Spectral separation of the cosmological SGWB for LISA in context of galactic and astrophysical background

Gravitational Wave Orchestra

by Guillaume Boileau , Postdoc Universiteit Antwerpen
on September 9, 2022

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Introduction

- LISA
- Stochastic Gravitational Wave Background (SGWB) in LISA band

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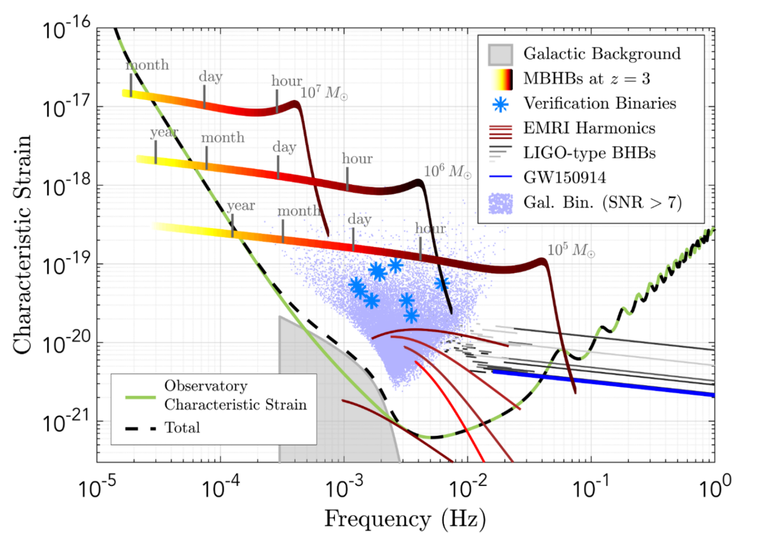
* LISA
* Stochastic Gravitational Wave Background (SGWB) in LISA band

» LISA source and noise



LISA proposal L3 (arXiv:1702.00786)

Introduction

* LISA
* Stochastic Gravitational Wave Background (SGWB) in LISA band

» Stochastic background in LISA band

Stochastic Background : Superposition of a large number of independent sources (unresolved sources):

* White dwarf binaries in our galaxy Lamberts *et al.* simulation of the waveform with galactic population of Double White Dwarf (DWD)
  $s(t) = \sum_{i=1}^{N} \sum_{P=+,\times} h_{P,i}(f_{orb,i}, M1_i, M2_i, X_i, Y_i, Z_i, t) \times F_P(\theta, \phi, t)\mathbf{D}(\theta, \phi, f)_P : \mathbf{e}_P$
  with a modulated waveform (Adams & Cornish)
* Binary Black Holes and Binary Neutron Stars from LIGO/Virgo Band
  * $\Omega_{GW} \simeq 1.8 \times 10^{-9} - 2.5 \times 10^{-9}$ at 25 Hz Chen *et al.* (2019)
  * $\Omega_{GW} \simeq 4.97 \times 10^{-9} - 2.58 \times 10^{-8}$ at 25 Hz Périgois *et al.* (2020)
* Cosmological sources: Phase transition, Preheating, Cosmic strings ... (early universe)

» Stochastic background in LISA band

Energy density spectrum:

Model of Energy density spectrum SGWB sources

$$\Omega_{GW}(f) = \frac{A_1 \left(\frac{f}{f_*}\right)^{\alpha_1}}{1 + A_2 \left(\frac{f}{f_*}\right)^{\alpha_2}} + \Omega_{Astro}\left(\frac{f}{f_*}\right)^{\alpha_{Astro}} + \Omega_{GW,Cosmo}(f)$$

with $\alpha = 2/3$ for the astrophysical component and low frequency DWD ($\Omega_{DWD,LF}(f) = \frac{A_1}{A_2}\left(\frac{f}{f_*}\right)^{\alpha_1 - \alpha_2}$) and different Cosmological models

Goal: Detecting a cosmological SGWB with LISA in the presence of an astrophysical background and Galactic foreground

Spectral separation methods

* LISA noise model
* MCMC: Markov chain Monte Carlo
* Fisher Information Matrix

Spectral separation methods

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» LISA noise model

Times-series XYZ to AET

$$A = \frac{1}{\sqrt{2}}(Z - X)$$
$$E = \frac{1}{\sqrt{6}}(X - 2Y + Z)$$
$$T = \frac{1}{\sqrt{3}}(X + Y + Z)$$

