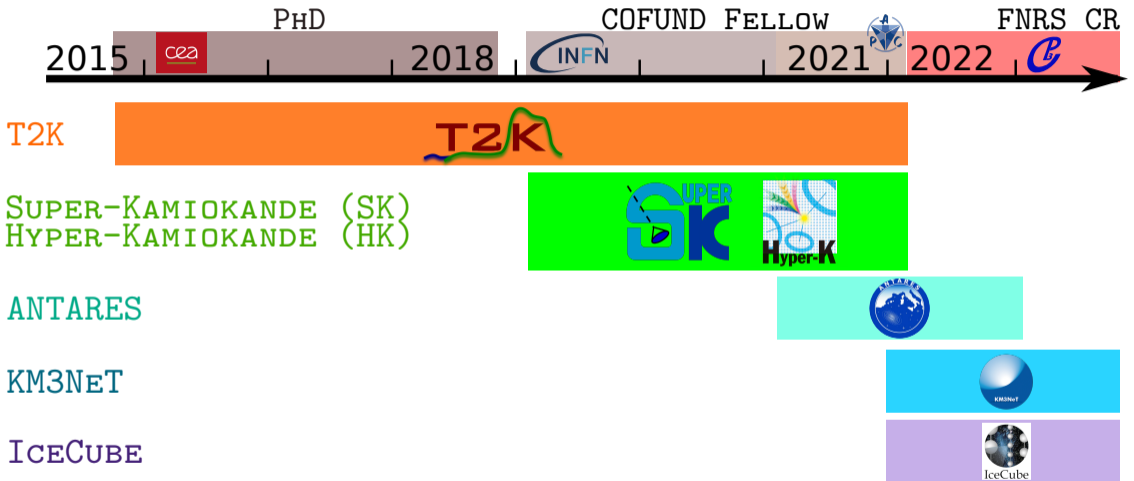


Neutrino emission from gravitational wave sources

CP3 seminar

Mathieu Lamoureux 

CP3, UCLouvain



Central theme: multimessenger astronomy with neutrinos and especially neutrinos from gravitational wave sources



T2K



SUPER-KAMIOKANDE (SK)
HYPER-KAMIOKANDE (HK)



ANTARES

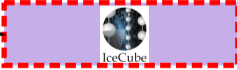


KM3NET

covered in this seminar



ICECUBE

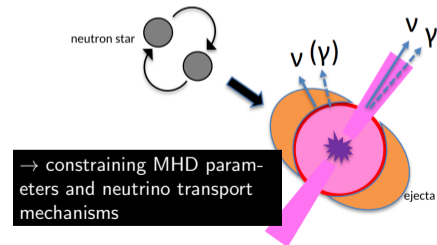
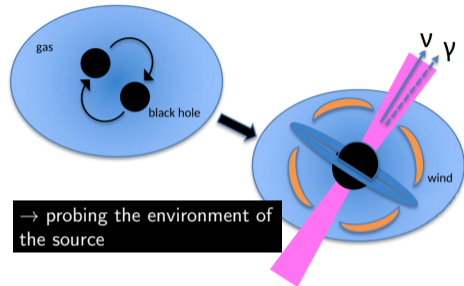


Central theme: multimessenger astronomy with neutrinos and especially neutrinos from gravitational wave sources

Introduction

Mergers of compact objects (Neutron Stars -NS-, Black Holes -BH-) are established gravitational wave (GW) emitters.

- **BNS** (NS+NS) or **NSBH** (NS+BH): may produce short Gamma-Ray Bursts with neutrino production
- **BBH** (BH+BH): neutrinos may be produced in the accretion disks of the BHs



<i>Spectrum</i>	$E^{-\gamma}$ often considered in searches and MeV/GeV emission?
<i>Shape</i>	isotropic (not realistic at high energy) or presence of directional jet?
<i>Timing</i>	GW170817 + GRB170817A observation hints to prompt signal for BNS

Total energy emitted in neutrinos?

Not well estimated, 10^{50} - 10^{53} erg? (1 erg \approx 624 GeV)



=



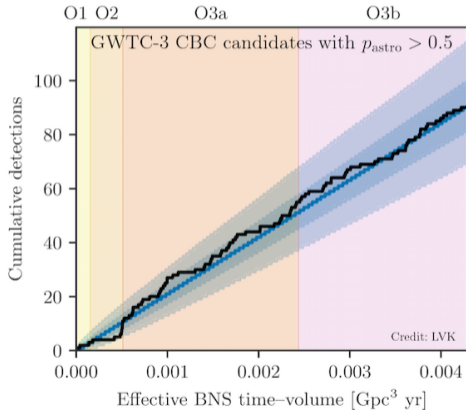
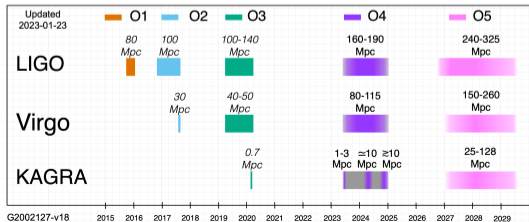
Belgium electricity production for $10^{25} - 10^{28}$ years!

Since 2015, ~ 100 detections distributed in 4 catalogs:

- **GWTC-1:** 11 events from O1 and O2
- **GWTC-2:** 39 events from O3a
- **GWTC-2.1:** 8 add. events from O3a
- **GWTC-3:** 35 events from O3b

Next observation period O4:

- expected to start **end of May 2023**
- **hundreds** of new detections per year



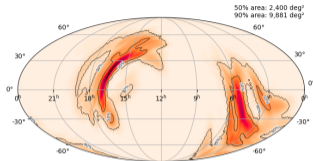
For each GW, we have:

- time of the event
- sky localisation
- estimated distance
- estimated masses

Classification:

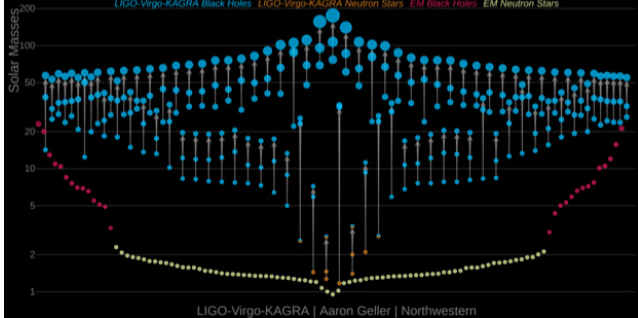
Events can be classified based on object masses:

- $m < 3 M_{\odot} = \text{NS}$
- $m > 3 M_{\odot} = \text{BH}$



Masses in the Stellar Graveyard

LIGO-Virgo-KAGRA Black Holes LIGO-Virgo-KAGRA Neutron Stars EM Black Holes EM Neutron Stars



In total:

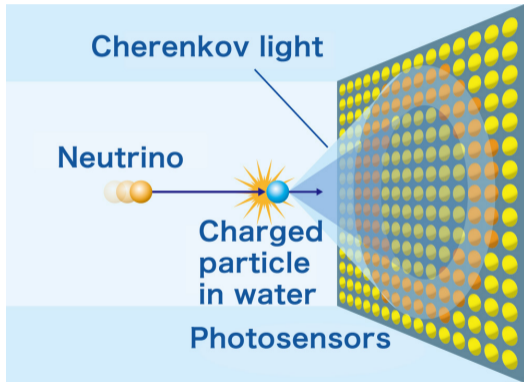
- 2 BNS (GW170817, GW190425)
- 5 NSBH
- the rest are BBH

Golden technique detection of Cherenkov light emitted by charged particles produced during neutrino interaction

Golden technology large water volume instrumented with photomultipliers (PMTs)

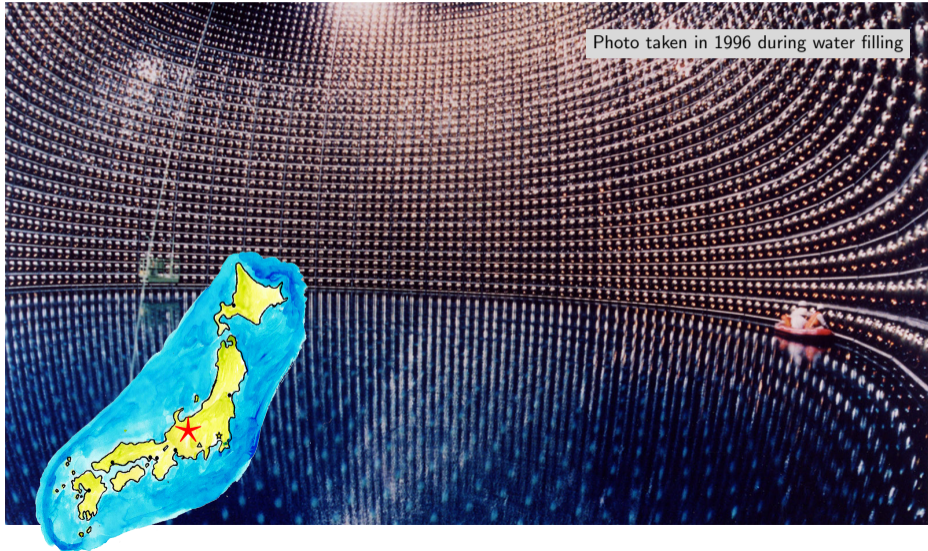
Three main strategies:

- *Super-Kamiokande/Hyper-Kamiokande*: water tank with PMTs on the walls of the tank
- *ANTARES/KM3NeT/Baikal-GVD...*: PMTs deployed on vertical lines in sea/ocean/lake
- *IceCube*: same as above, but in ice

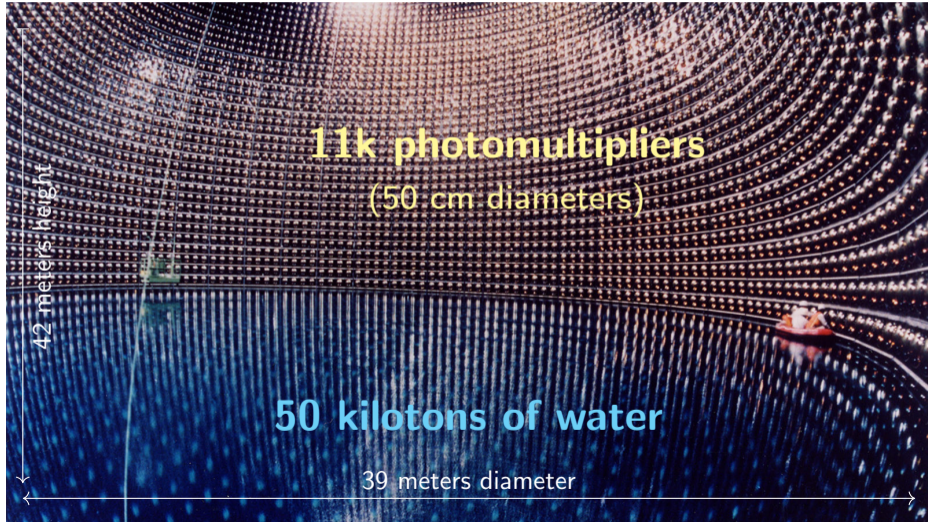


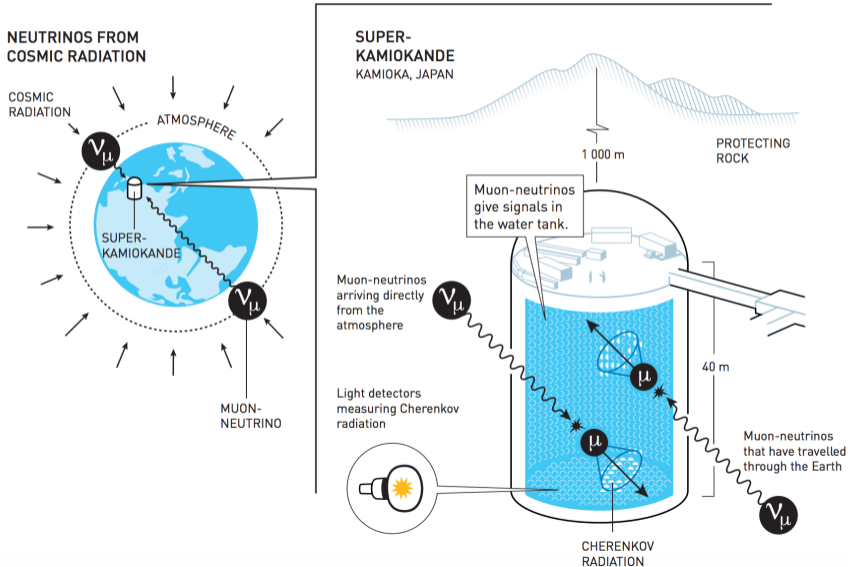
Follow-up with Super-Kamiokande

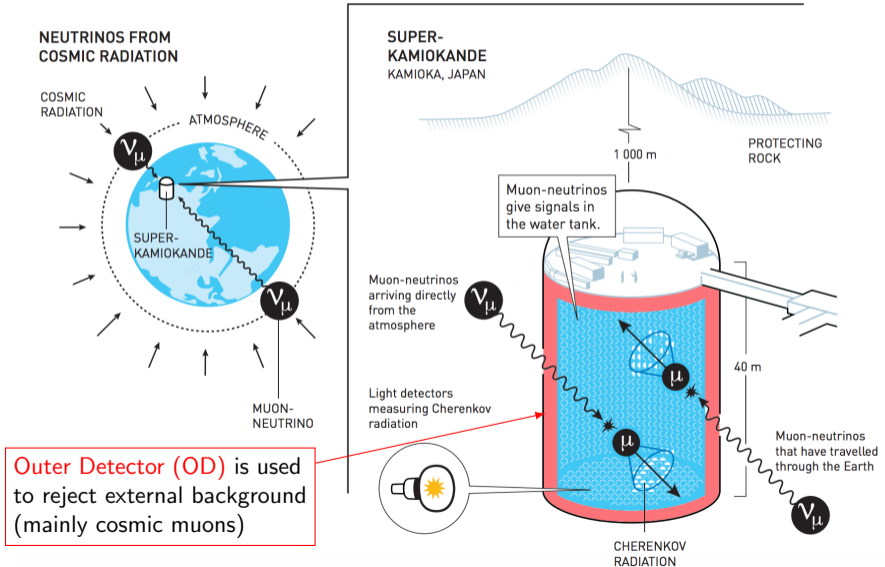
Experiment running since 1998, located in the Mozumi mine in Japan.



Experiment running since 1998, located in the Mozumi mine in Japan.

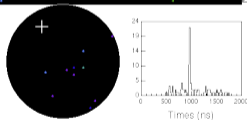
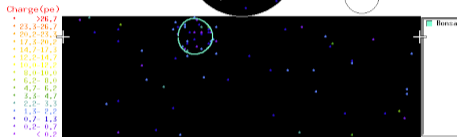
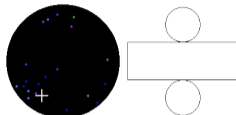






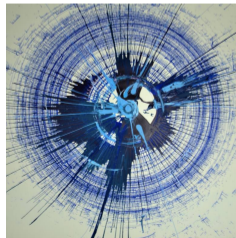
Low-energy events

Super-Kamiokande IV
 Run 61525 Sub 35 Event 66068270
 08-10-06 13:00:51
 Data: 186 hits, 247 pe
 Outer: 0 hits, 0 pe
 Trigger: null
 D_max: 483.5 cm
 E_vis: 3.8 MeV
 SelSel: E* 5.93 MeV cos(theta)*0.422
 selSel: gM* 0.65 Ricker* 0.17



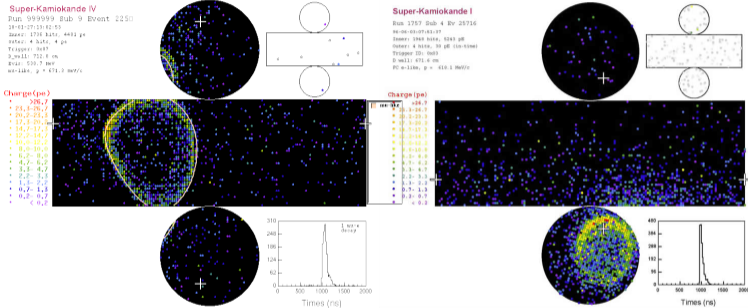
$$E_\nu = 3.5 - 100 \text{ MeV}$$

sparse PMT hits, faint ring



INCREASING ENERGY

Fully-Contained (FC) events



LOW-ENERGY EVENTS (LOWE)
 $E_\nu \sim 3.5 - 100 \text{ MeV}$

$$E_\nu = 0.1 - 10 \text{ GeV}$$

no activity in OD

μ : sharp ring

e: fuzzy ring



INCREASING ENERGY



LOW-ENERGY EVENTS (LOWE)
 $E_\nu \sim 3.5 - 100 \text{ MeV}$

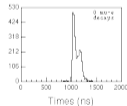
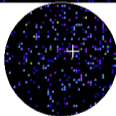
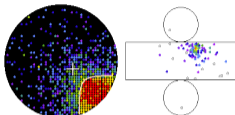
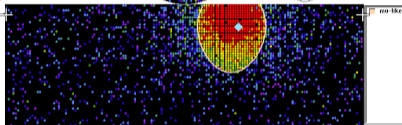
FULLY-CONTAINED EVENTS (FC)
 $E_\nu \sim 0.1 - 10 \text{ GeV}$

