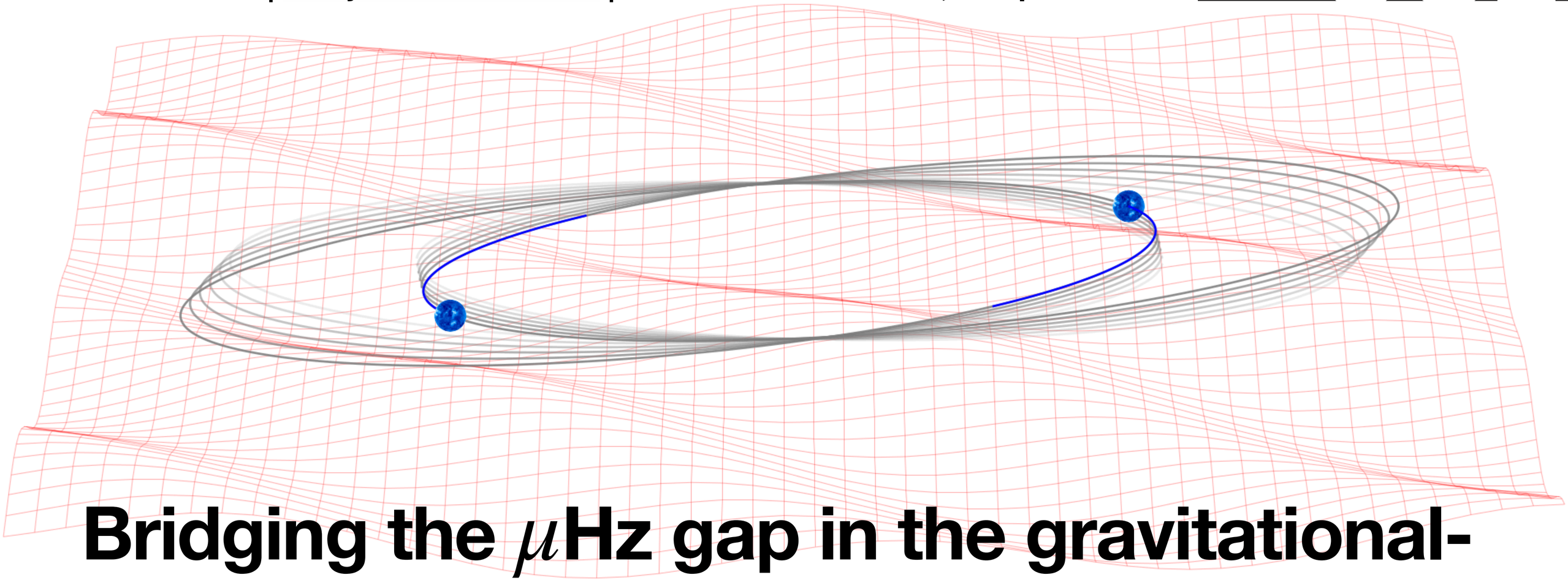
Alex Jenkins | alex.jenkins@ucl.ac.uk | GW Orchestra Louvain, 9 Sept 2022



Bridging the μHz gap in the gravitational-wave landscape with binary resonance

Based on 2107.04601 (PRL) + 2107.04063 (PRD) with Diego Blas (IFAE Barcelona)

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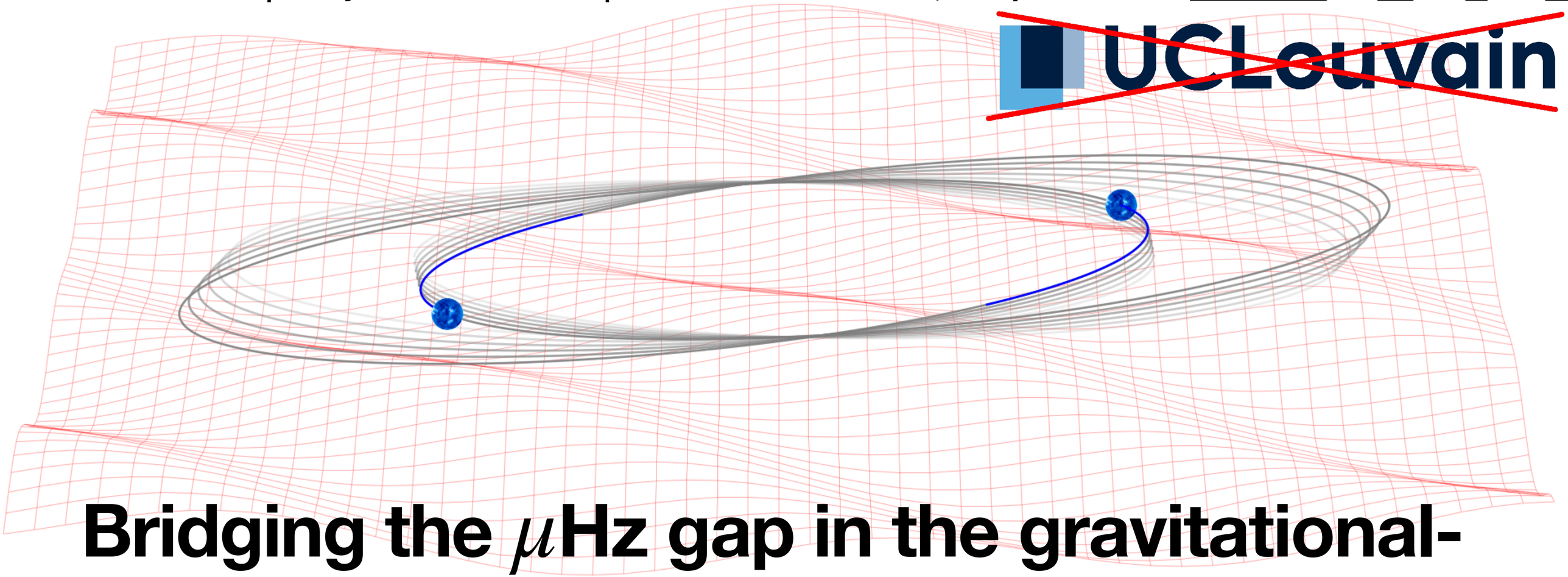

Bridging the μHz gap in the gravitational-wave landscape with binary resonance

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EXPRESS

Moon to become huge lab to detect 'hidden' waves from supermassive black holes

THE MOON could soon be turned into a giant experiment for detecting a presently "hidden" band of gravitational waves of the frequency generated by collisions between supermassive black holes, Express.co.uk can reveal.

FOCUS

The Moon as a Gravitational-Wave Detector

March 11, 2022 • *Physics* 15, 34

Thanks to a new analysis technique, precision measurements of the Earth-Moon distance should improve estimates of the size of the gravitational-wave background.

Physics

BIG THINK

HARD SCIENCE — APRIL 1, 2022

Using the Moon as a gravitational wave detector to study the origin of the Universe

To study the origin of the Universe, we could build a constellation of six expensive spacecraft — or we could just use the Moon.

SYFY

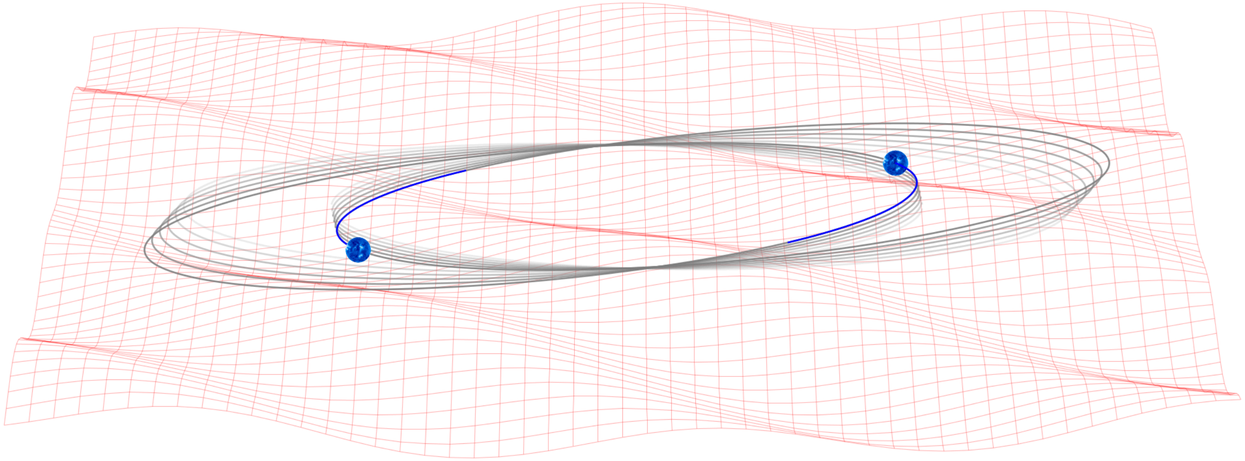
MARCH 22, 2022, 5:29 PM ET

TURNING THE MOON INTO A GRAVITATIONAL WAVE DETECTOR

That's no moon. It's a gravitational wave detector.

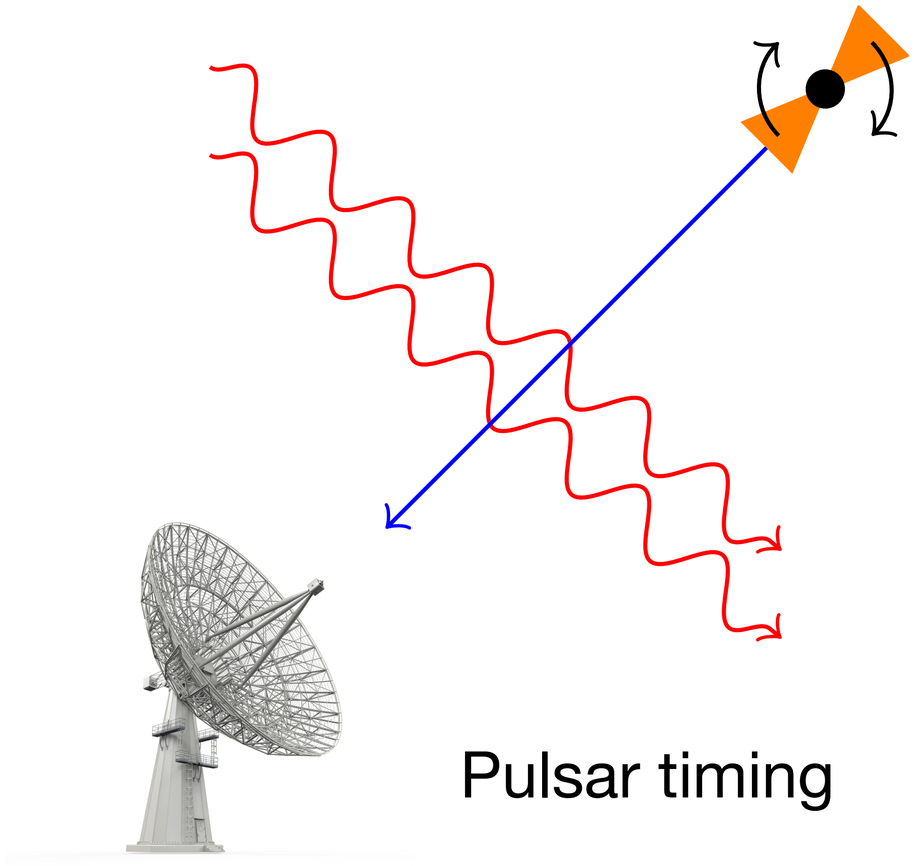
Plan for this talk

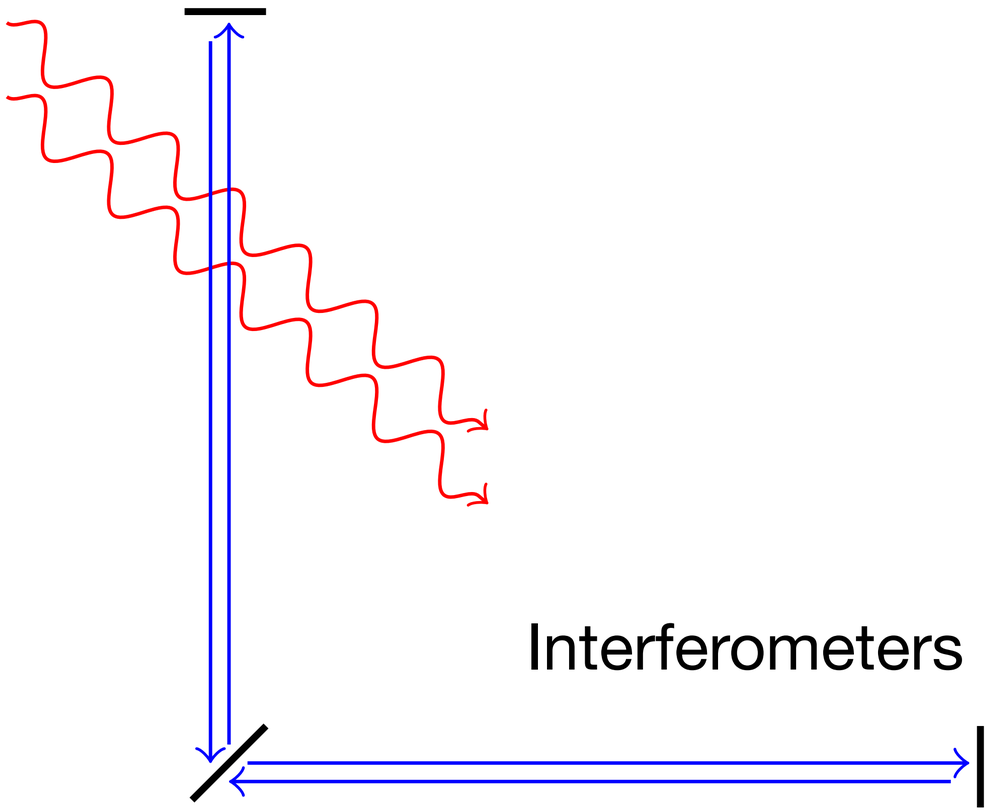
1. GW background constraints
2. Binary resonance
3. Our results



