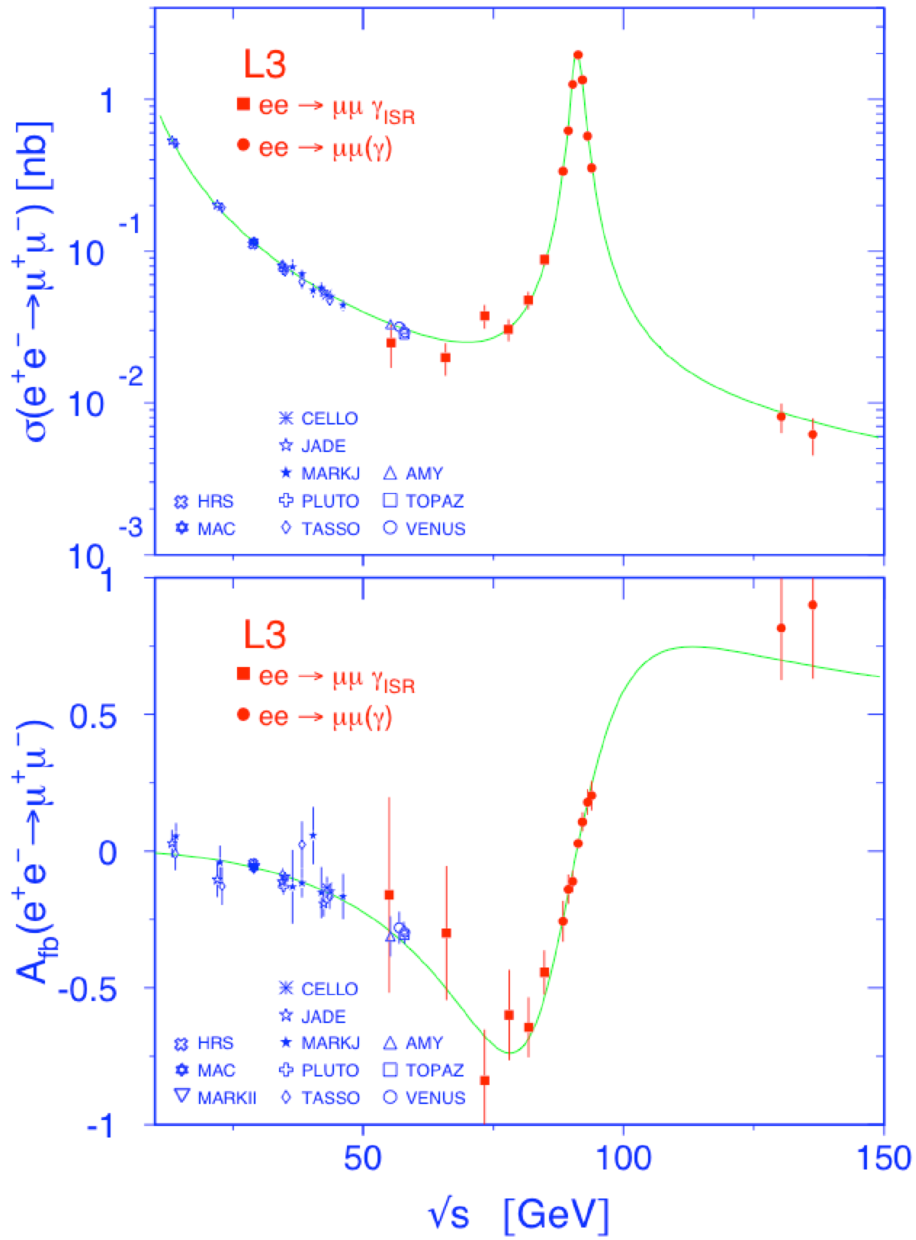
Z Boson Mass Measurement

$x^2_{max} \leftrightarrow$ cross section

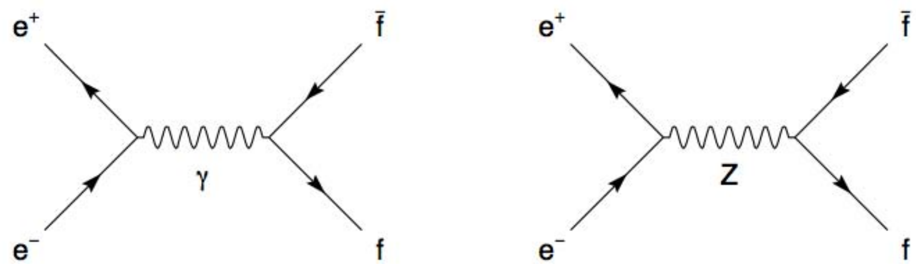


$\sin\phi \leftrightarrow$ angular asymmetry





Electron Positron Annihilation to Fermion Pairs



Photon exchange only ($k^2 \gg m_\mu^2$), unpolarized electrons:

$$\mathcal{M}_\gamma = -e^2(\bar{\psi}_\mu \gamma^\nu \psi_\mu)\frac{1}{k^2}(\bar{\psi}_e \gamma_\nu \psi_e) \quad ; \quad \overline{|\mathcal{M}_\gamma|^2} = 2e^4 \frac{t^2+u^2}{s^2}$$

Mandelstam variables:

$$t = -2p^2(1-\cos\theta) \quad ; \quad u = -2p^2(1+\cos\theta) \quad ; \quad s = -4p^2$$

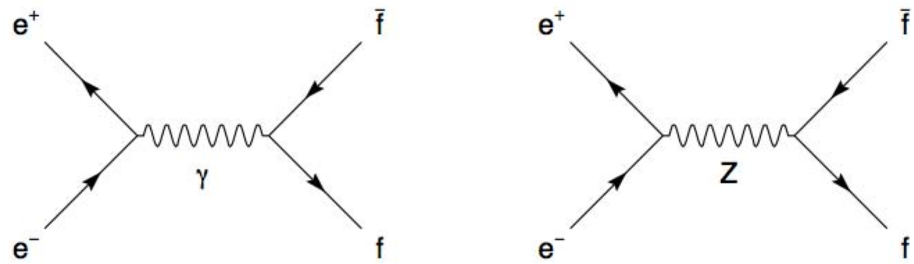
with the scattering angle θ between the incoming e^- and the outgoing μ^-.

Differential and total cross section:

$$\left.\frac{d\sigma_{QED}}{d\Omega}\right|_{cm} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}\overline{|\mathcal{M}_\gamma|^2} = \frac{\alpha^2}{4s}(1+\cos^2\theta)$$

$$\sigma_{QED} = \frac{4\pi\alpha^2}{3s}$$

Electron Positron Annihilation to Fermion Pairs



The Z exchange graph must be added at the amplitude level:

$$|\mathcal{M}|^2 = |\mathcal{M}_\gamma + \mathcal{M}_Z|^2 = |\mathcal{M}_\gamma|^2 + |\mathcal{M}_Z|^2 + |\mathcal{M}_I|$$

Universality means $g_V^e = g_V^\mu$ and $g_A^e = g_A^\mu$. Again for $s \gg m_\mu^2$:

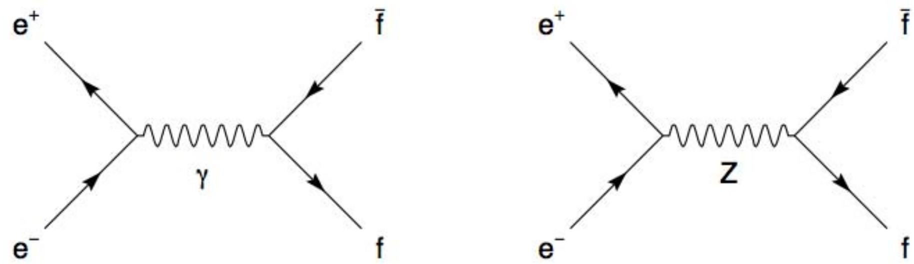
$$\mathcal{M}_Z = \frac{-\sqrt{2}G_F M_Z^2}{s - M_Z^2}\left[g_R^\mu(\bar{\psi}_{\mu_R}\gamma^\nu\psi_{\mu_R}) + g_L^\mu(\bar{\psi}_{\mu_L}\gamma^\nu\psi_{\mu_L})\right]$$
$$\left[g_R^e(\bar{\psi}_{e_R}\gamma_\nu\psi_{e_R}) + g_L^e(\bar{\psi}_{e_L}\gamma_\nu\psi_{e_L})\right]$$

with the helicity couplings:

$$g_R \equiv g_V - g_A \qquad ; \qquad g_L \equiv g_V + g_A$$

such that the left- and righthanded spinor components are explicit.

Electron Positron Annihilation to Fermion Pairs



Electrons and positrons polarized, polarization of final state measured:

$$\frac{d\sigma}{d\Omega}(\mathrm{e}_L^-\mathrm{e}_R^+ \to \mu_L^-\mu_R^+) = \frac{\alpha^2}{4s}(1+\cos\theta)^2|1+rg_L^\mu g_L^e|^2$$
$$\frac{d\sigma}{d\Omega}(\mathrm{e}_L^-\mathrm{e}_R^+ \to \mu_R^-\mu_L^+) = \frac{\alpha^2}{4s}(1-\cos\theta)^2|1+rg_R^\mu g_L^e|^2$$

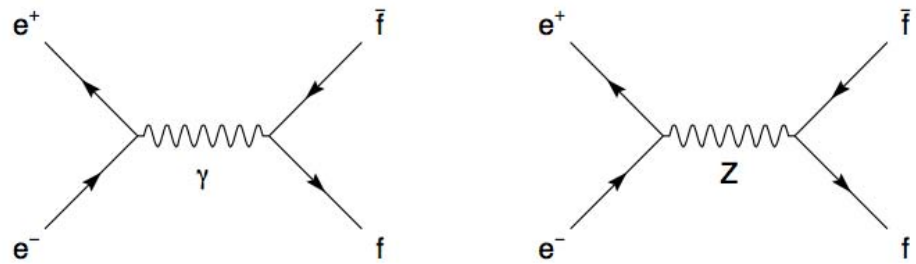
plus two more for $\mathrm{e}_R^-\mathrm{e}_L^+$. Attention: The final state fermions are polarized even if the incoming beams are not!

The pole term r, ratio of the Z propagator to the γ propagator:

$$r = \frac{\sqrt{2}G_F M_Z^2}{s - M_Z^2 + iM_Z\Gamma_Z}\left(\frac{s}{e^2}\right)$$

Only the real part $\Re(r)$ and $|r|^2$ contribute to the cross section.

Electron Positron Annihilation to Fermion Pairs



For the unpolarized case, we must average for cross sections:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left[A_0(1+\cos^2\theta) + A_1\cos\theta\right]$$

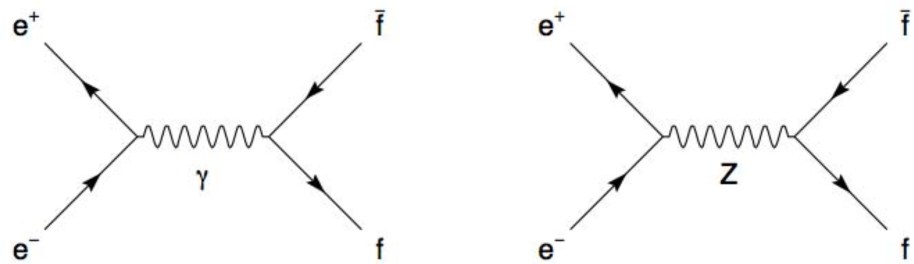
The coefficients are (for $g_i^\mu = g_i^e \equiv g_i$):

$$\begin{aligned} A_0 &= 1 + 2\Re(r)g_V^2 + |r|^2(g_V^2 + g_A^2)^2 \\ A_1 &= 4\Re(r)g_A^2 + 8|r|^2 g_V^2 g_A^2 \end{aligned}$$

The QED result $A_0 = 1$, $A_1 = 0$ is recovered far below the Z resonance, since $r \simeq 0$ at $s \ll M_Z^2$.

The pole term r is almost real below the resonance ($M_Z^2 - s \gg \Gamma_Z$), almost purely imaginary near the resonance ($M_Z^2 - s \ll \Gamma_Z$). Consequently, there are only small contributions of the Z graph, $\sim \Re(r)$, far below the resonance, while on resonance the large terms $\sim |r|^2$ dominate.

Electron Positron Annihilation to Fermion Pairs



The total cross section is:

$$\sigma(\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} A_0$$

It differs from the QED result only by the factor A_0. The angular asymmetry:

$$A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_F = \int_0^1 \frac{d\sigma}{d\Omega} d\cos\theta \, d\phi \qquad ; \qquad \sigma_B = \int_{-1}^0 \frac{d\sigma}{d\Omega} d\cos\theta \, d\phi$$

measures the relative strength of the asymmetric term $\sim \cos\theta$ relative to the symmetric term $\sim (1+\cos\theta)^2$:

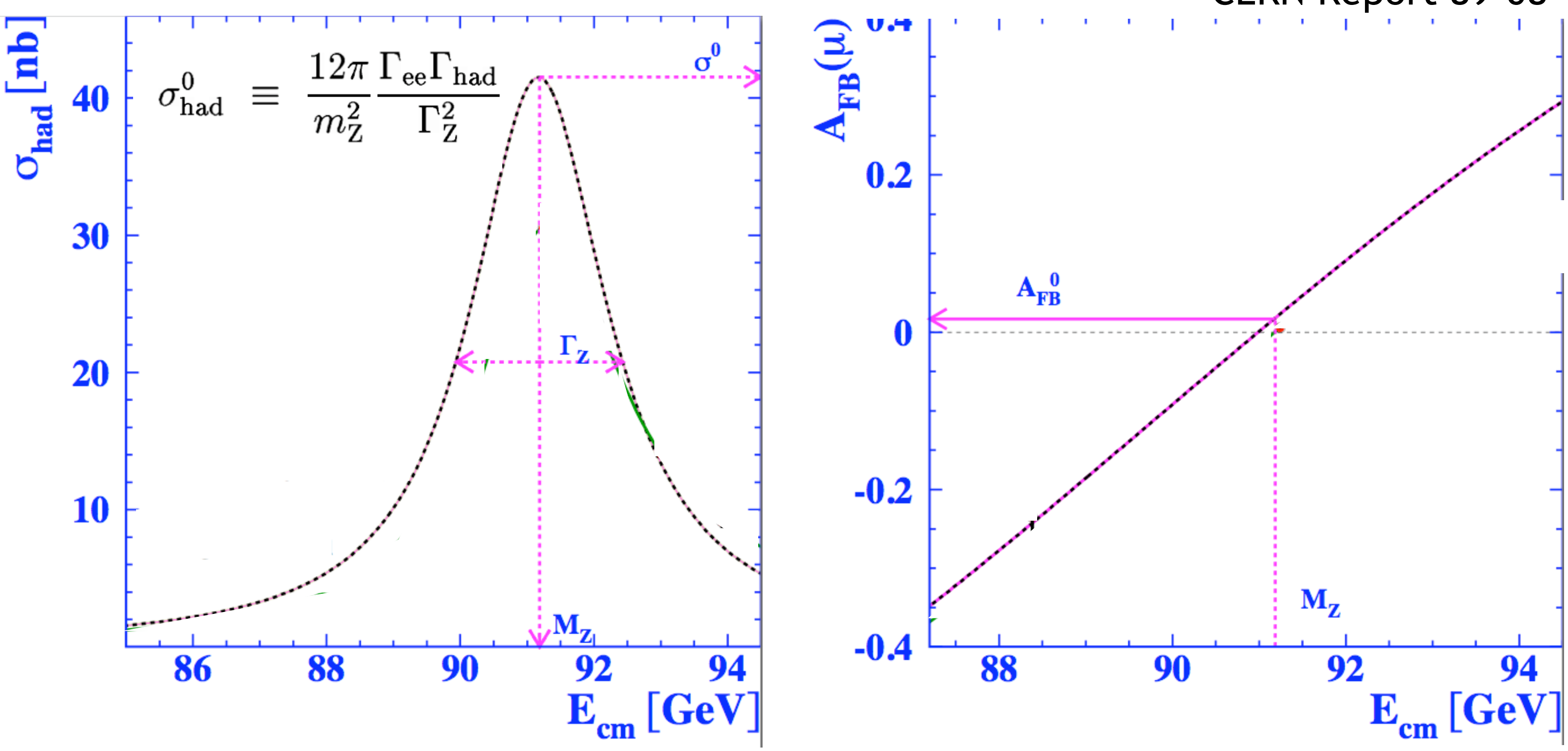
$$A_{FB} = \frac{3A_1}{8A_0} \simeq \begin{cases} \frac{3}{2}\Re(r) g_A^2 & \text{far below the resonance} \\ \frac{3g_V^2 g_A^2}{(g_V^2+g_A^2)^2} & \text{on the resonance} \end{cases}$$

Useful Quantifies in Z Physics

m_Z	Z-boson mass
Γ_Z	Z-boson width
σ^0_{had}	ee->Z->qq cross section
$R^0_e = \Gamma_{had}/\Gamma_{ee}$ $R^0_\mu = \Gamma_{had}/\Gamma_{\mu\mu}$ $R^0_\tau = \Gamma_{had}/\Gamma_{\tau\tau}$	Leptonic widths (expressed as fraction of hadronic)
$R^0_l = \Gamma_{had}/\Gamma_{ll}$	As above, with "leptonic universality"
$A^0_{FB}{}^e$, $A^0_{FB}{}^\mu$, $A^0_{FB}{}^\tau$	Leptonic forward-backward asymmetries
$A^0_{FB}{}^l$	As above, with "leptonic universality"

Z Boson Line Shape

CERN Report 89-08



Fundamental properties of the Z boson:

- Mass and width: M_Z, Γ_Z, from position and width of resonance peak, i.e. r.
- Couplings: g_{Af}, g_{Vf}, $f =$ e, μ, τ, q, from partial widths and asymmetries, σ^0_{had}, $R^0_l = \Gamma_{had}/\Gamma_{ll}$, A^0_{fb} and polarization.

Why not Invariant Mass of Decaying Z?

