Joint cosmological and gravitational-wave population inference using dark sirens and galaxy catalogues

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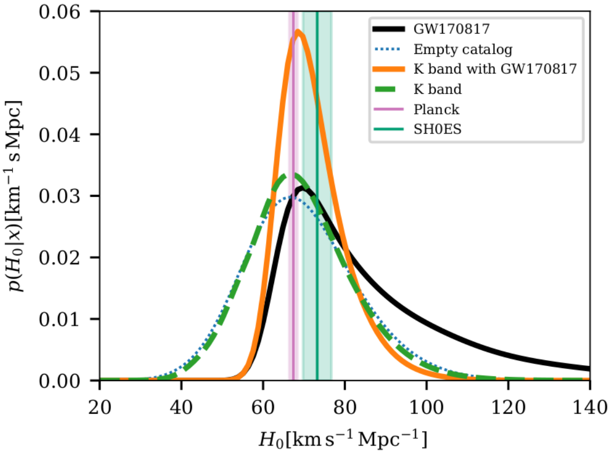
O3 result LVK Cosmology group



Constraints on the cosmic expansion history from the third LIGO–Virgo–KAGRA Gravitational-Wave Transient Catalog (using `gwcosmo`)

Why did we need to rewrite the method?

The previous version of `gwcosmo` was slow

- We expect more events during Run O4
- The resulting H_0 posterior depends on
  - CBC mass distribution
  - CBC merger rate (with z)

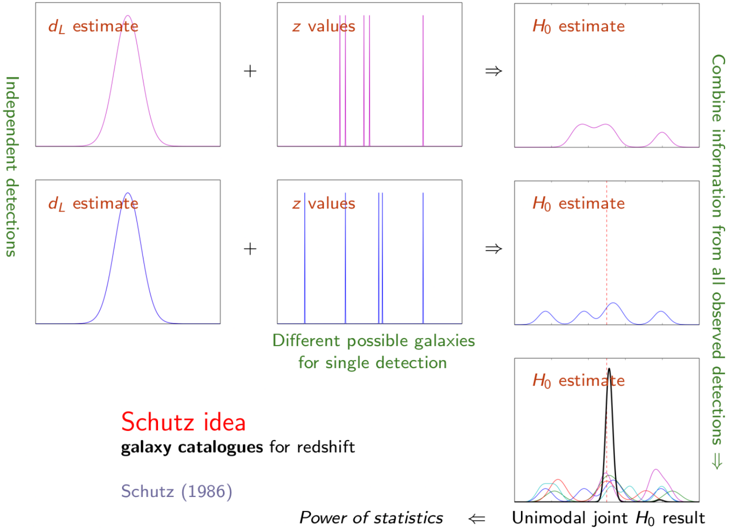
$\longrightarrow$ informative priors impact the result and introduce a bias

Before, `gwcosmo` was too computationally expensive to vary these inputs. Thus, many hyperparameters were fixed

The new version (1000x faster) allows joint estimation of cosmological and compact binary population parameters

Content

‘Dark Siren’ method



Schutz idea

galaxy catalogues for redshift

Schutz (1986)

Theory

Old method:

$$p(x_{\rm GW}|D_{\rm GW},H_0,I)=\frac{1}{N_{\rm pix}}\sum_i^{N_{\rm pix}}\left[\frac{p(x_{\rm GW}|\Omega_i,G,H_0,I)}{p(D_{\rm GW}|\Omega_i,G,H_0,I)}p(G|\Omega_i,D_{\rm GW},H_0,I)+\frac{p(x_{\rm GW}|\Omega_i,\bar{G},H_0,I)}{p(D_{\rm GW}|\Omega_i,\bar{G},H_0,I)}p(\bar{G}|\Omega_i,D_{\rm GW},H_0,I)\right]$$

New method:

$$p(x_{{\rm GW}i}|D_{{\rm GW}i},\Lambda)=\frac{\displaystyle\int\sum_j^{N_{\rm pix}}p(x_{{\rm GW}i}|z,\Omega_j,\Lambda,I)p(z|\Omega_j,\Lambda,I)dz}{\displaystyle\int p(D_{\rm GW}|z,\Lambda,I)\sum_j^{N_{\rm pix}}p(z|\Omega_j,\Lambda,I)dz}.$$

Theory

Old method:

Sum over galaxies

2 x 2D-integrals

1 x 2D-integrals

$$p(x_{\rm GW}|D_{\rm GW},H_0,I) = \frac{1}{N_{\rm pix}}\sum_i^{N_{\rm pix}}\left[\frac{p(x_{\rm GW}|\Omega_i,G,H_0,I)}{p(D_{\rm GW}|\Omega_i,G,H_0,I)}p(G|\Omega_i,D_{\rm GW},H_0,I)+\frac{p(x_{\rm GW}|\Omega_i,\bar{G},H_0,I)}{p(D_{\rm GW}|\Omega_i,\bar{G},H_0,I)}p(\bar{G}|\Omega_i,D_{\rm GW},H_0,I)\right]$$

Total: ~ Npix x 4 x 2D-integrals for every H_0 value.