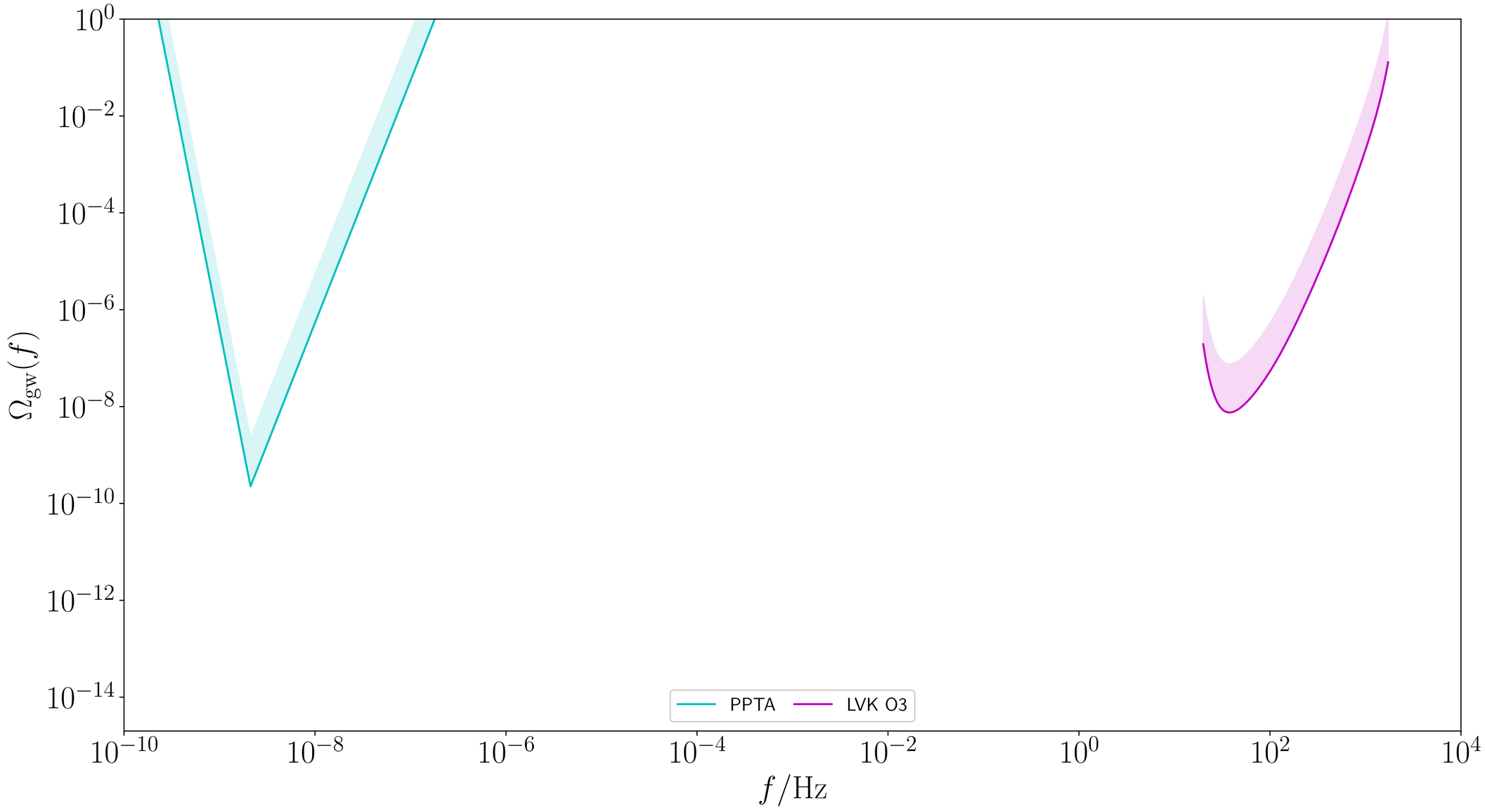
How do we measure the GW background?

Two key methods (currently), both search for GW perturbations to photon travel times

