Theory of stochastic gravitational wave backgrounds from the perspective of cosmology

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Outline

- Generalities of cosmological SGWB production
  - Wave equation & general solutions
  - Characteristic frequency
  - Discovery potential

- Example of sources
  - First oder phase transition: more detailed
  - Inflation: just the basic picture

Stochastic gravitational wave background

GENERAL: the superposition of sources that cannot be individually resolved

- Astrophysical: binaries too numerous and with too low SNR to be identified (talk by Irina, poster by Jésus…)

- Cosmological:

  - signals from the early universe with too small correlation scale with respect to the detector resolution

  - inflation: intrinsic, quantum fluctuations that become effectively classical (stochastic) due to the evolution

SGWB in cosmology:
probe of the early universe and high energy physics

SGWB from the early universe: generalities

$$\delta g_{00} = -2\phi$$

Perturbations on FRW universe

$$\delta g_{0i} = \delta g_{i0} = (\partial_i B + S_i)$$

$$\delta g_{ij} = -2\psi\delta_{ij} + (\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2)E + \partial_i F_j + \partial_j F_i + h_{ij} \qquad \partial_i h_{ij} = h_{ii} = 0$$

Evolution equation:

$$\ddot{h}_{ij}(\mathbf{x},t) + 3\,H\,\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}\,h_{ij}(\mathbf{x},t) = 16\pi G\,\Pi_{ij}(\mathbf{x},t)$$

Source: tensor anisotropic stress

Perfect fluid

$$T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

In the cosmological context:
energy momentum tensor of the matter content of the universe (background + perturbations)

$$\delta T_{ij} = \bar{p}\,\delta g_{ij} + a^2[\delta p\,\delta_{ij} + (\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2)\sigma + 2\partial_{(i}v_{j)} + \Pi_{ij}]$$

$$(\partial_i v_i = 0,\ \partial_i\Pi_{ij} = 0,\ \Pi_{ii} = 0)$$

SGWB from the early universe: generalities

$$h_{ij}(\mathbf{x},t) = \sum_{r=+,\times} \int \frac{d^3\mathbf{k}}{(2\pi)^3}\, h_r(\mathbf{k},t)\, e^{-i\mathbf{k}\cdot\mathbf{x}}\, e^r_{ij}(\hat{\mathbf{k}})$$

The evolution equation decouples for each polarisation mode

$$h_r''(\mathbf{k},\eta) + 2\,\mathcal{H}\, h_r'(\mathbf{k},\eta) + k^2\, h_r(\mathbf{k},\eta) = 16\pi G\, a^2\, \Pi_r(\mathbf{k},\eta)$$

Solution of the **homogeneous** equation

$$H_r(\mathbf{k},\eta) = a\, h_r(\mathbf{k},\eta) \qquad H_r''(\mathbf{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_r(\mathbf{k},\eta) = 0$$

Power-law scale factor (it covers matter and radiation domination, and De Sitter inflation)

$$a''/a \simeq \mathcal{H}^2$$

CASE 1: Sub-Hubble modes, relevant for propagation after the source stops

$$k^2 \gg \mathcal{H}^2 \qquad h_r(\mathbf{k},\eta) = \frac{A_r(\mathbf{k})}{a(\eta)}\, e^{ik\eta} + \frac{B_r(\mathbf{k})}{a(\eta)}\, e^{-ik\eta}$$

In this limit, GWs are plane waves with redshifting amplitude

SGWB from the early universe: generalities

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$$a''/a \simeq \mathcal{H}^2$$

CASE 2: Super-Hubble modes, relevant for inflationary tensor perturbations

$$k^2 \ll \mathcal{H}^2 \qquad h_r(\mathbf{k},\eta) = A_r(\mathbf{k}) + B_r(\mathbf{k}) \int^{\eta} \frac{d\eta'}{a^2(\eta')}$$

the constant mode is a solution

SGWB from the early universe: generalities

$$h_r''(\mathbf{k},\eta) + 2\,\mathcal{H}\,h_r'(\mathbf{k},\eta) + k^2\,h_r(\mathbf{k},\eta) = 16\pi G\,a^2\,\Pi_r(\mathbf{k},\eta)$$

Solution of the **sourced** equation

First we analyse what can be said **in general** about the SGWB signal generated by a sourcing process occurring at a **given time t_*** in a phase of **standard cosmic expansion** of the universe (not inflation) and therefore operating causally

Why sources in the early universe produce SGWBs?

A GW source acting at time t_* in the early universe cannot produce a signal correlated on length/time scales larger than the causal horizon at that time

$$\boxed{\ell_* \leq H_*^{-1}}$$

ℓ_* characteristic length-scale of the source
(typical size of variation of the tensor anisotropic stresses)

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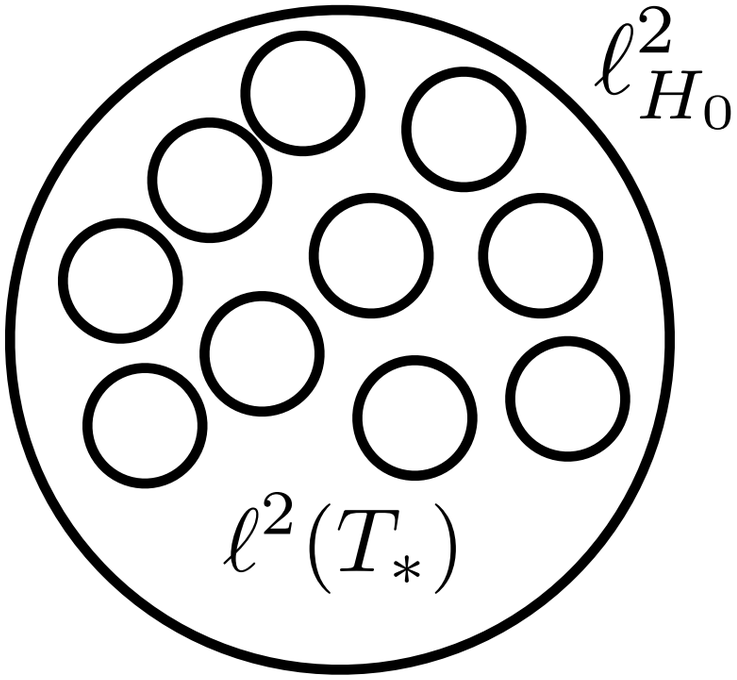


Angular size on the sky today of a region in which the SGWB signal is correlated

$$\Theta_* = \frac{\ell_*}{d_A(z_*)}$$

Angular diameter distance

Number of uncorrelated regions accessible today $\sim \Theta_*^{-2}$

Suppose a GW detector angular resolution of 10 deg $\longrightarrow$ $z_* \lesssim 17$

$$\Theta(z_* = 1090) \simeq 0.9\,\mathrm{deg} \qquad \Theta(T_* = 100\,\mathrm{GeV}) \simeq 10^{-12}\mathrm{deg}$$

Only the statistical properties of the signal can be accessed

Why sources in the early universe produce SGWBs?

- We access today the GW signal from many independent horizon volumes: $h_{ij}(\mathbf{x},t)$ must be treated as a random variable

- Notable exception: *SGWB from inflation* (intrinsic quantum fluctuations that become classical and stochastic outside the horizon)

- The universe is homogeneous and isotropic, so the GW source is operating everywhere more or less at the same time with the same average properties ("a-causal" initial conditions from inflation)

- Under the ergodic hypothesis, the ensemble average can be substituted with volume / time averages: we identify this average with the volume / time one necessary to define the GW energy momentum tensor

$$T^{GW}_{\mu\nu} = \frac{1}{32\pi G}\langle \nabla_\mu h_{\alpha\beta} \nabla_\nu h^{\alpha\beta} \rangle \qquad \rho_{GW} = \frac{\langle \dot{h}_{ij}\dot{h}^{ij} \rangle}{32\pi G}$$

Why sources in the early universe produce SGWBs?

The SGWB is in general homogenous and isotropic, unpolarised and gaussian

As the FLRW space-time

$$\langle h_{ij}(\mathbf{x},\eta_1)\, h_{lm}(\mathbf{y},\eta_2)\rangle = F_{ijlm}(|\mathbf{x}-\mathbf{y}|,\eta_1,\eta_2)$$

There are exceptions!

If the sourcing process preserves parity

$$\langle h_{+2}(\mathbf{k},\eta)h_{+2}(\mathbf{k},\eta) - h_{-2}(\mathbf{k},\eta)h_{-2}(\mathbf{k},\eta)\rangle = \langle h_{+}(\mathbf{k},\eta)h_{\times}(\mathbf{k},\eta)\rangle = 0$$

Helicity basis $\quad e_{ij}^{\pm 2} = \dfrac{e_{ij}^{+} \pm i\, e_{ij}^{\times}}{2}$

Central limit theorem: the signal comes from the superposition of many independent regions

Characterisation of a SGWB

Power spectrum of the GW amplitude $h_c(k,t)$

$$\langle h_r(\mathbf{k},\eta)\, h_p^*(\mathbf{q},\eta)\rangle = \frac{8\pi^5}{k^3}\, \underbrace{\delta^{(3)}(\mathbf{k}-\mathbf{q})}_{\text{Statistical homogeneity and isotropy}}\, \delta_{rp}\, h_c^2(k,\eta)$$

Unpolarised

Gaussianity: the two-point correlation function is enough to fully describe the SGWB

$$\langle h_{ij}(\mathbf{x},\eta)\, h_{ij}(\mathbf{x},\eta)\rangle = 2\int_0^{+\infty} \frac{dk}{k}\, h_c^2(k,\eta)$$

Related to the variance of the GW amplitude in real space

For *freely propagating sub-Hubble modes*, and taking the time-average:

$$\langle h_r(\mathbf{k},\eta)\, h_p^*(\mathbf{q},\eta)\rangle = \frac{1}{a^2(\eta)}\left[\langle A_r(\mathbf{k})\, A_p^*(\mathbf{q})\rangle + \langle B_r(\mathbf{k})\, B_p^*(\mathbf{q})\rangle\right]$$

$$h_c(k,\eta) \propto \frac{1}{a^2(\eta)}$$

Characterisation of a SGWB

Power spectrum of the GW energy density

$$\rho_{\rm GW} = \frac{\langle \dot{h}_{ij}(\mathbf{x},t)\,\dot{h}_{ij}(\mathbf{x},t)\rangle}{32\pi G} = \frac{\langle h'_{ij}(\mathbf{x},\eta)\,h'_{ij}(\mathbf{x},\eta)\rangle}{32\pi G\,a^2(\eta)} = \int_0^{+\infty} \frac{dk}{k}\,\frac{d\rho_{\rm GW}}{d\log k}$$

$$\langle h'_r(\mathbf{k},\eta)\,{h'_p}^*(\mathbf{q},\eta)\rangle = \frac{8\pi^5}{k^3}\,\delta^{(3)}(\mathbf{k}-\mathbf{q})\,\delta_{rp}\,{h'_c}^2(k,\eta)$$

For *freely propagating sub-Hubble modes*, and taking the time-average:

$${h'_c}^2(k,\eta) \simeq k^2\,h_c^2(k,\eta)$$

$$\frac{d\rho_{\rm GW}}{d\log k} = \frac{k^2\,h_c^2(k,\eta)}{16\pi G\,a^2(\eta)}$$

$$\rho_{\rm GW} \propto \frac{1}{a(\eta)^4}$$

GW energy density scales like radiation for freely propagating sub-Hubble modes (free massless particles)

Evolution of the SGWB in the FLRW universe

GW energy density parameter

Evaluated today, for a source that operated at time η_*

$$h^2\,\Omega_{\rm GW}(k,\eta_0) = \frac{h^2\rho_*}{\rho_c}\left(\frac{a_*}{a_0}\right)^4\left(\frac{1}{\rho_*}\frac{d\rho_{\rm GW}}{d\log k}(k,\eta_*)\right)$$