- Target resolution:

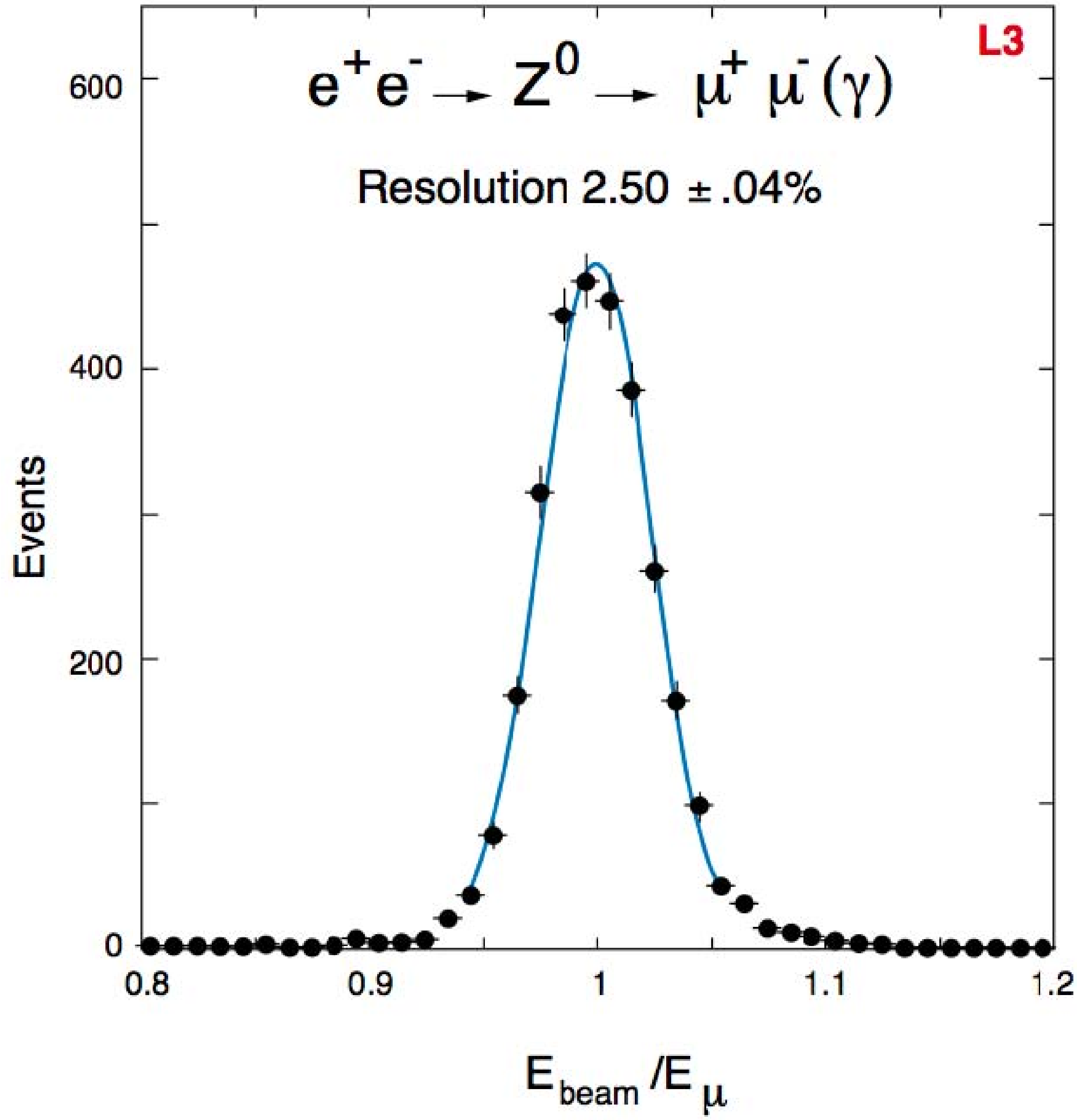
  $\sigma_M/M \simeq 2\times10^{-5}$

- Best momentum resolution:

  $\sigma_p/p \simeq 2 \times 10^{-2}$

  @ 50GeV

- At a lepton collider, initial state is better controlled than final state



Measurement Strategy I

- Scan the resonance by adjusting the beam energy

- Count Z bosons ”on peak” decaying into quarks:

$$\sigma^0_{had} \propto (N_{had} - N_{back})/\epsilon\mathcal{L}$$

- Count Z bosons ”on peak” decaying into leptons:

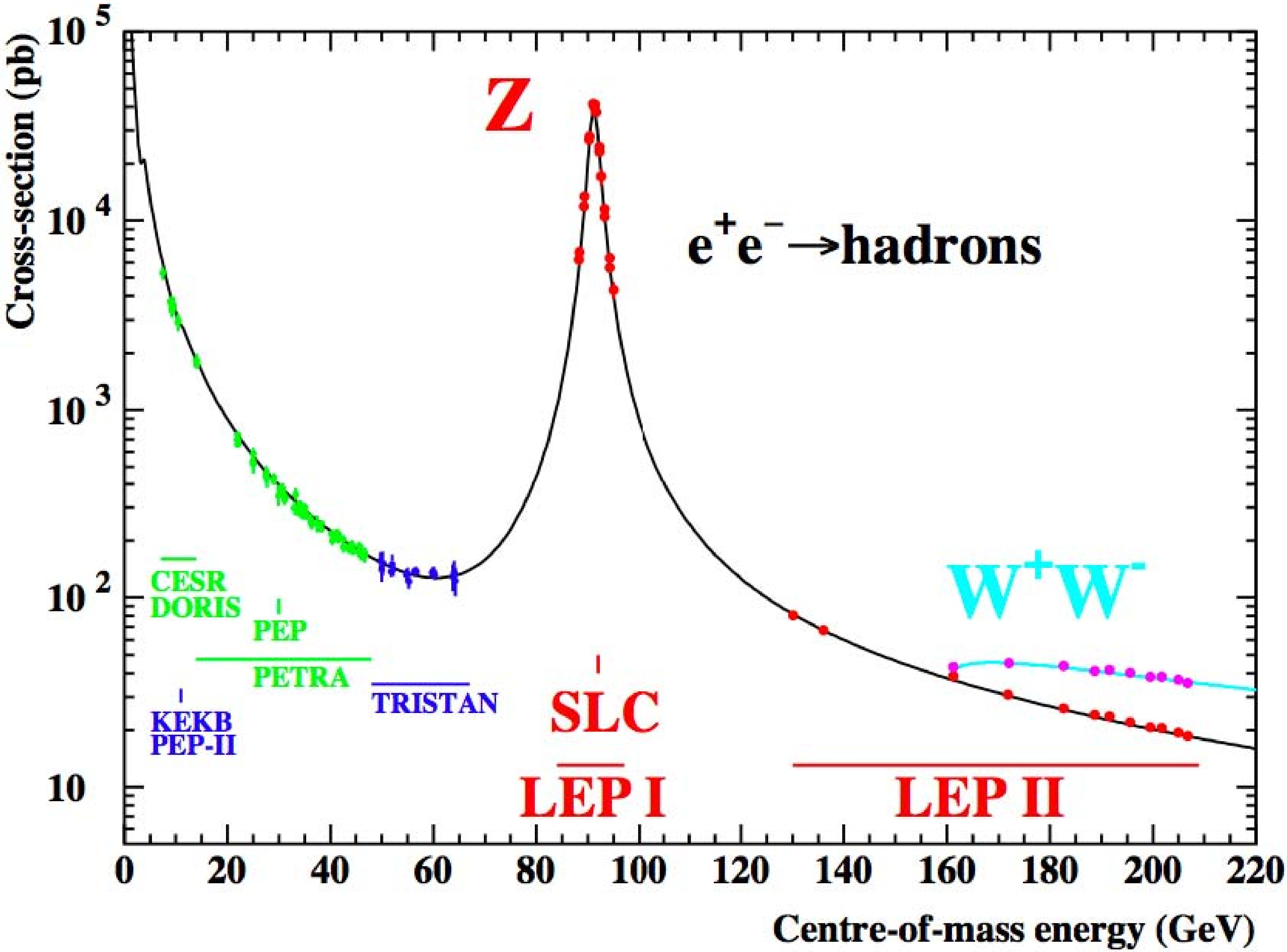
$$R^0_l \propto N_{had}/N_{ll}$$

Year	Centre-of-mass energy range [GeV]	Integrated luminosity [pb^{-1}]	
1989	88.2 – 94.2	1.7	
1990	88.2 – 94.2	8.6	} 7/pb off peak
1991	88.5 – 93.7	18.9	
1992	91.3	28.6	
1993	89.4, 91.2, 93.0	40.0	20/pb off peak
1994	91.2	64.5	
1995	89.4, 91.3, 93.0	39.8	20/pb off peak

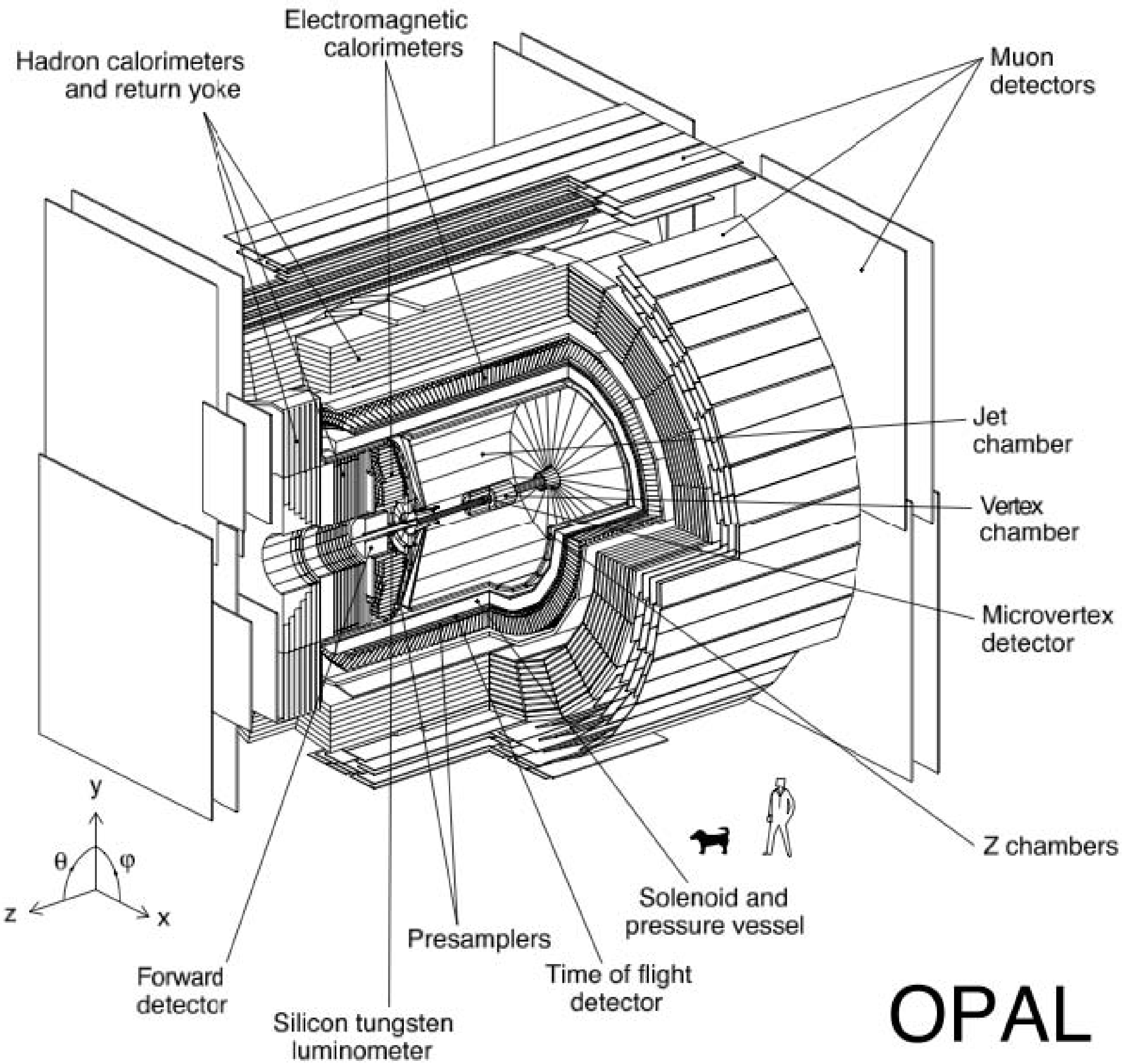
- Analyze angular distribution of decay products :

