Applications of the gauge/gravity duality

Black holes, fluid dynamics and instabilities

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Inter-University Attraction Pole meeting, *Louvain University*, 22/01/2009

Gravity / Fluid duality

- AdS / CFT duality:

  Certain strongly coupled field theories are dual to classical gravitational theories with negative cosmological constant

- Black holes as fluids: the membrane paradigm

  Laws of black hole mechanics $dM = T dA$

  $dA \geq 0$

  $\Updownarrow$

  Fluid with surface tension $T \sim \kappa$

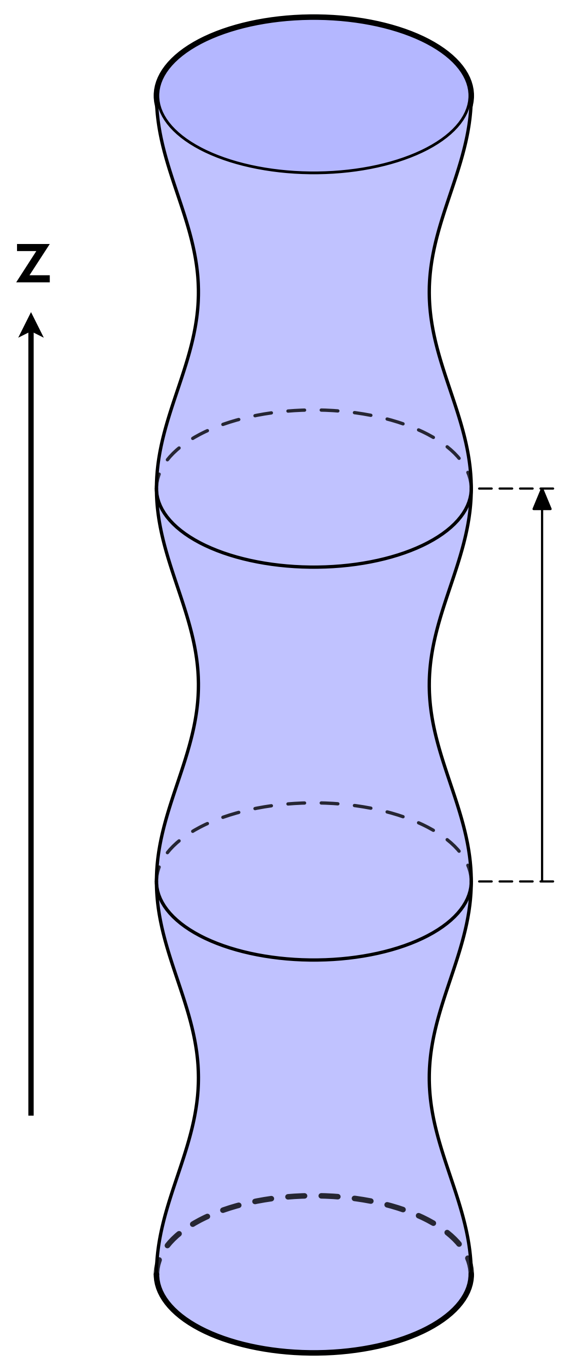
- Allows to study thermodynamic and transport properties of strongly coupled QFT...

Gregory-Laflamme Instability

Gregory-Laflamme,hep-th/9404071

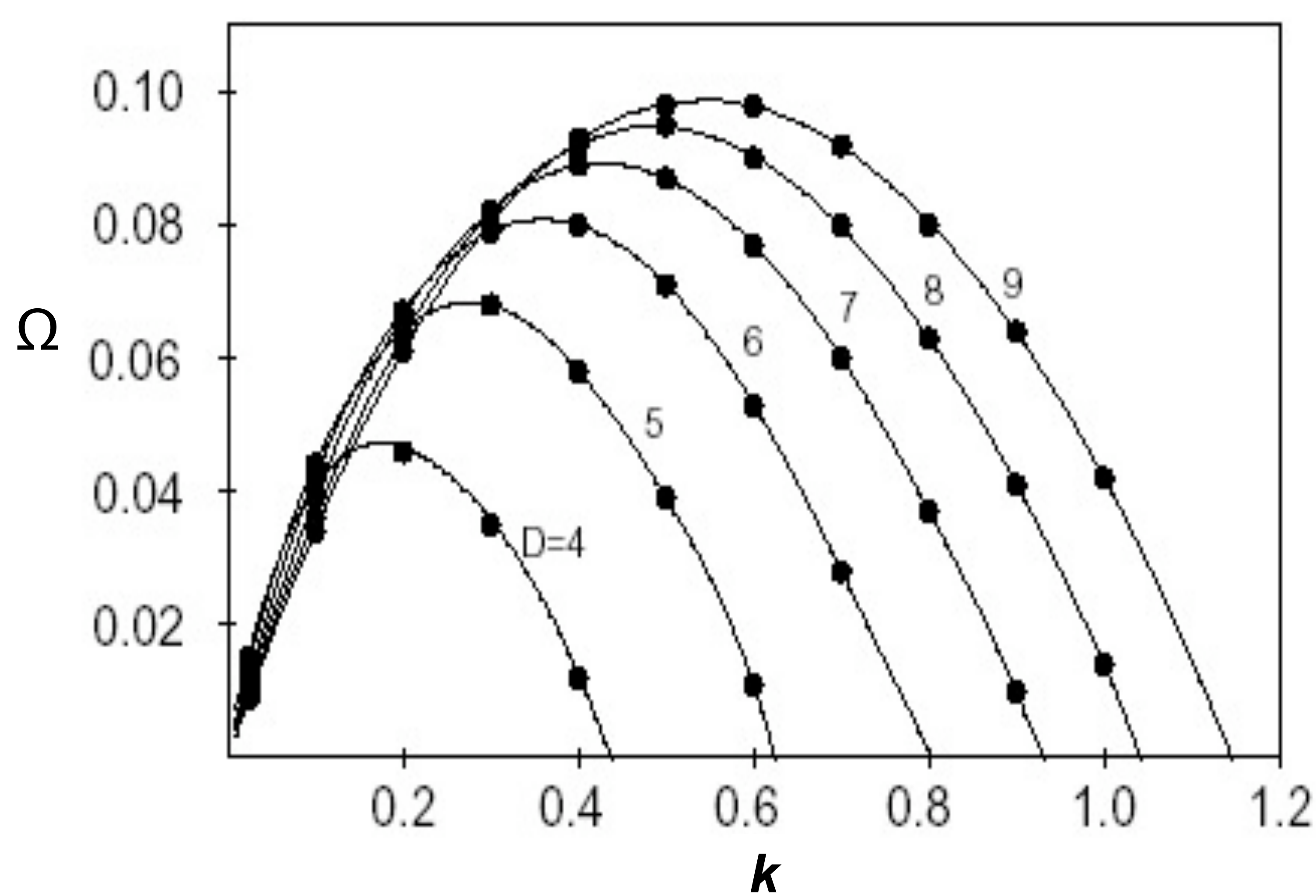
perturbations $g_{\mu\nu} + \epsilon h_{\mu\nu}$

$$h_{\mu\nu} = \mathrm{Re}\left[e^{\frac{\Omega t}{r_h} + i\frac{kz}{r_h}} P_{\mu\nu(r)} \right]$$



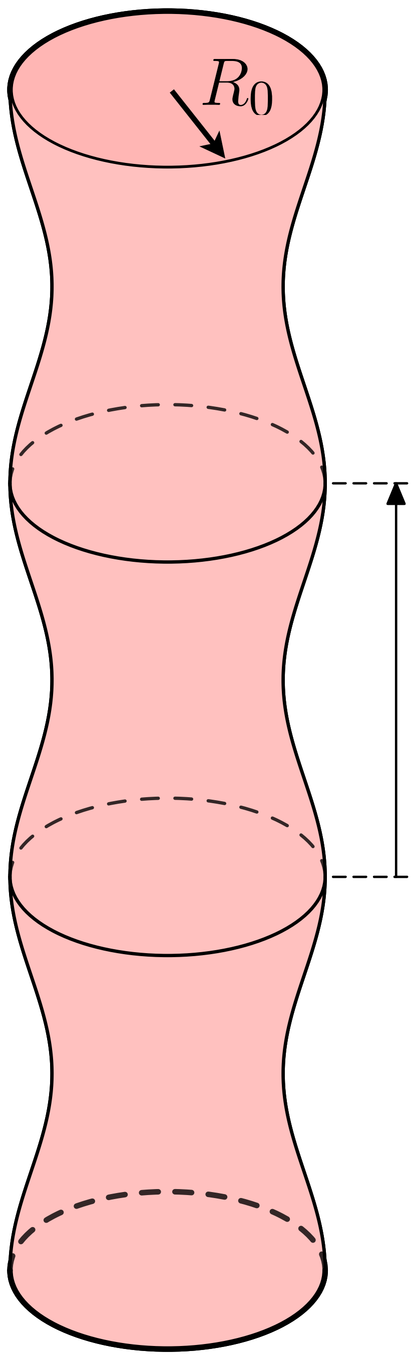
$\lambda = \frac{2\pi}{k}$

instability for $\lambda > \lambda_{GL} = \dfrac{2\pi r_h}{k_c}$



Rayleigh-Plateau Instability

Plateau, 1849



Fluctuations with $\lambda > 2\pi R_0$ decrease the energy and grow exponentially in time