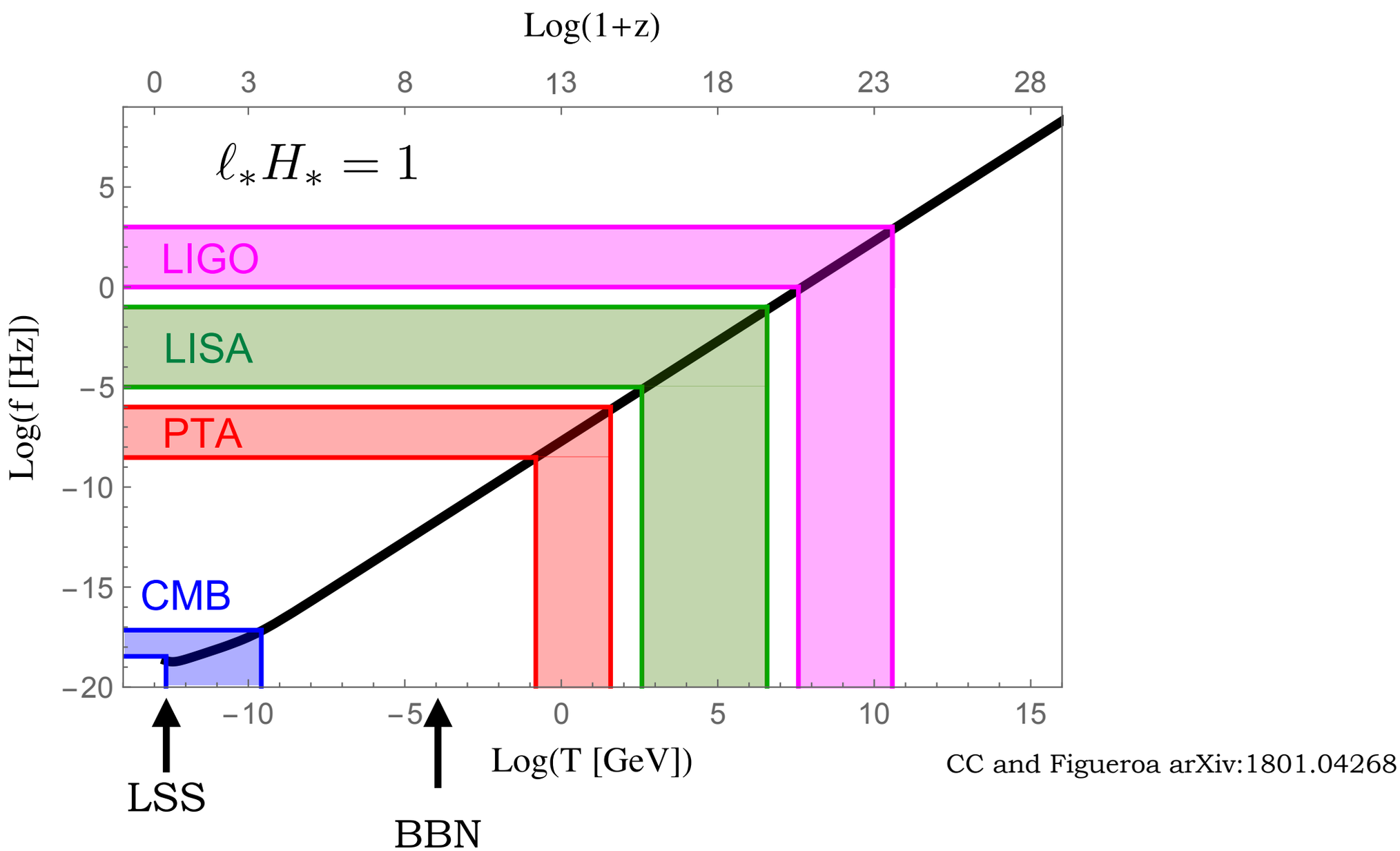
characteristic frequency of the GW signal

$$\boxed{f_* = \frac{1}{\ell_*} \geq H_*}$$

$\ell_* H_*$ Ratio of the typical length-scale of the GW sourcing process (size of the anisotropic stresses) and the Hubble scale at the generation time

$$f = f_*\frac{a_*}{a_0} = \frac{1.65\times 10^{-7}}{\ell_* H_*}\left(\frac{g(T_*)}{100}\right)^{1/6}\frac{T_*}{\rm GeV}\,{\rm Hz}$$

Characteristic frequency of the GW signal

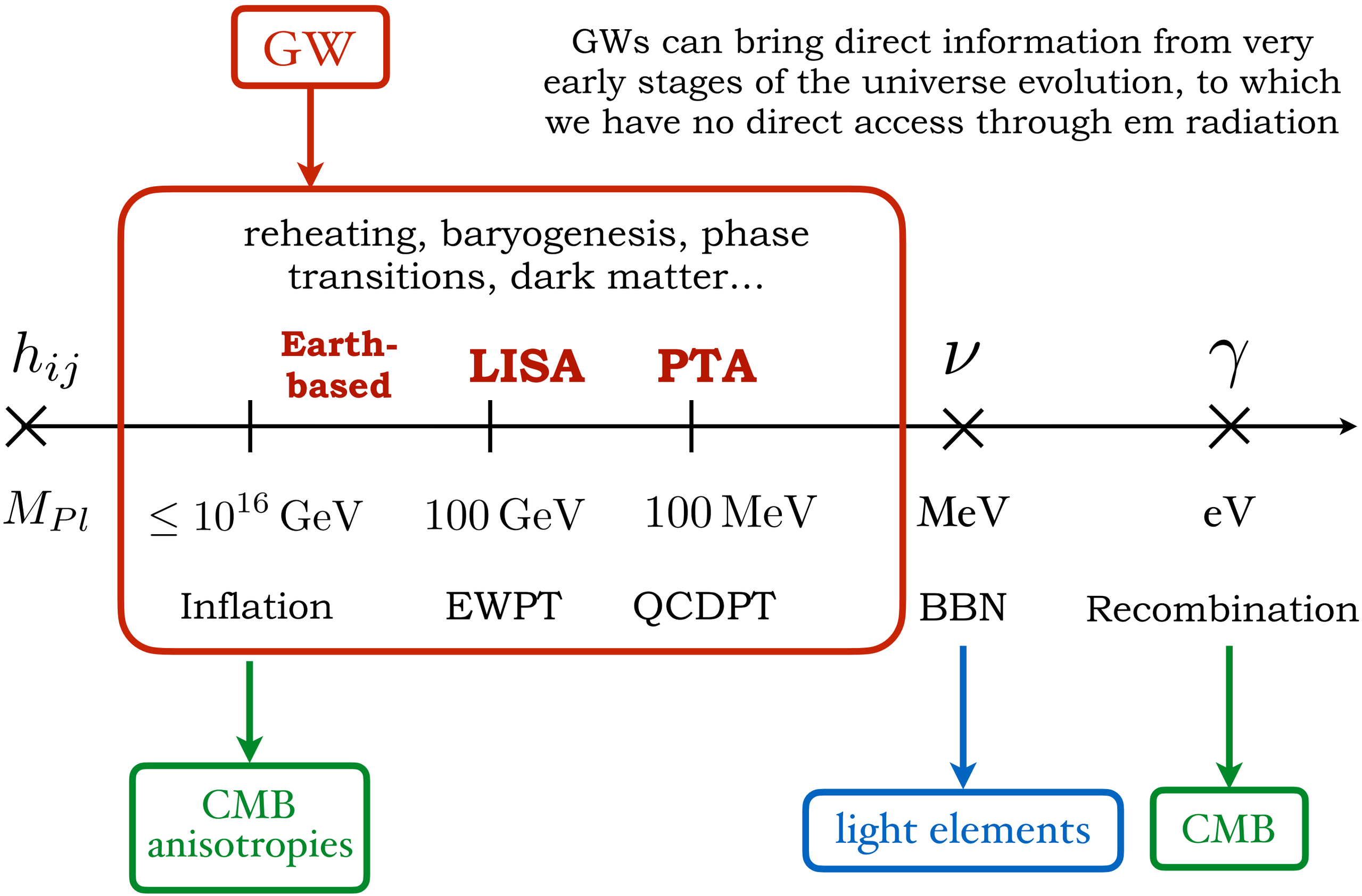


CC and Figueroa arXiv:1801.04268

$T_{\rm QCD} \sim 100\,{\rm MeV}$ $\quad \ell_* H_* \sim 0.1 \longrightarrow f \sim 10\,{\rm nHz}$ PTA

$T_{\rm EW} \sim 100\,{\rm GeV}$ $\quad \ell_* H_* \sim 0.01 \longrightarrow f \sim {\rm mHz}$ LISA

Discovery potential of primordial SGWB detection



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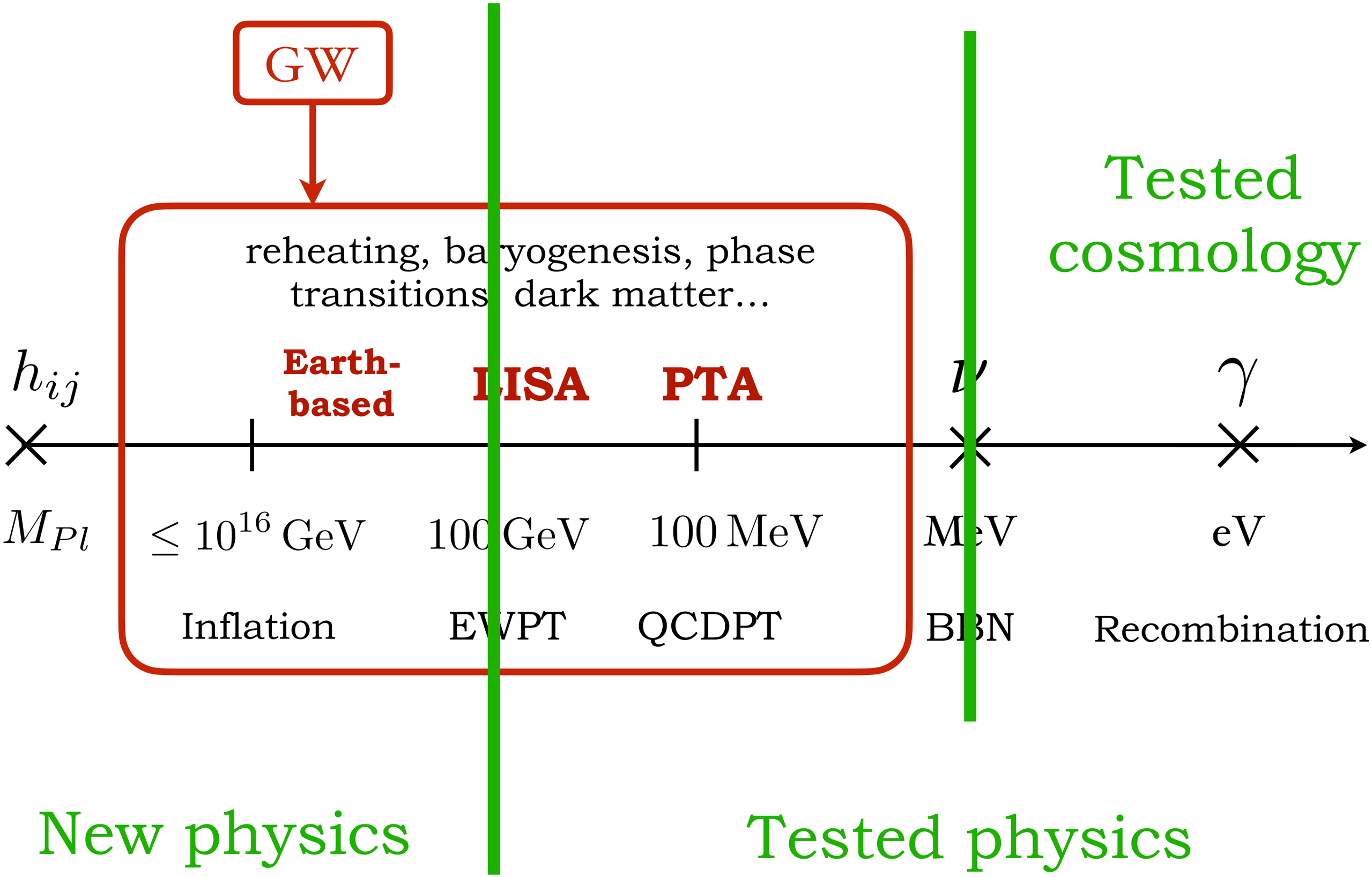
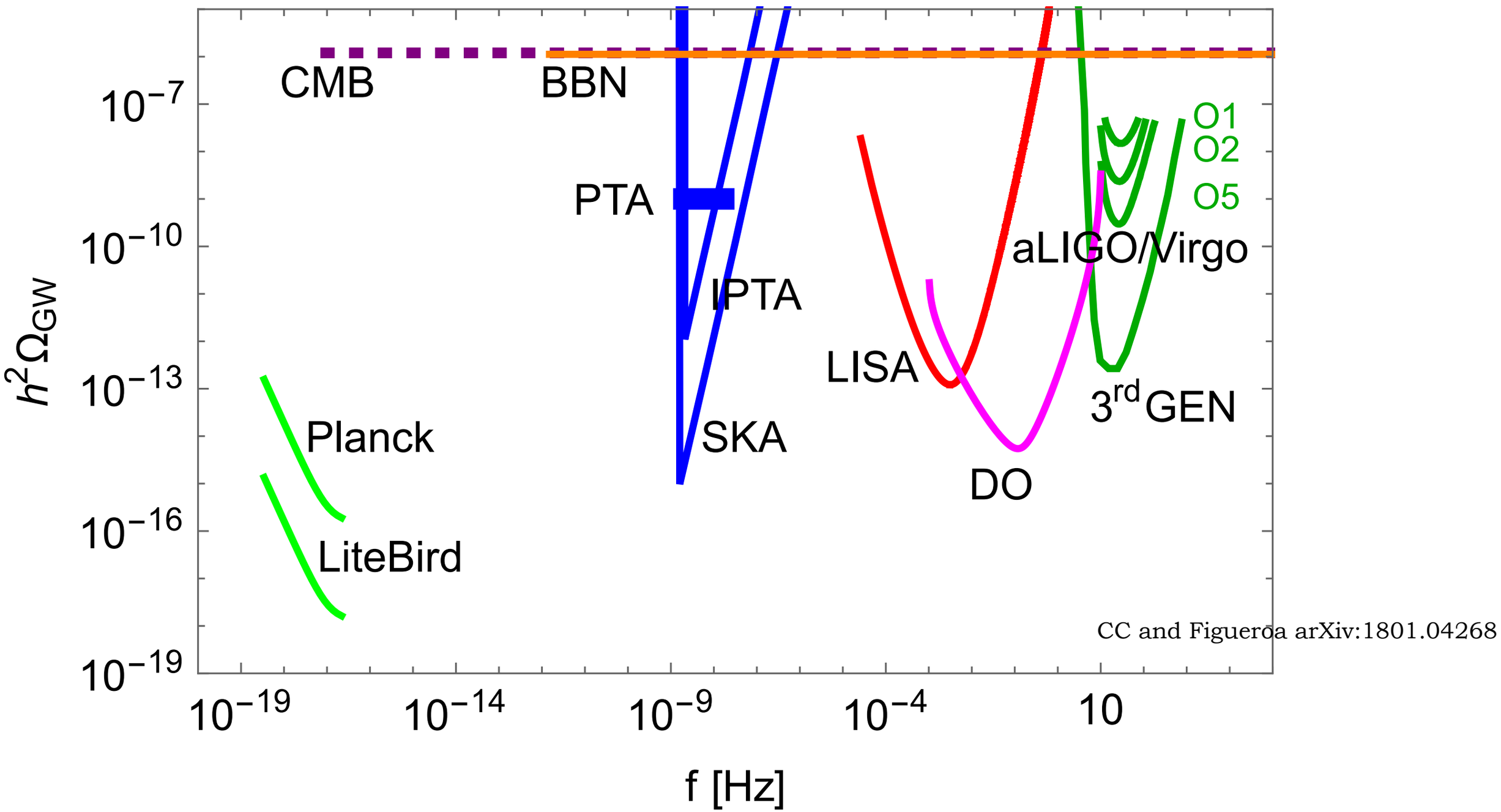


What is/will be known about a stochastic GW background:



Are there signals to populate this diagram?