$$A^{0l}_{fb} \propto (N_F - N_B)/(N_F + N_B)$$

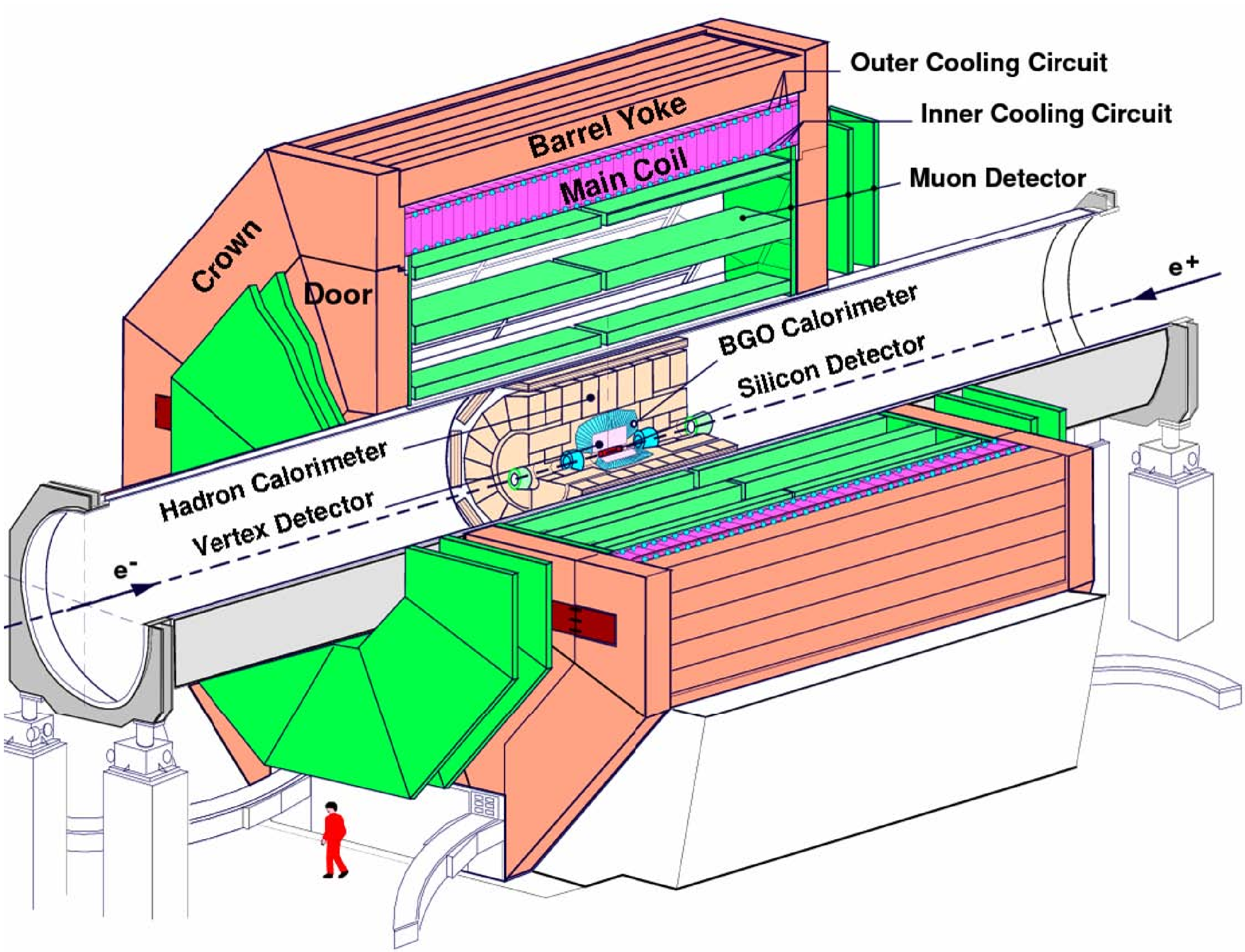
Measurements on the Z Boson Resonance



Typical Experiment: OPAL

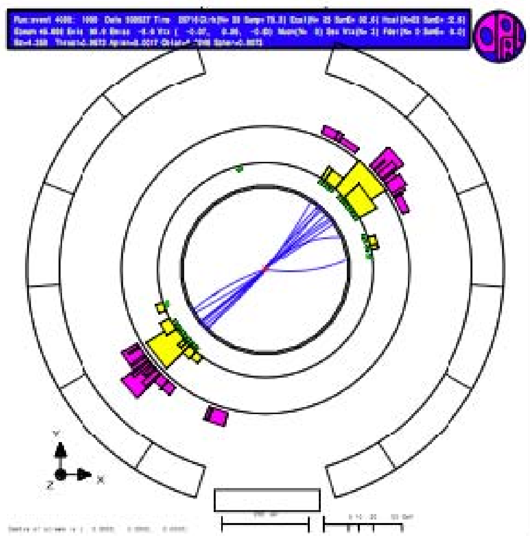


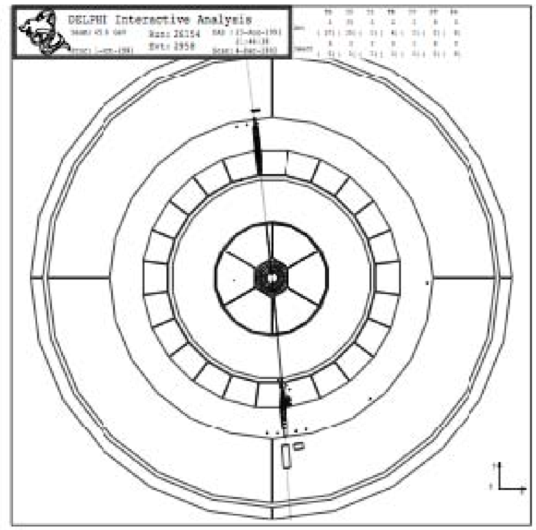
Atypical Experiment: L3



Fermion Pair Selection

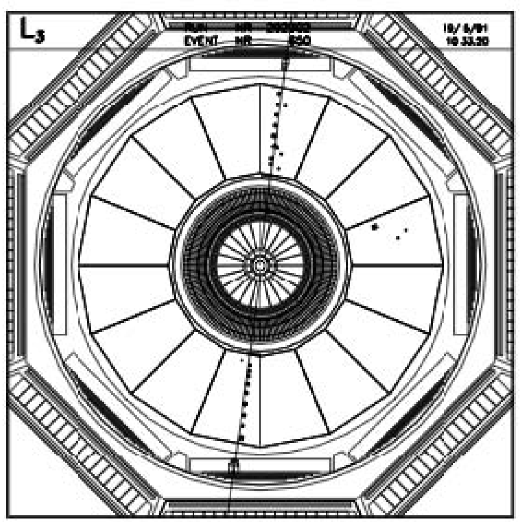
qq, 70%

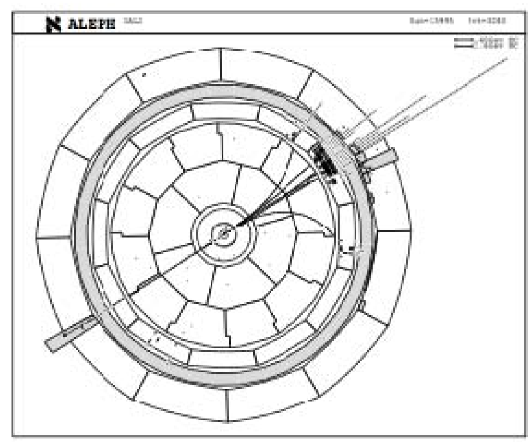




ee, 3%

$\mu\mu$, 3%





$\tau\tau$, 3%

+ 21% $\nu\nu$

Event Selection

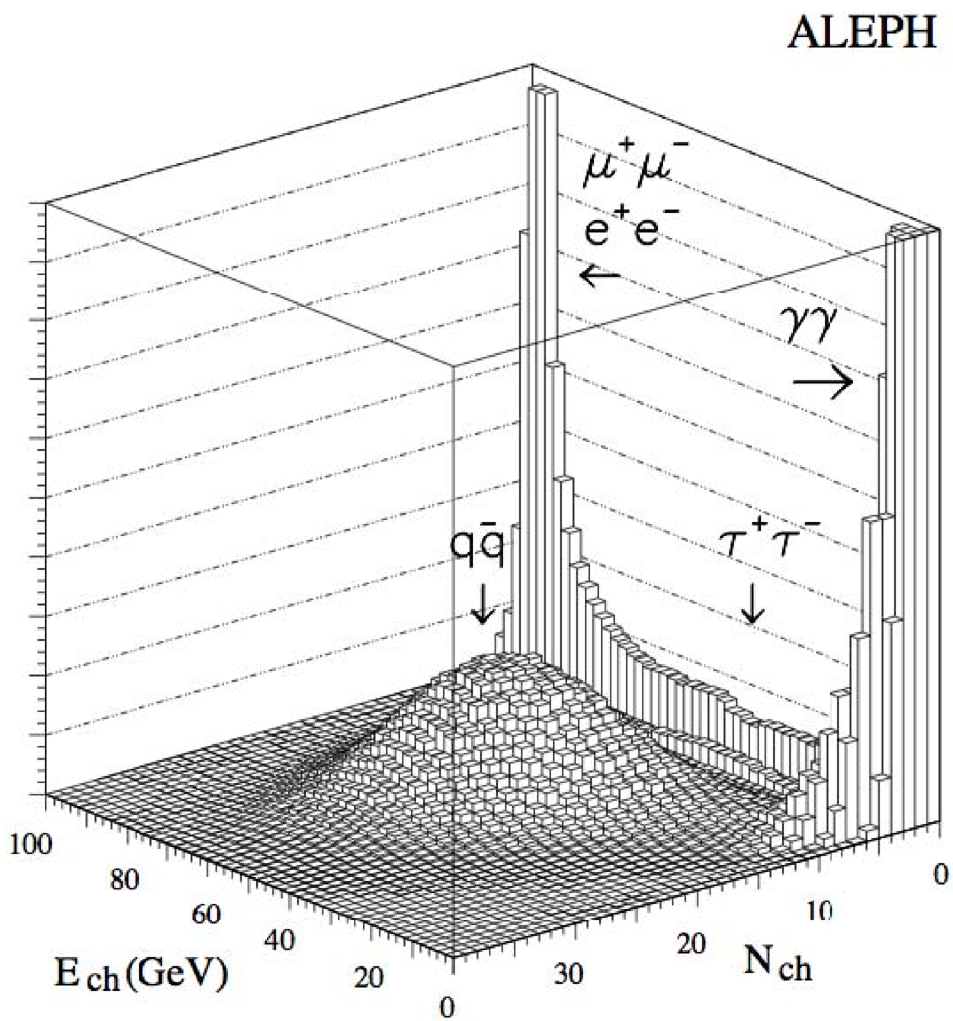


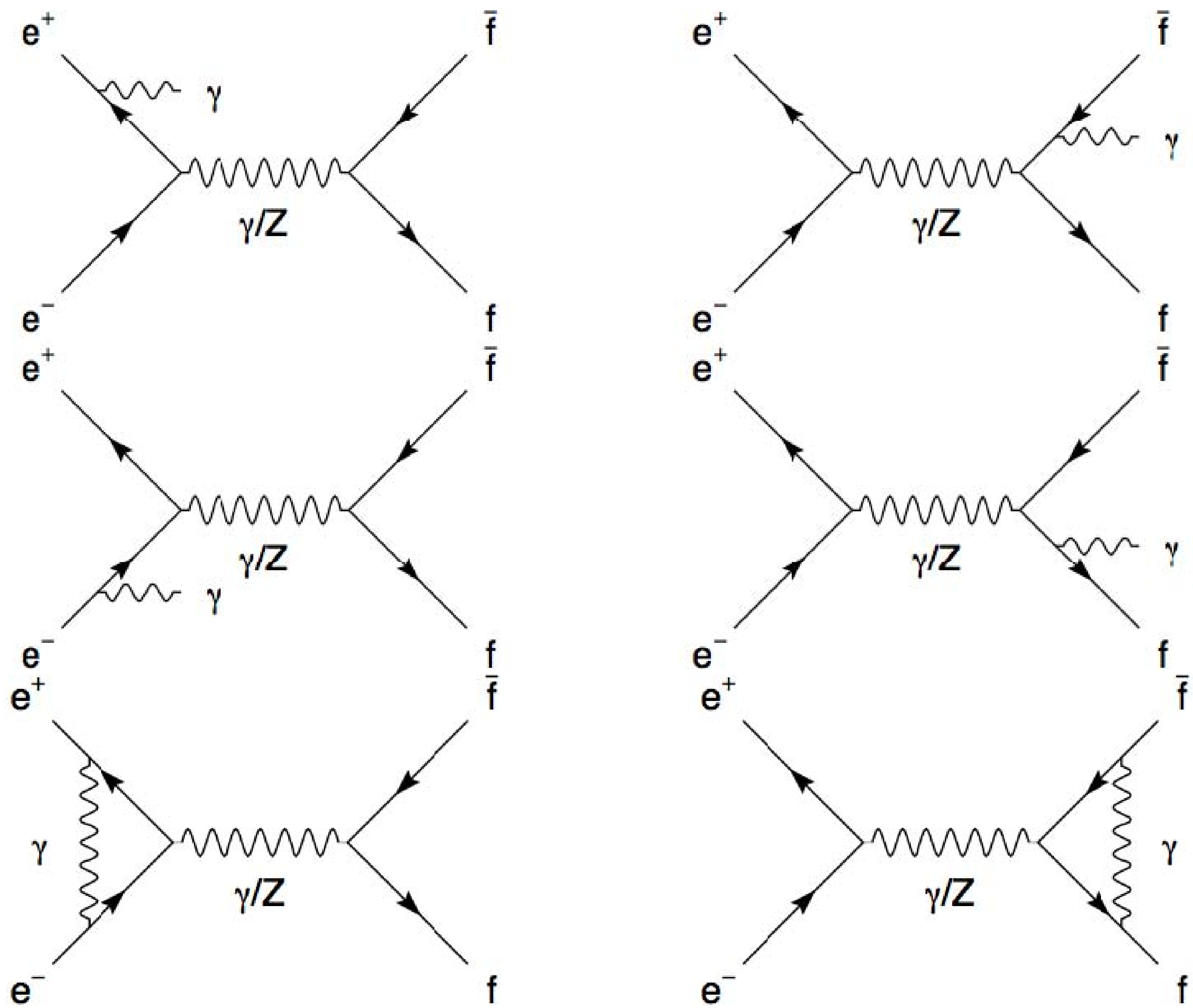
Figure 2.1: Experimental separation of the final states using only two variables, the sum of the track momenta, E_{ch}, and the track multiplicity, N_{ch}, in the central detector of the ALEPH experiment.

Event Selection

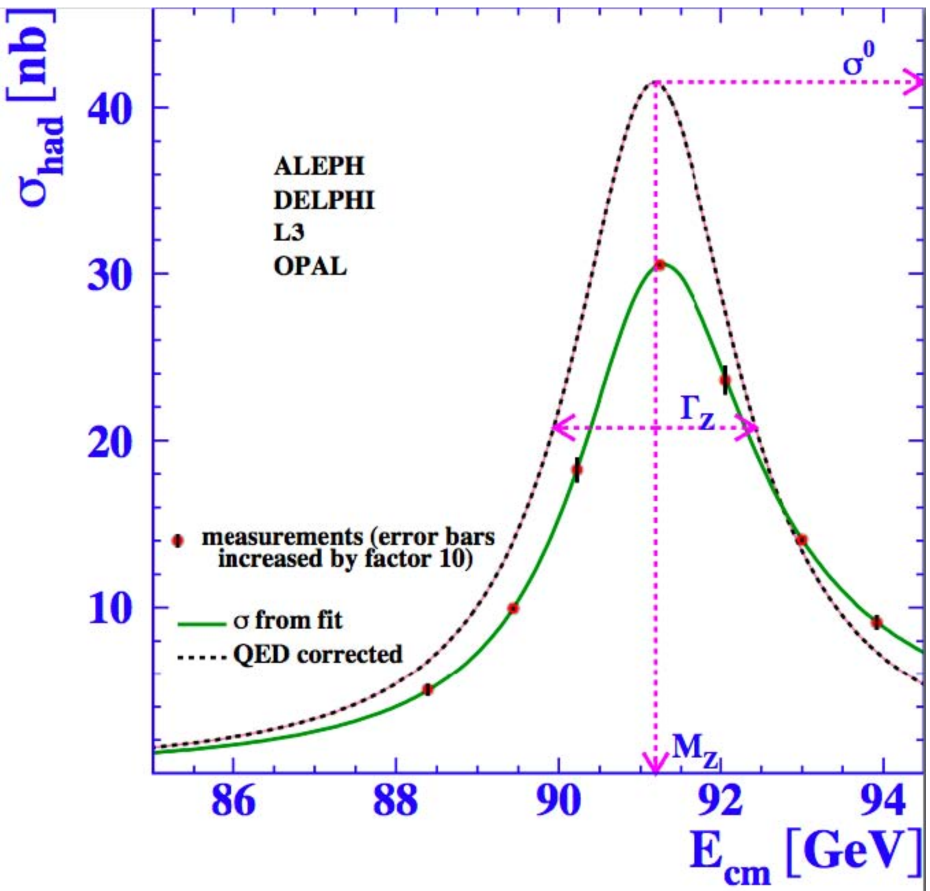
	ALEPH	DELPHI	L3	OPAL
$q\bar{q}$ final state				
acceptance	$s'/s > 0.01$	$s'/s > 0.01$	$s'/s > 0.01$	$s'/s > 0.01$
efficiency [%]	99.1	94.8	99.3	99.5
background [%]	0.7	0.5	0.3	0.3
e^+e^- final state				
acceptance	$-0.9 < \cos\theta < 0.7$ $s' > 4m_\tau^2$	$\lvert\cos\theta\rvert < 0.72$ $\eta < 10^\circ$	$\lvert\cos\theta\rvert < 0.72$ $\eta < 25^\circ$	$\lvert\cos\theta\rvert < 0.7$ $\eta < 10^\circ$
efficiency [%]	97.4	97.0	98.0	99.0
background [%]	1.0	1.1	1.1	0.3
$\mu^+\mu^-$ final state				
acceptance	$\lvert\cos\theta\rvert < 0.9$ $s' > 4m_\tau^2$	$\lvert\cos\theta\rvert < 0.94$ $\eta < 20^\circ$	$\lvert\cos\theta\rvert < 0.8$ $\eta < 90^\circ$	$\lvert\cos\theta\rvert < 0.95$ $m_{f\bar{f}}^2/s > 0.01$
efficiency [%]	98.2	95.0	92.8	97.9
background [%]	0.2	1.2	1.5	1.0
$\tau^+\tau^-$ final state				
acceptance	$\lvert\cos\theta\rvert < 0.9$ $s' > 4m_\tau^2$	$0.035 < \lvert\cos\theta\rvert < 0.94$ $s' > 4m_\tau^2$	$\lvert\cos\theta\rvert < 0.92$ $\eta < 10^\circ$	$\lvert\cos\theta\rvert < 0.9$ $m_{f\bar{f}}^2/s > 0.01$
efficiency [%]	92.1	72.0	70.9	86.2
background [%]	1.7	3.1	2.3	2.7

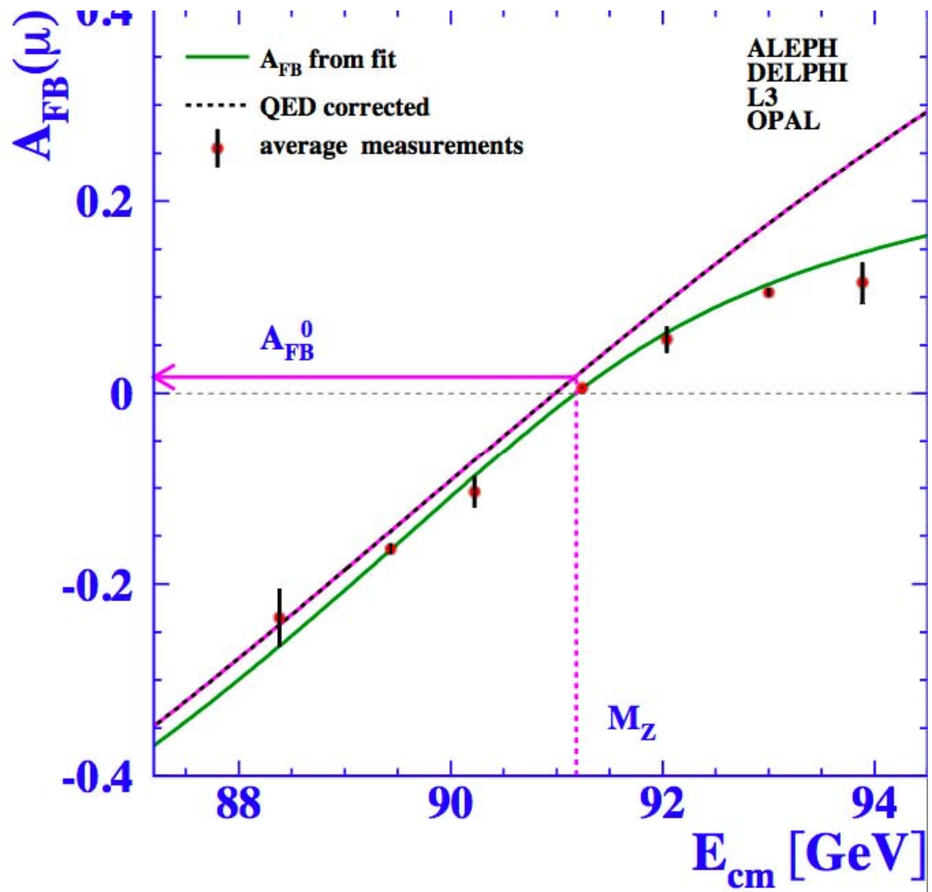
Number of Events										
	$Z \to q\bar{q}$					$Z \to \ell^+\ell^-$				
Year	A	D	L	O	LEP	A	D	L	O	LEP
1990/91	433	357	416	454	1660	53	36	39	58	186
1992	633	697	678	733	2741	77	70	59	88	294
1993	630	682	646	649	2607	78	75	64	79	296
1994	1640	1310	1359	1601	5910	202	137	127	191	657
1995	735	659	526	659	2579	90	66	54	81	291
Total	4071	3705	3625	4096	15497	500	384	343	497	1724