Partially-Contained (PC) events

Super-Kamiokande IV
 Run 999999 Sub 6 Event 1170
 18-01-27 13:09:59
 Tower: 0845 bits, 50482 pe
 Outer: 71 bits, 159 pe
 Trigger: 0009
 D_max: 922.6 cm
 D_min: 3.8 649
 no-like, p = 3030.3 meV/c

Charge (pe)

- >26.7
- 23.3-26.7
- 20.0-23.3
- 17.5-20.0
- 14.7-17.5
- 12.5-14.7
- 10.0-12.5
- 8.0-10.0
- 6.0-8.0
- 4.7-6.0
- 3.5-4.7
- 2.0-3.5
- 1.5-2.0
- 0.7-1.5
- 0.0-0.7
- < 0.0



$E_\nu = 0.1 - 100 \text{ GeV}$

exit activity in OD

INCREASING ENERGY

LOW-ENERGY EVENTS (LOWE)
 $E_\nu \sim 3.5 - 100 \text{ MeV}$

FULLY-CONTAINED EVENTS (FC)
 $E_\nu \sim 0.1 - 10 \text{ GeV}$

PARTIALLY-CONTAINED EVENTS (PC)
 $E_\nu \sim 0.1 - 100 \text{ GeV}$

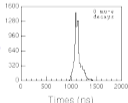
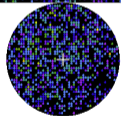
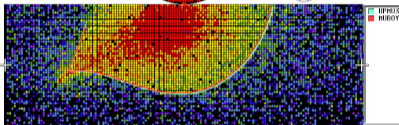
INCREASING ENERGY 

Upward-going muons (UPMU)

Super-Kamiokande IV
 Run 76804 Sub 27 Event 261998870
 17-09-04:14:36:26
 Date: 8399 hits, 52941 pe
 Offset: 134 hits, 602 pe
 Trigger: 0c100001e
 D_max: 1630.0 cm
 EvtId: 3.0 NE7
 PhotoGating: none

Charge(pe)

- >26.7
- 23.3-26.7
- 20.0-23.3
- 17.5-20.0
- 14.7-17.3
- 12.0-14.7
- 10.0-12.7
- 8.0-10.0
- 6.0-8.0
- 4.7-6.0
- 3.5-4.7
- 2.0-3.3
- 1.3-2.0
- 0.7-1.3
- 0.0-0.7
- < 0.2

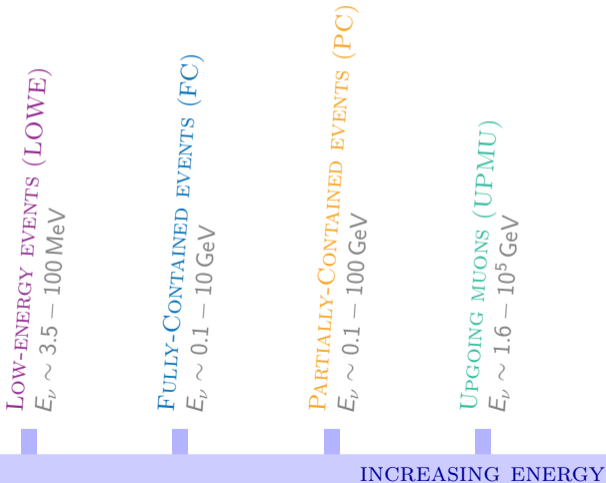


$$E_\nu = 1.6 \text{ GeV} - 10 \text{ TeV}$$

entering activity in OD

stopping ($p_\mu > 1.6 \text{ GeV}$) or through-going ($L > 7 \text{ m}$)

Upgoing only to reject atmospheric muons

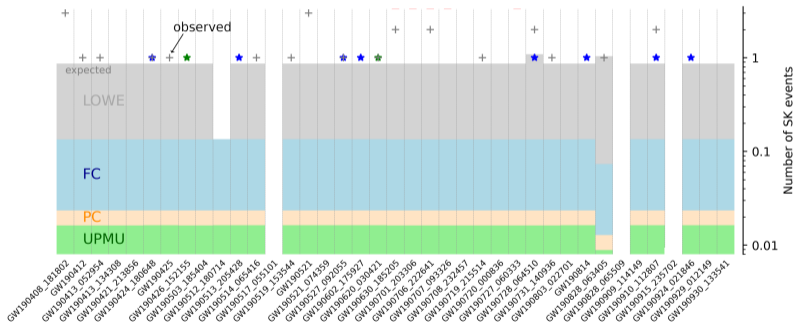


- Four samples
- Energies ranging from MeV to TeV
 - * solar neutrinos
 - * supernova neutrinos
 - * accelerator neutrinos (T2K)
 - * atmospheric neutrinos
 - * astrophysical neutrinos?

- Focus on **GWTC-2** catalog
- Define a ± 500 s centered on GW time
- Search for events within this time window, in the **four SK samples**
- Compare observation with expected background and extract **constraints** on neutrino emission
- Compute signal significance by comparing neutrino directions and GW localisation

Low-energy sample	High-energy samples		
	FC	PC	UPMU
Standard solar/SRN selection + 7 MeV energy threshold to ensure stable bkg rate	Standard atmospheric selection		
expected background in 1000 seconds = 0.729	0.112	0.007	0.016

Performed the analysis for 39 GWs in GWTC-2. Three of them were associated to SK downtime (due to calibration) (one less for low-energy due to HV issues).

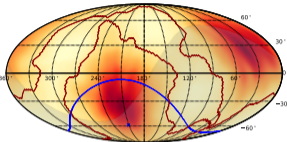


In total:

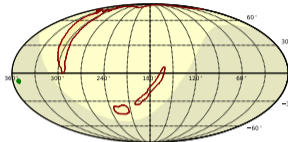
Sample	N_{obs}	N_{exp}
LOWE+	24	24.97
FC*	8	3.95
PC*	0	0.26
UPMU*	2	0.58

No significant excess was observed in any of the samples.

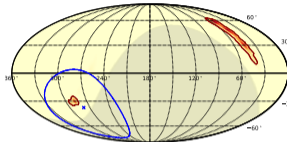
GW190424_180648



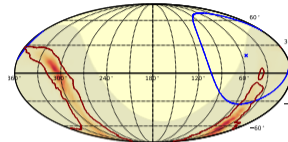
GW190426_152155



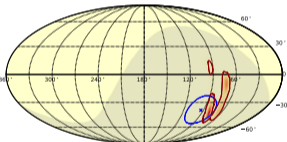
GW190513_205428



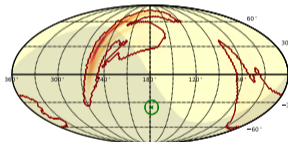
GW190527_092055



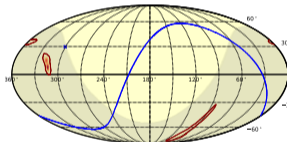
GW190602_175927



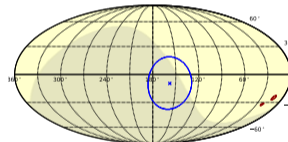
GW190620_030421



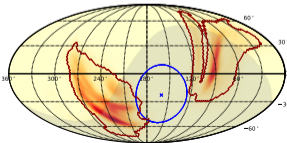
GW190728_064510



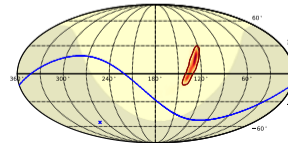
GW190814



GW190910_112807



GW190924_021846



Skymaps in equatorial coordinates

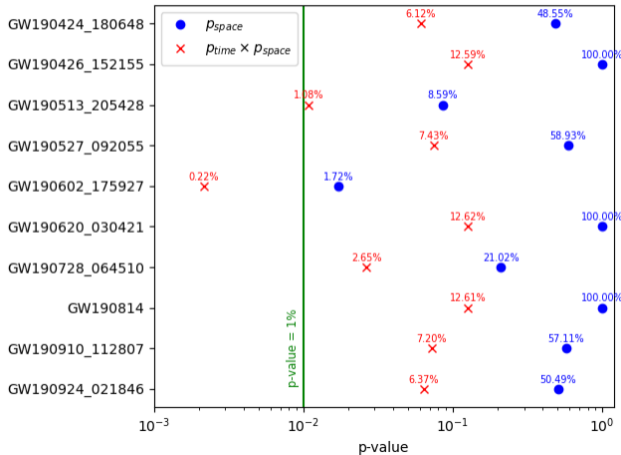
Red: GW localisation and 90% contour

Blue: SK FC events with 1σ angular uncertainty

Green: SK UPMU events.

