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The impact of non-equilibrium effects on the relic density of dark matter

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Based on **2204.07078** and **2104.05684** in collaboration

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CP3 Seminar @ UCLouvain

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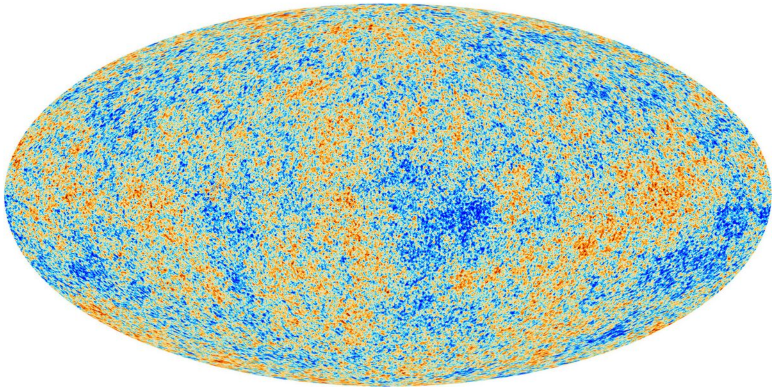
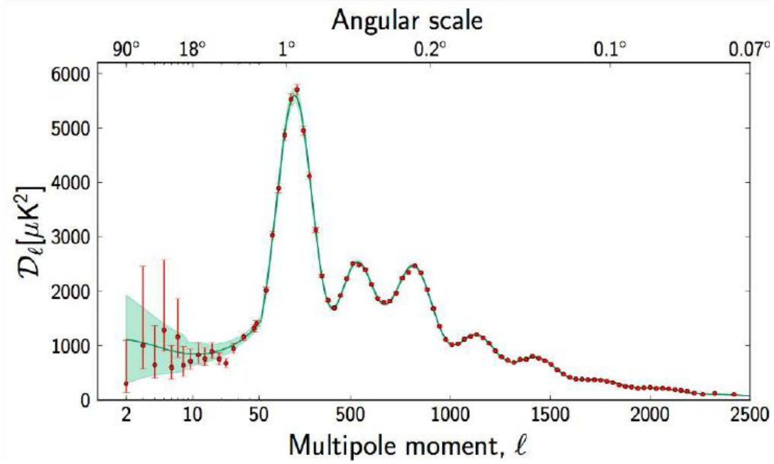
UNIVERSITY OF WARSAW

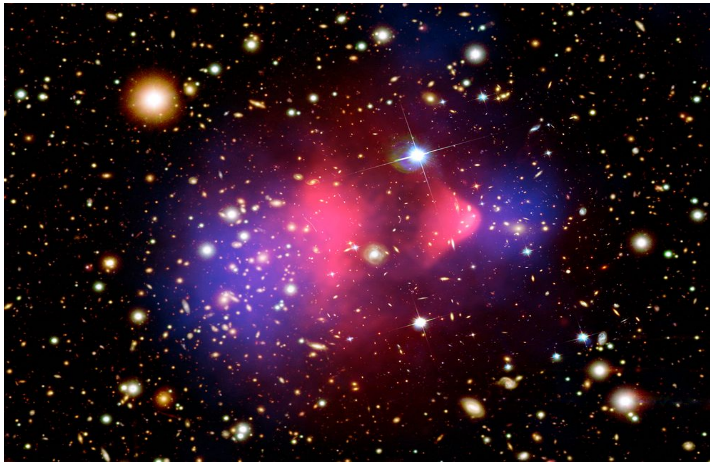
Introduction in a nutshell

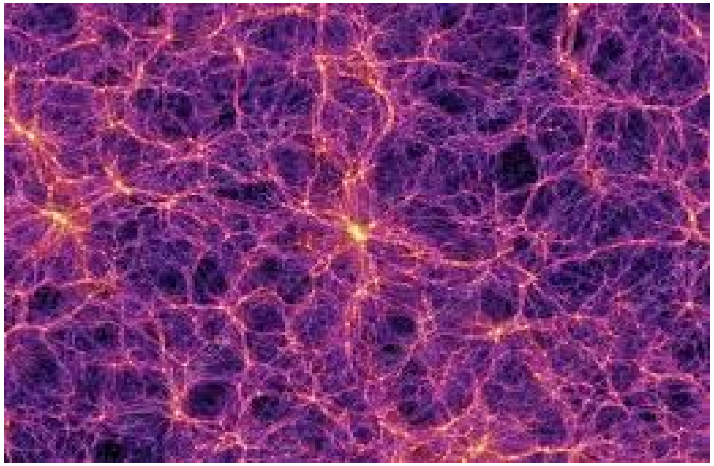
- The standard method of calculating the relic density of dark matter is based on several assumptions that are not valid in some cases
- I outline a more general approach to this problem and discuss the importance of this approach
- I will show a few examples

Dark matter (DM) and its relic density









Planck measurement 1807.06209

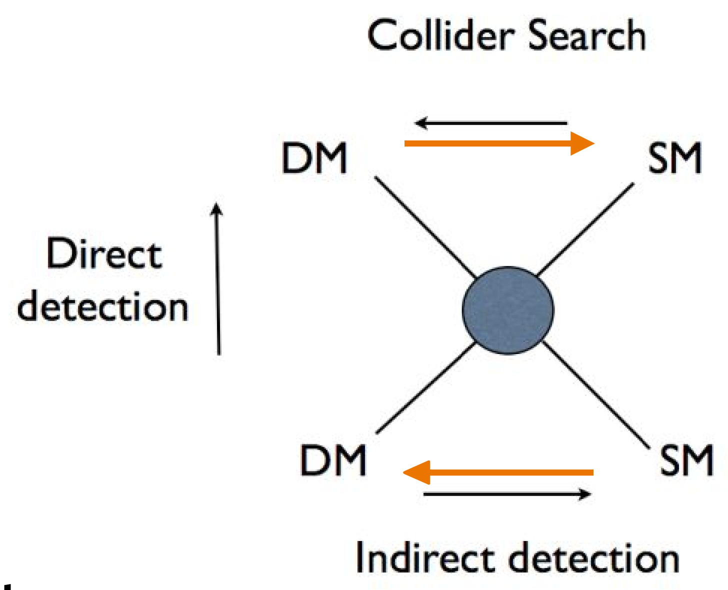
$$\Omega_{\rm dm} h^2 = \frac{\rho_{\rm dm}}{\rho_{\rm crit}} h^2 = 0.12$$

Interactions with the SM particles

If DM is coupled to the SM states (even weakly) → different possible detection strategies + production in the early Universe

Common production channels:

