ÖAW AUSTRIAN ACADEMY OF SCIENCES

HEPHY INSTITUT FÜR HOCHENERGIEPHYSIK

Learning likelihood with tree boosting for extracting EFT parameters

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HEPHY, Austrian Academy of Sciences, Vienna

21/11/2023

Lunch Seminar,
Université catholique de Louvain

Physics@LHC: What's the status?

Newest fundamental particle discovered: Last missing piece in standard model (SM)

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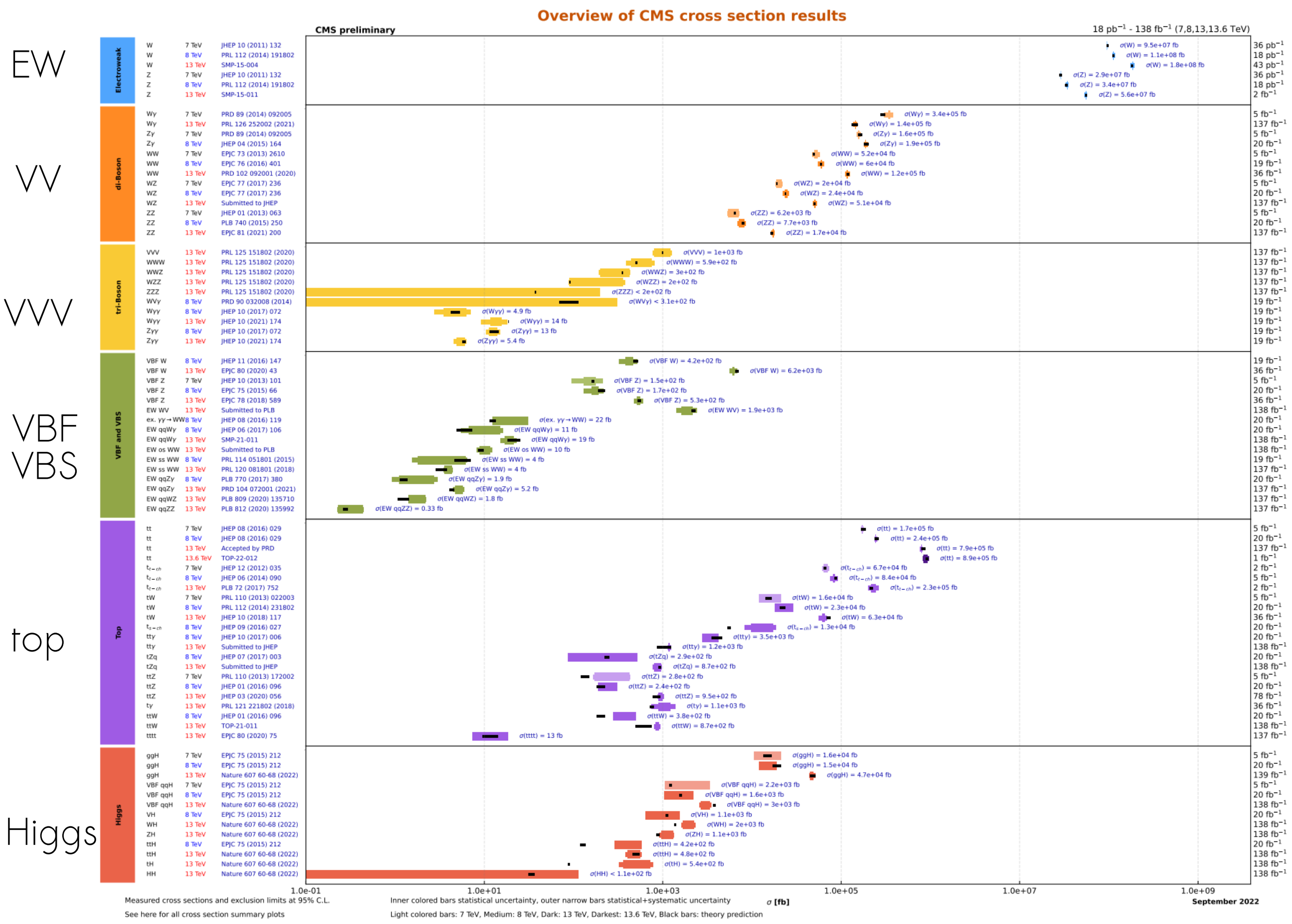
CMS summary plots



Precision measurement of plethora of processes
→ In general, good agreement with SM prediction

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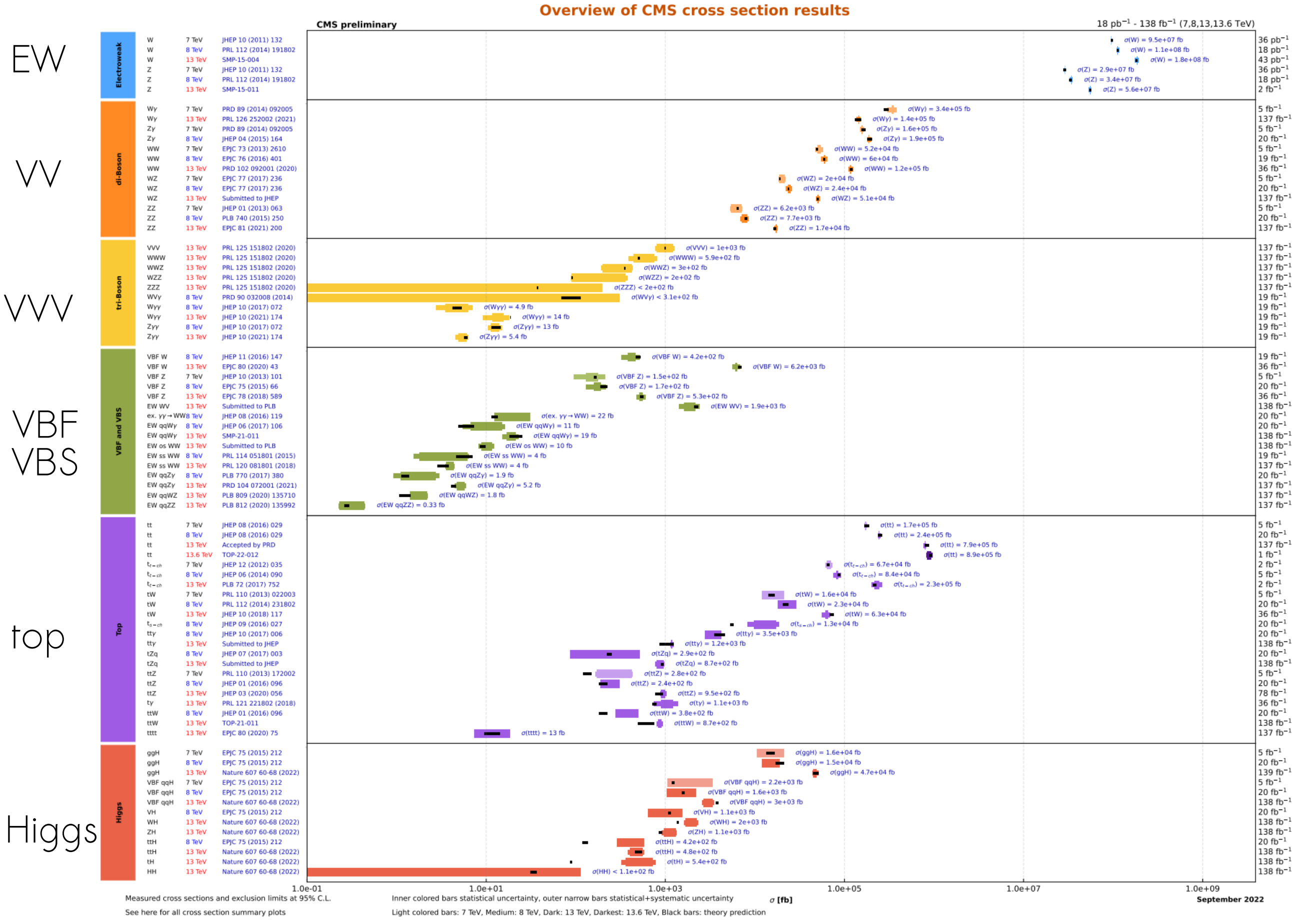
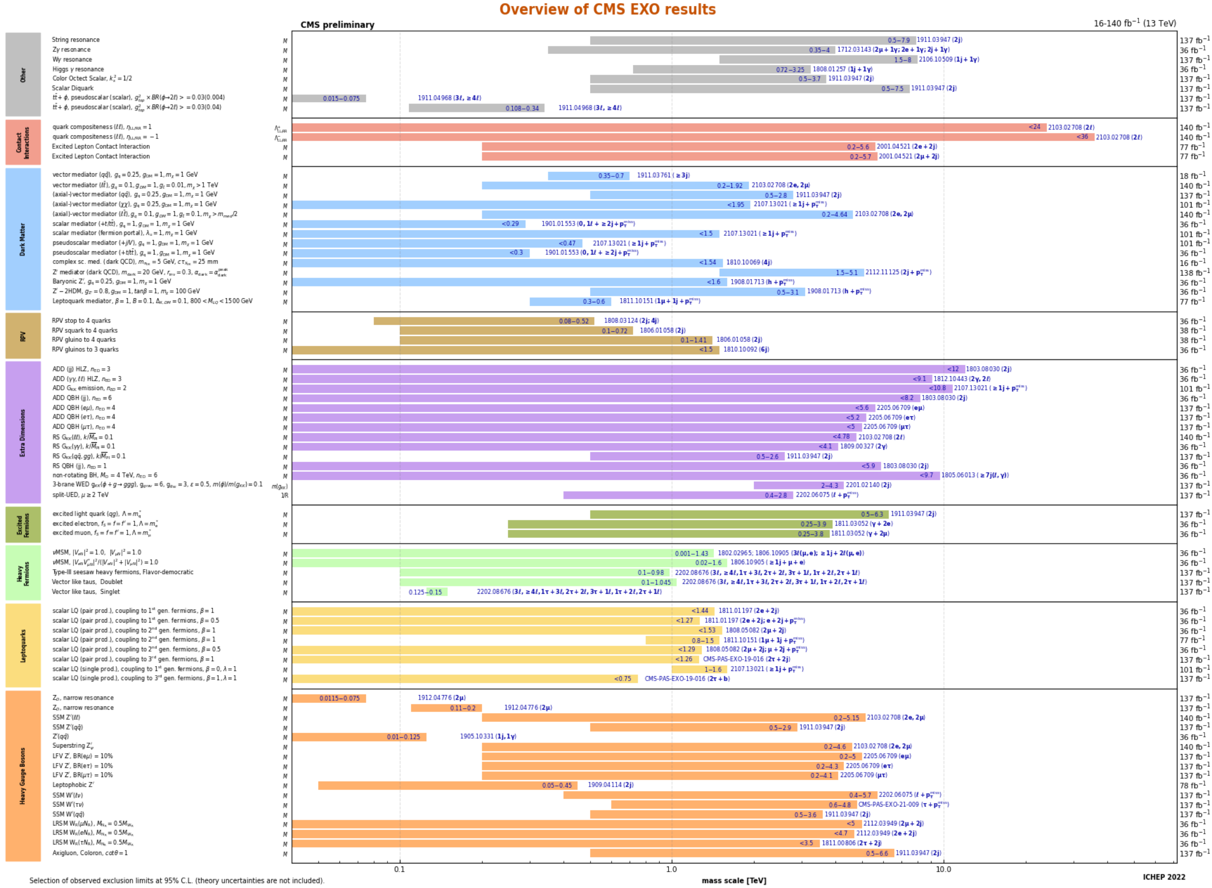


CMS-EXO summary plots



Precision measurement of plethora of processes
→ In general, good agreement with SM prediction

No smoking gun signature of new physics yet from LHC data

But, there are small interesting deviations (publicly available) ;)

Standard model effective field theory (SMEFT)

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'Model-independent' way to look for new physics signatures

Assumptions:

- → Unitarity, locality, Poincaré symmetry
- → Field content (relevant at EW scale) same as in SM
- → SM Gauge symmetries $SU(3)_C \times SU(2)_L \times U(1)_Y$ respected

SM ← A low-energy approximation of a more general theory

Physics at high scale appears as a correction to low energy theory

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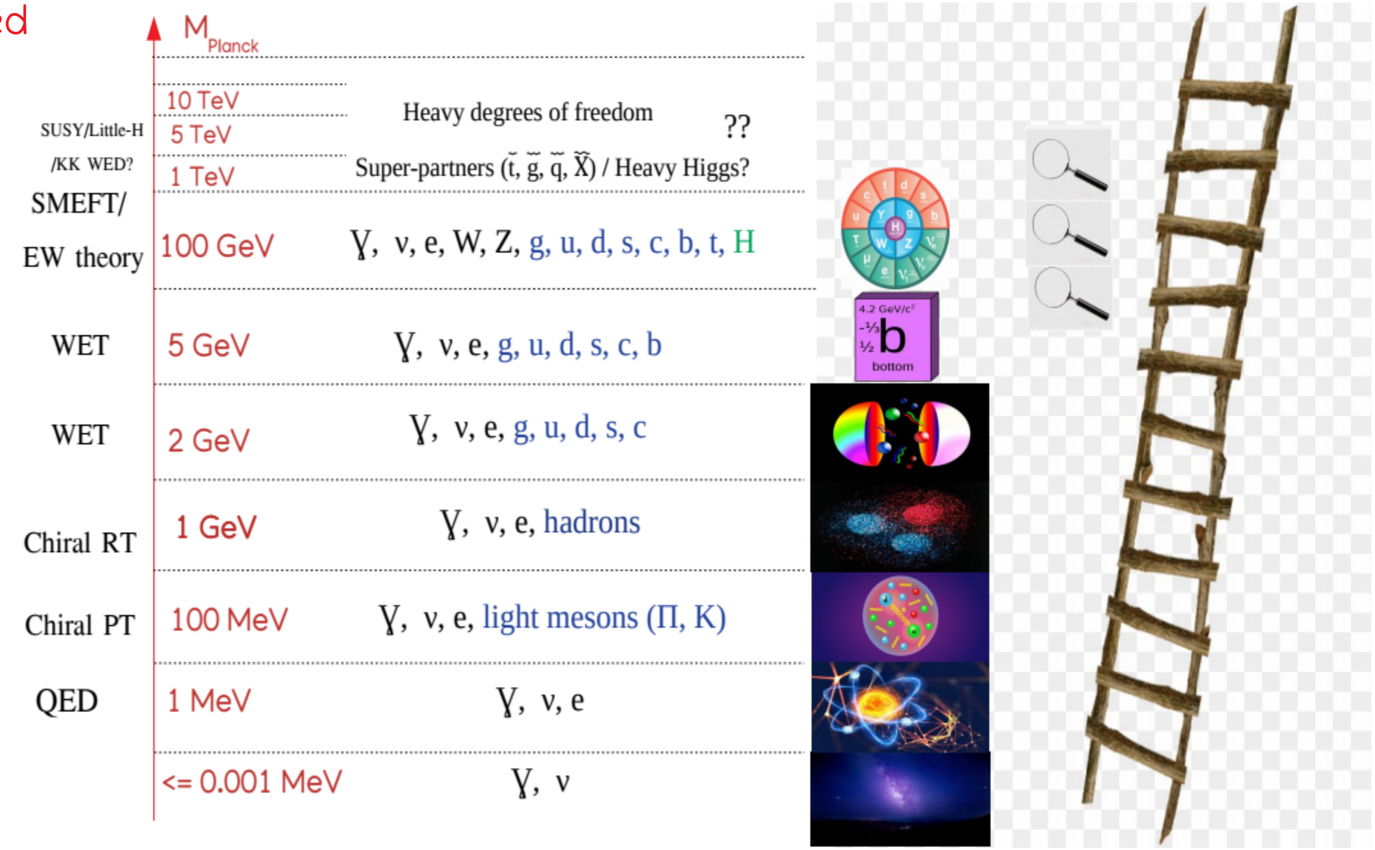

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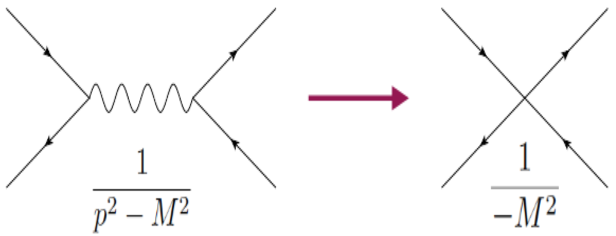
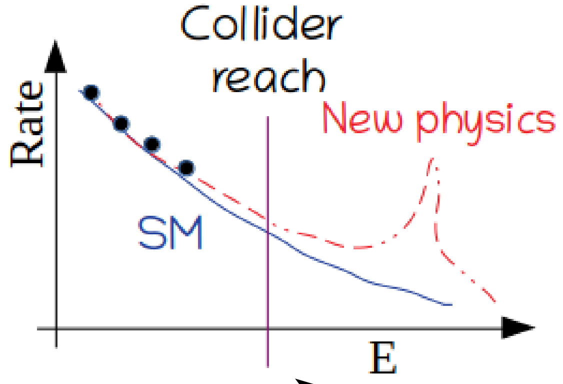
New (heavy) d.o.f. modify SM interactions





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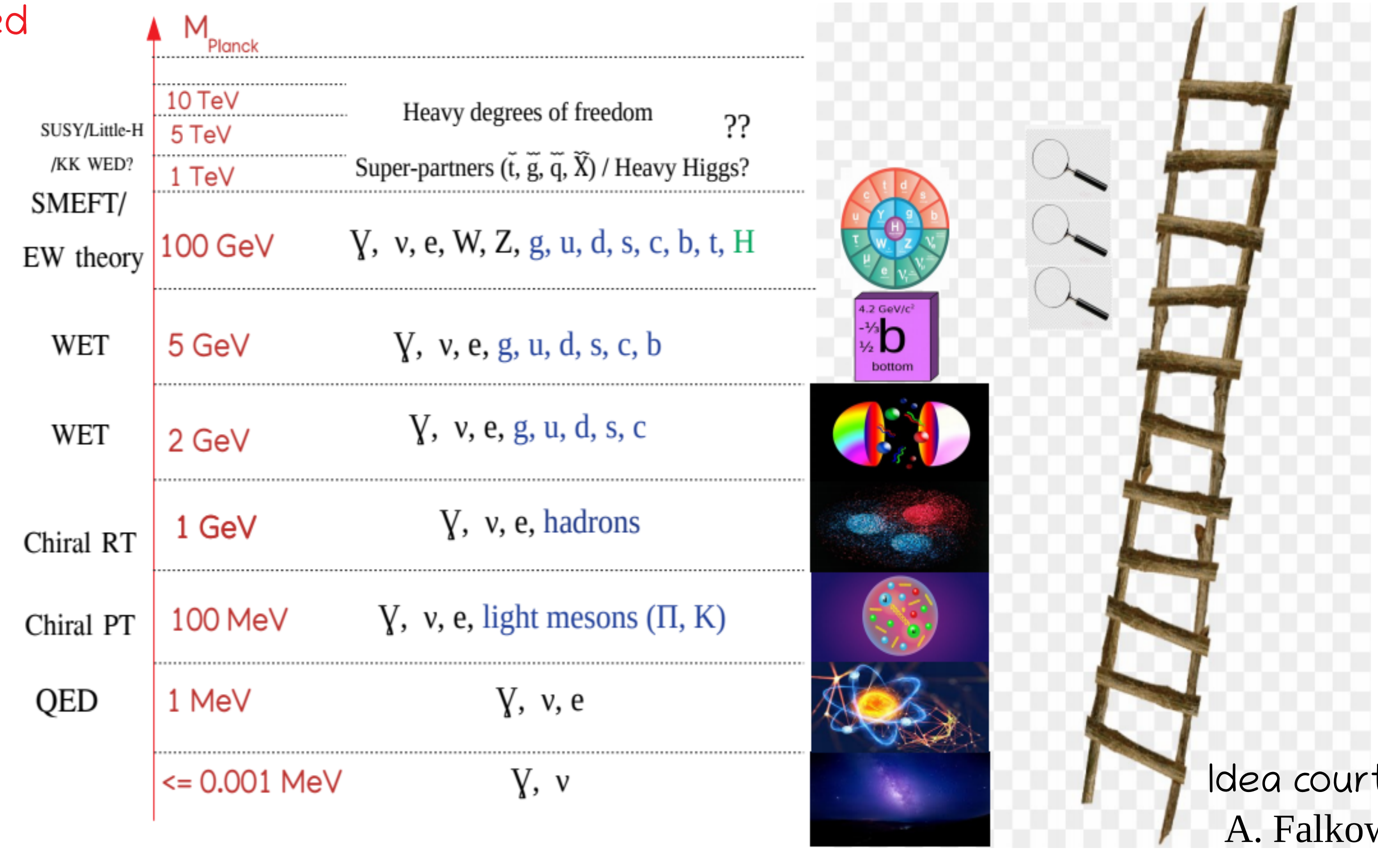