Shaded area: SK upgoing sky.

Test statistic (TS) has been built to separate signal (point-source) from background (full-sky). It is used to compute p-values (compared observed TS to background distribution).



The most significant $GW+\nu$ coincidence is for GW190602_175927:

$$p = 0.22\%$$

Considering the number of trials ($N = 36$ follow-ups), we get a **post-trial** p-value:

$$P = 7.8\%$$

The neutrino flux is assumed as $\frac{dN}{dE_\nu} = \phi_0 E_\nu^{-2}$ and

$$N_{\text{expected signal}} = \int_{E_{\min}}^{E_{\max}} dE_\nu A_{\text{eff}}^{s,f}(E_\nu, \theta) \times \frac{dN}{dE_\nu}.$$

Sample-by-sample flux limits

For each sample/flavour ($\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$), we define the likelihood:

$$\mathcal{L}(\phi_0; n_B, N) = \int \frac{(a(\Omega)\phi_0 + n_B)^N}{N!} e^{-(a(\Omega)\phi_0 + n_B)} \mathcal{P}_{\text{GW}}(\Omega) d\Omega$$

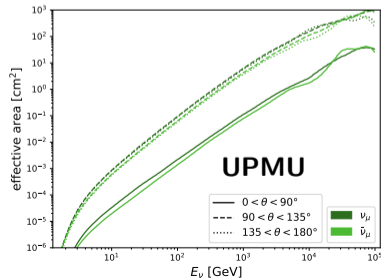
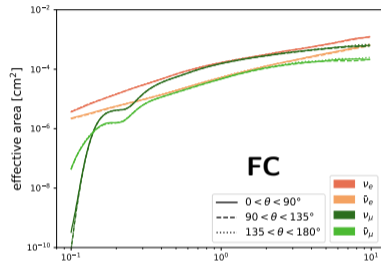
where $a(\Omega) = \int_{E_{\min}}^{E_{\max}} dE_\nu A_{\text{eff}}(E_\nu, \theta) E_\nu^{-2}$ is the **acceptance**.

90% limit on flux is obtained by solving $\int_0^{\phi^{\text{up}}} \mathcal{L}(\phi) d\phi = 0.9$

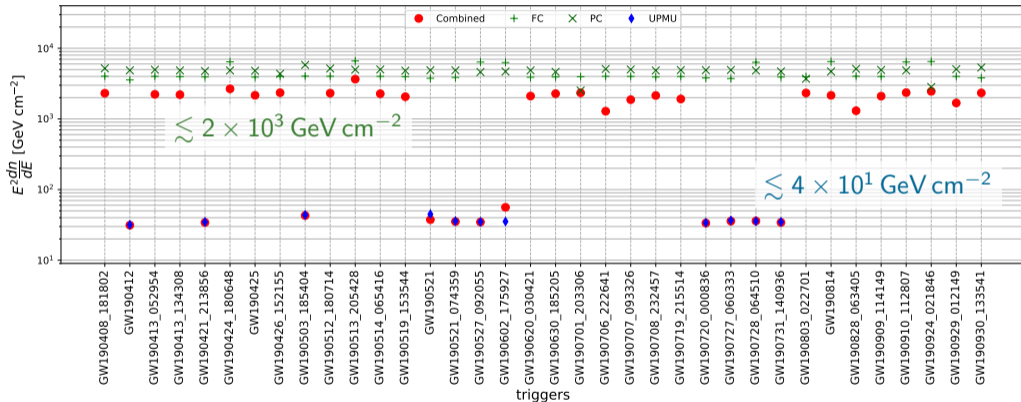
Combined flux limits

Limits combining FC, PC and UPMU are obtained by using the combined TS used for significance computation.

Effective area A_{eff}



Example of limits for ν_μ flavour:

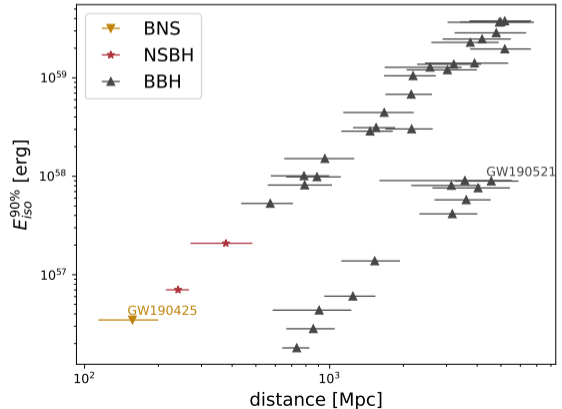


Better limits with the UPMU sample when the GW is below the local horizon. Combined limits are close to the best individual one.

- We assume isotropic emission of high-energy neutrinos around the source.
- The distance to the GW source is estimated by LIGO/Virgo from GW observations.
- The total energy emitted in neutrinos from the source is related to the flux at Earth:

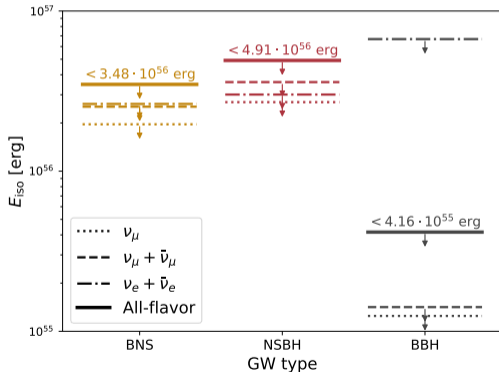
$$E_{\text{iso}} = 4\pi d^2 \int \frac{dN}{dE} \times E dE$$

- Limits on E_{iso} are computed in the same way as the ones on the flux.
- They range between 2×10^{56} erg and 4×10^{59} erg.

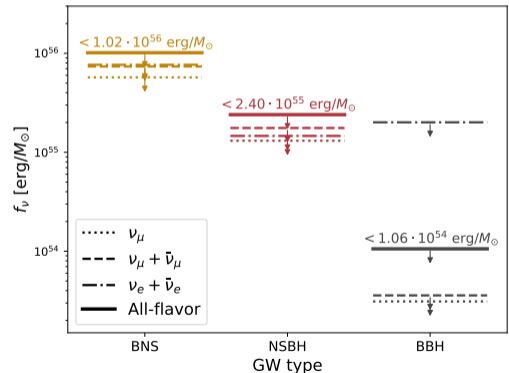


- We may expect all BBH mergers will have similar behaviour (same for NSBH and BNS).
- *Hypothesis 1*: they all the same neutrino emission and we want to constrain E_{iso}
- *Hypothesis 2*: the emission scales with the “size” of the binary system e.g., $E_{\text{iso}} = f \times M_{\text{initial}}$.

Typical emission per source population



Constraint on scaling factor f (hyp.2)

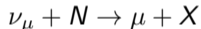


Improvement of a **factor 5** for BBH with respect to individual limits.

Follow-up with ANTARES

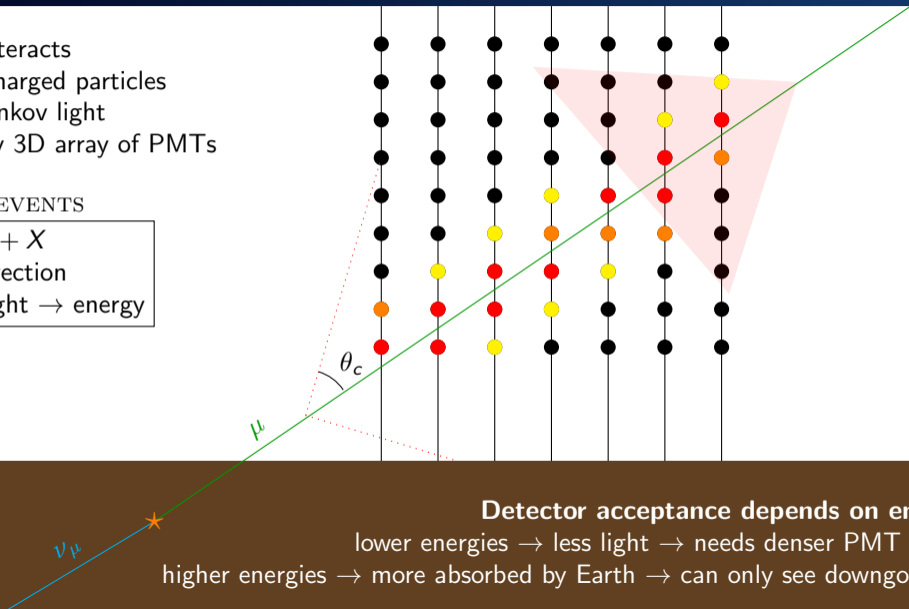
1. Neutrino interacts
2. Produces charged particles
3. Emit Cherenkov light
4. Detected by 3D array of PMTs