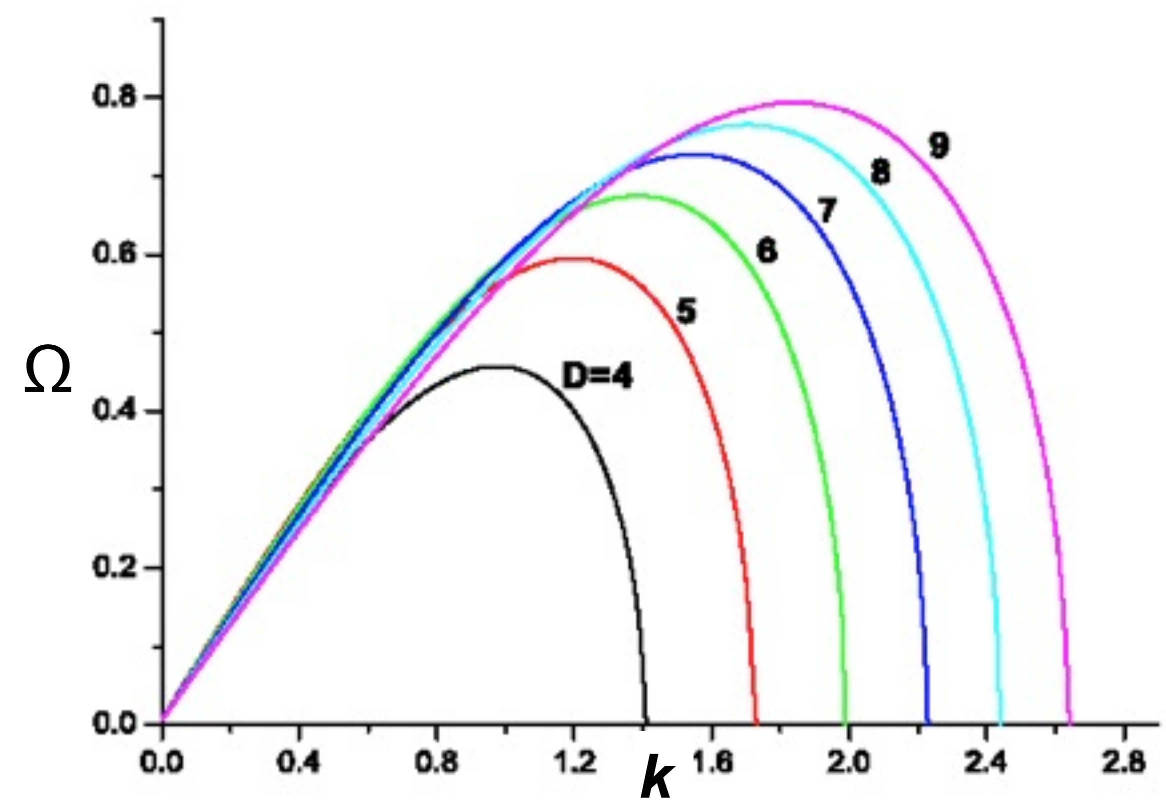
$$R_0 k_c = \sqrt{D-2}$$

$\left(R_0 k_{GL} = \sqrt{D}, \text{large } D \right)$

Kol & Sorkin,hep-th/0407058

Qualitatively RP=GL!

V. Cardoso, O.J.C. Dias,hep-th/0602017



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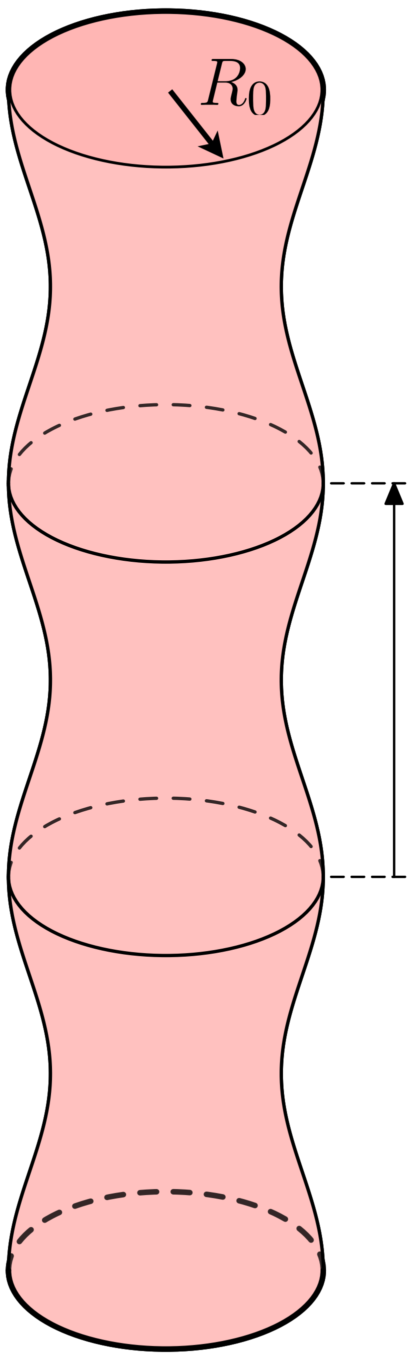
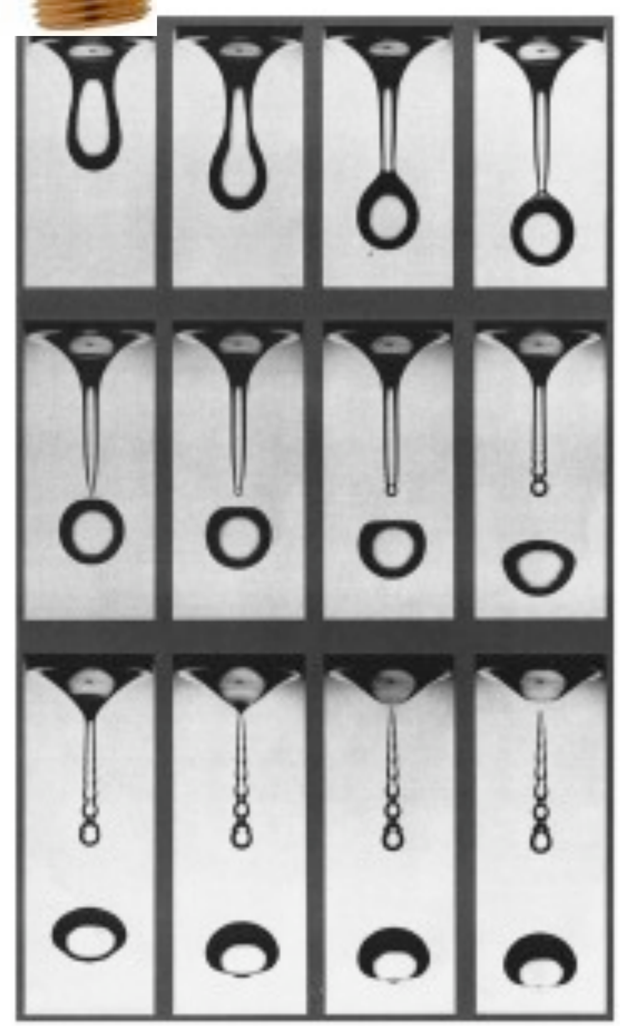