Idea courtesy:
A. Falkowski

Use data from precision electroweak, top quark, Higgs boson measurements together

Standard model effective field theory (SMEFT)

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$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i^{(5)}}{\Lambda}\mathcal{O}_{5,i} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_{6,i} + \sum_i \frac{c_i^{(7)}}{\Lambda^3}\mathcal{O}_{7,i} + \sum_i \frac{c_i^{(8)}}{\Lambda^4}\mathcal{O}_{8,i} + \ldots\ .$$

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Lepton number violation

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Lepton number violation

Lepton & Baryon number violation

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Lepton number violation

Lepton & Baryon number violation

59 SMEFT operators @ dim=6

Grzadkowski, Iskrzyński, Misiak, Rosiek (2010)

$1: X^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$Q_{\widetilde G}$	$f^{ABC}\widetilde G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$
$Q_{\widetilde W}$	$\epsilon^{IJK}\widetilde W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^*\left(H^\dagger D_\mu H\right)$

$5: \psi^2H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar l_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar q_p u_r \widetilde H)$
Q_{dH}	$(H^\dagger H)(\bar q_p d_r H)$

$4: X^2H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
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Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
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Q_{HWB}	$H^\dagger\tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde WB}$	$H^\dagger\tau^I H\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2XH + \text{h.c.}$	
Q_{eW}	$(\bar l_p\sigma^{\mu\nu}e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar l_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$
Q_{uG}	$(\bar q_p\sigma^{\mu\nu}T^A u_r)\widetilde H\, G_{\mu\nu}^A$
Q_{uW}	$(\bar q_p\sigma^{\mu\nu}u_r)\tau^I\widetilde H\, W_{\mu\nu}^I$
Q_{uB}	$(\bar q_p\sigma^{\mu\nu}u_r)\widetilde H\, B_{\mu\nu}$
Q_{dG}	$(\bar q_p\sigma^{\mu\nu}T^A d_r)H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar q_p\sigma^{\mu\nu}d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar q_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$

$7: \psi^2H^2D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar l_p\gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar l_p\tau^I\gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar e_p\gamma^\mu e_r)$
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Q_{Hud} + h.c.	$i(\widetilde H^\dagger D_\mu H)(\bar u_p\gamma^\mu d_r)$

$8: (\bar LL)(\bar LL)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu\tau^I l_r)(\bar q_s\gamma^\mu\tau^I q_t)$

$8: (\bar RR)(\bar RR)$	
Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$
Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$
Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$
Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$
Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$
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$8: (\bar LL)(\bar RR)$	
Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
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Alonso, Jenkins, Manohar, Trott (2013)

Ideally,
2499 numbers to measure

Class	N_{op}	CP-even n_g	1	3	CP-odd n_g	1	3
1	4	2	2	2	2	2	2
2	1	1	1	1	0	0	0
3	2	2	2	2	0	0	0
4	8	4	4	4	4	4	4
5	3	$3n_g^2$	3	27	$3n_g^2$	3	27
6	8	$8n_g^2$	8	72	$8n_g^2$	8	72
7	8	$\frac{1}{2}n_g(9n_g+7)$	8	51	$\frac{1}{2}n_g(9n_g-7)$	1	30
$8: (\bar{L}L)(\bar{L}L)$	5	$\frac{1}{4}n_g^2(7n_g^2+13)$	5	171	$\frac{7}{4}n_g^2(n_g-1)(n_g+1)$	0	126
$8: (\bar{R}R)(\bar{R}R)$	7	$\frac{1}{8}n_g(21n_g^3+2n_g^2+31n_g+2)$	7	255	$\frac{1}{8}n_g(21n_g+2)(n_g-1)(n_g+1)$	0	195
$8: (\bar{L}L)(\bar{R}R)$	8	$4n_g^2(n_g^2+1)$	8	360	$4n_g^2(n_g-1)(n_g+1)$	0	288
$8: (\bar{L}R)(\bar{R}L)$	1	n_g^4	1	81	n_g^4	1	81
$8: (\bar{L}R)(\bar{L}R)$	4	$4n_g^4$	4	324	$4n_g^4$	4	324
8 : All	25	$\frac{1}{8}n_g(107n_g^3+2n_g^2+89n_g+2)$	25	1191	$\frac{1}{8}n_g(107n_g^3+2n_g^2-67n_g-2)$	5	1014
Total	59	$\frac{1}{8}(107n_g^4+2n_g^3+213n_g^2+30n_g+72)$	53	1350	$\frac{1}{8}(107n_g^4+2n_g^3+57n_g^2-30n_g+48)$	23	1149

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Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2 X H + \mathrm{h.c.}$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

$8: (\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$

$8: (\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$
Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$
Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p \gamma_\mu u_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$

$8: (\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{d}_s \gamma^\mu T^A d_t)$

$8: (\bar{L}R)(\bar{R}L) + \mathrm{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

$8: (\bar{L}R)(\bar{L}R) + \mathrm{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

Flavor assumptions → Different # of coefficients

Alonso, Jenkins, Manohar, Trott (2013)

Ideally,
2499 numbers to measure

Class	N_{op}	CP-even n_g	1	3	CP-odd n_g	1	3
1	4	2	2	2	2	2	2
2	1	1	1	1	0	0	0
3	2	2	2	2	0	0	0
4	8	4	4	4	4	4	4
5	3	$3n_g^2$	3	27	$3n_g^2$	3	27
6	8	$8n_g^2$	8	72	$8n_g^2$	8	72
7	8	$\frac{1}{2}n_g(9n_g+7)$	8	51	$\frac{1}{2}n_g(9n_g-7)$	1	30
$8: (\bar{L}L)(\bar{L}L)$	5	$\frac{1}{4}n_g^2(7n_g^2+13)$	5	171	$\frac{7}{4}n_g^2(n_g-1)(n_g+1)$	0	126
$8: (\bar{R}R)(\bar{R}R)$	7	$\frac{1}{8}n_g(21n_g^3+2n_g^2+31n_g+2)$	7	255	$\frac{1}{8}n_g(21n_g+2)(n_g-1)(n_g+1)$	0	195
$8: (\bar{L}L)(\bar{R}R)$	8	$4n_g^2(n_g^2+1)$	8	360	$4n_g^2(n_g-1)(n_g+1)$	0	288
$8: (\bar{L}R)(\bar{R}L)$	1	n_g^4	1	81	n_g^4	1	81
$8: (\bar{L}R)(\bar{L}R)$	4	$4n_g^4$	4	324	$4n_g^4$	4	324
8 : All	25	$\frac{1}{8}n_g(107n_g^3+2n_g^2+89n_g+2)$	25	1191	$\frac{1}{8}n_g(107n_g^3+2n_g^2-67n_g-2)$	5	1014
Total	59	$\frac{1}{8}(107n_g^4+2n_g^3+213n_g^2+30n_g+72)$	53	1350	$\frac{1}{8}(107n_g^4+2n_g^3+57n_g^2-30n_g+48)$	23	1149

Standard model effective field theory (SMEFT)

HEPHY INSTITUT FÜR HOCHENERGIEPHYSIK

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i^{(5)}}{\Lambda}\mathcal{O}_{5,i} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_{6,i} + \sum_i \frac{c_i^{(7)}}{\Lambda^3}\mathcal{O}_{7,i} + \sum_i \frac{c_i^{(8)}}{\Lambda^4}\mathcal{O}_{8,i} + \dots$$

Lepton number violation

Lepton & Baryon number violation

59 SMEFT operators @ dim=6

Grzadkowski, Iskrzyński, Misiak, Rosiek (2010)

$1: X^3$		$2: H^6$		$3: H^4D^2$		$5: \psi^2H^3 + \text{h.c.}$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	Q_{eH}	$(H^\dagger H)(\bar l_p e_r H)$
$Q_{\widetilde G}$	$f^{ABC}\widetilde G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$			Q_{HD}	$(H^\dagger D_\mu H)^*(H^\dagger D_\mu H)$	Q_{uH}	$(H^\dagger H)(\bar q_p u_r \widetilde H)$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$					Q_{dH}	$(H^\dagger H)(\bar q_p d_r H)$
$Q_{\widetilde W}$	$\epsilon^{IJK}\widetilde W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$						

$4: X^2H^2$		$6: \psi^2XH + \text{h.c.}$		$7: \psi^2H^2D$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p\sigma^{\mu\nu}e_r)\tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar l_p\gamma^\mu l_r)$
$Q_{H\widetilde G}$	$H^\dagger H\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar l_p\tau^I\gamma^\mu l_r)$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p\sigma^{\mu\nu}T^A u_r)\widetilde H\, G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar e_p\gamma^\mu e_r)$
$Q_{H\widetilde W}$	$H^\dagger H\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p\sigma^{\mu\nu}u_r)\tau^I\widetilde H\, W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar q_p\gamma^\mu q_r)$
Q_{HB}	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar q_p\sigma^{\mu\nu}u_r)\widetilde H\, B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{H\widetilde B}$	$H^\dagger H\, \widetilde B_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar q_p\sigma^{\mu\nu}T^A d_r)H\, G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar u_p\gamma^\mu u_r)$
Q_{HWB}	$H^\dagger\tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p\sigma^{\mu\nu}d_r)\tau^I H\, W_{\mu\nu}^I$	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar d_p\gamma^\mu d_r)$
$Q_{H\widetilde WB}$	$H^\dagger\tau^I H\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$	Q_{Hud} + h.c.	$i(\widetilde H^\dagger D_\mu H)(\bar u_p\gamma^\mu d_r)$

$8: (\bar LL)(\bar LL)$		$8: (\bar RR)(\bar RR)$		$8: (\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu\tau^I l_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

$8: (\bar LR)(\bar RL) + \text{h.c.}$		$8: (\bar LR)(\bar LR) + \text{h.c.}$	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\epsilon_{jk}(\bar q_s^k d_t)$
		$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\epsilon_{jk}(\bar q_s^k T^A d_t)$
		$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\epsilon_{jk}(\bar q_s^k u_t)$
		$Q_{lequ}^{(3)}$	$(\bar l_p^j\sigma_{\mu\nu}e_r)\epsilon_{jk}(\bar q_s^k\sigma^{\mu\nu}u_t)$

Alonso, Jenkins, Manohar, Trott (2013)

Class	$N_{\rm op}$	CP-even n_g	1	3	CP-odd n_g	1	3
1	4	2	2	2	2	2	2
2	1	1	1	1	0	0	0
3	2	2	2	2	0	0	0
4	8	4	4	4	4	4	4
5	3	$3n_g^2$	3	27	$3n_g^2$	3	27
6	8	$8n_g^2$	8	72	$8n_g^2$	8	72
7	8	$\frac12 n_g(9n_g+7)$	8	51	$\frac12 n_g(9n_g-7)$	1	30
$8: (\bar LL)(\bar LL)$	5	$\frac14 n_g^2(7n_g^2+13)$	5	171	$\frac74 n_g^2(n_g-1)(n_g+1)$	0	126
$8: (\bar RR)(\bar RR)$	7	$\frac18 n_g(21n_g^3+2n_g^2+31n_g+2)$	7	255	$\frac18 n_g(21n_g+2)(n_g-1)(n_g+1)$	0	195
$8: (\bar LL)(\bar RR)$	8	$4n_g^2(n_g^2+1)$	8	360	$4n_g^2(n_g-1)(n_g+1)$	0	288
$8: (\bar LR)(\bar RL)$	1	n_g^4	1	81	n_g^4	1	81
$8: (\bar LR)(\bar LR)$	4	$4n_g^4$	4	324	$4n_g^4$	4	324
8 : All	25	$\frac18 n_g(107n_g^3+2n_g^2+89n_g+2)$	25	1191	$\frac18 n_g(107n_g^3+2n_g^2-67n_g-2)$	5	1014
Total	59	$\frac18(107n_g^4+2n_g^3+213n_g^2+30n_g+72)$	53	1350	$\frac18(107n_g^4+2n_g^3+57n_g^2-30n_g+48)$	23	1149

Ideally,
2499 numbers to measure

Flavor assumptions → Different # of coefficients

$U(3)^5$

$U(3)_q \times U(3)_u \times U(3)_d \times U(3)_l \times U(3)_e$

$U(2)_q \times U(2)_u \times U(2)_d$

$U(1)_e \times U(1)_\mu \times U(1)_\tau$ $\quad U(3)_l \times U(3)_e$

	general		U35		MFV		top		topU3l	
	all	$\not{CP}$	all	$\not{CP}$	all	$\not{CP}$	all	$\not{CP}$	all	$\not{CP}$
$\mathcal{L}_6^{(1)}$	4	2	4	2	2	-	4	2	4	2
$\mathcal{L}_6^{(2,3)}$	3	-	3	-	3	-	3	-	3	-
$\mathcal{L}_6^{(4)}$	8	4	8	4	4	-	8	4	8	4
$\mathcal{L}_6^{(5)}$	54	27	6	3	7	-	14	7	10	5
$\mathcal{L}_6^{(6)}$	144	72	16	8	20	-	36	18	28	14
$\mathcal{L}_6^{(7)}$	81	30	9	1	14	-	21	2	15	2
$\mathcal{L}_6^{(8a)}$	297	126	8	-	10	-	31	-	16	-
$\mathcal{L}_6^{(8b)}$	450	195	9	-	19	-	40	2	27	2
$\mathcal{L}_6^{(8c)}$	648	288	8	-	28	-	54	4	31	4
$\mathcal{L}_6^{(8d)}$	810	405	14	7	13	-	64	32	40	20
tot	2499	1149	85	25	120	-	275	71	182	53

Brivio (2020)

Standard model effective field theory (SMEFT)

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$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i^{(5)}}{\Lambda}\mathcal{O}_{5,i} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_{6,i} + \sum_i \frac{c_i^{(7)}}{\Lambda^3}\mathcal{O}_{7,i} + \sum_i \frac{c_i^{(8)}}{\Lambda^4}\mathcal{O}_{8,i} + \dots$$

Lepton number violation

Lepton & Baryon number violation

59 SMEFT operators @ dim=6

Grzadkowski, Iskrzyński, Misiak, Rosiek (2010)

$1: X^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$(H^\dagger D_\mu H)^*(H^\dagger D_\mu H)$

$5: \psi^2H^3 + \text{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$
Q_{HWB}	$H^\dagger\tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger\tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2XH + \text{h.c.}$	
Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$

$7: \psi^2H^2D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p\gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p\tau^I\gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p\gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p\tau^I\gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p\gamma^\mu d_r)$

$8: (\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$

$8: (\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$
Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8: (\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8: (\bar{L}R)(\bar{R}L) + \text{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

$8: (\bar{L}R)(\bar{L}R) + \text{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$

Alonso, Jenkins, Manohar, Trott (2013)

Class	$N_{\rm op}$	CP-even n_g	1	3	CP-odd n_g	1	3
1	4	2	2	2	2	2	2
2	1	1	1	1	0	0	0
3	2	2	2	2	0	0	0
4	8	4	4	4	4	4	4
5	3	$3n_g^2$	3	27	$3n_g^2$	3	27
6	8	$8n_g^2$	8	72	$8n_g^2$	8	72
7	8	$\frac{1}{2}n_g(9n_g+7)$	8	51	$\frac{1}{2}n_g(9n_g-7)$	1	30
$8: (\bar{L}L)(\bar{L}L)$	5	$\frac{1}{4}n_g^2(7n_g^2+13)$	5	171	$\frac{7}{4}n_g^2(n_g-1)(n_g+1)$	0	126
$8: (\bar{R}R)(\bar{R}R)$	7	$\frac{1}{8}n_g(21n_g^3+2n_g^2+31n_g+2)$	7	255	$\frac{1}{8}n_g(21n_g+2)(n_g-1)(n_g+1)$	0	195
$8: (\bar{L}L)(\bar{R}R)$	8	$4n_g^2(n_g^2+1)$	8	360	$4n_g^2(n_g-1)(n_g+1)$	0	288
$8: (\bar{L}R)(\bar{R}L)$	1	n_g^4	1	81	n_g^4	1	81
$8: (\bar{L}R)(\bar{L}R)$	4	$4n_g^4$	4	324	$4n_g^4$	4	324
8 : All	25	$\frac{1}{8}n_g(107n_g^3+2n_g^2+89n_g+2)$	25	1191	$\frac{1}{8}n_g(107n_g^3+2n_g^2-67n_g-2)$	5	1014
Total	59	$\frac{1}{8}(107n_g^4+2n_g^3+213n_g^2+30n_g+72)$	53	1350	$\frac{1}{8}(107n_g^4+2n_g^3+57n_g^2-30n_g+48)$	23	1149

Ideally,
2499 numbers to measure

Automated in

SMEFTsim @LO
(all D-6 SMEFT operators)

SMEFT@NLO
(CP-even D-6 SMEFT operators)

Flavor assumptions → Different # of coefficients

$U(3)^5$

$U(3)_q \times U(3)_u \times U(3)_d \times U(3)_l \times U(3)_e$

$U(2)_q \times U(2)_u \times U(2)_d$

$U(1)_e \times U(1)_\mu \times U(1)_\tau$ $U(3)_l \times U(3)_e$

	general all	general $\cancel{CP}$	U35 all	U35 $\cancel{CP}$	MFV all	MFV $\cancel{CP}$	top all	top $\cancel{CP}$	topU3l all	topU3l $\cancel{CP}$
$\mathcal{L}_6^{(1)}$	4	2	4	2	2	-	4	2	4	2
$\mathcal{L}_6^{(2,3)}$	3	-	3	-	3	-	3	-	3	-
$\mathcal{L}_6^{(4)}$	8	4	8	4	4	-	8	4	8	4
$\mathcal{L}_6^{(5)}$	54	27	6	3	7	-	14	7	10	5
$\mathcal{L}_6^{(6)}$	144	72	16	8	20	-	36	18	28	14
$\mathcal{L}_6^{(7)}$	81	30	9	1	14	-	21	2	15	2
$\mathcal{L}_6^{(8a)}$	297	126	8	-	10	-	31	-	16	-
$\mathcal{L}_6^{(8b)}$	450	195	9	-	19	-	40	2	27	2
$\mathcal{L}_6^{(8c)}$	648	288	8	-	28	-	54	4	31	4
$\mathcal{L}_6^{(8d)}$	810	405	14	7	13	-	64	32	40	20
tot	2499	1149	85	25	120	-	275	71	182	53

Brivio (2020)

Polynomial parameterization in SMEFT

Polynomial parameterization in SMEFT

$$\mathcal{M}_{\text{SMEFT}} = \mathcal{M}_{\text{SM}} + \sum_i \frac{c_i}{\Lambda^2}\mathcal{M}_{6,\text{i}}$$

$$\begin{aligned}\sigma &\sim |\mathcal{M}_{\text{SMEFT}}|^2 \\ &\sim |\mathcal{M}_{\text{SM}}|^2 + \sum_i \frac{c_i}{\Lambda^2} 2\text{Re}(\mathcal{M}^\dagger_{\text{SM}}\mathcal{M}_{6,\text{i}}) + \sum_i \frac{c_i^2}{\Lambda^4}|\mathcal{M}_{6,\text{i}}|^2 + \sum_i\sum_{j,j>i}\frac{c_i c_j}{\Lambda^4}\mathcal{M}_{6,\text{i}}\mathcal{M}_{6,\text{j}}\end{aligned}$$

σ is a quadratic function of coefficients !

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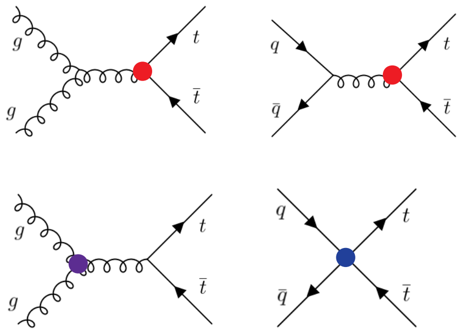
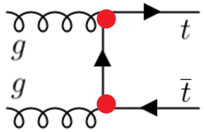
Example





$\mathcal{O}_G$ $\mathcal{O}_{\widetilde{G}}$

$\mathcal{O}_G^{(33)}$

4-fermion ops (~8)

~10 parameters to measure

Polynomial parameterization in SMEFT

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INSTITUT FÜR HOCHENERGIEPHYSIK

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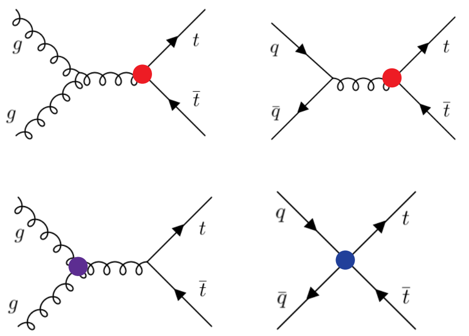

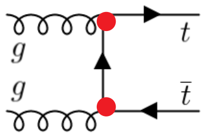

$\mathcal{O}_G$ $\mathcal{O}_{\tilde{G}}$

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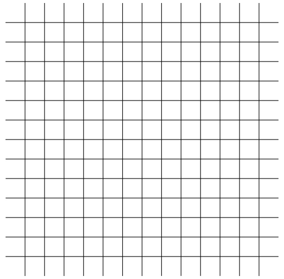
4-fermion ops (~8)

~10 parameters to measure

Curse of dimensionality

In a typical search, generate signal in 10-dim grids

If only 10 values / coefficient → 10^{10} signal samples!!!



