

Gravitational wave observations with PTA, can cosmic strings explain the signal?

(A cosmologist point of view)

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Cosmology, Universe and Relativity at Louvain (CURL)

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Introduction to pulsar timing
(Cosmologist point of view)

Hellings and Downs correlations

Data analysis and results

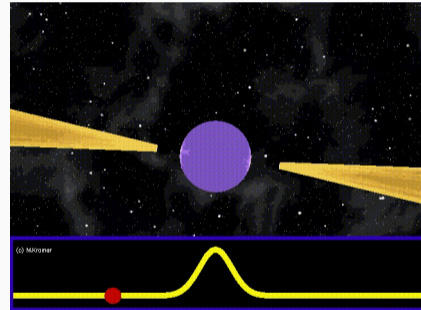
Cosmic string interpretation

Conclusion

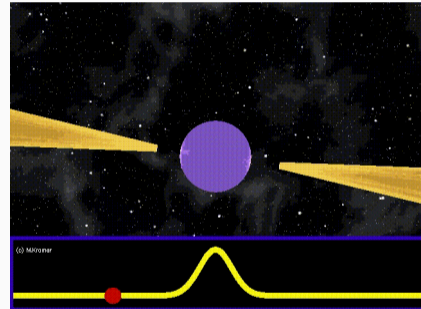
Introduction to pulsar timing

(Cosmologist point of view)

- Neutron stars are **compact stars** with very **short rotational period** and extreme **magnetic fields**

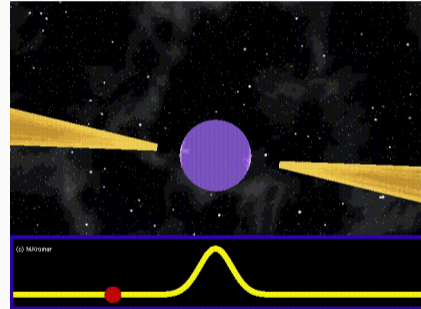


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⇒ Analogous to a **lighthouse**

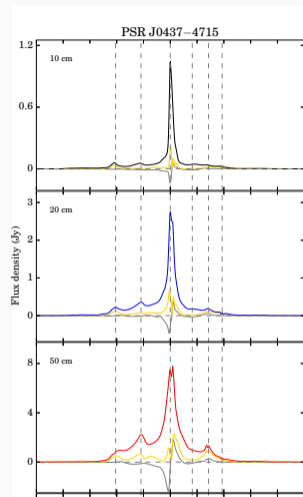


Introduction to pulsars

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- Generally, the magnetic axis is not aligned with the spin axis, so radiation is swept through space.
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- They appear to the observer as pulses, separated by a fixed period that equals the **spin**



Pulse profiles vary across observing frequencies

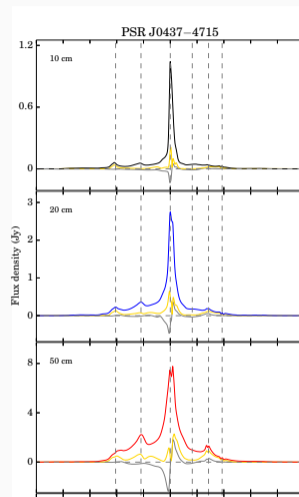


Pulsars studied in Parkes Pulsar Timing Array Dai et al. 2015

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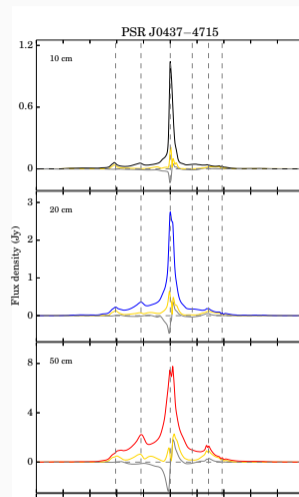
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Most pulsar timing are carried around 1.4 GHz