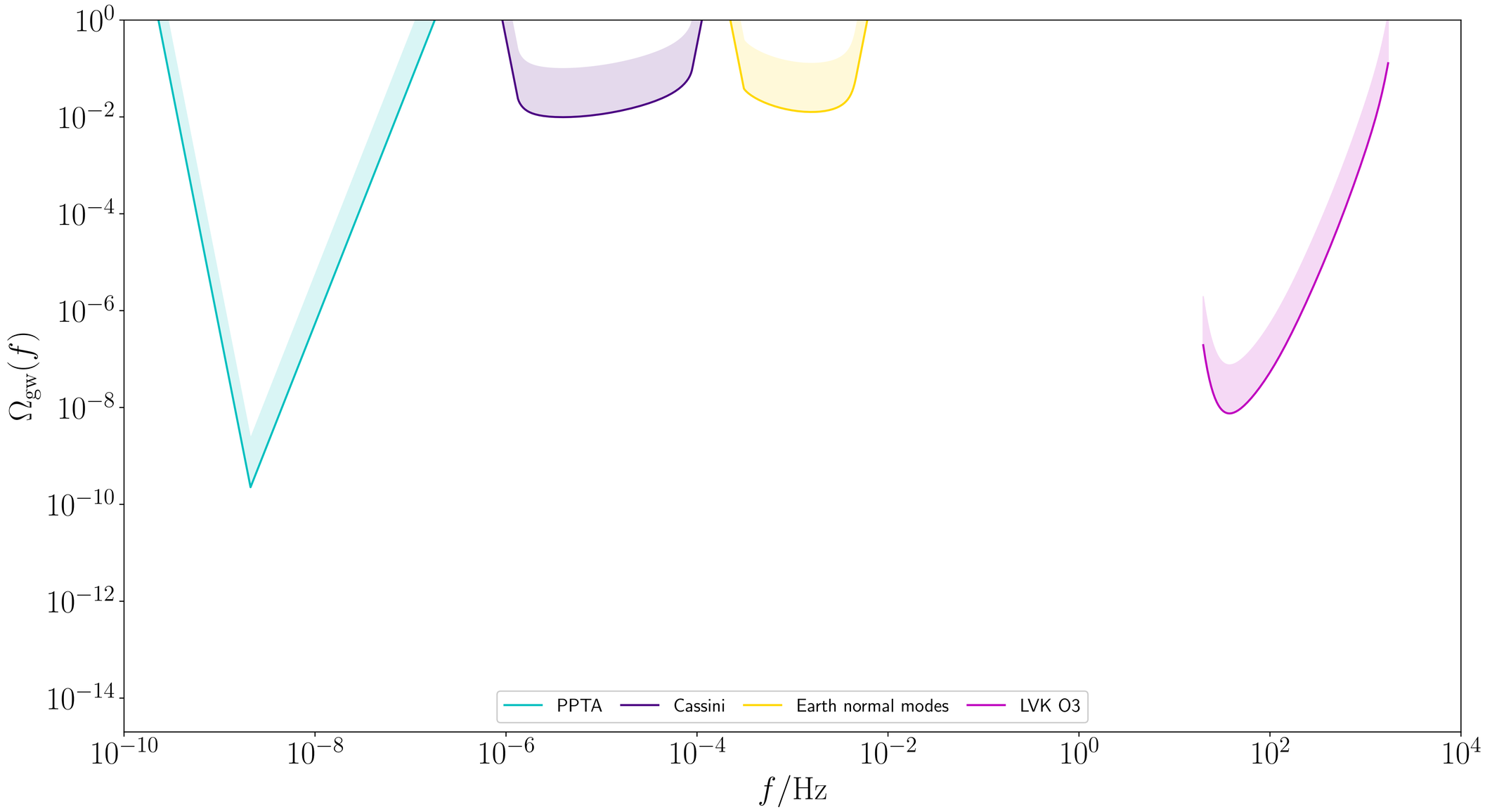




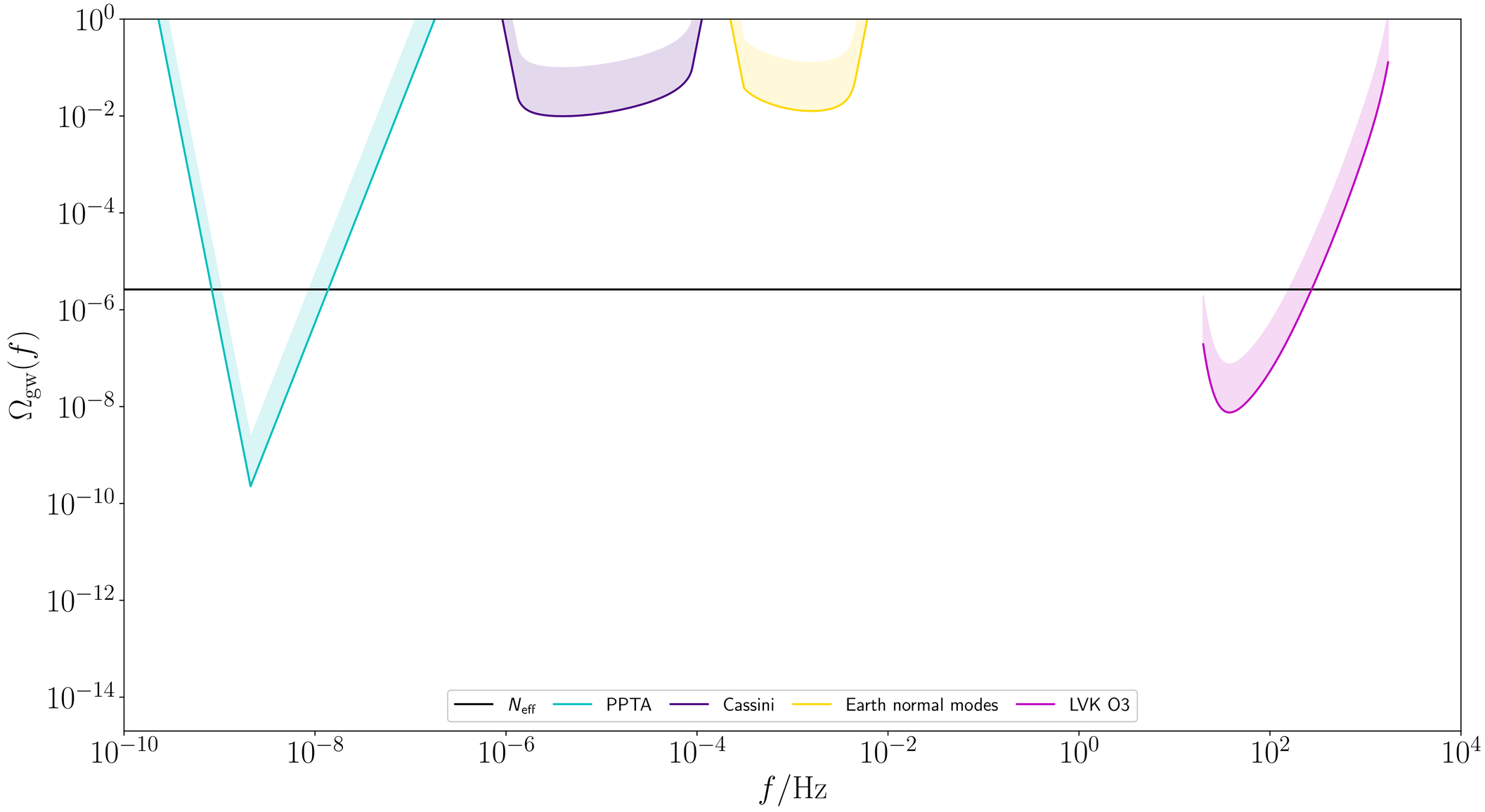
Current GWB constraints



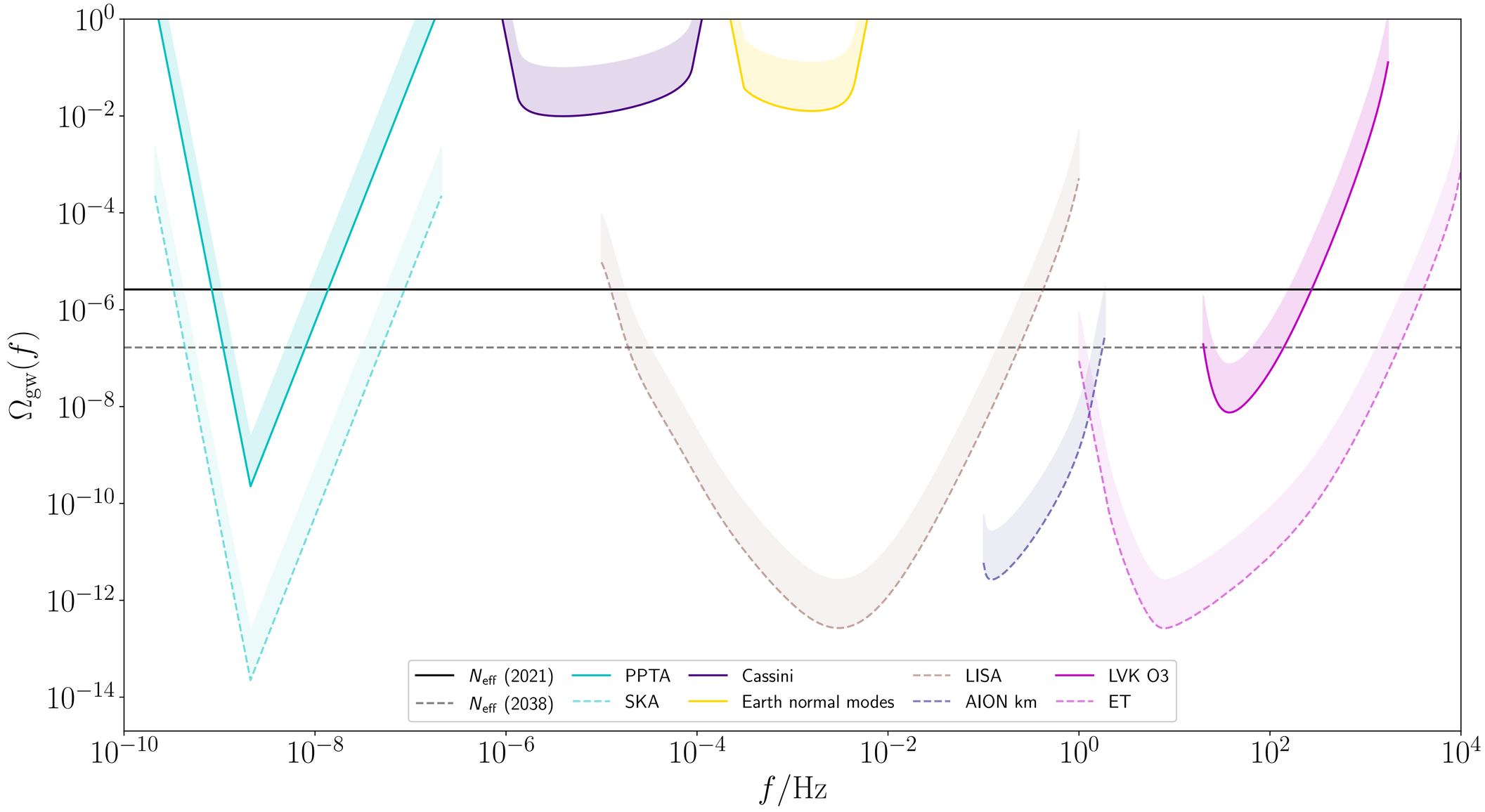
Current GWB constraints



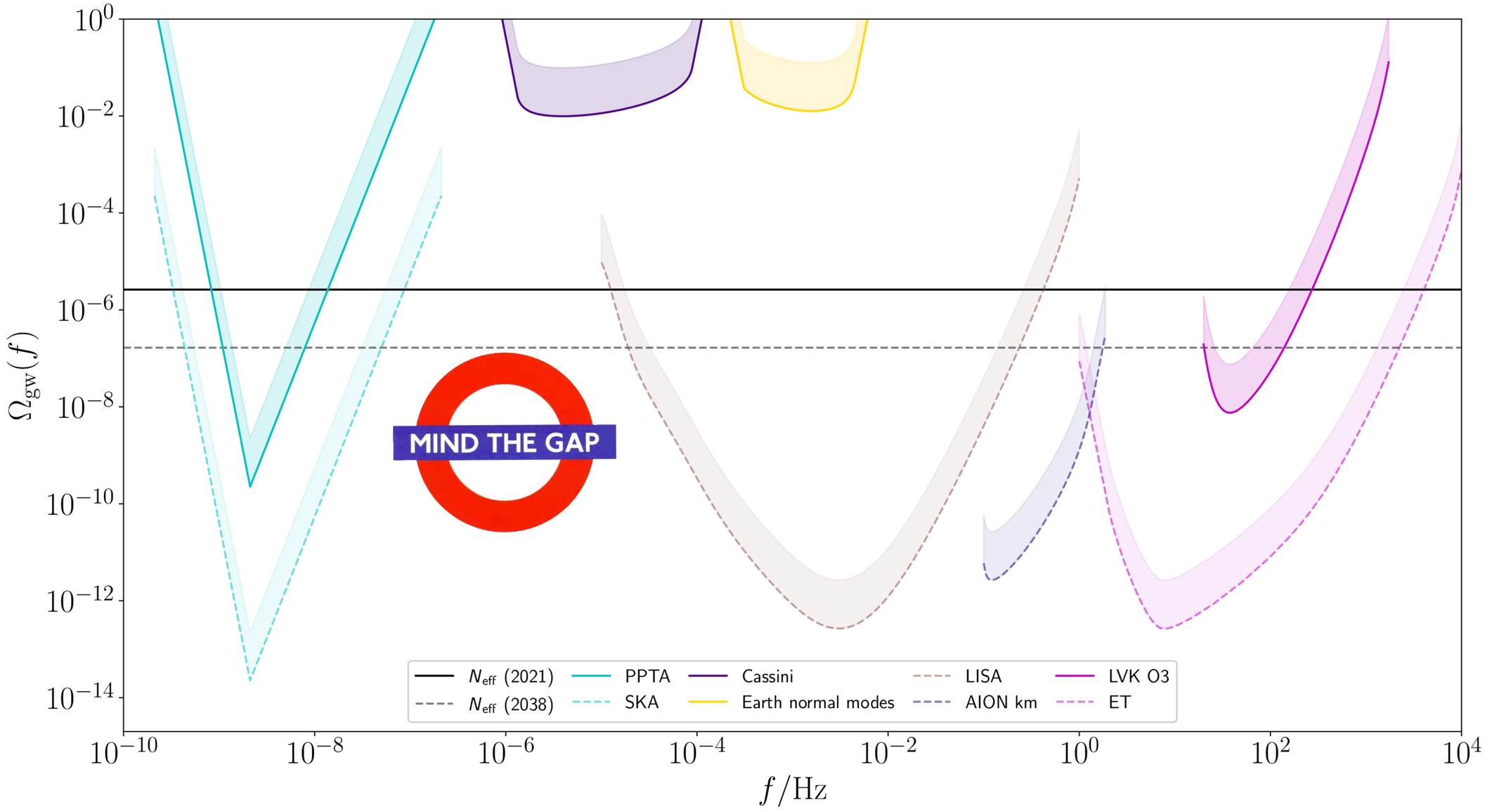
Current GWB constraints



Forecast GWB constraints

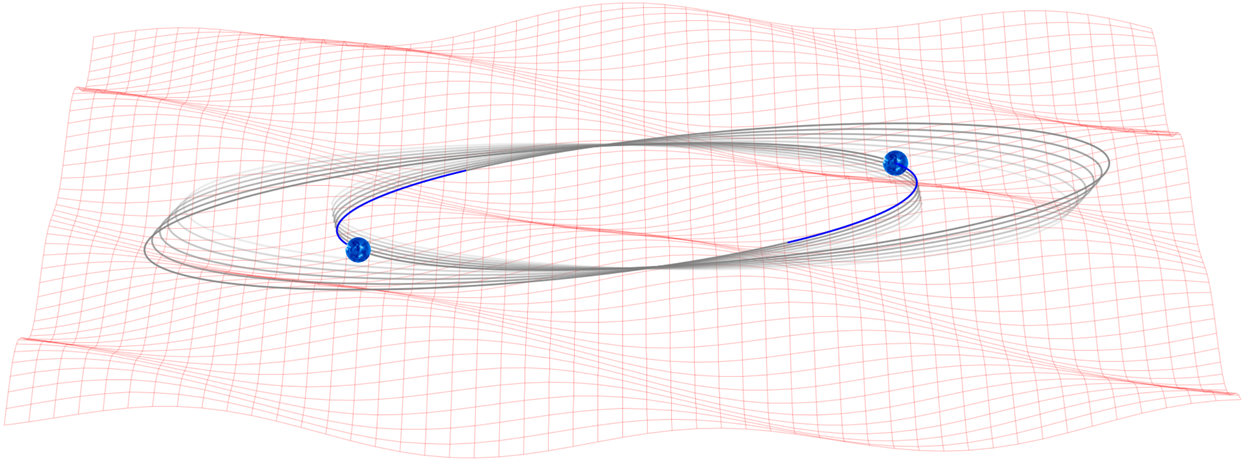


Forecast GWB constraints

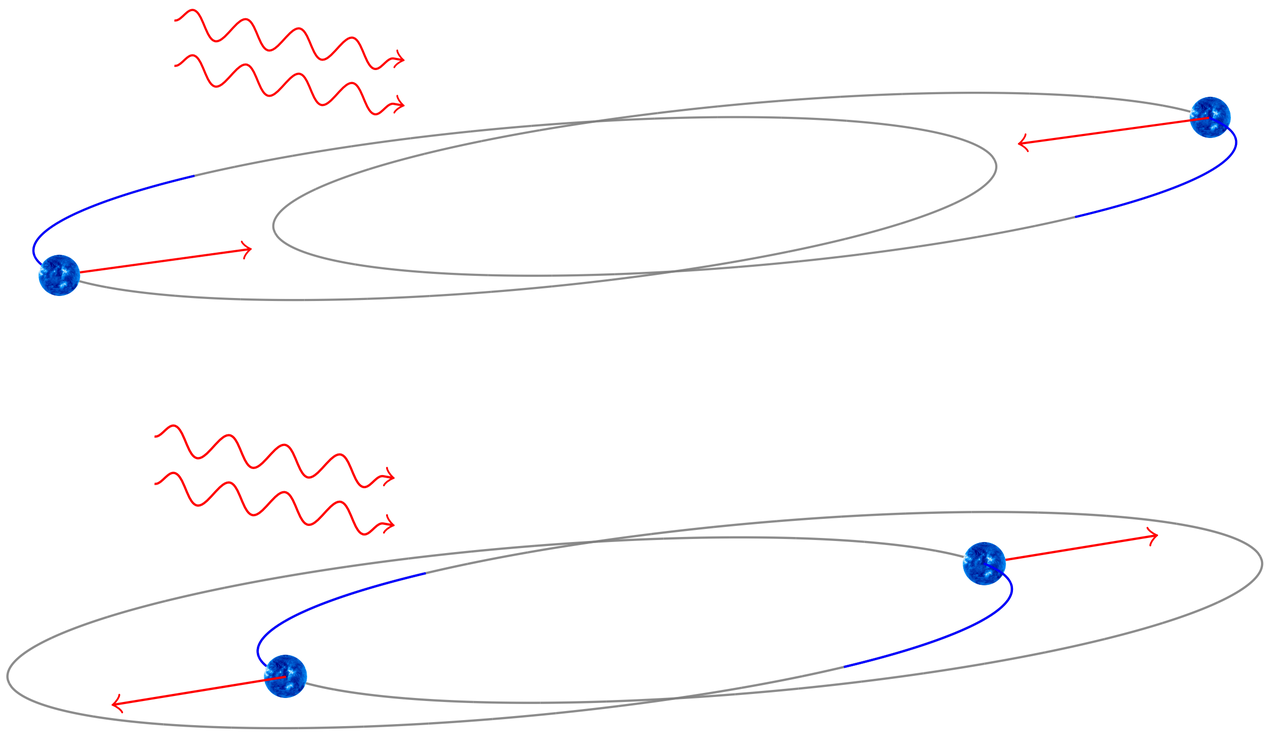


Plan for this talk

1. GW background constraints
2. Binary resonance
3. Our results



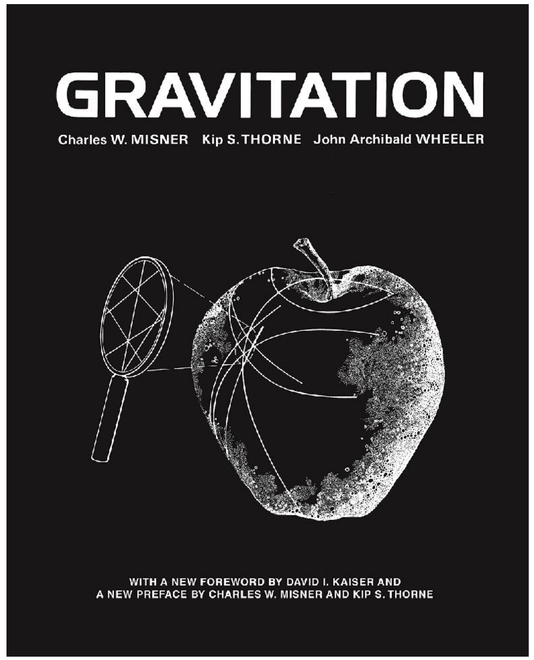
A way forward: binary resonance



- GWs cause oscillations between orbiting bodies
- Resonance for frequencies $f = n/P$ (where P is the period)
- Imprints on orbit accumulate over time

Binary resonance: a brief history

- Similar idea discussed (rather pessimistically…) in MTW (1973)



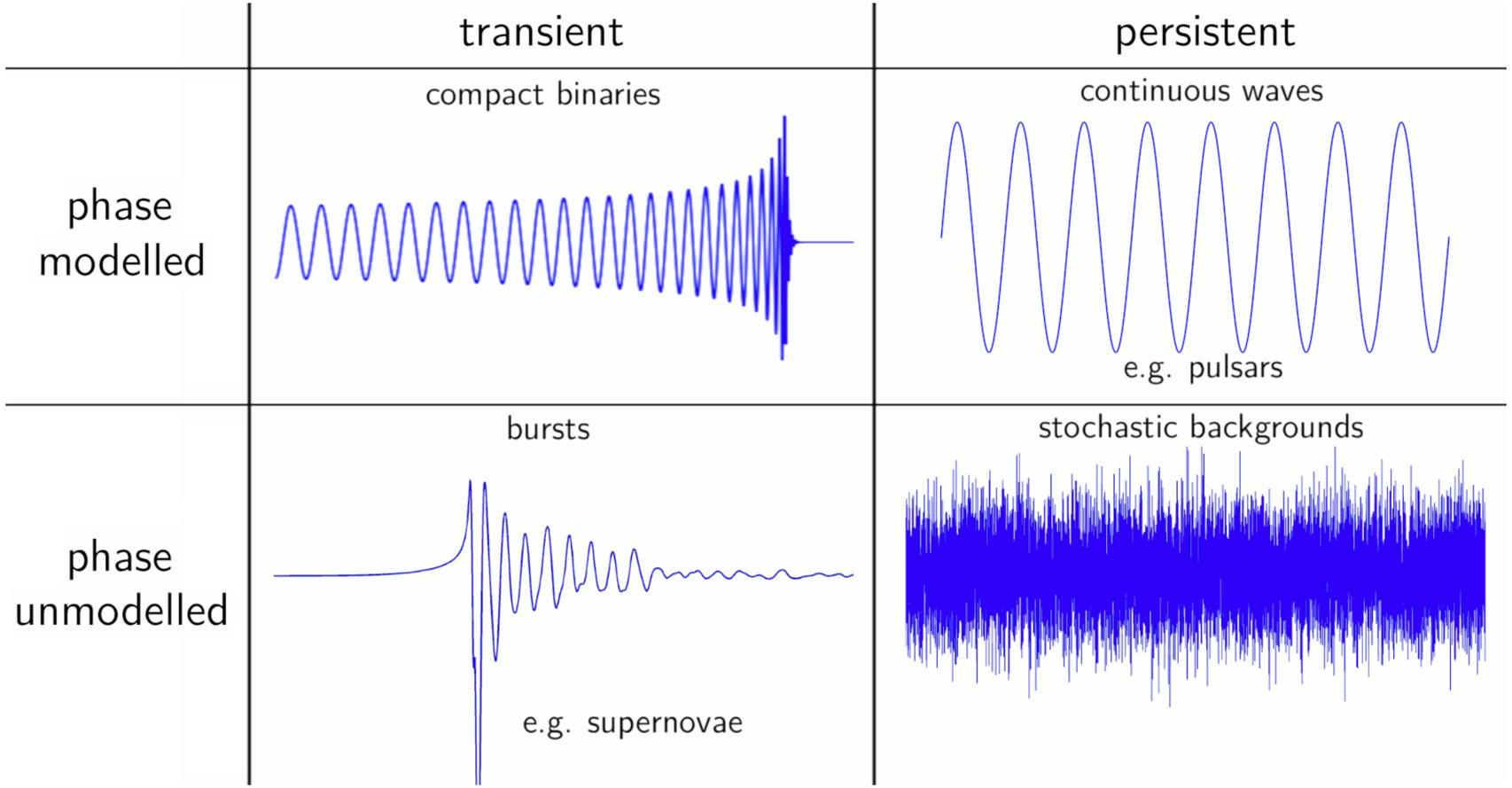
1. The Relative Motions of Two Freely Falling Bodies

As a gravitational wave passes two freely falling bodies, their proper separation oscillates (Figure 37.3). This produces corresponding oscillations in the redshift and round-trip travel times for electromagnetic signals propagating back and forth between the two bodies. Either