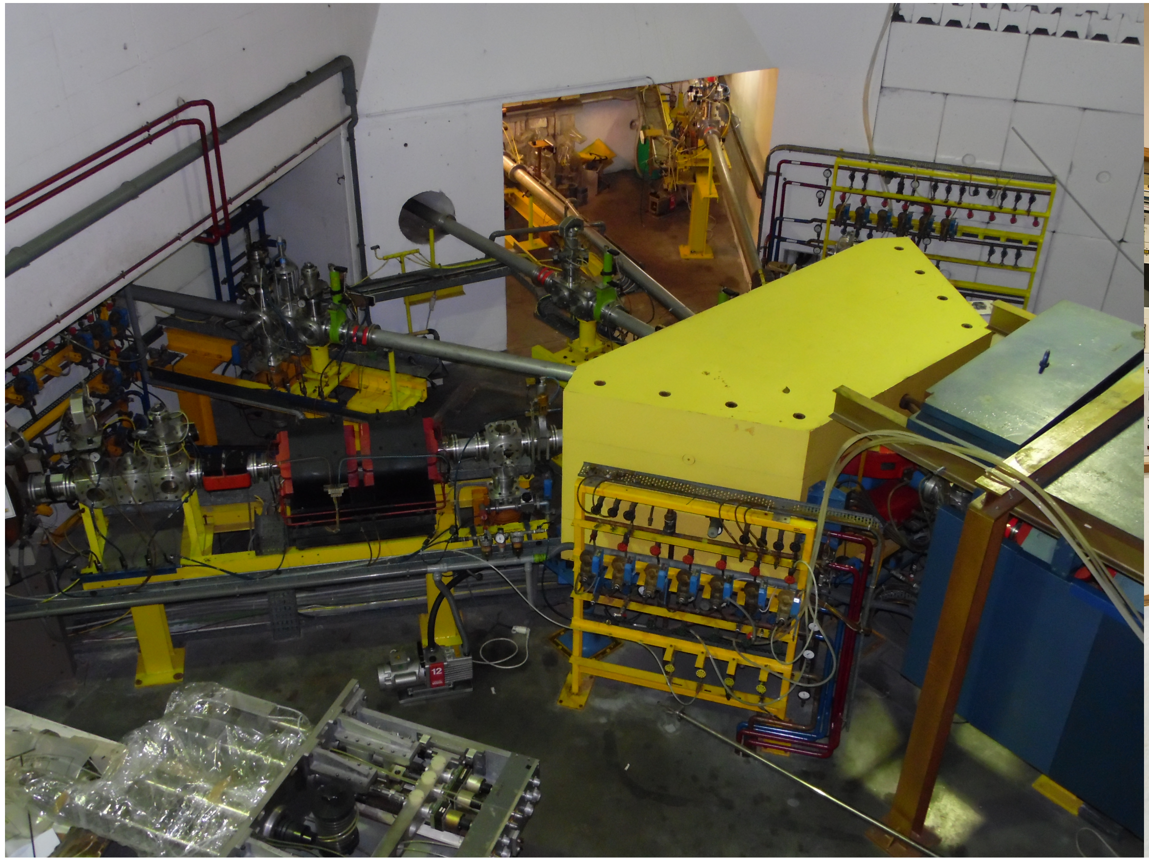
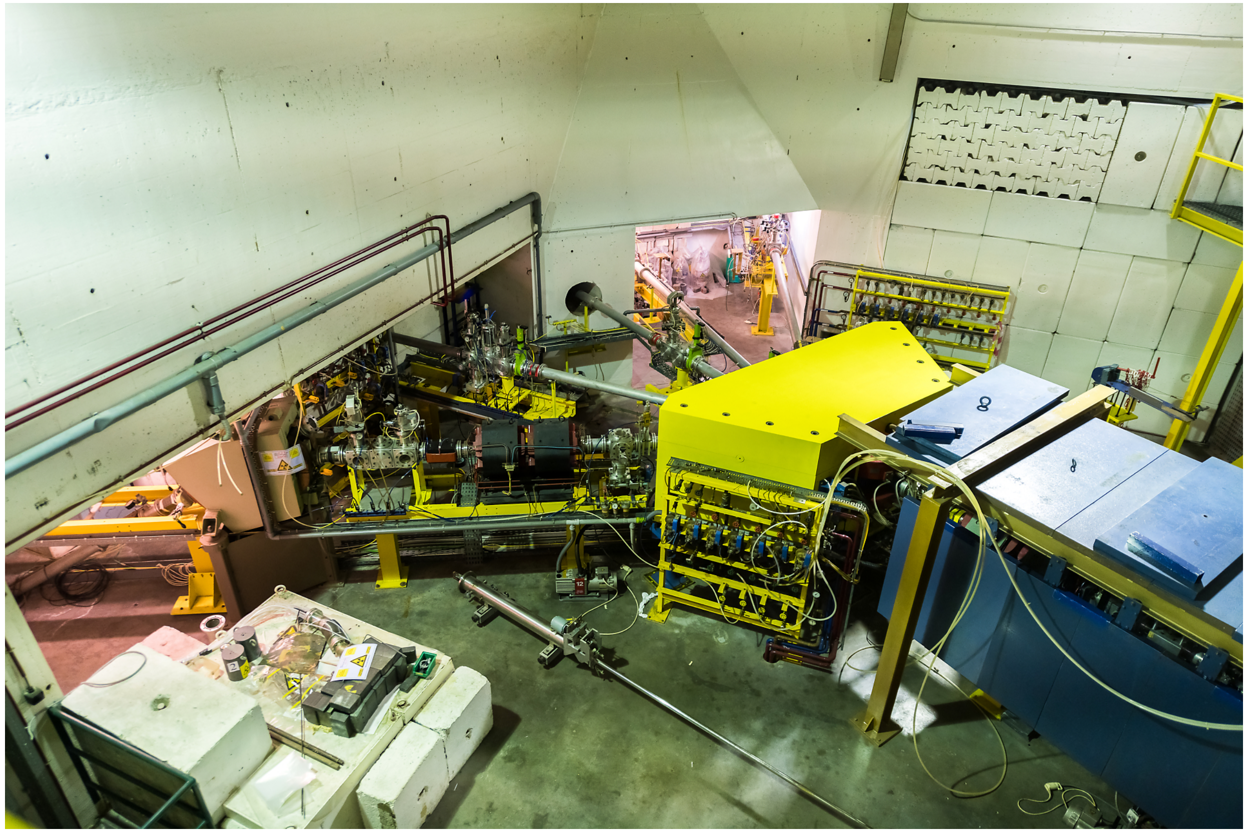