Examples of SGWB sources in the early universe

- Inflation:
  - quantum tensor fluctuations (at first and second order)
  - tensor modes from additional fields (scalar, gauge…)
  - GWs linked to primordial BHs
  - preheating
  - modifications of gravity
  - …

- Other phase transitions:
  - stable topological defects (in particular strings)
  - *first order* phase transitions
    - bubble wall collisions
    - bulk fluid motion (compressional and vortical)
    - magnetic fields

- Foreground from astrophysical sources (galactic binaries, stellar origin BHB…)
  - *to be accounted for or subtracted of the spectral shape is known*

SGWB from a stochastic source in the radiation era

$$H_r''(\mathbf{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_r(\mathbf{k},\eta) = \boxed{16\pi G\, a^3\, \Pi_r(\mathbf{k},\eta)}$$

Possible sources of tensor anisotropic stress in the early universe:

- Scalar field gradients $\Pi_{ij} \sim [\partial_i\phi\partial_j\phi]^{TT}$
- Bulk fluid motion $\Pi_{ij} \sim [\gamma^2(\rho+p)v_i v_j]^{TT}$
- Gauge fields $\Pi_{ij} \sim [-E_iE_j - B_iB_j]^{TT}$
- Second order scalar perturbations, Π_{ij} from a combination of $\partial_i\Psi, \partial_i\Phi$
- ...

First, solution of the GW propagation equation,
keeping the GW source as general as possible

SGWB from a stochastic source in the radiation era

For most processes in the early universe, the source must be treated as a stochastic variable

$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi_p^*(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\frac{\delta^{(3)}(\mathbf{k}-\mathbf{q})}{k^3}\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

Anisotropic stress power spectrum at unequal time

Suppose the source operates in a time interval $\eta_{\rm fin}$ - $\eta_{\rm in}$ in the radiation dominated era

$$H_r^{\rm rad}(\mathbf{k},\eta<\eta_{\rm fin}) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta} d\tau\, a(\tau)^3\,\sin[k(\eta-\tau)]\,\Pi_r(\mathbf{k},\tau)$$

Matching at $\eta_{\rm fin}$ with the homogeneous solution to find the GW signal today

$$H_r^{\rm rad}(\mathbf{k},\eta>\eta_{\rm fin}) = A_r^{\rm rad}(\mathbf{k})\cos(k\eta) + B_r^{\rm rad}(\mathbf{k})\sin(k\eta)$$

$$A_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a(\tau)^3\,\sin(-k\tau)\,\Pi_r(\mathbf{k},\tau),$$

$$B_r^{\rm rad}(\mathbf{k}) = \frac{16\pi G}{k}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a(\tau)^3\,\cos(k\tau)\,\Pi_r(\mathbf{k},\tau)$$

SGWB from a stochastic source in the radiation era

GW amplitude power spectrum today for modes $k\eta_0 \gg 1$

$$\langle h_r(\mathbf{k},\eta_0)\, h_p^*(\mathbf{q},\eta_0)\rangle = \frac{1}{a_0^2}[\langle A_r(\mathbf{k})\, A_p^*(\mathbf{q})\rangle + \langle B_r(\mathbf{k})\, B_p^*(\mathbf{q})\rangle]$$

GW energy density power spectrum today for modes $k\eta_0 \gg 1$

$$\frac{d\rho_{\rm GW}}{d\log k} = \frac{k^2\, h_c^2(k,\eta_0)}{16\pi G\, a_0^2}$$

(freely propagating sub-Hubble modes)

$$\frac{d\rho_{\rm GW}}{d\log k}(k,\eta_0) = \frac{4}{\pi}\,\frac{G}{a_0^4}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, a^3(\tau)\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\zeta\, a^3(\zeta)\, \cos[k(\eta-\zeta)]\,\Pi(k,\tau,\zeta)$$

SGWB from a **FAST** stochastic source in the radiation era

GW amplitude power spectrum today for modes $k\eta_0 \gg 1$

$$\langle h_r(\mathbf{k},\eta_0)\, h_p^*(\mathbf{q},\eta_0)\rangle = \frac{1}{a_0^2}[\langle A_r(\mathbf{k})\, A_p^*(\mathbf{q})\rangle + \langle B_r(\mathbf{k})\, B_p^*(\mathbf{q})\rangle]$$

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(freely propagating sub-Hubble modes)

$$\frac{d\rho_{\rm GW}}{d\log k}(k,\eta_0) = \frac{4}{\pi}\frac{G}{a_0^4}\int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\tau\, \underbrace{\cancel{a^3(\tau)}}_{a_*^3} \int_{\eta_{\rm in}}^{\eta_{\rm fin}} d\zeta\, \underbrace{\cancel{a^3(\zeta)}}_{a_*^3}\, \underbrace{\cancel{\cos[k(\eta-\zeta)]}}_{\simeq 1}\, \underbrace{\cancel{\Pi(k,\tau,\zeta)}}_{\Pi(k)}$$

SUPPOSE:

$\Delta\eta = \eta_{\rm fin} - \eta_{\rm in} \ll \mathcal{H}_*^{-1}$ $\quad k\eta_{\rm in} \ll 1$ $\quad \Pi(k,\tau,\eta)$ constant over $\Delta\eta$

SGWB from a **FAST** stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

$$h^2\Omega_{\rm GW}(k,\eta_0) = \frac{3}{2\pi^2}\, h^2\Omega^0_{\rm rad}\left(\frac{g_0}{g_*}\right)^{\frac{1}{3}} (\Delta\eta\mathcal{H}_*)^2 \underbrace{\left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 \tilde{P}_{\rm GW}(k)}$$

$$\Pi(k) = \rho_\Pi \tilde{P}_{\rm GW}(k)$$

From the time integrals

SGWB from a **FAST** stochastic source in the radiation era

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$\mathcal{O}(10^{-11})$

Value that would guarantee a detection in a not so far future

$\mathcal{O}(10^{-6})$

Factor depending slightly on the generation epoch through the number of relativistic d.o.f.

$\mathcal{O}(10^{-5})$

Only slow, very anisotropic processes have the chance to generate detectable SGWB signals for sub-Hubble sources

SGWB from a **FAST** stochastic source in the radiation era

GW energy density parameter today for modes $1/\eta_0 \ll k \ll 1/\eta_{in}$

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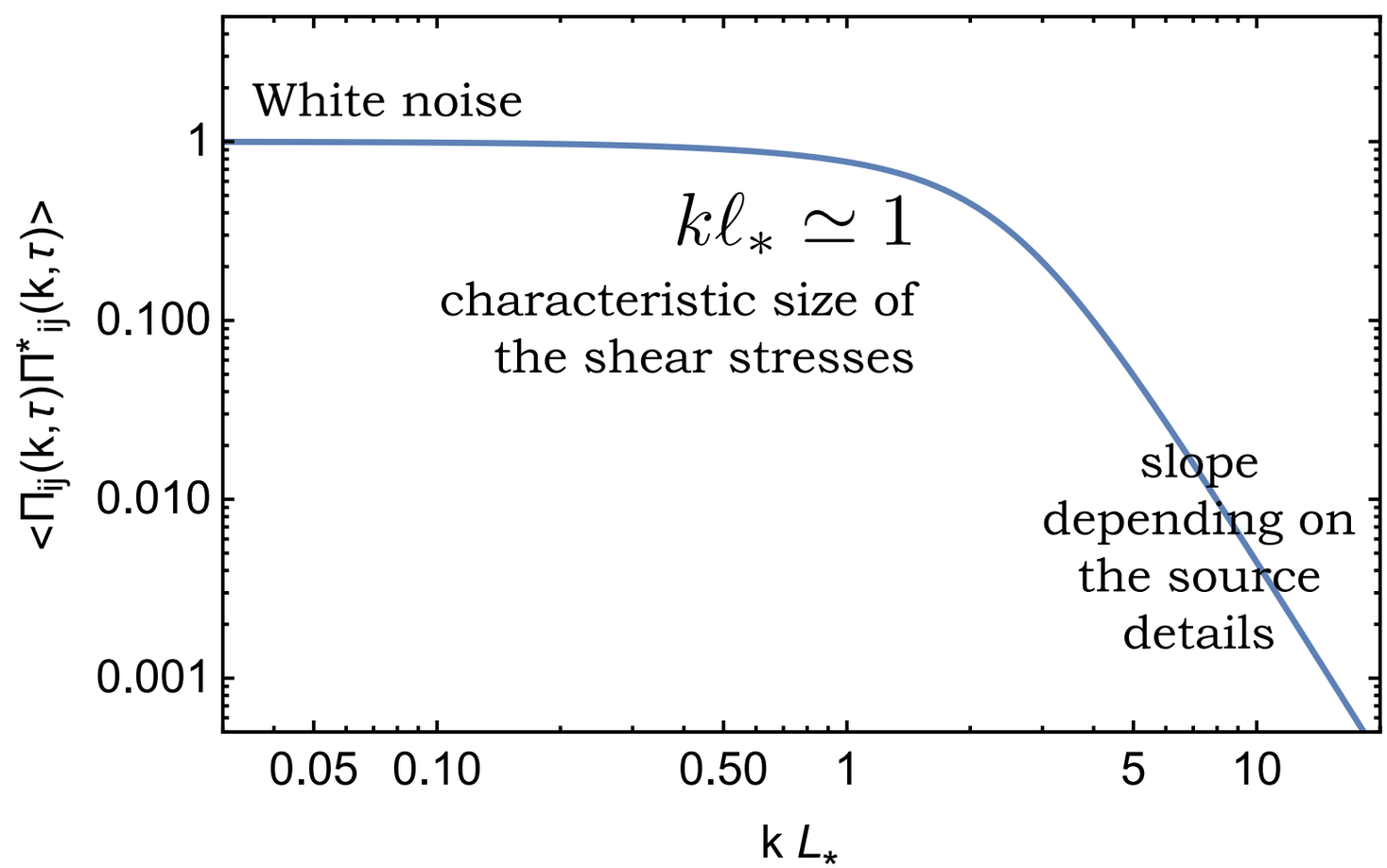
$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi^*_p(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\frac{\delta^{(3)}(\mathbf{k}-\mathbf{q})}{k^3}\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

Independent on k for large enough scales (uncorrelated)

$$\ell_* \leq H_*^{-1}$$



SGWB from a **FAST** stochastic source in the radiation era

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$$\langle \Pi_r(\mathbf{k},\tau)\,\Pi^*_p(\mathbf{q},\zeta)\rangle = \frac{(2\pi)^3}{4}\,\frac{\delta^{(3)}(\mathbf{k}-\mathbf{q})}{k^3}\,\delta_{rp}\,\Pi(k,\tau,\zeta)$$

$$1/\eta_0 \ll k \ll \mathcal{H}_* \ll 1/(a_*\ell_*)$$

Range of validity of the solution

Causality of the sourcing process

$$\Omega_{\rm GW}(k) \propto \tilde{P}_{\rm GW}(k) \propto (k\ell_*)^3$$

SGWB from a stochastic source in the radiation era

- Characteristic time of the source evolution $\delta t_c = \dfrac{\ell_*}{v_{\rm rms}}$

- Characteristic time of the GW production from the Green's function: $\delta t_{\rm gw} \sim \dfrac{1}{k}$

- GW production goes faster than source evolution for all relevant wave-numbers including the spectrum peak $k > \dfrac{v_{\rm rms}}{\ell_*}$

- One assumes that the source is constant in time for a finite time interval which can be larger than the Hubble time $\delta t_{\rm fin} \sim \mathcal{N} \delta t_c$

- One can then easily integrate to find the GW spectrum

$$h^2 \Omega_{\rm GW}(k, \eta_0) \propto h^2 \Omega^0_{\rm rad} \left(\frac{g_0}{g_*}\right)^{\frac{1}{3}} \left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 \tilde{P}_{\rm GW}(k) \begin{cases} \ln^2[1 + \mathcal{H}_* \delta t_{\rm fin}] & \text{if } k\, \delta t_{\rm fin} < 1 \\ \ln^2[1 + (k/\mathcal{H}_*)^{-1}] & \text{if } k\, \delta t_{\rm fin} \geq 1 \end{cases}$$

