

# Goals

- Discuss thermal relic benchmarks
- Discuss signal computation - J-factors, spectra

## Indirect detection

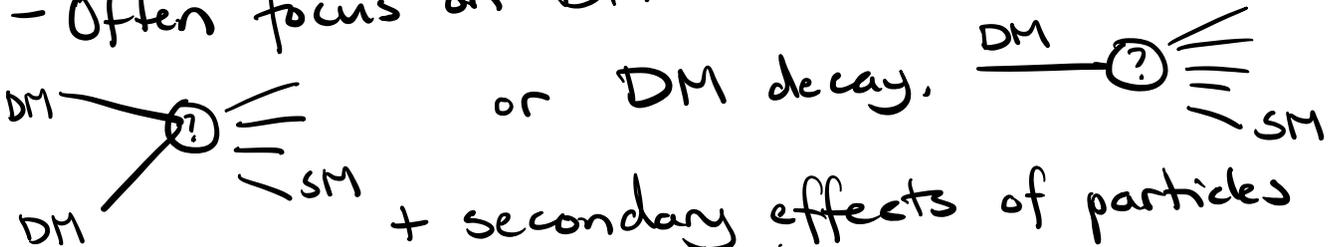
- Aim: identify visible products of DM interactions, involving DM already present in the cosmos

Does the interaction involve ambient DM?

N: accelerator searches, Y: in/direct detection

Does the interaction occur inside the detector? Y = direct detection, N = indirect detection

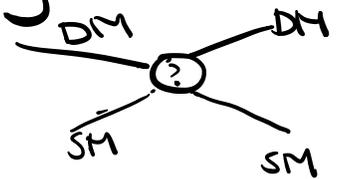
- Often focus on DM annihilation,



+ secondary effects of particles produced this way

but can also include scattering

oscillation, , etc



- Benefits from huge volumes/times to integrate  $n \cdot n$  (interactions occurring anywhere in

- past lightcone may be observable)
- Can piggyback on telescopes built for astronomy/astrophysics
  - Studies of the early universe, compact objects (stars/black holes/planets/etc), etc can probe conditions of density, temperature, field strength impossible to achieve on Earth
  - Wide range of target regions & signal channels
    - can cross-check apparent signals & probe many different models
  - Unique sensitivity to lifetime of DM, & to origin of DM in models where annihilation fixes the DM abundance
  - Downsides: not controlled experiments & backgrounds can be large + complicated

### Some central questions:

- How large a signal do we expect?
  - Unknown, but can consider benchmark scenarios
    - classic one is annihilation of thermal relic DM
- What does the signal look like as a function of energy, time, position?

## Thermal dark matter (benchmark scenario)

Starting point: Suppose DM was once in thermal equilibrium with the Standard Model (requires interactions beyond gravity).

What happens as the universe expands?

Specifically, suppose DM has a number-changing "annihilation" reaction,  $DM + DM \rightarrow X$  (in SM). We will assume for now there is only one type of DM (e.g. not DM & anti-DM).

- If we set the annihilation cross-section to zero, so total DM number is conserved, then the DM no. density  $n$  dilutes with redshift as

$$\frac{d}{dt}(na^3) = 0 \Rightarrow a^3 \frac{dn}{dt} + 3a^2 n \frac{da}{dt} = 0 \Rightarrow \frac{dn}{dt} + 3Hn = 0$$

- When we turn on annihilation, this adds two more terms to  $\frac{dn}{dt}$ : depletion from  $DM DM \rightarrow X$  & production from  $X \rightarrow DM DM$ .

$$\frac{dn}{dt} + 3Hn = -\frac{n^2}{2} \langle \sigma v \rangle \times 2 + \langle \sigma v \rangle x$$

counts # of DM-DM pairs
annihilation cross section
2 particles removed per annihilation
independent of  $n$   
unknown parameter

We can relate the production term to the depletion term via detailed balance: in equilibrium, the rates should cancel. Writing the RHS of the equation as

$$\langle \sigma v \rangle (x - n^2), \Rightarrow x = n_{eq}^2$$

(Equivalently: as  $\langle \sigma v \rangle \rightarrow \infty$  the DM & SM will come into equilibrium, to preserve the LHS of the equation we must have  $\langle \sigma v \rangle (x - n^2) \rightarrow \text{constant}$ , i.e.  $x - n^2 \rightarrow 0$  as  $n \rightarrow n_{eq}$ .)