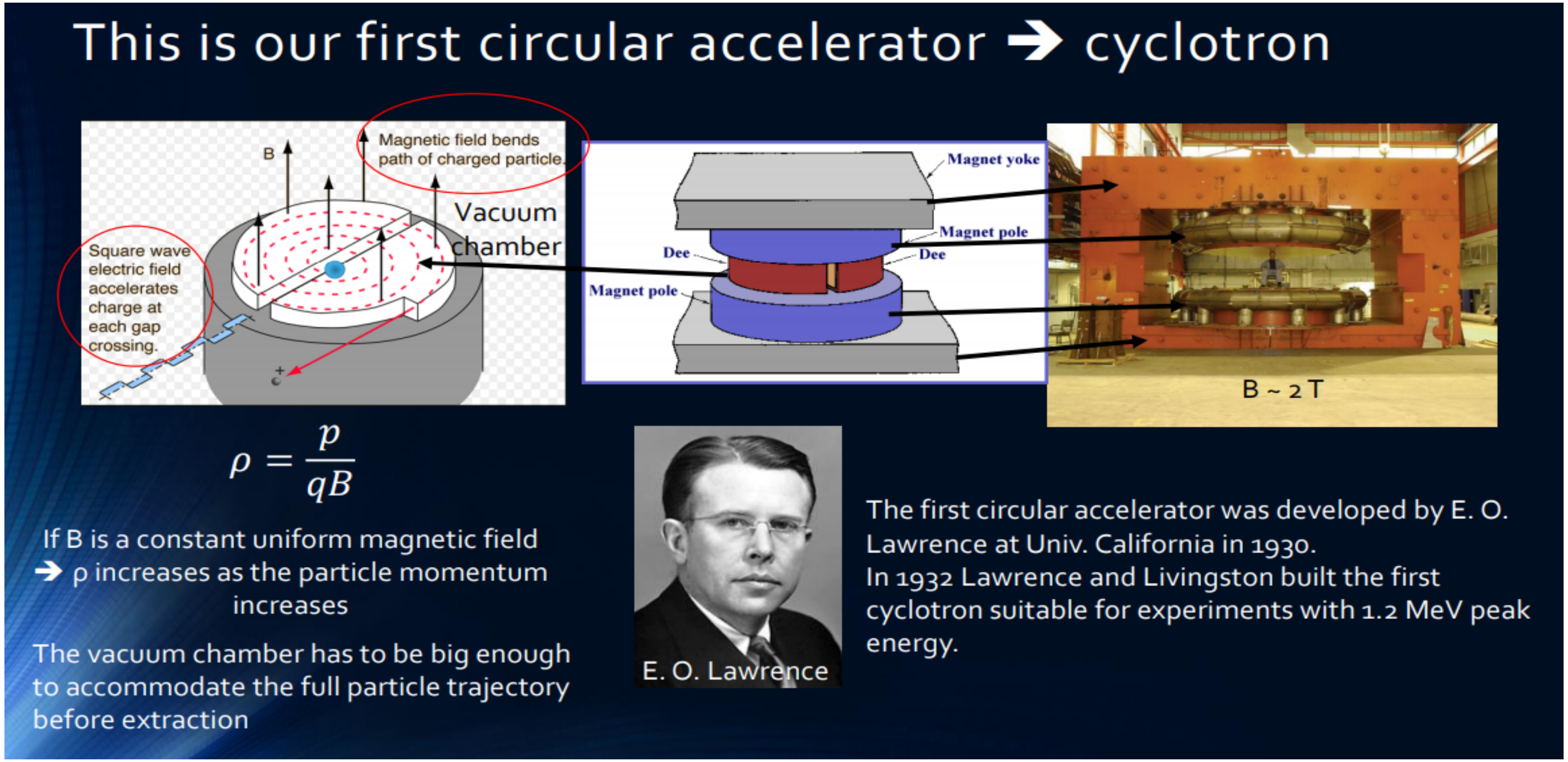
Characterization of the transfer line of the Louvain cyclotron