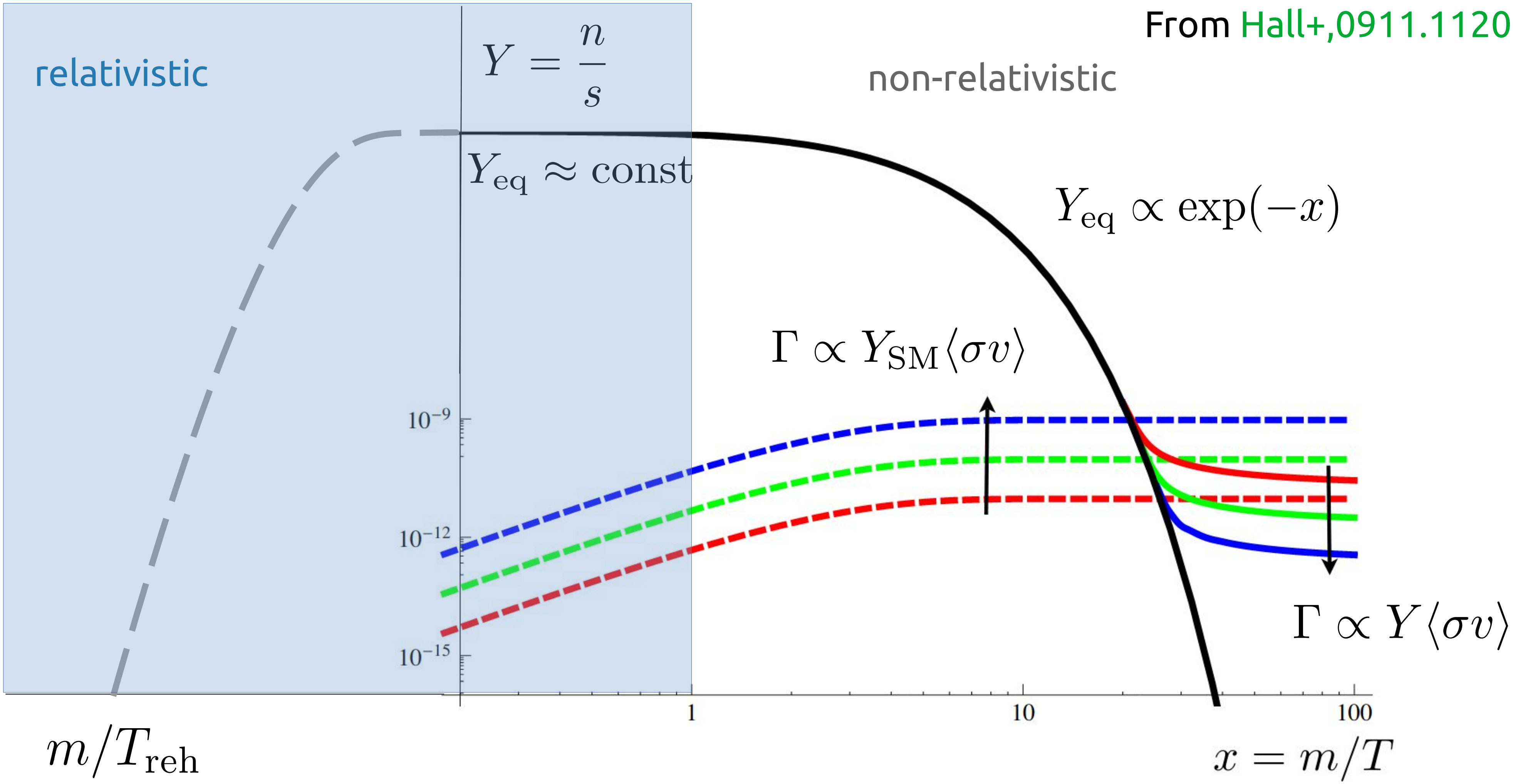
- Annihilation (*in the pic*)
- Decay



Interaction rate in the early Universe determines the relic density

Freeze in/out

From Hall+,0911.1120



The rates Γ are for 2→ 2 annihilation

Freeze in/out

Freeze in

- DM never reaches full thermal equilibrium
- Stronger interactions → larger relic density
- Typical rate for annihilation

$$\langle \sigma v \rangle \sim 10^{-40} \mathrm{cm}^3/\mathrm{s}$$

- Relic density is established around at

x ~ 2 – 3

- Depends on initial conditions

Freeze out

- DM starts in the full thermal equilibrium
- Stronger interactions → smaller relic density
- Typical rate for annihilation

$$\langle \sigma v \rangle \sim 10^{-26} \mathrm{cm}^3/\mathrm{s}$$

- Relic density is established at

x ~ 25 – 30

- Independent of initial conditions

Standard approach

Boltzmann equation for the number density (**nBE**)

$$\frac{dY}{dx} = \frac{s(x)}{x\,\tilde{H}(x)} \langle \sigma v \rangle(x) \left[Y_{\rm eq}^2(x) - Y^2 \right]$$

(for 2→ 2 annihilation)

Hubble parameter $\tilde{H} = H/[1+\tilde{g}]$

Correction for the change of entropy dof $\tilde{g} \equiv \frac{1}{3} \frac{T}{g_{\rm eff}^s} \frac{dg_{\rm eff}^s}{dT}$

Velocity averaged cross section

Equilibrium density of DM

Can also incorporate decays, co-annihilations, etc. Belanger+, 1402.0787 Arina+, 2012.09016

Used in many numerical packages, e.g. micrOMEGAs or MadDM

What's under the hood?

In fact, the Boltzmann equation for the number density is derived from the general **Full Boltzmann equation** (**fBE**)

$$2E_i\left(\partial_t - H\, p\partial_p\right) f_i(p) = C\left[f_i\right]$$

Collision term

DM **distribution function**

Integrating over the momentum of i $\quad g_i \int \frac{d^3 p_i}{(2\pi)^3}$

$$\frac{g_i}{s}\int \frac{d^3 p_i}{(2\pi)^3} f_i = Y$$

$$g_i \int \frac{d^3 p_i}{(2\pi)^3}\, C[f_i] = ?$$

Structure of collision term

Takes into account all the processes in which the particle participates

Number-changing processes – impact on the density*

$$C[f_\chi] = C_{\text{ann}} + C_{\text{dec}} + C_{\text{el}} + C_{\text{self}} + \ldots$$

Equilibrating processes – impact on the shape of the distribution

Elastic and self-scatterings parts are integrated out for the nBE

* have an impact of the shape of the distribution too

Collision term for annihilation

For DM + DM → SM + SM :

$$C_{\mathrm{ann}}[f_\chi] = \frac{1}{2g_\chi}\int \frac{d^3\tilde{p}}{(2\pi)^3\tilde{E}}\int \frac{d^3k}{(2\pi)^3\omega}\int \frac{d^3\tilde{k}}{(2\pi)^3\tilde{\omega}}$$

All momenta configurations

$$\times(2\pi)^4\delta^{(4)}(E+\tilde{E}-\omega-\tilde{\omega})$$

Energy and momenta conservation

$$\times\Big[|\mathcal{M}|^2_{\mathrm{SM}\to\mathrm{DM}}\, f_{\mathrm{SM}}(\omega)f_{\mathrm{SM}}(\tilde{\omega})[1\pm f_\chi(E)][1\pm f_\chi(\tilde{E})]$$

$$-|\mathcal{M}|^2_{\mathrm{DM}\to\mathrm{SM}}\, \underline{f_\chi(E)f_\chi(\tilde{E})}[1\pm f_{\mathrm{SM}}(\omega)][1\pm f_{\mathrm{SM}}(\tilde{\omega})]\Big]$$

Probability x number of states

Quantum corrections (final states)

Leads to an integro-differential equation

Assumptions

- DM is in **kinetic equilibrium** with the SM

$$f_i = \frac{n_i}{n_i^{\rm eq}}\, f_i^{\rm eq}(E_i, T_{\rm SM})$$

$$f_i^{\rm eq} = \exp(-E_i/T_{\rm SM})$$

Maxwell-Boltzmann
(for non-relativistic or very dilute gasses)

$$f_i^{\rm eq} = \frac{1}{\exp(E_i/T_{\rm SM}) \pm 1}$$

Fermi-Dirac/Bose-Einstein
(for dense relativistic gases)

- Quantum corrections are often neglected, because DM is *non-relativistic* and *dilute* around the formation of relic density

From fBE to nBE

$$f_{\rm SM}(\omega) f_{\rm SM}(\tilde{\omega}) = \exp(-\omega/T) \cdot \exp(-\tilde{\omega}/T) =$$

Energy conservation

$$= \exp\left(-\frac{E+\tilde{E}}{T}\right) = f_\chi^{\rm eq}(E) f_\chi^{\rm eq}(\tilde{E})$$

Thus, one eventually gets the nBE (Gelmini-Gondolo approach)

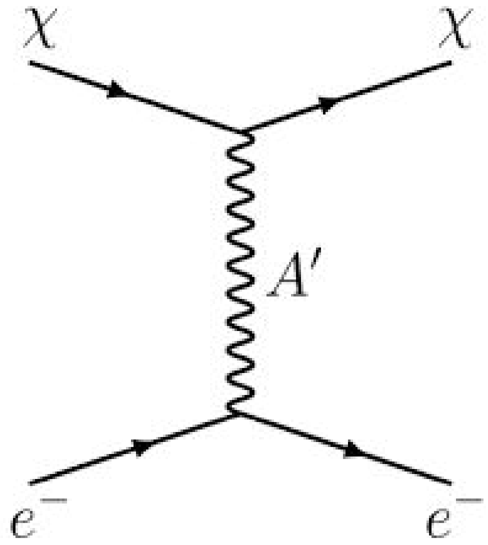
$$\frac{dY}{dx} = \frac{s(x)}{x\,\tilde{H}(x)} \langle\sigma v\rangle(x)\, [Y_{\rm eq}^2(x) - Y^2]$$

Gelmini, Gondolo, Nucl.Phys.B 360 (1991)

$$\langle\sigma v\rangle = \frac{g_\chi^2}{n_\chi^{\rm eq}\cdot n_\chi^{\rm eq}} \int \frac{d^3p}{(2\pi)^3} \frac{d^3\tilde{p}}{(2\pi)^3} \sigma_{\rm ann} v\, f_\chi^{\rm eq}(p) f_\chi^{\rm eq}(\tilde{p})$$

Elastic scatterings

The kinetic equilibrium is maintained mostly by the elastic scatterings of DM on the particles in the SM plasma



WIMPs typically go out of kinetic equilibrium much later than the FO

T ~ 1-10 MeV (x >> 100)