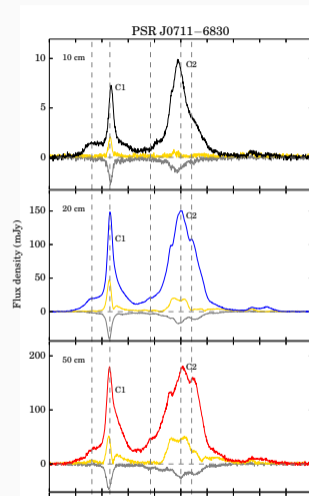
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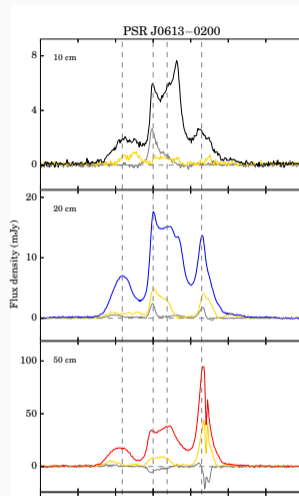
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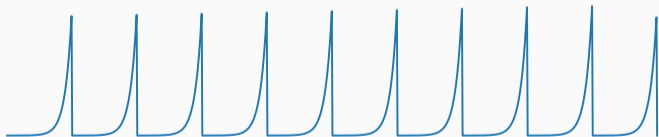
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Radio emission from pulsars

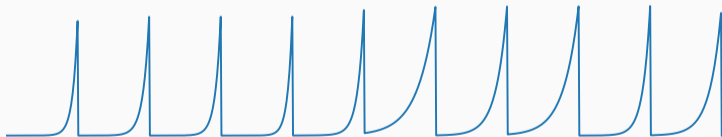
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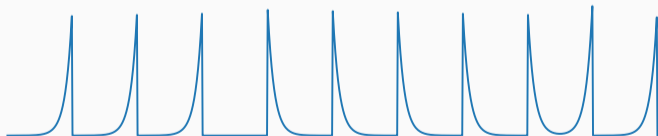
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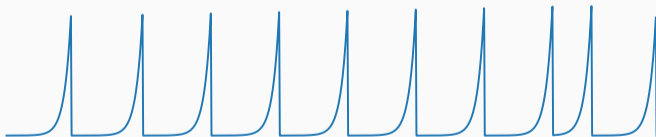
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- Nulling: Pulsars that turn off only to reappear at some point after
- Moding: Pulsars that arbitrarily change between different fingerprints
- **Drifting:** Pulses appear to come a bit late after each rotation only to reset after a few dozen rotations



Different types of pulsars

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- large angular momentum \implies far more **stable clocks** than slow pulsars

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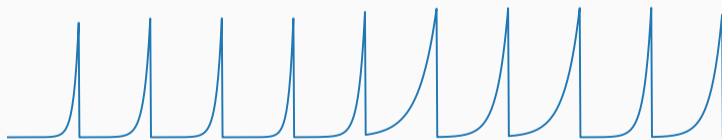
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- Template profiles may take advantage of the **frequency-dependence** of the pulses

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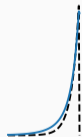
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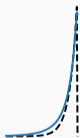


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The ToA combines the observation time stamp with the phase measurement



Transferring the observed times to the Pulsar

Accounting for all known propagation and geometric delays

$$t_{\text{PSR}} = t_{\text{obs}} - \Delta_{\odot} - \Delta_{\text{ISM}} - \Delta_{\text{Bin}}$$

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- Δ_{Bin} , for pulsars that are in **binary** systems

Once the time of emission is determined, it can be converted to a rotational phase

$$\phi(t_{\text{PSR}}) = \nu(t_{\text{PSR}} - t_0) + \frac{1}{2}\dot{\nu}(t_{\text{PSR}} - t_0)^2 + \dots$$

- ν is the pulsar's frequency
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In practice, there is an **interplay** between

- Construction of the template profile
- Determination of the timing model
- Knowledge of the propagation/geometric delays

Difference between observed and predicted time of arrivals of the pulsars' radio pulses

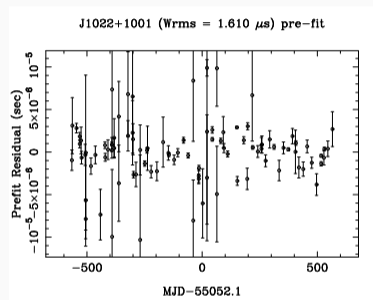
$$\delta t_i = t_i^{\text{obs}} - t_i^{\text{TM}}$$

Timing residuals

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Errors have an impact on timing residuals