Thus we can write the evolution equation for  $n$  as:

$$\frac{dn}{dt} + 3Hn = -\langle \sigma v \rangle (n^2 - n_{eq}^2)$$

We typically approximate  $n_{eq}$  by the Boltzmann distribution w/  $\mu=0$  (as in this scenario the DM is its own antiparticle):

$$n_{eq} \sim \begin{cases} (m_{DM} T)^{3/2} e^{-m_{DM}/T}, & m_{DM} \gg T \quad \text{non-relativistic} \\ T^3 & , m_{DM} \ll T \quad \text{relativistic} \end{cases}$$

(There are many variations on this scenario - e.g. with both DM & anti-DM, DM annihilates against a partner particle, there are more than two DM particles in the initial state, etc.)

For detailed calculations of  $n$  at late times, i.e. the DM abundance at the CMB, we solve this equation numerically

(see e.g. arXiv: 1204.3622)

Let us understand the general features of the solution.

Two regimes: (1)  $n^2 \langle \sigma v \rangle \ll 3Hn \Rightarrow$  dilution from cosmic expansion dominates  $\Rightarrow n \propto 1/a^3$

(2)  $n^2 \langle \sigma v \rangle \gg 3Hn \Rightarrow n \rightarrow n_{eq} \Rightarrow n$  follows equilibrium distribution

Crossover occurs at  $n \langle \sigma v \rangle \sim H$  - decoupling criterion

For annihilation, at early times  $n \sim n_{eq}$ , when  $n \langle \sigma v \rangle \sim H$  then  $n$  stops tracing  $n_{eq}$  and  $na^3 \sim \text{constant}$  - we say the comoving abundance freezes out.

(Note: if the initial condition is far from  $n_{eq}$  then  $n \rightarrow n_{eq}$  early on - if the decoupling happens before  $n$  reaches  $n_{eq}$  we say the DM is "freezing in".)

Suppose freezeout occurs at  $T = T_f$ , then for  $T \leq T_f$

$$\text{we can write } n \sim \frac{n_f a_f^3}{a^3} \sim \frac{n_{eq,f} a_f^3}{a^3} \sim n_{eq,f} \left(\frac{T}{T_f}\right)^3$$

$f$  subscripts indicate freezeout

$n$  follows  $n_{eq}$  down to  $T_f$

ignoring  $g_{DM}$  changes - OK for rough estimates

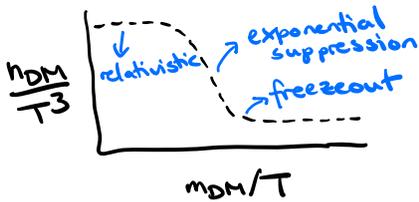
Two major cases to consider:  $T_f \gg m_{DM}$  (freezeout while relativistic) &  $T_f \ll m_{DM}$  (freezeout while non-relativistic)

First case: "hot relic" ( $T_f \gg m_{DM}$ )

- $n_{eq,f} \sim T_f^3 \Rightarrow n \sim n_{eq,f} \left(\frac{T}{T_f}\right)^3 \sim T^3$  after freezeout
- abundance similar to photon,  $\sim 10^9 \times$  baryon no. density
- only  $6 \times$  baryon mass density  $\Rightarrow$  mass must be  $\sim 10^8 \times$  baryon mass  $\sim O(10 \text{ eV})$
- will overproduce DM if  $m_{DM} \gtrsim 10 \text{ eV}$  - but for  $m_{DM} \leq 10 \text{ eV}$ , temperature implies too high velocity to form small halos, can be ruled out (this is the "hot dark matter" scenario)

Second case: "cold relic" ( $T_f \ll m_{DM}$ )

- DM is non-relativistic at freezeout,  $n_f \sim (m_{DM} T_f)^{3/2} e^{-m_{DM}/T_f}$
- $n$  exponentially drops as  $T$  falls until freezeout occurs.



The rapid suppression of  $n$  for  $T \leq m_{DM}$  drives  $n \langle \sigma v \rangle$  quickly below  $H$   
 $\rightarrow T_f \sim m_{DM}$  up to log factors  
 more correctly,  $T_f \sim m_{DM}/20$  so  $e^{-m_{DM}/T_f} \sim 10^{-9}$  gives roughly  $\eta$ , for DM similar in mass to baryons

Freezeout depends on size of  $\langle \sigma v \rangle$

- let us estimate what  $\langle \sigma v \rangle$  we need to get the correct relic density

- we can parameterize DM/matter density by  $T$  at matter-radiation equality, denote  $T_{MRE}$

- we want  $m_{DM} n_{DM}(T_{MRE}) \sim T_{MRE}^4$   
matter density radiation density assuming freezeout occurs before MRE