PSD AET

$$N_A = N_E = N_X(f) - N_{XY}(f)$$
$$N_T = N_X(f) + 2N_{XY}(f)$$

$$\begin{cases} N_X(f) = \left(4S_s(f) + 8\left(1 + \cos^2\left(\frac{f}{f_*}\right)\right) S_a(f)\right) |1 - e^{-\frac{2if}{f_*}}|^2 \\ N_{XY}(f) = -\left(2S_s(f) + 8S_a(f)\right) \cos\left(\frac{f}{f_*}\right) |1 - e^{-\frac{2if}{f_*}}|^2 \end{cases}$$

$$\begin{cases} S_s(f) = N_{Pos} \\ S_a(f) = \frac{N_{acc}}{(2\pi f)^4}\left(1 + \left(\frac{f_1}{f}\right)^2\right) \end{cases}$$

with $f_* = \frac{c}{2\pi L}$, $L = 2.5^9$m, $f_1 = 0.4$ mHz
Acceleration noise : $N_{Acc} = 1.44 \times 10^{-48}$ s^{-4}Hz^{-1}
Optical Metrology System noise : $N_{Pos} = 3.6 \times 10^{-41}$ Hz^{-1}

Spectral separation methods

* LISA noise model
* MCMC: Markov chain Monte Carlo
* Fisher Information Matrix

» MCMC (Markov chain Monte Carlo)

Likelihood function, (d = data, θ = parameter)

likelihood

$$\mathcal{L}(\mathbf{d}|\theta) = -\frac{1}{2}\sum_{k=0}^{N}\left[\frac{d_A^2}{S_A+N_A} + \frac{d_E^2}{S_E+N_E} + \frac{d_T^2}{N_T} + \left(8\pi^3(S_A+N_A)(S_E+N_E)N_T\right)\right]$$

with N_I the LISA noise of channel $I = [A, E, T]$ and $S_I(f) = \frac{3H_0^2}{4\pi^2}\frac{\sum_i \Omega_{GW,i}}{\mathcal{R}_{\mathcal{I}}(f)f^3}$ the SGWB.

* posterior distribution $p(\theta|d) \propto p(\theta)\mathcal{L}(\mathbf{d}|\theta)$
* using log uniform and uniform prior $p(\theta) = \prod_i U(\theta_i, a_i, b_i)$
* Estimation parameters $\theta_{LISA} + \theta_{Astro} + \theta_{Galac} + \theta_{Cosmo}$.

$\Longrightarrow$ using a Metropolis-Hasting sampler

» MCMC (Markov chain Monte Carlo)

Adaptive MCMC using a Metropolis-Hasting with a sampler distribution target proposal (Σ_n current empirical estimate of the covariance matrix, $\beta = 0.25$, d the number of parameters, N the multi-normal distribution)

Adaptive McMC

$$Q_n(x) = (1-\beta)N(x, (2.28)^2\Sigma_n/d) + \beta N(x, (0.1)^2 I_d/d)$$

$\Longrightarrow$ sampling from the joint posterior distribution of the parameters $\theta_{LISA} + \theta_{Astro} + \theta_{Galac} + \theta_{Cosmo}$

Spectral separation methods

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» Fisher Information Matrix

Fisher Information Matrix

$$F_{ab} = \frac{1}{2}\mathrm{Tr}\left(\mathcal{C}^{-1}\frac{\partial \mathcal{C}}{\partial \theta_a}\mathcal{C}^{-1}\frac{\partial \mathcal{C}}{\partial \theta_b}\right)$$
$$= \frac{1}{2}\sum_{I=A,E,T}\sum_{k=0}^{N}\frac{\frac{\partial S_I(f)+N_I(f)}{\partial \theta_a}\frac{\partial S_I(f)+N_I(f)}{\partial \theta_b}}{(S_I(f)+N_I(f))^2}$$

Co-variance Matrix

$$\mathcal{C}(\theta, f) = \begin{pmatrix} S_A + N_A & 0 & 0 \\ 0 & S_E + N_E & 0 \\ 0 & 0 & N_T \end{pmatrix}$$

with N_I is the LISA noise of the channel $I = [A, E, T]$ and $S_I \propto \sum_\alpha \Omega_\alpha \left(\frac{f}{f_{ref}}\right)^{\alpha-3}$ the SGWB.