Bringmann, 0903.0189

If annihilation rate is much larger than the rate of scatterings → early kinetic decoupling → the assumption about f <u>doesn't hold!</u>

Example: Scalar singlet DM

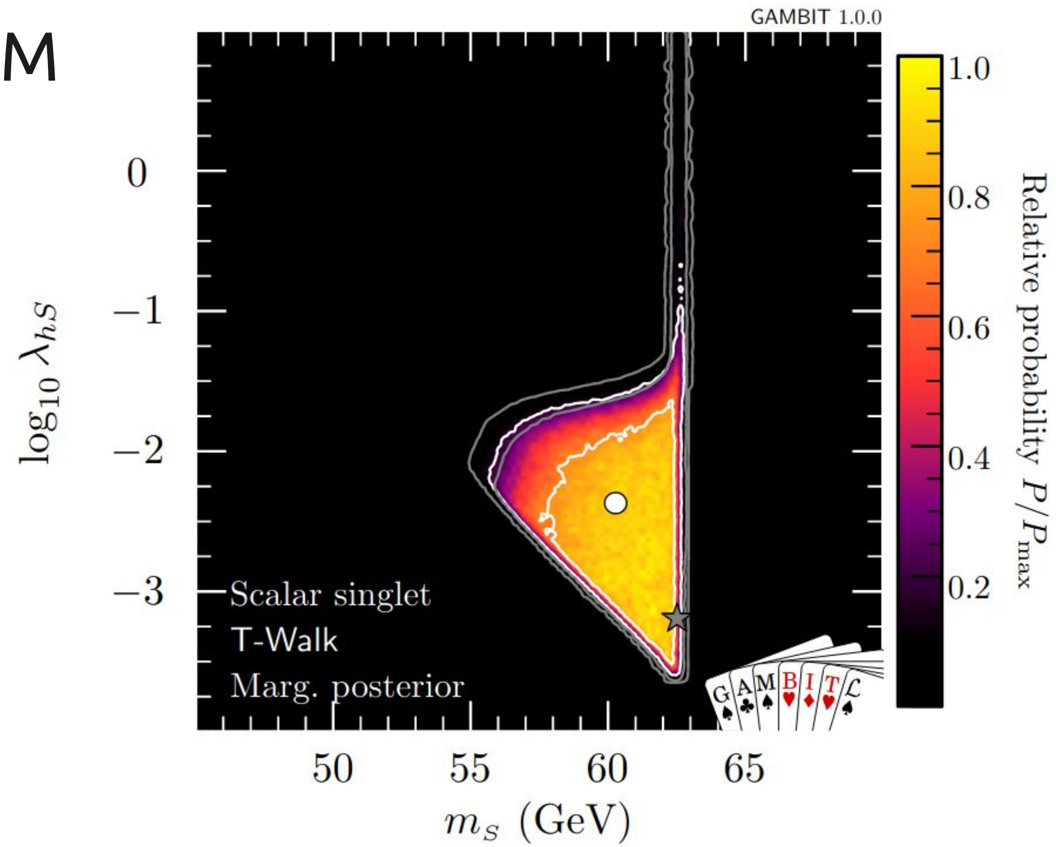
Resonant annihilation into SM fermions ($2m_{DM} \sim m_{Higgs}$)

$$\sigma v \propto \frac{\lambda_S^2}{(s - m_h^2)^2 + m_h^2 \Gamma_h^2}$$

Elastic scattering

$$|\mathcal{M}|^2 \propto \frac{\lambda_S^2}{(t - m_h^2)^2}$$

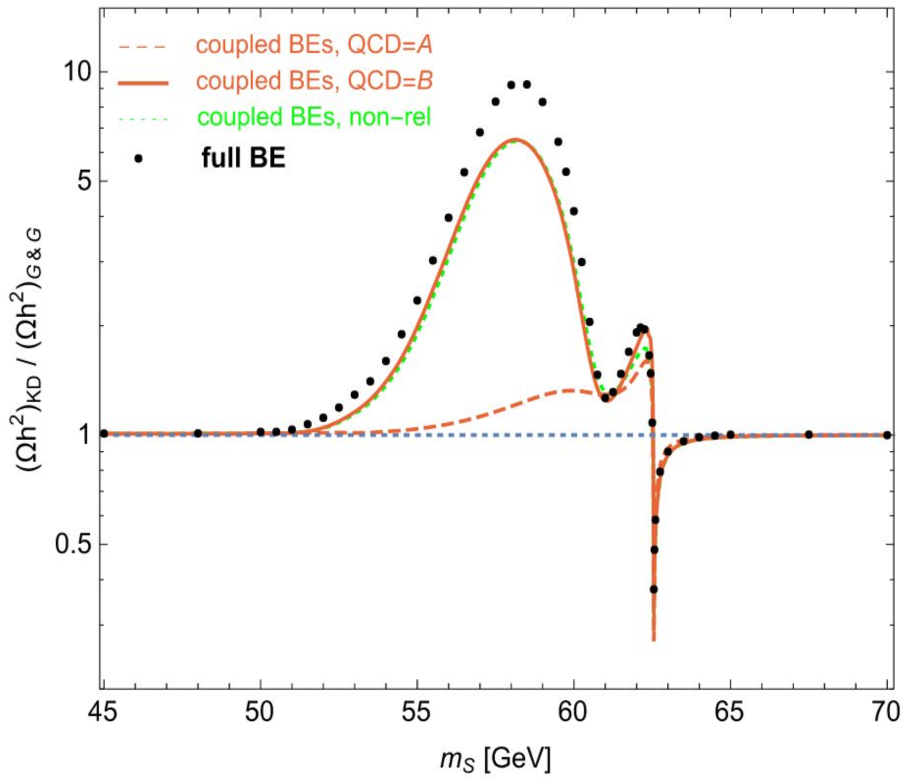
Leads to early **kinetic decoupling**

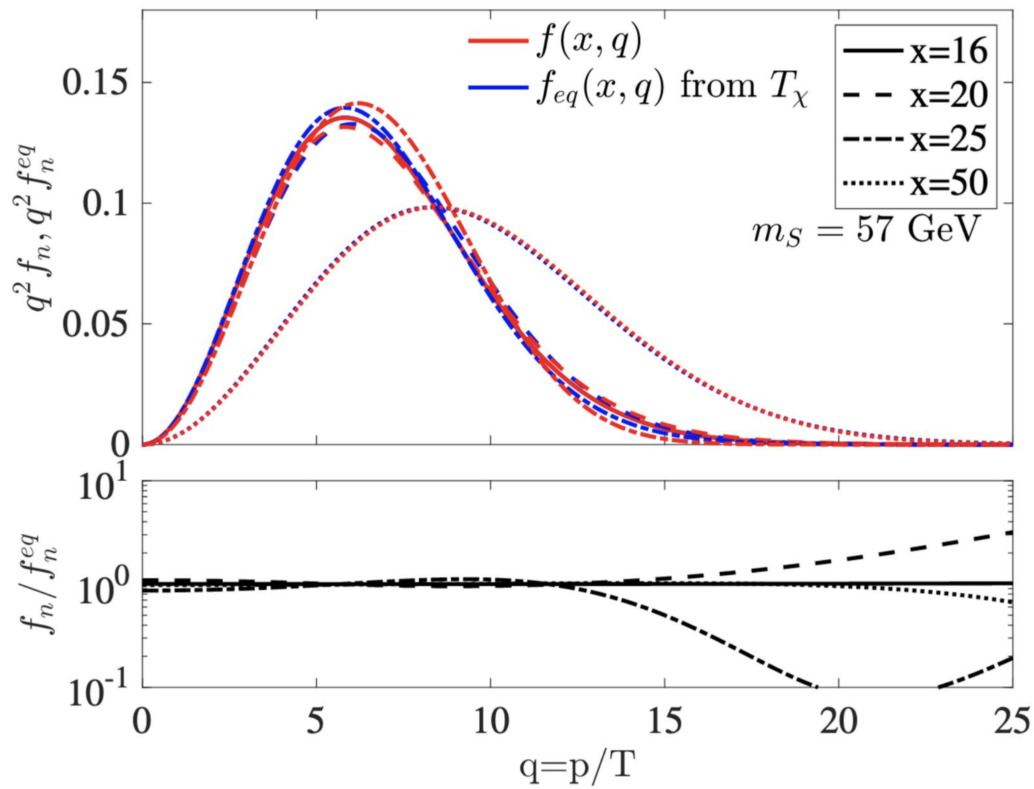


The best-fit region obtained by GAMBIT Collaboration, 1705.07931

Example: Scalar singlet DM

Binder+, 1706.07433





Deviation of relic density from the Gelmini-Gondolo approach

Distribution functions $q^2 f(q)$ and their ratio to the equilibrium ones f/f^{eq}