effect, oscillating redshift or oscillating travel time, could be used in principle to detect the passage of the waves. Examples of such detectors are the Earth-Moon separation, as monitored by laser ranging [Fig. 37.2(a)]; Earth-spacecraft separations as monitored by radio ranging; and the separation between two test masses in an Earth-orbiting laboratory, as monitored by redshift measurements or by laser interferometry. Several features of such detectors are explored in exercises 37.6 and 37.7. As shown in exercise 37.7, such detectors have so low a sensitivity that they are of little experimental interest.

- This *does not* account for the resonance, which boosts detectability
- More recently investigated by Hui+, 1212.2623 — Hulse-Taylor binary pulsar bounds
- Similar ideas used to search for ultralight dark matter by Blas+, 1612.06789

Why a stochastic background?

- *Cumulative* effect on orbit, so we need a *persistent* source
- *Narrow* resonance band(s), so we need a *broadband* source

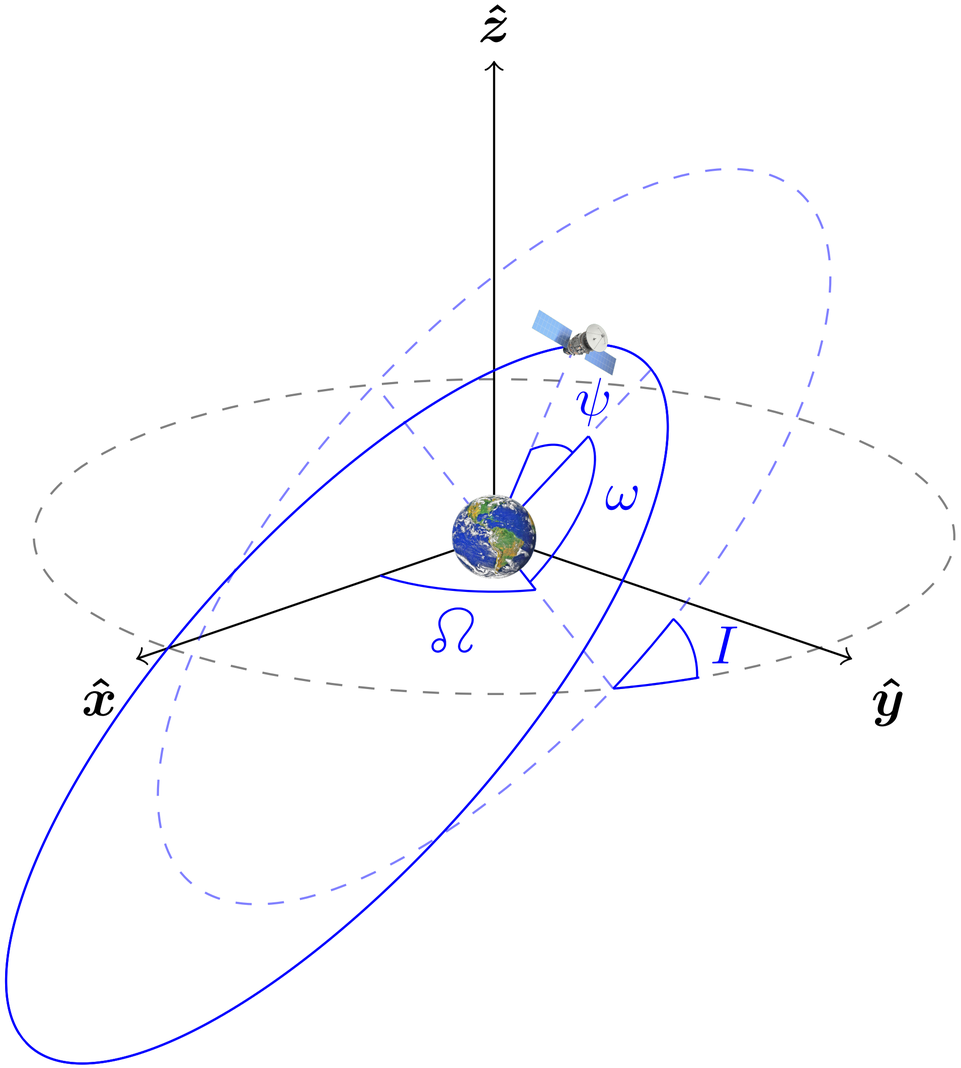


Osculating orbital elements

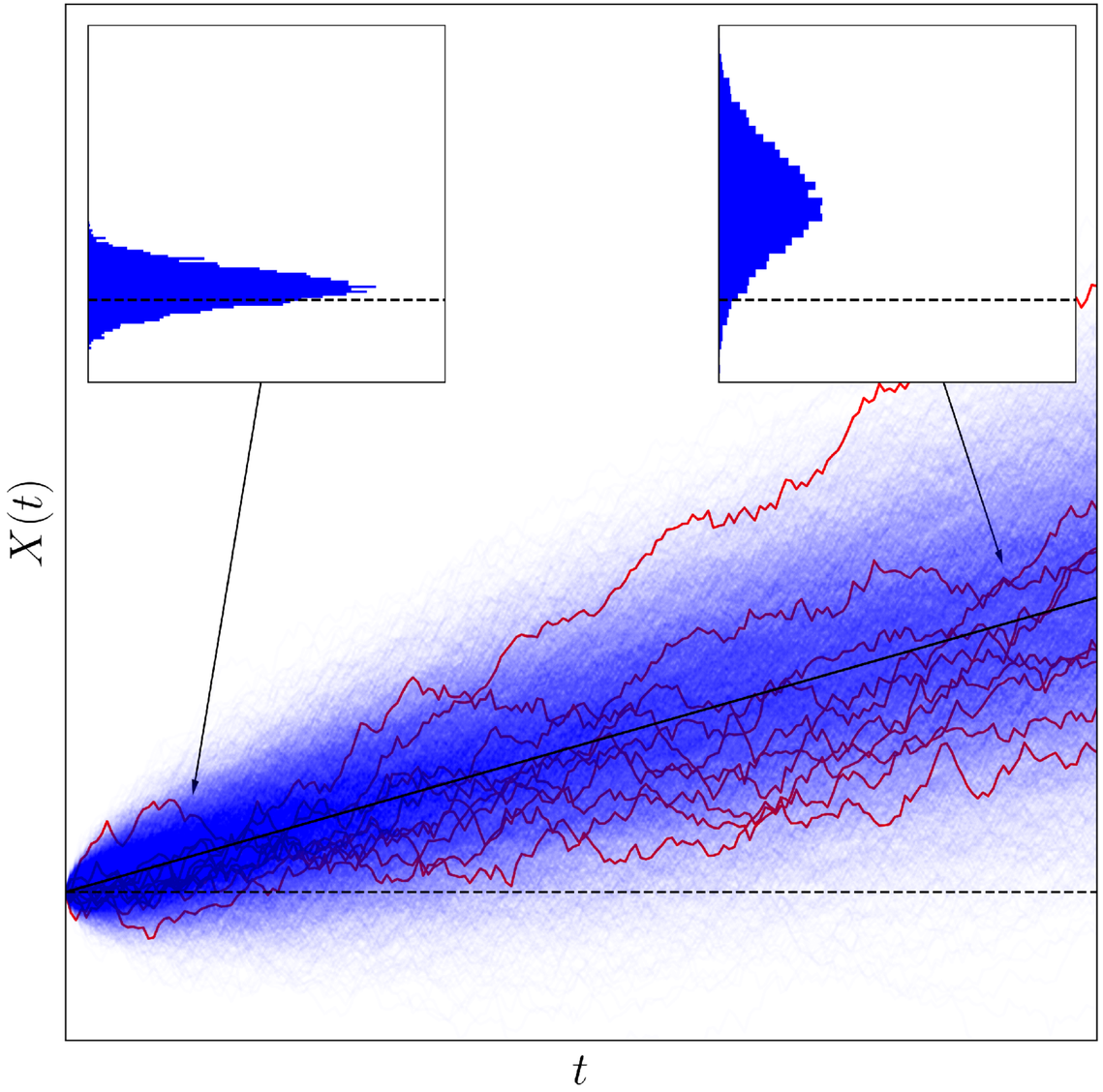
- Period P, eccentricity e:
  size and *shape* of orbit