A. Roper Pol et al, arXiv:2201.05630

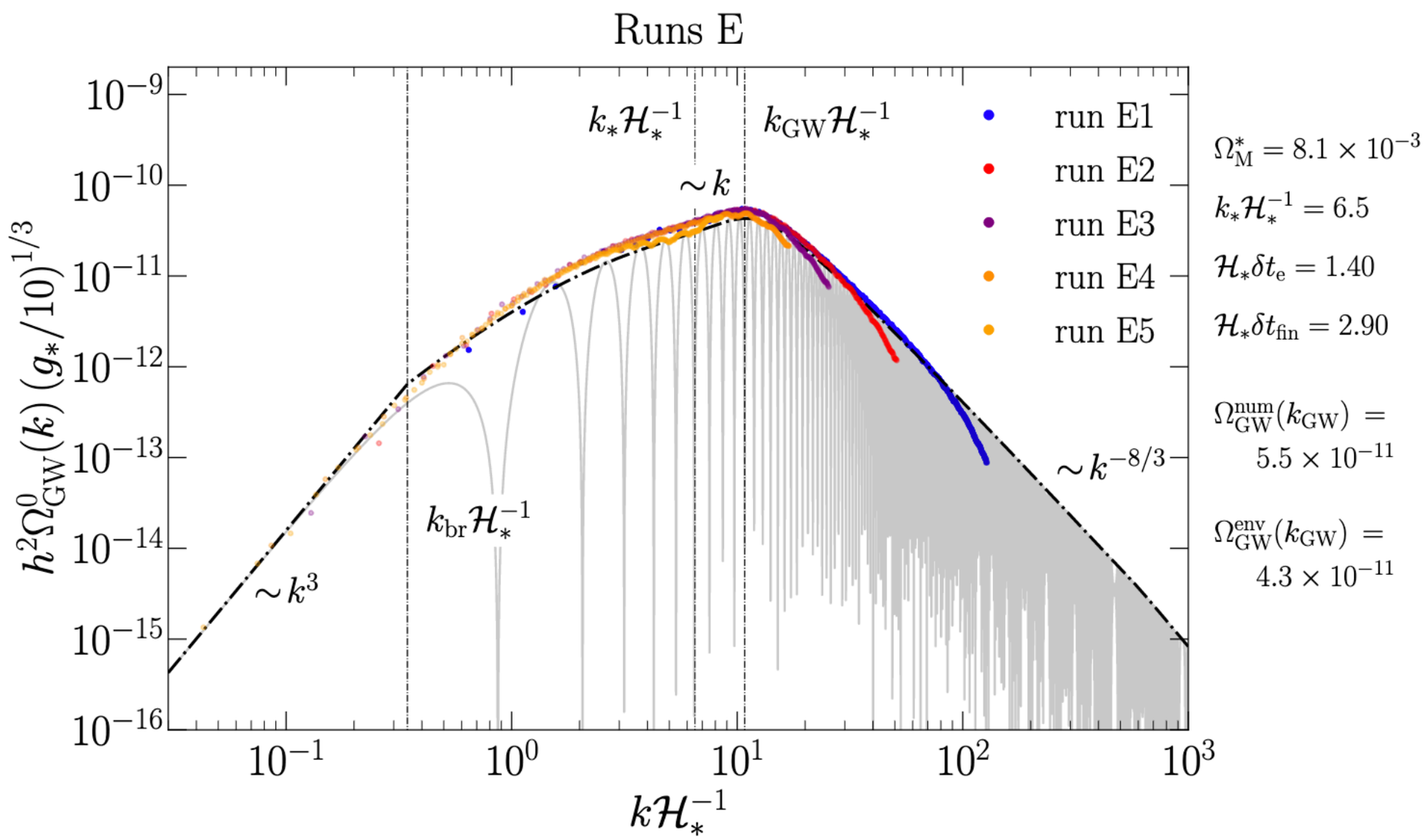
SGWB from a stochastic source in the radiation era: example

$$k_{\rm peak} \simeq 4\pi/\ell_*$$

$$\Omega_{\rm gw,\,peak} \propto \left(\frac{\rho_\Pi}{\rho_{\rm rad}}\right)^2 (\mathcal{H}_*\ell_*)^2$$

Transition from k^3 to k^1 at $k \simeq 1/\delta t_{\rm fin}$

Can be smoother if $\delta t_{\rm fin} > 1/\mathcal{H}_*$



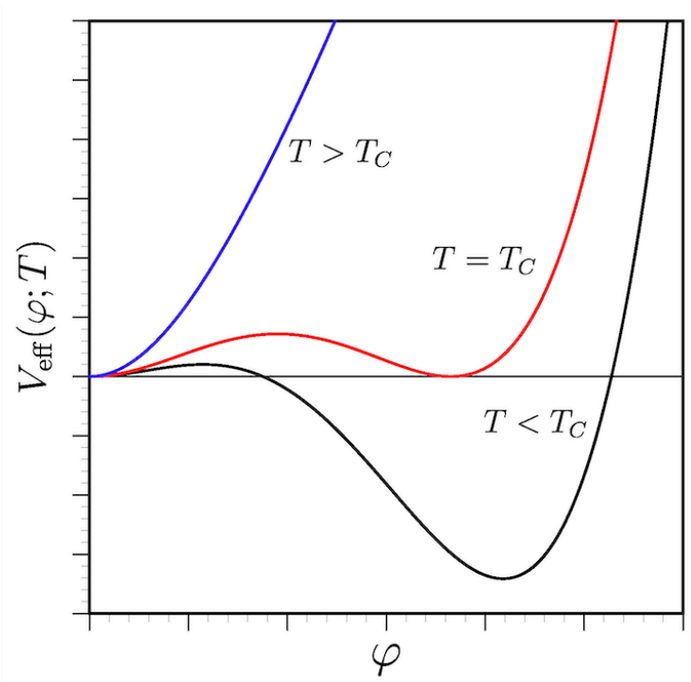
A. Roper Pol et al, arXiv:2201.05630

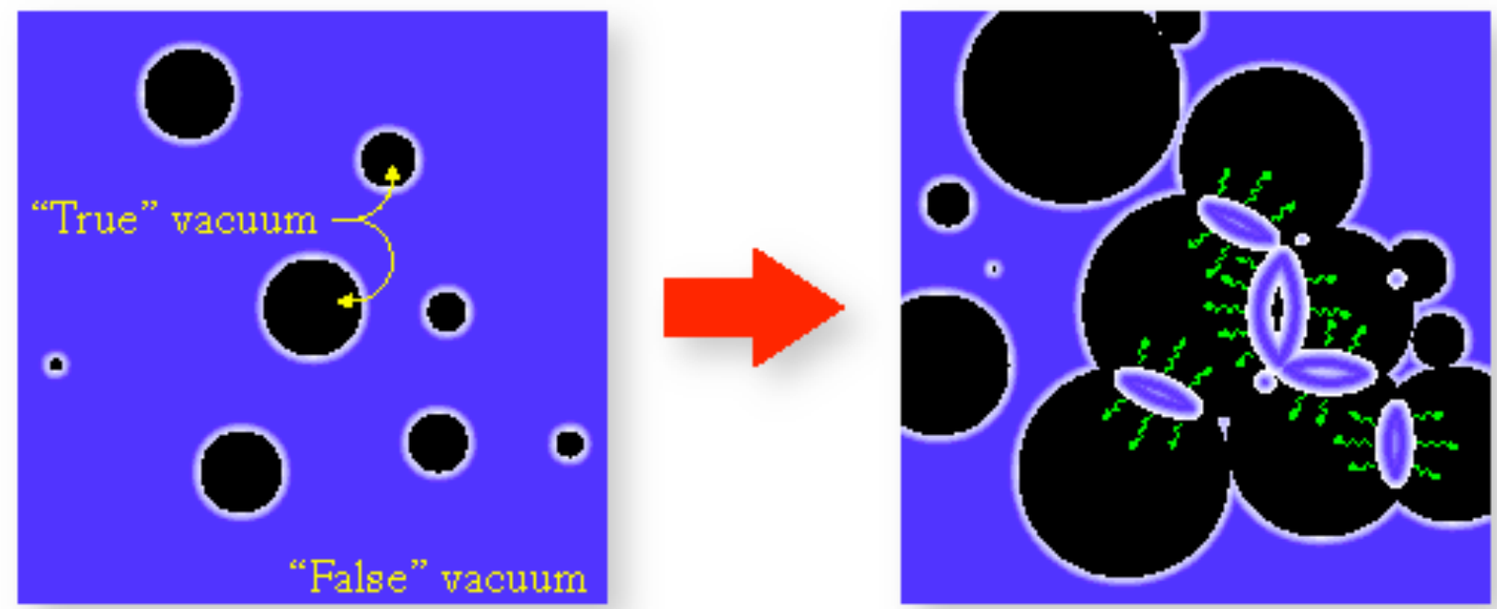
Examples of signals

- First oder phase transitions
- Inflation

Sources of tensor anisotropic stress at a first order phase transition:

GW sourcing process $$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = \boxed{16\pi G\,\Pi_{ij}^{TT}}$$

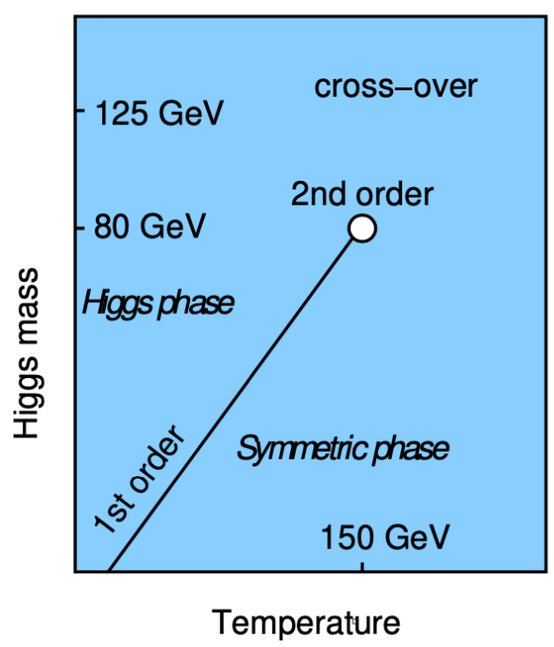




- Bubble collision (scalar field gradients) $\Pi_{ij}^{TT} \sim [\partial_i\phi\partial_j\phi]^{TT}$
- Bulk fluid motion $\Pi_{ij}^{TT} \sim [\gamma^2(\rho+p)v_iv_j]^{TT}$
- Electromagnetic fields $\Pi_{ij}^{TT} \sim [-E_iE_j - B_iB_j]^{TT}$

Electroweak phase transition: phase transition of the Higgs field, driven by the temperature decrease as the universe expands

Standard Model
of particle physics:
Cross-over
Negligible GW production



M. Hindmarsh et al,
arXiv:2008.09136

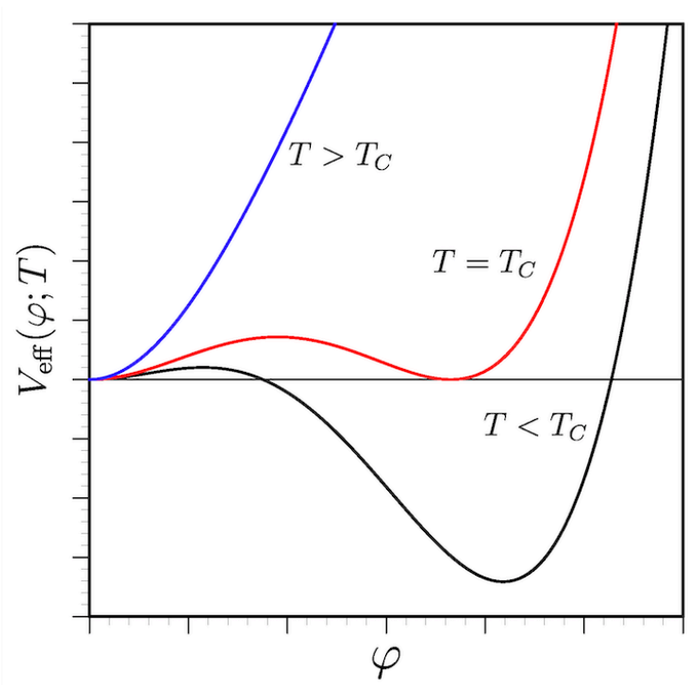
Beyond the Standard Model:
First order phase transition
Possibly observable GW production

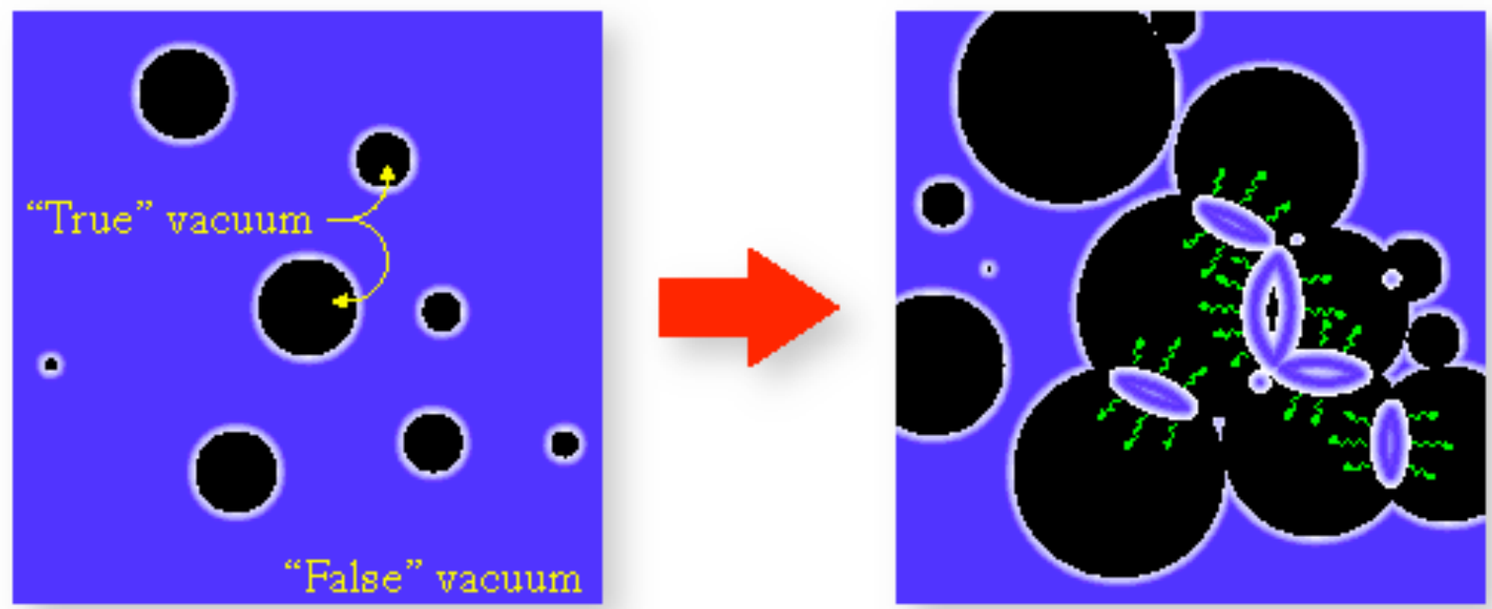
Examples of scenarios leading to observable signals:

- singlet/multiplet extensions of SM or MSSM (SUSY motivated or not)
- SM plus dimension six operator (EFT approach)
- Dark Matter sector uncoupled to the SM
- Warped extra dimensions
- ...