Velocity dependence

If number-changing process is strongly velocity dependent:

- Resonant annihilation
- Sommerfeld enhancement
- Etc.
- Threshold annihilation
- Inelastic scattering / bound state formation

Its rate is sensitive to the shape of the distribution

$$\langle \sigma v \rangle = \frac{g_\chi^2}{n_\chi^2} \int \frac{d^3p}{(2\pi)^3} \frac{d^3\tilde{p}}{(2\pi)^2} \sigma_{\rm ann} v \, f_\chi(p) f_\chi(\tilde{p})$$

What about DM self-scatterings?

$$C[f_\chi] = C_{\text{ann}} \ + \ C_{\text{dec}} \ + \ C_{\text{el}} \ + \ C_{\text{self}} + \ldots$$

Self-scatterings are usually neglected in the calculations of the relic abundance

Two (extreme) cases:

- The shape of the distribution is established by elastic scatterings (elastic scatterings are more frequent ← a lot of light SM states in the plasma) *considered before*

- Self-scatterings are very efficient in establishing DM self-equilibrium (with a $T \neq T_{SM}$) → approximation to fBE, solving for the density + DM temperature (*next slide*)

cBE – coupled system of Boltzmann equations

Assumption: DM in **self-scattering equilibrium**

$$f_i = \frac{n_i}{n_i^{\rm eq}}\, f_i^{\rm eq}(E_i, T_{\rm DM}) \qquad T_{\rm DM} \neq T_{\rm SM}$$

Integrating the fBE over $\frac{g_i}{3n_i}\int \frac{d^3p}{(2\pi)^3}\,\frac{p_i^2}{E_i}$ (2nd moment)

$$\frac{g_i}{3n_i}\int \frac{d^3p}{(2\pi)^3}\,\frac{p_i^2}{E_i}\exp(-E_i/T) = T$$

cBE – coupled system of Boltzmann equations

We get a system of coupled Boltzmann equations (**cBE**) for the density and temperature

$$\begin{cases} \dfrac{Y'}{Y} = \dfrac{m_\chi}{x\tilde{H}} C_0 \,, \\ \dfrac{y'}{y} = \dfrac{m_\chi}{x\tilde{H}} C_2 - \dfrac{Y'}{Y} + \dfrac{H}{x\tilde{H}} \dfrac{\langle p^4/E^3 \rangle}{3T_\chi} \end{cases}$$

C_0, C_2 – the corresponding moments of the collision term

$$T_\chi = y s^{2/3} / m_\chi$$

y – temperature parameter

$$\langle p^4/E^3 \rangle \equiv n_\chi^{-1} \, g_\chi \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}^4}{E^3} f_\chi(\mathbf{p})$$

Are self-scatterings important?

Self-scatterings can be more important than elastic scatterings in ***shaping*** the distribution:

- Momentum transfer $\Delta p/p \sim 1$ (for elastic $\Delta p/p \ll 1$) → less collisions required

- Couplings and vertices can be different (enhanced)

- Less constrained by observations

$$\sigma_{\rm el}/m \lesssim 10^{-34}\ {\rm cm}^2/{\rm GeV}$$

On electrons, from structure formation
Nguyen+, 2107.12380

$$\sigma_{\rm self}/m \lesssim 10^{-24}\ {\rm cm}^2/{\rm GeV}$$

From cluster collisions
Kim+, 1608.08630

The role of self-scatterings

We studied the impact of self-scatterings on the relic density and momentum distribution of DM in **2204.07078**

- We **compared** the use of different approaches (nBE, cBE, fBE without scatterings) to the full solution that includes self-scatterings

- We considered an *example model* in which the inclusion of self-scatterings is **crucial** for the correct evaluation of the relic density

(see further)

The model

Decaying scalar singlet + DM fermion + dark U(1)*

$$\mathscr{D}_\mu = \partial_\mu - ie' A'_\mu$$

$$\mathscr{L} = \mathscr{L}_{\rm SM} + \frac{1}{2}(\partial_\mu S)^2 - V(S,H) + yS\bar{\chi}\chi + m_{\rm DM}\bar{\chi}\chi + \bar{\chi} i \mathscr{D}_\mu \gamma^\mu \chi - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{\epsilon}{2}F'_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_A^2 A'_\mu A'^\mu$$

$$V(H,S) = -\mu_H^2 |H|^2 - \frac{1}{2}m_S^2 S^2 + \lambda_H |H|^4 + \frac{\lambda_S}{4}S^4 + \frac{\lambda_{HS}}{2}|H|^2 S^2$$

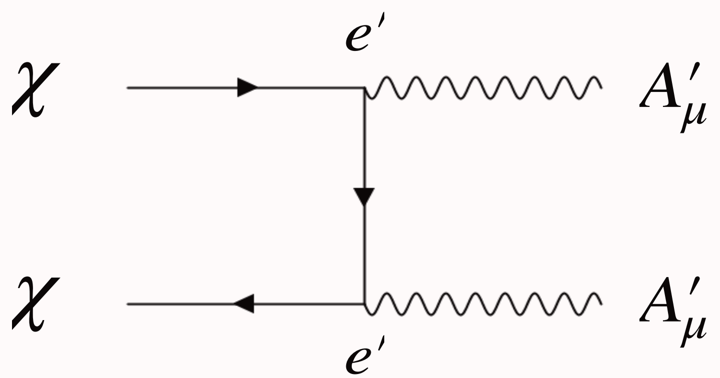
Scalar decays → non-thermal component of DM

Dark U(1) → self- and elastic-scatterings + annihilation into SM

* this model without dark U(1) was studied in a similar context in Ala-Mattinen+, 2201.06456

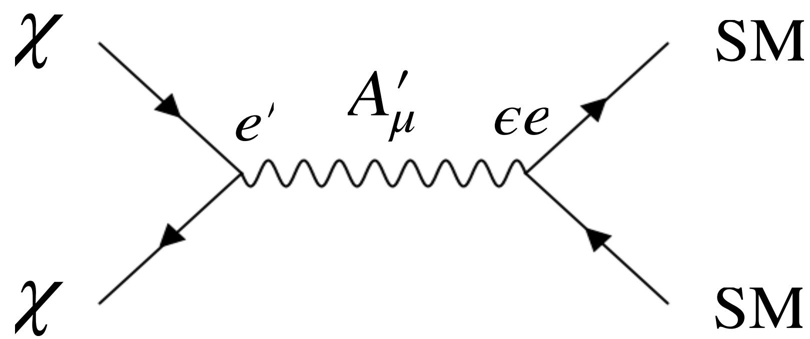
The model

Decaying scalar singlet + DM fermion + dark U(1)