- Inclination I, ascending node Ω:
  orientation in space

- Pericentre ω, mean anomaly at epoch ε:
  radial and angular *phases*



Our new approach



- Track distribution function $W(\mathbf{X}, t)$ of orbital elements $\mathbf{X} = (P, e, I, \dots)$

$$\Pr(\mathbf{X} \in \mathcal{X} \,|\, t) = \int_{\mathcal{X}} \mathrm{d}\mathbf{X}\, W(\mathbf{X}, t)$$

- Evolves through *Fokker-Planck eqn.*

$$\frac{\partial W}{\partial t} = -\frac{\partial}{\partial X_i}(D_i^{(1)} W) + \frac{\partial}{\partial X_i}\frac{\partial}{\partial X_j}(D_{ij}^{(2)} W)$$

Find the code on GitHub!
github.com/alex-c-jenkins/gw-resonance

Some general results

- Noise-induced drift for the period P is *positive* — net *absorption* of energy which competes with energy loss from radiating GWs

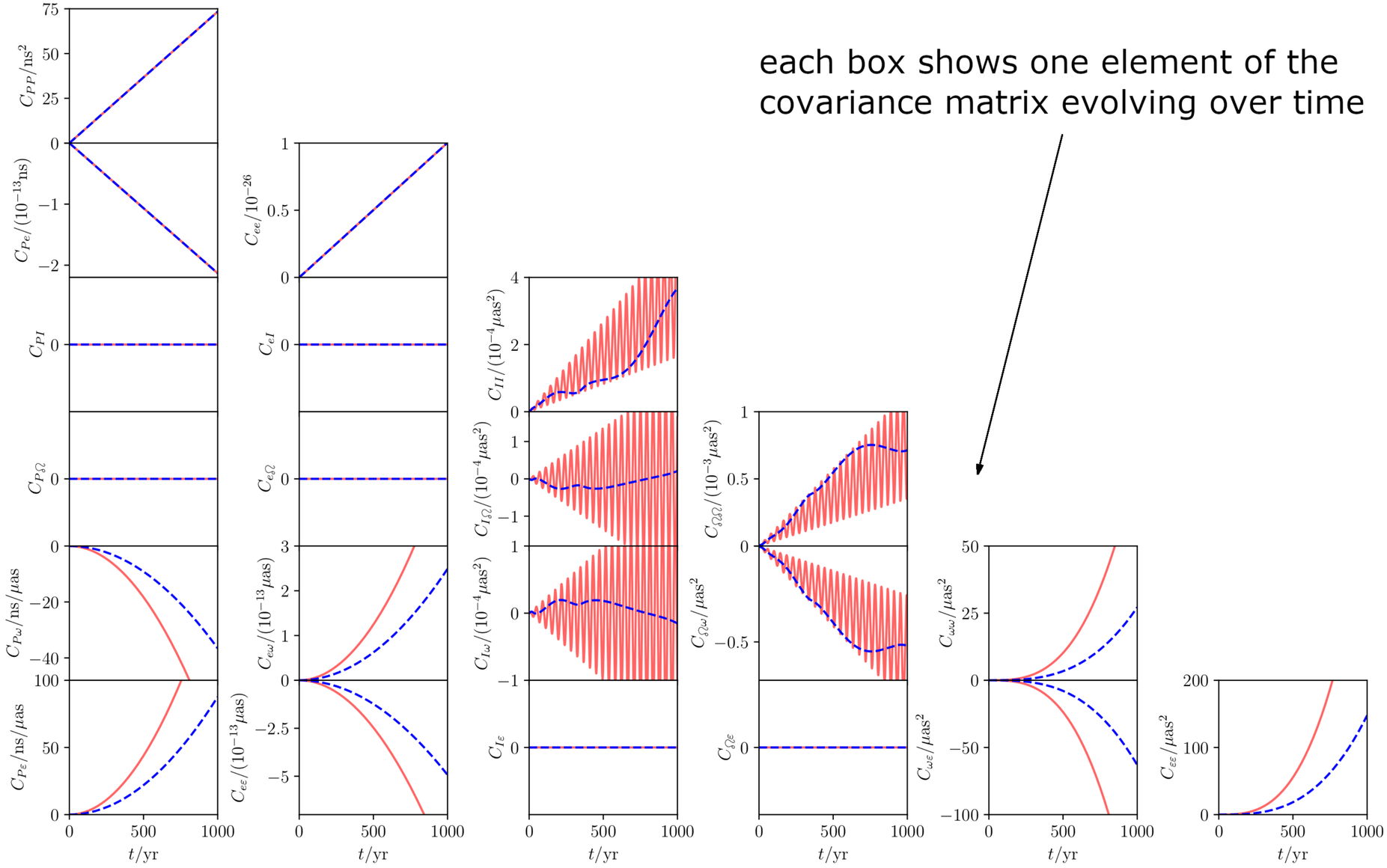
$$P_{\rm crit} \approx 95\,{\rm yr} \times \left(\frac{\Omega_{\rm gw}}{10^{-6}} \frac{1/4}{\eta} \right)^{-3/11} \left(\frac{M}{M_\odot} \right)^{5/11}$$

- Drift and diffusion of eccentricity e are generically *positive* in the limit $e \to 0$, so GW absorption drives binaries away from being circular

- GW *emission* makes binaries *hard* and *circular*,
  GW *absorption* makes them *soft* and *eccentric*!

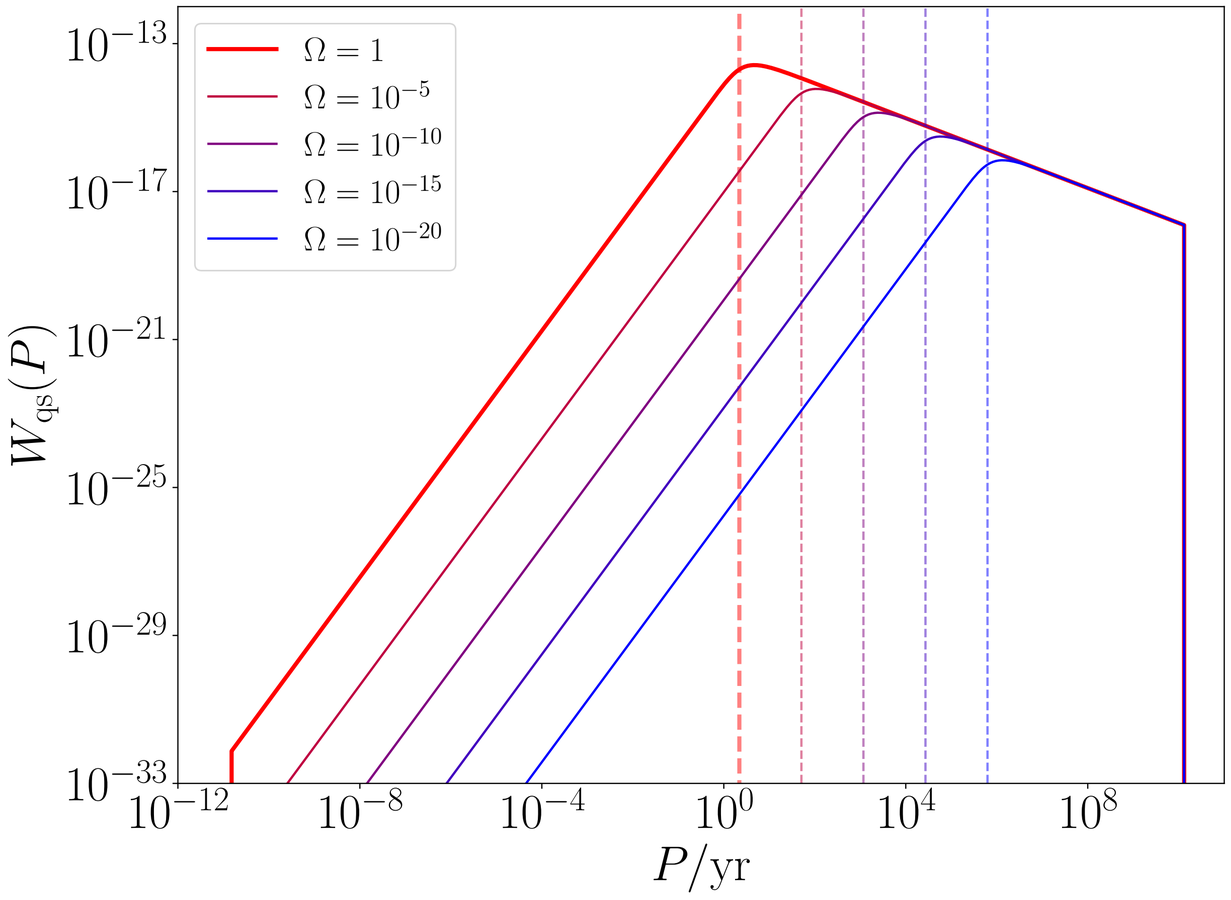
Example: the Hulse-Taylor binary pulsar

each box shows one element of the covariance matrix evolving over time



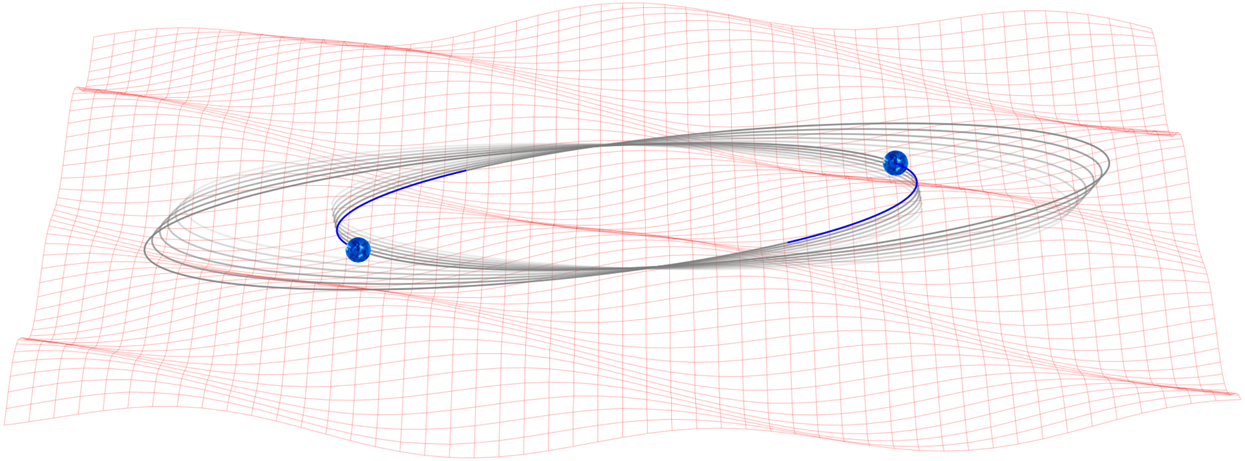
What happens on long timescales?