$\implies \sqrt{F_{aa}^{-1}} = \sigma_a$

Results 6 parameter A-MCMC

» Context: Stochastic background Astrophysical and flat cosmological components

Energy density spectrum:

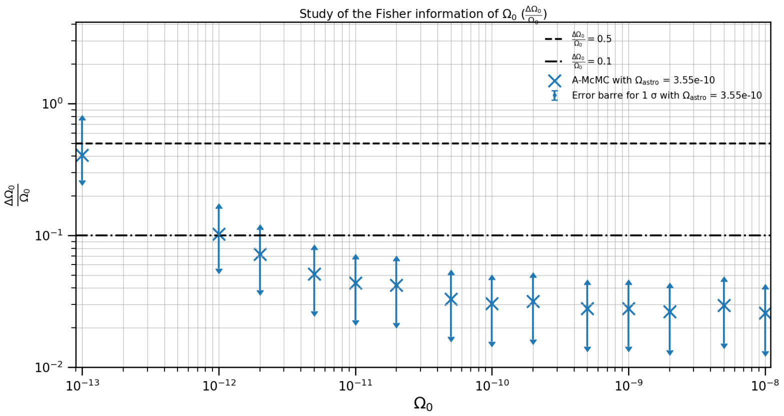
Model of Energy density spectrum SGWB sources

$$\Omega_{GW}(f) = \Omega_{Astro}\left(\frac{f}{f_*}\right)^{\alpha_{Astro}} + \Omega_{Cosmo}\left(\frac{f}{f_*}\right)^{\alpha_{Cosmo}}$$

with $\alpha_{Astro} = 2/3$ for the astrophysical component and $\alpha_{Cosmo} = 0$ for the cosmological component

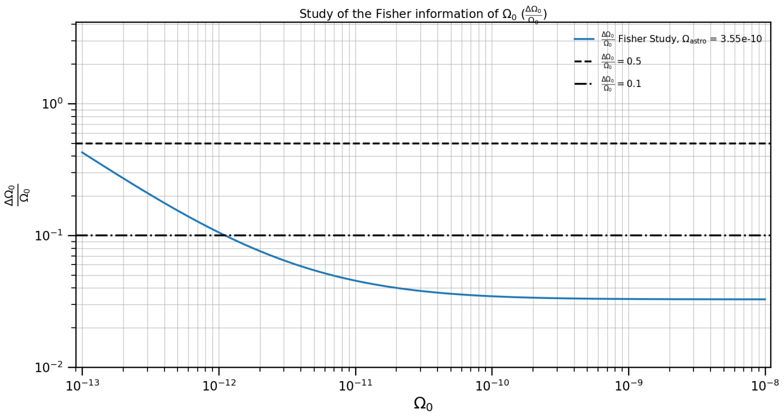
Goal: Detecting a cosmological SGWB with LISA in the presence of an astrophysical background

» Results 6 parameter A-MCMC



Uncertainty of the estimation of the Cosmological Amplitude from the Fisher Information study in line (with the Cramer-Rao calculation) and the parametric estimation from the A-MCMC in scatters for the channel A with the noise channel T. The upper horizontal dash line represents the error level 50%. In fact, above the line, the error is greater than 50%.

» Results 6 parameter A-MCMC



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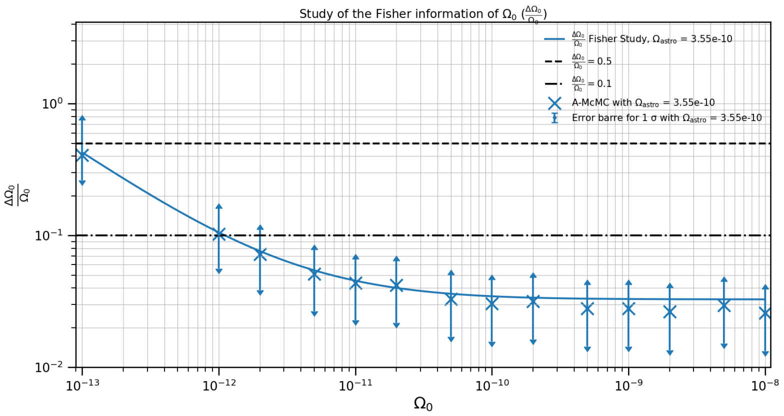