Not needed for SMEFT :D

# of signal samples (for 'n' coefficients): $1 + n + n(n+1)/2$ ← Sufficient

If polynomial is of order 'k', # of minimum signal points N(n,k) = $\frac{k+1}{n}\binom{n+k}{k+1}$

For n=10, k=2 → N(10,2) = 66

Polynomial parameterization in SMEFT

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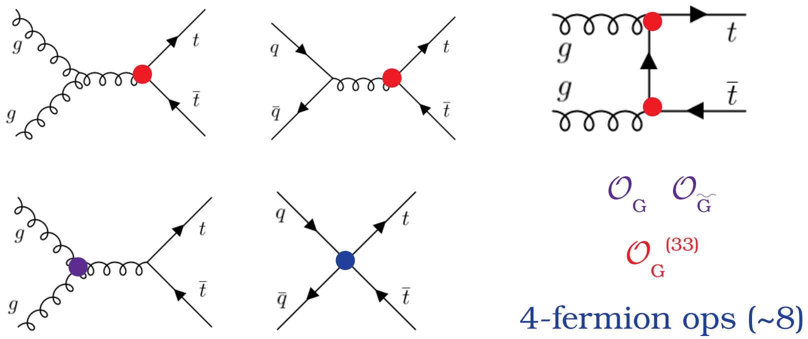
Example



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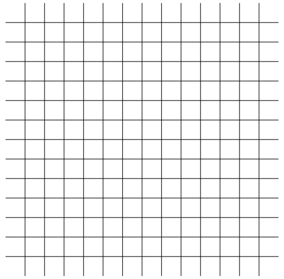

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Difference between SMEFT & SM is small → Possible to reweight SM sample to obtain SMEFT prediction

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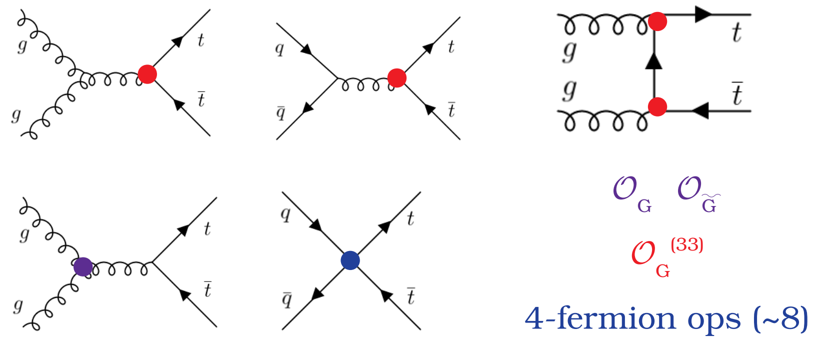
Example



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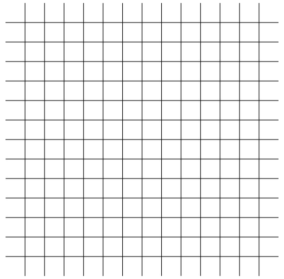
Curse of dimensionality



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For n=10, k=2 → N(10,2) = 66

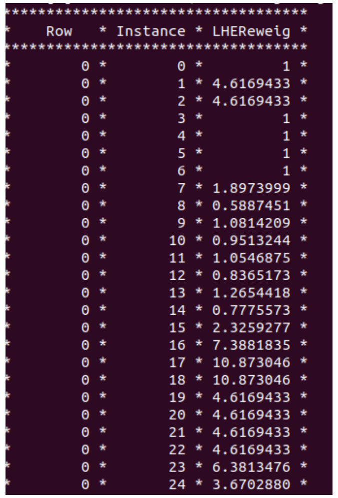
Difference between SMEFT & SM is small → Possible to reweight SM sample to obtain SMEFT prediction

$$w = \frac{|\mathcal{M}_{SMEFT}(c=c_1)|^2}{|\mathcal{M}_{SMEFT}(c=c_0)|^2}$$

Storing N(n,k) weights per event is sufficient!

Curse of dimensionality is lifted !

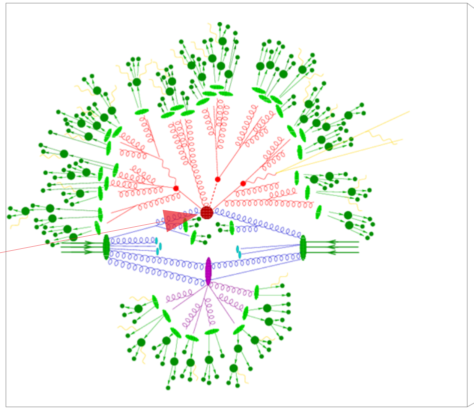
Possible to truncate non-polynomial cases (option available in SMEFTsim)

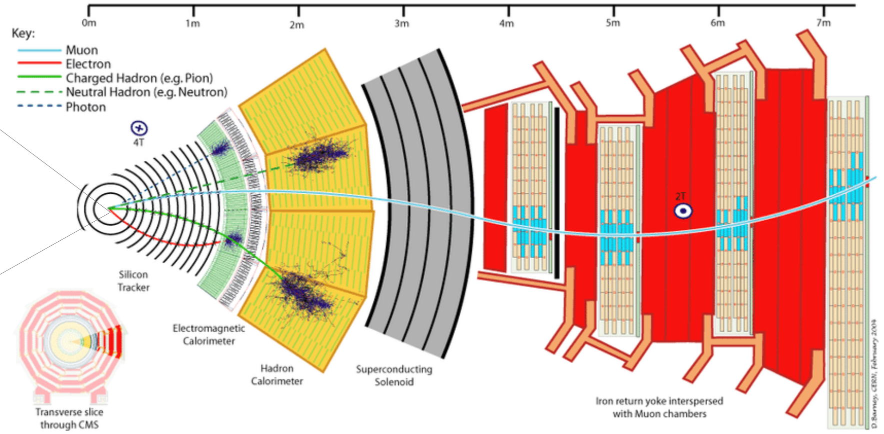


Row	Instance	LHEReweig
0	0	1
0	1	4.6169433
0	2	4.6169433
0	3	1
0	4	1
0	5	1
0	6	1
0	7	1.8973999
0	8	0.5887451
0	9	1.0814209
0	10	0.9513244
0	11	1.0546875
0	12	0.8365173
0	13	1.2654418
0	14	0.7775573
0	15	2.3259277
0	16	7.3881835
0	17	10.873046
0	18	10.873046
0	19	4.6169433
0	20	4.6169433
0	21	4.6169433
0	22	4.6169433
0	23	6.3813476
0	24	3.6702880

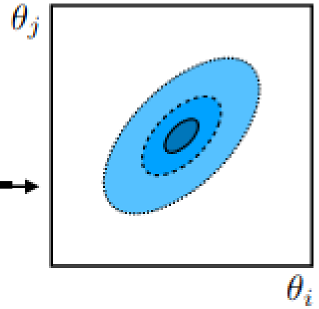
Optimal observable for SMEFT analysis

$\mathcal{L}_{\text{SMEFT}}$





??



Likelihood ratio trick in classification:

$$L = \int dx \sum_{z \in 0,1} p(\boldsymbol{x}, z) \, (z - f(\boldsymbol{x}))^2$$

$$f^*(\boldsymbol{x}) = \frac{p(\boldsymbol{x}|\text{SM})}{p(\boldsymbol{x}|\text{SM}) + p(\boldsymbol{x}|\theta)} = \frac{1}{1 + \frac{\sigma(\theta)}{\sigma(\text{SM})} r(\boldsymbol{x})}$$

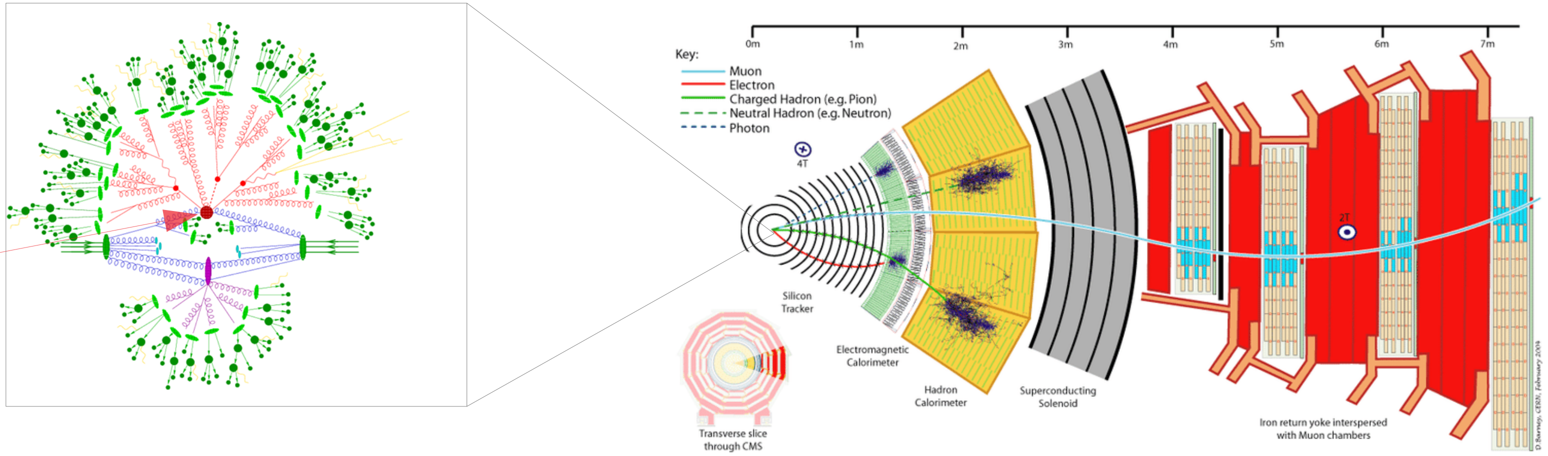
← Optimal test statistic

$$0 \to \text{SM}, \; 1 \to \theta$$

$$r(\boldsymbol{x}) = \frac{p(\boldsymbol{x}|\theta)}{p(\boldsymbol{x}|\text{SM})}$$

Optimal observable for SMEFT analysis

$\mathcal{L}_{\text{SMEFT}}$



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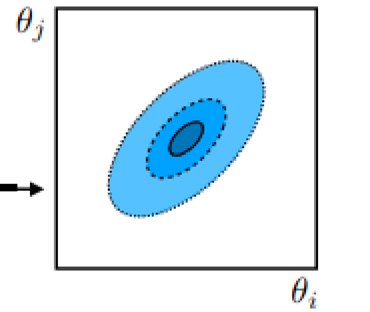

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$$r(\boldsymbol{x}) = \frac{p(\boldsymbol{x}|\theta)}{p(\boldsymbol{x}|\text{SM})}$$

$$p(\boldsymbol{x}|\boldsymbol{\theta}) = \int p(\boldsymbol{x}|\boldsymbol{z})p(\boldsymbol{z}|\boldsymbol{\theta})d\boldsymbol{z}$$

$$p(\boldsymbol{x}|\boldsymbol{\theta}) = \int\int\int p(\boldsymbol{x}|z_{\text{Det}})p(z_{\text{Det}}|z_{\text{Had}})p(z_{\text{Had}}|z_{\text{PS}})p(z_{\text{PS}}|z)dz_{\text{Det}}dz_{\text{Had}}dz_{\text{PS}}p(\boldsymbol{z}|\boldsymbol{\theta})d\boldsymbol{z}$$

Hard to model transfer function

Observables used for SMEFT analysis

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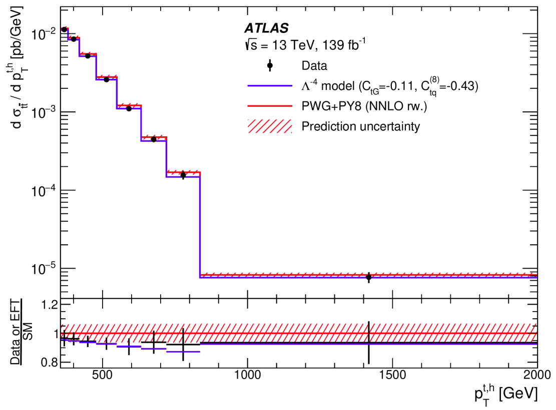

Differential cross sections: Straightforward

Feasible only with a few kinematic observables
EFT effects ignored in remaining variables

E.g. JHEP 04 (2023) 80

Observables used for SMEFT analysis

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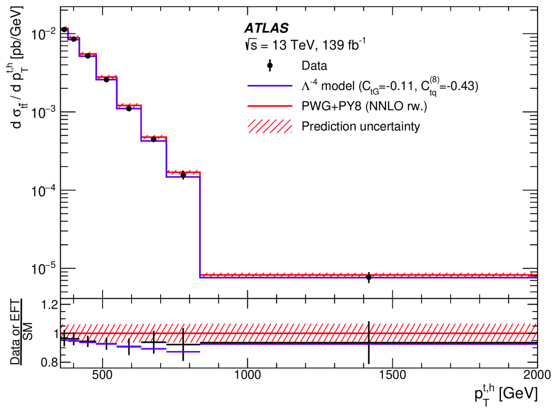

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Machine learning based discriminators: Enough experience in community

Construction depends on signal points in EFT space
~ exploits effects due to quadratic terms in EFT expansion

E.g. JHEP 12 (2021) 083

Observables used for SMEFT analysis

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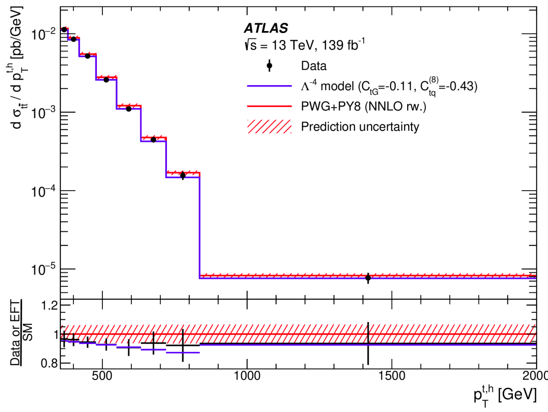

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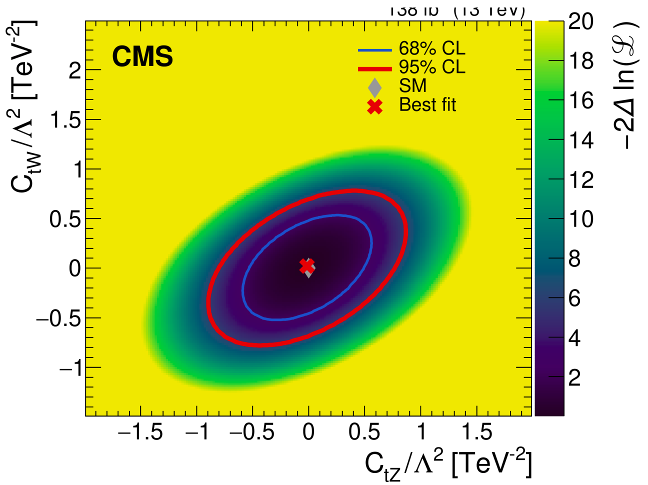

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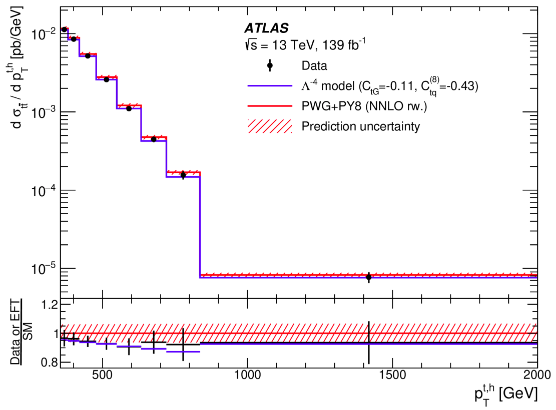

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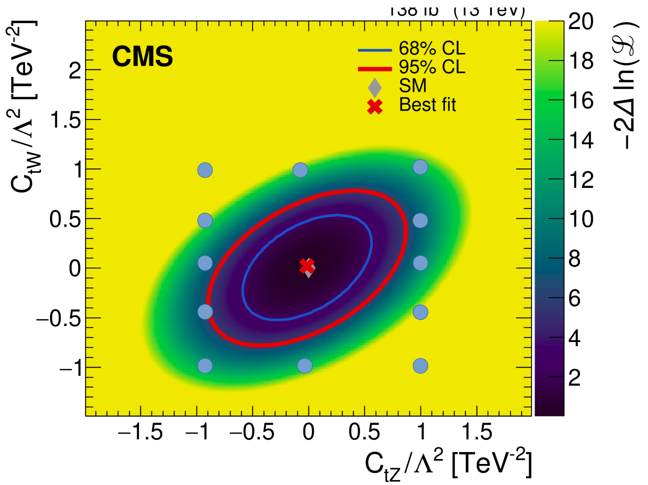

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E.g. JHEP 12 (2021) 083

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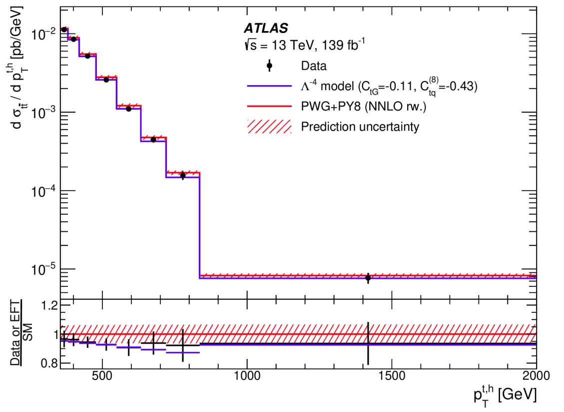
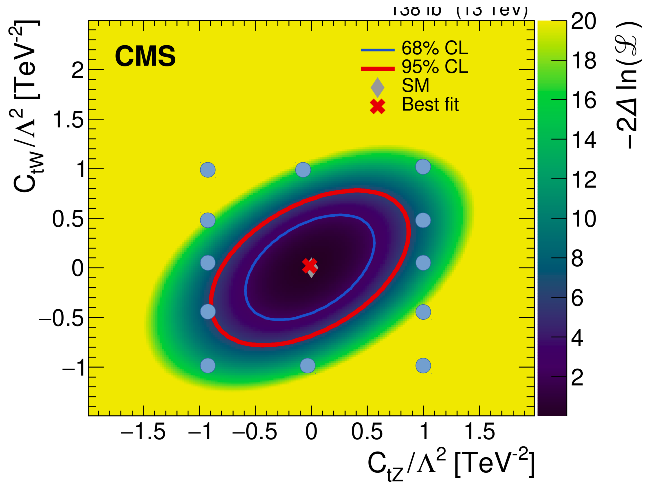


Differential cross sections: Straightforward

Feasible only with a few kinematic observables
EFT effects ignored in remaining variables

E.g. JHEP 04 (2023) 80



Machine learning based discriminators: Enough experience in community

$$r(\boldsymbol{x}) = \frac{\sum_{\mathcal{B}} p(\boldsymbol{x}|\theta)}{p(\boldsymbol{x}|\mathrm{SM})}$$

Construction depends on signal points in EFT space
~ exploits effects due to quadratic terms in EFT expansion

E.g. JHEP 12 (2021) 083

Matrix element method: Observable ~ Likelihood ratio ~ optimal

if transfer function is modeled well

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Observables used for SMEFT analysis

$$L = \int d\boldsymbol{x} \sum_{z \in 0,1} p(\boldsymbol{x}, z)\,(z - f(\boldsymbol{x}))^2$$