New method:

$$p(x_{{\rm GW}i}|D_{{\rm GW}i},\Lambda) = \frac{\int\sum_j^{N_{\rm pix}} p(x_{{\rm GW}i}|z,\Omega_j,\Lambda,I)p(z|\Omega_j,\Lambda,I)dz}{\int p(D_{\rm GW}|z,\Lambda,I)\sum_j^{N_{\rm pix}} p(z|\Omega_j,\Lambda,I)dz}.$$

Theory

Constants that will cancel in the main computation (therefore ignorable)

Sum over galaxies in the catalogue(assuming e.g. Gaussian uncertainty - no need to sample), weighted by luminosity

$$p(z|\Omega_i, H_0, s, I) = \frac{1}{p(s|\Omega_i, H_0, I)p(s|I)} \times p(s|z, I)\left[p(G|\Omega_i, H_0, I)\frac{1}{N_{\rm gal}(\Omega_i)}\sum_k^{N_{\rm gal}(\Omega_i)} \mathcal{G}(z - z_k; \sigma_k)p(s|M(z, m_k, H_0), I) + p(z|I)\int_{M(z, m_{\rm th}(\Omega_i), H_0)}^{M_{\rm max}} p(M|H_0, I)p(s|M, I)\, dM\right]$$

Merger rate evolution with redshift

Probability that a galaxy (in this pixel) is in the catalogue

"Out of catalogue" part

$$p(G|\Omega_i, H_0, I) = \int_0^\infty \int_{M_{\rm min}}^{M(z, m_{\rm th}(\Omega_i), H_0)} p(z|I)p(M|H_0, I)\, dM dz.$$

Theory

$$p(z|\Omega_i, H_0, s, I) = \frac{1}{p(s|\Omega_i, H_0, I)p(s|I)}$$

$$\times\, \boxed{p(s|z,I)} \boxed{\left[p(G|\Omega_i, H_0, I)\frac{1}{N_{\rm gal}(\Omega_i)} \sum_{k}^{N_{\rm gal}(\Omega_i)} \mathcal{G}(z - z_k; \sigma_k) p(s|M(z, m_k, H_0), I) + p(z|I) \int_{M(z, m_{\rm th}(\Omega_i), H_0)}^{M_{\rm max}} p(M|H_0, I) p(s|M, I)\, dM \right]}$$

Precompute this part

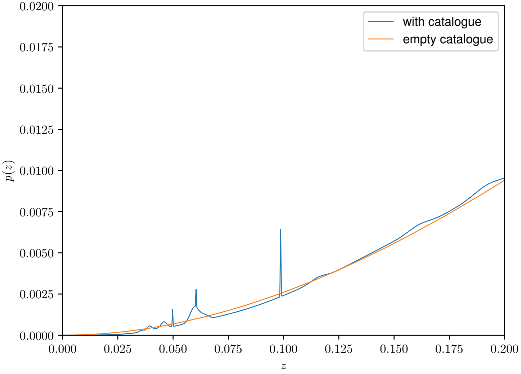
Can include the rates term afterwards (ie GW merger rate model can be applied during likelihood evaluation)

LOS redshift prior

One healpy pixel

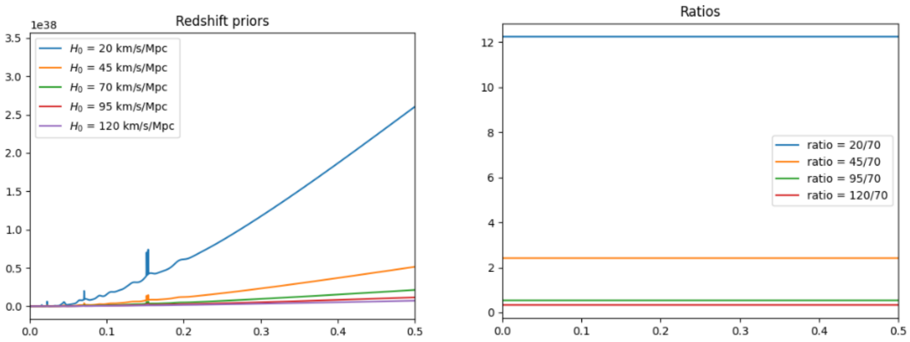
In-catalogue contribution: Gaussian peaks corresponding to galaxies