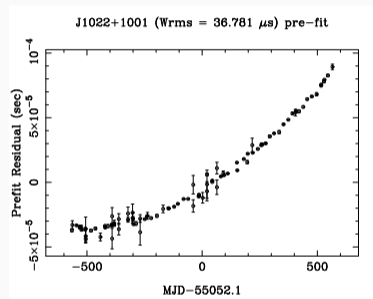
Typical PTA dataset Verbiest, Osłowski, and Burke-Spolaor 2021

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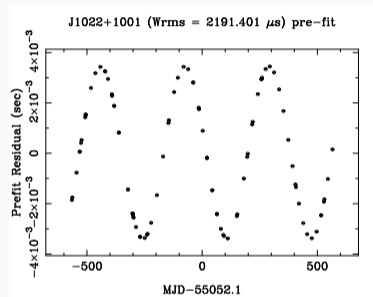
1% error on the spindown Verbiest, Osłowski, and Burke-Spolaor 2021

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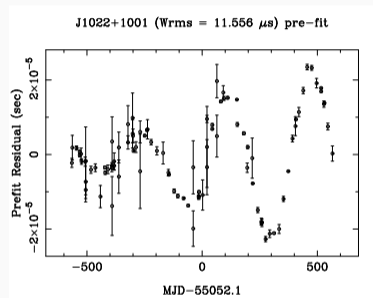
Positional offset of 0.1 arcsec in right ascension and declination Verbiest, Osłowski, and Burke-Spolaor 2021

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Proper motion is 10% incorrect Verbiest, Osłowski, and
Burke-Spolaor 2021

Hellings and Downs correlations

Good reviews: Jenet and Romano 2015; Romano and Allen 2023

- Time delay due to the passing of a GW

$$\Delta T(t) = \frac{1}{2c} u^i u^j \int_0^L ds h_{ij}[\tau(s), \vec{x}(s)]$$

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- Plane-wave decomposition of the GW

$$h_{ij}(t, \vec{x}) = \int_{-\infty}^{+\infty} df \int d\hat{\mathbf{k}} \sum_{A=+, \times} h_A(f, \hat{\mathbf{k}}) e_{ij}^A(\hat{\mathbf{k}}) \exp[i2\pi f(t - \hat{\mathbf{k}} \cdot \vec{x}/c)]$$

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- At zeroth order, the photon propagates on a straight line

$$\vec{x}(s) = \vec{r}_1 + s\hat{\mathbf{u}}, \quad \tau(s) = t + (s - L)/c \quad \vec{r}_2 = \vec{r}_1 + L\hat{\mathbf{u}},$$

pulsar at $\hat{\mathbf{p}} = -\hat{\mathbf{u}}$

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Response function

$$R^A(f, \hat{\mathbf{k}}) \equiv \frac{1}{2} u^i u^j e_{ij}^A(\hat{\mathbf{k}}) \frac{1}{i2\pi f} \frac{1}{1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{u}}} \left[\textcircled{1} - \exp\left(-\frac{i2\pi f L}{c}(1 - \hat{\mathbf{k}} \cdot \hat{\mathbf{u}})\right) \right]$$

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- Earth term
- Pulsar term
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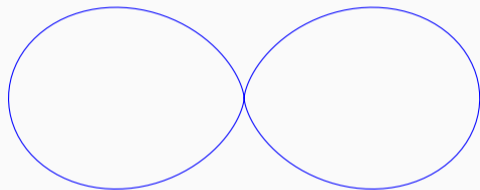
- Earth term
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- Breaks the $\hat{\mathbf{u}} \rightarrow -\hat{\mathbf{u}}$ symmetry, there is a difference if the photon is *surfing* the GW or *fight upstream*
- Interaction between the **photon and the GW polarizations**

- The response function reduces to

$$R^A(f, \hat{\mathbf{k}}) = u^i u^j e_{ij}^A(\hat{\mathbf{k}}) \frac{L}{2c}$$

- Take a pulsar in the $\hat{\mathbf{z}}$ direction and $\cos(\theta) = \hat{\mathbf{k}} \cdot \hat{\mathbf{u}}$, then

$$\left| R^+(f, \hat{\mathbf{k}}) \right| = \frac{L}{2c} \sin^2(\theta), \quad \left| R^\times(f, \hat{\mathbf{k}}) \right| = 0$$



Response function $|R^+|$ for a pulsar located in the $+\hat{\mathbf{z}}$ direction.