TRACK EVENTS



- fit line \rightarrow direction

- amount of light \rightarrow energy



Detector acceptance depends on energy:

lower energies \rightarrow less light \rightarrow needs denser PMT layout

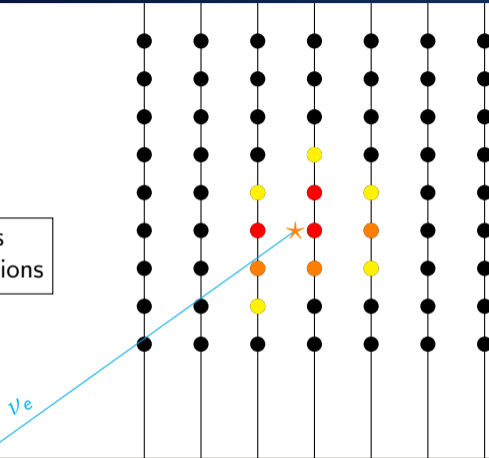
higher energies \rightarrow more absorbed by Earth \rightarrow can only see downgoing ν_s

1. Neutrino interacts
2. Produces charged particles
3. Emit Cherenkov light
4. Detected by 3D array of PMTs

SHOWER EVENTS

$\nu_e + \nu_\tau$ charged current interactions

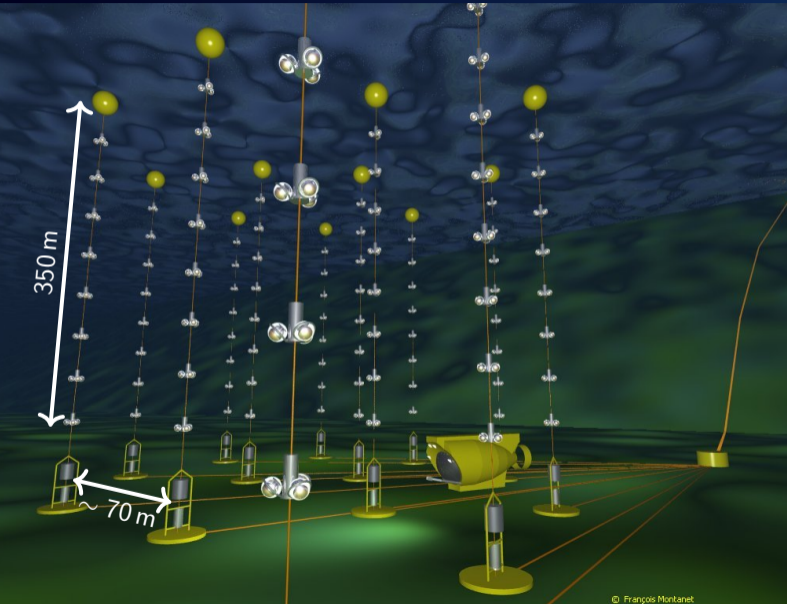
$\nu_e + \nu_\mu + \nu_\tau$ neutral current interactions



Detector acceptance depends on energy:

lower energies \rightarrow less light \rightarrow needs denser PMT layout

higher energies \rightarrow more absorbed by Earth \rightarrow can only see downgoing ν_s



- In operation since 2006 (completed in 2008)
- Off the coast of Toulon
- 12 lines
- 25 storeys/line
- 3 PMTs / storey

*Total instrumented
volume:
10 Mt*

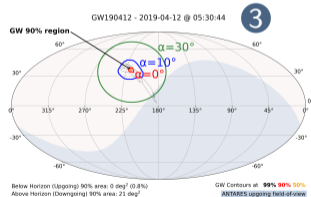
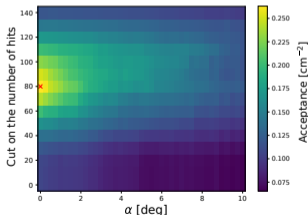
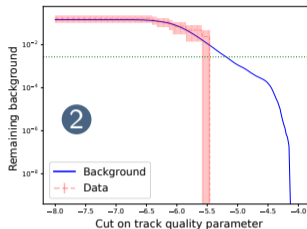
Using both track and shower events

For each GW:

- ① Find corresponding data period
- ② **Estimate background**
- ③ **Optimize search region and cuts** to ensure low background and maximise signal acceptance
- ④ Compute detector acceptance
- ⑤ Estimate systematic uncertainties
- ⑥ Count number of events in data
- ⑦ **Constrain neutrino emission**

For BBH population (same for NSBH):

- Constrain typical emission



$$\textcircled{7} \quad \mathcal{L} = \prod_c \text{Poisson} \left[N_{\text{obs}}^{(c)}; N_{\text{bkg}}^{(c)} + \phi a^{(c)} \mathcal{A}^{\gamma, (c)}(\Omega)/6 \right]$$

Assumption: $\left. \frac{dN}{dE} \right|_{\text{all-flavours}} = \phi_0 E^{-\gamma} + \text{equipartition between flavours}$

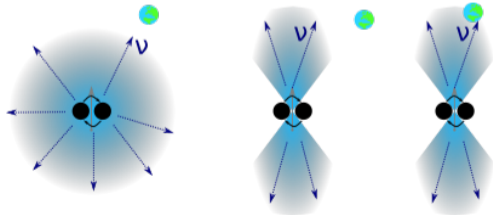
- Constraints on the flux normalisation $\phi_0 = E^\gamma dN/dE$
- Constraints on the total energy emitted in neutrino:
 - assuming **isotropic** emission
 - assuming **jetted** emission [new w.r.t. SK paper]
- Constraints on $f_\nu = E_{\text{tot},\nu}/E_{\text{rad}}$ [different w.r.t. SK paper]
 ($E_{\text{rad}} = \sum M_{\text{init}} - M_{\text{final}}$ is the total energy radiated in GWs)

Method

- Poisson counting in each event category:

$$\mathcal{L} = \prod_{c \in C} \text{Poisson} \left(N^{(c)}; B^{(c)} + \phi_0 \cdot a_\gamma^{(c)} \cdot f_\gamma^{(c)}(\Omega) \right)$$
- Priors on background and signal to cover systematics, flat prior on ϕ_0 , $E_{\text{tot}} \dots$
- Posterior distribution:

$$P(\phi_0) \propto \sum_t \mathcal{L} \left(\{N^{(c)}\}; \{B^{(c)}\}_t, \{a_\gamma^{(c)}\}_t, \Omega_t, \phi_0, \gamma \right)$$



- Follow-up of **80 GW events** reported during O3 (April 2019 – March 2020).
- **No excess observed** in any channels.
- Upper limits on neutrino emission:
 - typical flux limits:

$$E^2 dN/dE < 4 - 60 \text{ GeV cm}^{-2}$$

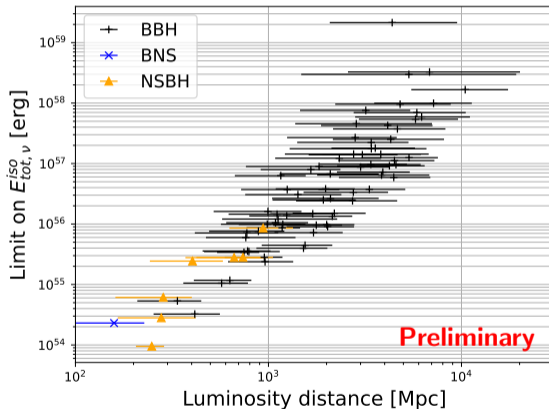
- limits on total energy emitted in neutrinos (assuming isotropic emission):

$$E_{\text{tot},\nu}^{\text{iso}} < 10^{54} - 2 \times 10^{59} \text{ erg}$$

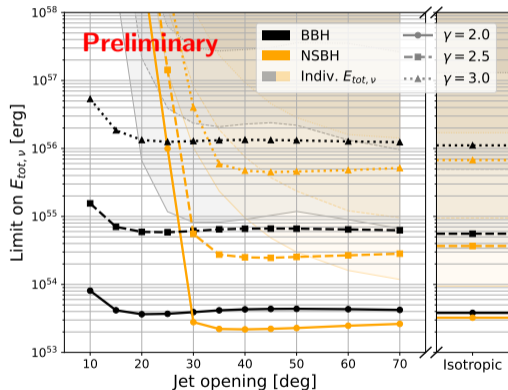
- limits on $f_{\nu}^{\text{iso}} = E_{\text{tot},\nu}^{\text{iso}}/E_{\text{rad}}$:

$$f_{\nu}^{\text{iso}} < 2 - 50\,000$$

Individual limits on total isotropic energy

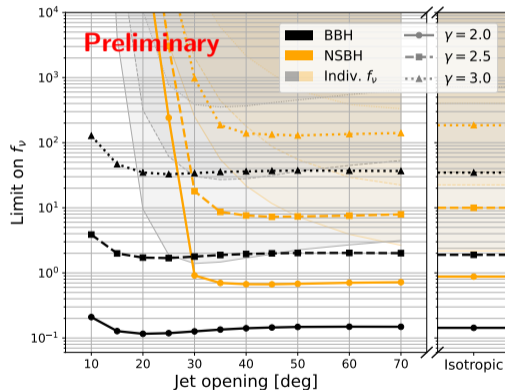


Same strategy as for Super-K, except that we also consider non-isotropic emissions and $E^{-2.5}$, $E^{-3.0}$ spectra.