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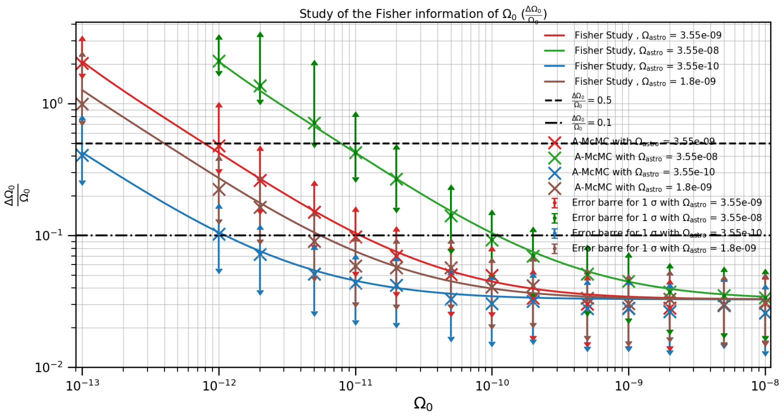

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» Results 6 parameter A-MCMC

Prediction of the measurement limit of Cosmological Amplitude in 4 cases of isotropic astrophysical background with 2 noise parameters (acceleration noise : $N_{Acc} = 1.44 \times 10^{-48}$ s^{-4}Hz^{-1} and Optical Metrology System noise $N_{Pos} = 3.6 \times 10^{-41}$ Hz^{-1}) of 4 years mission data measurement:

Limit for BBH/BNS + Cosmo + LISA noise

- $\Omega_{astro} = 3.55 \times 10^{-8}$ (25 Hz): $\Omega_{Cosmo,lim} = 7.8 \times 10^{-12}$
- $\Omega_{astro} = 3.55 \times 10^{-9}$ (25 Hz): $\Omega_{Cosmo,lim} = 7.8 \times 10^{-13}$
- $\Omega_{astro} = 1.8 \times 10^{-9}$ (25 Hz): $\Omega_{Cosmo,lim} = 3.6 \times 10^{-13}$
- $\Omega_{astro} = 3.55 \times 10^{-10}$ (25 Hz): $\Omega_{Cosmo,lim} = 7.6 \times 10^{-14}$

$\Longrightarrow$ paper G. Boileau *et. al.* (PhysRevD.103.103529).

@ Guillaume Boileau, Nelson Christensen, Renate Meyer, Neil J. Cornish

Spectral separation with a modulated galactic foreground

* Orbital Modulation of the white dwarf binaries in our Galaxy
* 10 parameter runs of A-MCMC (LISA noise + BBH/BNS + DWD + Cosmo)

Spectral separation with a modulated galactic foreground

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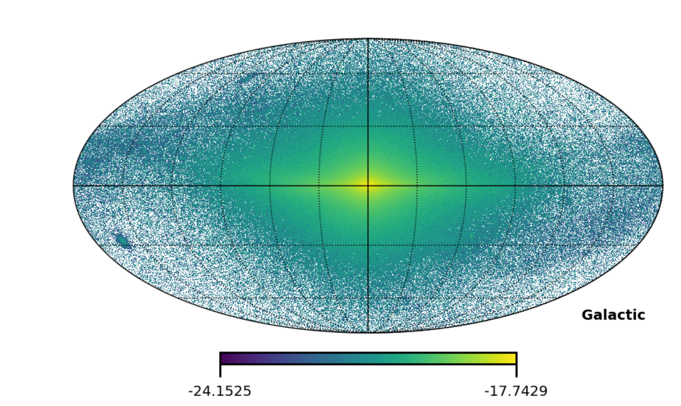
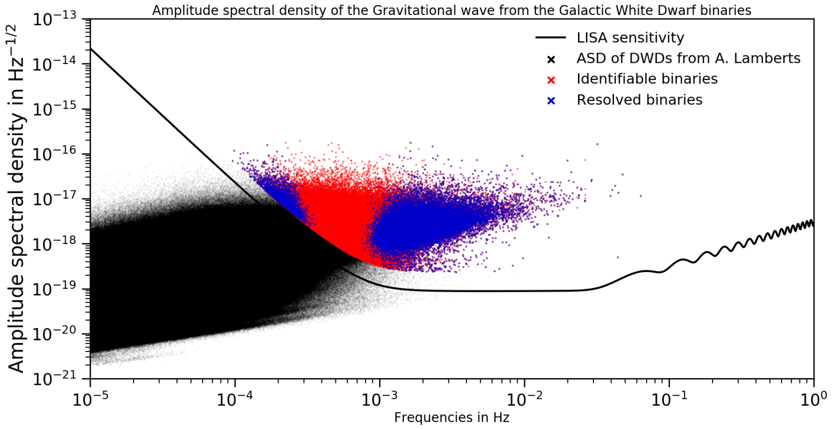
» Population of the white dwarf binaries in our Galaxy