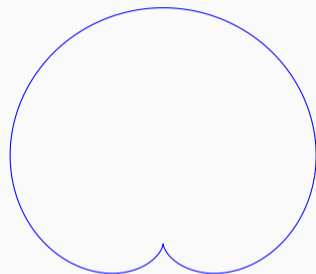
Long-arm limit $fL/c \ll 1$ (PTA)

- We neglect the oscillatory pulsar term, provided $\hat{\mathbf{k}} \cdot \hat{\mathbf{u}} \neq 1$

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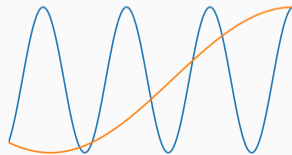
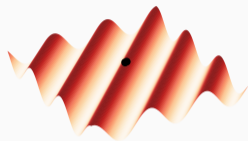
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Stochastic background of GW

$$\langle h_A(f, \hat{\mathbf{k}}) \rangle = 0, \quad \langle h_A(f, \hat{\mathbf{k}}) h_{A'}(f', \hat{\mathbf{k}}') \rangle = \frac{1}{8\pi} H(f) \delta(f' - f) \delta_{AA'} \delta^2(\hat{\mathbf{k}}, \hat{\mathbf{k}}')$$

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- Stationary
- Unpolarized

Hellings and Downs correlation (1/2)

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$$\Gamma_{ab}(f) = \frac{1}{12\pi^2 f^2} \Gamma_{ab}$$

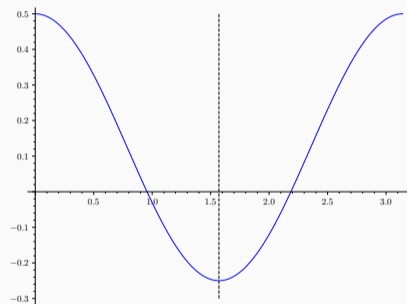
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$$\Gamma_{ab} = \frac{1}{2} P_2(\cos \gamma_{ab}) + \frac{\delta_{ab}}{2}$$



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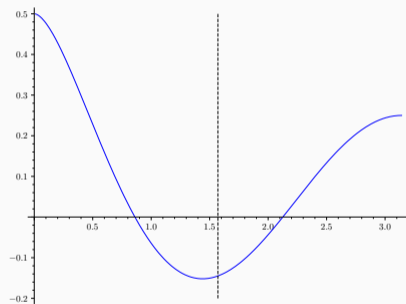
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- In the **long-arm** limit

$$\Gamma_{ab} = \frac{1}{2} + \frac{3}{2} \left(\frac{1 - \cos \gamma_{ab}}{2} \right) \left[\ln \left(\frac{1 - \cos \gamma_{ab}}{2} \right) - \frac{1}{6} \right] + \frac{\delta_{ab}}{2}$$



Data analysis and results

In addition to the deterministic components of the timing model, we have non-deterministic effects

Uncorrelated noises

- Pulse jitter
- Intrinsic spin noise
- Orbital Irregularities
- ISM Propagation Effects

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- Imperfections in the reference clock
⇒ **monopole**
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All these noises can either be White (Uncorrelated in time) or Chromatic (Correlated in time)

Phenomenological noise models

Instead of modeling every noise individually, PTA construct a **phenomenological** model summarized in terms of the covariance matrix

$$C_{(ai)(bj)} = \mathcal{N}_{a,i} \delta_{ij} \delta_{ab} + C_{a,ij}^{\text{PSR}} \delta_{ab} + \Gamma_{ab} C_{ij}^{\text{CRN}}$$

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- $C_{a,ij}^{\text{PSR}}$ Intrinsic red noise covariant matrix
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$$p(\delta t|\eta) = \frac{1}{\sqrt{|2\pi C|}} \exp\left(-\frac{1}{2}\delta t^T C^{-1}\delta t\right)$$