Sub-threshold annihilation

$m_\chi < m_{A'_\mu}$

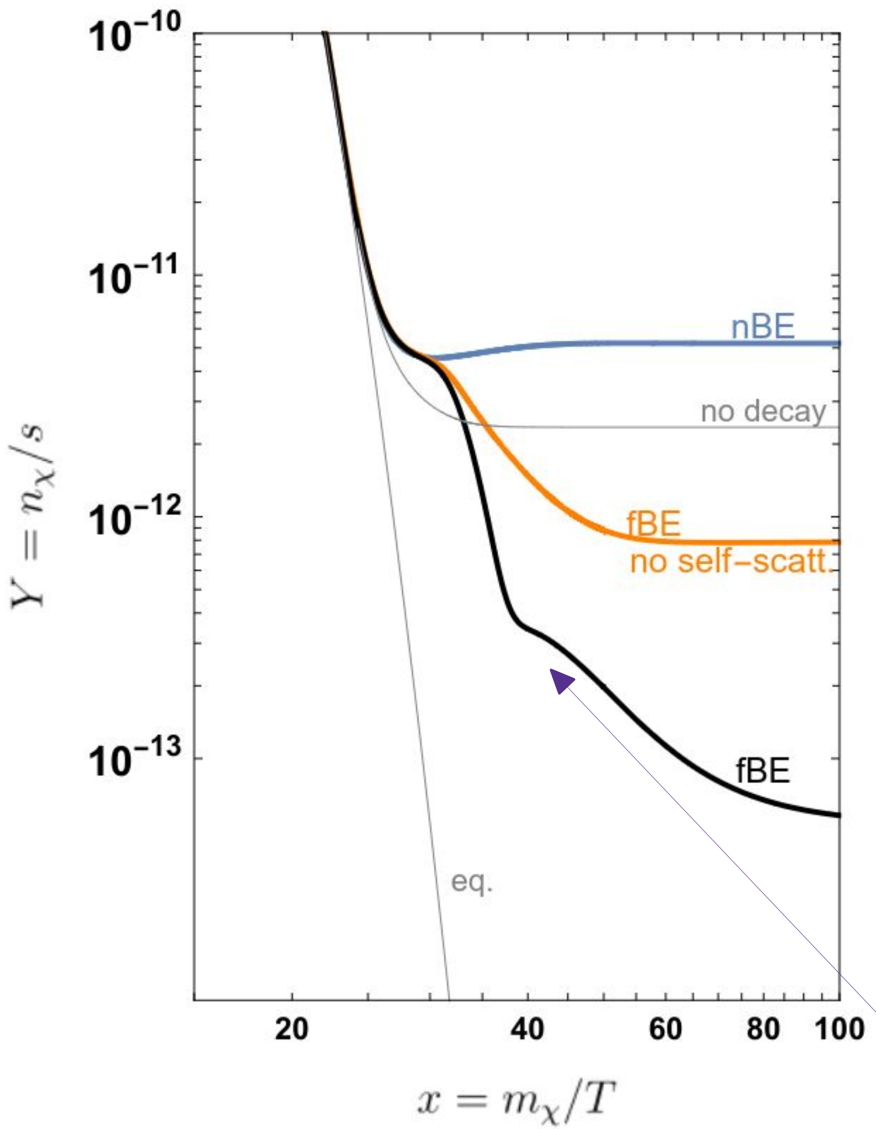


Resonant annihilation
(suppressed by the small ϵ)

Two DM annihilation processes are possible

In our case the strong velocity dependence comes from sub-threshold

Density evolution



Kinetic equilibrium:

No decay – standard freeze-out

nBE – the density is increased by the late-time decay products

sub–threshold + $S \to \bar{\chi}\chi$
$m_\chi = 100\,\mathrm{GeV}$
$m_A = 108\,\mathrm{GeV}$
$m_S = 400\,\mathrm{GeV}$
$e' = 1.$
$\epsilon = 0.001$

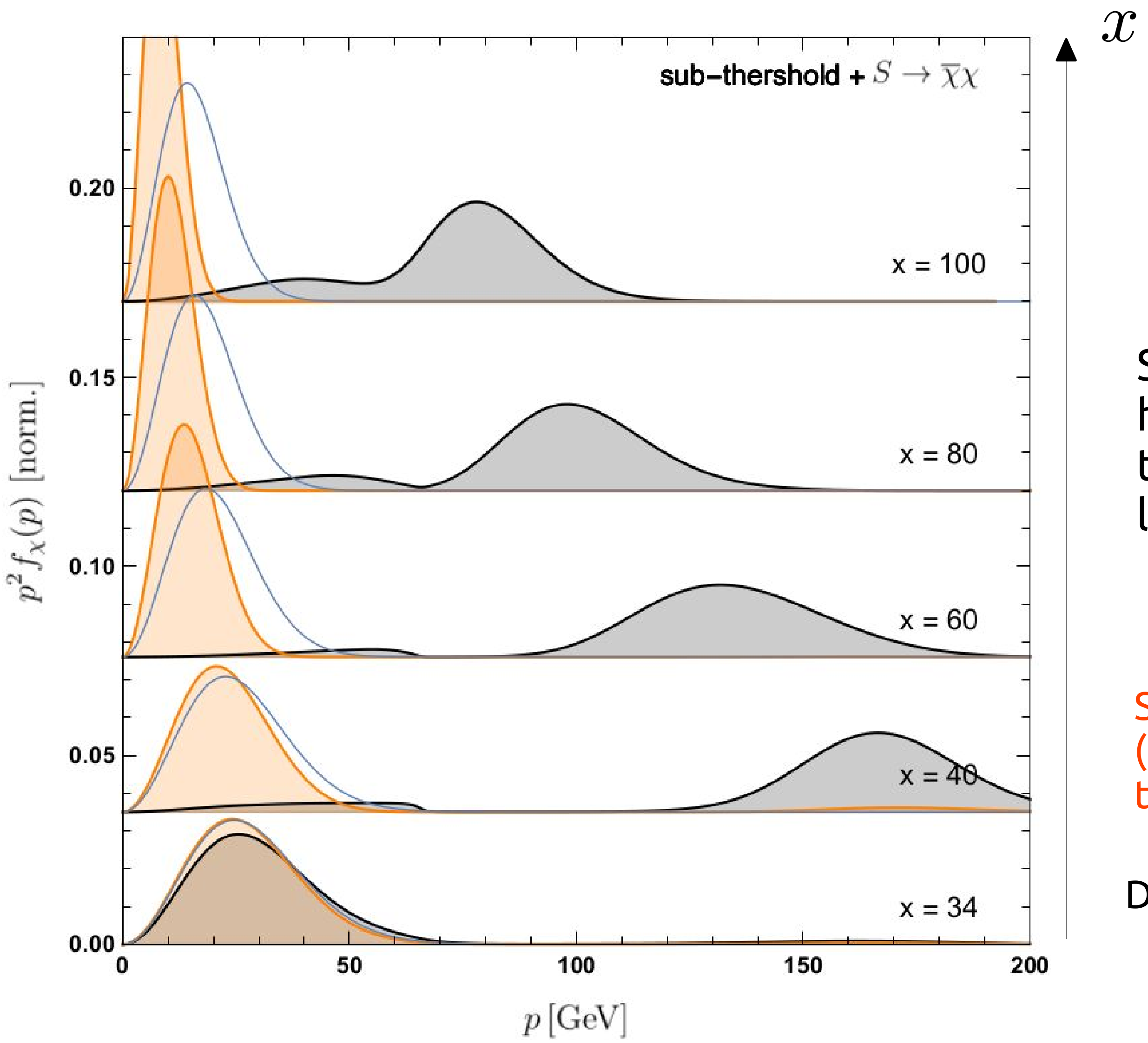
Early kinetic decoupling (fBE):

No self-scatterings – hot particles from decays extend the annihilation into SM and deplete the density

Self-scatterings *redistribute the energy* from decaying particles and extend the annihilation even longer

Decays essentially stop contributing

Distribution function



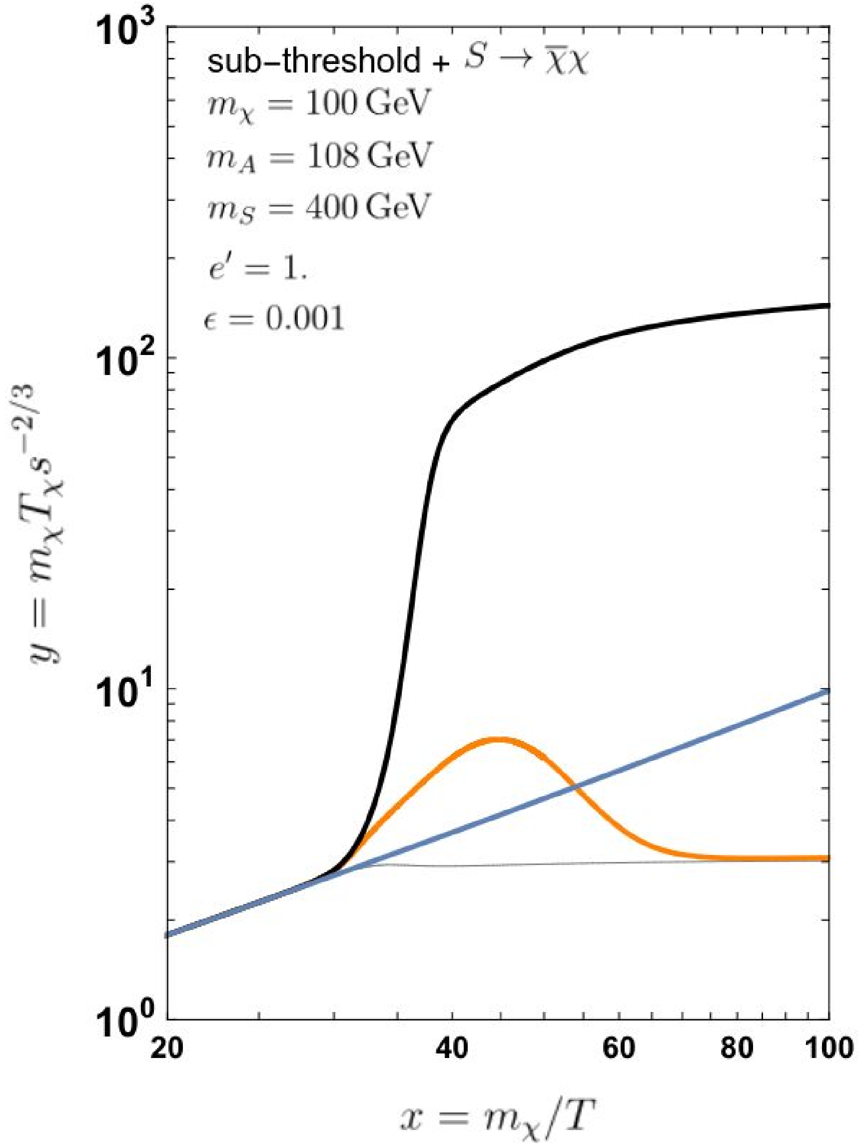
- nBE
- fBE (no scatterings)
- fBE (active scatterings)