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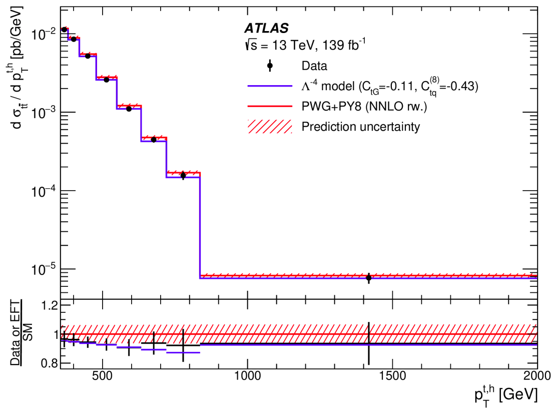

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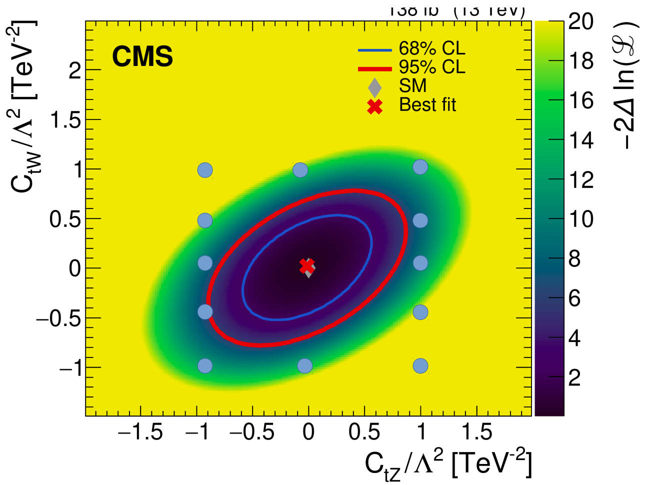

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Likelihood-free inference: Learn full likelihood ratio ← optimal — More in this talk

Learning SMEFT likelihood (I)

Starting point: Augmented dataset D ← list of events with a number of SMEFT weights / event

Description: Extended likelihood $L(\mathcal{D}(N_{\text{obs}}, \boldsymbol{x})|\boldsymbol{\theta}) = P_{\mathcal{L}\sigma(\boldsymbol{\theta})}(N_{\text{obs}}) \prod_{i=1}^{N_{\text{obs}}} p_i(\boldsymbol{x}|\boldsymbol{\theta}) = \frac{(\mathcal{L}\sigma(\boldsymbol{\theta}))^{N_{\text{obs}}}}{N_{\text{obs}}!} e^{-\mathcal{L}\sigma(\boldsymbol{\theta})} \prod_{i=1}^{N_{\text{obs}}} p_i(\boldsymbol{x}|\boldsymbol{\theta})$ $p_i(\boldsymbol{x}, \boldsymbol{\theta}) = \frac{1}{\sigma(\boldsymbol{\theta})} \frac{d\sigma(\boldsymbol{x}|\boldsymbol{\theta})}{d\boldsymbol{x}}$

Neyman-Peasrson lemma: Optimal test statistic $\frac{L(\mathcal{D}(N_{\text{obs}}, \boldsymbol{x})|\boldsymbol{\theta}_1)}{L(\mathcal{D}(N_{\text{obs}}, \boldsymbol{x})|\boldsymbol{\theta}_0)}$

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Likelihood ratio

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SMEFT dependence of detector-level observables $p(\boldsymbol{x}|\boldsymbol{\theta}) = \int p(\boldsymbol{x}|\boldsymbol{z})p(\boldsymbol{z}|\boldsymbol{\theta})d\boldsymbol{z}$ ← intractable

parton showering + hadronization + detector simulation + reconstruction

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← tractable

Weights stored per event

Learning SMEFT likelihood (2)

Mean squared error (MSE) loss functions can be used to regress on joint LLR

Brehmer, Cranmer, Louppe, Pavez (2018)
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MadMiner

$$L[\hat{F}] = \int dx dz p(x, z|\theta_0)|F(x, z) - \hat{F}(x)|^2$$

G(x)

$$\frac{\delta}{\delta \hat{F}(x)} G(x) = 0$$

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Detector-level likelihood ratio!

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← Known from simulation

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← Goal to achieve

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Learning likelihood terms
order-by-order
in SMEFT coefficient

$$R_a(\boldsymbol{x}, \boldsymbol{z}) = \frac{\partial}{\partial \boldsymbol{\theta}_a} \frac{\sigma(\boldsymbol{\theta}) p(\boldsymbol{x}, \boldsymbol{z}|\boldsymbol{\theta})}{\sigma(\boldsymbol{\theta}_0) p(\boldsymbol{x}, \boldsymbol{z}|\boldsymbol{\theta}_0)}\bigg|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}$$

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in SMEFT coefficient

$$R_a(\boldsymbol{x}, \boldsymbol{z}) = \frac{\partial}{\partial \boldsymbol{\theta}_a} \frac{\sigma(\boldsymbol{\theta}) p(\boldsymbol{x}, \boldsymbol{z}|\boldsymbol{\theta})}{\sigma(\boldsymbol{\theta}_0) p(\boldsymbol{x}, \boldsymbol{z}|\boldsymbol{\theta}_0)}\Bigg|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}$$

$$R_{ab}(\boldsymbol{x}, \boldsymbol{z}) = \frac{\partial^2}{\partial \boldsymbol{\theta}_a \partial \boldsymbol{\theta}_b} \frac{\sigma(\boldsymbol{\theta}) p(\boldsymbol{x}, \boldsymbol{z}|\boldsymbol{\theta})}{\sigma(\boldsymbol{\theta}_0) p(\boldsymbol{x}, \boldsymbol{z}|\boldsymbol{\theta}_0)}\Bigg|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}$$

$$\hat{R}_a(x) = \frac{\int dz p(\boldsymbol{x}, \boldsymbol{z}|\boldsymbol{\theta}_0) R_a(\boldsymbol{x}, \boldsymbol{z})}{p(\boldsymbol{x}|\boldsymbol{\theta}_0)} \rightarrow R_a(\boldsymbol{x})$$

$$\hat{R}_{a,b}(x) = \frac{\int dz p(\boldsymbol{x}, \boldsymbol{z}|\boldsymbol{\theta}_0) R_{a,b}(\boldsymbol{x}, \boldsymbol{z})}{p(\boldsymbol{x}|\boldsymbol{\theta}_0)} \rightarrow R_{ab}(\boldsymbol{x})$$

In simulated events:

$$\sigma(\boldsymbol{\theta}) p(\boldsymbol{x}, \boldsymbol{z}|\boldsymbol{\theta}|\boldsymbol{\theta}) \rightarrow \omega(\boldsymbol{\theta})$$

$$R_a(\boldsymbol{x}_i, \boldsymbol{z}_i) = \frac{\frac{\partial}{\partial \boldsymbol{\theta}_a} \omega_i(\boldsymbol{\theta})\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}}{\omega_i(\boldsymbol{\theta}_0)} = \frac{\omega_{i,a}}{\omega_{i,0}}$$

$$R_{ab}(\boldsymbol{x}_i, \boldsymbol{z}_i) = \frac{\frac{\partial^2}{\partial \boldsymbol{\theta}_a \partial \boldsymbol{\theta}_b} \omega_i(\boldsymbol{\theta})\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}}{\omega_i(\boldsymbol{\theta}_0)} = \frac{\omega_{i,ab}}{\omega_{i,0}}$$

Regression targets

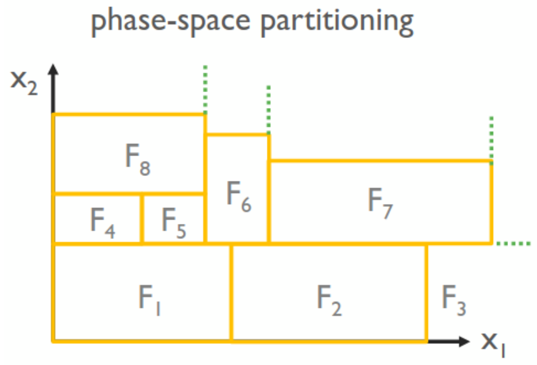
Learning SMEFT likelihood with decision trees (1)

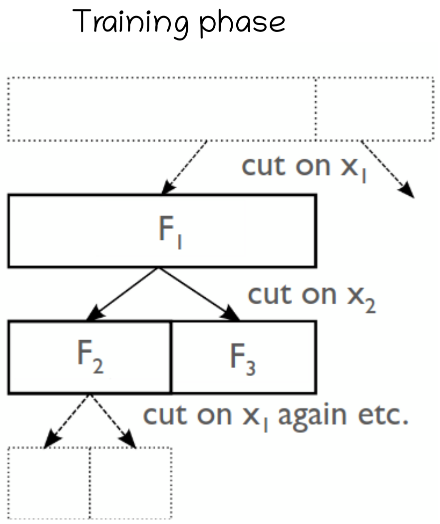
Tree prediction $\hat{F}(\mathbf{x}) = \sum_{j \in \mathcal{J}} \mathbb{1}_{\alpha_j}(\mathbf{x}) F_j$

Phase space partitioning → $\mathbb{1}_{\alpha_j}$; Prediction → F_j

x: Feature vectors
j: Terminal nodes
α_j : Requirements on x for node j
F_j : Prediction for node j

Minimization of loss function w.r.t. α_j and F_j





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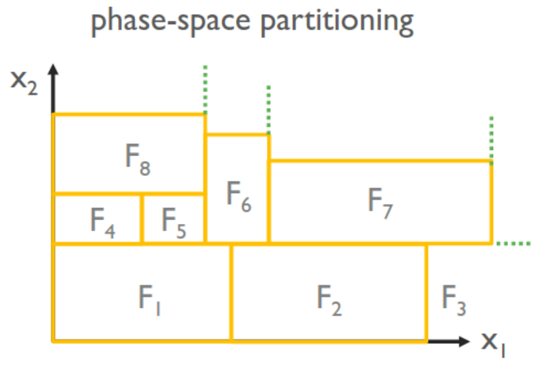

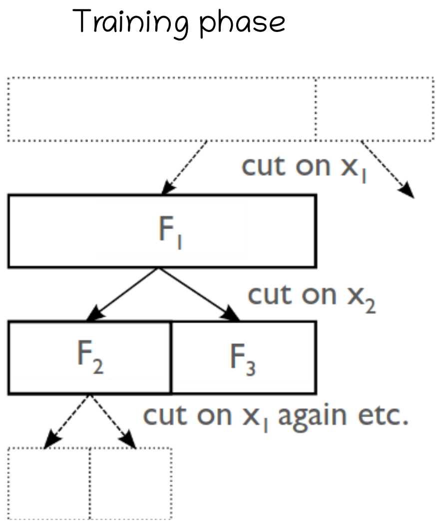

Minimization of loss function w.r.t. α_j and F_j

Linear term in SMEFT expansion

$$\mathrm{MSE}[\hat{F}_a] = \sum_{j\in\mathcal{J}} \sum_{i\in j} w_i \left| \frac{w_{i,a}}{w_i} - F_j \right|^2 = \sum_{i=1}^{N_{\mathrm{sim}}} \frac{w_{i,a}^2}{w_i} - 2\sum_{j\in\mathcal{J}} F_j \sum_{i\in j} w_{i,a} + \sum_{j\in\mathcal{J}} F_j^2 \sum_{i\in j} w_i.$$

Integral replaced by summation

$$\lambda_j = \sum_{i\in j} \omega_i$$

$$\frac{\partial}{\partial F_j}\mathrm{MSE}[\hat{F}_a] = 0 \longrightarrow \quad F_j = \frac{\sum_{i\in j} w_{i,a}}{\sum_{i\in j} w_i} = \left.\frac{\partial_a \lambda_j}{\lambda_j}\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} = \left.\partial_a \log \lambda_j\right|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}$$

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Fisher information for measurement of θ

Fisher information = Variance of score (= derivative of log-likelihood)

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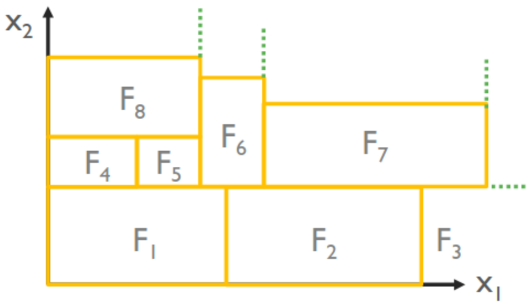
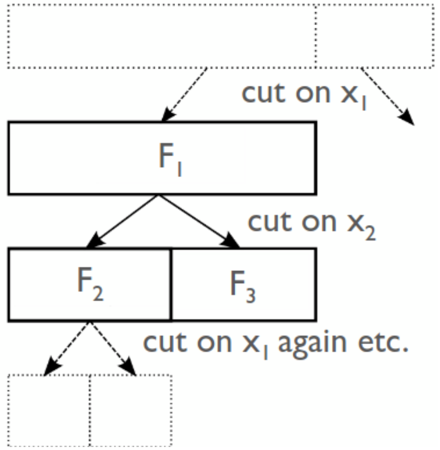
phase-space partitioning



Training phase



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Cramér-Rao bound: $\mathrm{Variance}(\hat{\theta}) \geq \frac{1}{I(\theta)}$ wiki

Node-split criterion maximizes Fisher information → Optimal in precision

Starting point of SC, N. Frohner, L. Lechner, R. Schoefbeck, D. Schwarz (2021)

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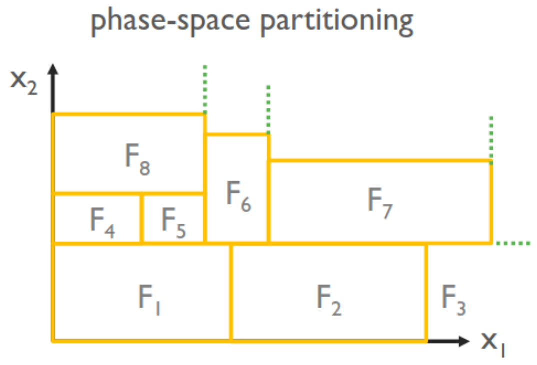

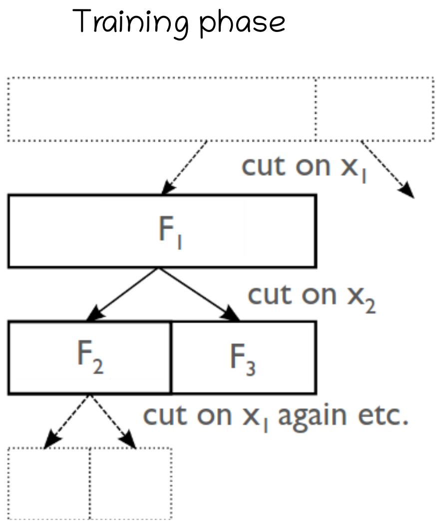

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$$L = -\sum_j \frac{(\partial_\theta\lambda_j)^2}{\lambda_j} = -\sum_j \frac{s_j^2}{b_j + \theta s_j} = -\frac{1}{\theta^2}\sum_j \lambda_j\rho^2 \equiv \sum_j \lambda_j \rho(1-\rho)$$

Gini index implemented in TMVA for classification

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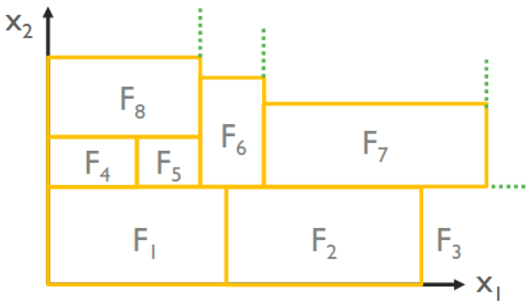

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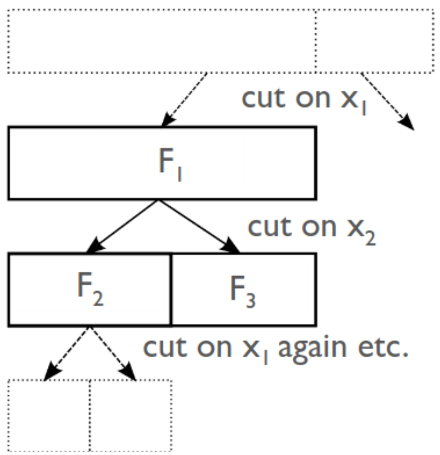

Linear term in SMEFT expansion

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Fisher information for measurement of θ

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Quadratic term in SMEFT expansion

$$F_j = \frac{\sum_{i \in j} w_{i,ab}}{\sum_{i \in j} w_i} = \frac{\partial_a \partial_b \lambda_j}{\lambda_j}\bigg|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}.$$

$$\mathrm{MSE}[\hat{F}_{ab}] = -\sum_{j \in \mathcal{J}} \frac{\left(\sum_{i \in j} w_{i,ab}\right)^2}{\sum_{i \in j} w_i} = -\sum_{j \in \mathcal{J}} \frac{(\partial_a \partial_b \lambda_j)^2}{\lambda_j}\bigg|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}$$

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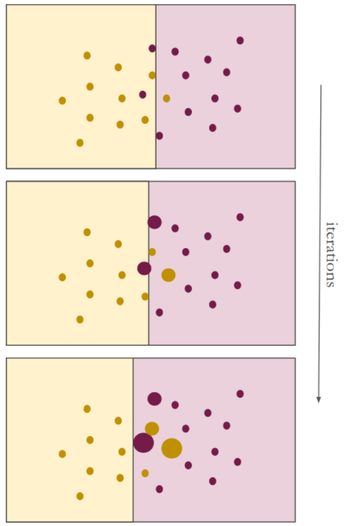
Learning SMEFT likelihood with decision trees (2)

Boosting: Provides a strong learner by iteratively training an ensemble of weak learners to pseudo-residuals of previous iteration

$$\hat{F}^b(x) = \hat{f}^b(x) + \eta \hat{F}^{b-1}(x)$$

Minimize loss function loss w.r.t. *f(x)*

← Goes on till a pre-defined number B



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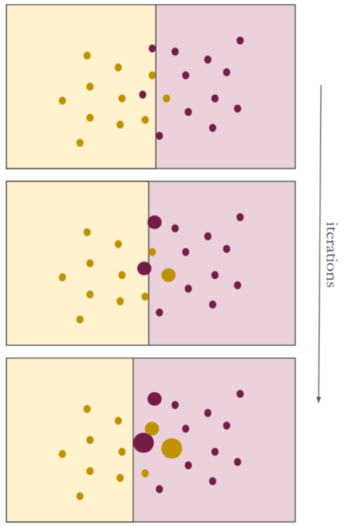
Weak learner needs to fit w - ηF ← Target needs to be updated in each iteration



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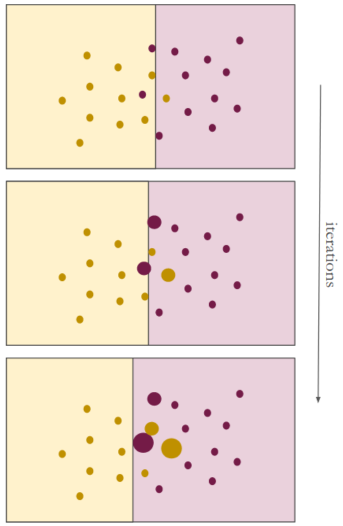

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Final outcome of algorithm $$\hat{R}(\boldsymbol{x}|\boldsymbol{\theta}, \boldsymbol{\theta}_0) = 1 + (\theta - \theta_0)_a \hat{F}_a^{(B)}(\boldsymbol{x}) + \frac{1}{2}(\theta - \theta_0)_a (\theta - \theta_0)_b \hat{F}_{ab}^{(B)}(\boldsymbol{x})$$ Boosted information tree (BIT)

SC, S. Roshap, R. Schoefbeck, D. Schwarz (2022)

Learning SMEFT likelihood with decision trees (2)

HEPHY INSTITUT FÜR HOCHENERGIEPHYSIK

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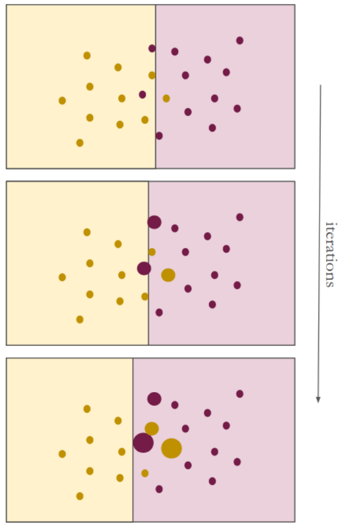

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Final outcome of algorithm

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Boosted information tree (BIT)

SC, S. Roshap, R. Schoefbeck, D. Schwarz (2022)