To perform model comparison, we studied three approaches^a

- **Full method:** sample parameters assuming a given SGWB spectrum

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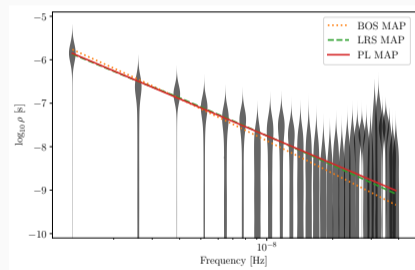
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- Full method: sample parameters assuming a given SGWB spectrum
- Resampling method: first sample parameters assuming $\Gamma_{ab} = \delta_{ab}$. Likelihood is factorized for each pulsar Then resample the posteriors assuming Γ_{ab}
- **Free spectrum**: assume that all frequency bins are independent and obtain a posterior distribution for each of them. Adding new models is inexpensive

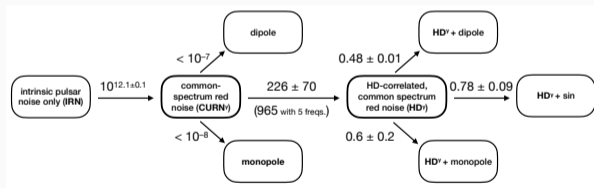
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Posterior distributions in the 30 frequency bins Quelquejay Leclere et al. 2023

Increasing evidence for GWs

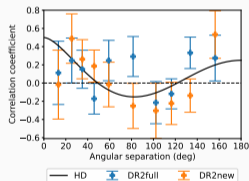
- **NANOGrav** claims $3.5 - 4\sigma$ with 67 pulsars Agazie et al. 2023



Bayes factors between models of correlated red noise in the NANOGrav 15-year data set Agazie et al. 2023

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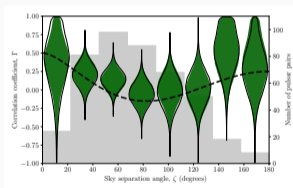
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Constraints on the overlap reduction function from the optimal statistic Antoniadis et al. 2023a

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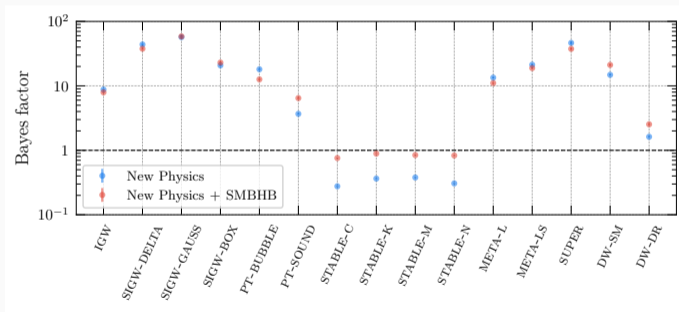
Measured spatial correlations as a function of the angular separation angle [Reardon et al. 2023](#)

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 - CPTA claims 4.6σ with 57 pulsars Xu et al. 2023 but...
-
- Only 3 years of data
 - Analysis carried at single frequencies
 - Frequentist method (and argue that Bayesian method is biased)
 - They cannot distinguish HD at 4.6σ and dipole at 4.1σ

Interpretations

- Inspiring supermassive black hole binaries (SMBHBs)
- Scalar-induced GWs
- First-order phase transitions
- Cosmic strings
- Domain walls



Bayes factors for the model comparisons between the new-physics interpretations and the interpretation in terms of SMBHBs alone Afzal et al. 2023

Cosmic string interpretation

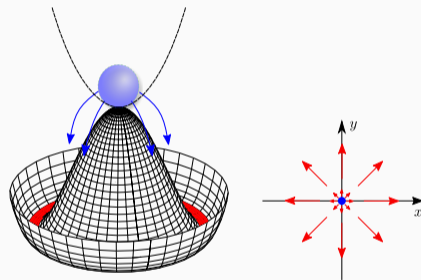
What is a cosmic string ?

Cosmic string

A **cosmic string** is a one dimensional topological defect¹. May form when the vacuum manifold has non-contractible loops.