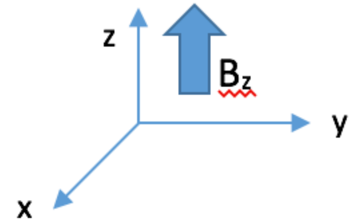


Cyclotron principle

Cyclotrons solved the problem of linear accelerators, i.e. in order to increase the energy of the particle in a linear accelerator longer and longer linacs are needed ➔ increase in cost and length. E.g. CLIC is 50 km linac. The best is to drive particles in a circular orbit and reuse the accelerating structure many times. The first circular accelerator was a **CYCLOTRON** proposed by Lawrence in 1930. The first built cyclotron had a kinetic energy of 1.2 MeV.



Let's derive the equation of motion in a cyclotron. The coordinate system is the following:



The magnetic field has only one component:

$$B = \begin{pmatrix} 0 \\ 0 \\ Bz \end{pmatrix}$$

We can get the equation of motion from the Lorentz force:

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = q(vxB) = q\begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ v_x & v_y & 0 \\ 0 & 0 & B_z \end{vmatrix} = q(\boldsymbol{v_y B_z}\vec{\imath} - \boldsymbol{v_x B_z}\vec{\jmath})$$

We assume that the motion is confined in the (x,y) plane, therefore:

$$p = \begin{pmatrix} p_x \\ p_y \\ 0 \end{pmatrix} = \begin{pmatrix} mv_x \\ mv_y \\ 0 \end{pmatrix} \Rightarrow \frac{dp}{dt} = \dot{p} = \begin{pmatrix} \boldsymbol{m\dot{v}_x} \\ \boldsymbol{m\dot{v}_y} \\ 0 \end{pmatrix}$$

$$\dot{p_x} = m\dot{v_x} = qv_yB_z$$

$$\dot{p_y} = m\dot{v_y} = -qv_xB_z$$

Differentiating the equations versus time we get:

$$\ddot{v_x} + \frac{q^2}{m^2} B_z^2 v_x = 0$$

$$\ddot{v_y} + \frac{q^2}{m^2} B_z^2 v_y = 0$$

with solutions

$$v_x(t) = v_0 cos\omega_z t$$

$$v_y(t) = v_0 sin\omega_z t$$

with

$$\omega_z = \frac{q}{m} B_z = \frac{q}{\gamma m_0} B_z$$

➔ **CYCLOTRON FREQUENCY OR LARMOR FREQUENCY**

The cyclotron frequency does not depend on the particle velocity. Since the particle is **NON-RELATIVISTIC**, γ = 1, and m = m_0, and since B is also constant, an increase of energy cannot be compensated by an increase of revolution frequency, therefore is compensated by an increase of bending radius, ρ. Therefore, the particle describes a spiral.

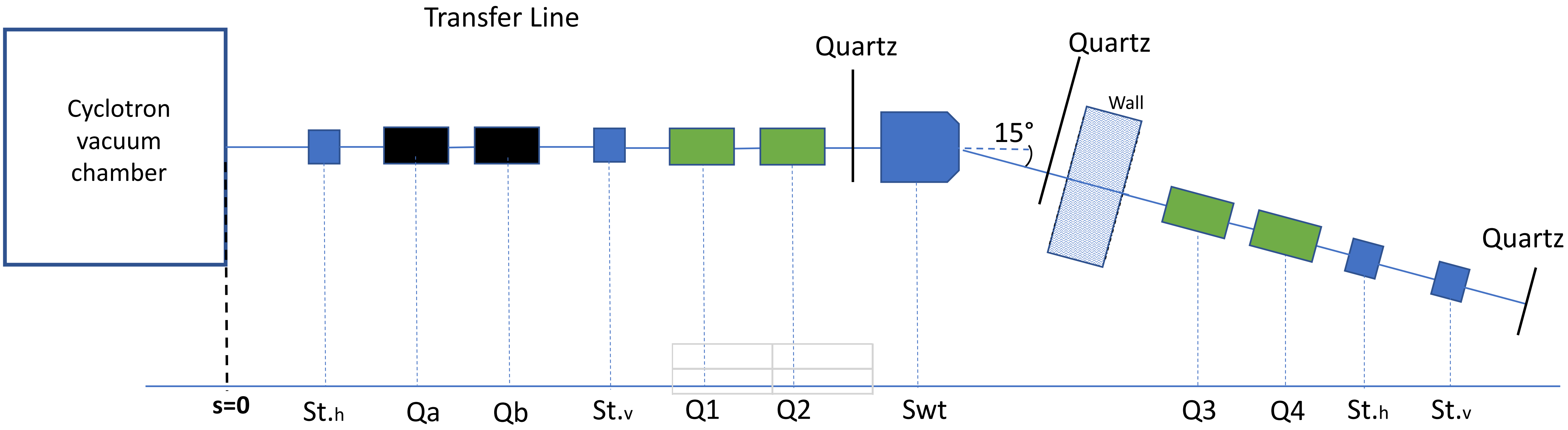
Synchronous condition ➔ $\omega_z = \omega_{RF}$ such every time the particle crosses the gap sees an accelerating electric field.