Out-of-catalogue contribution: Galaxies uniformly distributed in comoving volume and luminosities following a Schechter function distribution



LOS redshift prior

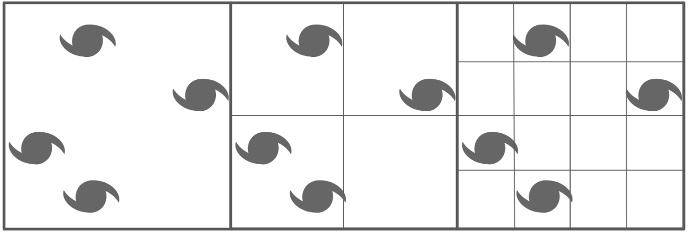
The H_0 dependence comes down to a normalization constant



The same prior is used to evaluate the GW likelihood and GW selection effects, so this dependence cancels

LOS redshift prior - angular resolution

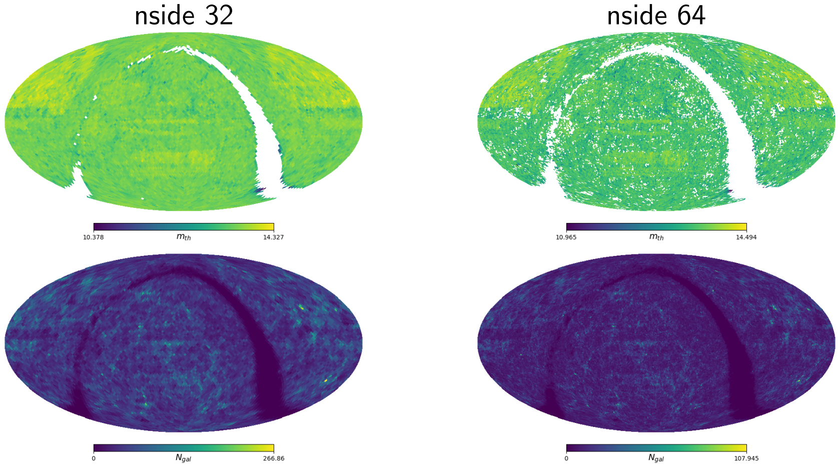
Implicit angular resolution dependence in galaxy weighting and magnitude thresholds



Ensure convergence for increasing angular resolution
Define 2 levels of resolution: n_{map} and n_{high}
Calculate the effective number of galaxies at n_{map} and adjust to n_{high}

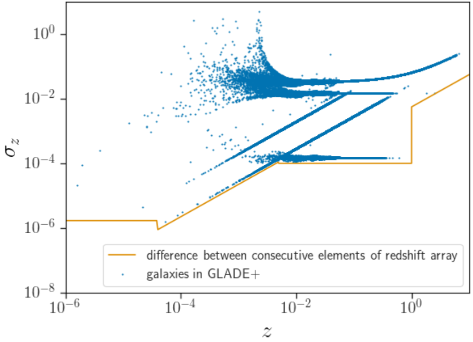
LOS redshift prior

Angular resolution: control galaxy weighting using precomputed skymaps



LOS redshift prior

z resolution

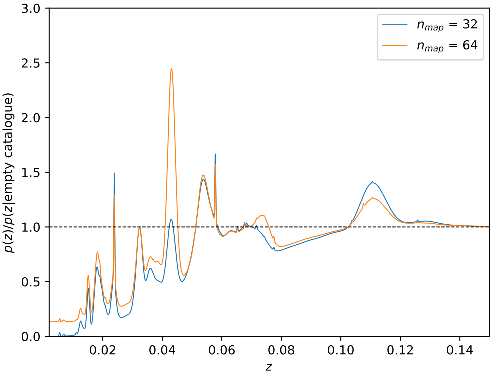


LOS redshift prior

GW190814

Weighted sum of LOS zpriors of multiple pixels

Illustrates over- and underdensities compared to the empty catalogue distribution



Joining spectral sirens

- **Spectral sirens:** features of the mass distribution of the compact object population break the mass-redshif degeneracy.
- Have been always carried out separately due to computational difficulty.
- Novel method for carrying out cosmological and GW population parameters jointly.