Example: Mexican hat potential

- Vacuum manifold is a circle $\mathcal{M} = S^1$
- Fundamental group $\Pi_1(\mathcal{M}) = \mathbb{Z}$
- We expect strings to be formed in most models of spontaneous symmetry breaking²



¹Kibble 1976

²Jeannerot, Rocher, and Sakellariadou 2003

Energy scale	Width	Linear density
GUT : 10^{16} GeV	2×10^{-32} m	$G\mu \approx 10^{-6}$
3×10^{10} GeV	5×10^{-27} m	$G\mu \approx 10^{-17}$
10^8 GeV	2×10^{-24} m	$G\mu \approx 10^{-22}$
EW : 100 GeV	2×10^{-18} m	$G\mu \approx 10^{-34}$

Nambu-Goto strings: one dimensional limit

- Width of the string very small compared to other length scales in the problem.
- String modeled as a line with mass per unit length $\mu \propto \eta^2$
- The Nambu-Goto action which minimizes the area swept by the string

$$\mathcal{S} = -\mu \int d\tau d\sigma \sqrt{-\det \gamma}$$

γ_{ab} : the induced metric on the string, τ is a time-like and σ a space-like coordinate along the string

Nambu-Goto strings in flat spacetime

Cosmic string dynamics

In flat spacetime, it satisfies a wave equation whose solution is

$$\mathbf{X}(t, \sigma) = \frac{1}{2}[\mathbf{a}(t - \sigma) + \mathbf{b}(t + \sigma)], \quad \mathbf{a}'^2 = \mathbf{b}'^2 = 1.$$

For a **closed loop** $\mathbf{X}(t, \sigma + \ell) = \mathbf{X}(t, \sigma)$: it oscillates with a period $T = \frac{\ell}{2}$.

Cosmic strings emit gravitational waves:

- **Oscillation**



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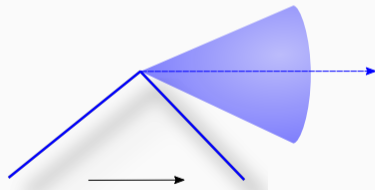
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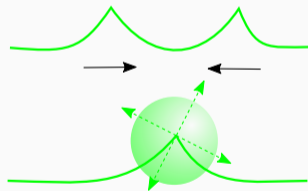
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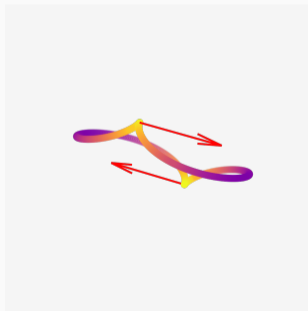
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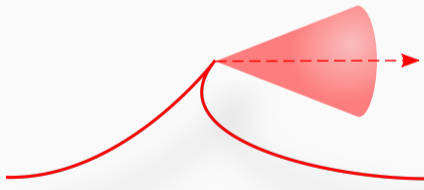
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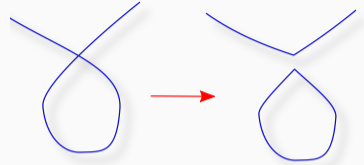
The number of cosmic string loops $\mathcal{N}(\ell, t)$

Model for the population of loops

- Long strings are stretched by the expansion of the Universe: $a(t)$
- They intersect each other and produce loops : $\mathcal{P}(\ell, t)$
- Loops decay by emitting gravitational waves : $\dot{E} = -\Gamma G\mu^2$

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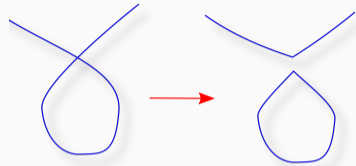
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- One-scale model Kibble 1985

$$t^5 \mathcal{P}(\ell, t) = C \delta\left(\frac{\ell}{t} - \alpha\right)$$

- Power-law loop production function Polchinski and Rocha 2006

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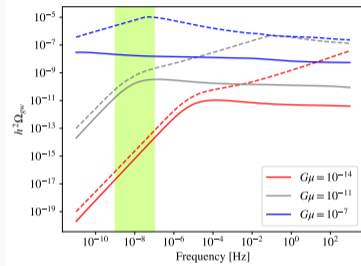
The stochastic background of gravitational waves