When the particle becomes **RELATIVISTIC** we cannot assume $m = m_0$, but $m=\Upsilon m_0$ which increases with energy. In this case, ω_Z, decreases and therefore, ω_{RF} will have to be decreased accordingly. This gives rise to the **SYNCHROCYCLOTRONS.**

$$\omega_z = \frac{q}{\gamma m_0} B_z \neq cte$$

SynchroCyclotrons only work for a limited range of energies and provide with pulsed beam, therefore they provide with less intensity than cyclotrons, but a bit more energy. The solution to this intensity limitation is the use of cyclotrons in which the magnetic field increases as the radius increases ➔ **ISOCYCLOTRONS**:

$$\omega_z = \frac{q}{\gamma m_0} B_z(\rho) = cte$$



Quadrupoles :

Qa, Qb : $l_{eff} = 310\ mm$
$B = 23\ [Gauss/A]\ = 23\ 10\text{-}4\ [T/A]$ at poles
$r = 50\ mm$

Q1, Q2, Q3, Q4 : $l_{eff} = 292\ mm$
$B = 76{,}4\ [Gauss/A]\ = 76{,}4\ 10\text{-}4\ [T/A]$
$r = 38{,}1\ mm$

Bending magnet length (for beam line R) :

Swt : $l_{eff} = 958{,}1\ mm$

Emittance as measured in the 90's

Values for a 65MeV proton beam at s=0

(P =momentum central path)

x	12,9	mm
x'	8	mr
y	1,5	mm
y'	8	mr
l	0	mm
d	0	%
P	0,355	GeV/c

Sequence Position of elements = position of center (rem: wall width unknown at this point, I took 1000 mm)

Elements	[mm]	Typical current used for protons (65MeV) [A]							
Ouput cyclo	0								
St1.h	355	not used							
Qa	982	121,3							
Qb	1397	128,8							
St2.v	2960	3,68							
Q1	4602	38,9							
Q2	5016	35,5							
quartz	5578								
Swt*	6492	85,6							
quartz	10878								
Q3	14880	29,8							
Q4	15264	31,6							
St3.h	16014	3,5							
St4.v	16403	0,6							
quartz	18608								

* The center for the bending magnet was considered as the half of the effective path length of a central particule in the bending magnet.

Question 1: quadrupole polarities

Using a magnetic probe, power each quadrupole of the line, one by one, to the same current, e.g. 30 A and find the north pole. Determine if the quadrupole is focusing or defocusing.

A **focusing quadrupole**, Figure 1, is defined when for a positive current injected into the circuit by the power converter, the north is on the top-right pole. For a positive charged particle coming into the picture, like an injected beam from a source, the force is focusing (blue lines) in the horizontal plane and defocusing (red lines) in the vertical plane.

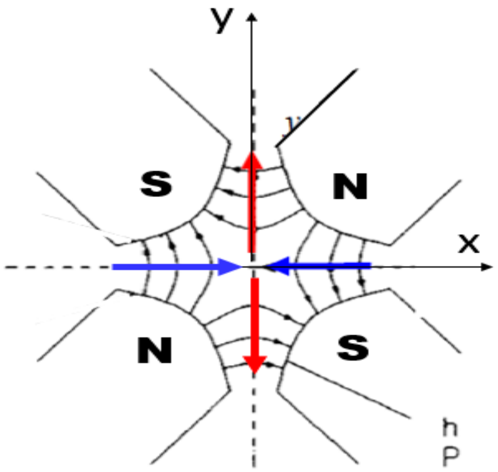


Fig. 1

A **defocusing quadrupole**, Figure 2, is defined when for a positive current injected into the circuit by the power converter, the south is on the top-right pole. For a charged particle coming into the picture, like an injected beam from a source, the force is defocusing (red lines) in the horizontal plane and focusing (blue lines) in the vertical plane.