$\Rightarrow m_{DM} n_{eq,f} \left(\frac{T_{MRE}}{T_f}\right)^3 \sim T_{MRE}^4$

But also  $n_{eq,f} \langle \sigma v \rangle \sim H_f \sim \frac{T_f^2}{M_{Pl}}$  by definition of freezeout

$\Rightarrow m_{DM} \frac{T_f^2}{M_{Pl} \langle \sigma v \rangle} \times \frac{1}{T_f^3} \sim T_{MRE} \Rightarrow \langle \sigma v \rangle \sim \left(\frac{m_{DM}}{T_f}\right) \frac{1}{M_{Pl} T_{MRE}}$

For  $T_f \sim m_{DM}$ ,  $\Rightarrow \langle \sigma v \rangle \sim \frac{1}{(100 \text{ TeV})^2} \sim 10^{-27} \text{ cm}^3/\text{s}$   $\sim 10^{19} \text{ GeV}$   $\sim 1 \text{ eV}$

Picks out a characteristic cross section - more careful calculation gives  $\langle\sigma v\rangle \approx 2-3 \times 10^{-26} \text{ cm}^3/\text{s}$ , weak dependence "thermal relic cross section" on  $m_{\text{DM}}$

- in a weakly-coupled theory, tree-level annihilation expected to be parametrically  $\langle\sigma v\rangle \sim \frac{\alpha^2}{m_{\text{DM}}} \Rightarrow m_{\text{DM}} \sim \alpha(100 \text{ TeV})$
- for weak-scale couplings  $\alpha \sim 10^{-2}$ , predicts  $m_{\text{DM}} \sim 1 \text{ TeV}$ 
  - "WIMP miracle" (WIMP = Weakly Interacting Massive Particle)
- $\alpha \leq 1 \Rightarrow m_{\text{DM}} \leq 100 \text{ TeV}$

More formally, unitarity bounds on the maximum annihilation cross-section for a given mass require  $m_{\text{DM}} \leq 200 \text{ TeV}$  to get correct  $\langle\sigma v\rangle$ , e.g. [arXiv:1904.11503](#)

(Note: this limit can relax if the DM is composite, or if there is a non-standard cosmological evolution that dilutes its abundance)

- works OK down to  $m_{\text{DM}} \sim 1 \text{ MeV}$ , when bounds on new species relativistic during BBN become relevant
- if BBN limits are evaded, can extend down to  $m_{\text{DM}} \sim$  several keV, where "warm dark matter" bounds on velocity become relevant - in thermal case,  $m_{\text{DM}} \rightarrow T_f \rightarrow$  temperature at late times,  $\Rightarrow$  can recast upper limit on velocity as lower bound on mass,  $m_{\text{DM}} \gtrsim 5 \text{ keV}$  ([arXiv:1702.01764](#), [1908.06983](#), [1911.02663](#))

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In any case, this predicts a specific annihilation signal:

$\text{DM} + \text{DM} \rightarrow X$  may still occur today.

↓  
could be photons, neutrinos,  
stable charged particles,  
or SM particles that decay  
to any of the above

This argument can be easily generalized to the

case with  $N$  DM particles in the initial state:

Demand as before  $m_{DM} n_{eq,f} \left( \frac{T_{HRE}}{T_f} \right)^3 \sim T_{HRE}^4$

But now  $n_{eq,f}^{N-1} \langle \sigma v^{N-1} \rangle \sim H_f \sim \frac{T_f^2}{m_{PL}}$

Now taking  $T_f \sim m_{DM}$  for an order-of-magnitude estimate as before,  $\Rightarrow \langle \sigma v^{N-1} \rangle \sim \frac{m_{DM}^2}{m_{PL} (T_{HRE} m_{DM}^2)^{N-1}}$

If parametrically we write  $\langle \sigma v^{N-1} \rangle \sim \frac{\alpha^N}{m_{DM}^{3N-4}}$ ,

$$\Rightarrow \frac{\alpha^N}{m_{DM}^{3N-4}} \sim \frac{1}{m_{PL} T_{HRE}^{N-1} m_{DM}^{2N-4}} \Rightarrow m_{DM} \sim (\alpha m_{PL} T_{HRE}^{N-1})^{1/N}$$

Thus higher  $N$  leads to lower characteristic mass scales. Many other variations are also possible - e.g. with velocity-dependent rates, or other states with non-trivial evolution mediating the interactions, or asymmetric DM, etc.