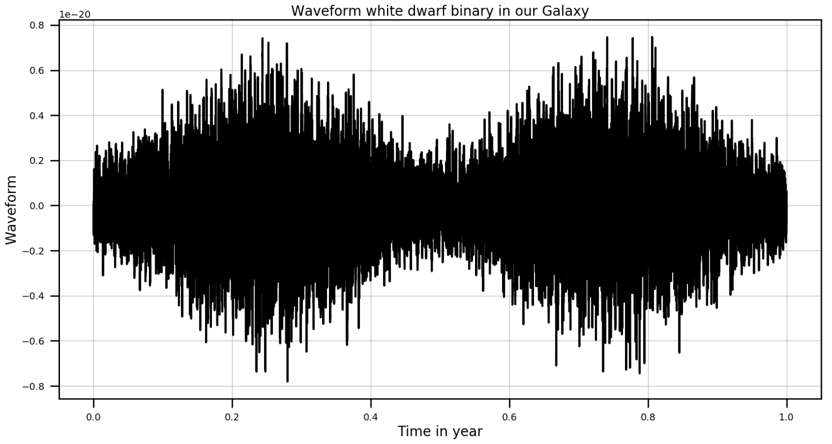
Map of the distribution of the log-amplitude of GW from the Galactic WD binaries for $f_{GW} \geq 1 \times 10^{-5}$ Hz. The DWD distribution in position (X, Y, Z) is from the simulation of Lamberts *et. al.* This map is made with the Galactic coordinates *GLON*, *GLAT* with $N_{side} = 256$.

» Population of the white dwarf binaries in our Galaxy



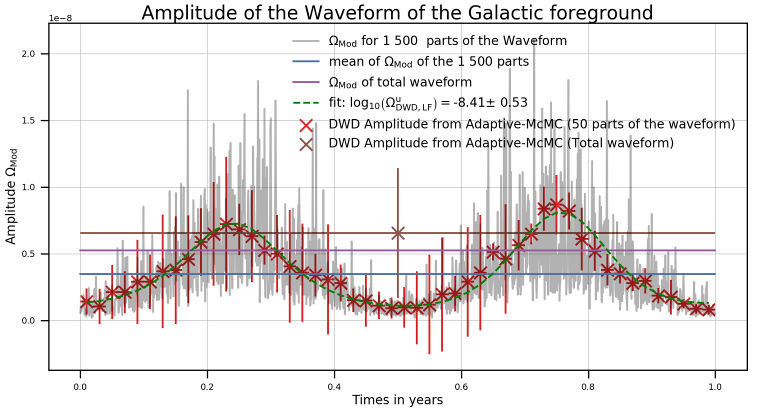
ASD of the galactic White dwarf binaries from A. Lamberts (2019), **black**: $\simeq$ 35 000 000 binaries, **red**: binaries $SNR > 7$ and **blue**: $SNR > 7$ + LISA bin ($\simeq$ 32 000 binaries)

» Orbital Modulation of the white dwarf binaries in our Galaxy



Total Gravitational signal of the white-Dwarf binaries seen by LISA

» Orbital Modulation of the white dwarf binaries in our Galaxy



Measurement of the orbital Modulation of the DWD amplitude. In grey: $\Omega_{Mod,i} = \frac{4\pi^2}{3H_0}\left(\frac{c}{2\pi L}\right)^2 A_i^2$. In **red** scatter : 50 A-MCMC of 8 parameters (BBH + WD + LISA noise) for small sections of the year. In green, fit on the 50 runs to estimate the modulation. Modulation model : $\Omega_{Mod,i} = \Omega^{u}_{DWD,LF}\left(F^2_{+,i} + F^2_{\times,i}\right)$.