Separate training for each linear (‘a’) & quadratic terms (‘ab’) → Total # of trainings = n + n(n+1)/2

LLR to achieve

$$q(\mathcal{D}(N_{\rm obs},\boldsymbol{x})|\boldsymbol{\theta}_1,\boldsymbol{\theta}_0) = -2\left[\mathcal{L}(\sigma(\boldsymbol{\theta}_1)-\sigma(\boldsymbol{\theta}_0)) - \sum_{i=1}^{N_{\rm obs}} \ln R(\boldsymbol{x}|\boldsymbol{\theta}_1,\boldsymbol{\theta}_0)\right]$$

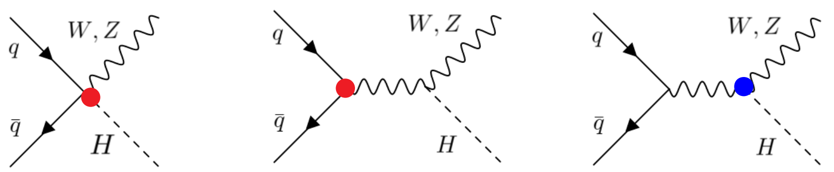
= (in large sample limit)

LLR obtained

$$\hat{q}(\mathcal{D}(N_{\rm obs},\boldsymbol{x})|\boldsymbol{\theta}_1,\boldsymbol{\theta}_0) = -2\left[\mathcal{L}(\sigma(\boldsymbol{\theta}_1)-\sigma(\boldsymbol{\theta}_0)) - \sum_{i=1}^{N_{\rm obs}} \ln \hat{R}(\boldsymbol{x}|\boldsymbol{\theta}_1,\boldsymbol{\theta}_0)\right]$$

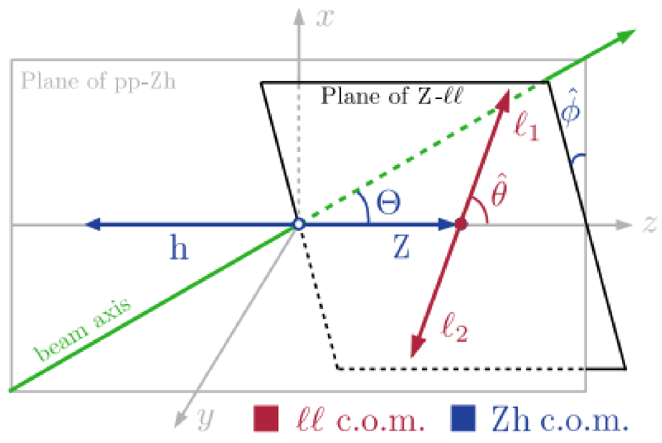
Application of algorithm (I)

Z(→ $l^+ l^-$) H (→ bb) production



3 SMEFT operators

$$\mathcal{O}_{Hq}^{(3)} = iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H \bar{q}\sigma^a \gamma^\mu q$$
$$\mathcal{O}_{HW} = (H^\dagger H) W_{\mu\nu} W^{\mu\nu}$$
$$\mathcal{O}_{H\widetilde{W}} = (H^\dagger H) W^a_{\mu\nu} \widetilde{W}^{a\mu\nu}$$



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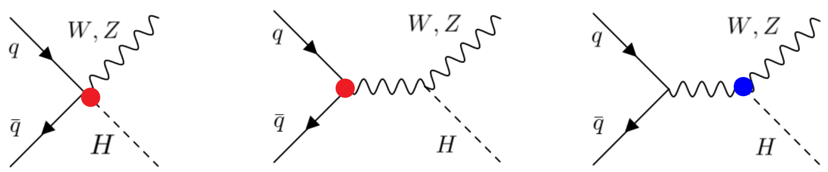

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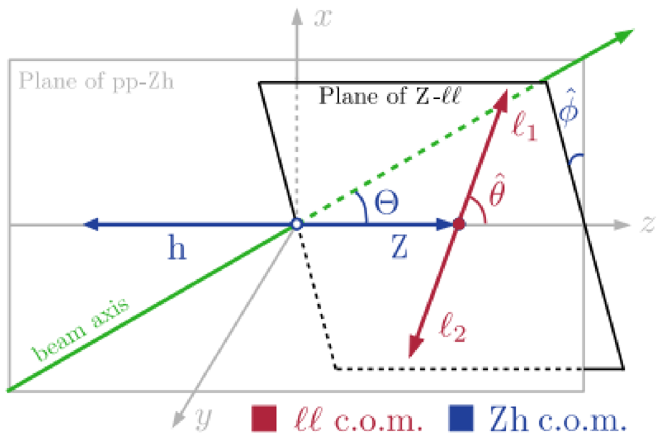

Analytic calculation for EFT-dependence available

Nakamura (2017)

Banerjee, Gupta, Reiness, Seth, Spannowsky (2019)

Thanks to S. Banerjee & R. S. Gupta for providing translation between refs

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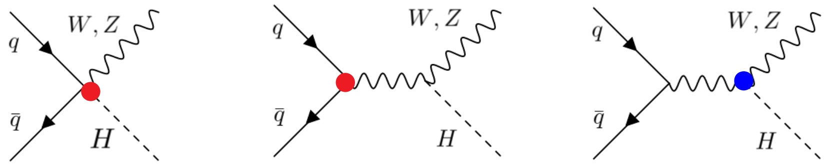
$Z(\to l^+ l^-)\,H(\to bb)$ production



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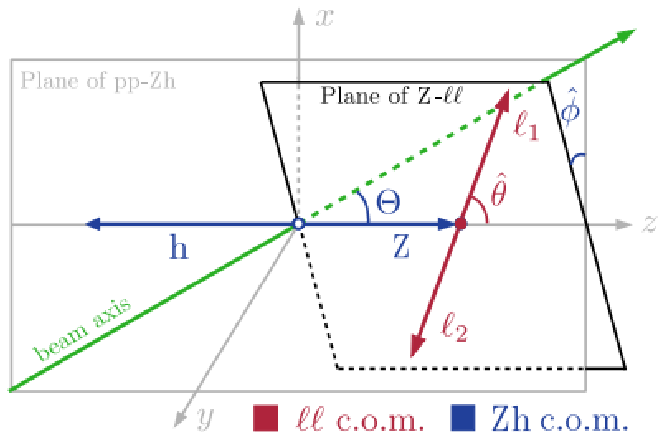

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$$\mathcal{M}^{\lambda=\pm}_\sigma(q\bar{q}) = \sigma\frac{1+\sigma\lambda\cos\Theta}{\sqrt{2}}\hat{\mathcal{M}}^{\lambda=\pm}_\sigma,$$
$$\mathcal{M}^{\lambda=0}_\sigma(q\bar{q}) = \sin\Theta\,\hat{\mathcal{M}}^{\lambda=0}_\sigma,$$

$$\hat{\mathcal{M}}^{\lambda=\pm}_\sigma = g_Z m_Z\sqrt{\hat{s}}\Bigg[\frac{g_{Z\sigma}}{\hat{s}-m_Z^2} + c_{\theta_W}\left(1+\frac{\hat{s}-m_h^2}{m_Z^2}\right)\left(\frac{g_{Z\sigma}c_{\theta_W}}{\hat{s}-m_Z^2}+\frac{Q_q e s_{\theta_W}}{\hat{s}}\right)\frac{v^2}{\Lambda^2}C_{\rm HW}$$
$$-\frac{2i\lambda k\sqrt{\hat{s}}}{m_Z^2}c_{\theta_W}\left(\frac{g_{Z\sigma}c_{\theta_W}}{\hat{s}-m_Z^2}+\frac{Q_q e s_{\theta_W}}{\hat{s}}\right)\frac{v^2}{\Lambda^2}C_{\rm H\widetilde{W}}\Bigg] + g_Z^2\frac{\sqrt{\hat{s}}}{m_Z}T_q^{(3)}\frac{v^2}{\Lambda^2}C_{{\rm HQ}^{(3)}},$$

$$\hat{\mathcal{M}}^{\lambda=0}_\sigma = -g_Z w\sqrt{\hat{s}}\Bigg[\frac{g_{Z\sigma}}{\hat{s}-m_Z^2}$$
$$+c_{\theta_W}\left(1+\frac{\hat{s}-m_h^2}{m_Z^2}-\frac{2k^2\sqrt{\hat{s}}}{m_Z^2 w}\right)\left(\frac{g_{Z\sigma}c_{\theta_W}}{\hat{s}-m_Z^2}+\frac{Q_q e s_{\theta_W}}{\hat{s}}\right)\frac{v^2}{\Lambda^2}C_{\rm HW}\Bigg]$$
$$-g_Z^2 T_q^{(3)}\frac{w\sqrt{\hat{s}}}{m_Z^2}\frac{v^2}{\Lambda^2}C_{{\rm HQ}^{(3)}}.$$

$$\mathcal{A}(\hat{s},\Theta,\theta,\varphi) = \frac{-ig^V_\ell}{\Gamma_V}\sum_\lambda \mathcal{M}^\lambda_\sigma(\hat{s},\Theta)d^{J=1}_{\lambda,1}(\theta)e^{i\lambda\hat{\varphi}},$$

$$\sum_\tau|\mathcal{A}(\hat{s},\Theta,\theta,\varphi)|^2 = \sum_i a_i(\hat{s})f_i(\Theta,\theta,\varphi),$$

$$k = w \to \frac{\sqrt{s}}{2}$$

$$\begin{aligned}
f_{LL} &= S^2_\Theta S^2_\theta,\\
f^1_{TT} &= C_\Theta C_\theta,\\
f^2_{TT} &= (1+C^2_\Theta)(1+C^2_\theta),\\
f^1_{LT} &= C_\varphi S_\Theta S_\theta,\\
f^2_{LT} &= C_\varphi S_\Theta S_\theta C_\Theta C_\theta,\\
\tilde{f}^1_{LT} &= S_\varphi S_\Theta S_\theta,\\
\tilde{f}^2_{LT} &= S_\varphi S_\Theta S_\theta C_\Theta C_\theta,\\
f_{TT'} &= C_{2\varphi}S^2_\Theta S^2_\theta,\\
\tilde{f}_{TT'} &= S_{2\varphi}S^2_\Theta S^2_\theta,
\end{aligned}$$

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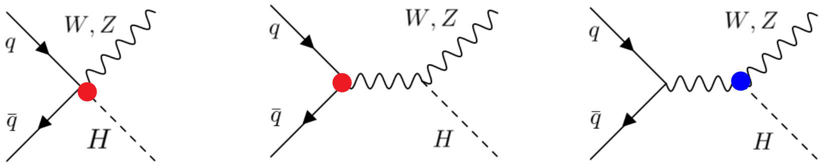
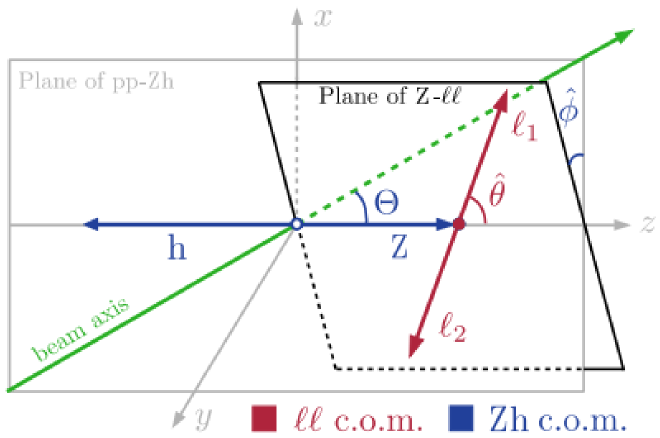
Z(→ l⁺ l⁻) H (→ bb) production



3 SMEFT operators

$$\mathcal{O}_{Hq}^{(3)} = iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H \bar{q}\sigma^a \gamma^\mu q$$
$$\mathcal{O}_{HW} = (H^\dagger H)\, W_{\mu\nu} W^{\mu\nu}$$
$$\mathcal{O}_{H\widetilde{W}} = (H^\dagger H)\, W^a_{\mu\nu} \widetilde{W}^{a\mu\nu}$$



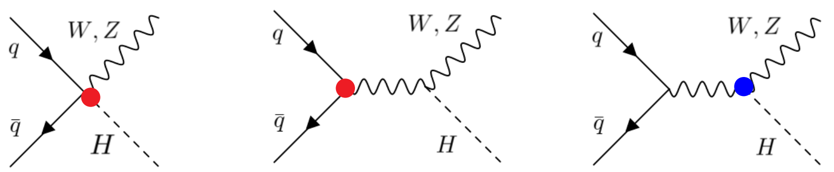
$$\frac{d\sigma^{\mathrm{Zh}}}{d\hat{s}\, dy\, d\cos\theta\, d\cos\hat{\theta}\, d\hat{\phi}} = \frac{m_Z k}{12288\pi^3 \Gamma_Z s \hat{s}^{3/2}} \times$$
$$\sum_f \sum_\tau |g^\tau_{Z\ell\bar{\ell}}|^2 \sum_{q=u,d,c,s,b} \left(q(x_1)\bar{q}(x_2) d^{\tau\dagger} \sum_\sigma \rho_\sigma(q\bar{q}) d^\tau + \bar{q}(x_1) q(x_2) d^{\tau\dagger} \sum_\sigma \rho_\sigma(\bar{q}q) d^\tau \right)$$

← Expression for toy simulation

Application of algorithm (2)

$Z(\to l^+ l^-)\,H(\to bb)$ production

3 SMEFT operators



$$\mathcal{O}^{(3)}_{Hq} = iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H \bar{q}\sigma^a\gamma^\mu q$$
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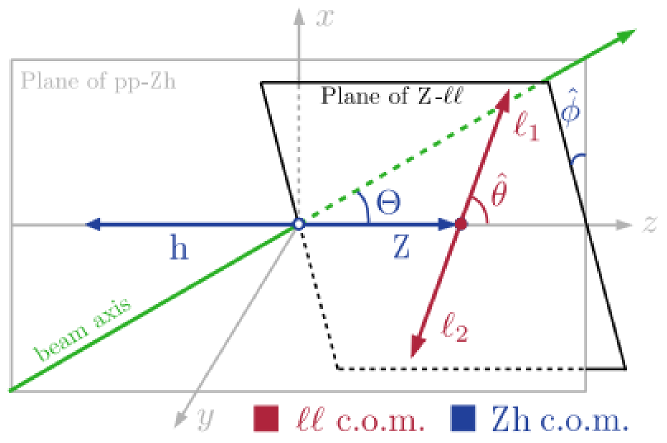

$$\frac{d\sigma^{Zh}}{d\hat{s}\,dy\,d\cos\theta\,d\cos\hat\theta\,d\hat\phi} = \frac{m_Z k}{12288\pi^3\Gamma_Z s\hat{s}^{3/2}} \times$$
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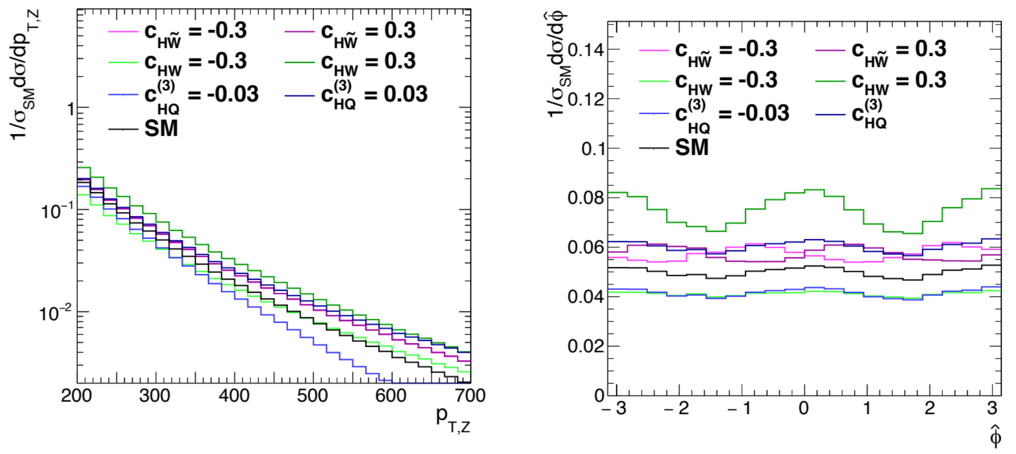
← Expression for toy simulation



EFT effects on kinematic distributions

Application of algorithm (2)

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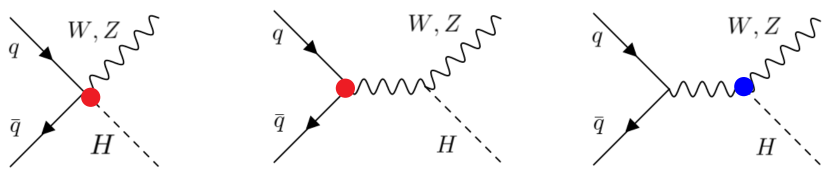
Z(→ l⁺ l⁻) H (→ bb) production



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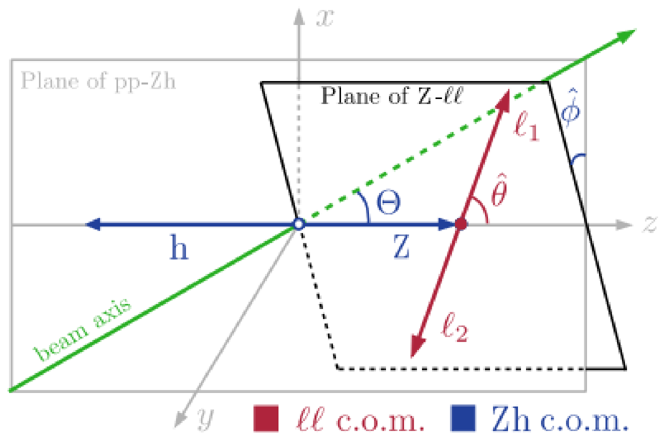
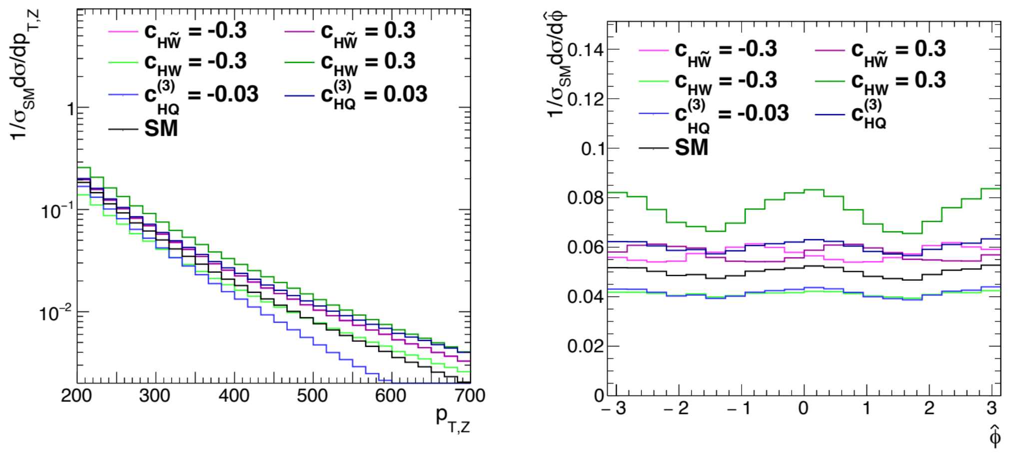