Radiative Corrections



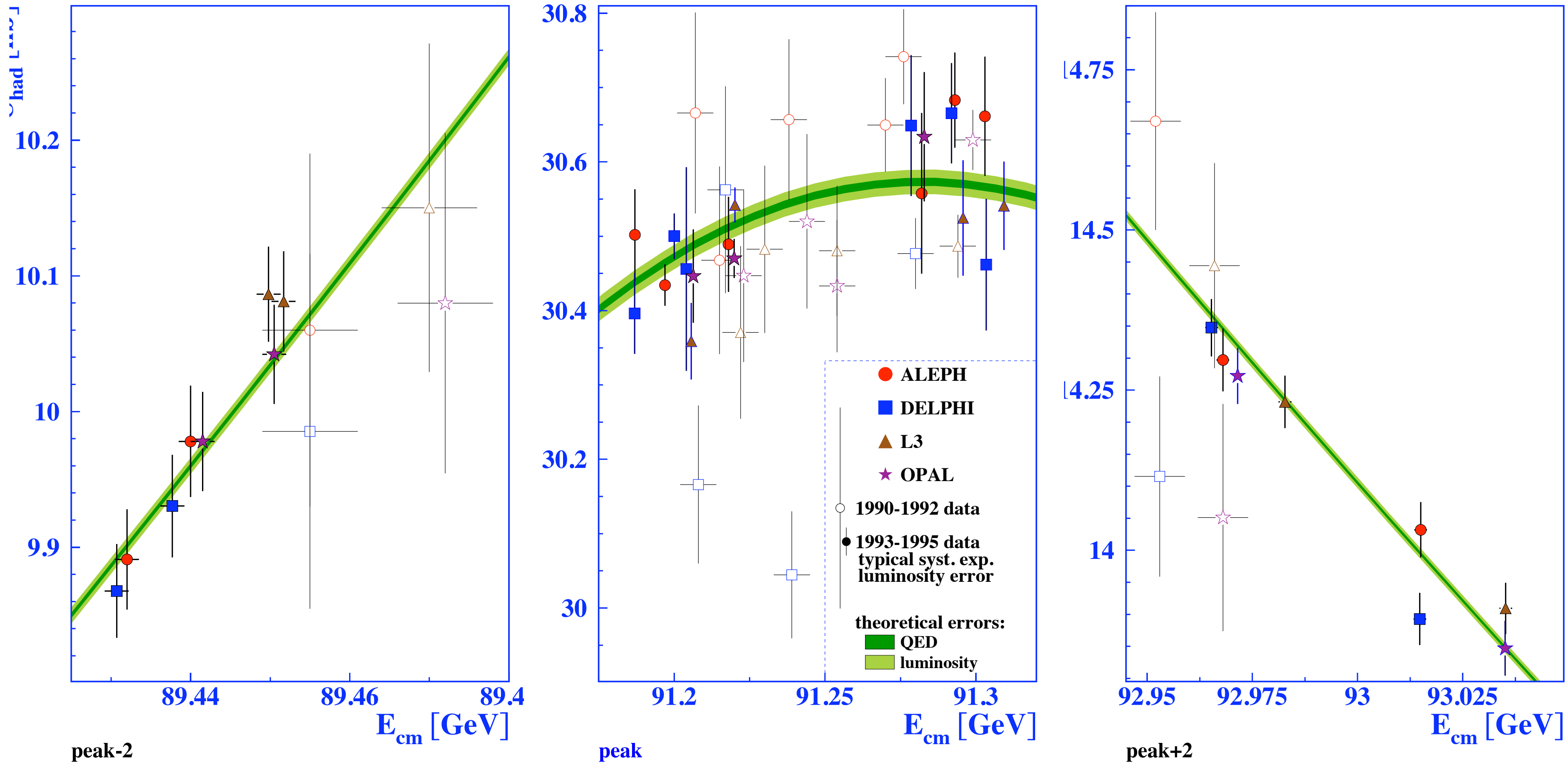
Radiative Corrections Modify Lineshape





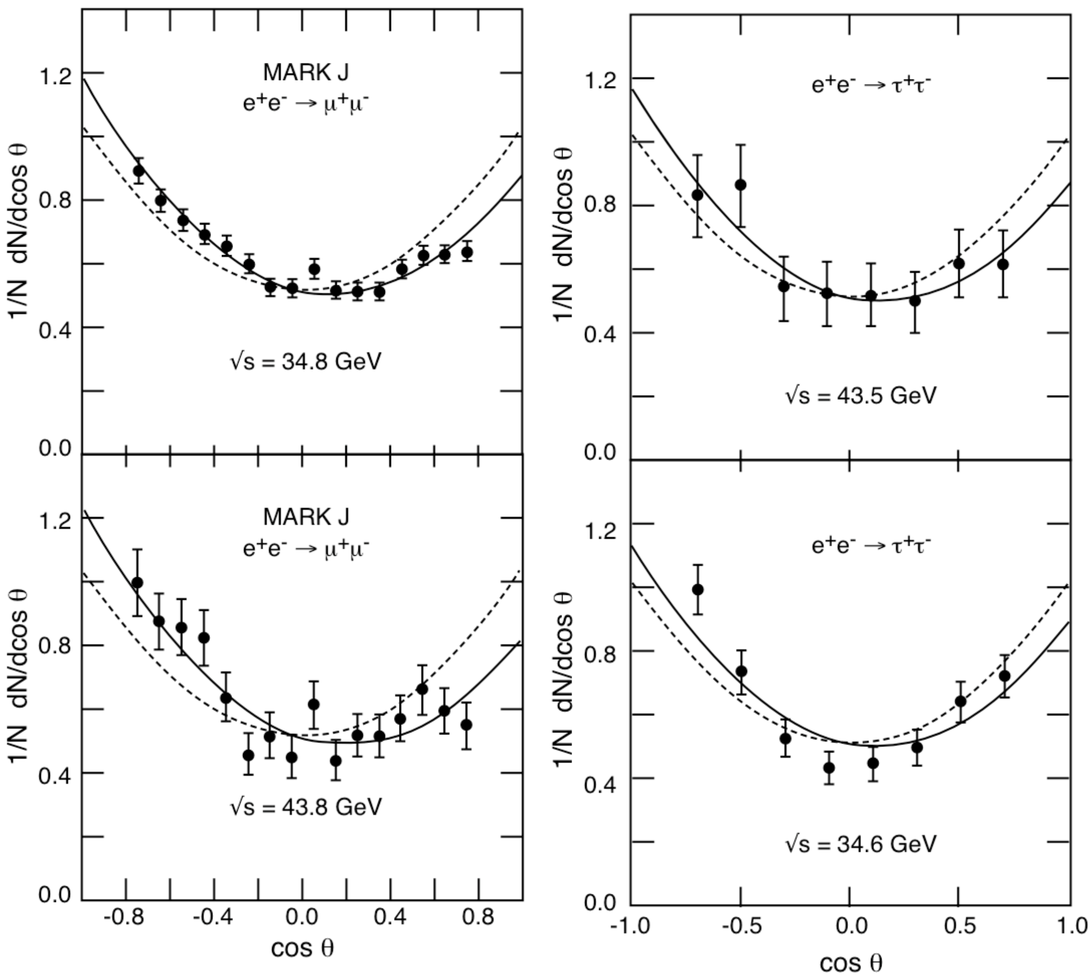
Results from 15 million Z bosons!

Details of the Total Cross Section Measurements



Good agreement among experiments, and with electroweak theory

Angular Distribution at Low Energies



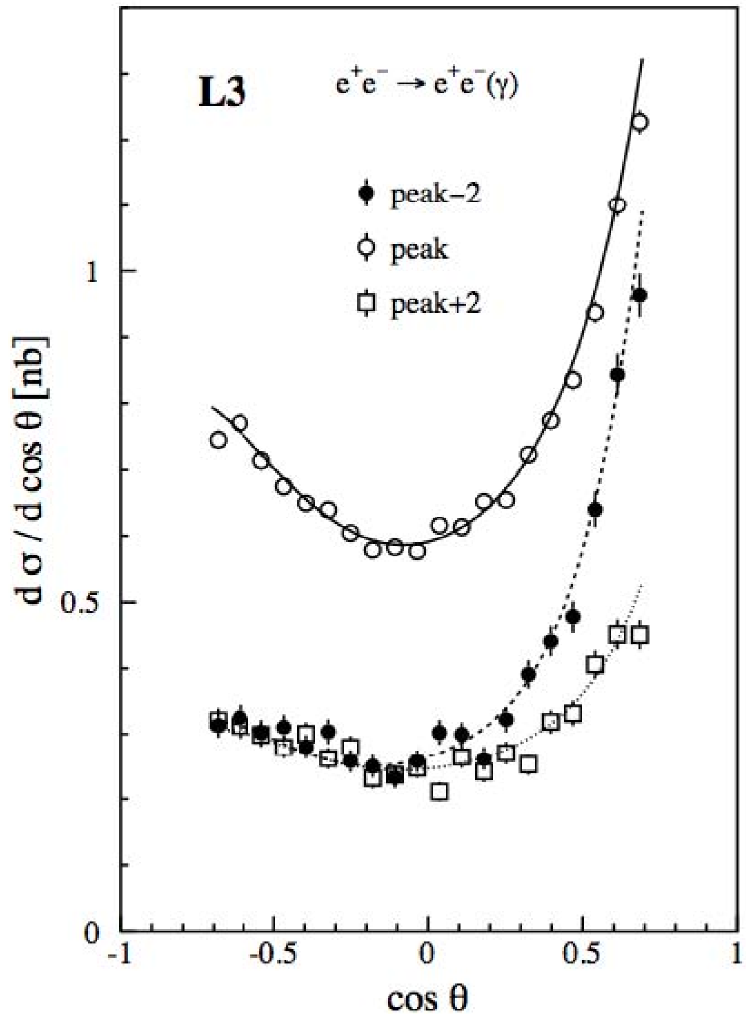
Far below the resonance:

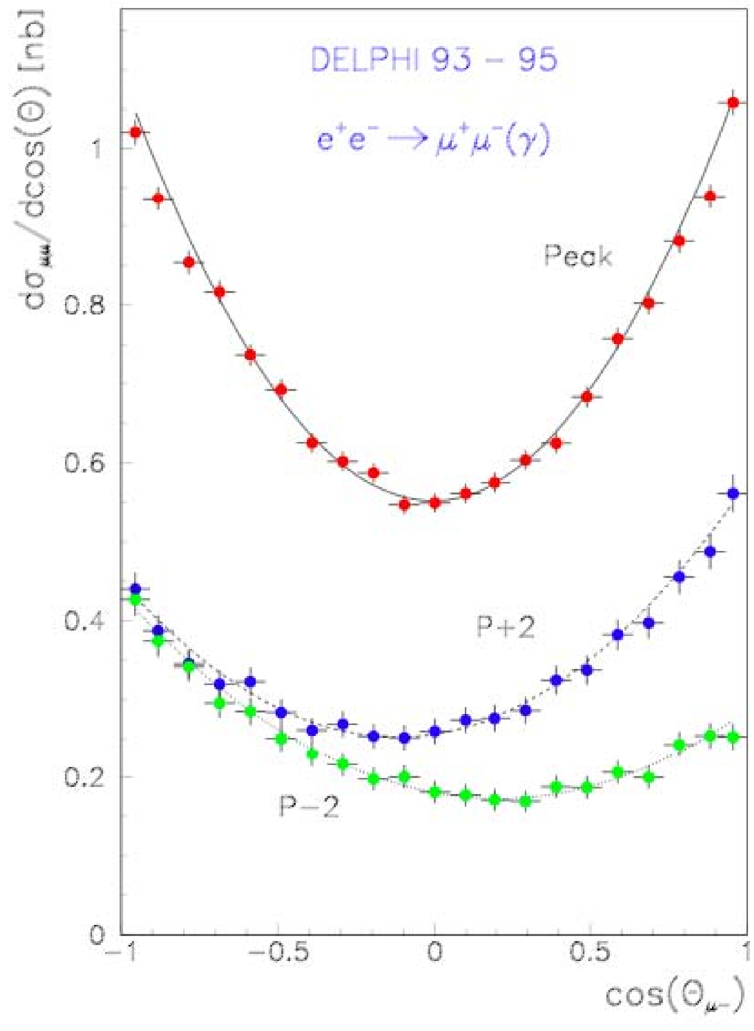
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}\left[A_0(1+\cos^2\theta)+ +A_1\cos\theta\right]$$

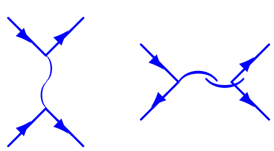
$$A_{FB} = \frac{3A_1}{8A_0} \simeq \frac{3}{2}\Re(r)g_A^2$$

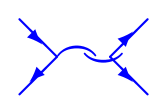
$$r = \frac{\sqrt{2}G_F M_Z^2}{s - M_Z^2 + iM_Z\Gamma_Z}\left(\frac{s}{e^2}\right)$$

Angular Distribution Close to the Resonance









Measurement Strategy II

- Count Z bosons "on peak" decaying into quarks:

$$\sigma_{had} \propto (N_{had} - N_{back})/\epsilon\mathcal{L}$$

- Correct σ_{had} to σ^0_{had}
- Count Z bosons "on peak" decaying into leptons (for each flavor):

$$R_l \propto N_{had}/N_{ll}$$

- Correct R_l to R^0_l
- Analyze angular distribution of decay products :

$$A^l_{fb} \propto (N_F - N_B)/(N_F + N_B)$$

- Correct A^l_{fb} to A^{0l}_{fb}
- Fit σ^0_{had}, R^0_l and A^{0l}_{fb} to get M_Z and Γ_Z
- Combine results from the experiments, taking into account correlations

Result Without Lepton Universality

		Correlations								
		m_Z	Γ_Z	σ^0_{had}	R^0_e	R^0_μ	R^0_τ	$A^{0,e}_{FB}$	$A^{0,\mu}_{FB}$	$A^{0,\tau}_{FB}$
$\chi^2/\text{dof} = 169/176$						ALEPH				
m_Z [GeV]	91.1891 ± 0.0031	1.000								
Γ_Z [GeV]	2.4959 ± 0.0043	0.038	1.000							
σ^0_{had} [nb]	41.558 ± 0.057	−0.091	−0.383	1.000						
R^0_e	20.690 ± 0.075	0.102	0.004	0.134	1.000					
R^0_μ	20.801 ± 0.056	−0.003	0.012	0.167	0.083	1.000				
R^0_τ	20.708 ± 0.062	−0.003	0.004	0.152	0.067	0.093	1.000			
$A^{0,e}_{FB}$	0.0184 ± 0.0034	−0.047	0.000	−0.003	−0.388	0.000	0.000	1.000		
$A^{0,\mu}_{FB}$	0.0172 ± 0.0024	0.072	0.002	0.002	0.019	0.013	0.000	−0.008	1.000	
$A^{0,\tau}_{FB}$	0.0170 ± 0.0028	0.061	0.002	0.002	0.017	0.000	0.011	−0.007	0.016	1.000
$\chi^2/\text{dof} = 177/168$						DELPHI				
m_Z [GeV]	91.1864 ± 0.0028	1.000								
Γ_Z [GeV]	2.4876 ± 0.0041	0.047	1.000							
σ^0_{had} [nb]	41.578 ± 0.069	−0.070	−0.270	1.000						
R^0_e	20.88 ± 0.12	0.063	0.000	0.120	1.000					
R^0_μ	20.650 ± 0.076	−0.003	−0.007	0.191	0.054	1.000				
R^0_τ	20.84 ± 0.13	0.001	−0.001	0.113	0.033	0.051	1.000			
$A^{0,e}_{FB}$	0.0171 ± 0.0049	0.057	0.001	−0.006	−0.106	0.000	−0.001	1.000		
$A^{0,\mu}_{FB}$	0.0165 ± 0.0025	0.064	0.006	−0.002	0.025	0.008	0.000	−0.016	1.000	
$A^{0,\tau}_{FB}$	0.0241 ± 0.0037	0.043	0.003	−0.002	0.015	0.000	0.012	−0.015	0.014	1.000
$\chi^2/\text{dof} = 158/166$						L3				
m_Z [GeV]	91.1897 ± 0.0030	1.000								
Γ_Z [GeV]	2.5025 ± 0.0041	0.065	1.000							
σ^0_{had} [nb]	41.535 ± 0.054	0.009	−0.343	1.000						
R^0_e	20.815 ± 0.089	0.108	−0.007	0.075	1.000					
R^0_μ	20.861 ± 0.097	−0.001	0.002	0.077	0.030	1.000				
R^0_τ	20.79 ± 0.13	0.002	0.005	0.053	0.024	0.020	1.000			
$A^{0,e}_{FB}$	0.0107 ± 0.0058	−0.045	0.055	−0.006	−0.146	−0.001	−0.003	1.000		
$A^{0,\mu}_{FB}$	0.0188 ± 0.0033	0.052	0.004	0.005	0.017	0.005	0.000	0.011	1.000	
$A^{0,\tau}_{FB}$	0.0260 ± 0.0047	0.034	0.004	0.003	0.012	0.000	0.007	−0.008	0.006	1.000
$\chi^2/\text{dof} = 155/194$						OPAL				
m_Z [GeV]	91.1858 ± 0.0030	1.000								
Γ_Z [GeV]	2.4948 ± 0.0041	0.049	1.000							
σ^0_{had} [nb]	41.501 ± 0.055	0.031	−0.352	1.000						
R^0_e	20.901 ± 0.084	0.108	0.011	0.155	1.000					
R^0_μ	20.811 ± 0.058	0.001	0.020	0.222	0.093	1.000				
R^0_τ	20.832 ± 0.091	0.001	0.013	0.137	0.039	0.051	1.000			
$A^{0,e}_{FB}$	0.0089 ± 0.0045	−0.053	−0.005	0.011	−0.222	−0.001	0.005	1.000		
$A^{0,\mu}_{FB}$	0.0159 ± 0.0023	0.077	−0.002	0.011	0.031	0.018	0.004	−0.012	1.000	
$A^{0,\tau}_{FB}$	0.0145 ± 0.0030	0.059	−0.003	0.003	0.015	−0.010	0.007	−0.010	0.013	1.000

Result With Lepton Universality

		Correlations				
		m_Z	Γ_Z	$\sigma^0_{\rm had}$	R^0_ℓ	$A^{0,\ell}_{\rm FB}$
$\chi^2/\text{dof} = 172/180$		**ALEPH**				
m_Z [GeV]	91.1893 ± 0.0031	1.000				
Γ_Z [GeV]	2.4959 ± 0.0043	0.038	1.000			
$\sigma^0_{\rm had}$ [nb]	41.559 ± 0.057	−0.092	−0.383	1.000		
R^0_ℓ	20.729 ± 0.039	0.033	0.011	0.246	1.000	
$A^{0,\ell}_{\rm FB}$	0.0173 ± 0.0016	0.071	0.002	0.001	−0.076	1.000
$\chi^2/\text{dof} = 183/172$		**DELPHI**				
m_Z [GeV]	91.1863 ± 0.0028	1.000				
Γ_Z [GeV]	2.4876 ± 0.0041	0.046	1.000			
$\sigma^0_{\rm had}$ [nb]	41.578 ± 0.069	−0.070	−0.270	1.000		
R^0_ℓ	20.730 ± 0.060	0.028	−0.006	0.242	1.000	
$A^{0,\ell}_{\rm FB}$	0.0187 ± 0.0019	0.095	0.006	−0.005	0.000	1.000
$\chi^2/\text{dof} = 163/170$		**L3**				
m_Z [GeV]	91.1894 ± 0.0030	1.000				
Γ_Z [GeV]	2.5025 ± 0.0041	0.068	1.000			
$\sigma^0_{\rm had}$ [nb]	41.536 ± 0.055	0.014	−0.348	1.000		
R^0_ℓ	20.809 ± 0.060	0.067	0.020	0.111	1.000	
$A^{0,\ell}_{\rm FB}$	0.0192 ± 0.0024	0.041	0.020	0.005	−0.024	1.000
$\chi^2/\text{dof} = 158/198$		**OPAL**				
m_Z [GeV]	91.1853 ± 0.0029	1.000				
Γ_Z [GeV]	2.4947 ± 0.0041	0.051	1.000			
$\sigma^0_{\rm had}$ [nb]	41.502 ± 0.055	0.030	−0.352	1.000		
R^0_ℓ	20.822 ± 0.044	0.043	0.024	0.290	1.000	
$A^{0,\ell}_{\rm FB}$	0.0145 ± 0.0017	0.075	−0.005	0.013	−0.017	1.000

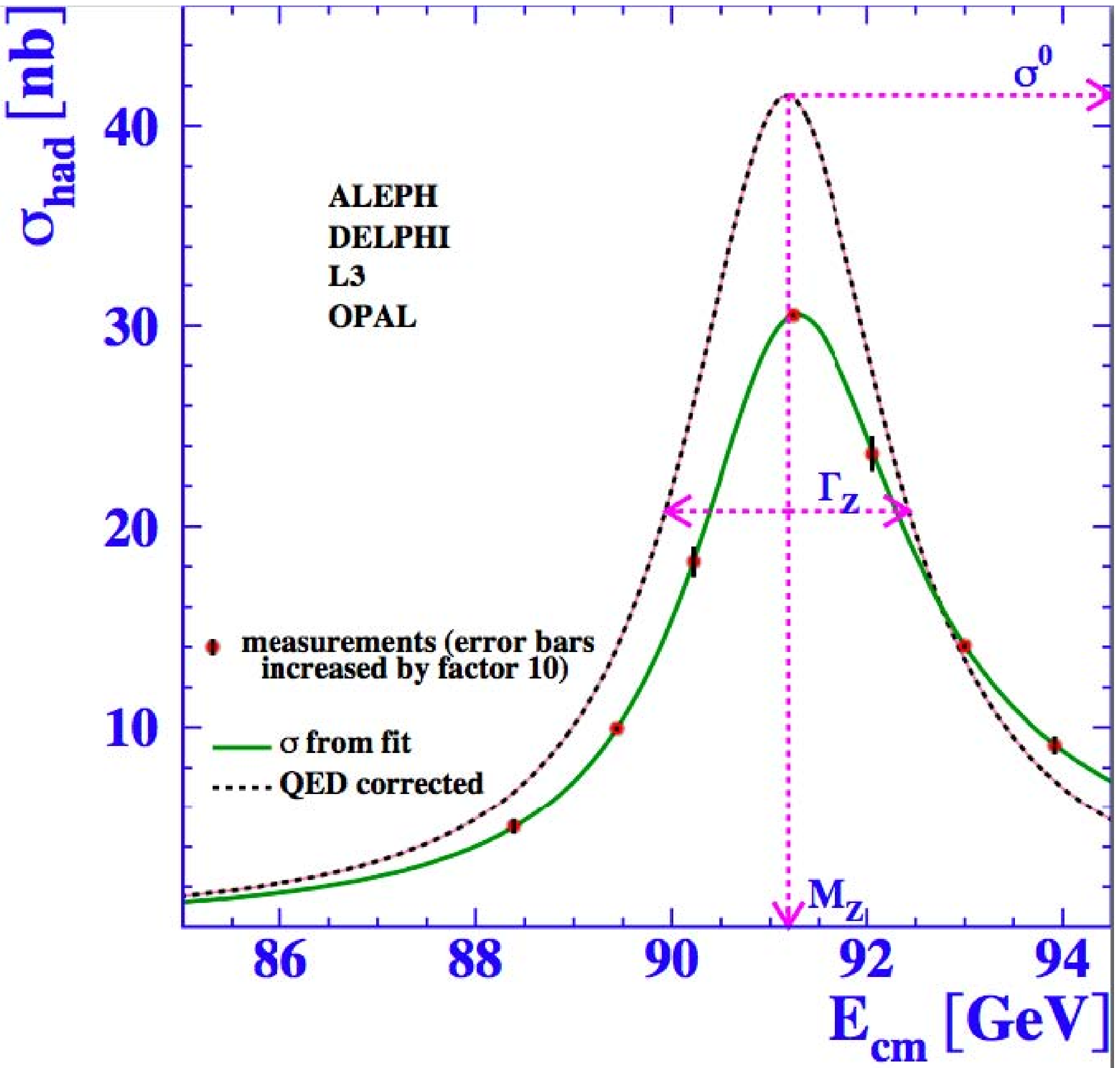
Result From Combined Experiments

Without lepton universality		Correlations								
$\chi^2/\mathrm{dof} = 32.6/27$		m_Z	Γ_Z	σ^0_{had}	R^0_{e}	R^0_{μ}	R^0_{τ}	$A^{0,\mathrm{e}}_{\mathrm{FB}}$	$A^{0,\mu}_{\mathrm{FB}}$	$A^{0,\tau}_{\mathrm{FB}}$
m_Z [GeV]	91.1876 ± 0.0021	1.000								
Γ_Z [GeV]	2.4952 ± 0.0023	−0.024	1.000							
σ^0_{had} [nb]	41.541 ± 0.037	−0.044	−0.297	1.000						
R^0_{e}	20.804 ± 0.050	0.078	−0.011	0.105	1.000					
R^0_{μ}	20.785 ± 0.033	0.000	0.008	0.131	0.069	1.000				
R^0_{τ}	20.764 ± 0.045	0.002	0.006	0.092	0.046	0.069	1.000			
$A^{0,\mathrm{e}}_{\mathrm{FB}}$	0.0145 ± 0.0025	−0.014	0.007	0.001	−0.371	0.001	0.003	1.000		
$A^{0,\mu}_{\mathrm{FB}}$	0.0169 ± 0.0013	0.046	0.002	0.003	0.020	0.012	0.001	−0.024	1.000	
$A^{0,\tau}_{\mathrm{FB}}$	0.0188 ± 0.0017	0.035	0.001	0.002	0.013	−0.003	0.009	−0.020	0.046	1.000