BBH stacking (72 events)

$$E_{tot, \nu}^{iso} < 4 \times 10^{53} \text{ erg}, f_{\nu} < 0.15$$

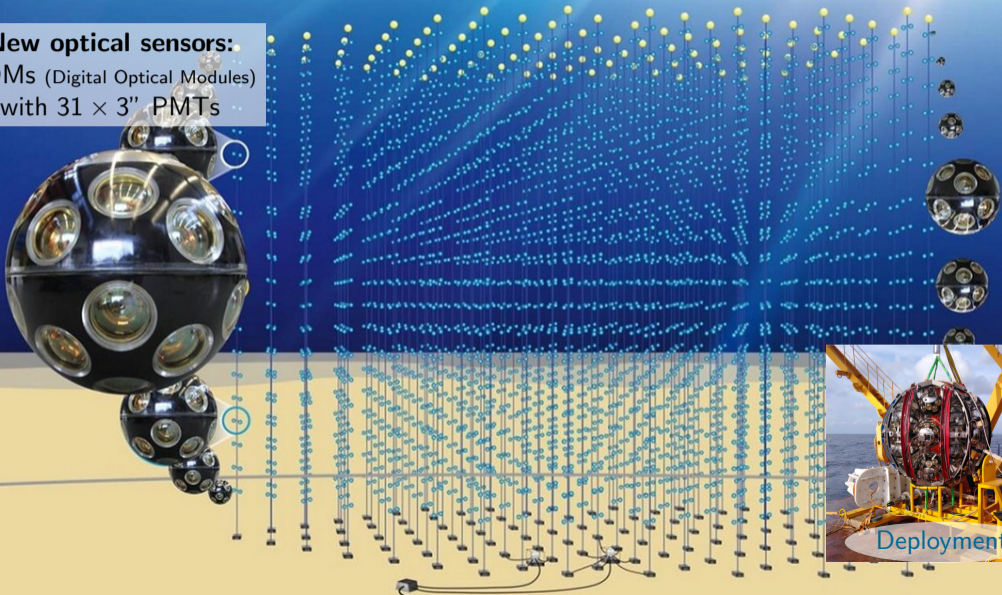


NSBH stacking (7 events)

$$E_{tot, \nu}^{iso} < 3.2 \times 10^{53} \text{ erg}, f_{\nu} < 0.88$$

Follow-up with KM3NeT

New optical sensors:
DOMs (Digital Optical Modules)
with $31 \times 3''$ PMTs



18 DOMs / line



New optical sensors:
DOMs (Digital Optical Modules)
with $31 \times 3''$ PMTs



1 building block
= 115 lines (DUs)



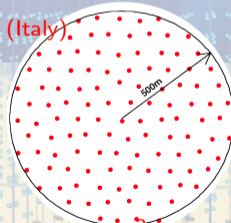
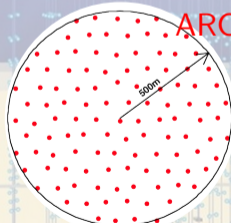
ORCA (France)

$E_\nu = \text{Gev-TeV}$

complementary

$E_\nu = \text{TeV-PeV}$

ARCA (Italy)

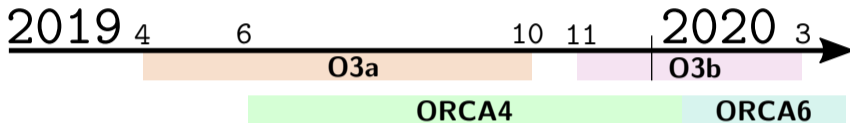


2 building blocks $\sim 1 \text{ km}^3$

18 DOMs / line



During the O3 run, KM3NeT/ORCA was taking data with 4-6 lines (ORCA4 / ORCA6)



TWO DIFFERENT SELECTIONS

Search in the 5-30 MeV energy range

- Search time window: $[t_{\text{GW}}, t_{\text{GW}} + 2 \text{ s}]$.
- Based on supernova search framework.
- Looking for an **excess of coincidental hits** in sliding 500 ms time windows.
- The maximum number of coincidences is converted to a p-value and limits on neutrino emission (quasi-thermal spectrum).

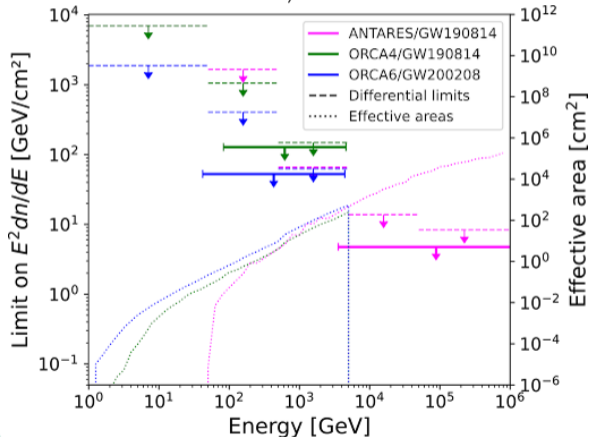
Search for **upgoing** tracks (GeV–TeV neutrinos)

- Search time window: $t_{\text{GW}} \pm 500 \text{ s}$.
- Background estimated using off-time events.
- Selection based on a **Boosted Decision Tree**.
- Cut on the score is optimized.
- Limits on neutrino emission (E^{-2} spectrum):

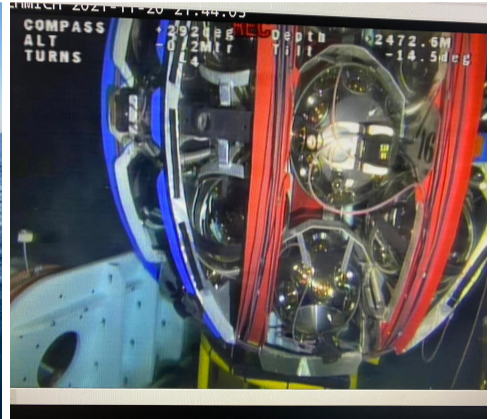
$$P(\phi) = \int \mathcal{L}(N_{\text{ON}} | b, \phi, a(\Omega)) \times \pi(b)\pi(a)\pi(\Omega)$$
assuming the source is in upgoing sky.