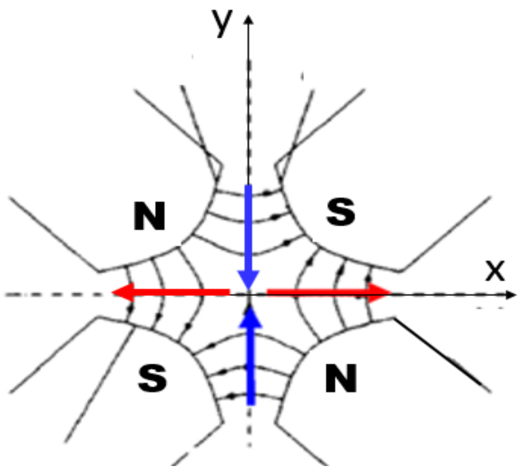


Fig. 2

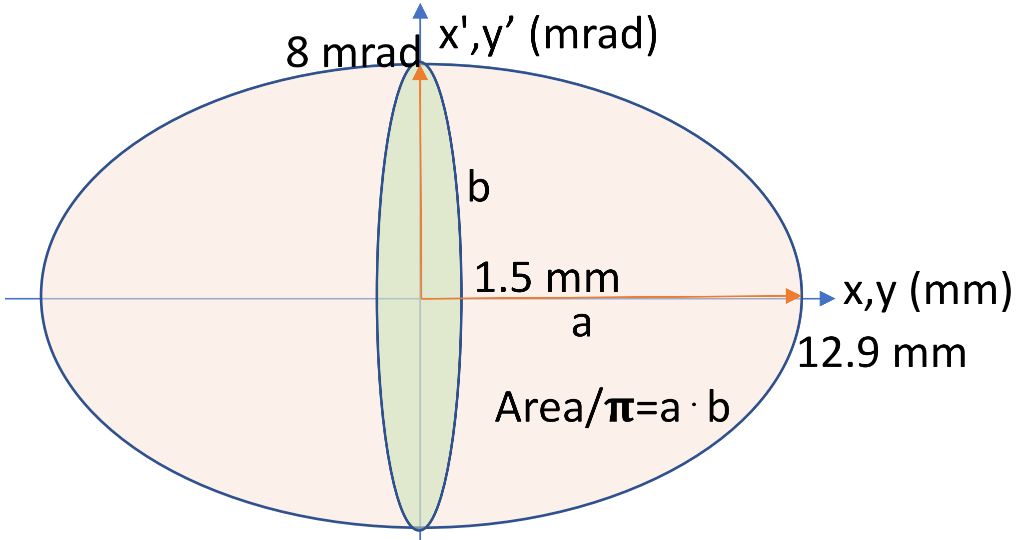
Question 2: determine the beam emittance

Emittance as measured in the 90's

Values for a 65MeV proton beam at s=0

(P =momentum central path)

x	12,9	mm
x'	8	mr
y	1,5	mm
y'	8	mr
l	0	mm
d	0	%
P	0,355	GeV/c



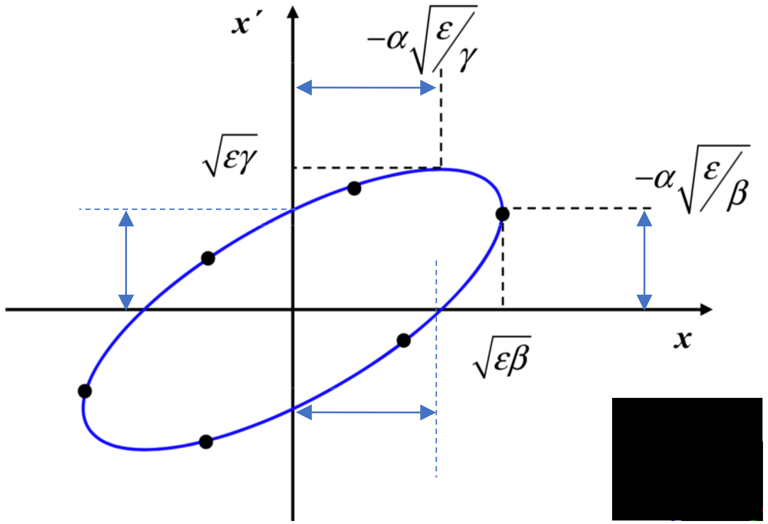
$\boldsymbol{\varepsilon}_x$=12.9 mm 8 mrad =103.2 mm mrad