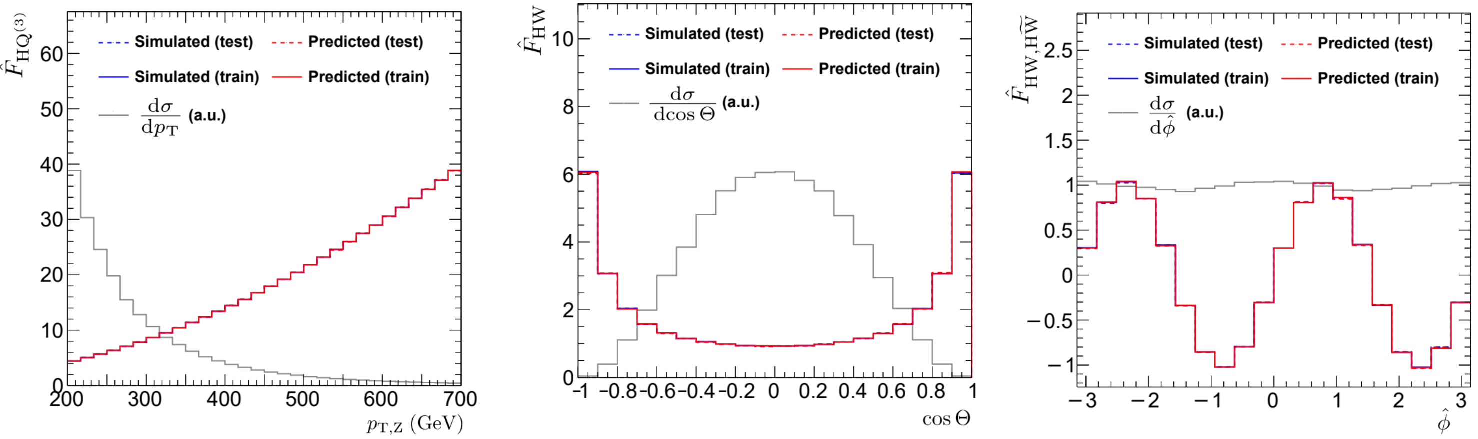
$$\frac{d\sigma^{Zh}}{d\hat{s}\, dy\, d\cos\theta\, d\cos\hat{\theta}\, d\hat{\phi}} = \frac{m_Z k}{12288\pi^3 \Gamma_Z s \hat{s}^{3/2}} \times$$

$$\sum_f \sum_\tau |g^\tau_{Z\ell\bar{\ell}}|^2 \sum_{q=u,d,c,s,b} \left(q(x_1)\bar{q}(x_2) d^{\tau\dagger} \sum_\sigma \rho_\sigma(q\bar{q}) d^\tau + \bar{q}(x_1) q(x_2) d^{\tau\dagger} \sum_\sigma \rho_\sigma(\bar{q}q) d^\tau \right)$$

← Expression for toy simulation



EFT effects on kinematic distributions



Coefficients are perfectly learned in toy simulation

Optimality in toy data

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Quantification of hypothesis testing $\bar{p}(t_{\boldsymbol{\theta}}, \boldsymbol{\theta}) = \int_{t_{\boldsymbol{\theta}}}^{\infty} \mathrm{d}t'_{\boldsymbol{\theta}}\, p(t'_{\boldsymbol{\theta}}|\boldsymbol{\theta})$ — 95% CL interval → $\bar{p}(t_{\mathrm{med}}(\boldsymbol{\theta}_{95\%}), \boldsymbol{\theta}_{95\%}) = 0.05$ $\quad t_{\mathrm{med}}(\boldsymbol{\theta}) = \mathrm{Median}(t_{\boldsymbol{\theta}}|\boldsymbol{\theta}_0)$

Type-2 error: $\beta = p(\bar{p} > 0.05|\boldsymbol{\theta}_0)$

Power = 1 - β

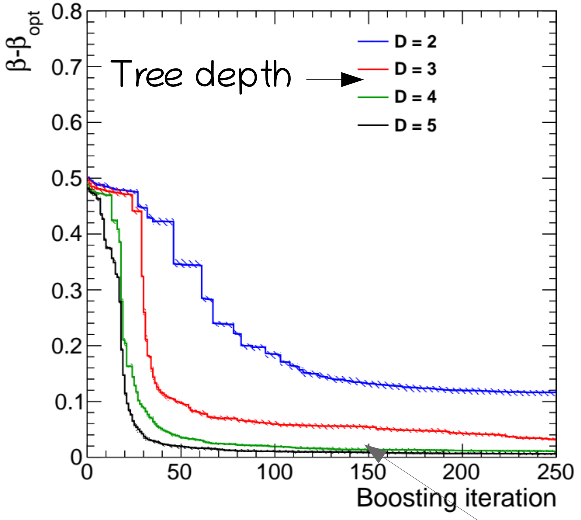
Type I and Type II Error		
Null hypothesis is ...	True	False
Rejected	Type I error False positive Probability = α	Correct decision True positive Probability = 1 - β
Not rejected	Correct decision True negative Probability = 1 - α	Type II error False negative Probability = β

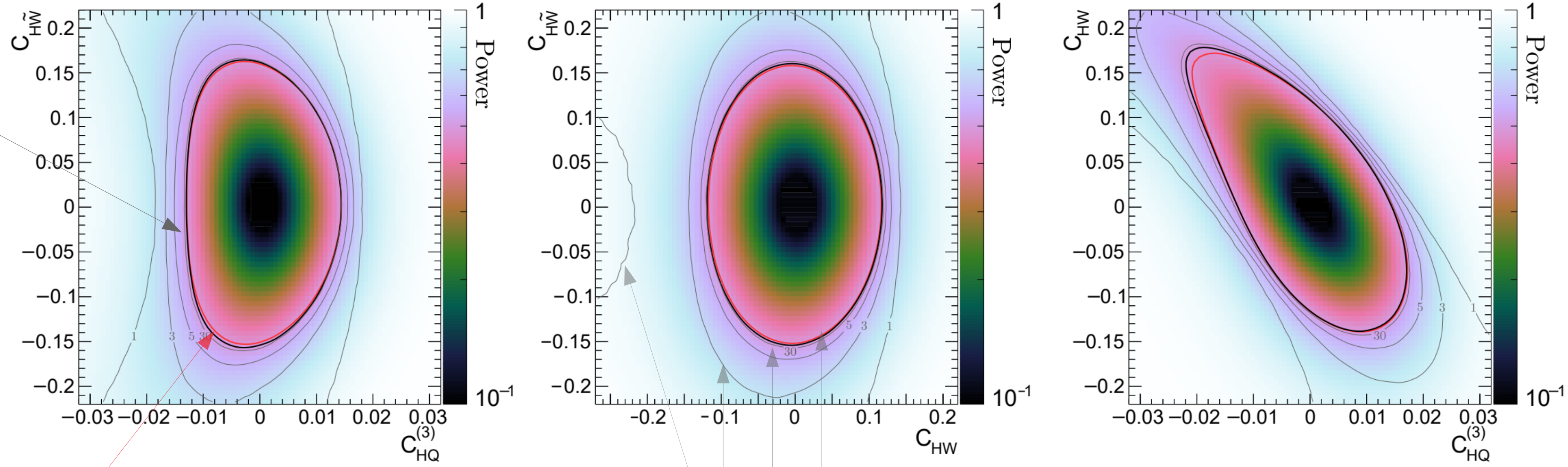
Scribbr

<u>Neyman-Pearson lemma:</u> Likelihood ratio test statistic gives maximum power (given a type-1 error)

BIT output $\quad \hat{R}(\boldsymbol{x}|\boldsymbol{\theta}, \boldsymbol{\theta}_0) = 1 + (\theta - \theta_0)_a \hat{F}_a^{(B)}(\boldsymbol{x}) + \frac{1}{2}(\theta - \theta_0)_a(\theta - \theta_0)_b \hat{F}_{ab}^{(B)}(\boldsymbol{x})$

Binned test statistic $\quad q_{\boldsymbol{\theta},\mathrm{binned}}(\mathcal{D}) = \sum_{j=1}^{M}\left(\lambda_j(\boldsymbol{\theta}) - \lambda_j(\boldsymbol{\theta}_0) - n_j \log \frac{\lambda_j(\boldsymbol{\theta})}{\lambda_j(\boldsymbol{\theta}_0)}\right)$ $\quad$ Distribution of R in 'M' bins





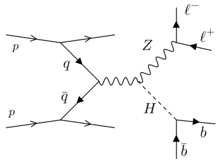
BIT achieves theoretically optimum constraints

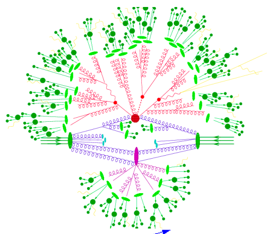
With sufficient # of bins, binned constraints converge to unbinned ones

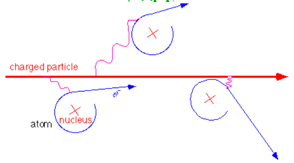
Performance in (real) simulation

SMEFTsim + Madgraph @LO → PYTHIA8 → DELPHES

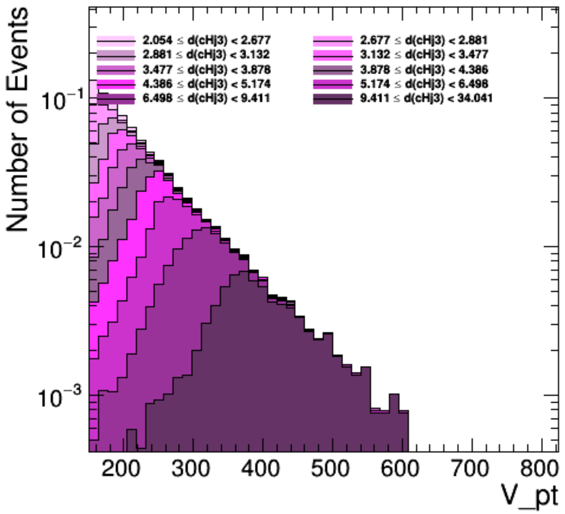
$\mathcal{O}^{(3)}_{Hq}, \mathcal{O}_{HW}, \mathcal{O}_{H\widetilde{W}}$

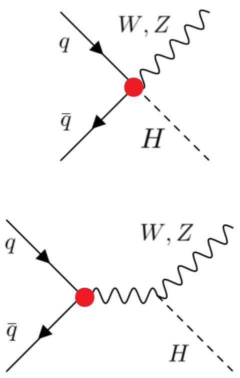


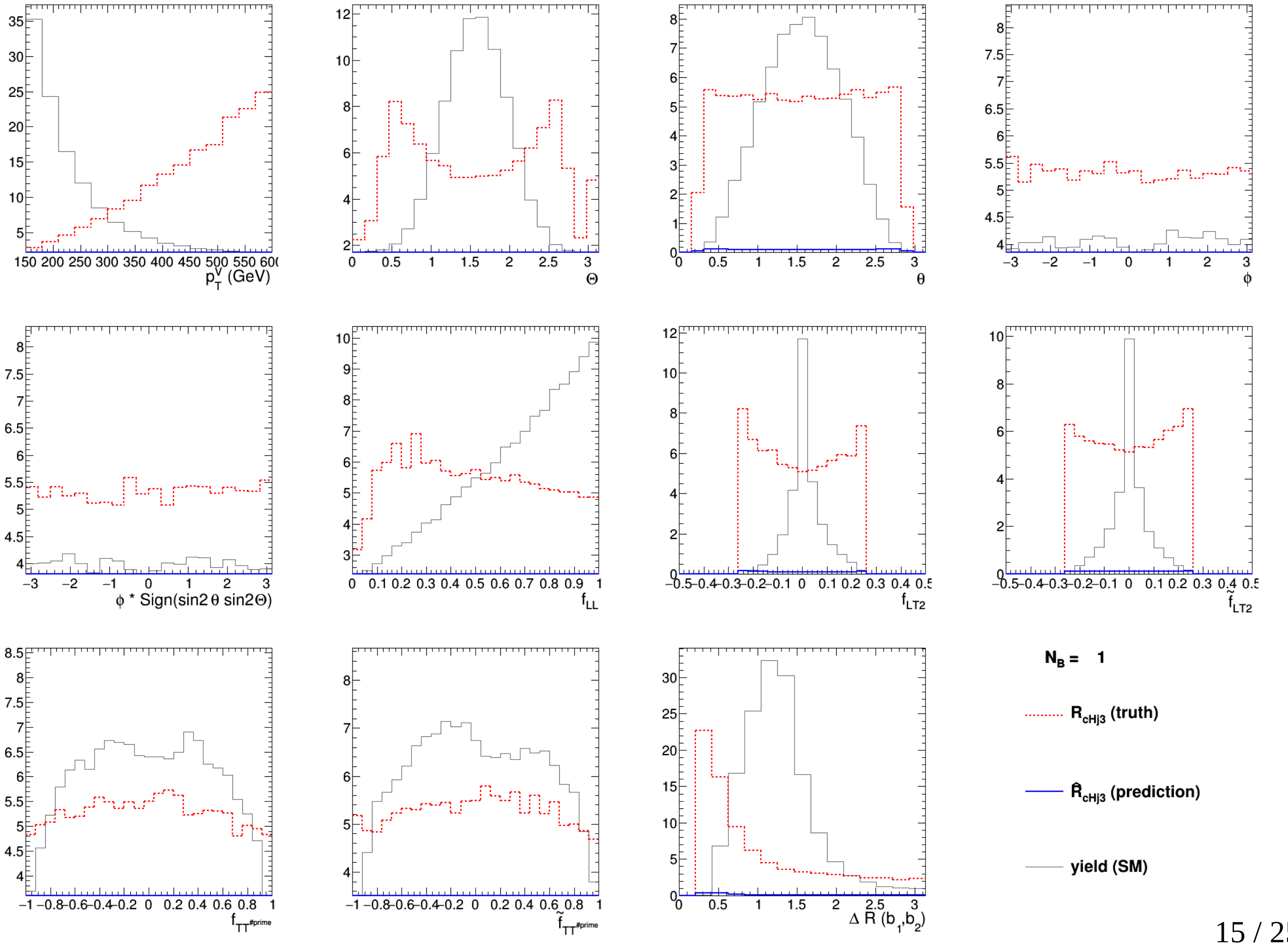




Learning linear term for $c_{Hq(3)}$







Performance in (real) simulation

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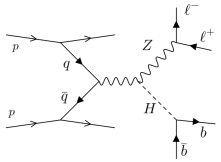
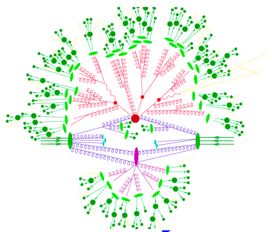
SMEFTsim $\mathcal{O}_{Hq}^{(3)}, \mathcal{O}_{HW}, \mathcal{O}_{H\widetilde{W}}$

+

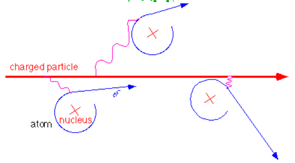
Madgraph @LO



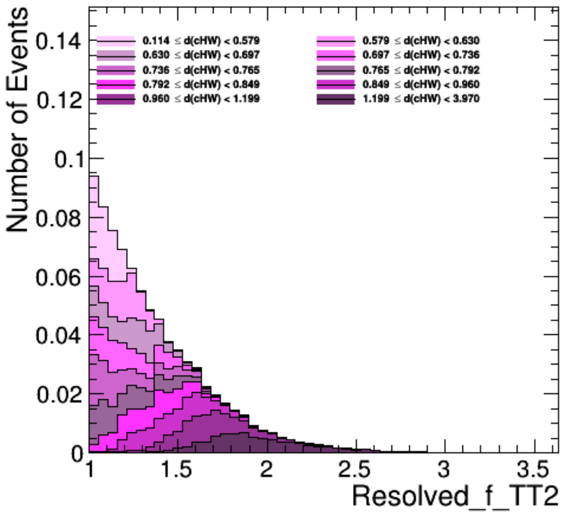
PYTHIA8

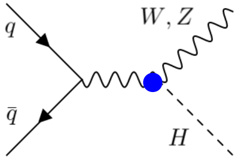


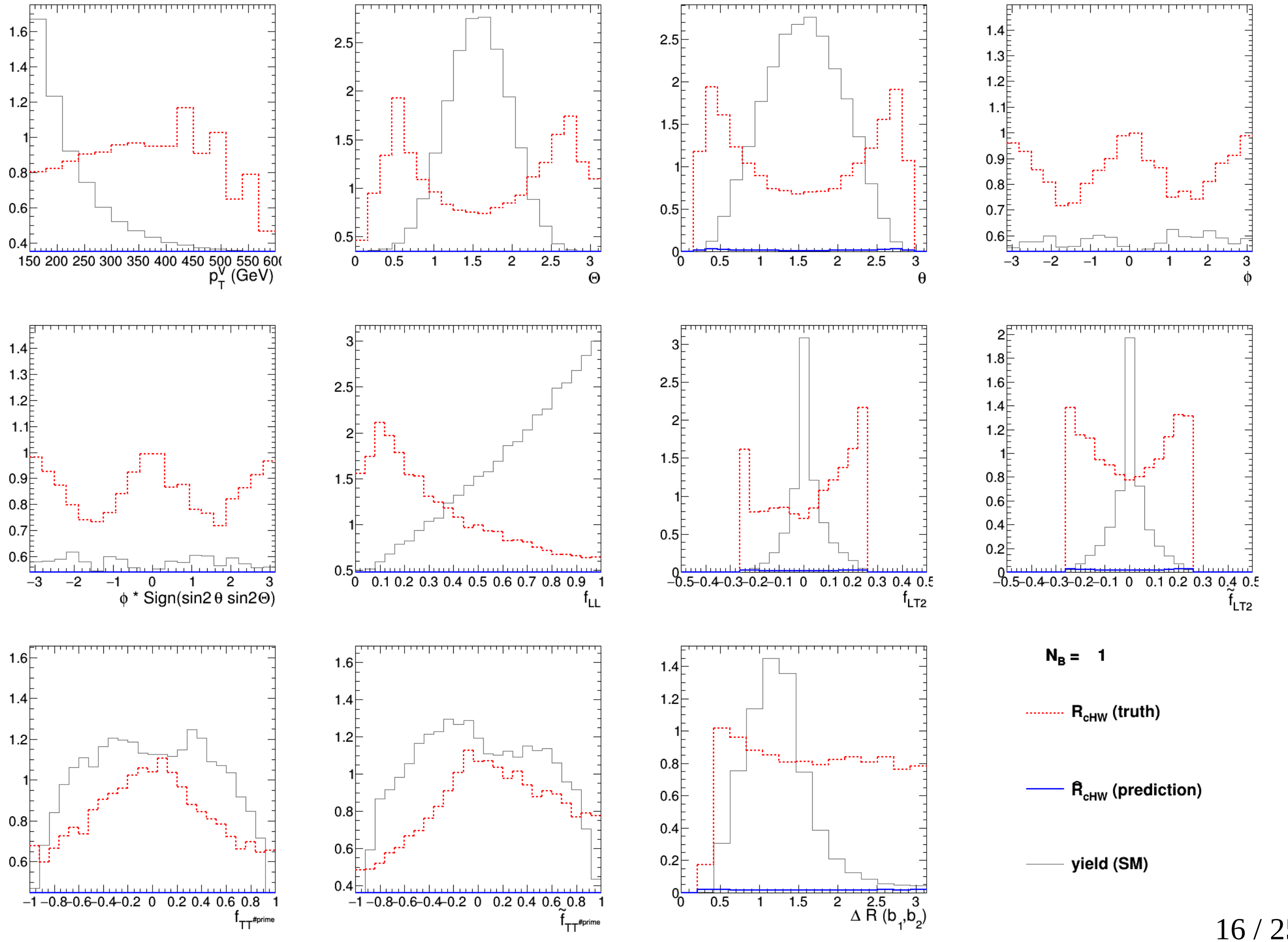
DELPHES



Learning linear term for c_{Hw}







Performance in (real) simulation

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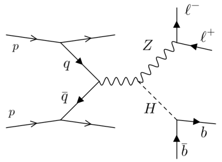
SMEFTsim

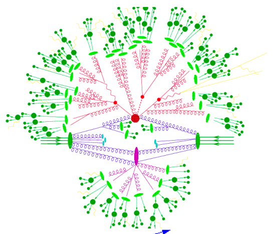
$\mathcal{O}_{Hq}^{(3)}, \mathcal{O}_{HW}, \mathcal{O}_{H\widetilde{W}}$