Fluctuatio h $\lambda > 2\pi R_0$ decrease the
energy an v exponentially in time

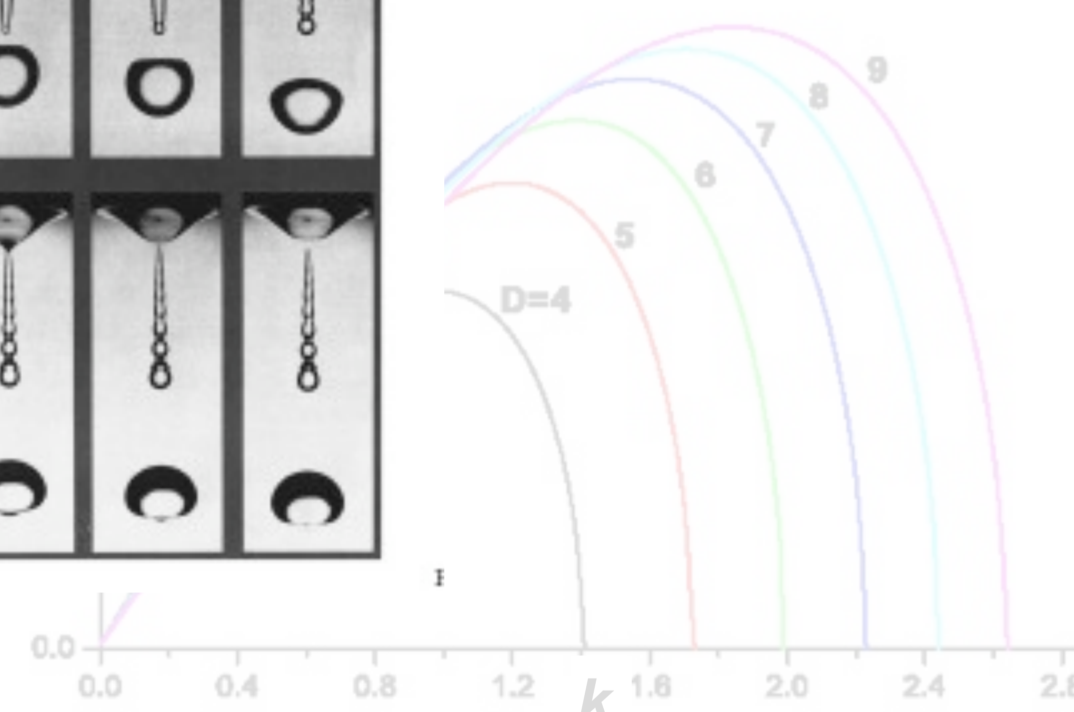
$= \frac{2\pi}{k}$



$R_0 k_c = \sqrt{D -$

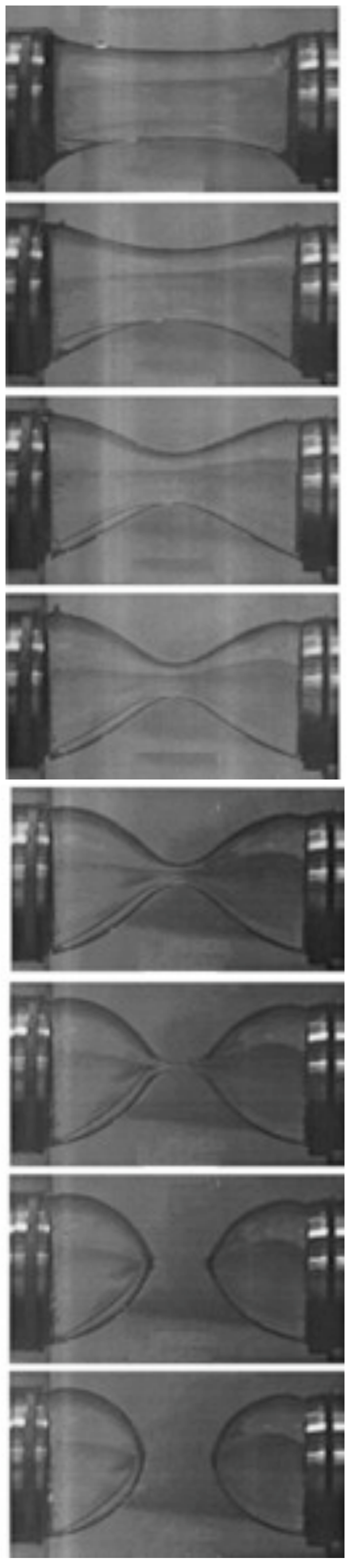
($R_0 k_{GL} = \sqrt{D}$, larg

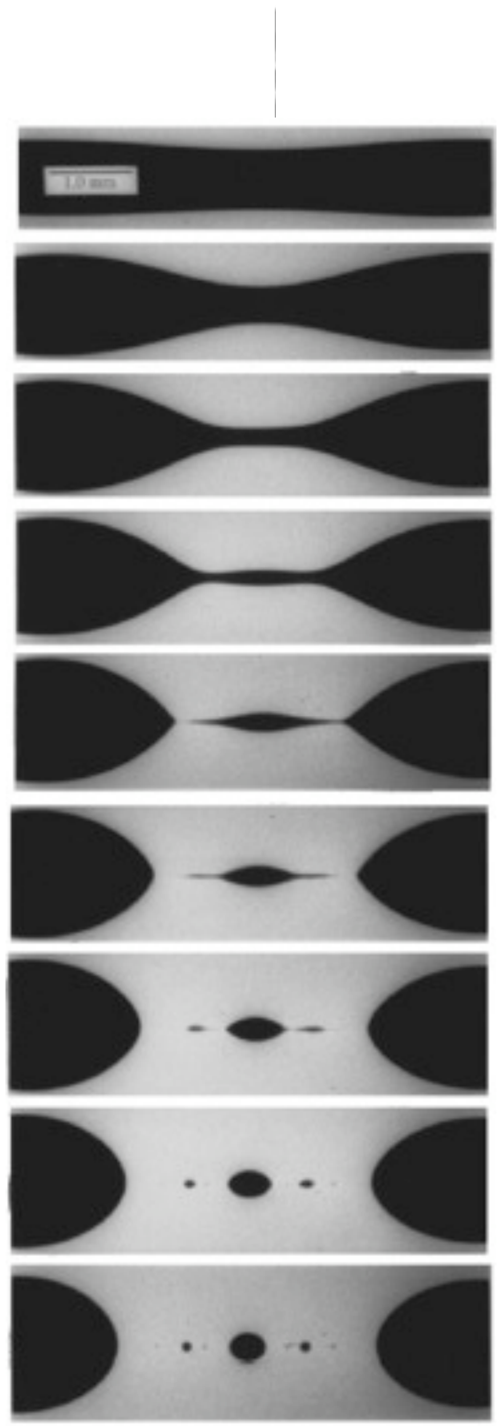
Kol & Sorkin,hep-th/04



Qualitatively RP= GL.

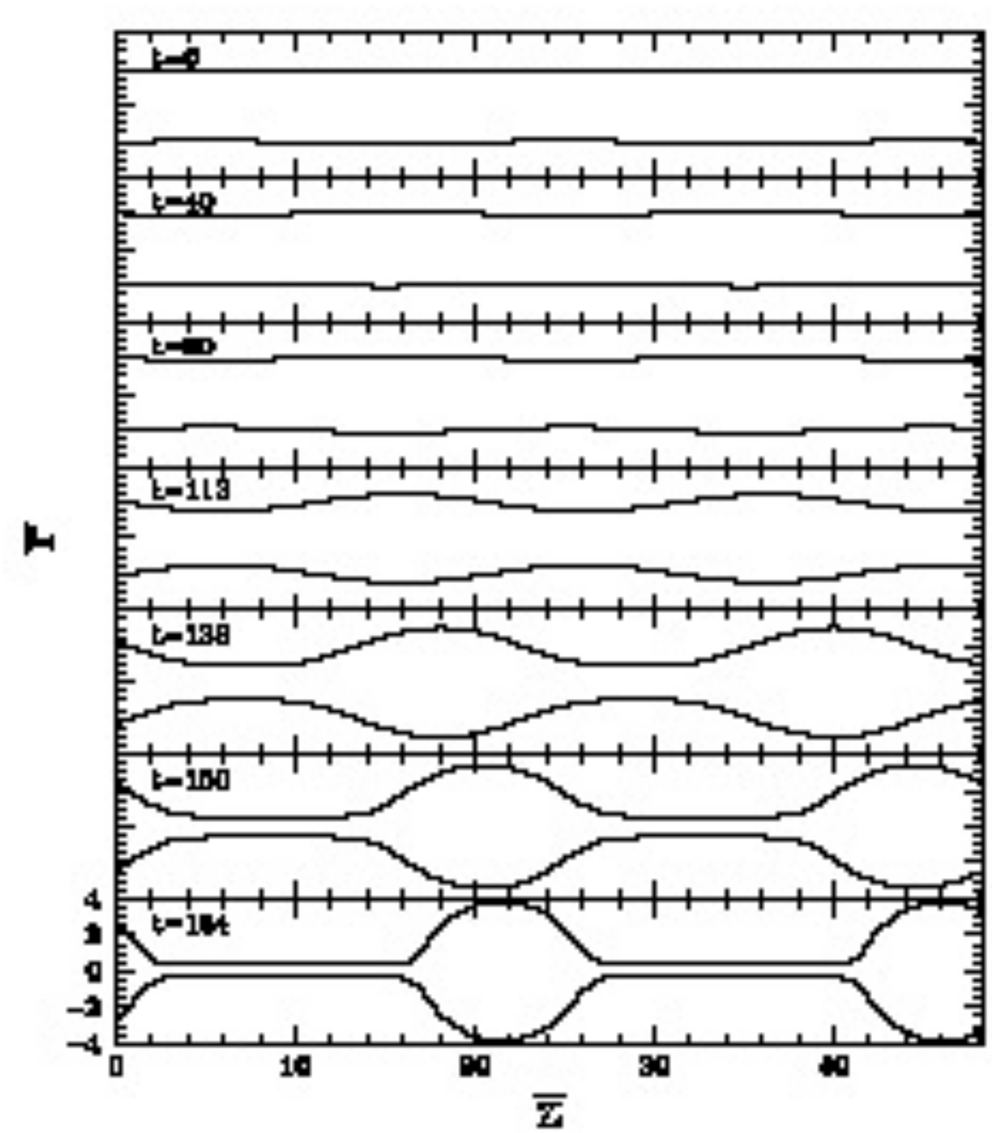
Rayleigh-Plateau Time Evolution

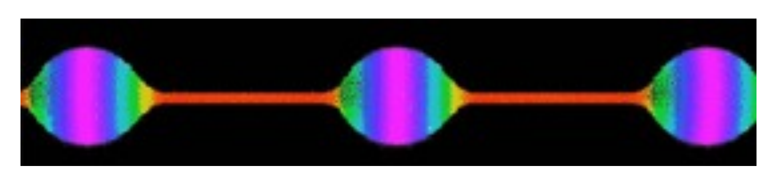




Time Evolution

Gregory-Laflamme Time evolution





Choptuik, Lehner, Olabarrieta, Petryk, Pretorius, Villegas, gr-qc/0304085

Black Holes in AdS/CFT

High T deconfined phase of gauge theory $\Longleftrightarrow$ Black Hole and Black Brane geometries