- Follow-up of ~ 50 **GW sources**.
- Again, **no significant excess** in both searches.
- Typical high-energy (GeV-TeV) flux limits:
 - **ORCA4**: $E^2 dN/dE < 100 - 500 \text{ GeV cm}^{-2}$
 - **ORCA6**: $E^2 dN/dE < 50 - 200 \text{ GeV cm}^{-2}$
- Typical total energy limits:
 - **MeV**: $E_{\text{tot},\nu}^{\text{iso}} < 6 \times 10^{59} - 10^{63} \text{ erg}$
 - **GeV-TeV**: $E_{\text{tot},\nu}^{\text{iso}} < 2 \times 10^{56} - 10^{59} \text{ erg}$
- Population studies for GeV-TeV analysis:
 - **BBH**: $E_{\text{tot},\nu}^{\text{iso}} < 3.1 \times 10^{55} \text{ erg}$
 - **NSBH**: $E_{\text{tot},\nu}^{\text{iso}} < 1.9 \times 10^{55} \text{ erg}$

Comparison between ANTARES and KM3NeT/ORCA4-6



ORCA: 15 lines, **ARCA:** 21 lines. **In 2023:** new lines in spring/autumn ([live blog](#) [🐦](#) [📺](#), [app](#))



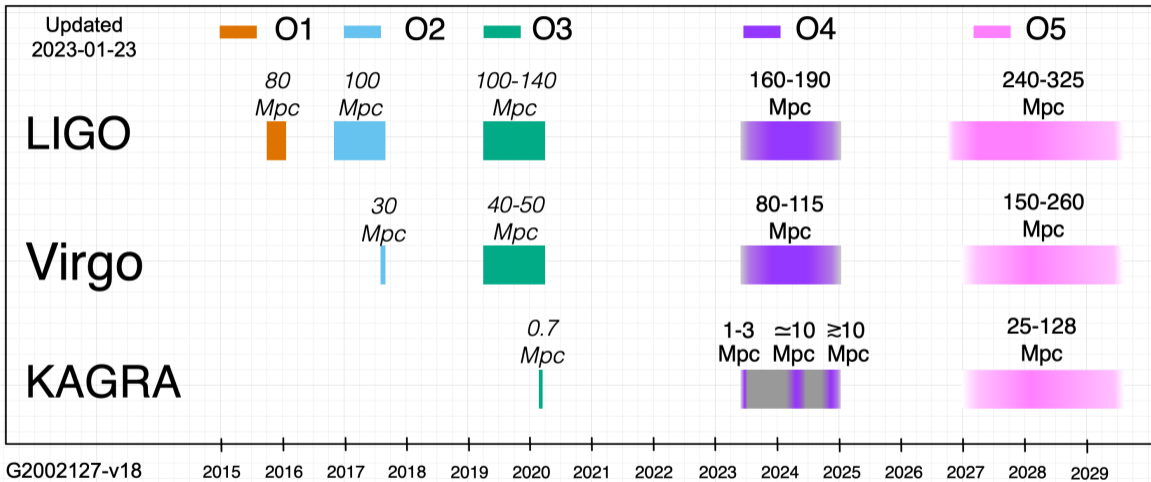


Type	Super-Kamiokande	ANTARES	KM3NeT
Energy range	7 – 100 MeV 0.1 GeV – TeV	100 GeV – PeV	5 – 30 MeV GeV – TeV
Time window	1000 s	1000 s	2 s and 1000 s
Flavours	$\bar{\nu}_e$ /all	all	all and $\nu_\mu + \bar{\nu}_\mu$
Flux limits (E^{-2})	40-2000 GeV cm^{-2}	4-60 GeV cm^{-2}	50-500 GeV cm^{-2}
$E_{\text{tot},\nu}^{\text{iso}}$, BBH stacking More details...	4×10^{55} erg Paper in ApJ	4×10^{53} erg Paper on arXiv	3×10^{55} erg Paper incoming!

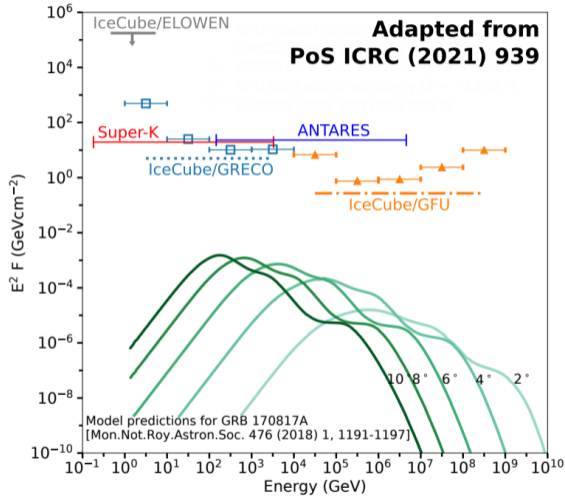
- Different selections, but same underlying statistical methods in all these searches.
- All observations are fully compatible with the expected background.



Outlooks



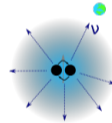
O4 expected to start on **May 24th, 2023**. Significant GW alert rate: **1 per day**



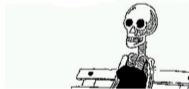
Be aware that this is a specific neutrino emission model, others may be more or less optimistic.

Despite many analyses, no excess found yet...

- 1 Waiting to get lucky for any significant neutrino detection?



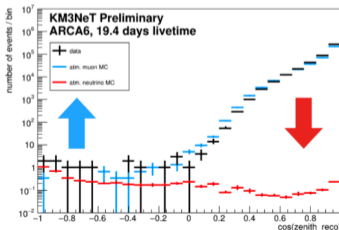
Waiting..



- 2 Extend the reach of current large telescopes (KM3NeT/IceCube) to the lowest energies.
- 3 Perform stacking analyses and population studies, taking benefit of the increasing catalog of GW sources.

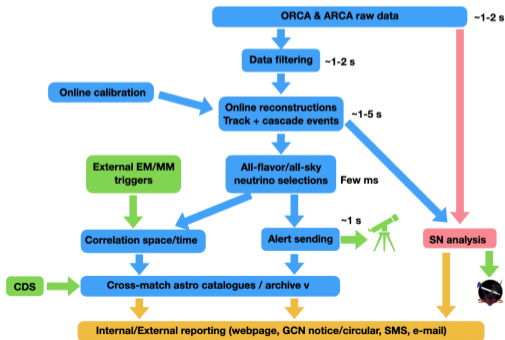
Ongoing development of new selections

- selection of GeV neutrinos using single-DOM events, to be used for transient searches **[Jonathan]**
- selection of downgoing track-like events:
 - needs to separate downgoing neutrinos from the dominant atmospheric muon background
 - development of dedicated boosted decision tree

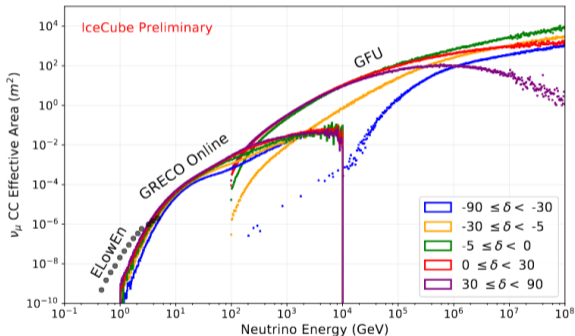


Getting prepared for O4

- new realtime framework under finalisation
- getting ready to follow O4 alerts quickly, with ORCA and ARCA, with low- and high-energy events

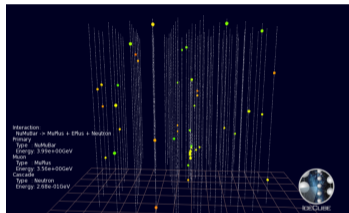


Papers: GW150914 (PRD 93, 122010), GW151226 (PRD 96, 022005), GW170817 (ApJ.Lett. 850 (2017) 2, L35), O1+O2 (ApJ.Lett. 898 (2020) 1, L10), O3 (PoS ICRC2021, 939, arXiv:2105.13160, arXiv:2208.09532)



Different analyses:

- GFU, > 100 GeV (ν_μ), $b = 6.7$ mHz
- GRECO, 5 – 100 GeV (ν_μ), $b = 4.5$ mHz
- ELOWEN, 0.5 – 5 GeV (all), $b = 20$ mHz



Example of GeV neutrino event (+noise)

Ongoing works with [Gwen] and [Karlijn]:

- improvement of the ELOWEN selection (0.5-5 GeV)
- studies about adding directionality information to the ELOWEN events
- getting prepared for quick follow-up of O4 real-time alerts!

All experiments publish independent results. How complementary are they?



Joint Analysis of Neutrinos and Gravitational waves

New Bayesian framework aiming to perform quick analysis for single detector and combination of different samples in different detectors to exploit complementarity.

Inputs:

- **Neutrino:** observation, background, Instrumental Response Functions
- **GW:** posterior samples, skymap

Configuration:

- Neutrino spectrum (E^{-2} , E^{-3} , $E^{-2}e^{-E/E_{\text{cut}}}$...), emission model
- Priors on nuisance and signal parameters
- Poisson or point-source likelihood

Outputs:

- Limits on the flux, on the total energy...
- Stacked limits for sub-populations

Can be used to investigate synergies between different searches.

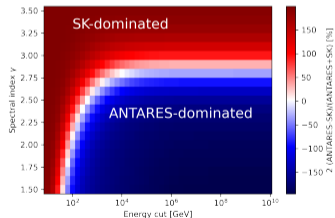
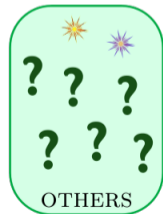
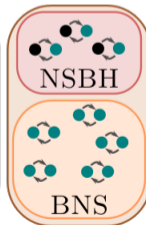
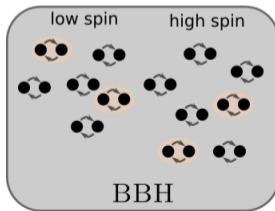
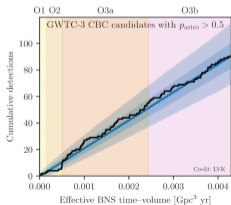
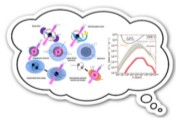


Fig: Example with ANTARES and Super-K, as a function of energy cut (x-axis) and spectral index (y-axis).