- The uncorrelated sum of all the GW signals produced by cosmic string loops constitutes a Stochastic Background of GW.
- We can estimate this background using energetic arguments

$$\Omega_{\text{GW}}(\ln f) = \frac{8\pi G}{3H_0^2} f \rho_{\text{GW}}$$

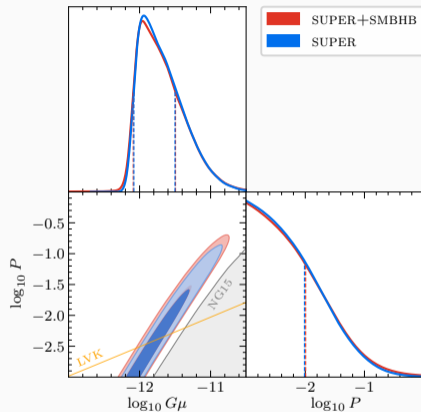
$$\rho_{\text{GW}}(f) = \int_0^{t_0} \frac{dt}{[1+z(t)]^4} P_{\text{gw}}(t, f') \frac{\partial f'}{\partial f}$$

$$P_{\text{gw}}[t, f'] = G\mu^2 \sum_m \frac{2m}{f'^2} P_m \mathcal{N}\left(\frac{2m}{f'}, t\right)$$



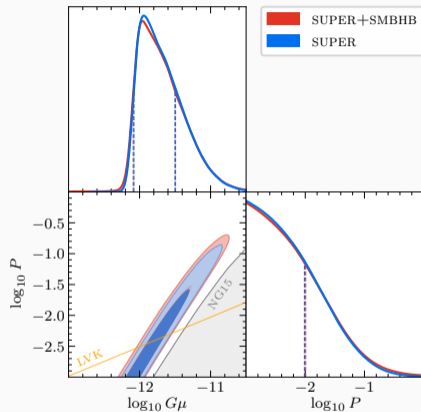
Quelquejay Leclere et al. 2023

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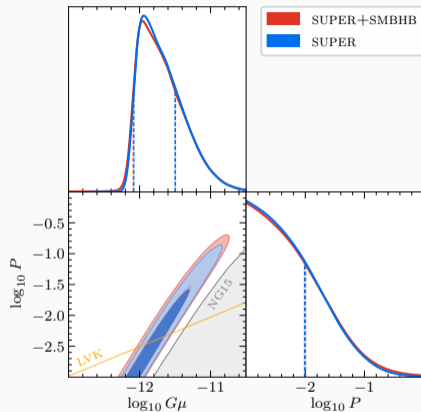
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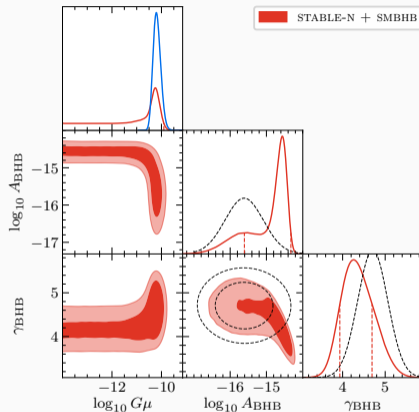
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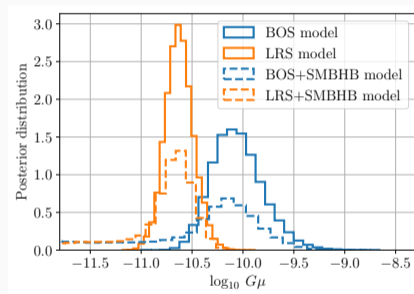
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- Scenario CS + SMBHB does not favor CS



Afzal et al. 2023

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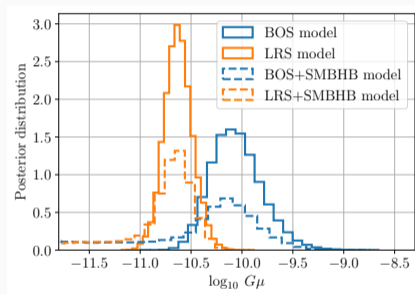
Antoniadis et al. 2023b

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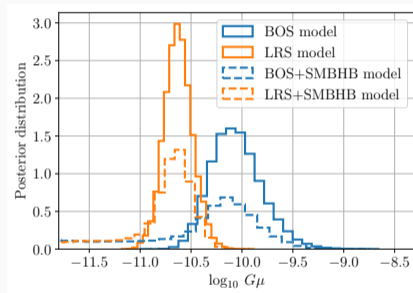
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- EPTA studied the BOS^a and the LRS^b models
- Values of $G\mu$ are comparable for both models
- EPTA does not favor a model in particular with $\mathcal{B} \approx 0.3$ ^c



Antoniadis et al. 2023b

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Conclusion

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- Looking forward to the International Pulsar Timing Array!
- Software and datasets are available online


Thank you

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