Sources of tensor anisotropic stress at a first order phase transition:

GW sourcing process $$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$





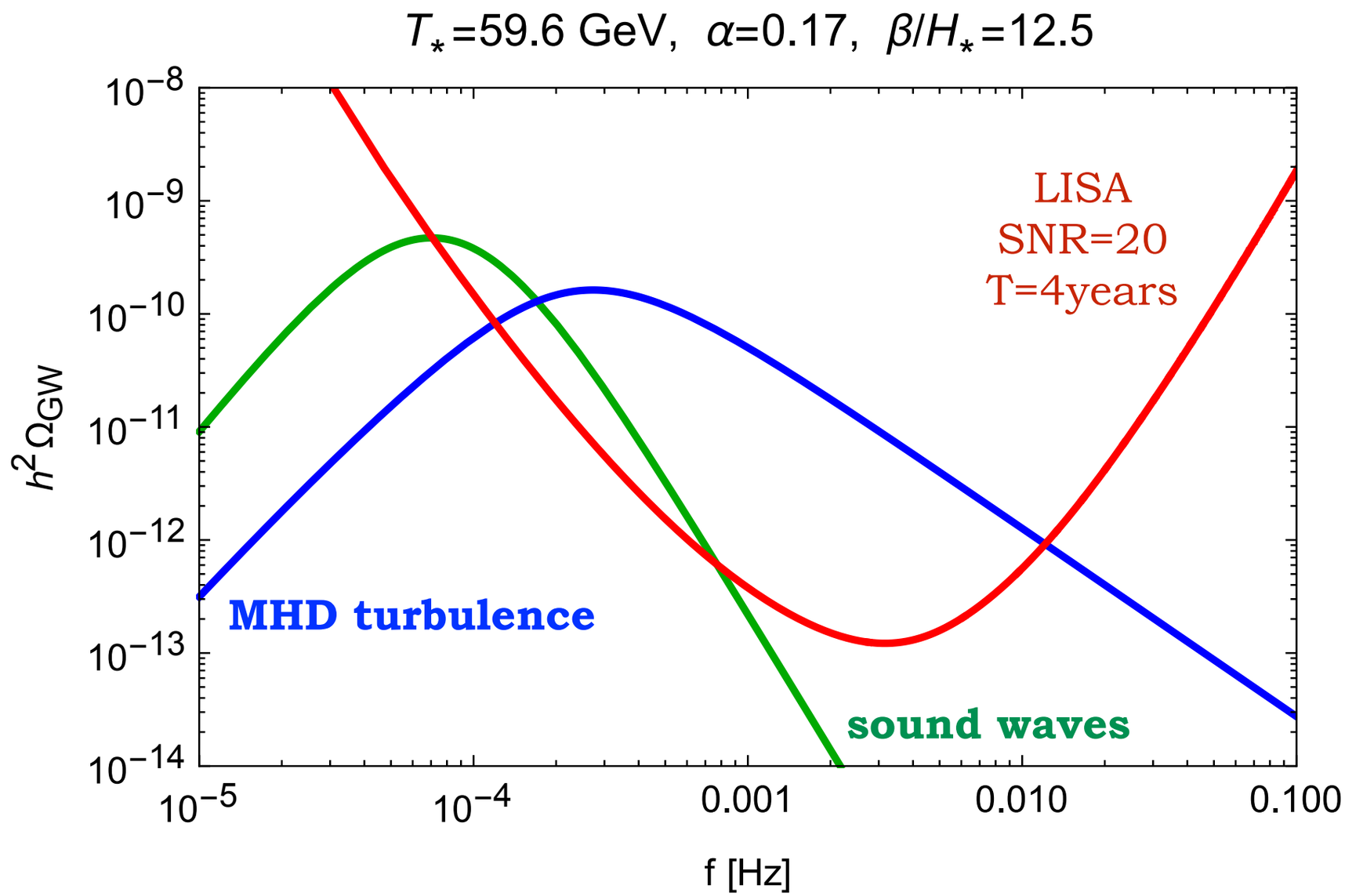
The characteristic scale of the tensor stresses determine the GW frequency:
connected to the bubble size

$$f = f_* \frac{a_*}{a_0} = \frac{1.65 \times 10^{-7}}{\ell_* H_*} \left(\frac{g(T_*)}{100}\right)^{1/6} \frac{T_*}{\rm GeV}\,{\rm Hz}$$

$T_{\rm EW} \sim 100\,{\rm GeV}$ $\quad \ell_* H_* \simeq 0.01$ $\longrightarrow$ $f \sim {\rm mHz}$ LISA

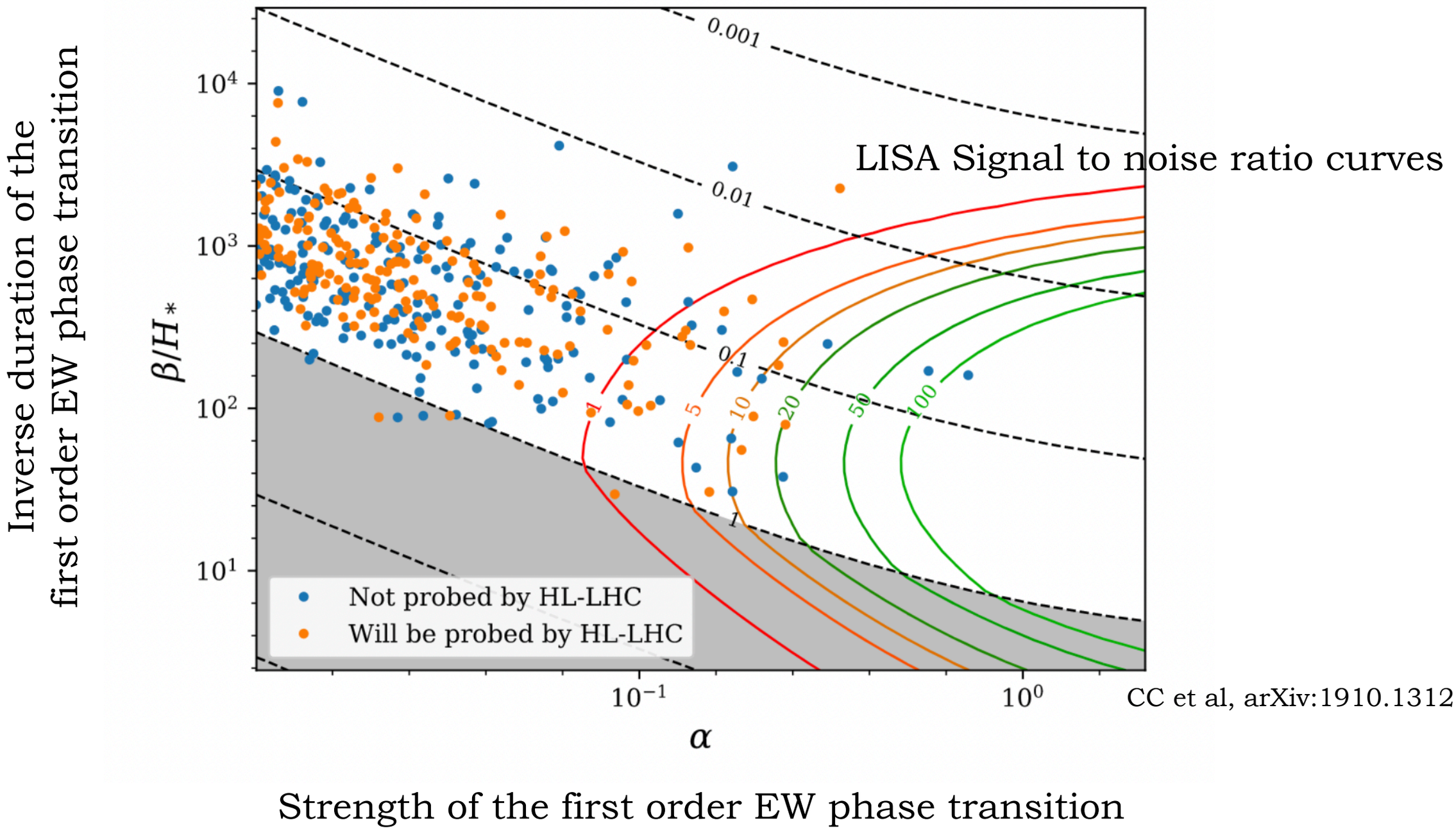
One example of GW signal from the EW phase transition
"Higgs portal" scenario

$$\tau_{\rm nl} \sim \frac{\ell_*}{v_{\rm rms}} = \frac{0.54}{\mathcal{H}_*}$$



$$\langle v^2 \rangle_{\rm turb} = 0.25\, \langle v^2 \rangle_{\rm sound} \qquad\qquad \delta t_{\rm fin} = 5\, \delta t_e$$

Can be probed both at LISA and at the High Luminosity LHC

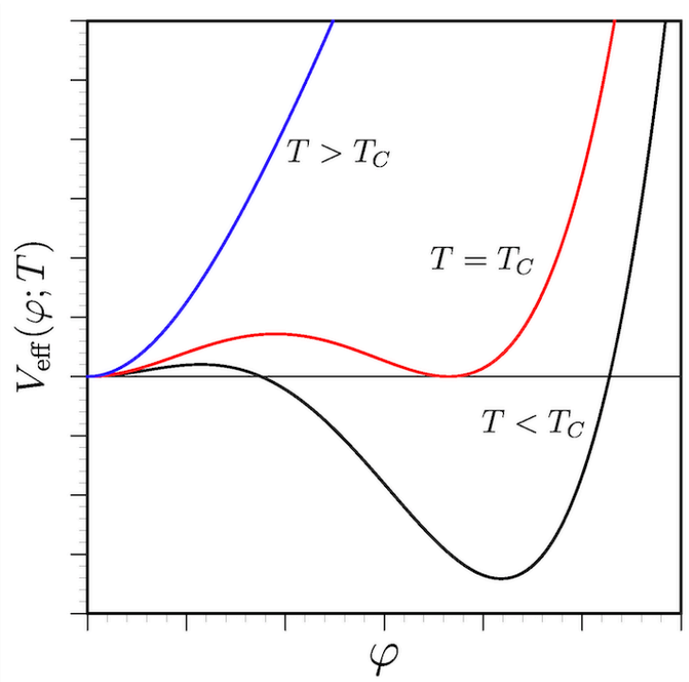


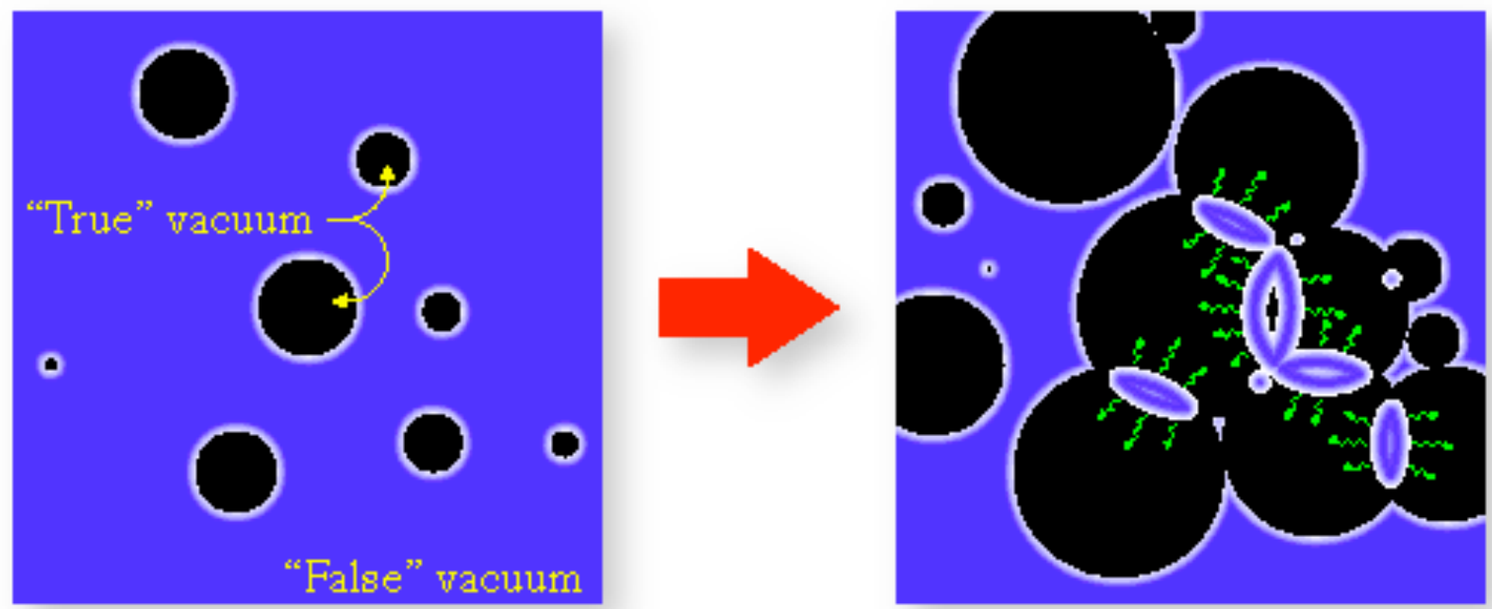
CC et al, arXiv:1910.13125

Strength of the first order EW phase transition

Sources of tensor anisotropic stress at a first order phase transition:

GW sourcing process $$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = \boxed{16\pi G\,\Pi_{ij}^{TT}}$$





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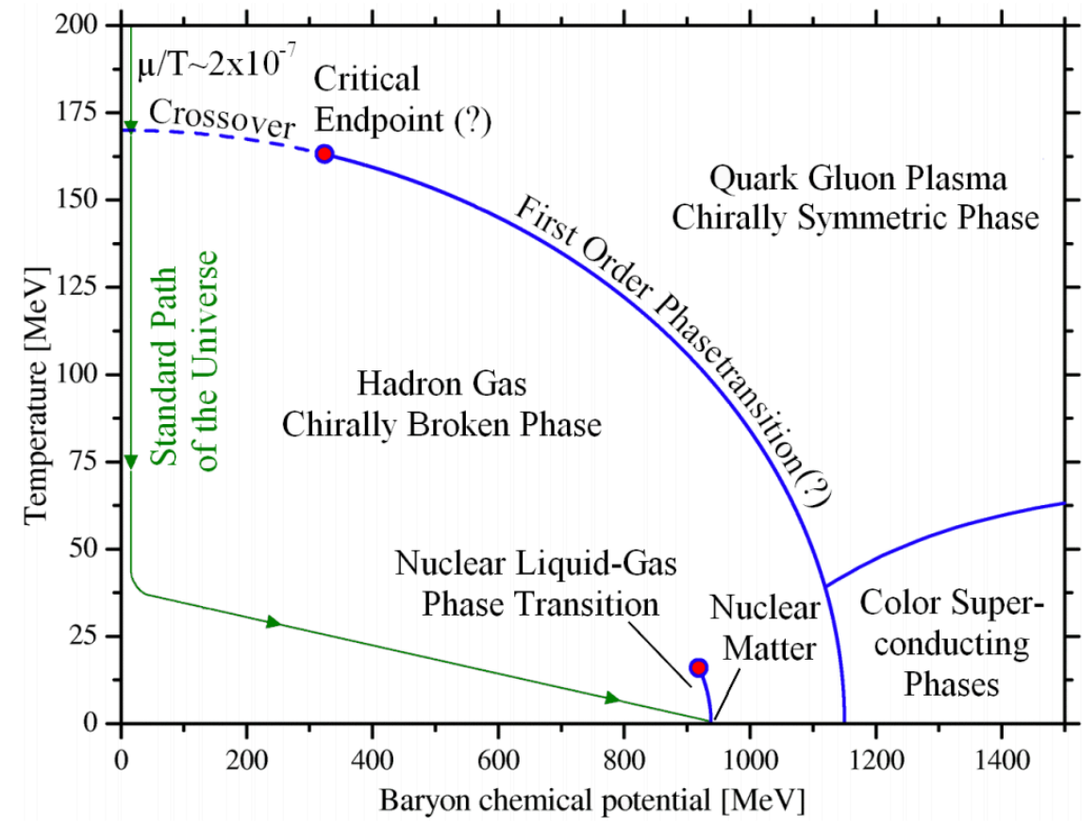
$\boxed{f \sim 10\,{\rm nHz}}$ PTA $\longrightarrow$ $\ell_* H_* \simeq 0.1$ $\quad T_* \sim 0.1\,{\rm GeV}$

QCD phase transition and Pulsar Timing Array noise excess

- In the Standard Model at zero baryon chemical potential it is a cross-over, negligible GW production
- It depends on the (uncertain) conditions of the early universe

D. Schwarz and Stuke, arXiv:0906.3434
M. Middeldorf-Wygas et al, arXiv:2009.00036

T. Boekel and J. Schaffner-Bielich, arXiv:1105.0832



PTA (nHz) are sensitive to energy scales around the **QCD scale**, so they can probe physical processes connected to the QCDPT IF it is first order

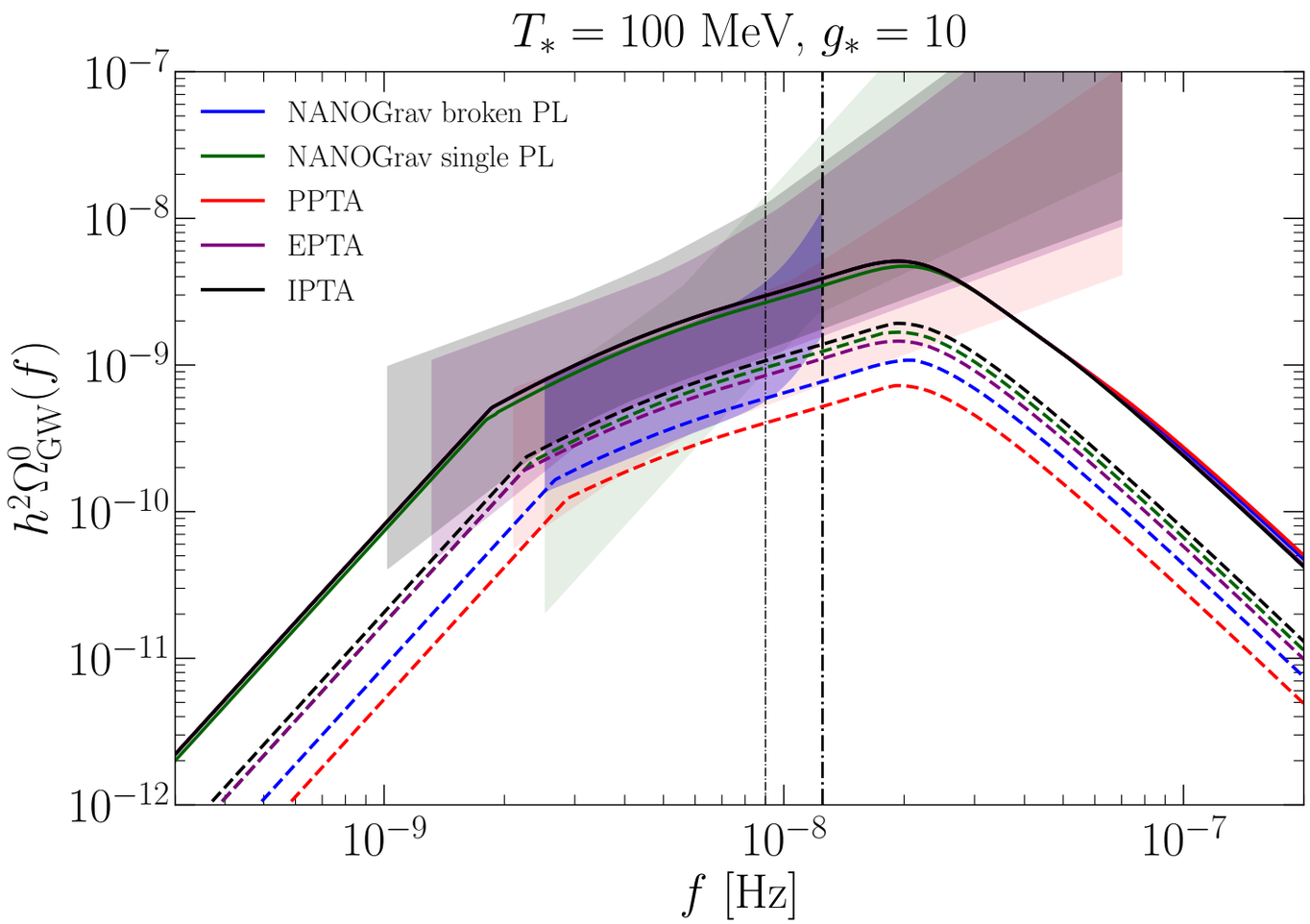
- PTA observatories (NANOGrav, Parkes, European) have recently measured common noise excess

Z. Arzoumanian et al, arXvi: 2009.04496, B. Goncharov et al, arXiv:2107.12112, S. Chen et al, arXiv:2110.13184

- It is compatible with the GW generated by fully developed MHD turbulence at the QCD scale

A. Neronov et al, arXiv:2009.14174

QCD phase transition and PTA noise excess: MHD turbulence from first order PT?

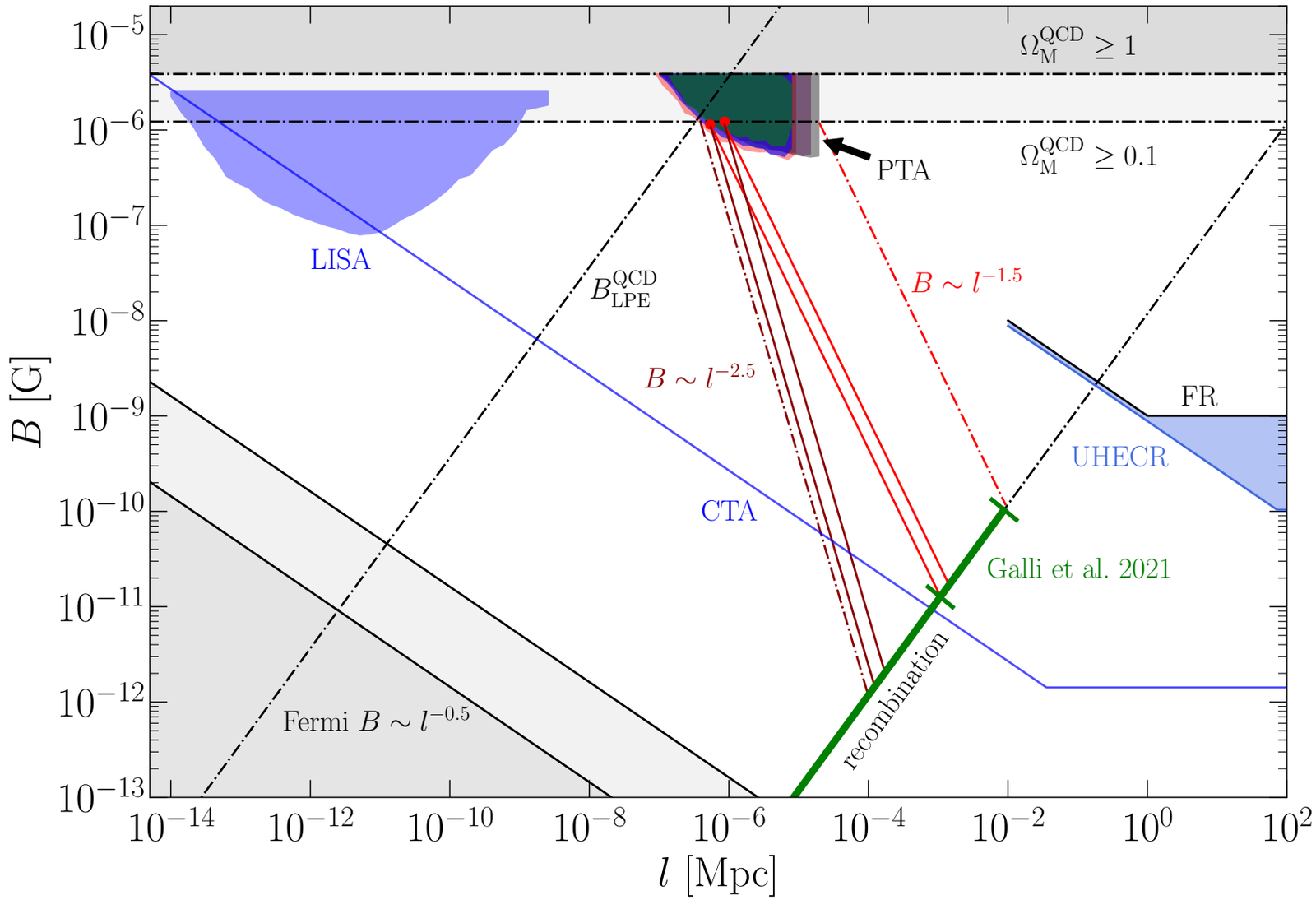


Regions compatible with the PTA observations, a GW spectrum must lie within them

The parameters are

$$(T_*,\ \Omega_B,\ \ell_* \mathcal{H}_*)$$

- For QCD temperature scales, the part of the GW spectrum falling in the region of best quality PTA data is the sub-peak one
  - Slopes (k^3 or k^1) fully compatible with PTA constraints
  - Visible break in the spectrum occurring at $k \sim \mathcal{H}_{QCD}$
- The temperature scale is constrained to 2 MeV < T_* < 200 MeV, the magnetic field energy density must be close to 10% of the radiation energy density and the magnetic correlation scale must be close to the horizon



- The magnetic field giving rise to the GW signal evolves in the radiation era

Banerjee and Jedamzik arXiv:0410032,
Durrer and Neronov, arXiv:1303.7121

- It might modify the CMB spectrum and ease the Hubble tension at recombination, seed the magnetic fields observed today in matter structures, and be constrained by future gamma-ray telescopes