Take-home message:

- Neutrino emission expected from binary mergers
- Many constraints from existing neutrino telescopes, but **no detection yet**
- Promising prospects with O4 (more events, more BNS...)
- ... but we should also benefit from new developments at neutrino telescopes:
 - extension to lower energies [GeV range explored here @ UCLouvain]
 - synergies between experiments
 - more advanced population studies



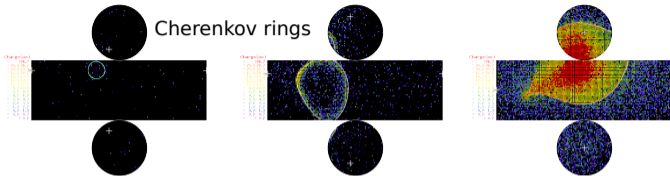
Backups

Energy ranges of neutrino telescopes

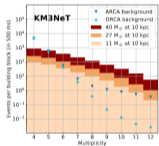
Super-K
Hyper-K

ANTARES
KM3NeT

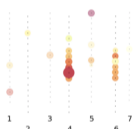
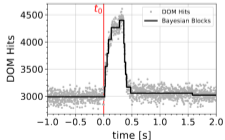
IceCube



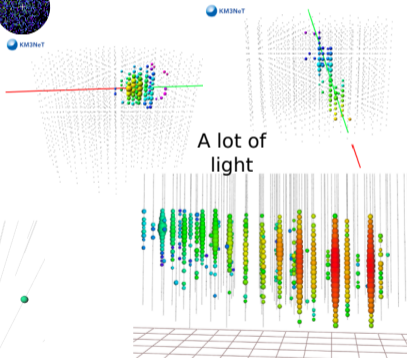
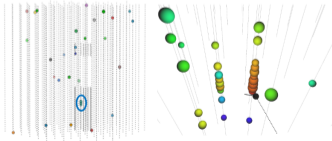
Cherenkov rings



Increase of PMT rate



Signal on few DOMs



A lot of light

MeV

GeV

TeV

PeV



How likely the SK observation is associated to background, given time+space correlations?

The p-value can be dissociated in $p = p_{\text{time}} \times p_{\text{space}}$, with:

- $p_{\text{time}} = \text{Prob}(N \geq 1) = 1 - e^{-n_B} \sim 12.6\%$ for $n_B = \text{total background}_{(\text{FC}+\text{PC}+\text{UPMU})} = 0.13$
- p_{space} is obtained by comparing neutrino direction and GW localisation.
 - For each sample ($k = \text{FC}, \text{PC}$ or UPMU), define the point-source likelihood $\mathcal{L}_\nu^{(k)}(n_S^{(k)}, \gamma; \Omega_S)$ that separates background from signal ($dn/dE \propto E^{-\gamma}$, direction Ω_S).
 - Compute the maximum log-likelihood ratio Λ (GW localisation \mathcal{P}_{GW} used as prior) and find the source direction Ω_S that maximises it:

$$\Lambda(\Omega_S) = 2 \sum_k \ln \left[\frac{\mathcal{L}_\nu(\widehat{n}_S^{(k)}, \widehat{\gamma}^{(k)}; \Omega_S)}{\mathcal{L}_\nu(n_S^{(k)} = 0; \Omega_S)} \right] + 2 \ln \mathcal{P}_{\text{GW}}(\Omega_S) \quad \text{and} \quad \boxed{\text{TS} = \max_{\Omega} [\Lambda(\Omega)]}$$

- Compare TS_{data} with the expected background distribution (with $N \geq 1$) to obtain p_{space} .

For each sample k , we define the likelihood:

$$\mathcal{L}_\nu^{(k)}(n_S^{(k)}, \gamma; \Omega_S) = \frac{e^{-(n_S^{(k)} + n_B^{(k)})} (n_S^{(k)} + n_B^{(k)})^{N^{(k)}}}{N^{(k)}!} \prod_{i=1}^{N^{(k)}} \frac{n_S^{(k)} \mathcal{S}^{(k)}(\vec{x}_i, E_i; \Omega_S, \gamma) + n_B^{(k)} \mathcal{B}^{(k)}(\vec{x}_i, E_i)}{n_S^{(k)} + n_B^{(k)}}$$

where $\mathcal{S}^{(k)}$ and $\mathcal{B}^{(k)}$ are the signal/background p.d.f. (characterizing detector response).

Then, we compute the log-likelihood ratio:

$$\Lambda(\Omega_S) = 2 \sum_k \ln \left[\frac{\mathcal{L}_\nu(\widehat{n_S^{(k)}}, \widehat{\gamma^{(k)}}; \Omega_S)}{\mathcal{L}_\nu(n_S^{(k)} = 0; \Omega_S)} \right] + 2 \ln \mathcal{P}_{GW}(\Omega_S)$$

The final test statistic and p-value are:

$$TS = \max_{\Omega} [\Lambda(\Omega)] \text{ and } p_{\text{space}} = \int_{TS_{\text{data}}}^{\infty} \mathcal{P}_{\text{bkg}}(TS) dTS$$

where $\mathcal{P}_{\text{bkg}}(TS)$ is the expected background distribution.

- **Flux:** We define the following likelihood by using the TS defined before:

$$\mathcal{L}(\phi_0; TS_{\text{data}}, \mathcal{P}_{GW}) = \int \sum_{k=0}^2 \left[\frac{(c(\Omega)\phi_0)^k}{k!} e^{-c(\Omega)\phi_0} \times \mathcal{P}_k(TS_{\text{data}}) \right] \times \mathcal{P}_{GW}(\Omega) d\Omega$$

where $P_i(TS)$ is the distribution of the test statistic assuming the signal consists in i events, assuming E^{-2} spectrum ($dn/dE = \phi_0 E^{-2}$). The 90% upper limit is obtained as above ($\int_0^{\phi_0^{\text{up}}} \mathcal{L}(\phi_0) d\phi_0 = 0.90$).

- **Total energy:** Same for E_{iso} limits:

$$\mathcal{L}(E_{\text{iso}}; TS_{\text{data}}^{(i)}, \mathcal{V}_{GW}^{(i)}) = \int \sum_{k=0}^2 \left[\frac{(c'(r, \Omega)E_{\text{iso}})^k}{k!} e^{-c'(r, \Omega)E_{\text{iso}}} \times \mathcal{P}_k^{(i)}(TS_{\text{data}}^{(i)}) \right] \times \mathcal{V}_{GW}^{(i)}(r, \Omega) d\Omega$$

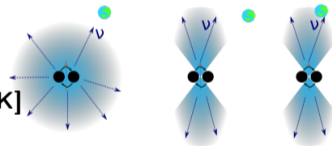
- Inputs
- detector acceptance as a function of direction
 - number of observed/expected events in track and shower channels
 - GW information (posterior samples, skymap)

Assumptions

$$\left. \frac{dN}{dE} \right|_{\text{all-flavours}} = \phi_0 E^{-\gamma} + \text{equipartition between flavours}$$

Outputs

- constraints on the flux normalisation $\phi_0 = E^\gamma dN/dE$
- constraints on the total energy emitted in neutrino:
 - assuming isotropic emission
 - assuming jetted emission [**DIFFERENT w.r.t. Super-K**]
- constraints on $f_\nu = E_{\text{tot},\nu}/E_{\text{rad}}$ [**DIFFERENT**]
 ($E_{\text{rad}} = \sum M_{\text{init}} - M_{\text{final}}$ is the total radiated energy = total energy budget of the merger process)



Method

- Poisson counting in each event category:

$$\mathcal{L} = \prod_{c \in \mathcal{C}} \text{Poisson} \left(N^{(c)}; B^{(c)} + \phi_0 \cdot a_\gamma^{(c)} \cdot f_\gamma^{(c)}(\Omega) \right)$$
- Priors on background and signal to cover systematics, flat prior on $\phi_0, E_{\text{tot}} \dots$
- Posterior distribution: $P(\phi_0) \propto \sum_{t \in \text{toys}} \mathcal{L} \left(\{N^{(c)}\}; \{B^{(c)}\}_t, \{a_\gamma^{(c)}\}_t, \Omega_t, \phi_0, \gamma \right)$