Spectral separation with a modulated galactic foreground

* Orbital Modulation of the white dwarf binaries in our Galaxy
* 10 parameter runs of A-MCMC (LISA noise + BBH/BNS + DWD + Cosmo)

» Context: Stochastic background In LISA paper of BBH/BNS prediction

Energy density spectrum:

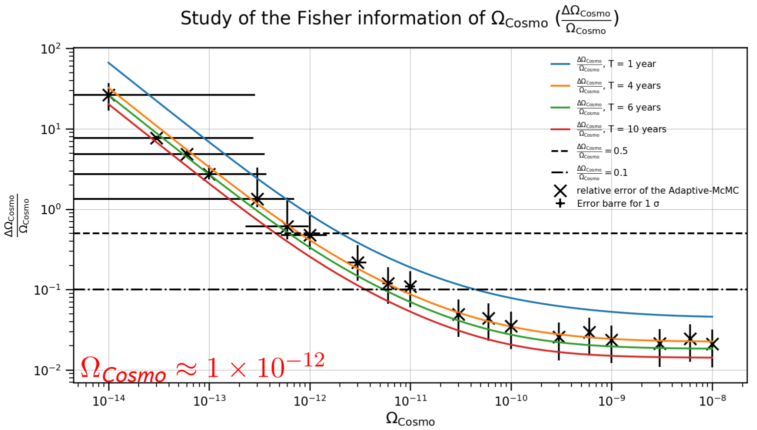
Model of Energy density spectrum SGWB sources

$$\Omega_{GW}(f) = \frac{A_1\left(\frac{f}{f_*}\right)^{\alpha_1}}{1 + A_2\left(\frac{f}{f_*}\right)^{\alpha_2}} + \Omega_{Astro}\left(\frac{f}{f_*}\right)^{\alpha_{Astro}} + \Omega_{Cosmo}\left(\frac{f}{f_*}\right)^{\alpha_{Cosmo}}$$

with $\alpha_{Astro} = 2/3$ for the astrophysical component and low frequency DWD ($\Omega_{DWD,LF}(f) = \frac{A_1}{A_2}\left(\frac{f}{f_*}\right)^{\alpha_1-\alpha_2}$). Cosmological model : $\alpha_{Cosmo} = 0$

Goal: Detecting a cosmological SGWB with LISA in the presence of an astrophysical background and Galactic foreground

» 10 parameter runs of A-MCMC (LISA noise + BBH/BNS + DWD + Cosmo)



$\Longrightarrow$ paper for MNRAS G. Boileau *et al.* (10.1093/mnras/stab2575)

@ Guillaume Boileau, Astrid Lamberts, Nelson Christensen, Neil J. Cornish, Renate Meyer

Spectral separation with cosmic strings

* Models
* Uncertainty
* Deviance Information Criterion (DIC)
* Result

Spectral separation with cosmic strings

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» Models : cosmic strings

Cosmic strings:

* predictions from field theory, stable topological defects
* formed during symmetry breaking phase transitions in the early Universe

Energy Spectral Density

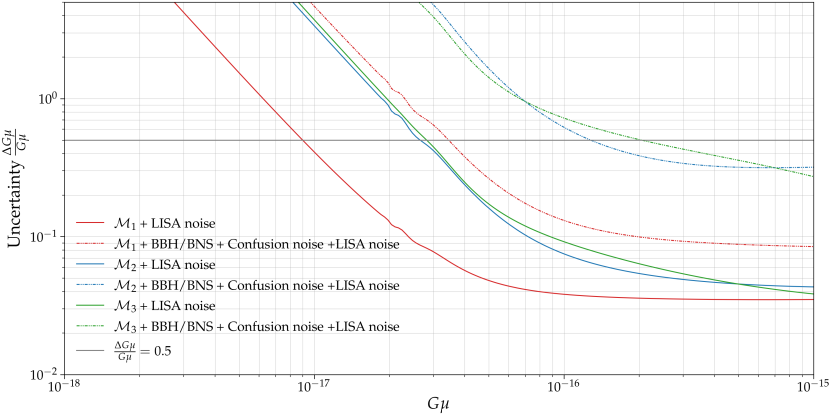
$$\Omega_{\mathrm{GW},G\mu}(G\mu, \mathcal{M}_i, f), i \in [1, 2, 3]$$

* $\mathcal{M}_1$: Auclair *et. al* analytical model of loops produced by the network of long chains described by a single free parameter (size of loops at the time of their formation) Kibble *et. al*
* $\mathcal{M}_2$: simulations of the Blanco-Pillado, Olum and Shlar (BOS) model. Networks of cosmic strings present between the eras of radiation and matter.
* $\mathcal{M}_3$: simulation of Lorenz, Ringeval and Sakellariadou (LRS), sister simulation to model 2, calculates and considers different quantities. (the power in the age of matter differs from that of model 2)