Self-scatterings redistribute the heat and "move" the distribution towards larger momenta → larger $\langle\sigma v\rangle$

Small component of high-energy particles (most of the energy is dumped into SM through annihilations)

Decays start to contribute to the density

Temperature evolution



$$y \propto T_\chi \cdot x^2$$

Significantly heated up

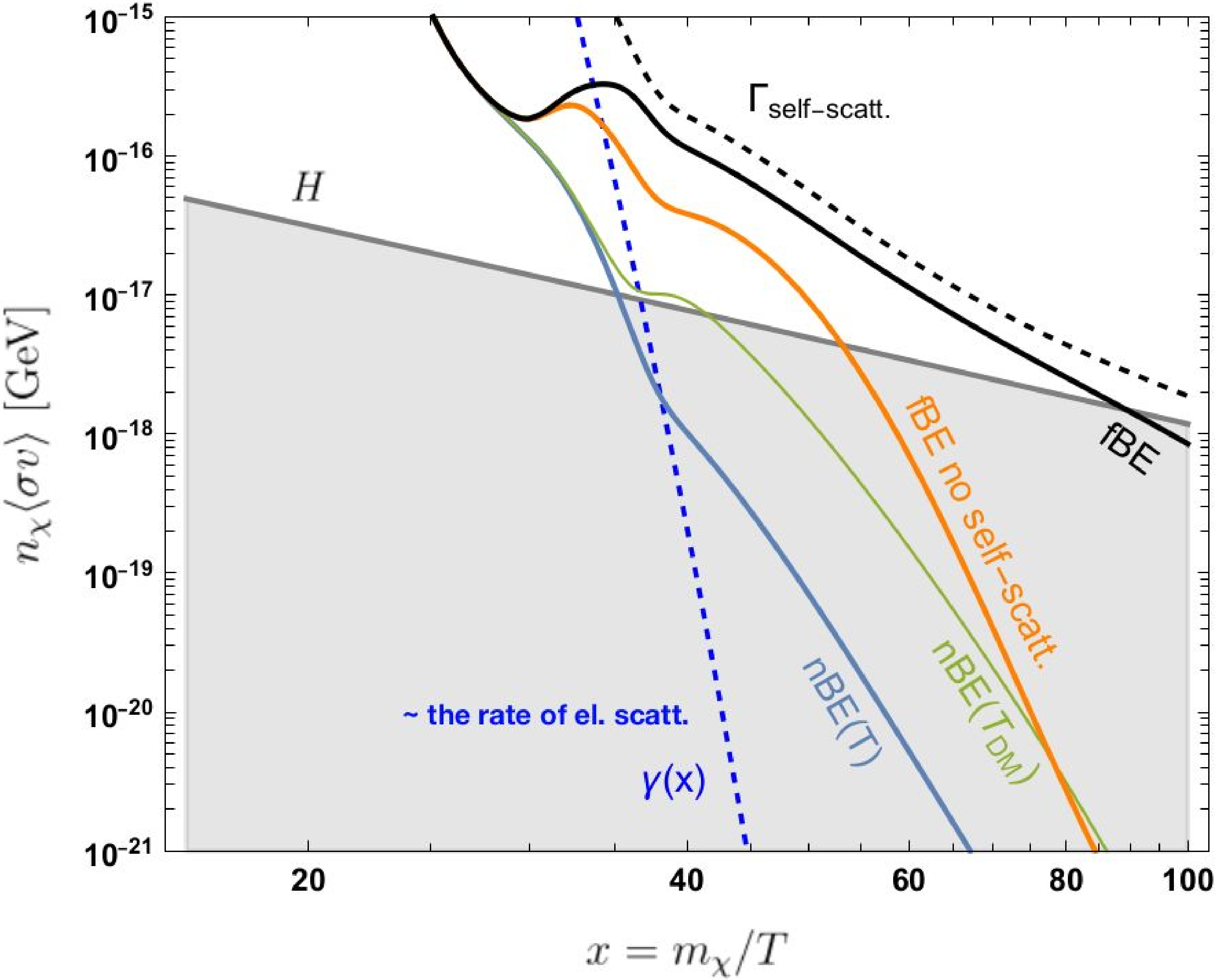
The heat from decays remains in the dark sector due to an efficient **redistribution by self-scatterings**

Temperature is equal to T_{SM} $\quad T_{\mathrm{SM}} \propto x^{-1}$

No self-scatterings – DM is slightly heated by decays and then the temperature decreases due to expansion

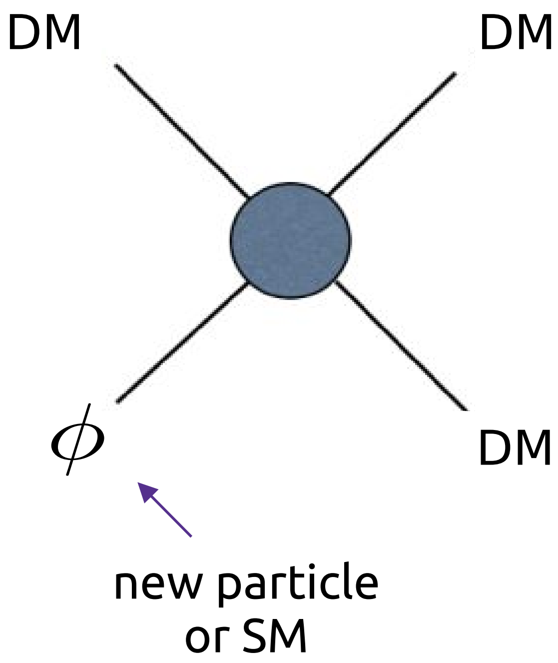
$$T_{\mathrm{non-rel}} \propto x^{-2}$$ (constant *y*)

Rates of processes



Semi-annihilation/production

Semi-production can appear from symmetry larger than Z_2



For example, scalar singlet + Z_3 complex scalar DM:

$$\mathcal{L}_{int} = \mathcal{L}_{SM} + \mathcal{L}_{\phi-SM} + \frac{\lambda}{2}\phi\left(\chi^3 + (\chi^*)^3\right)$$

Annihilation cross section is *moderately* velocity dependent, but the reaction efficiently redistributes the energy of DM → affects the rate → **affects the relic density**

Freeze-in from semi-production

We studied the deviation from equilibrium in the model with the semi-production freeze-in in **2104.05684**

- Early kinetic decoupling
- Both φ and χ have 0 initial abundances
- No VEVs
- No decays

Higgs portal interactions

$$\mathcal{L}_{\phi-SM} = A\phi H^\dagger H + \frac{\lambda_{h\phi}}{2}\phi^2 H^\dagger H - \mu_h^2 H^\dagger H + \frac{\lambda_h}{2}(H^\dagger H)^2$$

$$\mathcal{L}_{DS} = \frac{\mu_\phi^2}{2}\phi^2 + \frac{\mu_3^2}{3!}\phi^3 + \frac{\lambda_\phi}{4!}\phi^4 + \mu_\chi^2 \chi^*\chi + \frac{\lambda_\chi}{4}(\chi^*\chi)^2 + \frac{\lambda_1}{3!}\phi\left(\chi^3 + (\chi^*)^3\right) + \frac{\lambda_2}{2}\phi^2(\chi^*\chi)\,,$$