- Fokker-Planck formalism lets us find *(quasi-)stationary solutions*, which describe the distribution at late times
- Period distribution peaks at P_{crit}
- Inclination I becomes isotropic, other angles become uniform
- At the moment, we have only studied the case $e = 0$, but expect these features to be generic



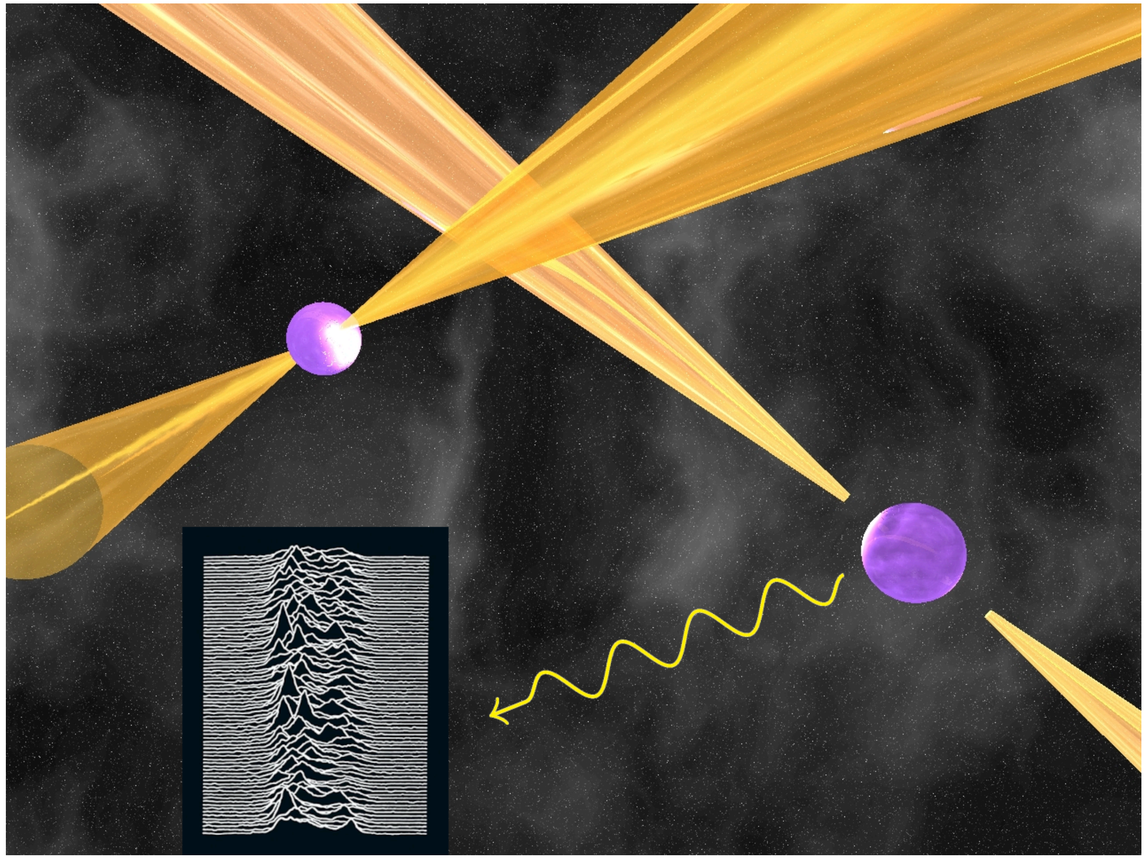
Plan for this talk

1. Stochastic GW background
2. Binary resonance
3. Our results



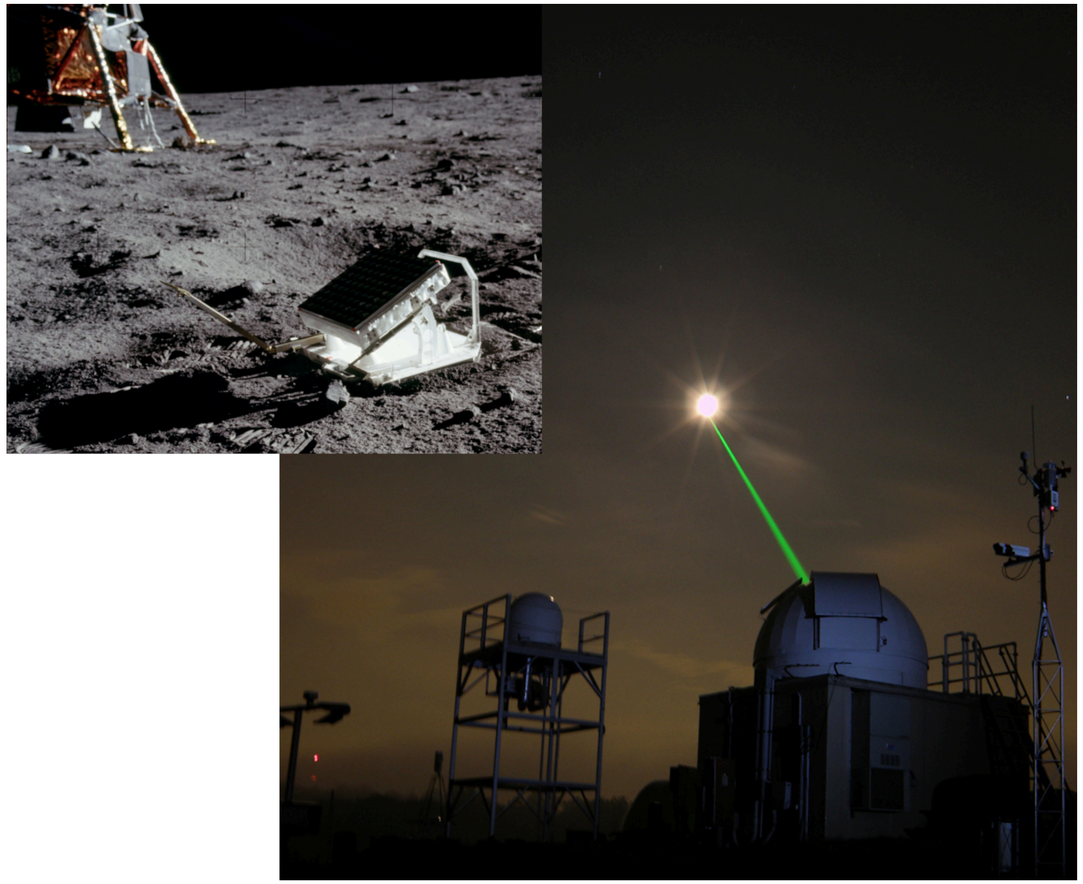
Two binary probes

Timing of binary pulsars



Michael Kramer

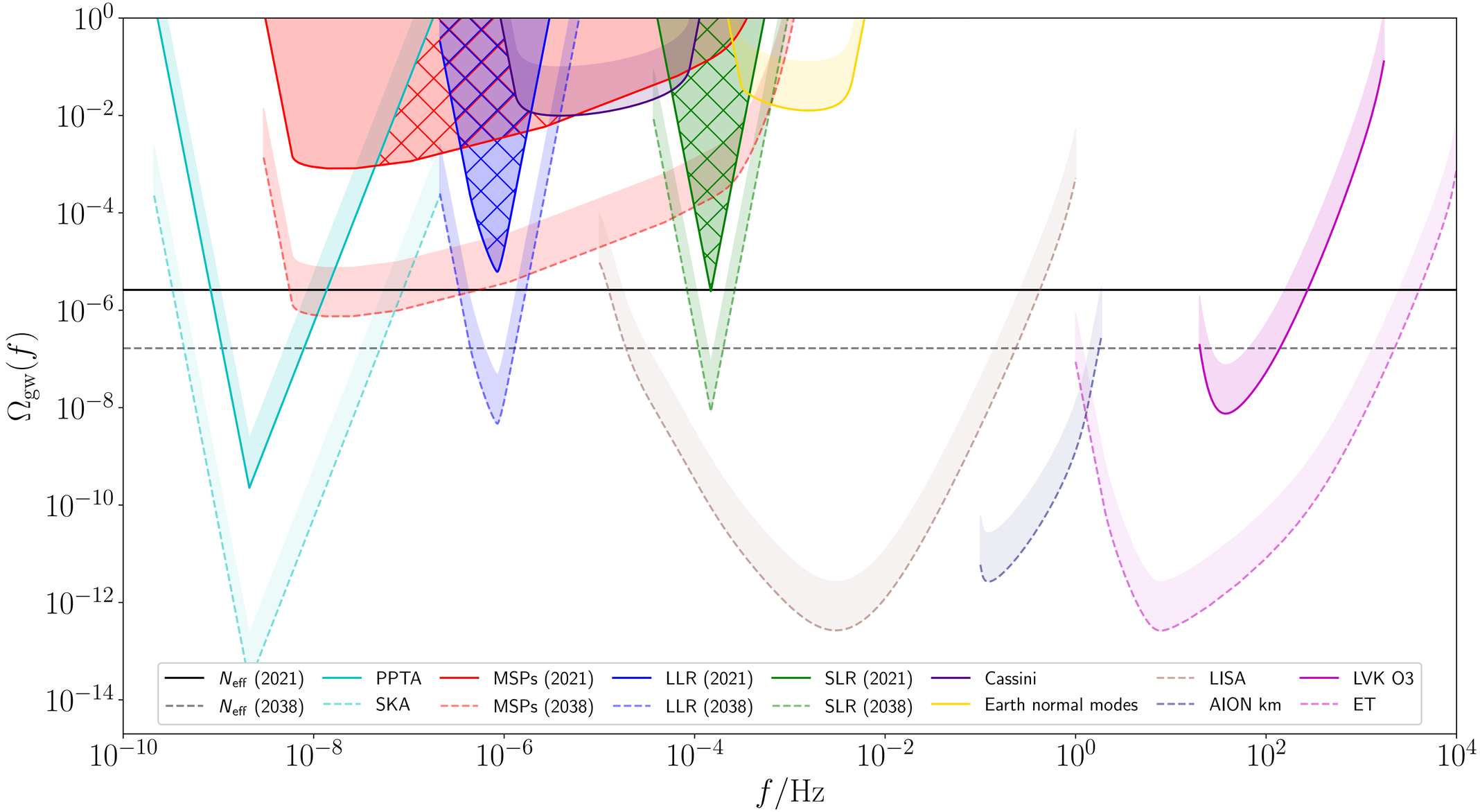
Lunar and satellite laser ranging



NASA

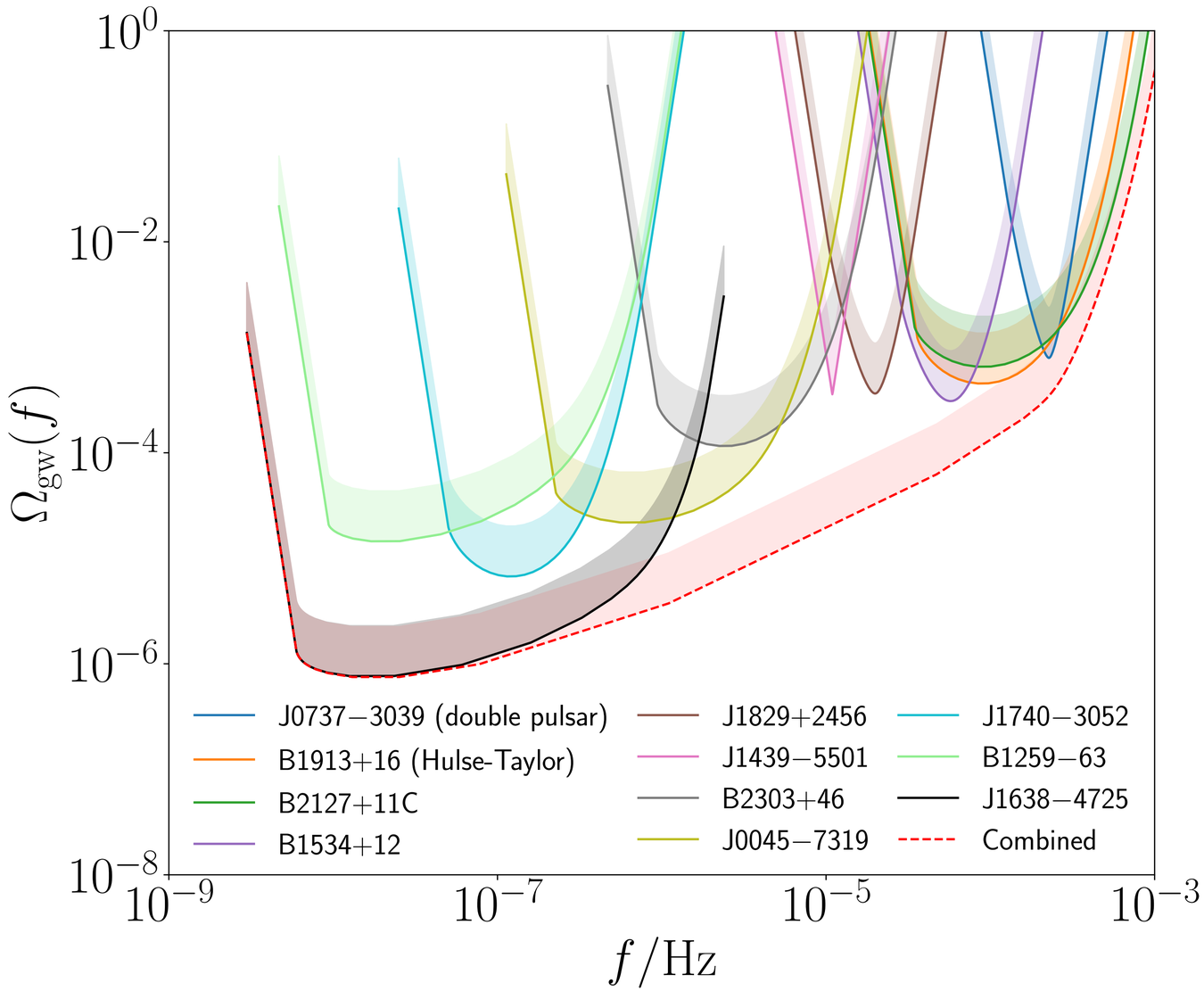
These data will be taken anyway — we get GW constraints “for free”!