Near equilibrium: Long wavelength fluctuations

derivative expansion $\quad l_{\rm var} \gg l_{\rm mfp}$

Black Holes in AdS/CFT

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Near equilibrium: Long wavelength fluctuations

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Fluid dynamics

$$T^{\mu\nu} = p(T)\eta^{\mu\nu} + \left(\varepsilon(T) + p(T)\right) u^\mu u^\nu + \mathcal{O}(\partial u) + \cdots$$

perfect fluid $\qquad$ dissipative effects

form dictated by symmetries + $\nabla_\mu s^\mu \geq 0$

coefficients from Einstein's equations

Fluid dynamics from gravity

Battacharrya, Hubeny, Minwalla, Rangamani, 0712.2456

Start with (planar, k=0) black brane solution of Einstein-AdS

$$ds^2 = -2dv\,dr - r^2 f(br)dv^2 + r^2 dx^i dx^i \qquad \text{(EF coord.)}$$

$$f(r) = 1 - \frac{1}{r^4} \qquad T = \frac{1}{\pi b}$$

Boost the brane β^i :

$$u^v = \frac{1}{\sqrt{1-\beta_i^2}} \qquad u^i = \frac{\beta^i}{\sqrt{1-\beta_i^2}}$$

$$ds^2 = -2u_\mu dx^\mu\,dr - r^2 f(br) u_\mu u_\nu\,dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu$$

$P_{\mu\nu} = u_\mu u_\nu + \eta_{\mu\nu}$ **projector** $\perp u^\mu$

Allow boost and temperature to vary slowly with x^μ

$$b = b(x^\mu) \qquad \beta^i = \beta^i(x^\mu)$$

Fluid dynamics from gravity

Battacharrya, Hubeny, Minwalla, Rangamani, 0712.2456

$$ds^2 = -2u_\mu dx^\mu\, dr - r^2 f(br) u_\mu u_\nu\, dx^\mu dx^\nu + r^2 P_{\mu\nu} dx^\mu dx^\nu$$

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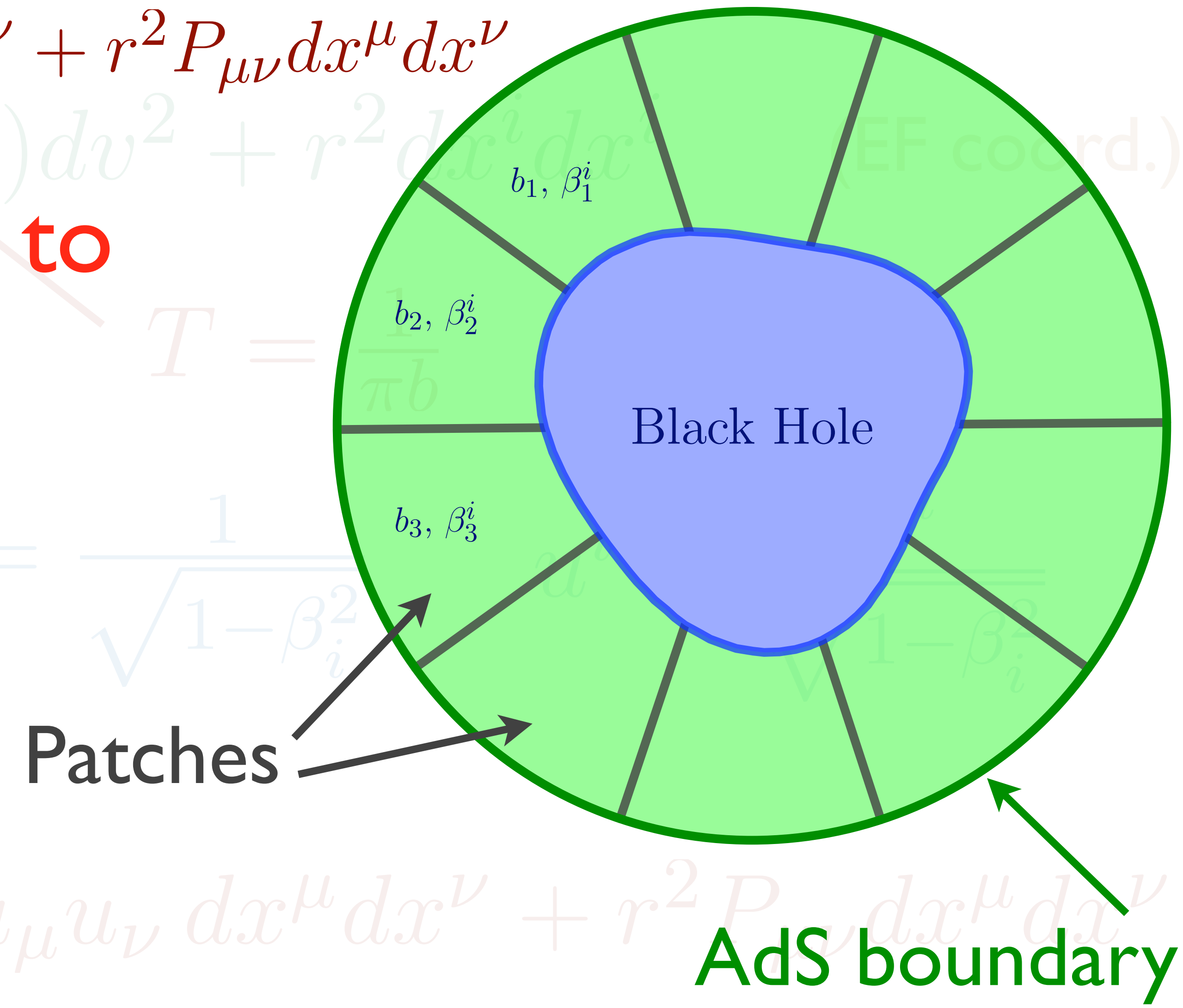


- ★ Einstein equations are solved order by order provided that the temperature and boost obey a set of equations which turn out to be the equations of ***boundary fluid dynamics*** $\nabla_\mu T^{\mu\nu} = 0$
- ★ Use the AdS/CFT dictionary to read the boundary stress tensor

Fluid dynamics from gravity

Battacharrya, Hubeny, Minwalla, Rangamani, 0712.2456

Boundary stress tensor (*conformal fluid*):

$$T^{\mu\nu} = (\pi T)^4 \left(\eta^{\mu\nu} + u^\mu u^\nu\right) \; -2(\pi T)^3 \sigma^{\mu\nu} + \mathcal{O}(\partial^2 u) + \cdots$$

order 0: perfect fluid

$$F = -(\pi T)^4$$

$$s = 4\pi^4 T^3$$

first order: shear viscosity $\eta = \pi^3 T^3$

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Policastro, Son & Starinets '01

Validity of the approximations:

fluid and curvature variations on scales $\gg l_{mfp}$ $\Longleftrightarrow$ Large Black Holes

Einstein-AdS gravity is dual to Fluid dynamics

Realization of the membrane paradigm

Example: The Viscosity Bound

shear viscosity ($-\eta\sigma^{\mu\nu}$ term in $T^{\mu\nu}$)

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Universal
(AdS gravity dual)

Tool to understand properties of strongly coupled quark-gluon plasma produced in heavy ion collisions at RHIC

KSS conjecture: $\frac{\eta}{s} \geq \frac{1}{4\pi}$

Kovtun, Son & Starinets '03

Violated in Gauss-Bonet, weaker bound

Brigante, Liu, Myers, Shenker, Yaida

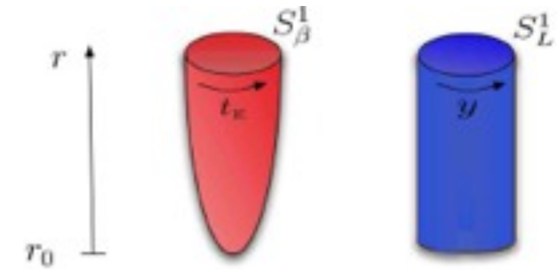
- ★ Magnetohydrodynamics / Einstein-Maxwell AdS gravity
    - Electrically charged plasma in an external B field
    - Conformal MHD equations (up to second order)
    - Black holes are dual to charged *diamagnetic* fluid
- ★ Conformal non-relativistic fluids (non-rel holography)
    - Schrödinger group
    - Fermions at unitarity, cold atoms
- ★ Other backgrounds
    - Quantum phase transitions
    - High Tc superconductivity
    - ...