Selection effects

- In the previous version of *gwcosmo* the only parameter being constrained was H_0.
- Now, we calculate GW selection effects for various hyperparameters using MCMC or Nested Sampling.

1. Theory
2. LOS redshift prior
3. Other improvements
4. **Results and comparison**

Results and comparison

GW Events from GWTC-3 catalogue with SNR > 11

- 42 BBHs
- 2 NSBH (GW200105 and GW200115)
- 2 BNS (GW170817 and GW190425)
- 1 asymmetric mass binary (GW190814), treated as NSBH

GLADE+ galaxy catalogue K-band

Compare results from the previous and new method

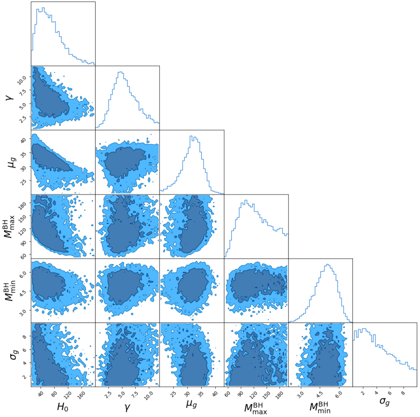
3 separate analyses

1. A pure population analysis using 42 BBHs
2. A galaxy catalogue analysis with fixed population assumptions using all the BBHs, NSBHs and BNSs
3. A population + galaxy catalogue analysis which uses all GW events and galaxy catalogue information, and jointly estimates cosmological and population parameters

Results and comparison

1) The population analysis of GWTC-3 BBHs

In agreement with `icarogw`



Results and comparison

2) The galaxy catalogue analysis of GWTC-3 with fixed population assumptions

Mass models

- BBH
  Powerlaw + peak
- NSBH
  m_1: Powerlaw + peak
  m_2: Uniform between $M_{\text{min}}^{\text{NS}}$ and $\min(M_{\text{max}}^{\text{NS}}, m_1)$

Merger rate evolution

Madau-Dickinson-like distribution

$$R(z) = R_0(1+z)^\gamma \frac{1+(1+z_p)^{-(\gamma+\kappa)}}{1+\left(\frac{1+z}{1+z_p}\right)^{\gamma+\kappa}}$$

Used parameters match the previous analysis

Results and comparison

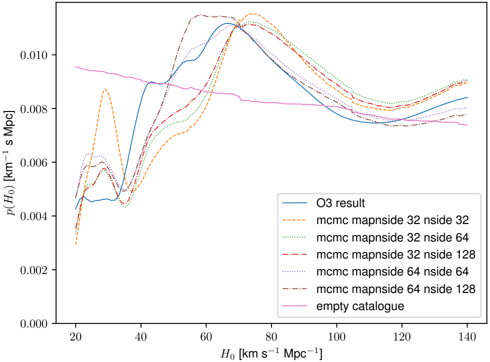
GW190814

Most informative event (nearby and small sky location area)

H_0 posterior shape more impacted by n_{map} than n_{high}

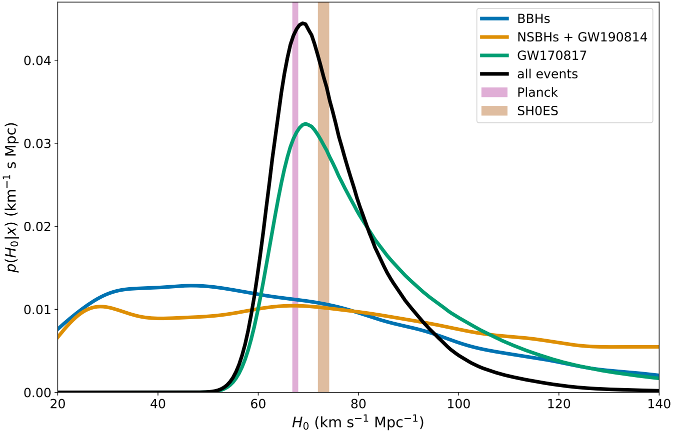
n_{map}=32 results are more robust (high number statistics)