Our forecast constraints



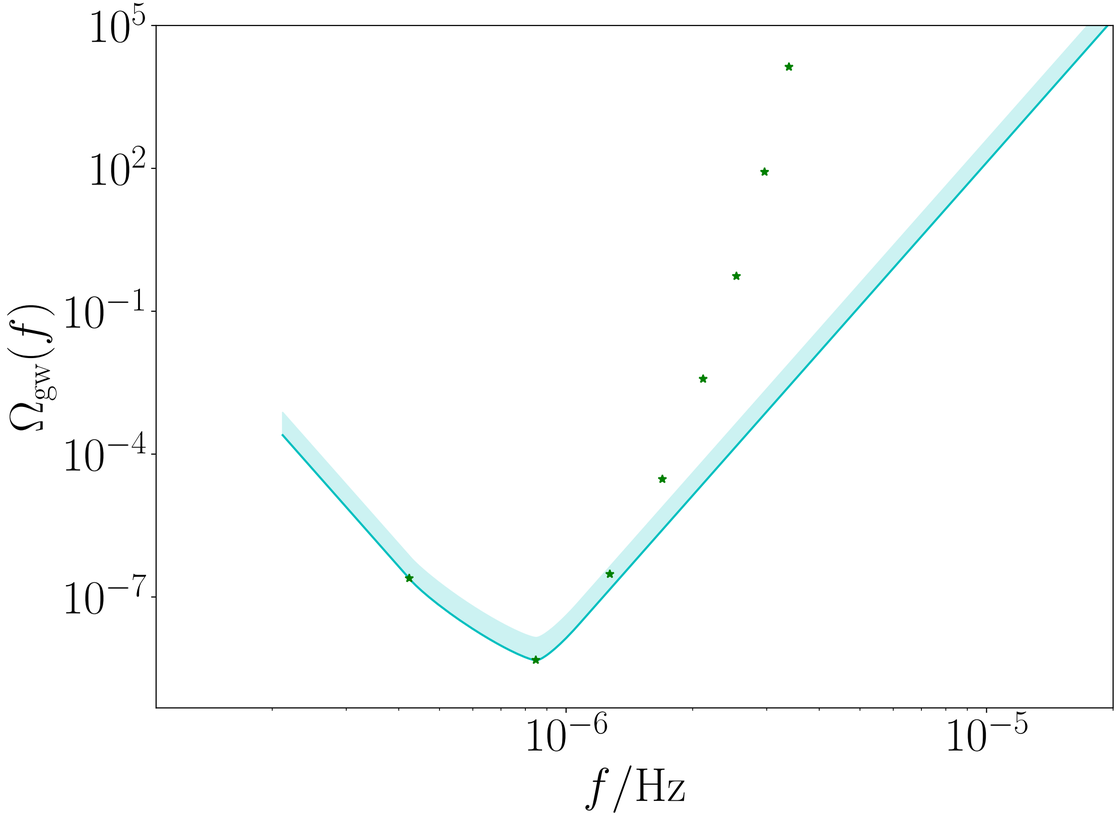
Combining binary pulsar bounds

- Red curve (same as previous slide) uses 215 binaries from the ATNF catalogue
- Constraint driven by just a handful of systems

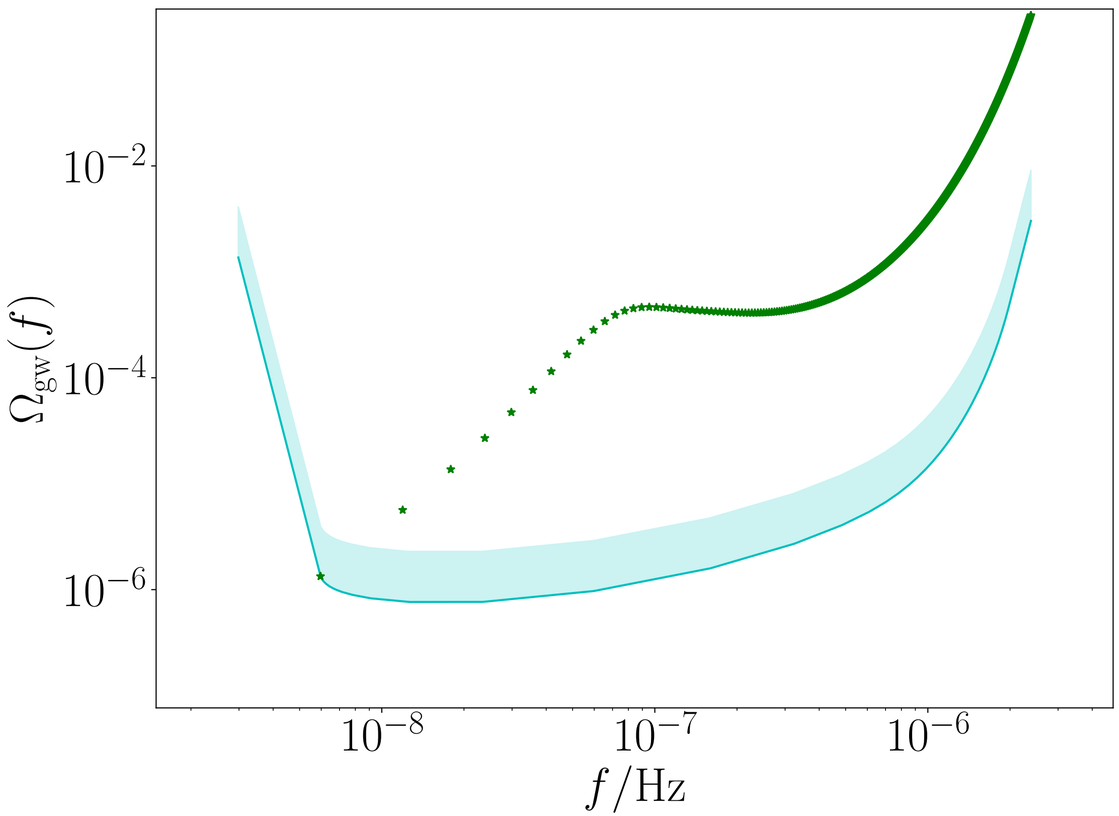


Power-law vs. monochromatic sensitivity

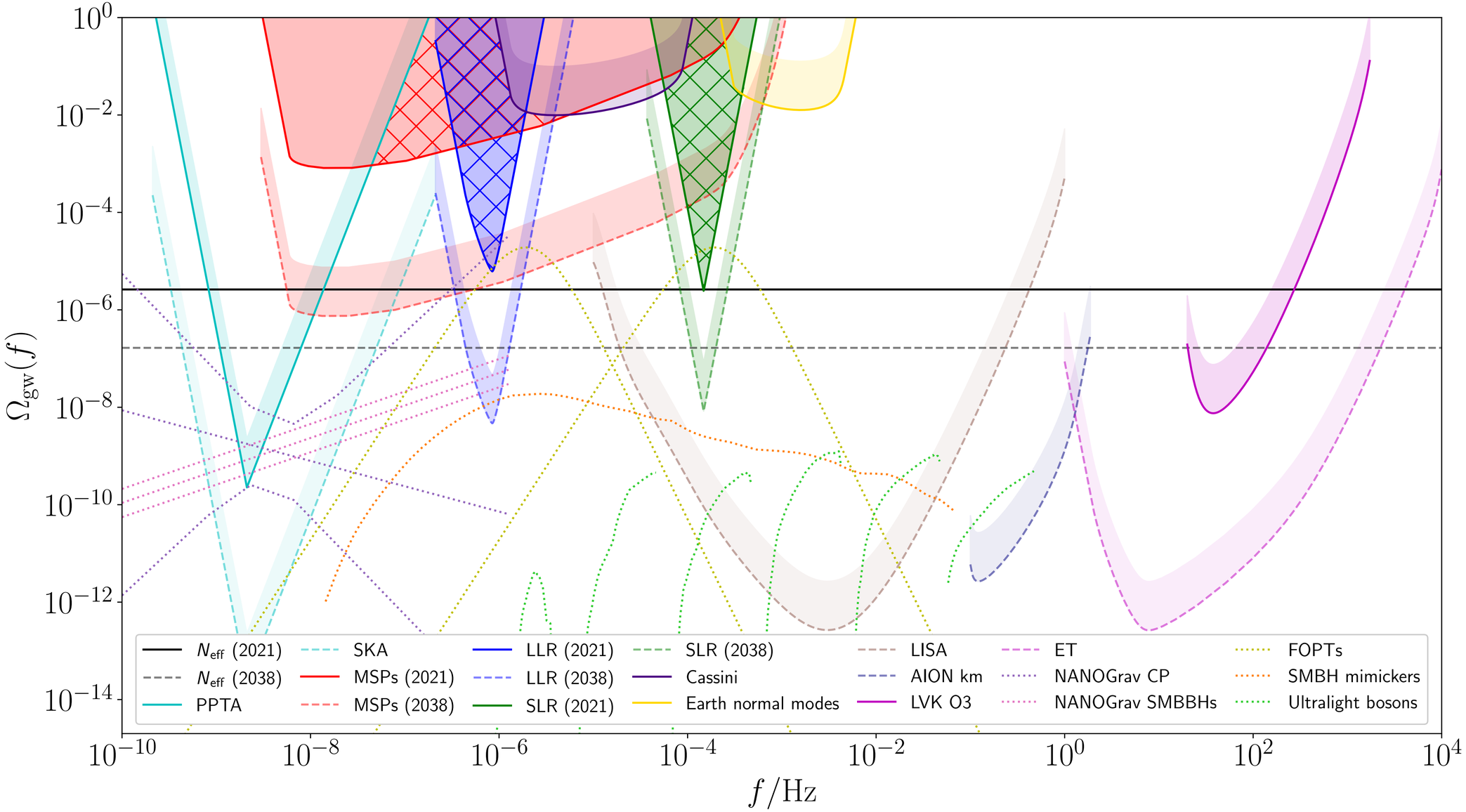
Lunar laser ranging, $e \approx 0.055$



Pulsar timing (J1638-4725), $e \approx 0.955$



Signals in the μHz band



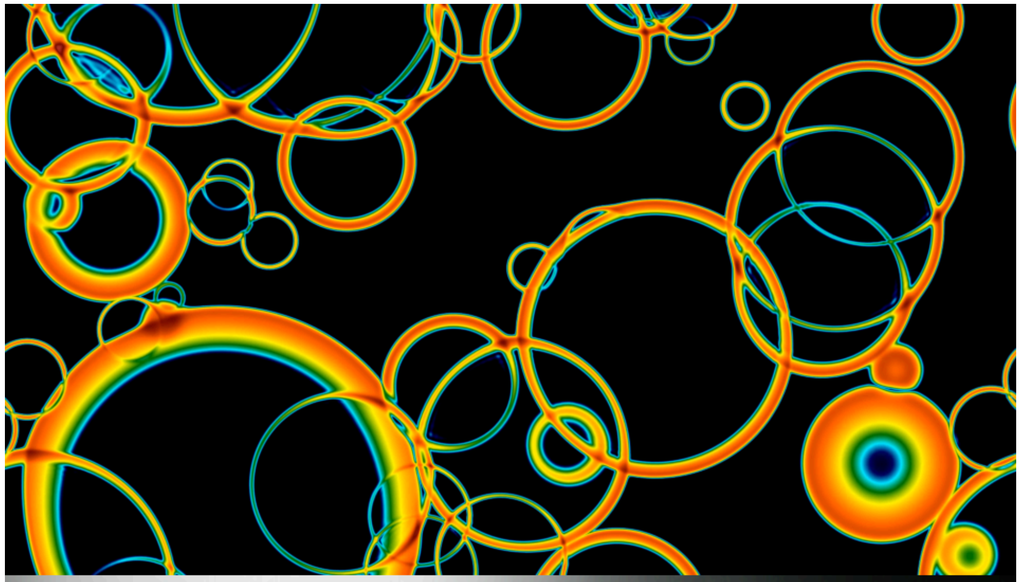
An example: cosmological phase transitions