Plasma lumps

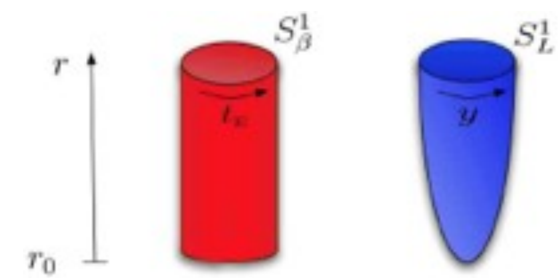
Lahiri, Minwalla, hep-th/0507219
Aharony, Minwalla, Wiseman, hep-th/0507219

Scherk-Schwarz compactification on a circle of AdS/CFT

T	Gravity	Gauge Theory
	Black Hole	**Plasma phase (deconfined)**
T_c		
	AdS soliton	**Vacuum phase (confined)**

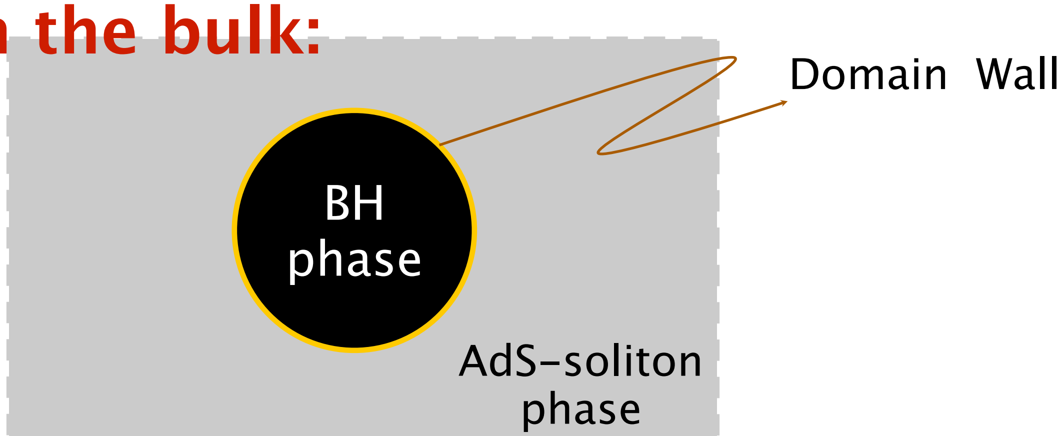


Confinement/deconfinement transition

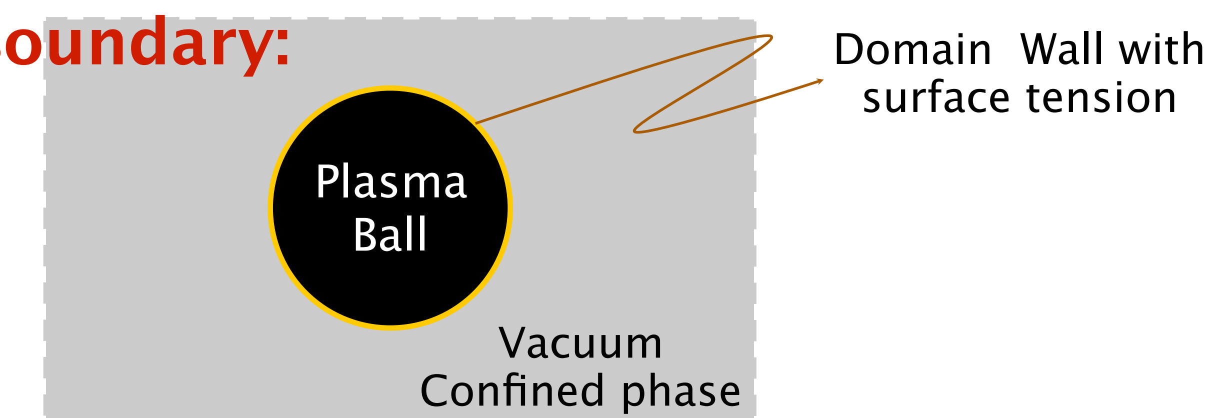


these phases can coexist near T_c,
separated by a domain wall

In the bulk:



On the AdS Boundary:



Plasma: $$T^{\mu\nu} = \left[(\rho + P)\, u^\mu u^\nu + P g^{\mu\nu}\right] \Theta(-f) - \sigma h^{\mu\nu} |\partial f|\, \delta(f)$$

mean curvature

surface tension

$$\nabla_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \boxed{P - P_0 = \sigma K \quad \text{Young-Laplace eq.}}$$

(relativistic Navier-Stokes + continuity in the bulk)

Stationary Configurations:

- Hydrodynamical and thermal equilibrium
- stationary fluid must be in rigid roto-translational motion

  $$u = \frac{\mathcal{T}}{T}(\xi - \Omega_I \chi_I)$$

  $T = \gamma \mathcal{T}$ is the **equilibrium plasma temperature**, dual to the **Hawking temperature of the horizon**
- Euler relation + Young-Laplace $\quad T = \frac{\sigma K + \rho}{\gamma s}$
- For a **static** fluid **K** is **constant** over the surface, but in a **stationary** fluid **K adjusts** to variations of fluid velocity near boundary.

Variational principles

Maximize entropy at constant energy and angular momentum

$$I[\mathcal{P}] = S[\mathcal{P}] - \beta E[\mathcal{P}] + \beta \Omega_I J_I[\mathcal{P}]$$

or equivalently, minimize the potential energy of the fluid

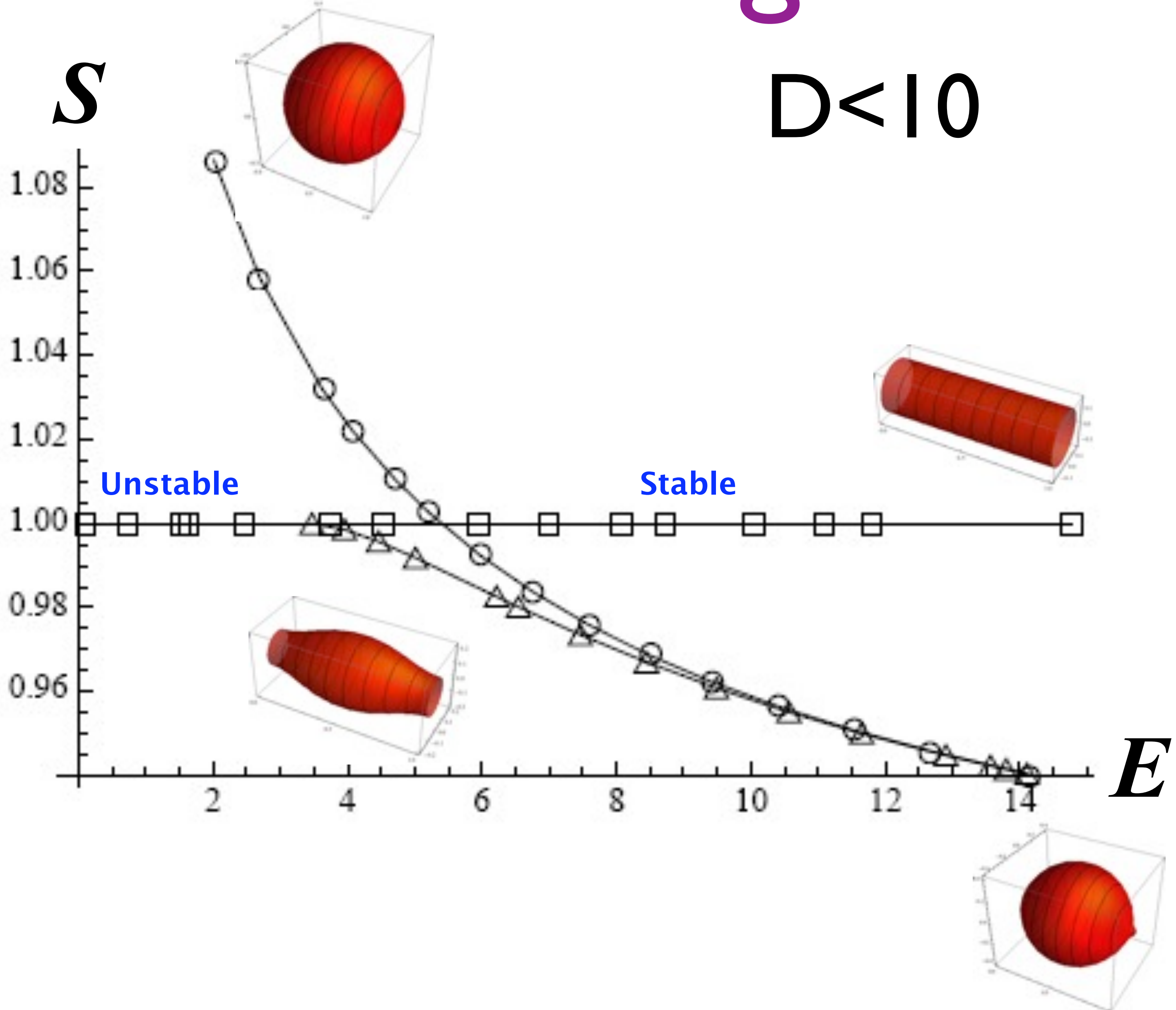
$$\hat{I}[\mathcal{P}] = \boxed{\sigma \mathcal{A}[\mathcal{P}] + U_{\rm cf}[\mathcal{P}]} - \eta V[\mathcal{P}] \qquad I[\mathcal{P}] = -\beta \hat{I}[\mathcal{P}]$$

(only assumes stationarity of background geometry and fluid, covers non-axisymmetric cases)