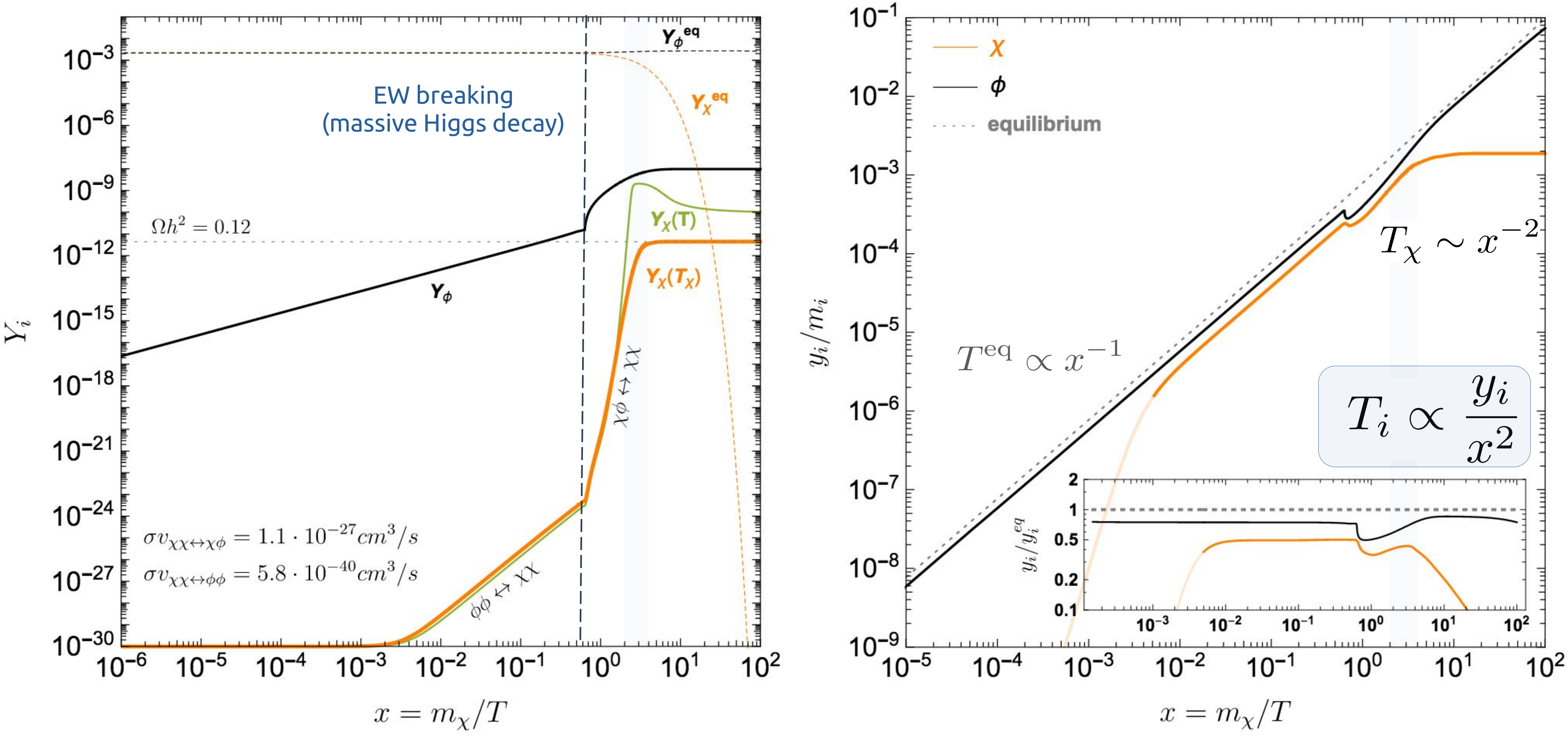
semi-production

Pair-production + elastic scatterings

We assume that self-scatterings are efficient and use cBE approach

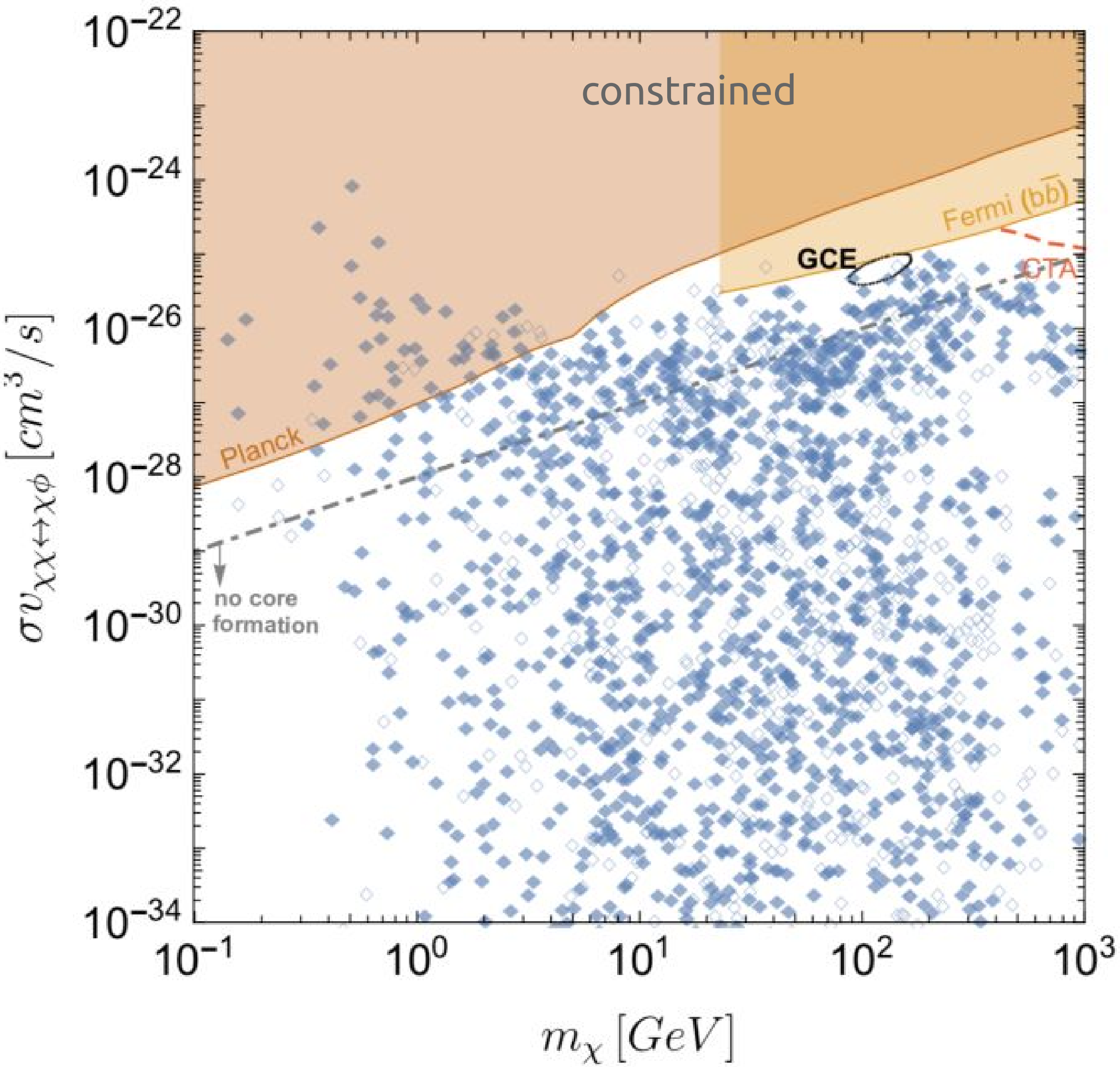
(a similar study was done by Bringmann+, 2206.10630)

Evolution of density and temperature



$$m_\chi = 100 \text{ GeV}, \ \mu_\phi = 1 \text{ GeV}, \ \lambda_1 = 1.1 \times 10^{-2}, \ \lambda_2 = 10^{-8}, \ \lambda_{h\phi} = 6 \times 10^{-11}$$

Indirect detection constraints and predictions



The results of the scan:

DM production **dominated** by semi-annihilation

Blue squares → within the reach of the future searches for φ

Potentially explain the galactic center excess (GCE)

1603.08228

Above the grey dot-dashed line → potentially explain the core formation in dSph

1803.09762

Numerical challenges in fBE

Collision term has the following structure

$$C[f_\chi] \propto \int \frac{d^3k}{(2\pi)^3 E_k} \cdots \Big[f_{\rm SM}(\tilde{p}) f_{\rm SM}(\tilde{k}) (1 \pm f_\chi(p))(1 \pm f_\chi(k)) \\ - f_\chi(p) f_\chi(k) (1 \pm f_{\rm SM}(\tilde{p}))(1 \pm f_{\rm SM}(\tilde{k})) \Big]$$

For 2→2 annihilation

We can expand the collision term in different terms in *orders of the distribution functions*

Terms of order O(f) > 2 are often ommitted

Numerical challenges in fBE

Collision terms have the following structure

$$C[f_\chi] \propto f_\chi(p) \int \ldots \int \frac{d^3k}{(2\pi)^3\, 2\omega} \ldots$$

Distribution function is not integrated over
Can be solved explicitly

- *Decay terms*
- *Quantum corrections*
- *Elastic scatterings*
- *Co-scatterings*
- *Etc.*

$$C[f_\chi] \propto \int \ldots \int \frac{d^3k}{(2\pi)^3\, 2\omega} f_\chi(k) \ldots$$