- Generic prediction of many theories beyond the standard model

- Four parameters:
    1. Temperature T_*
    2. Strength α
    3. Rate β/H_*
    4. Bubble-wall velocity v_w

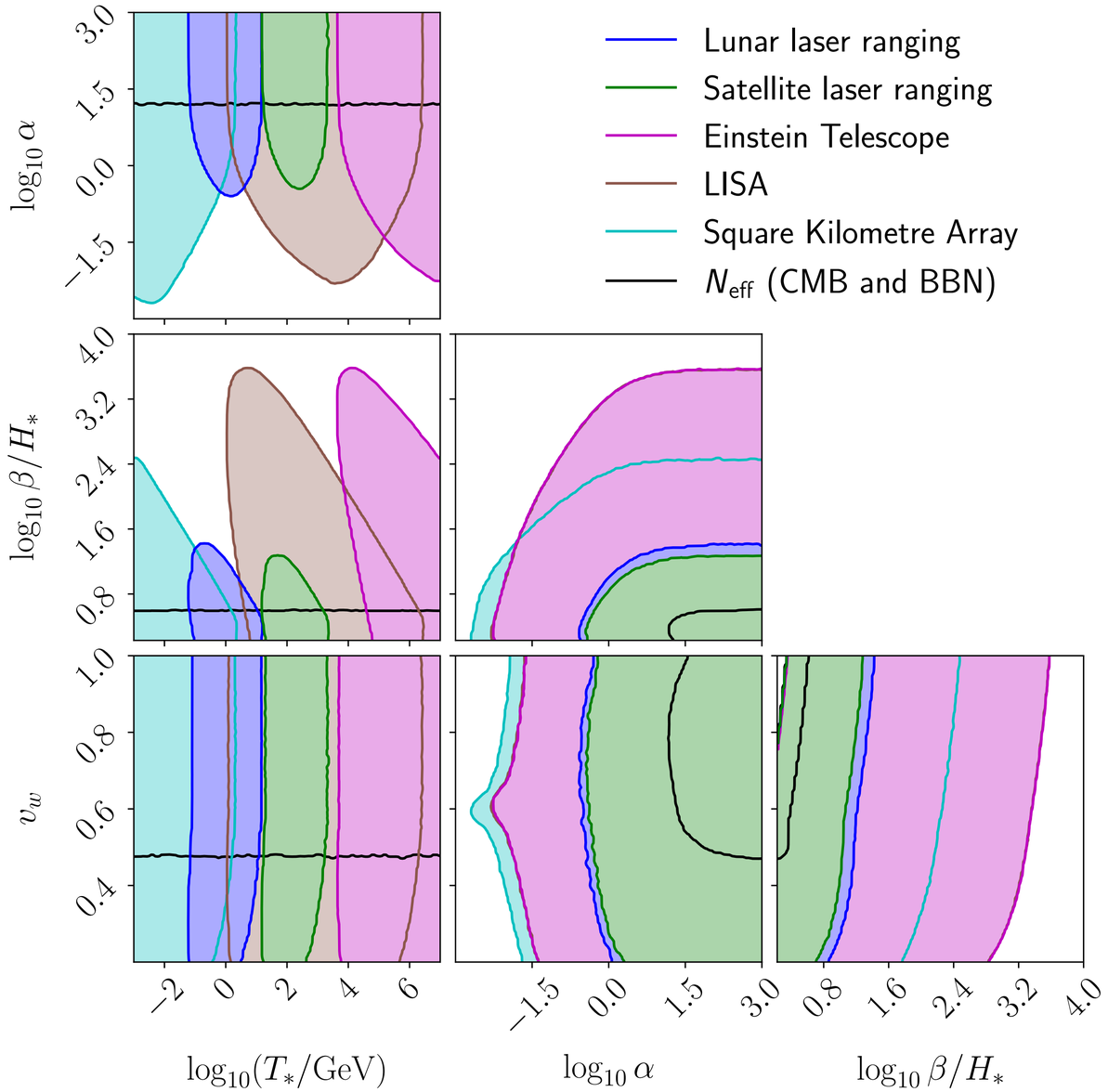
- Peak frequency

$$f_* \approx 19\,\mu\mathrm{Hz} \times \frac{T_*}{100\,\mathrm{GeV}} \frac{\beta/H_*}{v_w}$$



Phase transition constraints

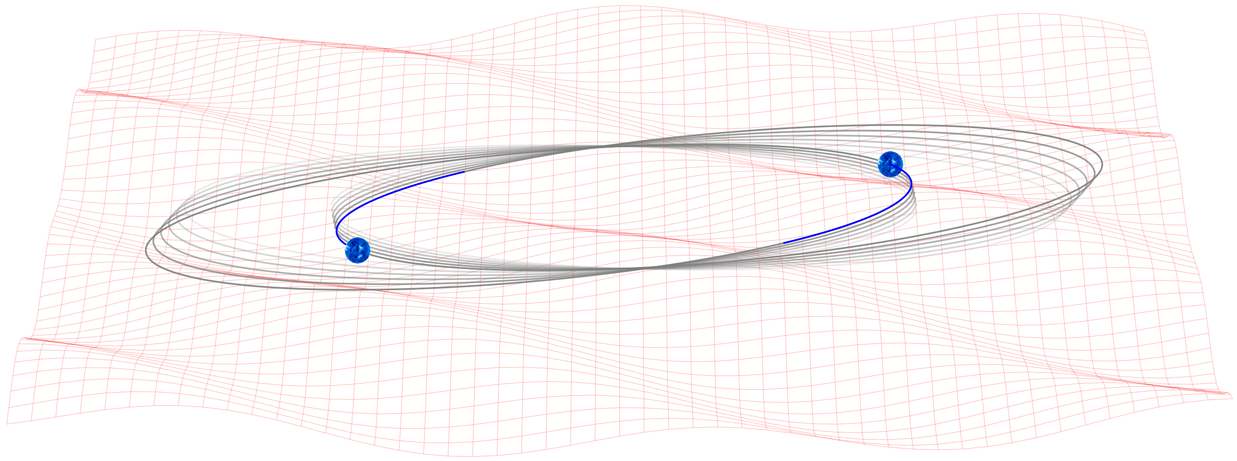
- LLR can access a unique region of parameter space!
- SLR could provide an independent confirmation of a LISA detection



Summary and outlook

- Binary resonance can probe a unique GW frequency band
- We have developed a powerful new formalism for studying this
- Unique constraints on phase transitions (and more)
- Plenty more work to do! More signals, more systems, plus using real data

Thanks for listening!



Backup slides

Cosmological bounds

- GWs contribute to the radiation energy density, and therefore the Hubble rate in the early Universe
- This is equivalent to increasing the effective number of neutrino species

$$N_{\rm eff} - N_{\rm eff}^{\rm (SM)} \approx 2.5 \times 10^5 \times \int_{\ln f_*}^{\infty} {\rm d}(\ln f) \Omega_{\rm gw}(f)$$

- This is an *integrated* constraint — no frequency info ($f < f_* \sim 10^{-15}\,{\rm Hz}$ corresponds to non-dynamical super-horizon modes)

Drift and diffusion coefficients

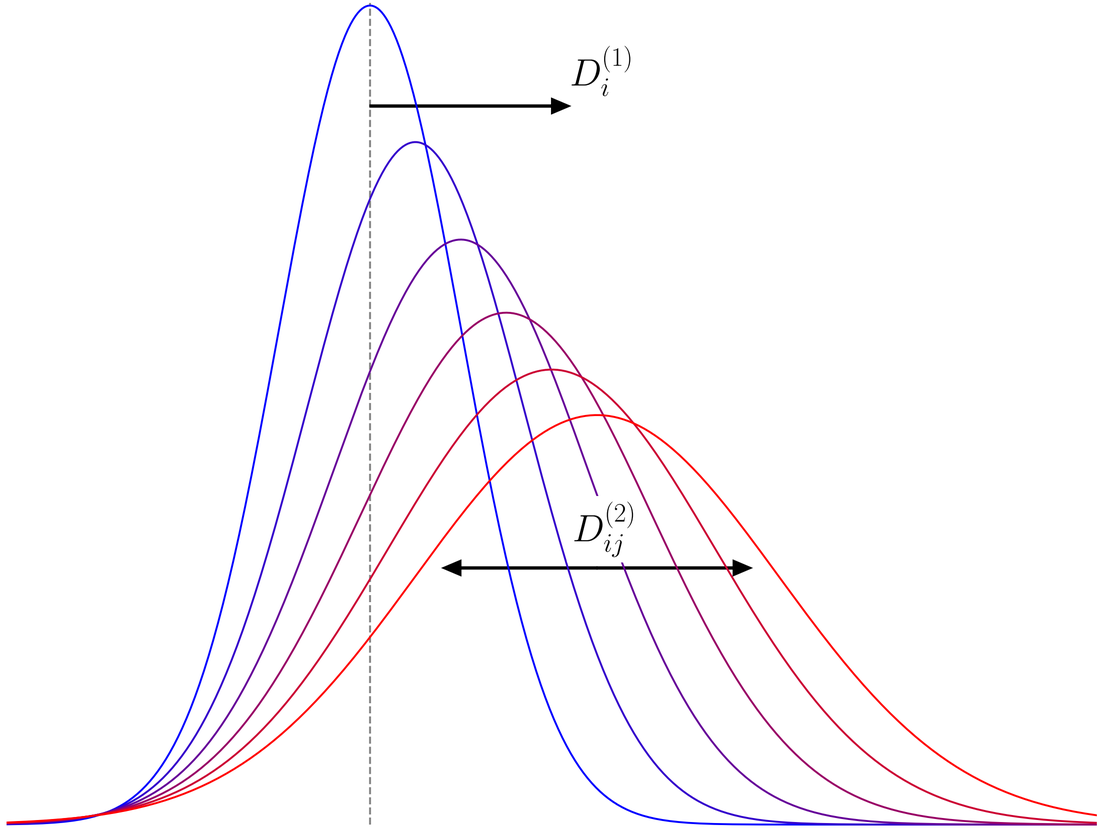
$$D_i^{(1)} = \lim_{\tau \to 0} \frac{1}{\tau} \langle X_i(t+\tau) - X_i(t) \rangle$$

$$D_{ij}^{(2)} = \lim_{\tau \to 0} \frac{1}{2\tau} \langle [X_i(t+\tau) - X_i(t)][X_j(t+\tau) - X_j(t)] \rangle$$

- We derive these in terms of SGWB spectrum, $\Omega_{\rm gw}(f)$
- Result is a sum over resonant frequencies $f = n/P$ (plus a deterministic piece):

$$D_i^{(1)}(\mathbf{X}) = V_i(\mathbf{X}) + \sum_{n=1}^{\infty} A_{n,i}(\mathbf{X}) \Omega_{\rm gw}(n/P)$$

$$D_{ij}^{(2)}(\mathbf{X}) = \sum_{n=1}^{\infty} B_{n,ij}(\mathbf{X}) \Omega_{\rm gw}(n/P)$$



Drift and diffusion

- Drift effect missing from previous studies!
- This is an example of “noise-induced drift”, which is generic in systems where the strength of the stochastic force depends on the state of the system
- This effect scales like $\langle \delta X(t) \rangle \sim D^{(1)} t \sim P H_0^2 \Omega_{\rm gw} t$
- Diffusion scales like $\langle \, | \delta X(t) |^2 \, \rangle^{1/2} \sim (D^{(2)} t)^{1/2} \sim (P H_0^2 \Omega_{\rm gw} t)^{1/2}$
- So drift grows faster with time, but is suppressed by a factor of $\Omega_{\rm gw}^{1/2} \ll 1$

Harmonic structure of the coefficients