With lepton universality		Correlations				
$\chi^2/\mathrm{dof} = 36.5/31$		m_Z	Γ_Z	σ^0_{had}	R^0_{ℓ}	$A^{0,\ell}_{\mathrm{FB}}$
m_Z [GeV]	91.1875 ± 0.0021	1.000				
Γ_Z [GeV]	2.4952 ± 0.0023	−0.023	1.000			
σ^0_{had} [nb]	41.540 ± 0.037	−0.045	−0.297	1.000		
R^0_{ℓ}	20.767 ± 0.025	0.033	0.004	0.183	1.000	
$A^{0,\ell}_{\mathrm{FB}}$	0.0171 ± 0.0010	0.055	0.003	0.006	−0.056	1.000

Measurement of M_Z to 23 ppm!

Uncertainties



Unusual level of accuracy, need to control systematics:

- Higher order corrections deform resonance curve: theory
- Luminosity shifts curve up and down: experiments
- Energy calibration shifts curve left and right: accelerator

Luminosity

Instantaneous luminosity from collider properties:

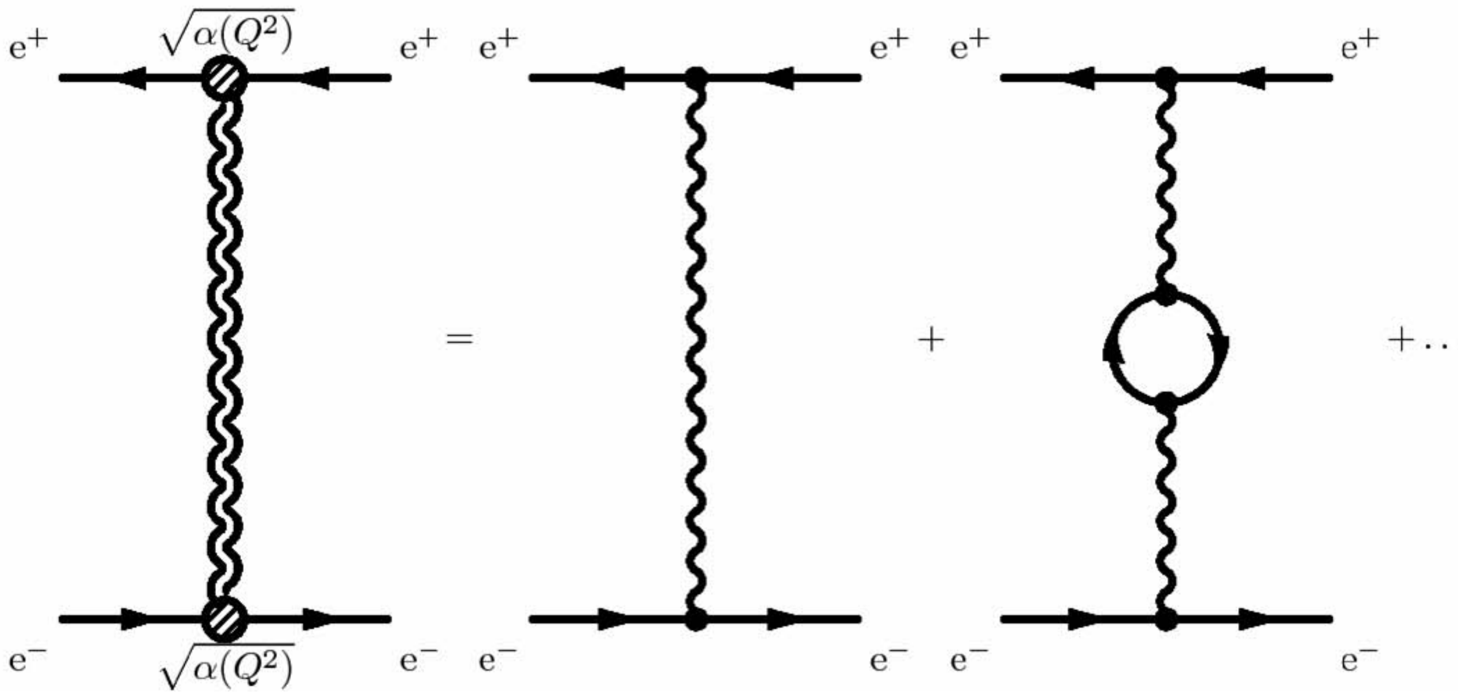
$$\mathcal{L} = \frac{N_+ N_- \nu H_d}{4\pi\sigma_x\sigma_y} \simeq 2 \times 10^{31}\text{cm}^{-2}\text{s}^{-1}$$

- Bunch population $N_+ \simeq N_- \simeq 3 \times 10^{12}$
- Crossing frequency $\nu = 11$kHz
- Beam-beam enhancement factor: $H_d \simeq 2$
- Horizontal and vertical beam sizes: $\sigma_x \simeq 100\mu$m, $\sigma_y \simeq 10\mu$m

Instantaneous luminosity not required (and not accurately known) for a counting experiment, what we need is integrated luminosity $\int \mathcal{L}\, dt$. Inferred from counting reactions of a reference process: $\int \mathcal{L}\, dt = (N - N_b)/(\sigma\epsilon)$

- Large cross section $\longrightarrow$ high N, small statistical error $\sigma_{\mathcal{L}}/\mathcal{L} \propto 1/\sqrt{N}$
- Calculable to high accuracy $\longrightarrow$ accurate σ
- Low background N_b, well known efficiency $\epsilon \longrightarrow$ small systematic error
- Choice at LEP: small angle Bhabha scattering

Small Angle Bhabha Scattering

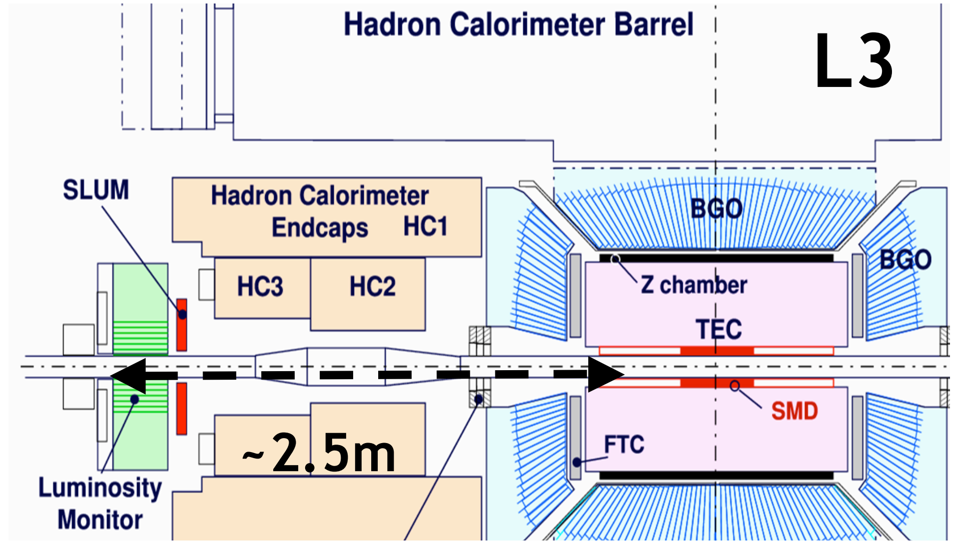


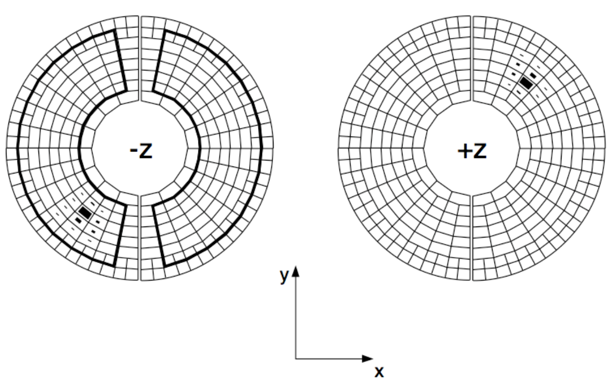
- Dominated by t-channel photon exchange at small angles: QED
- Theoretical uncertainty from uncalculated higher orders $\mathcal{O}(0.1\%)$
- Experimental uncertainty (acceptance, efficiency) $\mathcal{O}(0.1\%)$, mostly uncorrelated among experiments

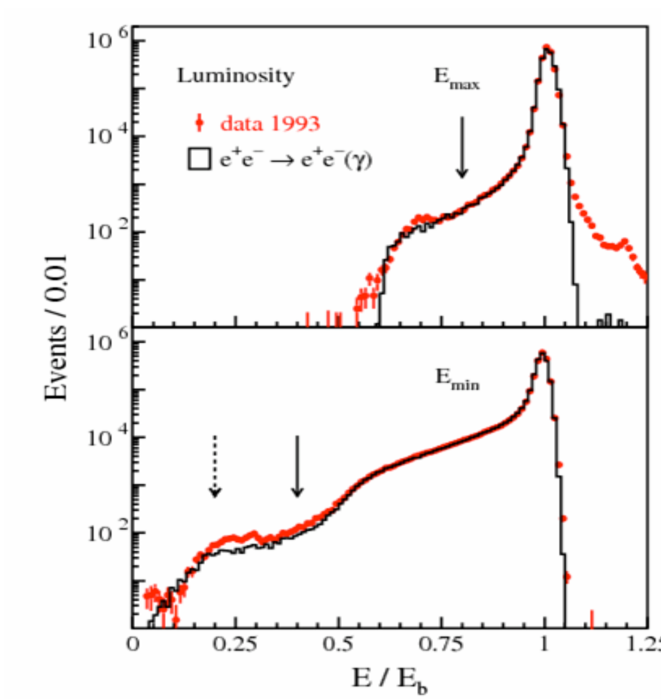
Luminosity Measurement at LEP

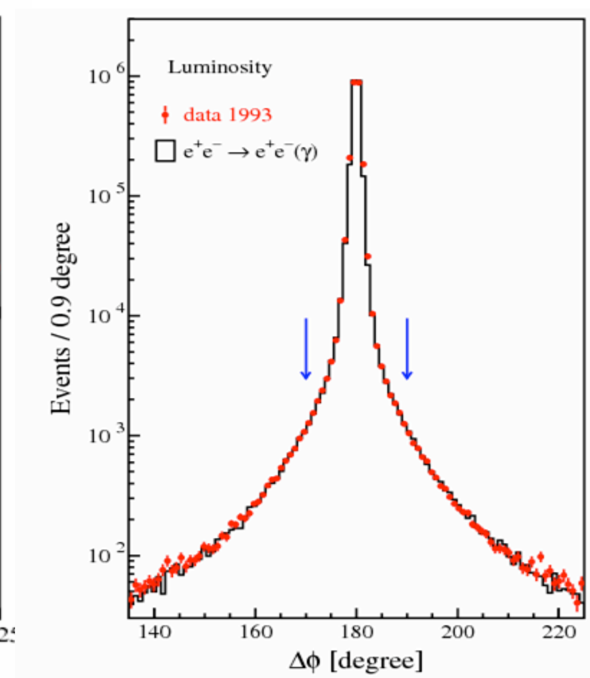
Luminosity monitors

- Small-angle calorimeters, possibly with tracker in front
- 1.4-1.8°<θ<3.1-3.3°
- Mechanical precision/position measurement crucial
- Select events, count events, control efficiency and background







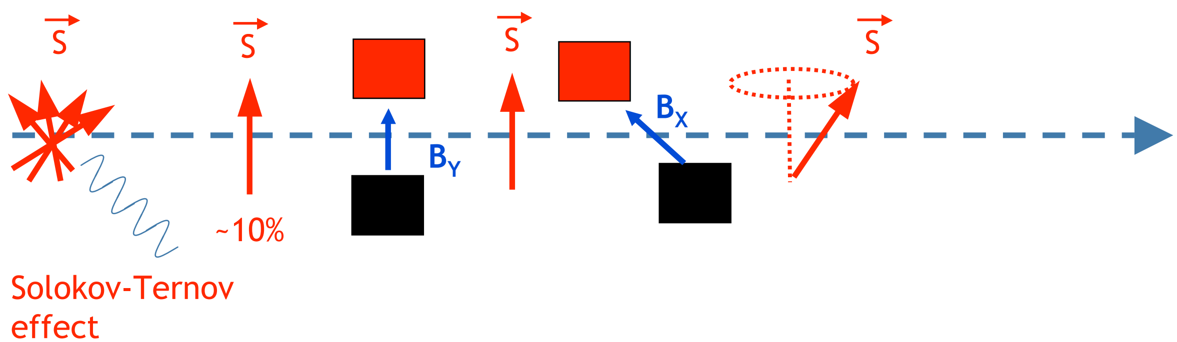


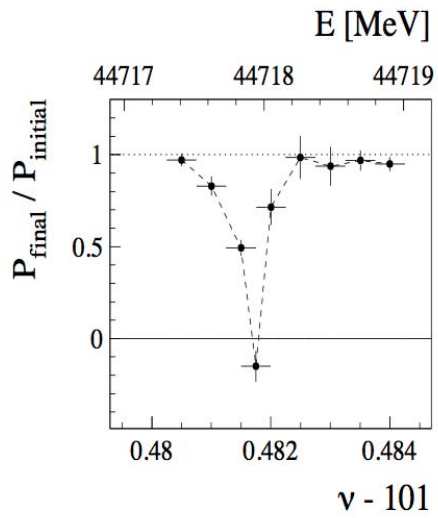
Measurement of Beam Energy

- Inferred from magnetic field measurement:

$$< E_b > = \frac{e}{2\pi} \oint_{orbit} B\, dl$$

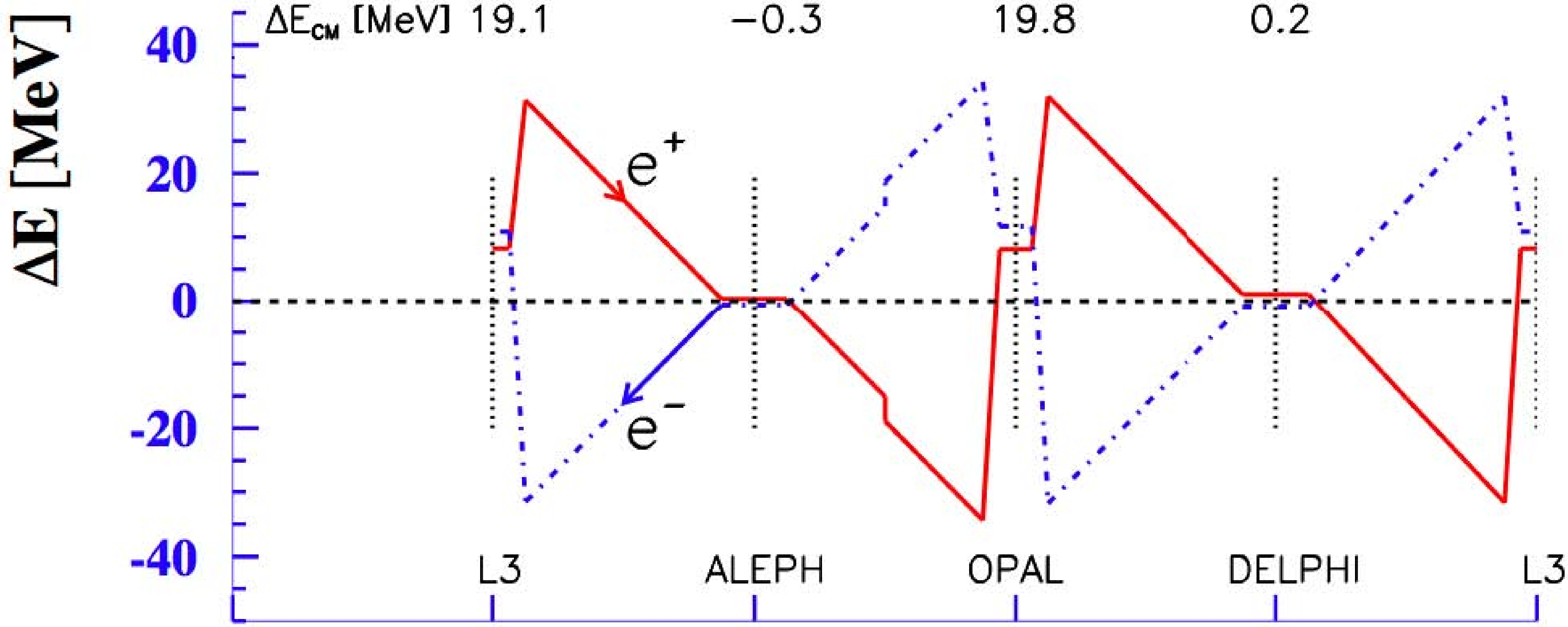
- Flux loop (Ampere's law) and NMR probes: relative precision 3×10^{-4}
- On order of magnitude more accurate by resonant depolarization:



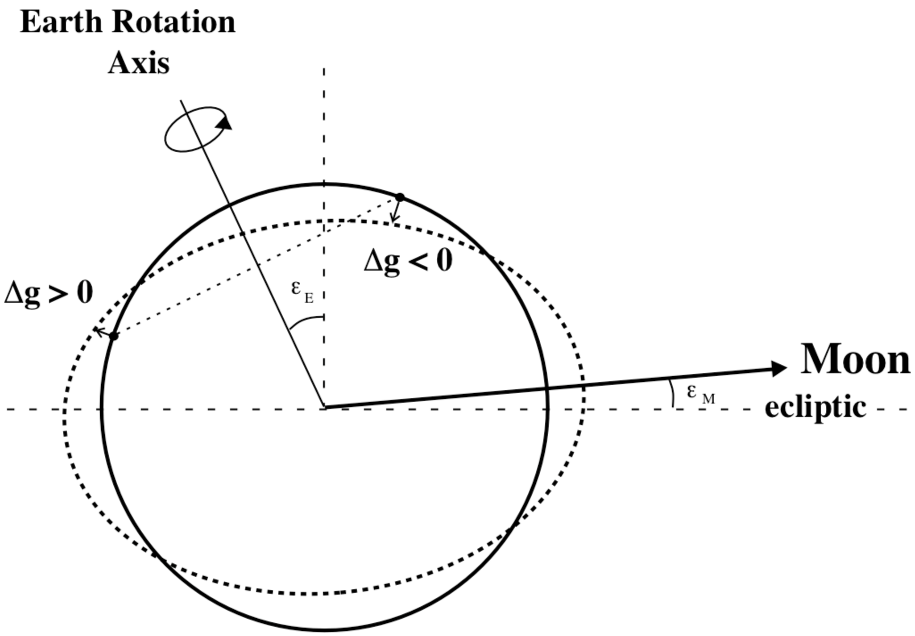


- Final precision $\Delta E_b / E_b = 2 \times 10^{-5}$

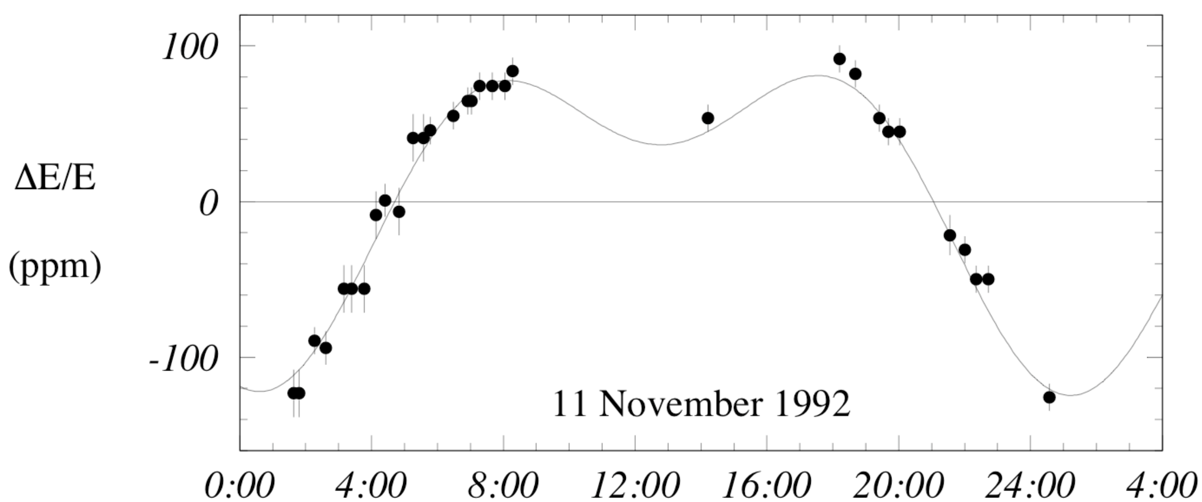
Attention: Energy Varies Around the Ring



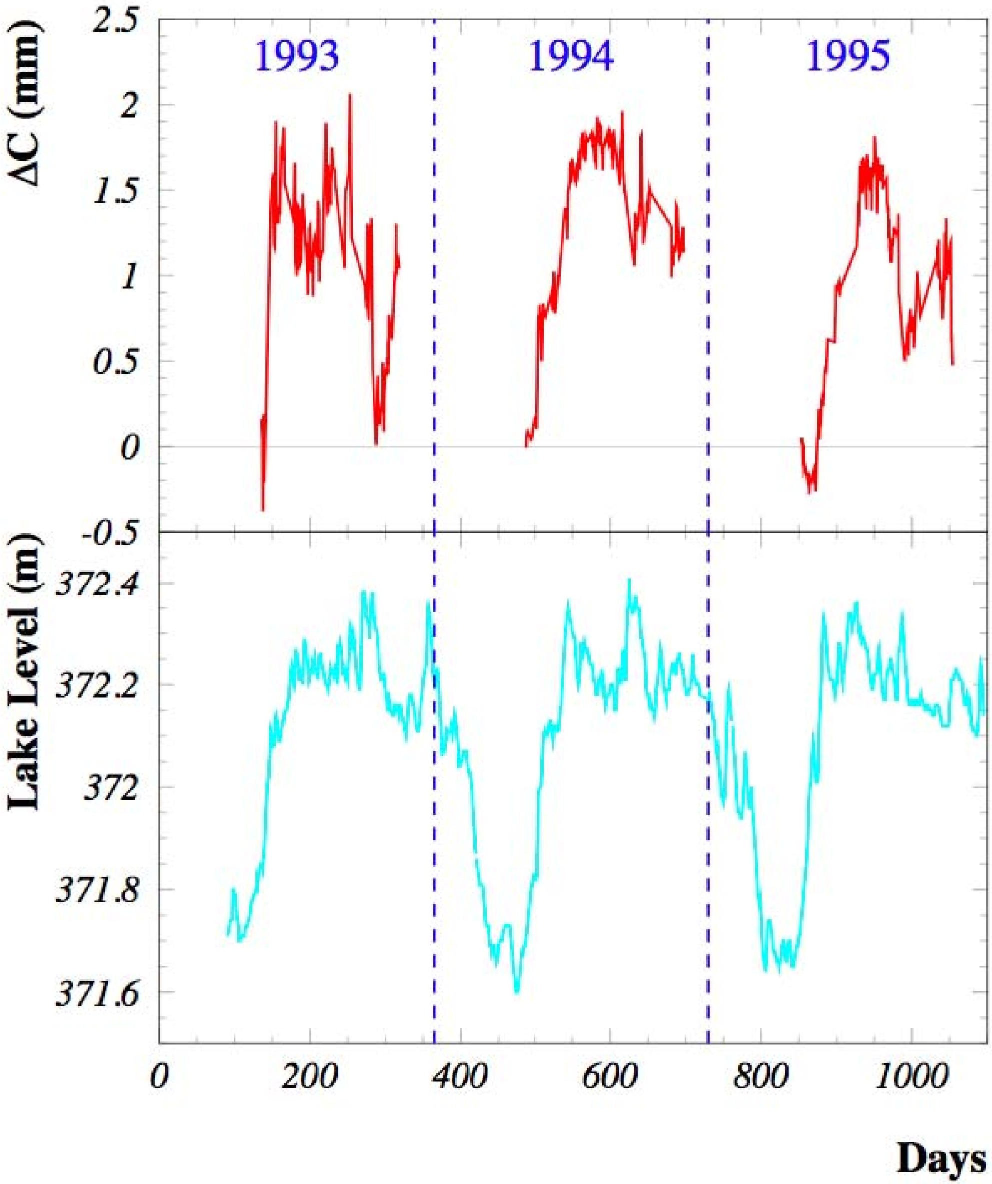
Attention: Energy Varies with Phase of the Moon



Like the Sea, the Earth crust has "tides" about 20cm high. These deform the accelerator by a few mm, compared to the 27km circinference. At constant frequency, this changes the energy. Strong focussing amplifies the effect. Effect of 10 MeV, corrected daily.

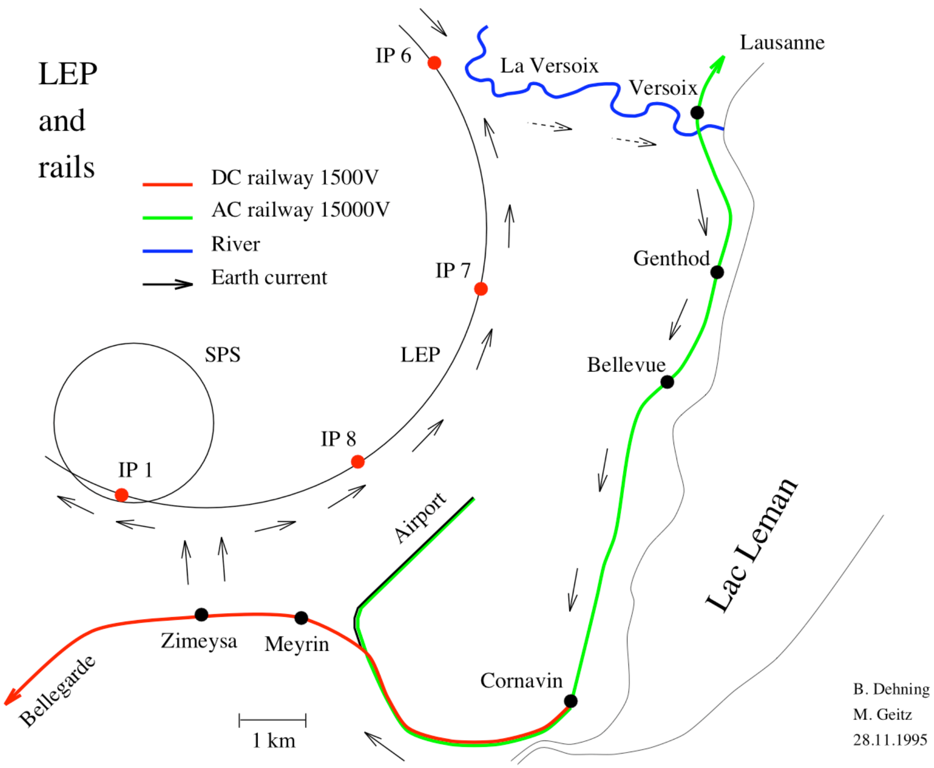


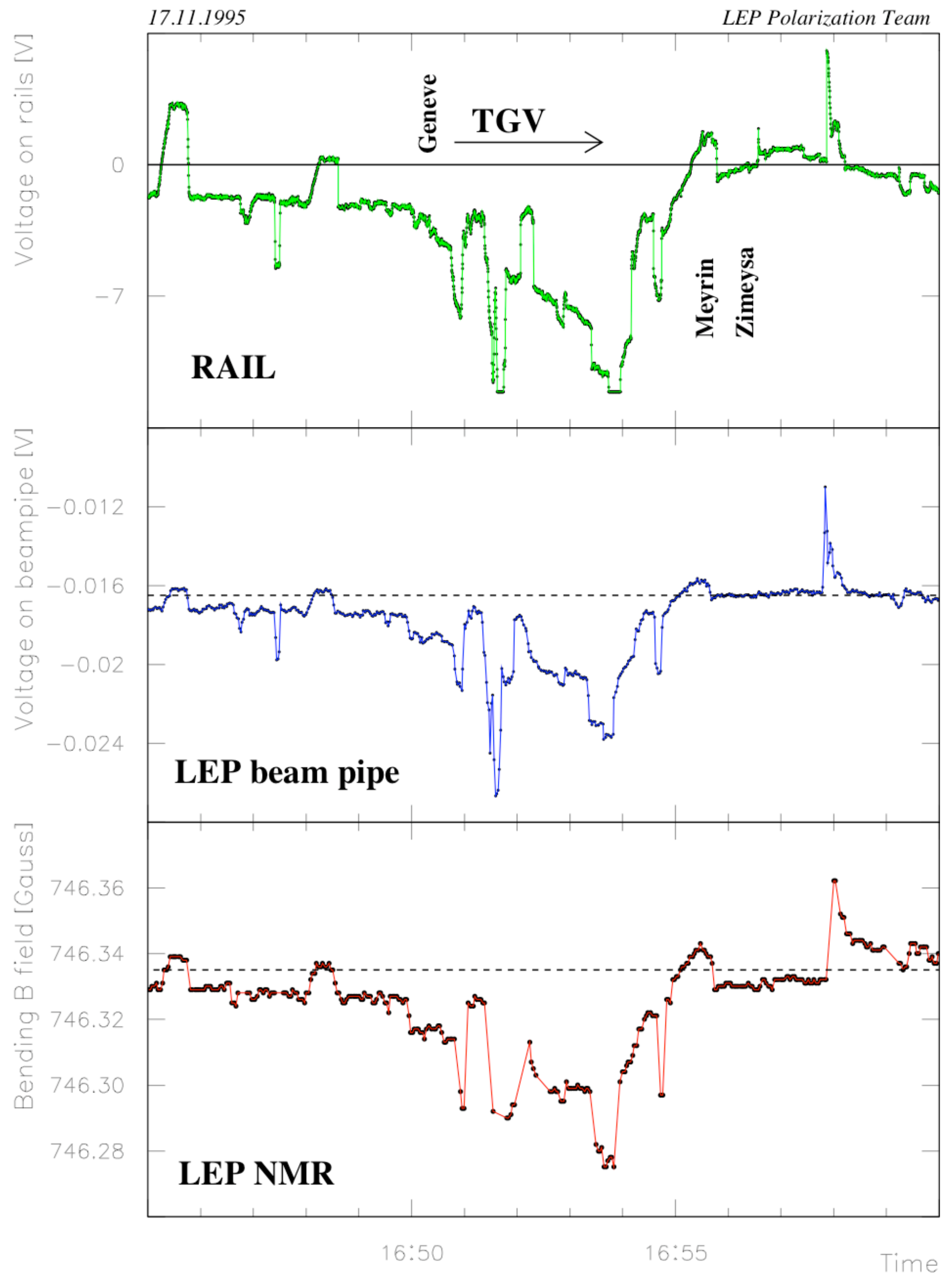
Attention: Energy Varies with Water Level in Lake Geneva



Water pressure from neighboring Lake Geneva also deform the accelerator by a few mm, compared to the 27km circinference. At constant frequency, this changes the energy. Strong focussing amplifies the effect. Effect of 10 MeV, corrected weekly.

Attention: Energy varies with TGV schedule



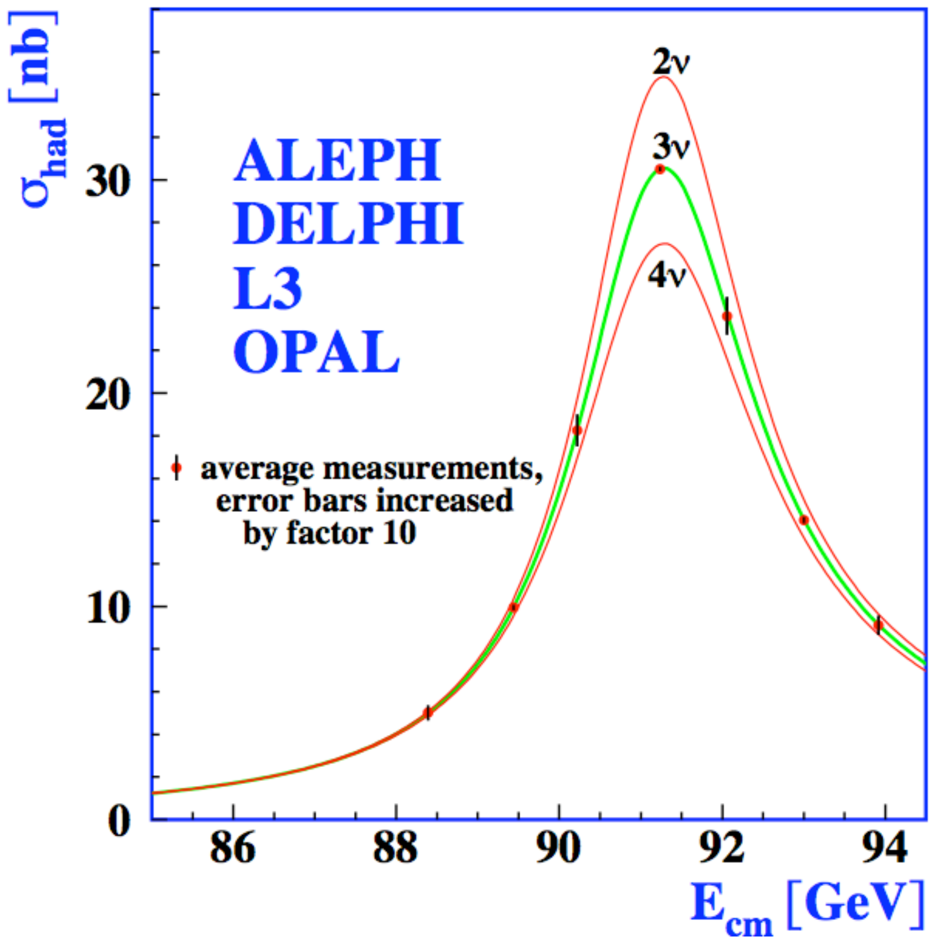


TGV induces parasitic currents in LEP magnets. Because of their hysteresis, they remember the train passage.