Spectral separation with cosmic strings

* Models
* Uncertainty
* Deviance Information Criterion (DIC)
* Result

» Uncertainty : cosmic strings



$G\mu$ uncertainty estimates from Fisher information for the three models $\mathcal{M}_i$. Solid lines present the results CS + LISA noise, case (I). The dot-dashed lines are the results considering CS + LISA noise + galactic foreground astrophysical SGWB; case (III). The horizontal gray line represents $\Delta G\mu/G\mu = 0.5$

Spectral separation with cosmic strings

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* Deviance Information Criterion (DIC)
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» Deviance Information Criterion (DIC)

Deviance Information Criterion (DIC) : analogous to AIC and BIC : criterion for model comparison (BF not sensible for improper prior). It combines a measure of model fit with a penalty for the number of independent parameters. It is easy to compute based on MCMC samples.

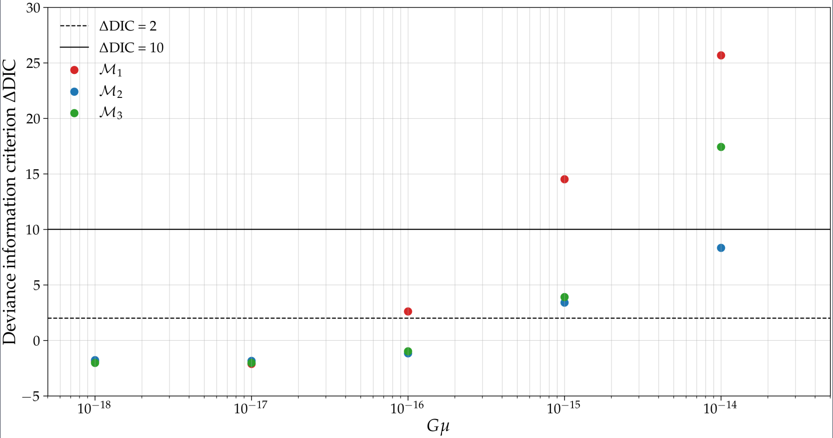
DIC

$$\begin{aligned} DIC &= D(\bar{\theta}) + 2p_d \\ D(\theta) &= -2 \log \mathcal{L}(\mathbf{d}|\theta) \\ p_d &= \overline{D}(\theta) - D(\bar{\theta}) \end{aligned}$$

Δ DIC

$$\Delta DIC = DIC_{\mathcal{M}_i} - DIC_{\varnothing \mathcal{M}_i}$$

» Deviance Information Criterion (DIC)



- $\Delta DIC < 2$: Not worth more than a bare mention
- ΔDIC 2 to 10 : positive
- $\Delta DIC > 10$: very strong

Spectral separation with cosmic strings

* Models
* Uncertainty
* Deviance Information Criterion (DIC)
* Result

Result

- Development of a discrete MCMC with a library (limit $\frac{\Delta G\mu}{G\mu}$)
- Good overlap between Fisher study and MCMC
- Development of DIC technic
- Future study: *add* MBHB and EMRIs GWB

LISA noise + Cosmic strings			
	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$
$G\mu_{lim}$	1×10^{-17}	3×10^{-17}	3×10^{-17}
LISA noise + DWD + BBH/BNS + Cosmic strings			
	$\mathcal{M}_1$	$\mathcal{M}_2$	$\mathcal{M}_3$
$G\mu_{lim}$	3×10^{-17}	1×10^{-16}	2×10^{-16}

Measurement limit

$G\mu \approx 10^{-16} \longrightarrow 10^{-15}$

$\Longrightarrow$ paper G. Boileau *et. al.* (PhysRevD.105.023510)

@ Guillaume Boileau, Alexander C. Jenkins, Mairi Sakellariadou, Renate Meyer, Nelson Christensen

Prospects for LISA to detect a gravitational-wave background from first order phase transitions

* Models
* Result

Prospects for LISA to detect a gravitational-wave background from first order phase transitions

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» First order phase transitions

- Quantum and thermal fluctuation from bubbles in the early Universe
- Production of GWs Collision of Bubble/ subsequent sound waves/Magneto hydrodynamic turbulence
- Simulation: Sound shell model (SSM) for GWs production Hindmarsh *et. al*
- SSM approximated by a double broken power law (Caprini *et. al*, Guo *et. al*, Gowling Hindmarsh)

Broken Power law first order phase transitions

$$\Omega_{\mathrm{GW},PT}(f) = \Omega_P \left(\frac{f}{f_\mathrm{p}}\right)^9 \left(\frac{1+r_\mathrm{b}^4}{r_\mathrm{b}^4 + \left(\frac{f}{f_\mathrm{p}}\right)^4}\right)^{(9-b)/4} \left(\frac{b+4}{b+4-m+m\left(\frac{f}{f_\mathrm{p}}\right)^2}\right)^{(b+4)/2}$$

Ω_P amplitude at the frequency peak f_p, r_b the ratio between the two breaks b the spectral slope between the two breaks and $m = \frac{9r_\mathrm{b}^4+b}{r_\mathrm{b}^4+1}$.