+

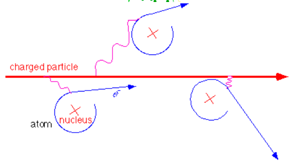
Madgraph @LO



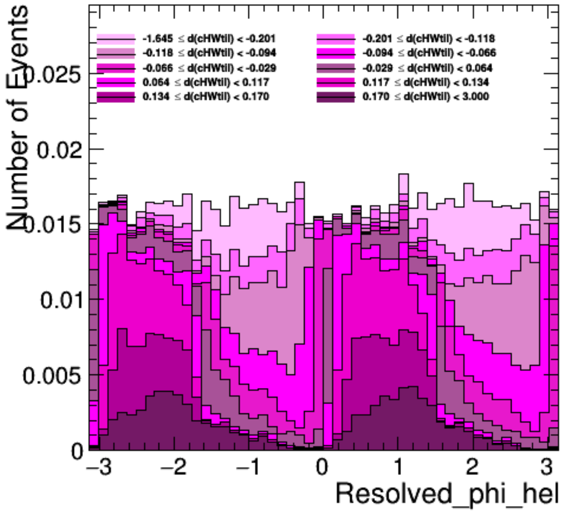
PYTHIA8

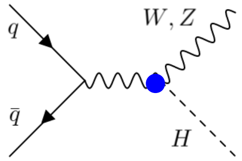


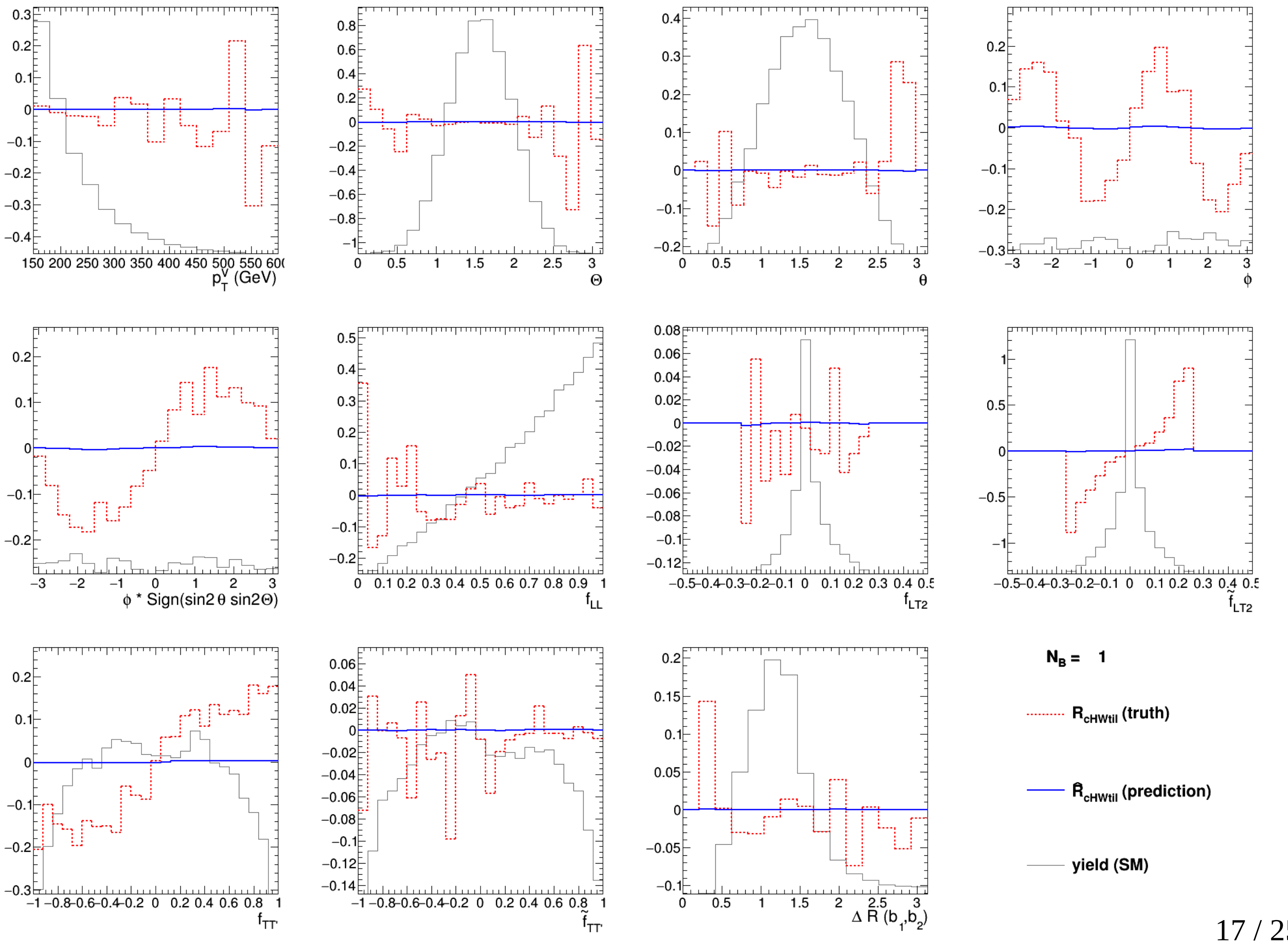
DELPHES



Learning linear term for $c_{H\widetilde{W}}$







Performance in (real) simulation

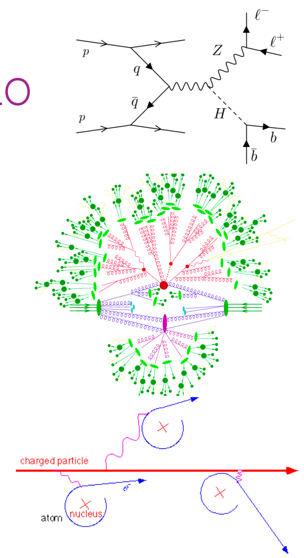
SMEFTsim $\mathcal{O}_{Hq}^{(3)}, \mathcal{O}_{HW}, \mathcal{O}_{H\widetilde{W}}$

+

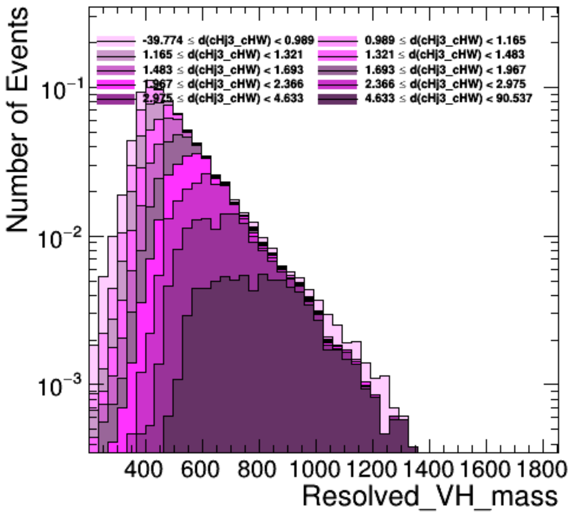
Madgraph @LO

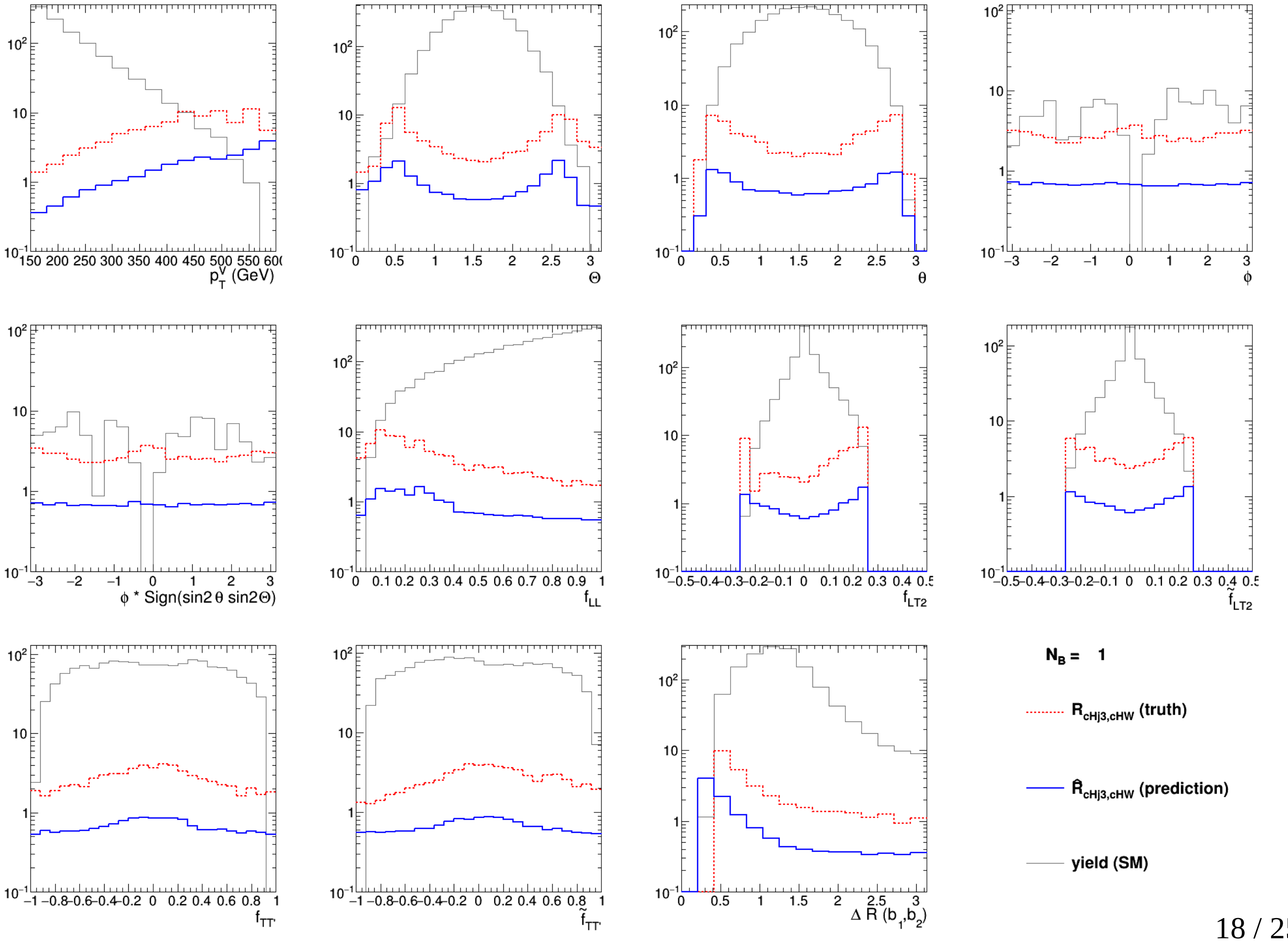
PYTHIA8

DELPHES



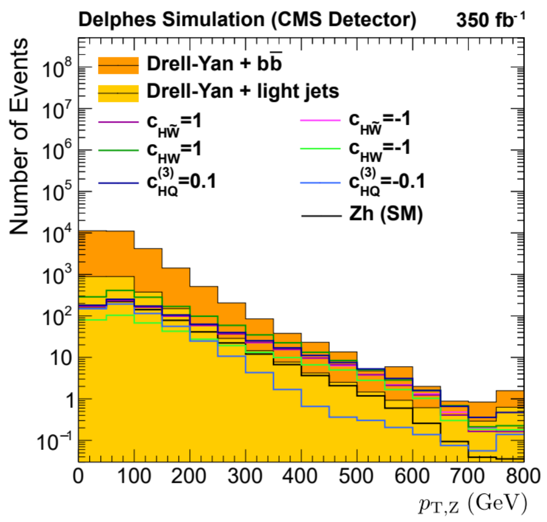
Learning x-term $c_{Hq(3)} c_{Hw}$

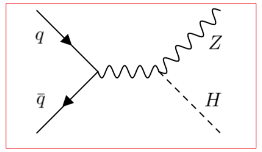




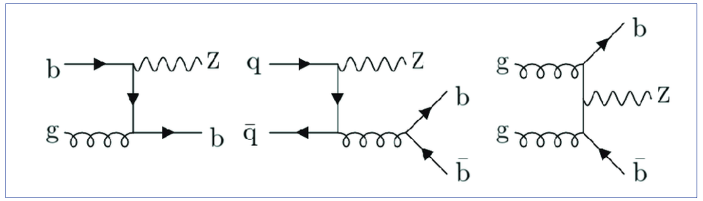
Performance in (real) simulation of signal + background

ZH SMEFT signal + Drell-Yan background





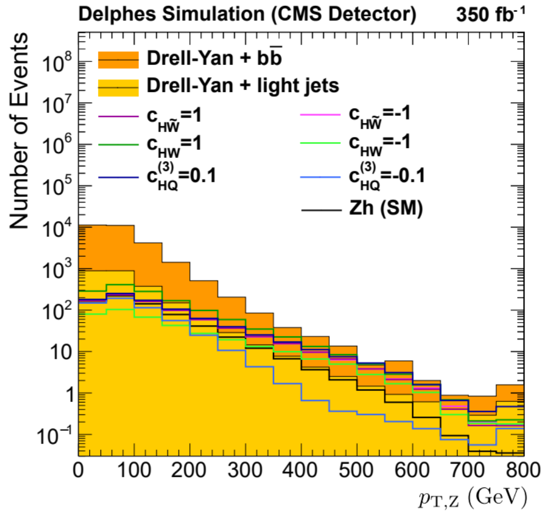
Signal



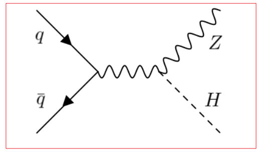
Background

Performance in (real) simulation of signal + background

ZH SMEFT signal + Drell-Yan background



Observable	Description
H_T	$\sum_{\text{jets}} p_T$
N_{jet}	Jet multiplicity
$p_T(j_1)$, $p_T(j_2)$, $p_T(j_3)$	p_T of the three highest p_T jets
$\|\eta(j_1)\|$, $\|\eta(j_2)\|$, $\|\eta(j_3)\|$	$\|\eta\|$ of the three highest p_T jets
$p_T(h)$, $\|\eta(h)\|$	p_T and $\|\eta\|$ from h candidate
$p_T(Z)$, $\|\eta(Z)\|$	p_T and $\|\eta\|$ from Z candidate
Θ, $\hat{\theta}$, $\hat{\phi}$	See Sec. 4.1 and Fig. 1
$f_{LL}, \ldots, \tilde{f}_{TT'}$	See Sec. 4.1 and Ref. [30]
$p_T(\ell_2)/p_T(\ell_1)$	Ratio of lepton p_T
$\Delta\phi(\ell_1,\ell_2)$, $\|\Delta\eta(\ell_1,\ell_2)\|$	Azimuthal and η difference of ℓ_1 and ℓ_2
$\Delta\phi(\text{b-jet}_1,\text{b-jet}_2)$, $\|\Delta\eta(\text{b-jet}_1,\text{b-jet}_2)\|$	Azimuthal and η difference of b jets
$m(\text{b-jet}_1,\text{b-jet}_2)$	Higgs candidate mass
$p_T(\text{b-jet}_2)/p_T(\text{b-jet}_1)$	Ratio of transverse b-jet momenta
$\Delta R(Z,h)$, $\|\Delta\eta(Z,h)\|$, $m(Z,h)$	Properties of the Zh system
$\Delta R(\text{non b-jet}, Z)$, $\Delta R(\text{non b-jet}, h)$	ΔR distances to non b-tagged jet
Thrust	See Ref. [51]



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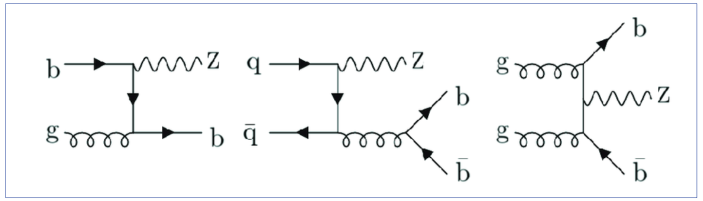

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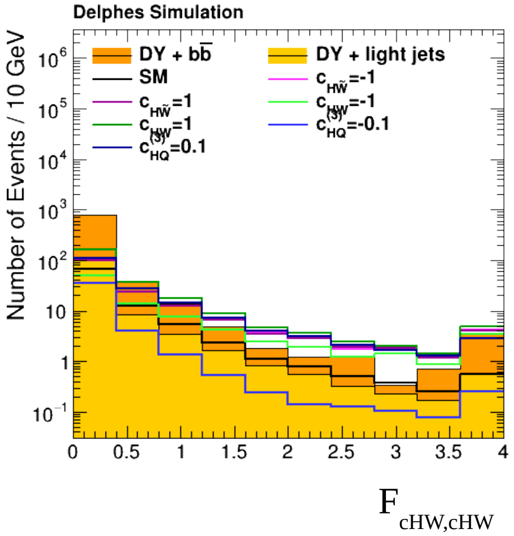
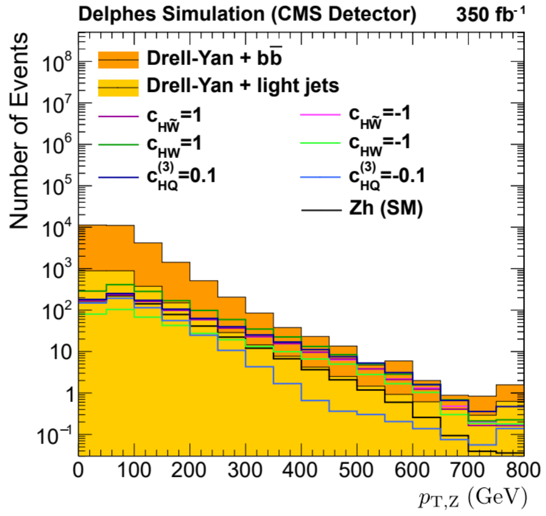


Performance in (real) simulation of signal + background

ZH SMEFT signal + Drell-Yan background



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$\lvert\eta(j_1)\rvert$, $\lvert\eta(j_2)\rvert$, $\lvert\eta(j_3)\rvert$	$\lvert\eta\rvert$ of the three highest p_T jets
$p_T(\text{h})$, $\lvert\eta(\text{h})\rvert$	p_T and $\lvert\eta\rvert$ from h candidate
$p_T(\text{Z})$, $\lvert\eta(\text{Z})\rvert$	p_T and $\lvert\eta\rvert$ from Z candidate
Θ, $\hat{\theta}$, $\hat{\phi}$	See Sec. 4.1 and Fig. 1
$f_{LL}, \ldots, \tilde{f}_{TT'}$	See Sec. 4.1 and Ref. [30]
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$\Delta\phi(\text{b-jet}_1, \text{b-jet}_2)$, $\lvert\Delta\eta(\text{b-jet}_1, \text{b-jet}_2)\rvert$	Azimuthal and η difference of b jets
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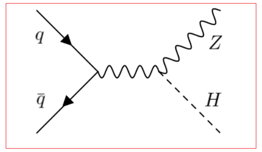
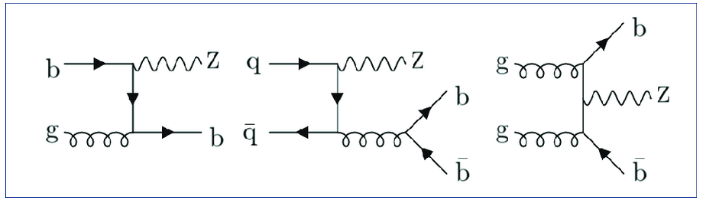
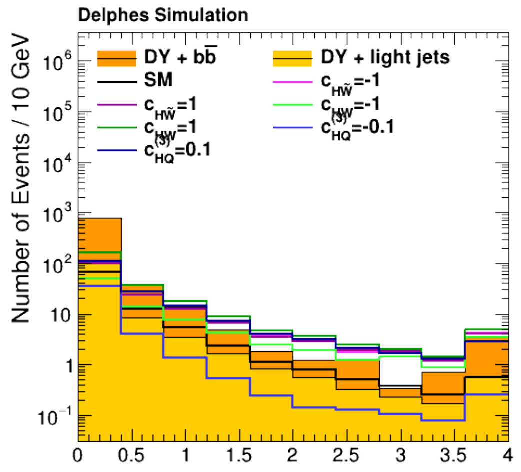
$\Delta R(\text{Z}, \text{h})$, $\lvert\Delta\eta(\text{Z}, \text{h})\rvert$, $m(\text{Z}, \text{h})$	Properties of the Zh system
$\Delta R(\text{non b-jet}, \text{Z})$, $\Delta R(\text{non b-jet}, \text{h})$	ΔR distances to non b-tagged jet
Thrust	See Ref. [51]