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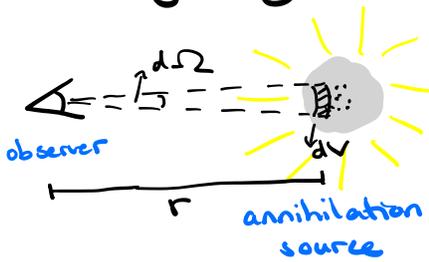
What would such an annihilation signal look like spatially?

- Photons/neutrinos travel on geodesics
- Charged particles are more complicated
- need to account for propagation effects
- Focus on 1st case (for purposes of this lecture)

## Directional information

For photons/neutrinos, we can look for signals from regions of high DM density. We can parametrize the signal

strength by the "J-factor" (annihilation)/"D-factor" (decay).



- Suppose our telescope at Earth has area  $dA$  and observes a solid angle  $d\Omega$  on the sky

- Consider the signal flux from a volume element  $dV = r^2 dr d\Omega$  at distance  $r$ .

- Annihilation (decay) rate in that volume is  $\frac{n^2 \langle \sigma v \rangle}{2} dV$  ( $n \Gamma dV$ )

- Suppose each annihilation/decay produces a photon/neutrino spectrum  $\Phi(E) = \frac{dN_{\gamma, \nu}}{dE}$

- Our telescope at Earth sees  $\frac{dA}{4\pi r^2}$  of the photons/neutrinos produced

$$\Rightarrow \frac{dN_{\text{observed}}}{dE dt} = \int dr \frac{dA}{4\pi r^2} \times r^2 d\Omega \times \Phi(E) \times \begin{cases} \frac{n(r)^2 \langle \sigma v \rangle}{2} & (\text{ann}) \\ n(r) \Gamma & (\text{decay}) \end{cases}$$

We neglect redshifting for now

$$\Rightarrow \frac{dN_{\text{obs}}}{dE dt d\Omega dA} = \Phi(E) \begin{cases} \frac{\langle \sigma v \rangle}{8\pi m_{\text{DM}}^2} \int \rho(r)^2 dr & (\text{ann.}) \\ \frac{\Gamma}{4\pi m_{\text{DM}}} \int \rho(r) dr & (\text{decay}) \end{cases}$$

here assuming  $\Phi, \langle \sigma v \rangle, \Gamma$  are position-independent

particle physics
astrophysics - mass density of DM is measured by gravitational probes

J-factor
D-factor

Note: some references include  $1/8\pi$  or  $1/4\pi$  factors in J/D-factors, others do not. Always check conventions!

These are differential J/D-factors, sometimes written  $dJ/d\Omega, dD/d\Omega$ ; we can integrate over  $d\Omega$  to get the total J/D-factor,  $J = \int \rho^2 dr d\Omega, D = \int \rho dr d\Omega$ .

Including redshifting: many searches are for targets at  $z \ll 1$  where cosmic expansion can be neglected, but when this is not the case, our signal prediction becomes:

$$\frac{dN_{\text{obs}}}{dE dA dt} = \int \frac{d\Omega}{4\pi} \int dz \frac{\Phi(E' = E(1+z))}{H(z) (1+z)^3} \begin{cases} \frac{\langle \sigma v \rangle}{2 m_{\text{DM}}^2} \rho^2(z, \theta, \phi) & (\text{ann}) \\ \frac{\Gamma}{m_{\text{DM}}} \rho(z, \theta, \phi) & (\text{dec}) \end{cases}$$

converts integral over  $r/t$  to  $z$ 
accounts for dilution of particles w. expansion

To unpack this:

- to see particles of energy  $E$  at  $z=0$ , need to start with particles at energy  $E' = E(1+z)$
- $H(z) = \frac{1}{a} \frac{da}{dt'} = \frac{d \ln a}{dt'} = -\frac{d \ln(1+z)}{dt'} = -\frac{1}{1+z} \frac{dz}{dt'}$
- $\Phi(E')$  so to convert from  $\frac{1}{dE'}$  to  $\frac{1}{dE}$ , need to multiply by  $\frac{dE'}{dE} = 1+z$
- $N$  particles injected at redshift  $z$  are diluted by a factor of  $\frac{1}{(1+z)^3}$  compared to particles injected today

- Overall: 
$$\frac{dN_{\text{obs}}}{dE dA dt} = \int \frac{d\Omega}{4\pi} \int_{t_{\text{init}}}^{t_{\text{today}}} \frac{c dt'}{H(z)} \Phi(E') \frac{dE'}{dE} \times \frac{1}{(1+z)^3}$$