Immediate Result: Number of Light Neutrino Species

$$R^0_{\rm inv} = \left(\frac{12\pi R^0_\ell}{\sigma^0_{\rm had} m^2_Z}\right)^{\frac{1}{2}} - R^0_\ell - (3+\delta_\tau) \qquad R^0_{\rm inv} \equiv \frac{\Gamma_{\rm inv}}{\Gamma_{\ell\ell}} = N_\nu \left(\frac{\Gamma_{\nu\bar\nu}}{\Gamma_{\ell\ell}}\right)_{\rm SM}$$

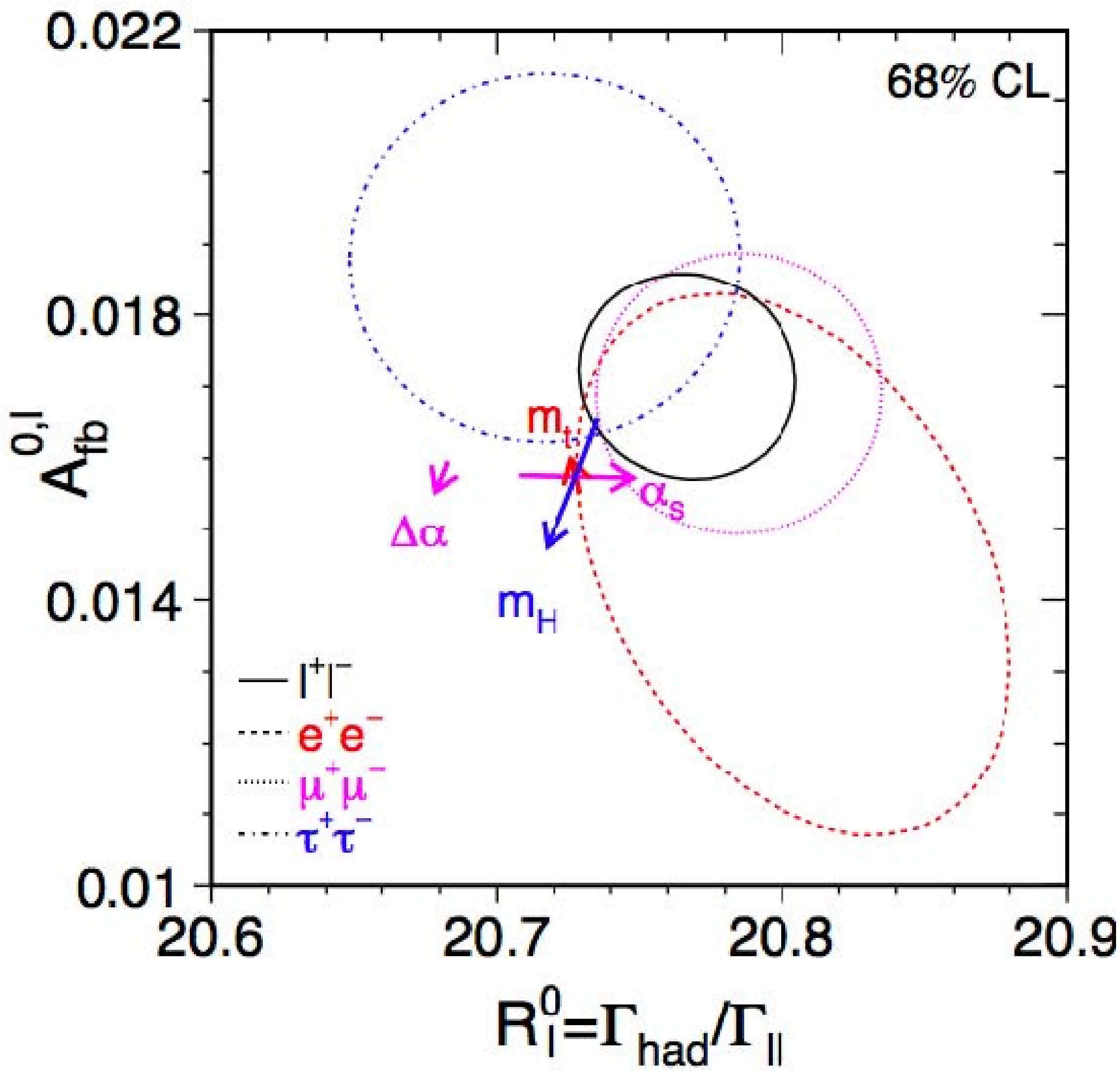
τ mass correction -0.23%



$$R^0_{\rm inv} = 5.943 \pm 0.016$$

$$\boxed{N_\nu = 2.9840 \pm 0.0082}$$

Immediate Result: Leptonic Universality



Asymmetry Parameters and Couplings

$$\mathcal{A}_\mathrm{f} = \frac{g_\mathrm{Lf}^2 - g_\mathrm{Rf}^2}{g_\mathrm{Lf}^2 + g_\mathrm{Rf}^2} = \frac{2 g_\mathrm{Vf} g_\mathrm{Af}}{g_\mathrm{Vf}^2 + g_\mathrm{Af}^2} = 2\frac{g_\mathrm{Vf}/g_\mathrm{Af}}{1 + (g_\mathrm{Vf}/g_\mathrm{Af})^2}$$

$$A_\mathrm{FB} = \frac{\sigma_\mathrm{F} - \sigma_\mathrm{B}}{\sigma_\mathrm{F} + \sigma_\mathrm{B}} \quad \Rightarrow \quad A_\mathrm{FB}^{0,\,\mathrm{f}} = \frac{3}{4}\mathcal{A}_\mathrm{e}\mathcal{A}_\mathrm{f}$$

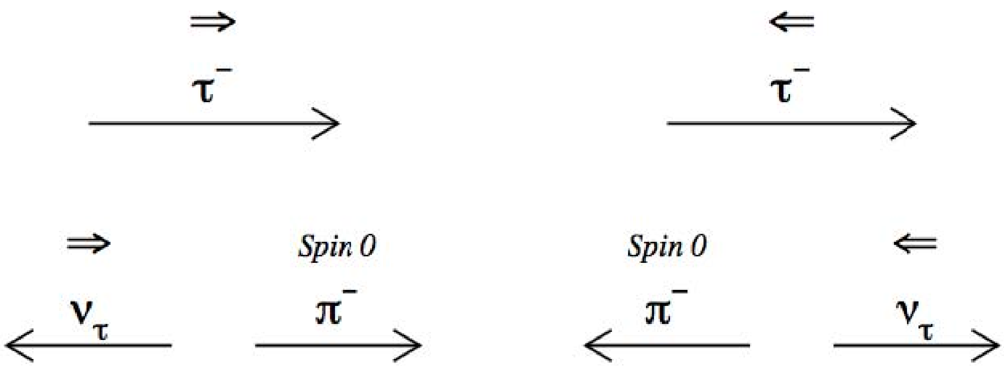
$$A_\mathrm{LR} = \frac{\sigma_\mathrm{L} - \sigma_\mathrm{R}}{\sigma_\mathrm{L} + \sigma_\mathrm{R}} \frac{1}{\langle|\mathcal{P}_\mathrm{e}|\rangle} \quad \Rightarrow \quad A_\mathrm{LR}^0 = \mathcal{A}_\mathrm{e}$$

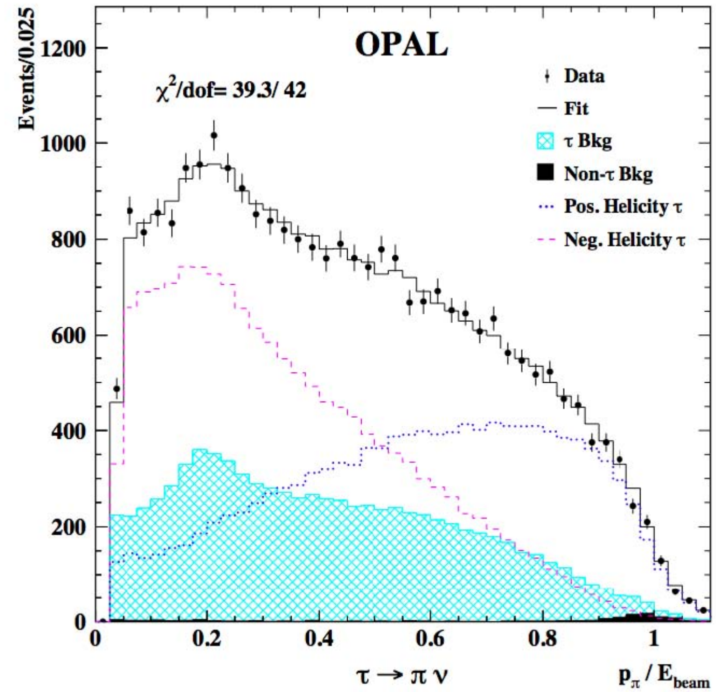
$$A_\mathrm{LRFB} = \frac{(\sigma_\mathrm{F} - \sigma_\mathrm{B})_\mathrm{L} - (\sigma_\mathrm{F} - \sigma_\mathrm{B})_\mathrm{R}}{(\sigma_\mathrm{F} + \sigma_\mathrm{B})_\mathrm{L} + (\sigma_\mathrm{F} + \sigma_\mathrm{B})_\mathrm{R}} \frac{1}{\langle|\mathcal{P}_\mathrm{e}|\rangle} \quad \Rightarrow \quad A_\mathrm{LRFB}^0 = \frac{3}{4}\mathcal{A}_\mathrm{f}$$

$$\mathcal{P}_\tau \equiv (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-) \quad \Rightarrow \quad \mathcal{P}_\tau = f(\mathcal{A}_\mathrm{e}, \mathcal{A}_\tau)$$

- All fermions for which charge can be measured: A_{fb}
- Beam polarization: A_{LR}, electrons only
- Final state polarization: $\mathcal{P}(\theta)$, τ leptons only

Tau Polarization Through Weak Decay

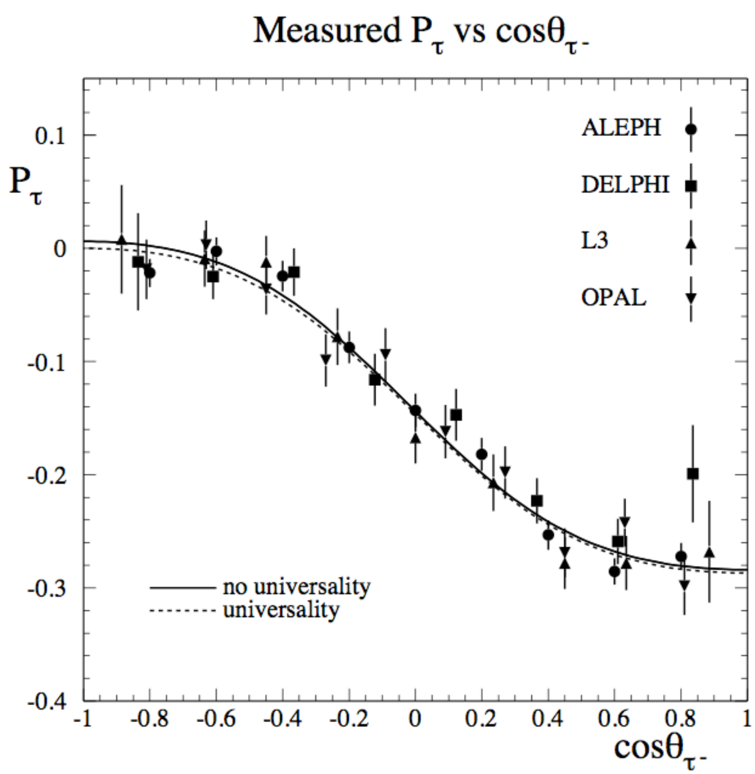




- V-A charged weak current violates parity maximally
- Left-handiness of neutrino implies a boost for the pion in the tau direction for positive helicity and a depletion othewise
- Decay spectrum fitted by combination of both helicities, amplitude ratio gives degree of polarizazion

%

Tau Polarization Results



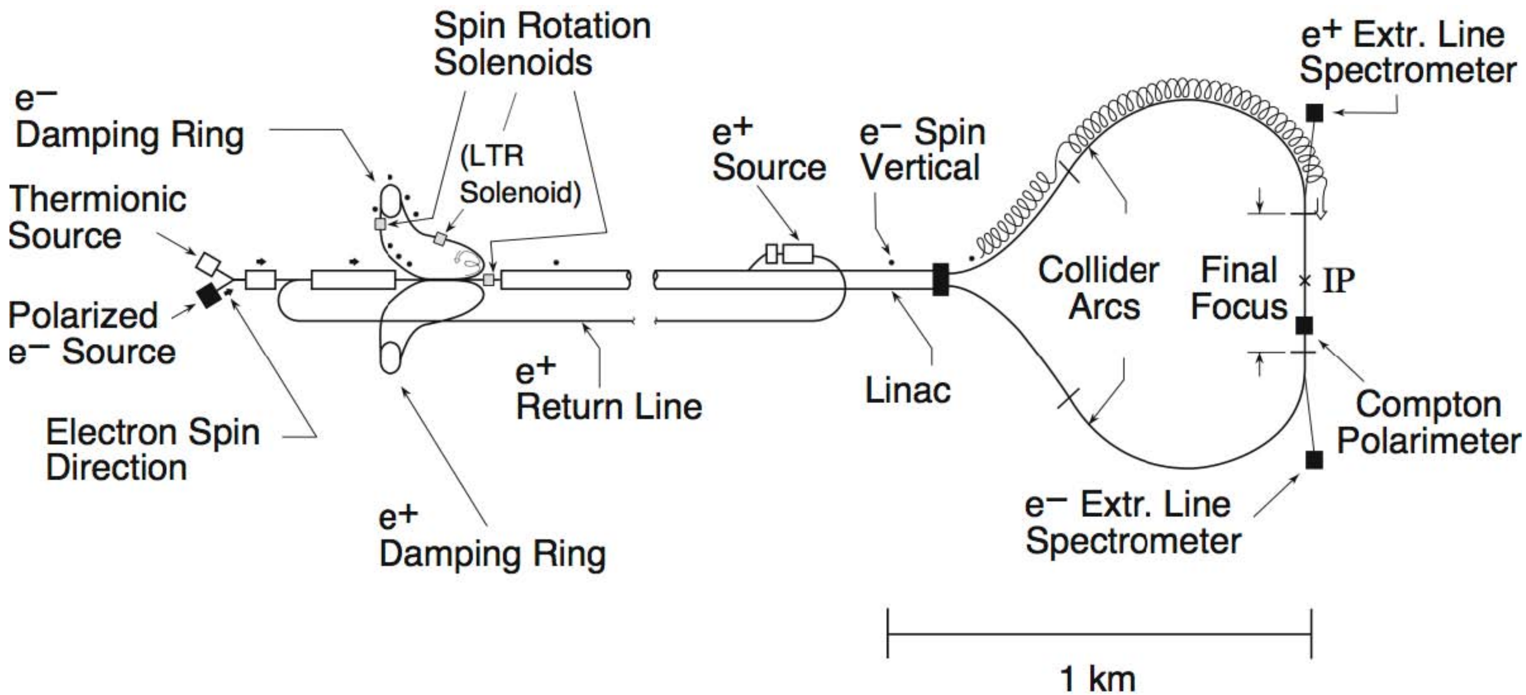
Experiment	$\mathcal{A}_\tau$	$\mathcal{A}_e$
ALEPH	$0.1451 \pm 0.0052 \pm 0.0029$	$0.1504 \pm 0.0068 \pm 0.0008$
DELPHI	$0.1359 \pm 0.0079 \pm 0.0055$	$0.1382 \pm 0.0116 \pm 0.0005$
L3	$0.1476 \pm 0.0088 \pm 0.0062$	$0.1678 \pm 0.0127 \pm 0.0030$
OPAL	$0.1456 \pm 0.0076 \pm 0.0057$	$0.1454 \pm 0.0108 \pm 0.0036$
LEP	$0.1439 \pm 0.0035 \pm 0.0026$	$0.1498 \pm 0.0048 \pm 0.0009$

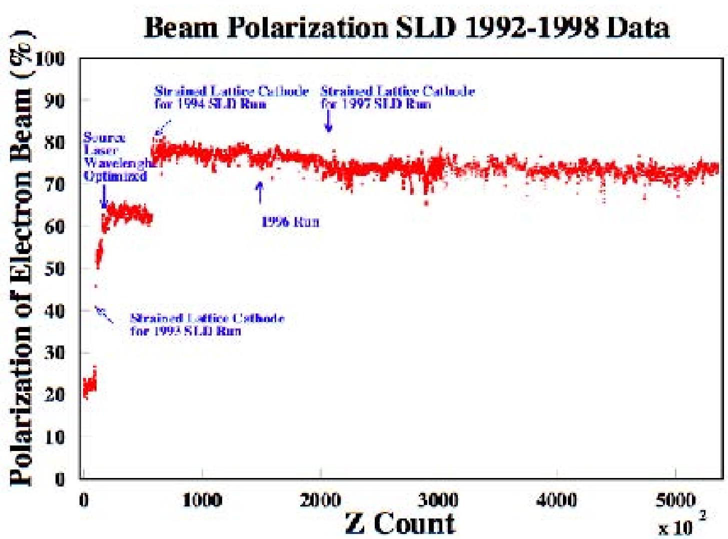
Combination

$$\boxed{\begin{aligned} \mathcal{A}_\tau &= 0.1439 \pm 0.0043 \\ \mathcal{A}_e &= 0.1498 \pm 0.0049 \end{aligned}}$$

$$\mathcal{P}_\tau(\cos\theta_{\tau^-}) = -\frac{\mathcal{A}_\tau(1+\cos^2\theta_{\tau^-}) + 2\mathcal{A}_e\cos\theta_{\tau^-}}{(1+\cos^2\theta_{\tau^-}) + \frac{8}{3}\mathcal{A}_{FB}^\tau\cos\theta_{\tau^-}}$$

Initial State Polarization



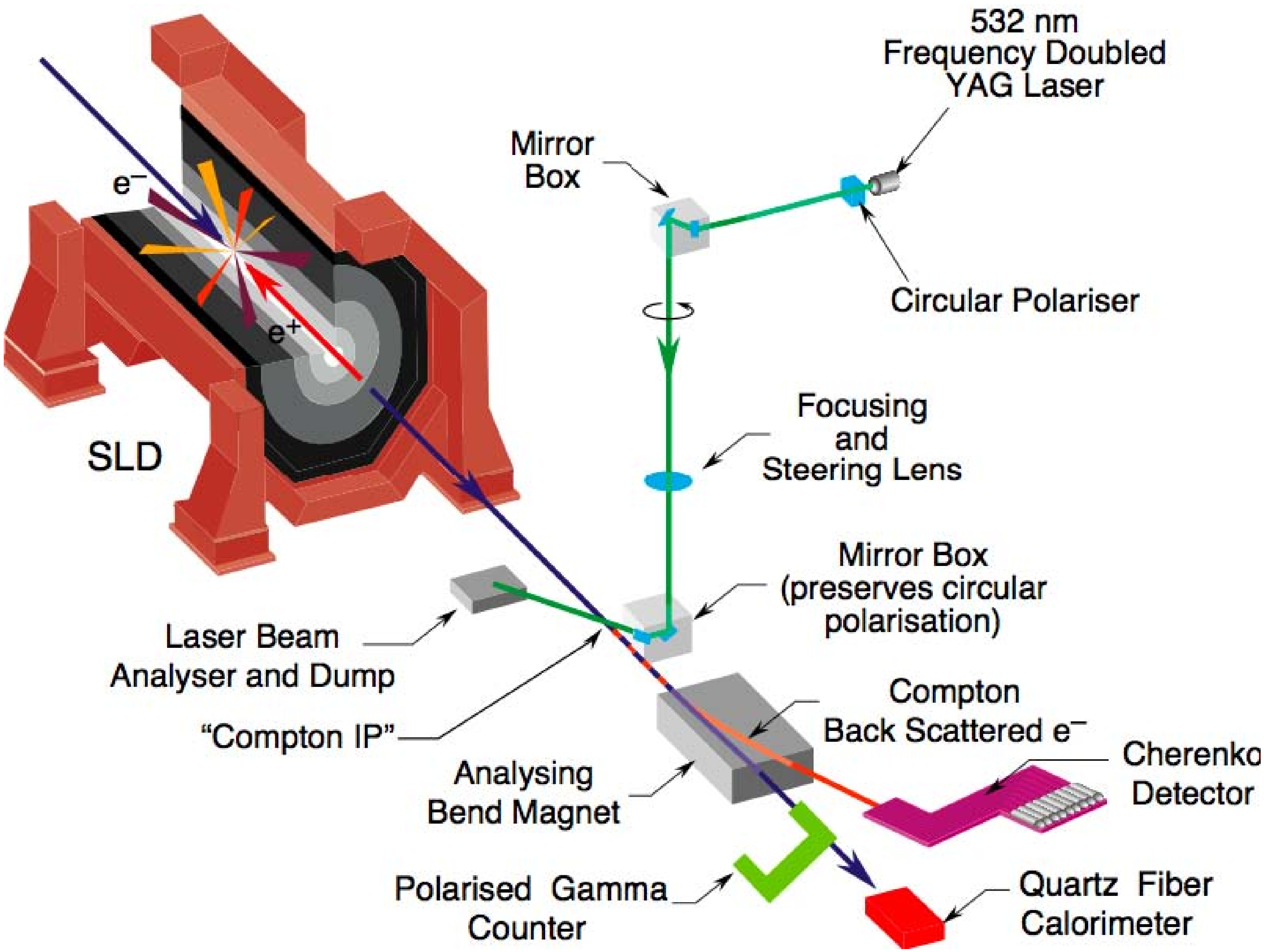


$$\mathcal{A}_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \frac{1}{< \mathcal{P}_e >}$$

$$= \frac{N_L - N_R}{N_L + N_R} \frac{1}{< \mathcal{P}_e >}$$

for $\mathcal{L}_L = \mathcal{L}_R$.