S. Galli et al, arXiv:2109.03816
Jedamzik and Pogosian, arXiv:2004.09487
Korochin et al, arXiv:2007.14331

Examples of signals

- First oder phase transitions
- Inflation

GW signal from inflation

Amplification of tensor metric vacuum fluctuations by the exponential expansion

$$h_r''(\mathbf{k},\eta) + 2\,\mathcal{H}\,h_r'(\mathbf{k},\eta) + k^2\,h_r(\mathbf{k},\eta) = 16\pi G\,a^2\,\cancel{\Pi_r(\mathbf{k},\eta)}$$

✓ canonically normalised free field $v_\pm = a\,M_{Pl}\,h_\pm$

✓ quantisation

✓ homogeneous wave equation: harmonic oscillator with *time dependent* frequency

$$v_\pm''(t) + (k^2 - a^2H^2)v_\pm(t) = 0$$

$k \gg a\,H$ sub-Hubble modes

$$\omega^2(t) = k^2$$

free field in vacuum
zero occupation number

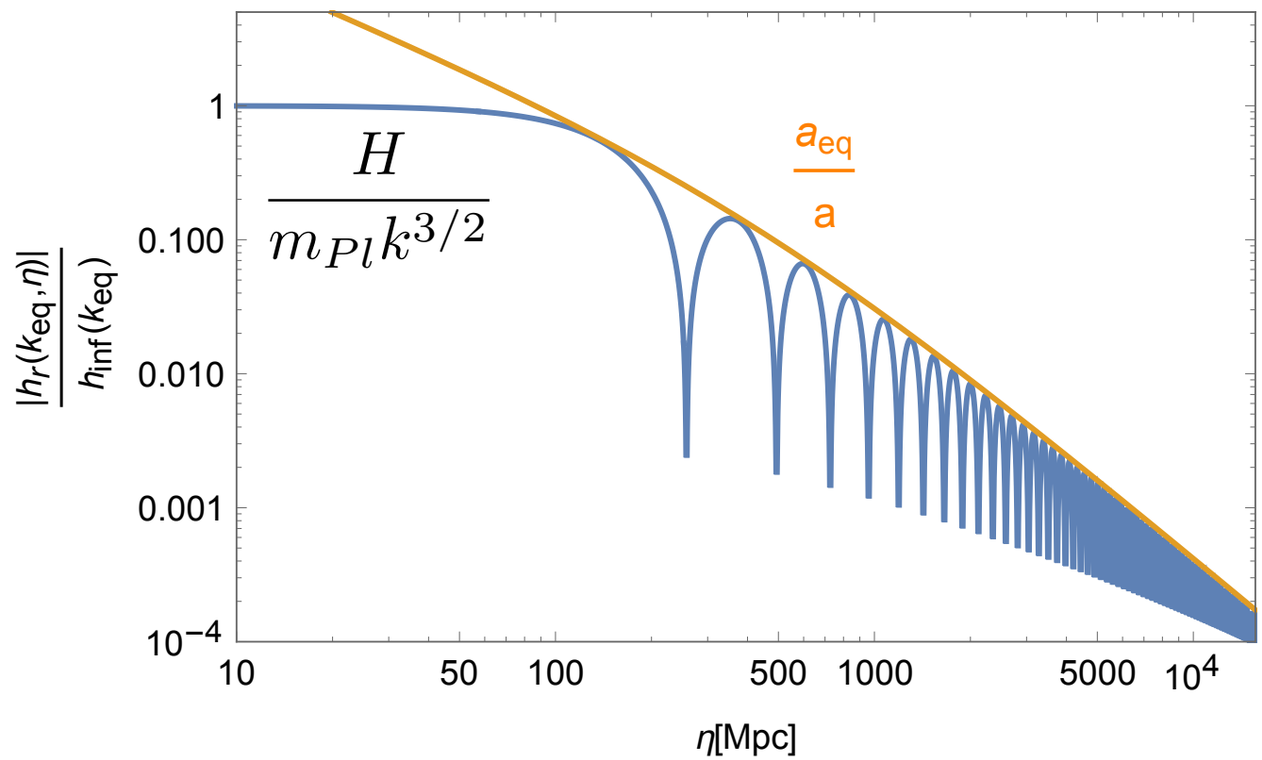
$k \ll a\,H$ super-Hubble modes

$$\omega^2(t) = -a^2H^2$$

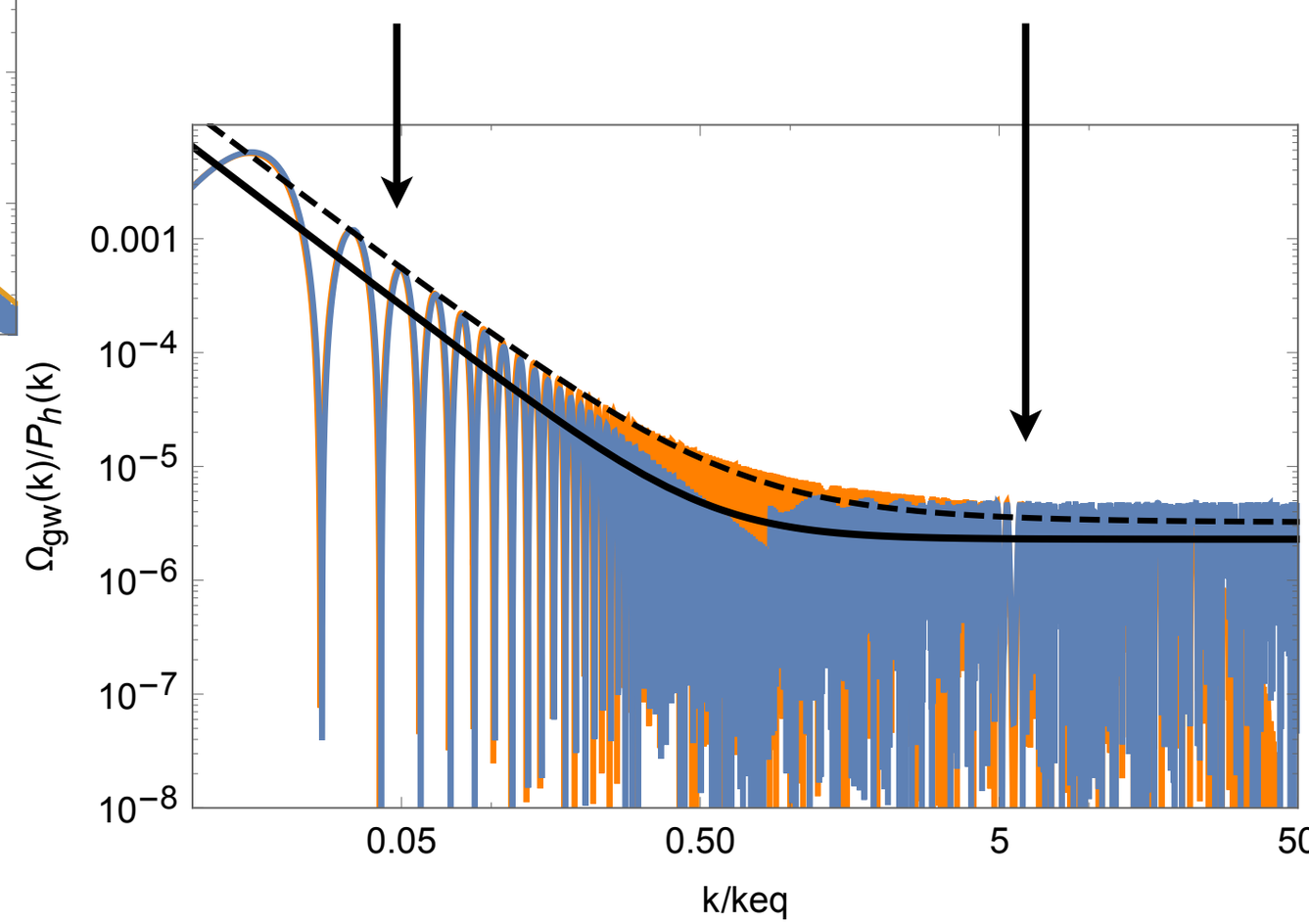
super-Hubble modes have very large occupation number

GW signal from (slow roll) inflation

- tensor spectrum $$\mathcal{P}_h = \frac{2}{\pi}\frac{H^2}{m_{Pl}^2}\left(\frac{k}{aH}\right)^{-2\epsilon} \qquad \epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$
- transfer function from inflation to today, as modes re-enter the Hubble horizon



$$\Omega_{\rm GW}(k,\eta_0) = \frac{[T'(k,\eta_0)]^2}{12a_0^2H_0^2}\mathcal{P}_h(k)$$



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$$\boxed{\Omega_{\rm GW}(f) = \frac{3}{128}\,\Omega_{\rm rad}\, r\, \mathcal{P}_{\mathcal{R}}^*\left(\frac{f}{f_*}\right)^{n_T}\left[\frac{1}{2}\left(\frac{f_{\rm eq}}{f}\right)^2 + \frac{16}{9}\right]}$$

- tensor to scalar ratio $r = \mathcal{P}_h/\mathcal{P}_{\mathcal{R}}$

- scalar amplitude at CMB pivot scale $\mathcal{P}_{\mathcal{R}}^* \simeq 2\cdot 10^{-9}$ $k_* = \frac{0.05}{\rm Mpc}$

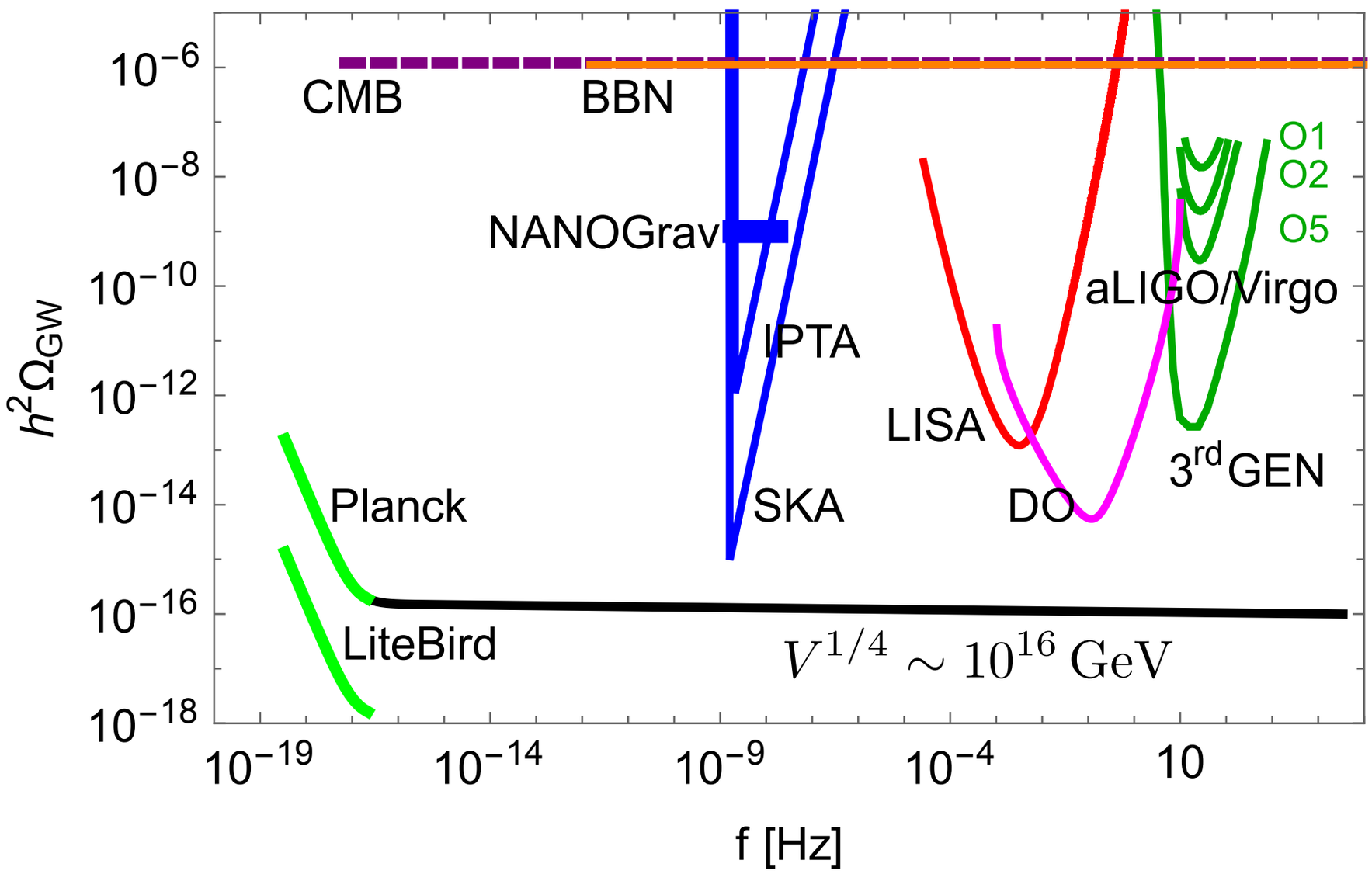
- GW signal extended in frequency: $H_0 \leq f \leq H_{\rm inf}$

continuous sourcing of GW as modes re-enter the Hubble horizon

GW signal from (slow roll) inflation

Gw detectors offer the amazing opportunity to probe the inflationary power spectrum (and the model of inflation) down to the tiniest scales