Distribution function has to be integrated over
Integro-differential equation

- *Annihilations*
- *Self-scatterings*
- *Quantum corrections to elastic scatterings*
- *N-state processes (N > 2)*

fBE → system of fBEs

$$f_\chi(k) \rightarrow \{ f_1(k_1), \ldots, f_n(k_n) \} \qquad \int \frac{dk}{E_k} \rightarrow \sum_i \frac{(\Delta k)}{E_k^i}$$

Numerical integration

Numerical challenges in fBE

Annihilation term has only one unknown function in the *lowest order* in f_χ

$$C_{\text{ann}}[f_\chi(p)] \propto \int \underline{d^3k} \int d^3\tilde{p} \int d^3\tilde{k} \quad f_\chi(p)\underline{f_\chi(k)}$$

Self-scattering inevitably has 2 unknown functions

$$C_{\text{self}}[f_\chi(p)] \propto \int d^3k \int \underline{d^3\tilde{p}} \int \underline{d^3\tilde{k}} \quad \underline{f_\chi(\tilde{k})f_\chi(\tilde{p})}$$

Backward term for self-scattering

Requires more summations over discretized distribution functions

Elastic scatterings

$$C_{\rm el} = \frac{1}{2g_\chi}\int \frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{E}}\int \frac{d^3k}{(2\pi)^3 2\omega}\int \frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}}$$
$$\times\,(2\pi)^4\,\delta^{(4)}(\tilde{p}+\tilde{k}-p-k)\,|\mathcal{M}|^2_{\chi f\leftrightarrow\chi f}$$
$$\times\,\big[\big(1\mp g^\pm(\omega)\big)\,g^\pm(\tilde{\omega})\underline{f_\chi(\tilde{E})} - (\omega\leftrightarrow\tilde{\omega}, E\leftrightarrow\tilde{E})\big]$$

Can be approximated in the limit of low momentum transfer ($\delta p/p \ll 1$)

$$C_{\rm el} \simeq \frac{E}{2}\gamma(T)\left[TE\partial_p^2 + \left(2T\frac{E}{p}+p+T\frac{p}{E}\right)\partial_p + 3\right]\underline{f_\chi}\,,$$

where the momentum exchange rate $\gamma(T)$ is given by

$$\gamma(T) = \frac{1}{48\pi^3 g_\chi m_\chi^3}\int d\omega\, g^\pm\partial_\omega\left(k^4\left\langle|\mathcal{M}|^2\right\rangle_t\right)$$

Fokker-Planck type approximation

As described in
Binder+, 1706.07433

How is fBE solved?

DRAKE code for the calculation of DM abundance

Written in *Mathematica* language

DRAKE
Dark matter Relic Abundance beyond Kinetic Equilibrium

https://drake.hepforge.org Binder+, 2103.01944

The current version solves **nBE**, **cBE** and **fBE** for the freeze out of 2-2 annihilation processes

For our studies we included:

- **Decays**
- **Self-scatterings** – implemented a C++ patch for *fast numerical intergration* of the collision term integrals
- Patches for *fast integration* of cBE moments of collision term

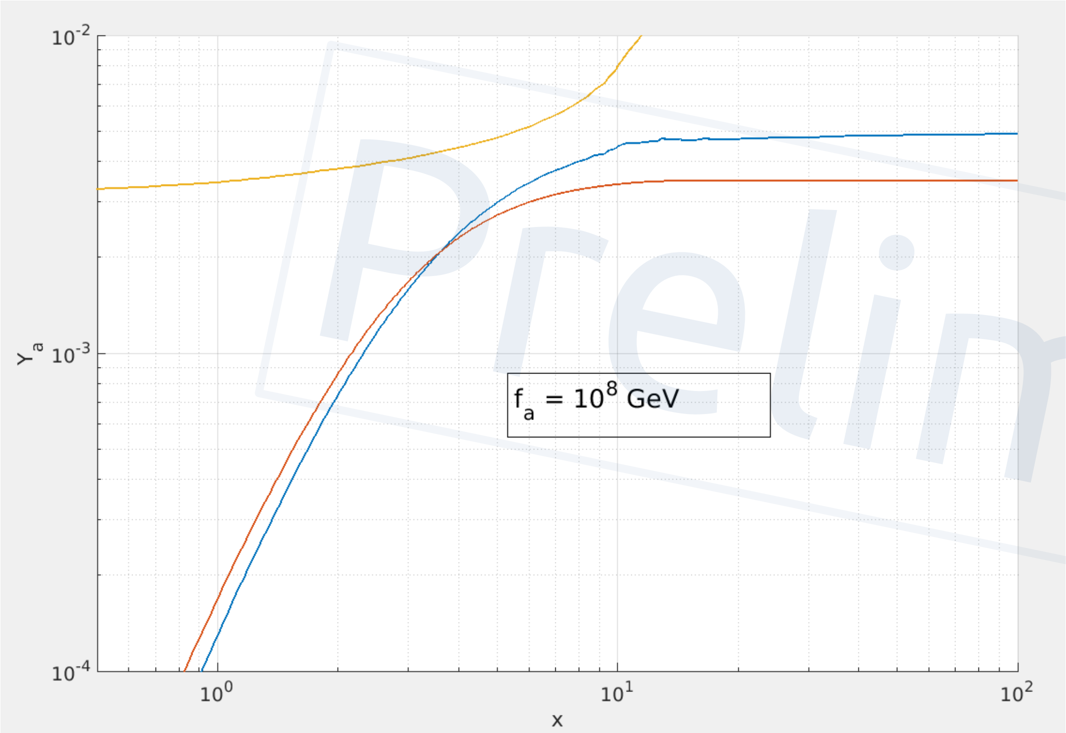
Conclusion

- The *interplay of different interactions* in some DM models can lead to a deviation of the DM energy distribution from the equilibrium shape
- The shape of the distribution affects the rate → affects **the relic density**. The effect is pronounced if:
  - Cross section is strongly velocity dependent
  - Weak elastic/self-scatterings *w.r.t. annihilation*
- To account for these effects one has to solve the **fBE** or **cBE** (if the dark sector is self-thermalized)

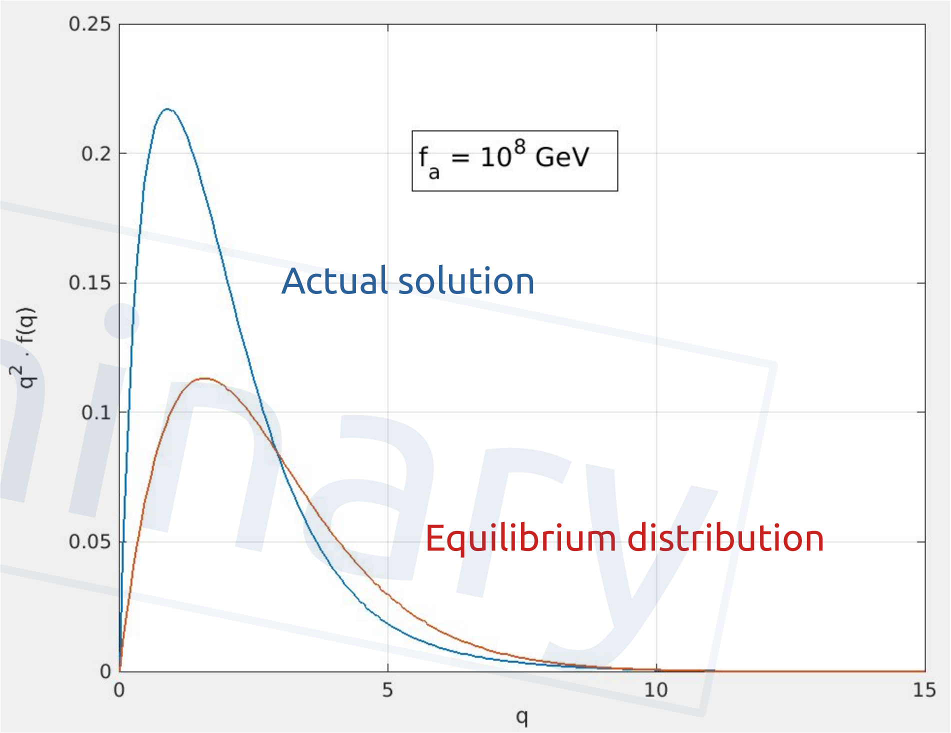
Axions and lepton-flavour violating decays

Axions contribute to the effective number of relativistic dof and can be constrained

In progress, with M. Badziak



Evolution of the number density of axions from tau decays



Distribution function of axions at the end of evolution