$\times \begin{cases} \frac{\langle \sigma v \rangle}{2m_{\text{DM}}^2} \rho^2(t', \theta, \phi) \text{ (ann)} \\ \frac{\Gamma}{m_{\text{DM}}} \rho(t', \theta, \phi) \text{ (dec)} \end{cases}$

$= \int \frac{d\Omega}{4\pi} \int_0^{z_{\text{max}}} \frac{1}{1+z} \frac{dz}{H(z)} \Phi(E') \frac{(1+z)}{(1+z)^3}$

$\times \begin{cases} \frac{\langle \sigma v \rangle}{2m_{\text{DM}}^2} \rho^2(t', \theta, \phi) \text{ (ann)} \\ \frac{\Gamma}{m_{\text{DM}}} \rho(t', \theta, \phi) \text{ (dec)} \end{cases}$

To also include absorption of particles on their way to us, we can insert a  $e^{-\tau(E, z)}$  term inside the integral.

- In simulations with only DM & no baryons, density profiles of halos can be well-described by Navarro-Frenk-White (NFW) or Einasto density profiles:

NFW: 
$$\rho(r) = \frac{\rho_0 (r/r_s)^{-1}}{(1 + r/r_s)^2}$$

$r_s$  = scale radius,  $\frac{d \ln \rho}{d \ln r} = -2$   
 Region  $w/r \sim r_s$  = flat part of rotation curve

For Milky-Way-sized galaxies,  $r_s \sim 20 \text{ kpc}$   
 At  $r \ll r_s$ ,  $\rho(r) \propto 1/r$ ; for  $r \gg r_s$ ,  $\rho(r) \propto 1/r^3$ .

$$\text{Einasto: } \rho(r) = \rho_{-2} \exp\left[-\frac{2}{\alpha} \left(\left(\frac{r}{r_2}\right)^\alpha - 1\right)\right]$$

$r_{-2}$ :  $\frac{d \ln \rho}{d \ln r} = -2$   
 analogous to  $r_s$

$\alpha$  estimated to be  $\alpha \approx 0.17$  for MW-sized halos

Let's do an estimate of signal size.

$$\frac{dN_{\text{obs}}}{dE dA dt} = \Phi(E) \frac{\langle \sigma v \rangle}{8\pi m_{\text{DM}}^2} J$$

Let's imagine we had a  $\gamma$ -ray line signal,

$$\Phi(E) = 2 \delta(E - m_{\text{DM}}), \text{ let's pick } m_{\text{DM}} = 100 \text{ GeV,}$$

&  $x_{\text{sec}} = 10^{-2} \times \text{thermal relic}$

$$= 2 \times 10^{-28} \text{ cm}^3/\text{s}$$

Suppose  $1 \text{ m}^2$  telescope, 1 year observation time

$$\Rightarrow N_{\text{obs}} = \int dE 2 \delta(E - m_{\text{DM}}) \times 1 \text{ m}^2 \times 3 \times 10^7 \text{ s} \\ \times 2 \times 10^{-28} \text{ cm}^3/\text{s} \times \frac{1}{8\pi (100 \text{ GeV})^2} \times J$$

$$\text{from GC} = 2 \times 10^4 \text{ cm}^2 \times 3 \times 10^7 \text{ s} \times 2 \times 10^{-28} \text{ cm}^3/\text{s}$$

$$\times \frac{10^{22} \text{ GeV}^2}{\text{cm s}} \times \frac{1}{8\pi} \times 10^{-4} \text{ GeV}^{-2}$$

$$\approx 10^{12} \times 10^{-6} \times \frac{1}{25} \times 10^{-4} \approx 4 \text{ photons}$$

Small signal - but not zero.

Uncertainties in DM distribution are a challenge,

but often a more severe challenge is...

## Astrophysical backgrounds

- Background-free channels: gamma-ray spectral lines / sharply peaked spectra, antideuterons, heavier antinuclei (lower-energy lines have backgrounds from atomic/nuclear transitions)
- Continuum photons & charged particles are produced abundantly - cosmic-ray electrons yield broad-spectrum EM signals via ICS & synchrotron; cosmic-ray protons collide with the gas & yield showers of particles including  $\pi^0$ 's ( $\rightarrow \gamma\gamma$ ), charged leptons, neutrons, antiprotons, etc
- Stars & thermal emission from our Galaxy are relevant at lower energies; supernovae, pulsars, active Galactic nuclei produce  $\gamma$ -rays & high-energy charged particles

All backgrounds are worse in Galactic plane & toward Galactic Center - dwarf satellite galaxies are relatively clean