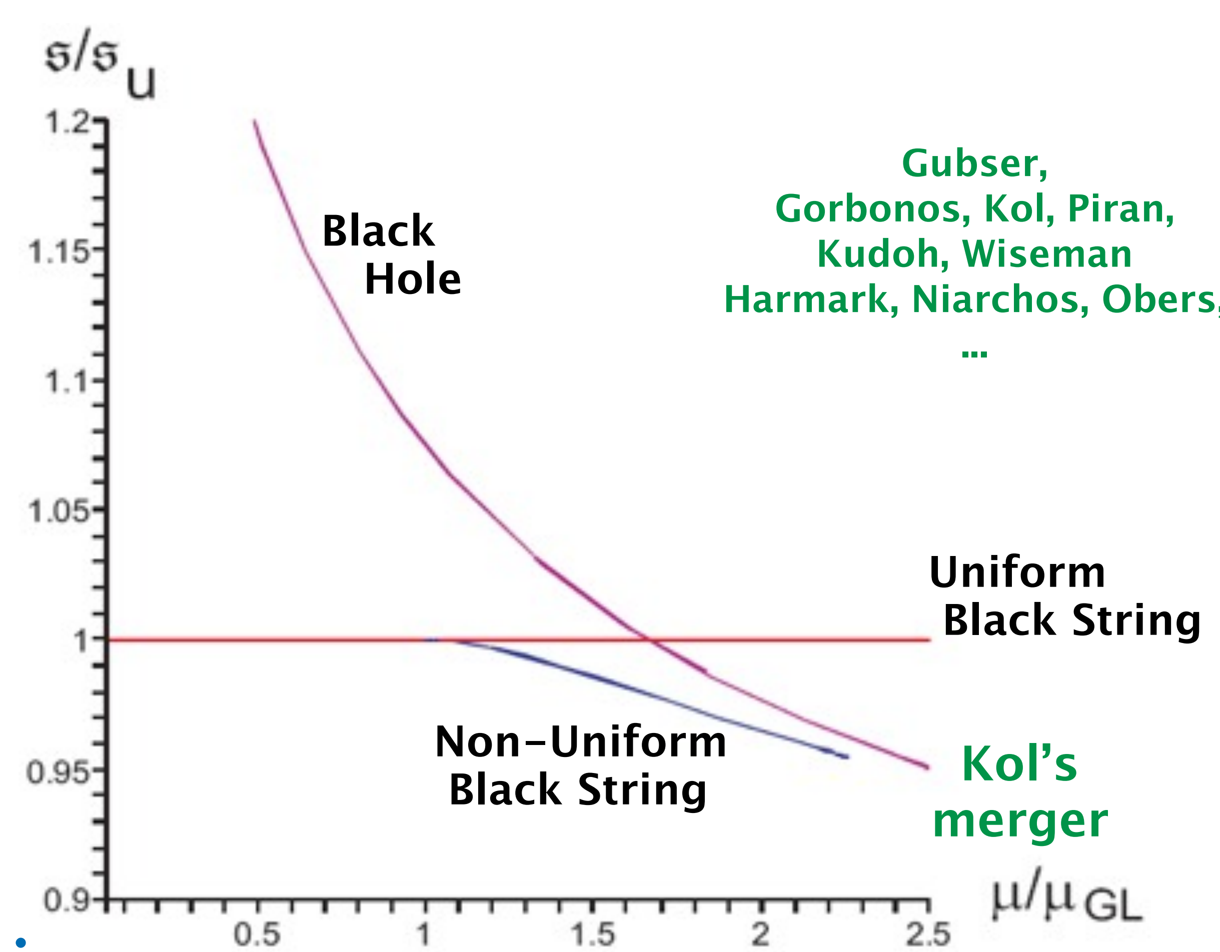
Fluid area minimization $\Leftrightarrow$ Black hole entropy maximisation

Note that the BH horizon is **not** mapped to the fluid boundary, but to the **entire** fluid **bulk**

Phase diagram for static plasma



Compare with **Gravitational phase diagram:**



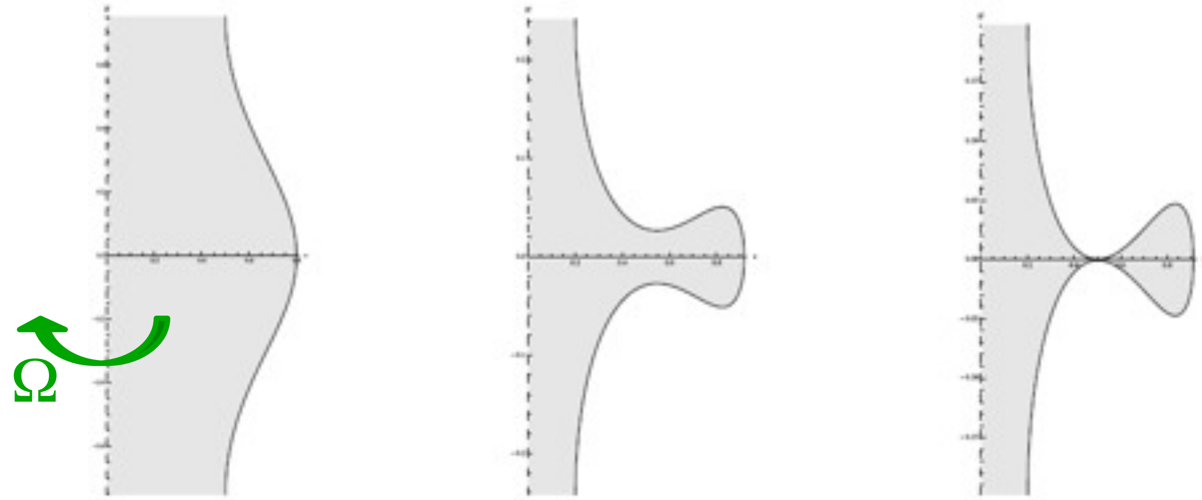
NOTE:

- These are available results for **Vacuum** BHs
- **Predictions** for **SS AdS** BHs

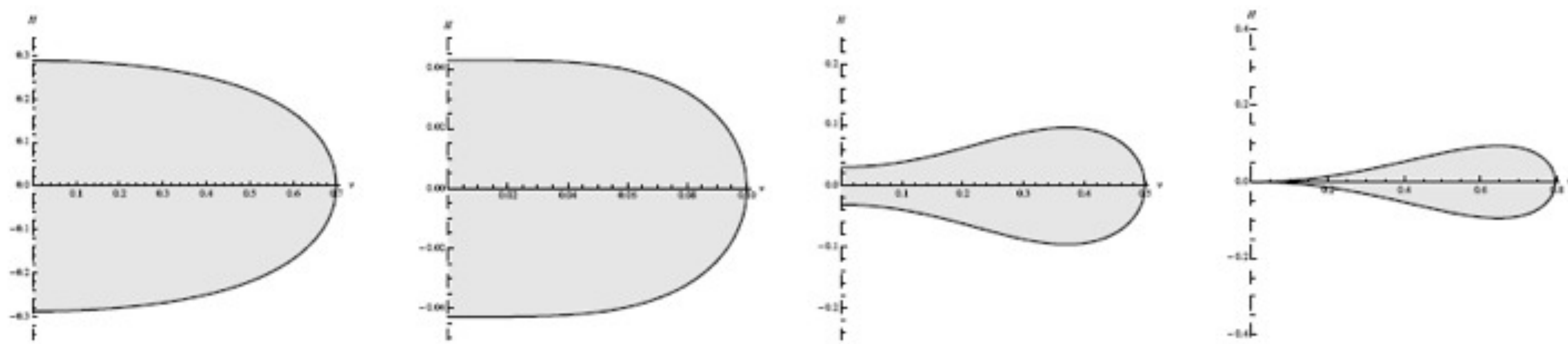
also agreement in the critical dimension

Rotating equilibrium plasmas

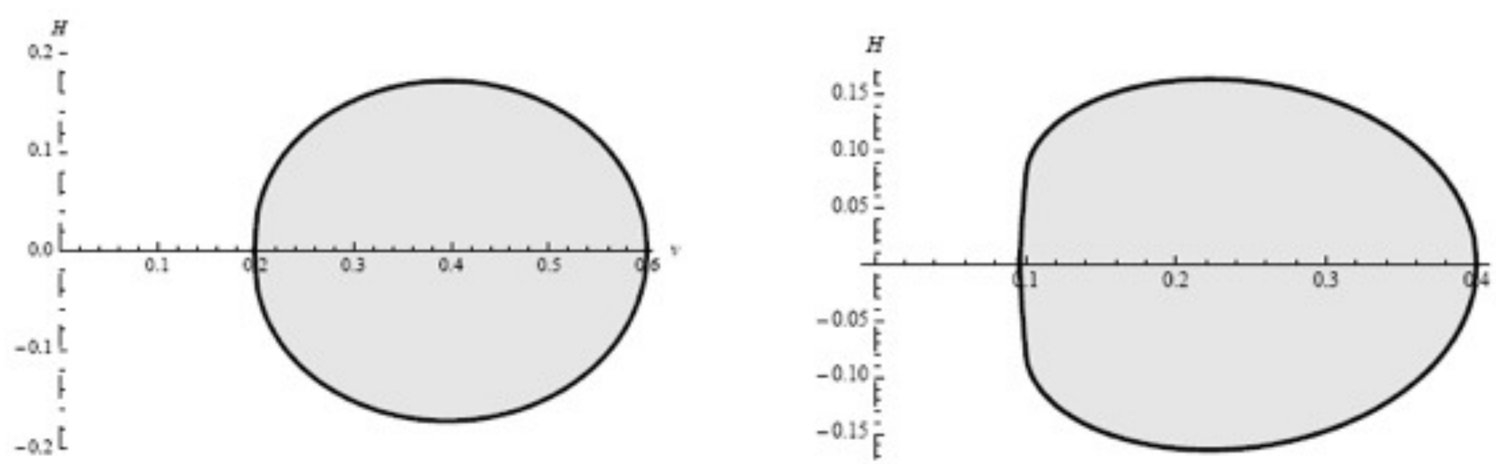
Rotating Non-Uniform Plasma Tubes



Rotating Plasma Balls



Rotating Plasma Rings



Gravitational dual:

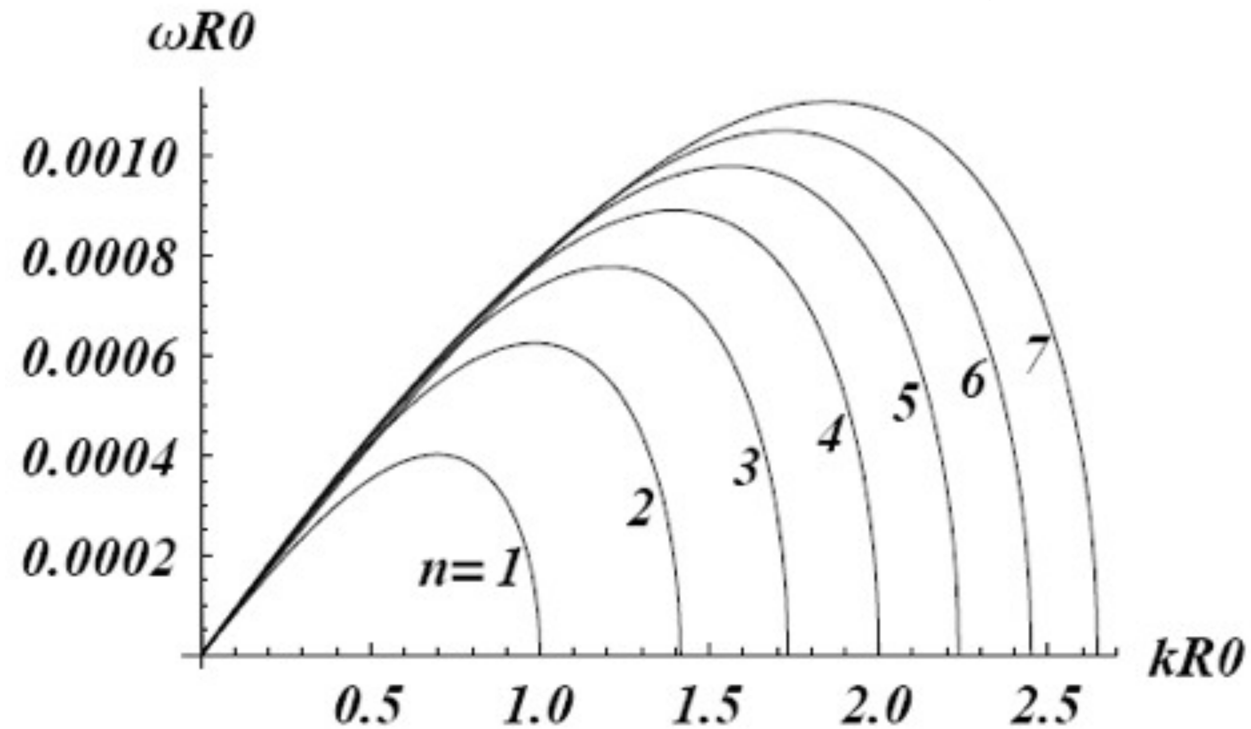
- BHs, ultra-spinning BHs, (Non-)Uniform Black strings, Black Rings
- Predictions for new BH phases in SS AdS and even in asymptotically flat BHs

RP instability of uniform plasma tubes

Threshold mode: perturb the equilibrium configuration
unstable modes for $\Delta U \leq 0$

$$kR_0 \leqslant \sqrt{n}$$

n=d−3



Dispersion relation $\omega(k)$:

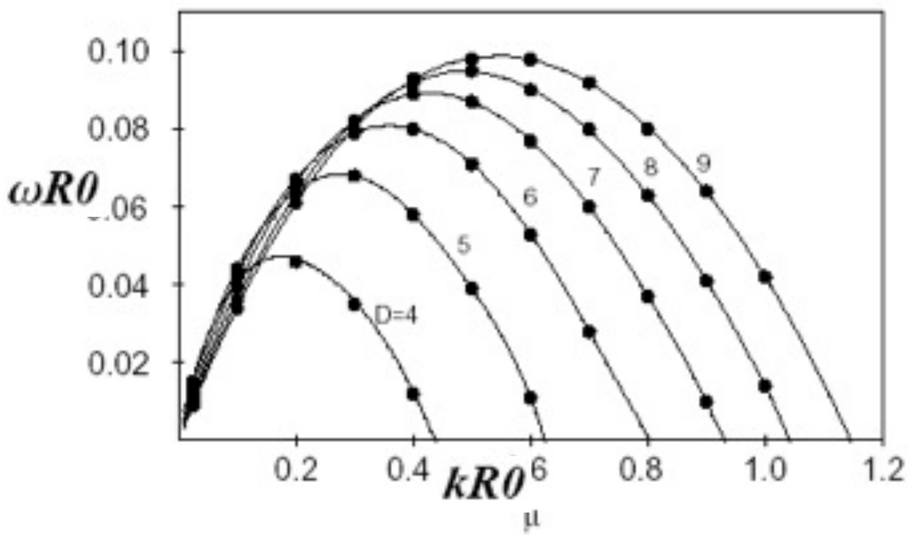
$$\omega^2 = \frac{n+3}{n+4}\frac{\sigma}{\rho_* R_o^3}\frac{pR_o\, I_{\frac{n+1}{2}}(pR_o)}{I_{\frac{n-1}{2}}(pR_o)}\left(n - k^2R_o^2 - \omega^2 R_o^2\right) \qquad p = k\left(1 + (n+3)\frac{\omega^2}{k^2}\right)^{\frac{1}{2}}$$

RP instability of uniform plasma tubes

Threshold mode: perturb the equilibrium configuration

unstable modes for $\Delta U \leq 0$

Compare with Gravitational Gregory-Laflamme dispersion relation:



Again, NOTE:

- These are available results for **Vacuum** BHs
- **Predictions** for **SS AdS** BHs

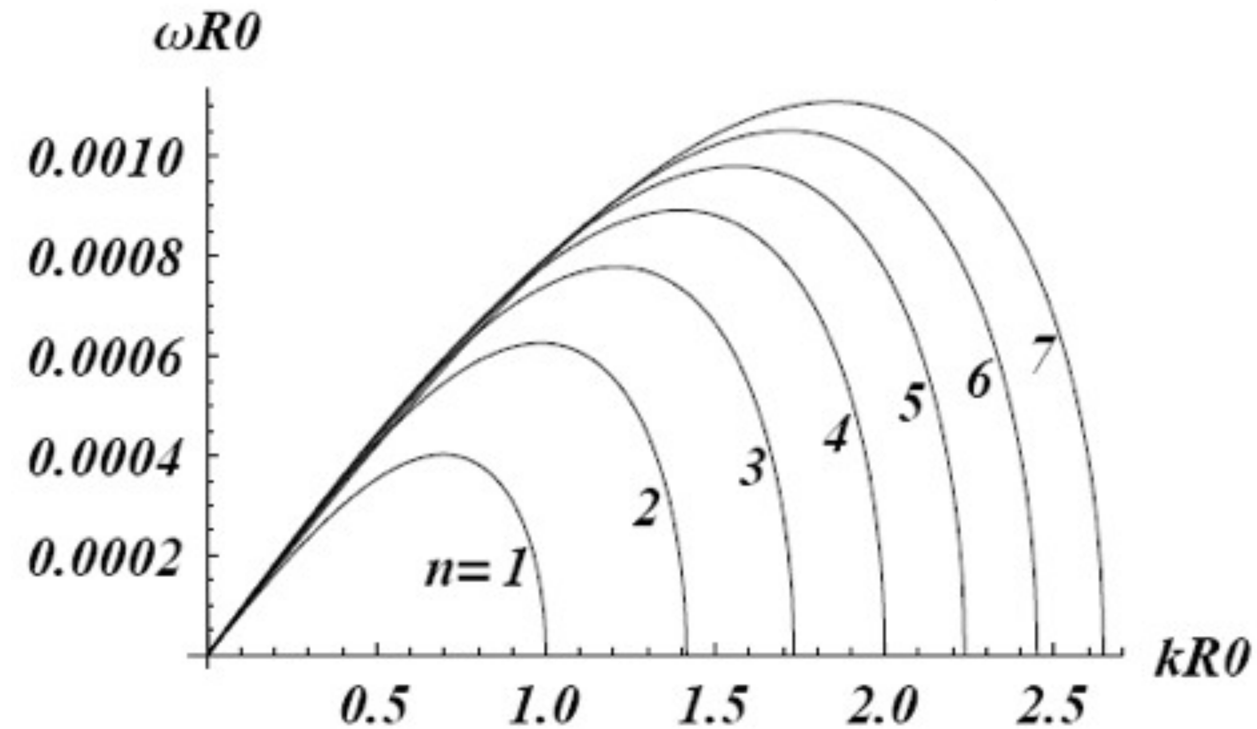
- Threshold wavenumber :

