

Goals:

- Discuss some ways that long-range forces/light mediators can affect dark matter phenomenology
 - Sommerfeld enhancement
 - Bound state formation

Last two lectures: somewhat generic search strategies for DM signals

How much does the model matter?

Lots of attention in recent years to dark sectors

- DM interacts with one or more partner particles,

one or more of which may interact with SM particles

- Lots of rich phenomenology

- One sub-case I find especially interesting -
dark sector with light or strongly-interacting force
carriers \rightarrow potentially observable dark-sector

interactions, enhanced annihilation, bound states

- Already a natural example in classic models of
weakly-interacting DM - for sufficiently heavy DM,
W & Z bosons can serve as "light force carriers".

Sommerfeld enhancement

- presence of a long-range attractive potential enhances annihilation for (kinetic energy)
 \leq (potential energy)
- can study via the behavior of 2-particle wavefunction $\psi(\vec{r})$ as $\vec{r} \rightarrow 0$ - describes enhancement of short-distance interactions

(assuming annihilation is short-distance)
e.g. for s-wave annihilation (\mathcal{L} momentum-independent),
enhancement factor is $|\psi(0)|^2$.

- for a Coulomb-like potential, $V(r) \sim \frac{\alpha_D}{r}$,
enhancement occurs for $mv^2 \leq \alpha_D^2 m$ i.e.
 $v \leq \alpha_D$. For $v_{\text{rel}} \ll \alpha_D$, $\langle v_{\text{rel}} \rangle = S \langle v_{\text{rel}} \rangle_0$
with $S \approx \frac{2\pi \alpha_D}{v_{\text{rel}}}$ for s-wave
annihilation ↓
tree-level
result
- For higher L , $S \propto \left(\frac{\alpha_D}{v_{\text{rel}}}\right)^{2L+1}$

- usually less constrained to consider a force carrier with finite mass $m_A \rightarrow$ Yukawa potential
 - still a large enhancement for $\frac{1}{m_A} \gtrsim$ Bohr radius,
i.e. $m_A \leq \alpha_D m_X$
 - if this condition is satisfied, enhancement is Coulomb-like for $v_{\text{rel}} \gtrsim \frac{m_A}{m_X}$, but
saturates for $v_{\text{rel}} \leq \frac{m_A}{m_X} \leq \alpha_D$.
 - at special values of $m_A/\alpha_D m_X$, resonances occur - zero-energy bound state →
 $S =$ scales as $\frac{1}{v_{\text{rel}}^2}$ instead of $\frac{1}{v_{\text{rel}}}$
 - can cause large enhancements of indirect signals at low temperatures/late times

Question: unitarity normally imposes

$$\sigma \leq \frac{4\pi}{k^2} (2L+1), \quad k = \text{momentum of initial-state DM particle}$$

$$= m_X v_{rel}/2 \text{ in non-relativistic limit}$$

$$\Rightarrow \sigma v_{rel} \leq \frac{4\pi}{m_X^2} (2L+1) \cdot \frac{4}{v_{rel}}$$

$\frac{1}{v_{rel}^2}$ scaling breaks unitarity on resonances?

Usual treatment: solve Schrödinger equation w/
desired long-range potential (often numerically)
to obtain $\psi(\vec{r})$

Standard BCs on SE: solution should be finite
at origin - no short-range physics

Unitarity \Rightarrow probability conservation in elastic
scattering, particles never disappear (i.e.
annihilate)

OK as a 1st-order expansion in annihilation coupling
- but on resonances, naive annihilation rate is
very large! Cannot ignore probability of this
outcome

Need to include short-range absorptive term when
solving for the wavefunction - can be viewed
as a modification to the short-distance BCs
for the Schrödinger equation

The same wavefunction calculation tells us about the scattering rate - can be of interesting (potentially observable) size, with non-trivial velocity dependence

- Bound states: unstable bound states can serve as an extra channel for annihilation. For Yukawa potential, the criterion for bound states to exist is $m_A \leq \alpha_Y m_X$ (same as for Sommerfeld enhancement)
- The criterion for radiative capture into bound states to be allowed = \exists a particle coupled to the DM with mass \leq binding energy + kinetic energy
- If the A is also the radiated particle (not always true), requires $m_A \leq \alpha_D^2 m_X$ - more stringent condition
- If allowed, dominant transition rate is single-photon dipole transition. For positronium, $\langle \sigma v_{rel} \rangle \propto \frac{\alpha^3}{m_e^2 v_{rel}} = \left(\frac{\alpha}{v_{rel}}\right) \frac{\alpha^2}{m_e^2}$, same scaling as SE-enhanced annihilation, factor of 3 larger
- Some transitions are forbidden by selection rules - e.g. if DM is a Majorana fermion, XX must be an antisymmetric state