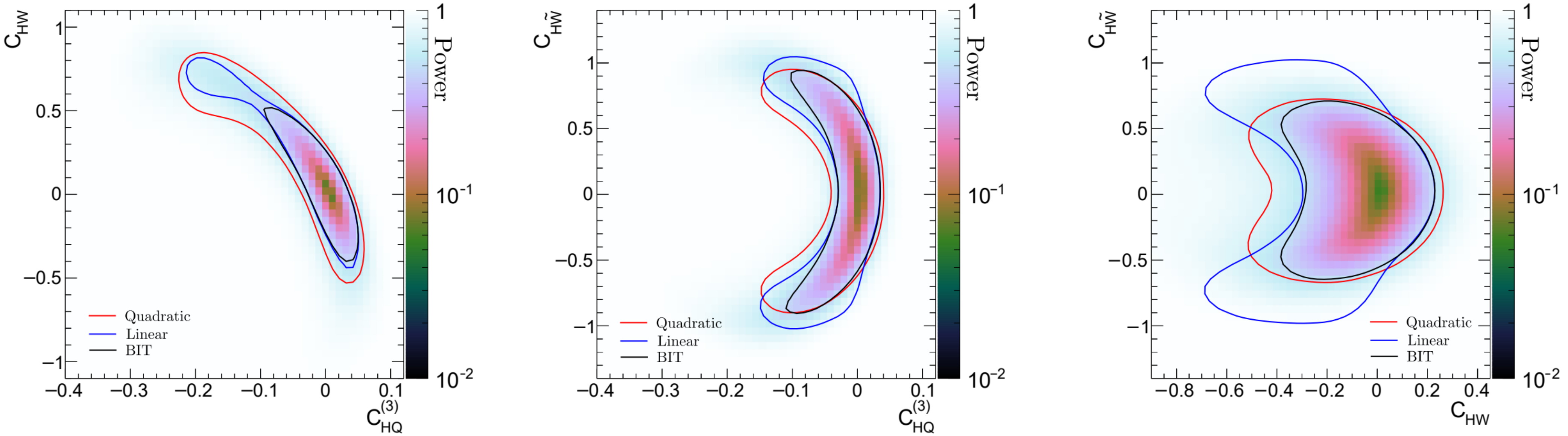
Signal



Background



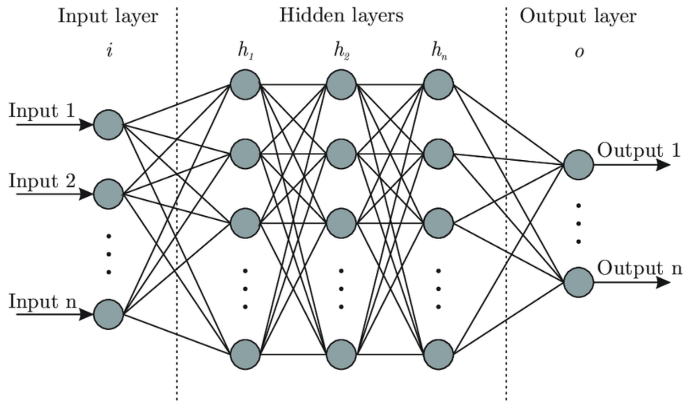
$F_{cHW,cHW}$



Comparison with 'traditional' approach

DNN to separate signal & bkg

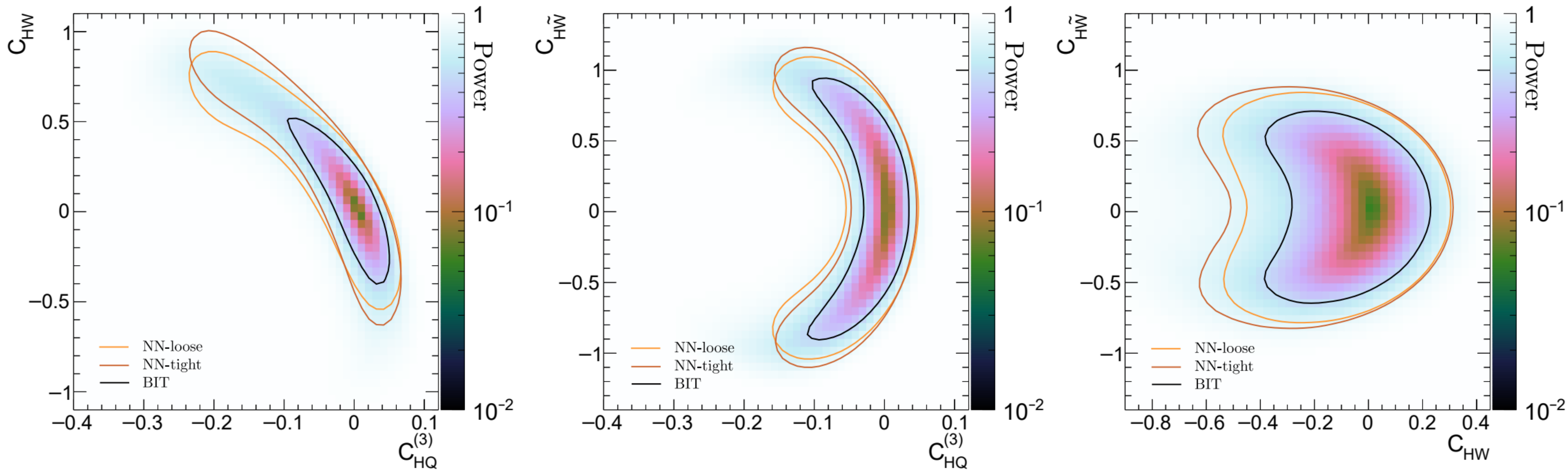
Multilayer perceptron with 2 layers with 100 & 50 nodes



Two thresholds:

Loose: 25% Bkg Eff 77% Sig Eff
Tight: 3.5% Bkg Eff 33% Sig Eff

Same variables used in BIT & DNN



Better constraints with BIT w.r.t. conventional 'reinterpretation' approach

BIT learns signal-background separation & SMEFT dependence for signal

Other works: Parameterized neural networks

Remember the likelihood ratio trick $f^*(\boldsymbol{x}) = \dfrac{p(\boldsymbol{x}|\mathrm{SM})}{p(\boldsymbol{x}|\mathrm{SM}) + p(\boldsymbol{x}|\theta)} = \dfrac{1}{1 + \frac{\sigma(\theta)}{\sigma(\mathrm{SM})} r(\boldsymbol{x})}$

Works with

Mean-squared error loss function $L = \int d\boldsymbol{x} \left(p(\boldsymbol{x}, \theta)(1 - f(\boldsymbol{x}))^2 + p(\boldsymbol{x}, \mathrm{SM})(f(\boldsymbol{x}))^2 \right)$

Cross-entropy loss function $L = \int d\boldsymbol{x} \left(p(\boldsymbol{x}, \theta) \log(1 - f(\boldsymbol{x})) + p(\boldsymbol{x}, \mathrm{SM}) \log(f(\boldsymbol{x})) \right)$

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Works with

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Cross-entropy loss function $L = \int dx \, (p(x,\theta)\log(1-f(x)) + p(x,\mathrm{SM})\log(f(x)))$

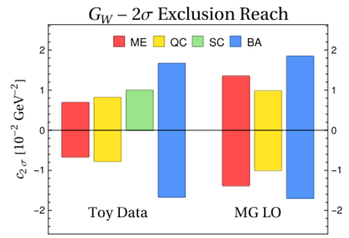
Parameterized classifiers with quadratic ansatz

Chen, Glioti, Panico, Wulzer (2020)

$$f(x,c) \equiv \frac{1}{1 + [1 + c\,\mathrm{n}_\alpha(x)]^2 + [c\,\mathrm{n}_\beta(x)]^2}$$

Minimizes MSE loss (same as BIT)

Applied on W(→ lν) Z (→ ll) sample
[toy simulation, realistic simulation @ particle-level]



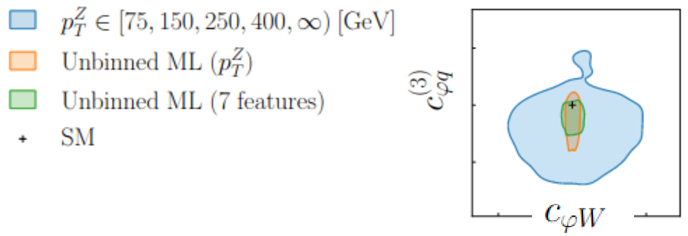
ML4EFT with unbinned likelihood

Ambrosio, Hoeve, Madigan, Rojo, Sanz (2022)

$$\hat{r}_\sigma(\boldsymbol{x}, \boldsymbol{c}) = 1 + \sum_{j=1}^{n_{\mathrm{eft}}} \mathrm{NN}^{(j)}(\boldsymbol{x}) c_j + \sum_{j=1}^{n_{\mathrm{eft}}} \sum_{k \geq j}^{n_{\mathrm{eft}}} \mathrm{NN}^{(j,k)}(\boldsymbol{x}) c_j c_k$$

Minimizes cross-entropy loss

Applied on tt, tt→2l+2ν+2b, Z(→ll)H(->bb) samples
[parton-level]



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Remember the likelihood ratio trick $f^*(\boldsymbol{x}) = \frac{p(\boldsymbol{x}|\mathrm{SM})}{p(\boldsymbol{x}|\mathrm{SM}) + p(\boldsymbol{x}|\theta)} = \frac{1}{1 + \frac{\sigma(\theta)}{\sigma(\mathrm{SM})} r(\boldsymbol{x})}$

Works with

Mean-squared error loss function $L = \int dx \left(p(\boldsymbol{x},\theta)(1 - f(\boldsymbol{x}))^2 + p(\boldsymbol{x},\mathrm{SM})(f(\boldsymbol{x}))^2 \right)$

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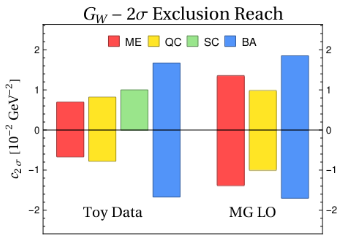

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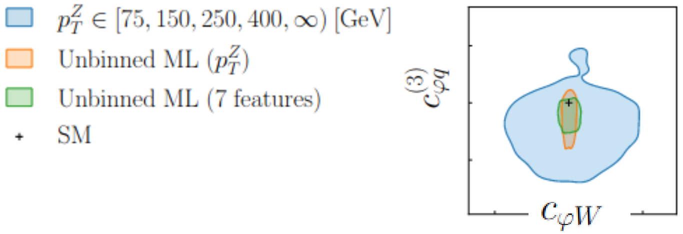
Minimizes cross-entropy loss

Applied on tt, tt→2l+2ν+2b, Z(→ll)H(->bb) samples
[parton-level]



Same philosophy as Madminer & BIT → Regress on joint likelihood ratio

Differences

Different event samples to learn individual linear & quadratic terms

BIT is much more speed-optimized

Summary & Outlook

- Effective field theory analysis coming to center stage of LHC research

- Usage of machine learning being explored in last few years to extract maximum information & probe EFT operators to the finest level

- Boosted decision tree based implementations offer attractive options
  - Simple
  - Fast
  - Needs a single sample (with EFT weights)

- Python-based framework publicly available [link]

- Neural network based strategies also useful

- Yet to see the first results from experiments on real data!

Summary & Outlook

- Effective field theory analysis coming to center stage of LHC research
- Usage of machine learning being explored in last few years to extract maximum information & probe EFT operators to the finest level
- Boosted decision tree based implementations offer attractive options
  - Simple
  - Fast
  - Needs a single sample (with EFT weights)
- Python-based framework publicly available [link]
- Neural network based strategies also useful
- Yet to see the first results from experiments on real data!

Extra Material

SMEFT operator at D=5

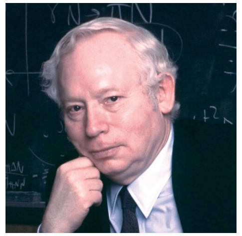
$$\mathcal{L}_5 = -\frac{C_5^{\ell\ell'}}{\Lambda}\left[\bar{L}_\ell^{\,c}\tilde{H}^*\right]\left[\tilde{H}^\dagger L_{\ell'}\right] + h.c.$$

$$L = \begin{pmatrix}\nu_L \\ \ell_L\end{pmatrix} \quad H = \begin{pmatrix}0 \\ v+h\end{pmatrix} \quad \tilde{H} = i\sigma_2 H^*$$

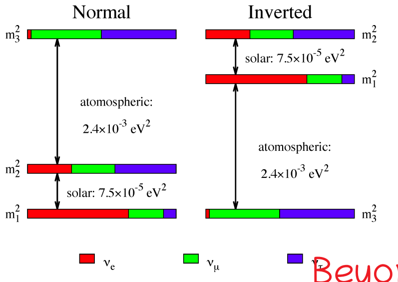
$$\longrightarrow -\frac{C_5^{\ell\ell'}}{2\Lambda}hh\bar{\nu}_\ell^{\,c}\nu_{\ell'} - \frac{C_5^{\ell\ell'}}{\Lambda}vh\bar{\nu}_\ell^{\,c}\nu_{\ell'} - \frac{C_5^{\ell\ell'}}{2\Lambda}v^2\bar{\nu}_\ell^{\,c}\nu_{\ell'} + h.c.$$

Majorana mass of neutrino

ΔL = 2 ← Lepton number violation



Phys. Rev. Lett. 43, 1566 (1979)



ν mass: 0.1-0.01 eV

→ $\Lambda > 10^{15}$ GeV!!

Beyond collider phenomenology

Solutions

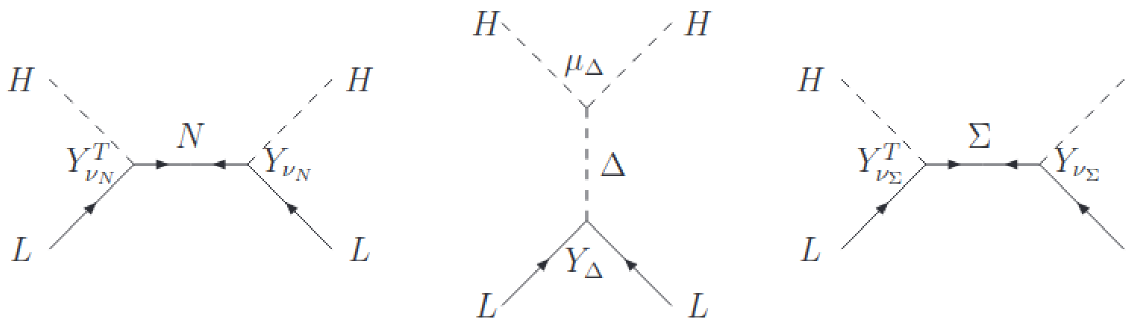
Same appears Λ @ all orders & c_5 is extremely small

New physics has multiple scales

Lepton (& baryon) number violating scales are very high
Other new physics at few TeV → offers sensitivity at colliders

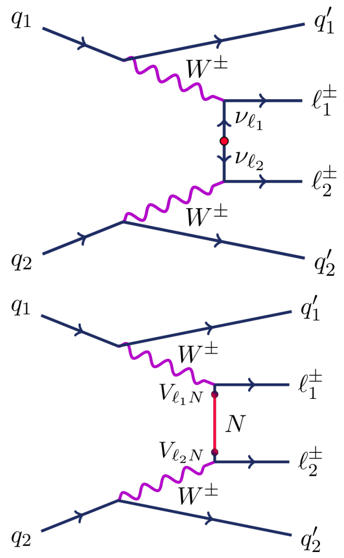
Theoretical models

Type-I, Type-II, & Type-III See-saw mechanisms

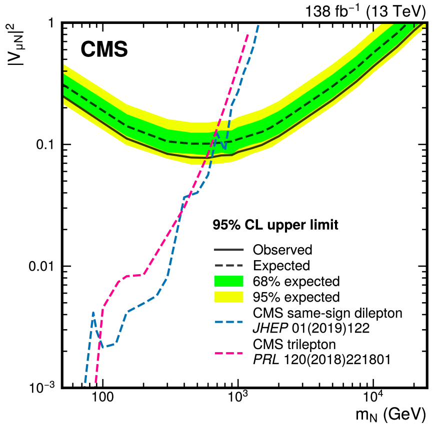


N: SU(2) singlet fermion   Δ : SU(2) triplet scalar   Σ : SU(2) triplet fermion

Collider signature



CMS-EXO-21-003



Standard model effective field theory (SMEFT)

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$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i^{(5)}}{\Lambda}\mathcal{O}_{5,i} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_{6,i} + \sum_i \frac{c_i^{(7)}}{\Lambda^3}\mathcal{O}_{7,i} + \sum_i \frac{c_i^{(8)}}{\Lambda^4}\mathcal{O}_{8,i} + \ldots\,.$$

Violate lepton number conservation

Violate baryon & lepton number conservation

$1: X^3$		$2: H^6$		$3: H^4D^2$		$5: \psi^2H^3 + \text{h.c.}$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$	Q_H	$(H^\dagger H)^3$	$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	Q_{eH}	$(H^\dagger H)(\bar l_p e_r H)$
$Q_{\widetilde G}$	$f^{ABC}\widetilde G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$			Q_{HD}	$(H^\dagger D_\mu H)^*(H^\dagger D_\mu H)$	Q_{uH}	$(H^\dagger H)(\bar q_p u_r \widetilde H)$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$					Q_{dH}	$(H^\dagger H)(\bar q_p d_r H)$
$Q_{\widetilde W}$	$\epsilon^{IJK}\widetilde W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$						

$4: X^2H^2$		$6: \psi^2XH + \text{h.c.}$		$7: \psi^2H^2D$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p\sigma^{\mu\nu}e_r)\tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar l_p\gamma^\mu l_r)$
$Q_{H\widetilde G}$	$H^\dagger H\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar l_p\tau^I\gamma^\mu l_r)$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p\sigma^{\mu\nu}T^A u_r)\widetilde H\, G_{\mu\nu}^A$	Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar e_p\gamma^\mu e_r)$
$Q_{H\widetilde W}$	$H^\dagger H\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p\sigma^{\mu\nu}u_r)\tau^I\widetilde H\, W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar q_p\gamma^\mu q_r)$
Q_{HB}	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$	Q_{uB}	$(\bar q_p\sigma^{\mu\nu}u_r)\widetilde H\, B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{H\widetilde B}$	$H^\dagger H\, \widetilde B_{\mu\nu}B^{\mu\nu}$	Q_{dG}	$(\bar q_p\sigma^{\mu\nu}T^A d_r)H\, G_{\mu\nu}^A$	Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar u_p\gamma^\mu u_r)$
Q_{HWB}	$H^\dagger\tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p\sigma^{\mu\nu}d_r)\tau^I H\, W_{\mu\nu}^I$	Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar d_p\gamma^\mu d_r)$
$Q_{H\widetilde WB}$	$H^\dagger\tau^I H\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$	Q_{Hud} + h.c.	$i(\widetilde H^\dagger D_\mu H)(\bar u_p\gamma^\mu d_r)$

Validity:

E<<Λ

→ EFT contributions power-suppressed

→ Deviation from SM prediction small

$8: (\bar LL)(\bar LL)$		$8: (\bar RR)(\bar RR)$		$8: (\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu\tau^I q_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu\tau^I l_r)(\bar q_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

$8: (\bar LR)(\bar RL) + \text{h.c.}$		$8: (\bar LR)(\bar LR) + \text{h.c.}$	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_{tj})$	$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\epsilon_{jk}(\bar q_s^k d_t)$
		$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\epsilon_{jk}(\bar q_s^k T^A d_t)$
		$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\epsilon_{jk}(\bar q_s^k u_t)$
		$Q_{lequ}^{(3)}$	$(\bar l_p^j\sigma_{\mu\nu}e_r)\epsilon_{jk}(\bar q_s^k\sigma^{\mu\nu}u_t)$

Standard model effective field theory (SMEFT)

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$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i^{(5)}}{\Lambda}\mathcal{O}_{5,i} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_{6,i} + \sum_i \frac{c_i^{(7)}}{\Lambda^3}\mathcal{O}_{7,i} + \sum_i \frac{c_i^{(8)}}{\Lambda^4}\mathcal{O}_{8,i} + \dots\,.$$

Violate lepton number conservation

Violate baryon & lepton number conservation

Assumptions:

→ Poincaré symmetry, locality
→ Field content (relevant at EW scale) same as in SM
→ SM Gauge symmetries SU(3) x SU(2) x U(1) respected

Validity:

E<<Λ

→ EFT contributions power-suppressed
→ Deviation from SM prediction small

$1 : X^3$	
Q_G	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK}W_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK}\widetilde{W}_\mu^{I\nu}W_\nu^{J\rho}W_\rho^{K\mu}$

$2 : H^6$	
Q_H	$(H^\dagger H)^3$

$3 : H^4D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$\left(H^\dagger D_\mu H\right)^*\left(H^\dagger D_\mu H\right)$

$5 : \psi^2H^3 + \mathrm{h.c.}$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4 : X^2H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu}B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu}B^{\mu\nu}$
Q_{HWB}	$H^\dagger\tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger\tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6 : \psi^2XH + \mathrm{h.c.}$	
Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I\widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p\sigma^{\mu\nu}u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p\sigma^{\mu\nu}d_r)H\, B_{\mu\nu}$

$7 : \psi^2H^2D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p\gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{l}_p\tau^I\gamma^\mu l_r)$
Q_{He}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p\gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i\overleftrightarrow{D}_\mu^I H)(\bar{q}_p\tau^I\gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p\gamma^\mu d_r)$

$8 : (\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$

$8 : (\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$
Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8 : (\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$8 : (\bar{L}R)(\bar{R}L) + \mathrm{h.c.}$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

$8 : (\bar{L}R)(\bar{L}R) + \mathrm{h.c.}$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\epsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$