Convergence at high n_{high}

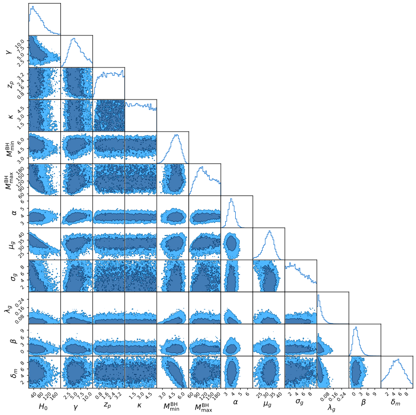


Results and comparison

2) The galaxy catalogue analysis of GWTC-3 with fixed population assumptions



3) Combined cosmological and population inference using GWTC-3
Less informative result but far more robust and no longer susceptible to bias due to fixed assumptions about the GW population



Appendix

Dictionary

Parameter	Definition
H_0	The Hubble constant
$x_{\rm GW}$	The GW data associated with some GW source, s.
$D_{\rm GW}$	Denotes that a GW signal was detected, *i.e.* that $x_{\rm GW}$ passed some detection statistic threshold $\rho_{\rm th}$.
g	Denotes that a galaxy is (G), or is not ($\bar{G}$), contained within the galaxy catalog.
s	Denotes that a GW signal was emitted.
M	Absolute magnitude.
z	Redshift.
Ω	Sky location (right ascension and declination).
d_L	Luminosity distance.
m	Apparent magnitude.
$m_{\rm th}$	Apparent magnitude threshold of the galaxy catalog.
$\rho_{\rm th}$	SNR threshold of the detector network.
Cosmological model	The cosmological model assumed for the analysis. Typically a Friedmann-Lemaître-Robertson-Walker universe.
Schechter parameters	The parameters which characterize the assumed absolute magnitude distribution of galaxies in the universe.
GW population	The assumed underlying population of GW sources.
Source frame parameters	Source frame parameters of a GW source, *e.g.* component masses, spins, inclination and polarization.
Detector frame parameters	As above, but redshifted into the detector frame.
Detector configuration	The network set up, including which detectors are included in the search and their noise floors.

Priors

POWER LAW + PEAK

Parameter	Description	Prior
$M_{\rm min}^{\rm BH}$	Minimum mass of the PL component of the black hole mass distribution.	$\mathcal{U}(2.0\mathrm{M}_\odot, 10.0\mathrm{M}_\odot)$
$M_{\rm max}^{\rm BH}$	Maximum mass of the PL component of the black hole mass distribution.	$\mathcal{U}(50.0\mathrm{M}_\odot, 200.0\mathrm{M}_\odot)$
α	Spectral index for the PL of the primary mass distribution.	$\mathcal{U}(1.5, 12.0)$
μ_g	Mean of the Gaussian component in the primary mass distribution.	$\mathcal{U}(20.0\mathrm{M}_\odot, 50.0\mathrm{M}_\odot)$
σ_g	Width of the Gaussian component in the primary mass distribution.	$\mathcal{U}(0.4\mathrm{M}_\odot, 10.0\mathrm{M}_\odot)$
λ_g	Fraction of the model in the Gaussian component.	$\mathcal{U}(0.0, 1.0)$
δ_m	Range of mass tapering on the lower end of the mass distribution.	$\mathcal{U}(0.0\mathrm{M}_\odot, 10.0\mathrm{M}_\odot)$
β	Spectral index for the PL of the secondary mass distribution.	$\mathcal{U}(-4.0, 12.0)$
$M_{\rm min}^{\rm NS}$	Minimum mass of the uniform neutron star mass distribution.	$\mathcal{U}(1.0\mathrm{M}_\odot, 1.5\mathrm{M}_\odot)$
$M_{\rm max}^{\rm NS}$	Maximum mass of the uniform neutron star mass distribution.	$\mathcal{U}(2.0\mathrm{M}_\odot, 5.0\mathrm{M}_\odot)$

Table 1. Summary of mass distribution parameters with the corresponding prior ranges.

MERGER RATE SHAPE PARAMETERS

Parameter	Description	Prior
R_0	Local merger rate.	$1/R_0$ (implicit)
γ	Power-law index describing the merger rate at low redshift.	$\mathcal{U}(0, 12.0)$
κ	Power-law index describing the merger rate at high redshift.	$\mathcal{U}(0, 6.0)$
z_p	The redshift where the slope of the merger rate changes.	$\mathcal{U}(0, 4.0)$

Table 2. Summary of merger rate shape parameters with the corresponding priors.