\Rightarrow constraints $L+S$ quantum number to be even
 But dipole transition has $\Delta L=1, \Delta S=0$
 \Rightarrow Cannot have $XX \rightarrow XX$ (bound) dipole transition

Commonly-used benchmark: pseudo-Dirac DM

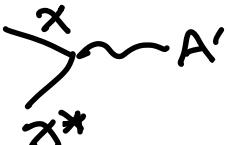
Two DM-like states, χ & χ^* , with $m_{\chi^*} > m_\chi$

but $|m_{\chi^*} - m_\chi| \ll m_\chi$ - nearly degenerate

At $T \gg |m_{\chi^*} - m_\chi|$, χ & χ^* form Dirac fermion charged under dark U(1)

Gauge boson of dark U(1) is A'

Interactions:



Sommerfeld:
 - can be solved semi-analytically

The diagram shows a horizontal line for χ and a horizontal line for χ^* with a curly line labeled A between them, representing a virtual A -boson exchange.

Bound-state formation:
 bound state

The diagram shows a horizontal line for χ and a horizontal line for χ^* with a curly line labeled A' between them, representing a virtual A' -boson exchange. A curly brace labeled "bound state" groups the two fermion lines.

Similar physics: with DM,

$$\begin{array}{c} \chi^+, \chi^- \\ \chi^0 \end{array}$$

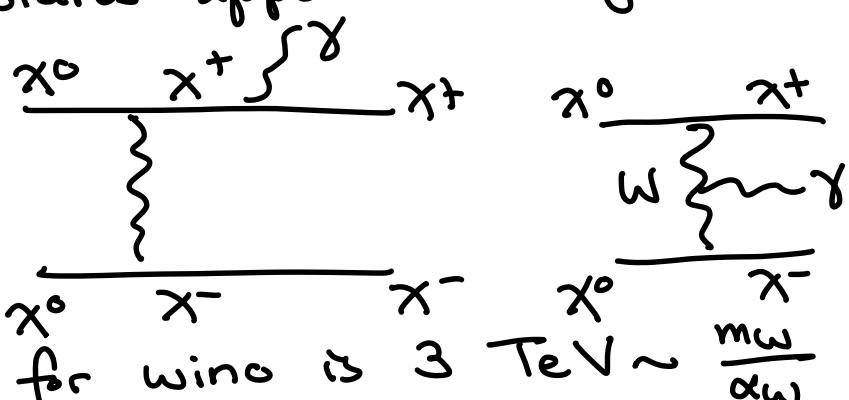
appears as W superpartner in SUSY models

Sommerfeld:

The diagram shows a horizontal line for χ^0 and a horizontal line for χ^- with a curly line labeled A between them, representing a virtual A -boson exchange. This exchange is shown interacting with a photon γ (curly line) and a χ^+ particle (wavy line).

Note: enhances $\chi^0 \chi^0 \rightarrow \gamma\gamma$ especially as charged states appear in diagrams

Bound-state formation:



Thermal mass for wino is $3 \text{ TeV} \sim \frac{m_W}{\alpha_W}$

Near threshold for 1st bound state

In fact \exists one BS, but as ground state, has $L=0$ - only accessible by dipole transition from $L=1$ initial state, suppressed at low velocities $V \leq \frac{m_W}{m_\chi}$

At higher masses, BS formation would become important - for SU(2) 5-plet, thermal mass is 14 TeV, high value is partly due to Sommerfeld / bound states enhancing annihilation during freezeout

At very high masses, can approximate SU(2) as unbroken $V(r)$ couples states, can be written as a matrix e.g. for wino, $V = \frac{\alpha_W}{r} \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 0 \end{pmatrix} \begin{matrix} \chi^+ \chi^- \\ \chi^0 \chi^0 \end{matrix}$

Subtlety: because of rules described above, this is only valid for $L+S$ even; $L+S$ odd cannot support $\chi^0\chi^0$ states, $V = \frac{\alpha w}{r}$ (for $\chi^+\chi^-$ states only)

For photon emission, only need to consider $\chi^0\chi^0$ & $\chi^+\chi^-$ states - with ω emission also $\chi^+\chi^0$ states.

Diagonalize potentials, finding:

$L+S$ even: attractive mode with $V = \frac{2\alpha w}{r}$

$L+S$ odd: " " " $V = \frac{\alpha w}{r}$

Leads to $L+S$ -dependent binding energies - $O(1)$ difference relative to hydrogen atom

(DM spectroscopy? But hard in practice.)

See e.g. 2007.13787

e.g. for w_{HO} , 1st few states are

4s singlet, 4d singlet, 2p singlet, 2s triplet, 4p triplet

3s singlet, 3d singlet, 3p triplet

2s singlet, 1s triplet, 2p triplet

1s spin-singlet