BUT! The signal in the standard slow roll scenario is too low

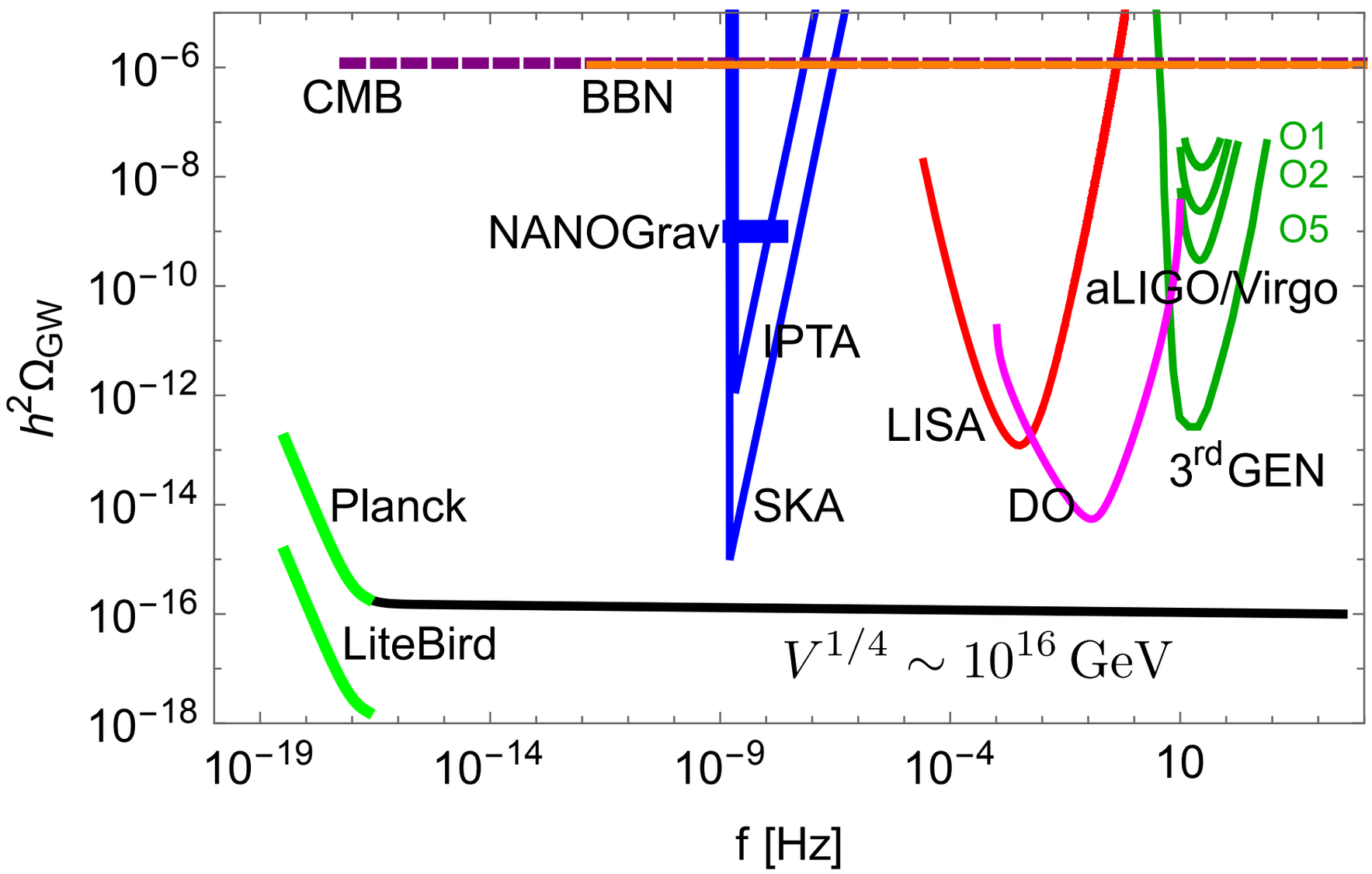


$$n_T \simeq -2\epsilon$$

GW signal from (slow roll) inflation

Gw detectors offer the amazing opportunity to probe the inflationary power spectrum (and the model of inflation) down to the tiniest scales

(P)reheating generates a signal, but unfortunately at very high frequencies



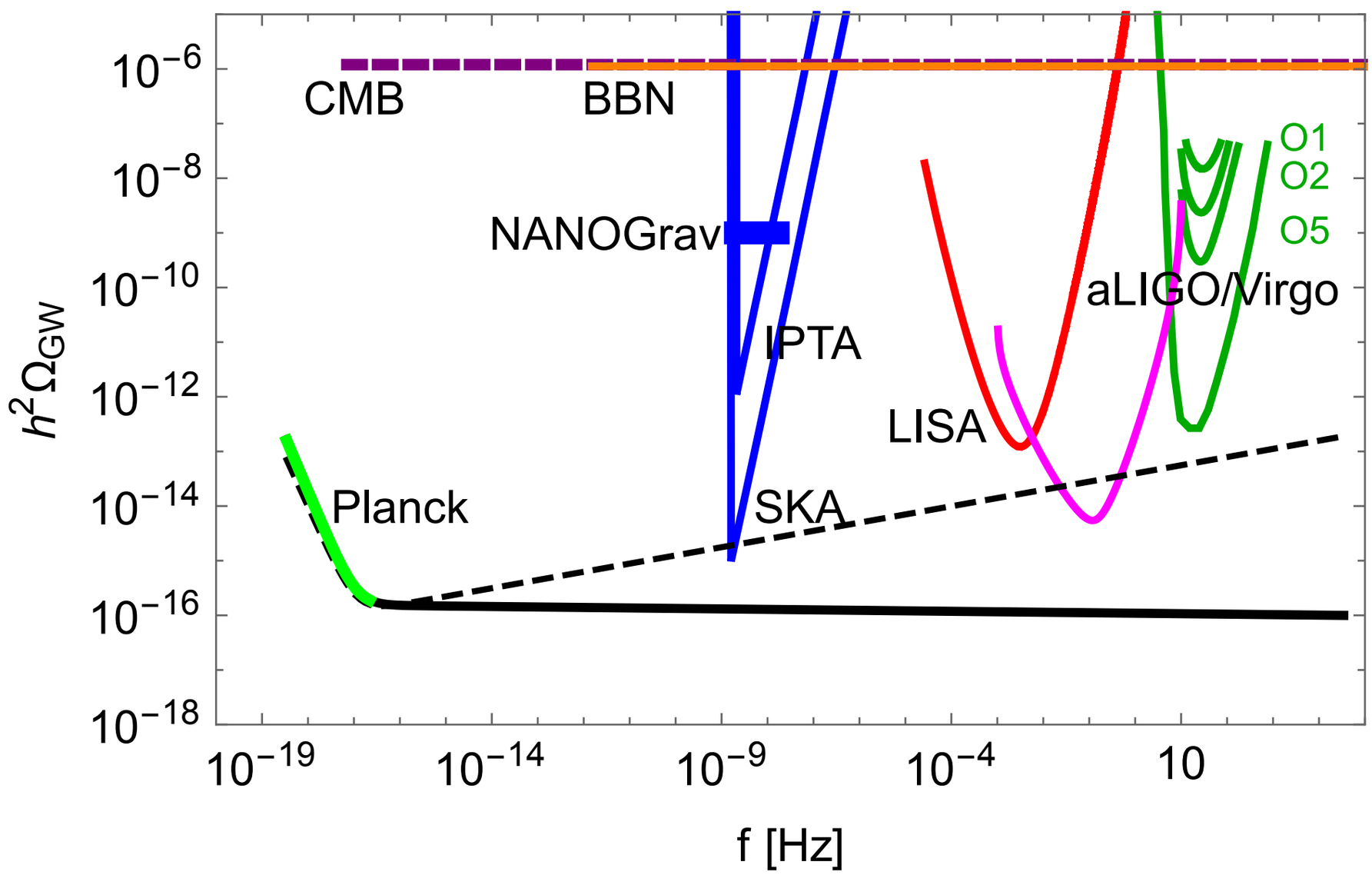
$V^{1/4} \sim 10^{16}\,\mathrm{GeV}$

$n_T \simeq -2\epsilon$

GW signal from (non-standard) inflation

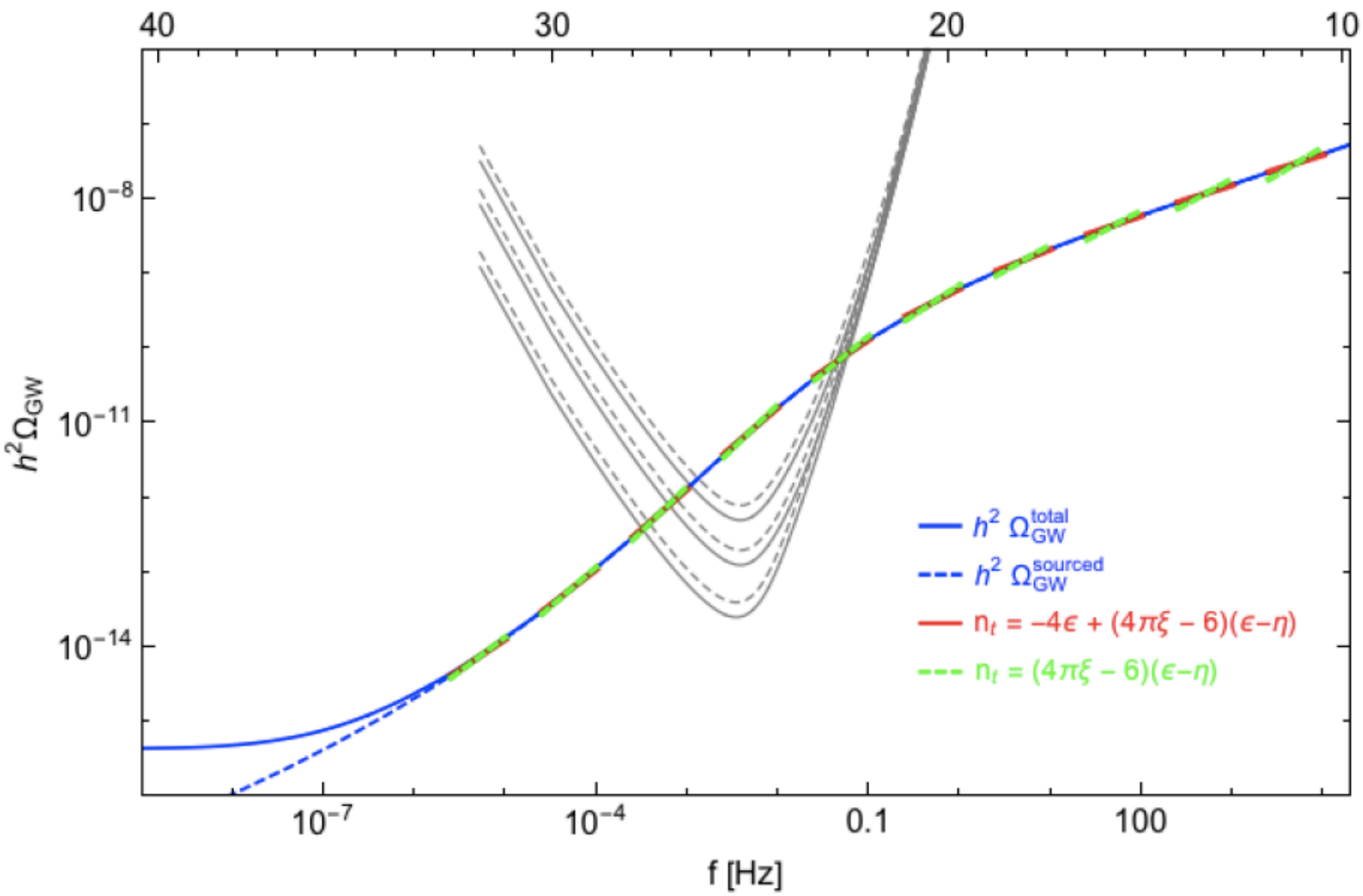
There is the possibility to enhance the signal going beyond the standard inflationary scenario: adding extra fields, modifying the inflaton potential, modifying the gravitational interaction, adding a phase with stiff equation of state...

$$H_r''(\mathbf{k},\eta) + \left(k^2 - \frac{a''}{a}\right) H_r(\mathbf{k},\eta) = 16\pi G\, a^3\, \Pi_r(\mathbf{k},\eta)$$



just one example: inflaton-gauge field coupling

$$\Delta\mathcal{L} = -\frac{1}{4\Lambda}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} \qquad \Pi_{ij} \sim [-E_i E_j - B_i B_j]^{TT}$$



$$\Lambda = \frac{M_{Pl}}{35}$$

quadratic inflaton potential

OTHER SIGNATURES:
non-gaussianity, chirality

N. Bartolo et al, arXiv:1610.06481
N. Bartolo et al, arXiv:1806.02819

To summarise:

- SGWB might reveal a powerful tool to probe the early universe and high energy physics
- The spectral shape must be predicted with good accuracy in order to disentangle the different sources (and also for foregrounds)
- General considerations about the characteristics of the spectral shape are possible in some cases, to pin down at least the class of SGWB sources
- Electroweak PT: at the limit of tested physics, GW signal can be accessed/ constrained by LISA only for models beyond the standard model of particle physics
- QCD PT: tested physics but difficult to predict, GW signal can be accessed/ constrained by PTA only for models beyond the standard model of particle physics
- Inflation: new physics but observationally compelling, extended GW signal in frequency, only accessible by CMB unless one goes beyond the standard slow roll scenario (there are well motivated scenarios!)
- **SGWBs from the primordial universe might seem speculative but their potential to probe fundamental physics is great and amazing discoveries can be around the corner**