Prospects for LISA to detect a gravitational-wave background from first order phase transitions

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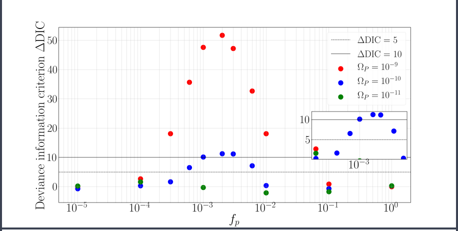
» Result

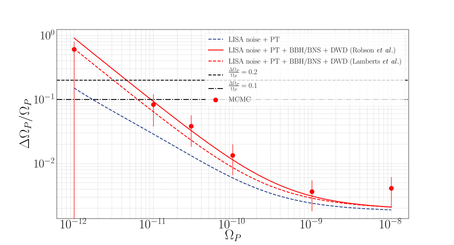
Context: Phase transition model $(r_b = 0.4, = 1)$ + Astrophysical background + DWD + LISA noise, Limit for $\Delta DIC > 5$:

- $\Omega_P = 1 \times 10^{-9}$ $\rightarrow f_p \in [2 \times 10^{-3}, 4 \times 10^{-2}]$ Hz
- $\Omega_P = 1 \times 10^{-10}$ $\rightarrow f_p \in [5 \times 10^{-3}, 7 \times 10^{-3}]$ Hz



Limit from Fisher $(r_b = 0.4, = 1$, $f_p = 1$mHz):

- $\frac{\Delta\Omega_P}{\Omega_P} = 0.01 \rightarrow \Omega_P \sim 1 \times 10^{-10}$
- $\frac{\Delta\Omega_P}{\Omega_P} = 0.1 \rightarrow \Omega_P \sim 1 \times 10^{-11}$
- similar result for Fisher (lines) and MCMC (scatter)



$\Longrightarrow$ Paper in preparation for JCAP.

Guillaume Boileau, Nelson Christensen, Chloe Gowling, Mark Hindmarsh, Renate Meyer.

Conclusion

» Conclusion

- We provide evidence that it is possible for LISA to measure the cosmological SGWB :

Measurement limit

$\Omega_{Cosmo,lim} = 8 \times 10^{-14} - 8 \times 10^{-12}$

Limitation

BBH and BNS principal limitation for the Cosmological background

- We use Lamberts *et al.* (10 October 2019), possibility to generate white dwarf waveform with other catalogs
- able to estimate more complex backgrounds, like broken power laws, or spectrum with peaks. Our method can be easily expanded with more complex cosmological backgrounds
$\Longrightarrow$ papers G. Boileau *et al.* (PhysRevD.103.103529) and MNRAS G. Boileau *et al.* (10.1093/mnras/stab2575)

» Conclusion Cosmological sources

Cosmic string

$G\mu \approx 10^{-16} \longrightarrow 10^{-15}$

$\Longrightarrow$ paper G. Boileau *et. al.* (PhysRevD.105.023510)

First order phase transition

- DIC : $\Omega_P = 1 \times 10^{-9} \rightarrow f_p \in [2 \times 10^{-3}, 4 \times 10^{-2}]$ Hz
  $\Omega_P = 1 \times 10^{-10} \rightarrow f_p \in [5 \times 10^{-3}, 7 \times 10^{-3}]$ Hz
- Fisher/MCMC $\frac{\Delta\theta}{\theta} \sim 1\%$: $\Omega_P = 1 \times 10^{-10}$ and $f_p = 3$ mHz
  $\frac{\Delta\theta}{\theta} \sim 10\%$: $r_b = 0.2$ and $b = 1$

Limitation of the study

- LISA noise amusing stationary, Gaussian and *AET* are uncorrelated
- Future study: add MBHB and EMRIs GWB or other cosmological sources

The End : Thank You !

[1] GB thanks the Centre national d'études spatiales (CNES) and Université de la Côte d'Azur (UCA) for support for this research.