$$k_c R_0 \sim \sqrt{D} \qquad (\text{large } D)$$

Kol, Sorkin, 2004

$$kR_0 \leqslant \sqrt{n}$$

n=d-3



RP for uniform rotating tubes

Addition of rotation increases the RP instability strength

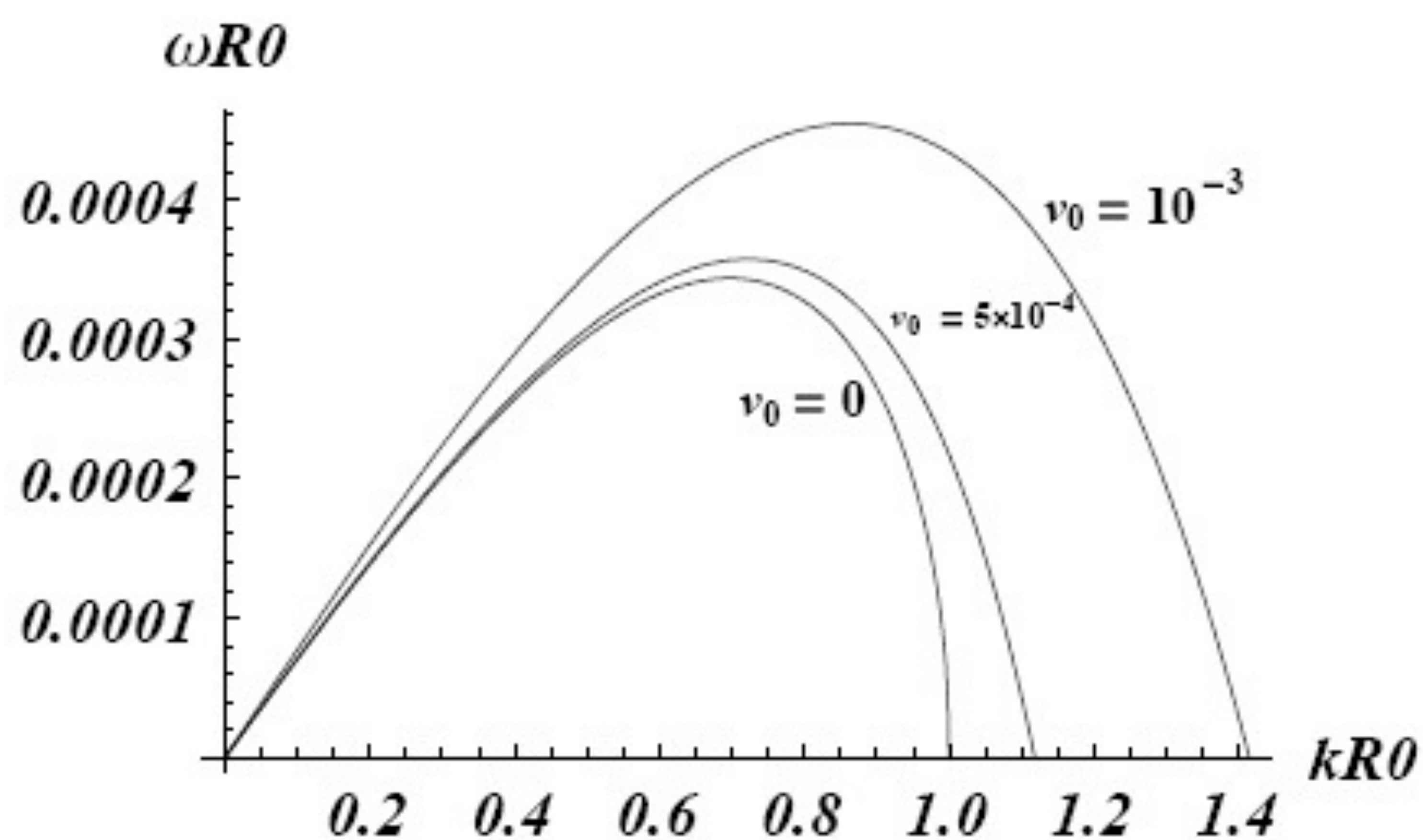
Threshold mode:

$$k^2 R_0^2 \leqslant \left[1 - m^2 + \frac{5}{4}\frac{\rho_c \omega_\phi^2 R_0^3}{\sigma}\left(1 - R_0^2\omega_\phi^2 - \omega_z^2\right)^{-7/2}\right]$$

Analytical dispersion relation for small angular velocities

- Increased growth rate

- Non-axisymmetric modes become unstable



Black string: Addition of rotation increases the GL instability strength

(Kleihaus, Kunz, Radu 2007)

Conclusions

Powerful tool to study classical aspects of horizon dynamics, black hole phases, stongly coupled gauge theories, relativistic Navier-Stokes....

- ★ **Rayleigh-Plateau** instability on fluid tubes is the holographic dual to **Gregory-Laflamme** instability of black strings
- ★ **MHD**: The magnetic field decreases the strength of the instability, but doesn't stabilize the plasma tube completely
- ★ Approximation valid for **large** plasma balls $\frac{\sigma}{\rho_0 R} \ll 1$
- ★ Do vacuum black holes have an hydrodynamic description?

  "New" dimensionless GR scale $\frac{\sigma}{\rho_0 R} = \frac{1}{D}$ ("large" black holes)

  Vacuum black holes in **high D** can have a fluid description

Time evolution of RP instability for a *rotating* tube

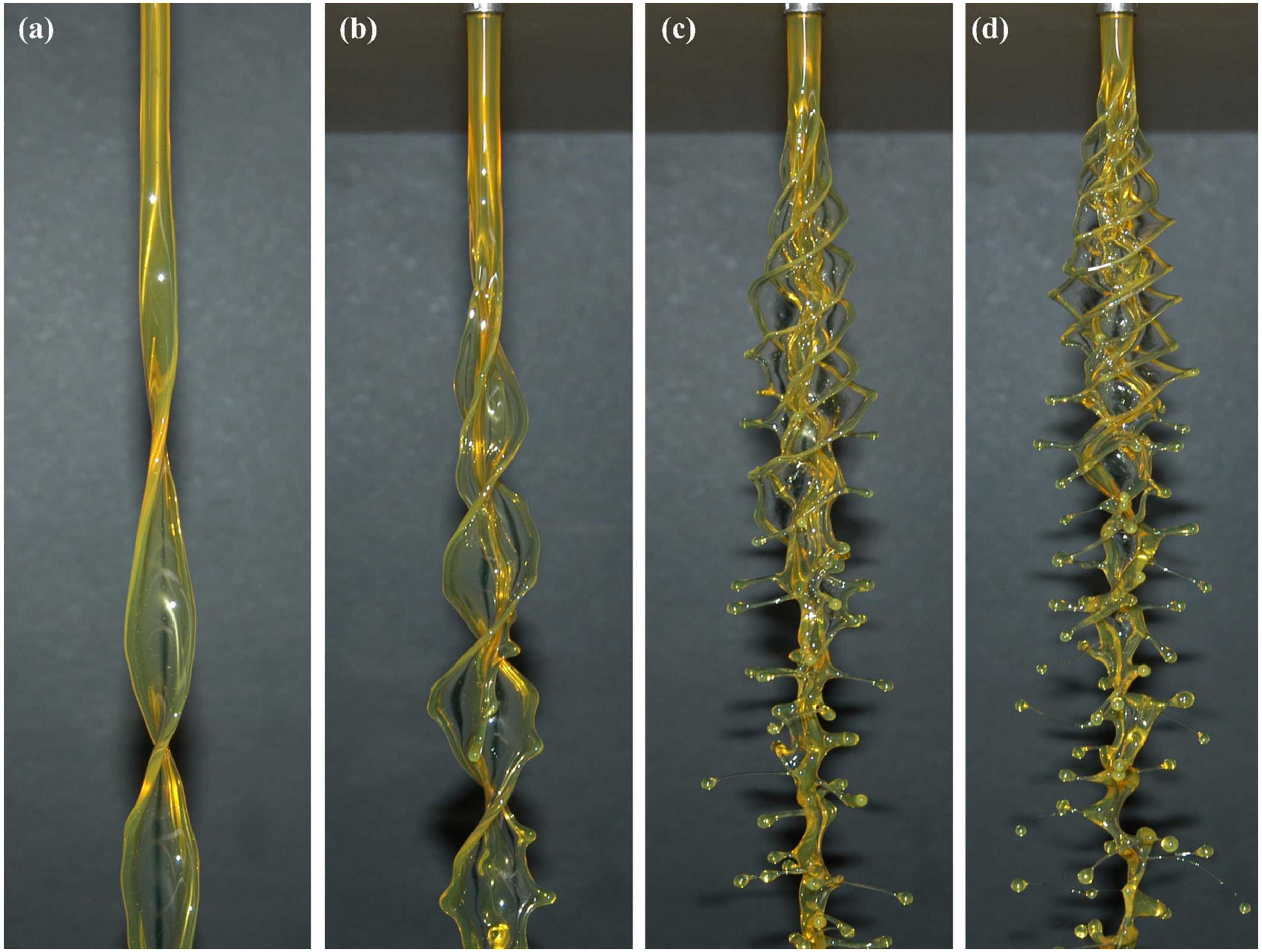


FIG. 4. (Color online) Still images of helical instabilities for SAE30 oil at approximately 35 °C: (a) $n=2$ at 157 rad/s, (b) $n=3$ at 241 rad/s, (c) $n=4$ at 346 rad/s, and (d) weak $n=5$ at 408 rad/s.

Kubitschek & Weidman (2007)