Beam Polarimeter

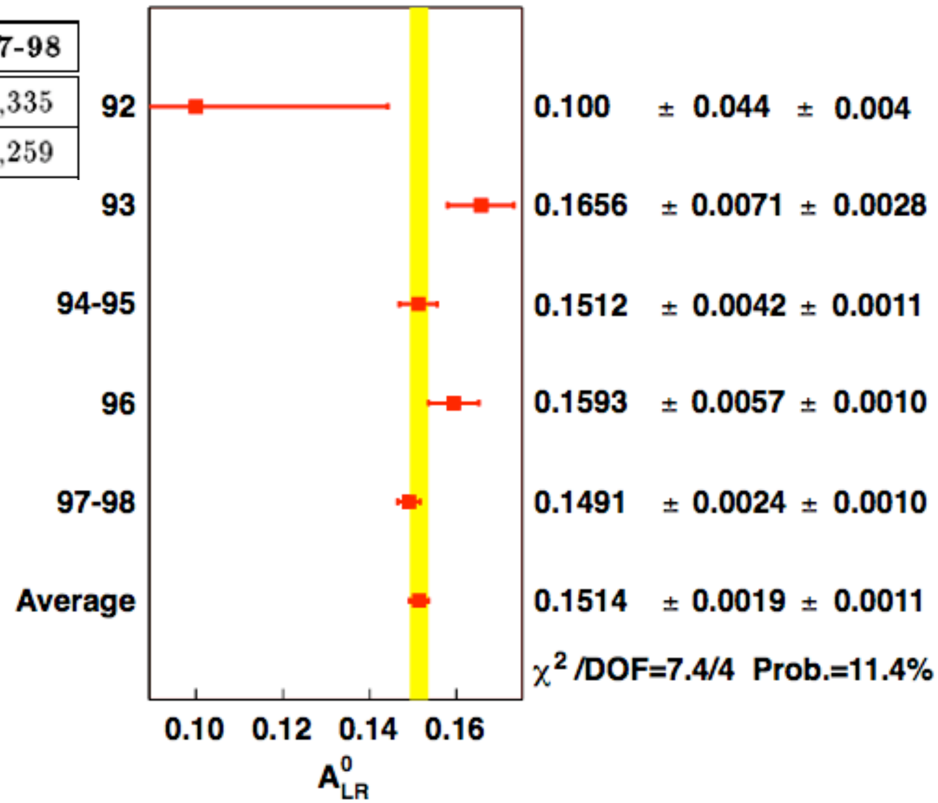


Initial State Polarization Results

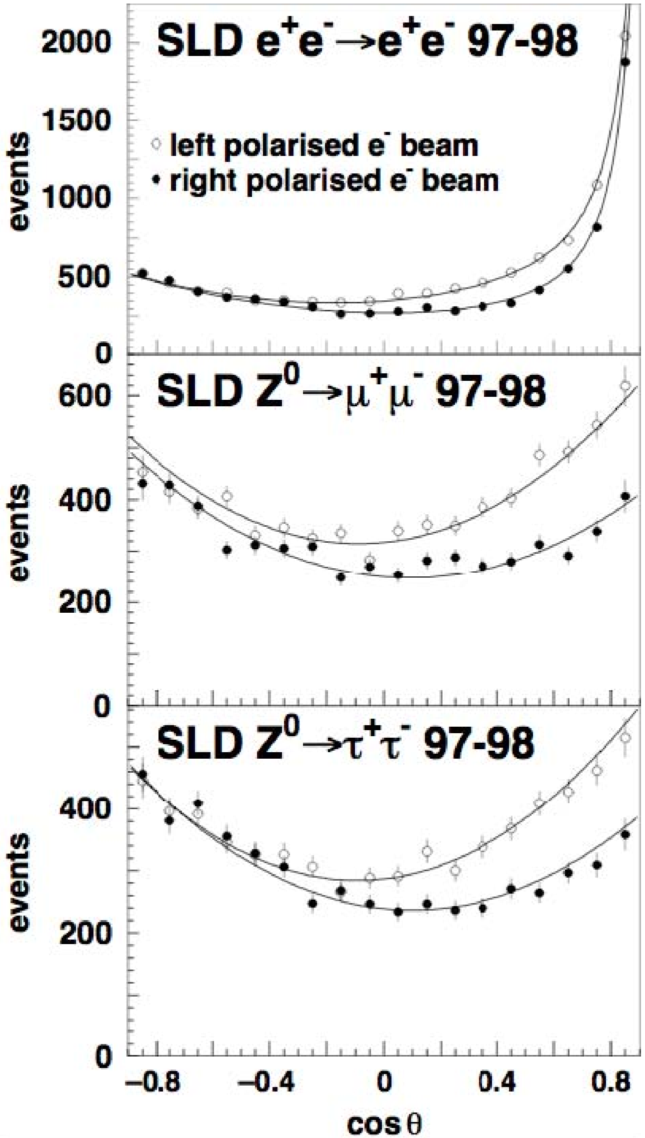
Source	1992	1993	1994-95	1996	1997-98
N_L	5,226	27,225	52,179	29,016	183,335
N_R	4,998	22,167	41,465	22,857	148,259

$$A_{LR} = \frac{N_L - N_R}{N_L + N_R} \frac{1}{\langle \mathcal{P}_e \rangle}$$

$$\boxed{A^0_{LR} = 0.1514 \pm 0.0022}$$



Polarized Forward-Backward Asymmetry



Forward-backward asymmetry for polarized beams:

$$\mathcal{A}_{LRFB} = \frac{(\sigma_F - \sigma_B)_L - (\sigma_F - \sigma_B)_R}{(\sigma_F + \sigma_B)_L + (\sigma_F + \sigma_B)_R} \frac{1}{< \mathcal{P}_e >}$$

measured with 22254 electron pairs, 16844 muon pairs and 16084 tau pairs.

Leptonic Asymmetry Results

$$A_{FB}^{0,l} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_l$$

$$A_{FB}^{0,b} = \frac{3}{4}\mathcal{A}_e\mathcal{A}_b$$

$$A_{LR}^{0} = \mathcal{A}_e$$

$$A_{LRFB}^{0} = \frac{3}{4}\mathcal{A}_f$$

$$\mathcal{P}_\tau = f(\mathcal{A}_e, \mathcal{A}_\tau)$$

Measurements

Parameter	$A_{FB}^{0,\ell}$	A_{LR}^{0}, A_{LRFB}^{ℓ}	$\mathcal{P}_\tau$
$\mathcal{A}_e$	0.139 ± 0.012	0.1516 ± 0.0021	0.1498 ± 0.0049
$\mathcal{A}_\mu$	0.162 ± 0.019	0.142 ± 0.015	—
$\mathcal{A}_\tau$	0.180 ± 0.023	0.136 ± 0.015	0.1439 ± 0.0043

Combinations (χ^2 = 3.6/5)

Parameter	Average	Correlations		
		$\mathcal{A}_e$	$\mathcal{A}_\mu$	$\mathcal{A}_\tau$
$\mathcal{A}_e$	0.1514 ± 0.0019	1.00		
$\mathcal{A}_\mu$	0.1456 ± 0.0091	-0.10	1.00	
$\mathcal{A}_\tau$	0.1449 ± 0.0040	-0.02	0.01	1.00

Leptonic universality

$$\mathcal{A}_\ell = 0.1501 \pm 0.0016$$

Leptonic Couplings from Asymmetries

$$\mathcal{A}_f = \frac{g_{Lf}^2 - g_{Rf}^2}{g_{Lf}^2 + g_{Rf}^2} = \frac{2 g_{Vf} g_{Af}}{g_{Vf}^2 + g_{Af}^2} = 2 \frac{g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2}$$

Parameter	Average	Correlations						
		$g_{A\nu}$	g_{Ae}	$g_{A\mu}$	$g_{A\tau}$	g_{Ve}	$g_{V\mu}$	$g_{V\tau}$
$g_{A\nu} \equiv g_{V\nu}$	$+0.5003 \pm 0.0012$	1.00						
g_{Ae}	-0.50111 ± 0.00035	−0.75	1.00					
$g_{A\mu}$	-0.50120 ± 0.00054	0.39	−0.13	1.00				
$g_{A\tau}$	-0.50204 ± 0.00064	0.37	−0.12	0.35	1.00			
g_{Ve}	-0.03816 ± 0.00047	−0.10	0.01	−0.01	−0.03	1.00		
$g_{V\mu}$	-0.0367 ± 0.0023	0.02	0.00	−0.30	0.01	−0.10	1.00	
$g_{V\tau}$	-0.0366 ± 0.0010	0.02	−0.01	0.01	−0.07	−0.02	0.01	1.00

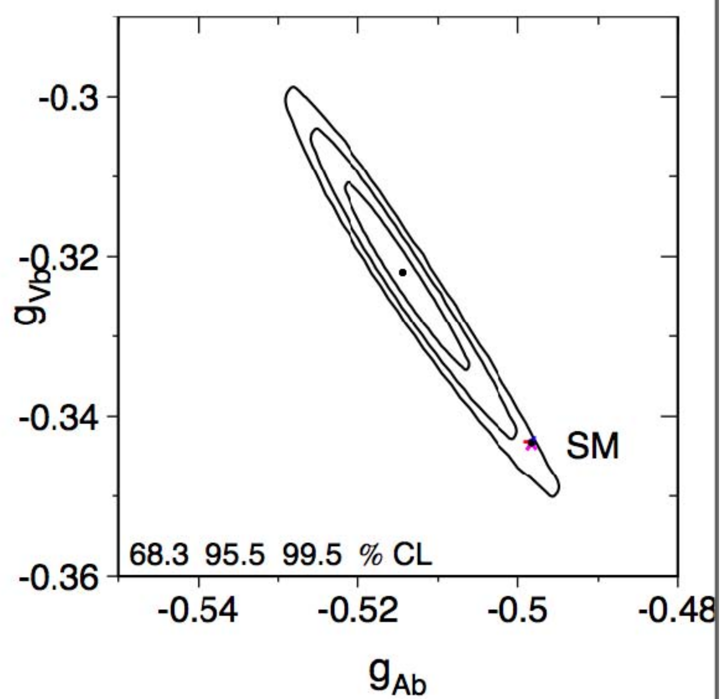
Universal weak neutral current couplings confirmed in the leptonic sector, at the permille level.

Heavy Quark Couplings from Asymmetries

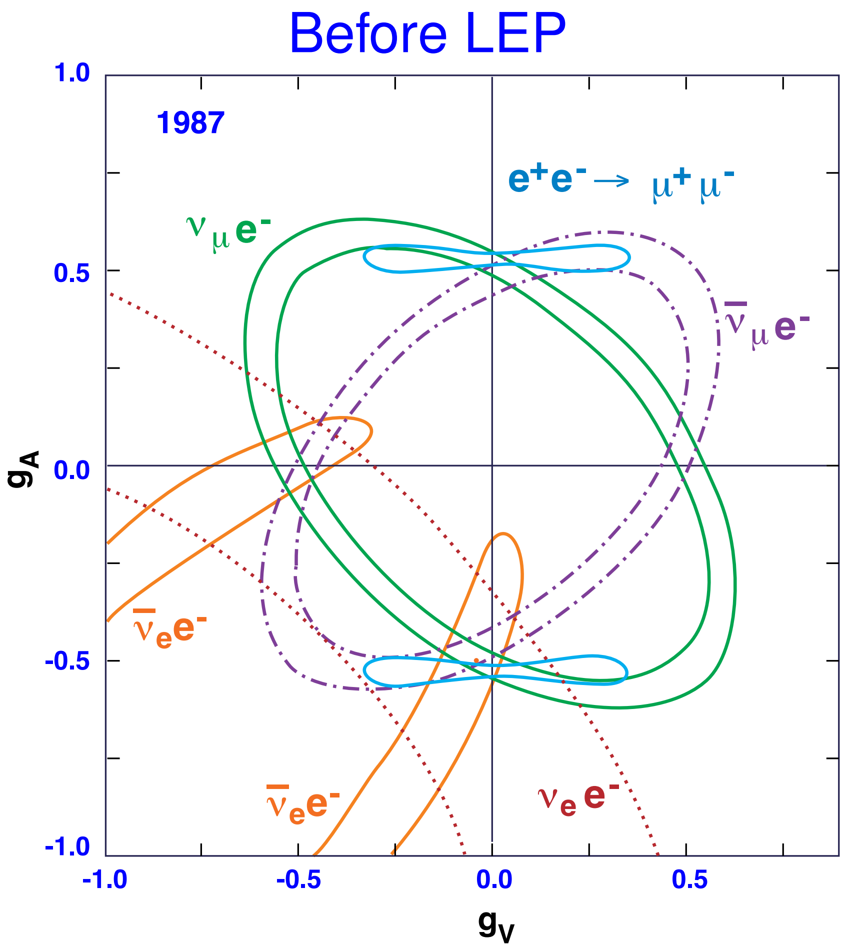
- Heavy quark analysis not covered here, rather complex
- Signatures: displaced vertices, energetic leptons from decay, fat jets
- Charge sign inferred from lepton or jet charge
- Results:

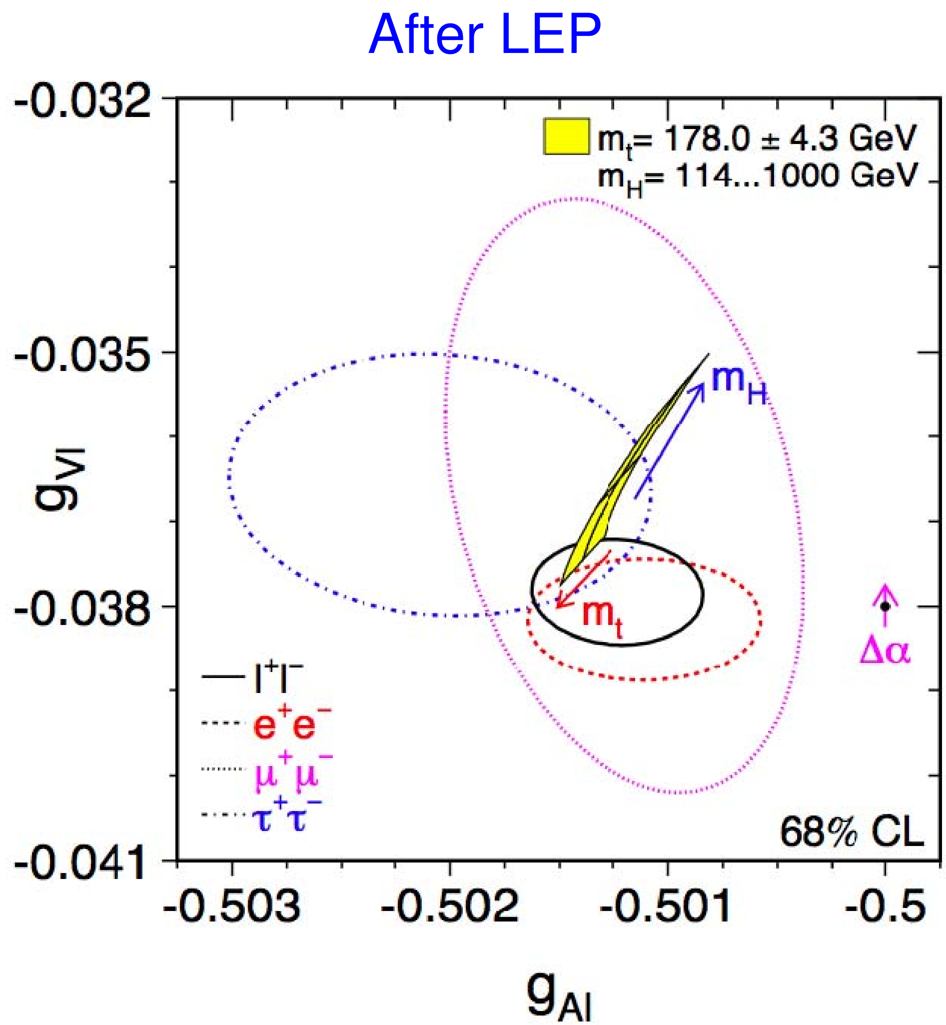
$$\mathcal{A}_\mathrm{f} = \frac{g_\mathrm{Lf}^2 - g_\mathrm{Rf}^2}{g_\mathrm{Lf}^2 + g_\mathrm{Rf}^2} = \frac{2 g_\mathrm{Vf} g_\mathrm{Af}}{g_\mathrm{Vf}^2 + g_\mathrm{Af}^2} = 2\frac{g_\mathrm{Vf}/g_\mathrm{Af}}{1 + (g_\mathrm{Vf}/g_\mathrm{Af})^2}$$

Parameter	Average	Correlations						
		$g_{\mathrm{A}\nu}$	$g_{\mathrm{A}\ell}$	g_Ab	g_Ac	$g_{\mathrm{V}\ell}$	g_Vb	g_Vc
$g_{\mathrm{A}\nu} \equiv g_{\mathrm{V}\nu}$	$+0.50075\pm0.00077$	1.00						
$g_{\mathrm{A}\ell}$	-0.50125 ± 0.00026	−0.49	1.00					
g_Ab	-0.5144 ± 0.0051	0.01	−0.02	1.00				
g_Ac	$+0.5034\pm0.0053$	−0.02	−0.02	0.00	1.00			
$g_{\mathrm{V}\ell}$	-0.03753 ± 0.00037	−0.04	−0.04	0.41	−0.05	1.00		
g_Vb	-0.3220 ± 0.0077	0.01	0.05	−0.97	0.04	−0.42	1.00	
g_Vc	$+0.1873\pm0.0070$	−0.01	−0.02	0.15	−0.35	0.10	−0.17	1.00



Leptonic Couplings from Asymmetries





Progress from qualitative to quantitative physics...