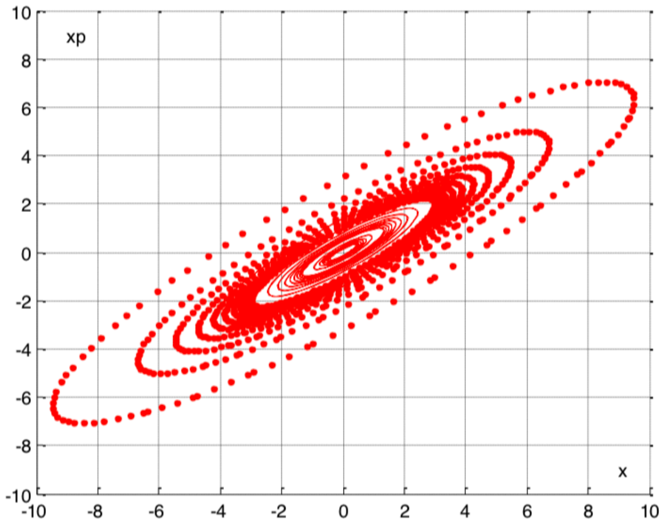
$\boldsymbol{\varepsilon}_y$=1.5 mm 8 mrad =12 mm mrad

Let's measure the emittance using the quadrupole scan method and compared with the measurements obtained in the 90's

Measurement of the beam emittance

$$\frac{Area}{\pi} = \varepsilon$$





We see that all particles travel along their individual ellipses in phase space. If we now choose one with the largest phase ellipse within a particular beam, we know all particles within that ellipse will stay within that ellipse. Therefore we are able to describe the collective behavior of a beam formed by many particles by the dynamics of a single particle.

Since all particles enclosed by a phase space ellipse stay within that ellipse, we only need to know how the ellipse parameters transform along the beam line to be able to describe the beam.

Let’s define the beam matrix with the well known Twiss parameters:

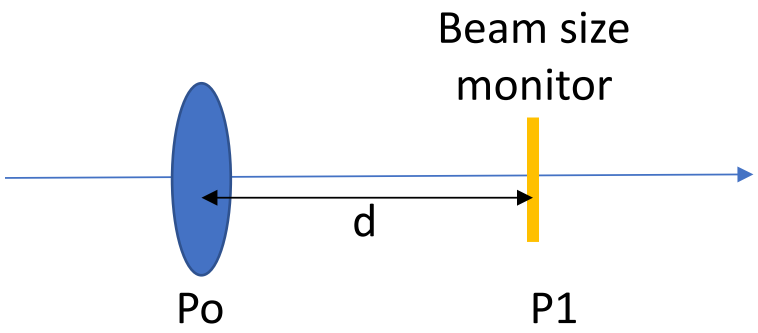
$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \varepsilon \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

$$\left.\begin{aligned} \sigma_{11} &= \langle x_i^2 \rangle = \varepsilon\beta \\ \sigma_{22} &= \langle x_i'^2 \rangle = \varepsilon\gamma \\ \sigma_{12} &= \langle x_i x'_i \rangle = -\varepsilon\alpha \end{aligned}\right\} \quad \varepsilon^2 = \sigma_{11}\,\sigma_{22} - \sigma_{12}^2$$

If we find a way to determine the beam matrix, then we can measure the emittance

QUADRUPOLE SCAN METHOD TO MEASURE BEAM EMITTANCE

To determine the beam matrix at a place Po, we consider a beam transport line with one quadrupole at Po and a beam size monitor at P1. We vary the strength of the quadrupole and measure the beam size at P1 as a function of the quadrupole strength. This is equivalent to measure the beam size at a different locations in the line.



$$\begin{pmatrix} 1 - d/f & d \\ -1/f & 1 \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$$

Matrix in thin lens

It can be demonstrated (H. Wiedemann, Particle Accelerator Physics, Chapter 5.1 Measurement of beam emittance) that form the beam matrix at P0, one can get the beam matrix element 11 at P1, i.e. the beam size at P1:

This is what we measure with the beam diagnostic device

This we vary in steps

$$\sigma_{1,11}(k) = \left(d^2\ell_q^2\sigma_{0,11}\right)k^2 + \left(-2d\,\ell_q\sigma_{0,11} - 2d^2\ell_q\sigma_{0,12}\right)k + \left(\sigma_{0,11} + 2d\,\sigma_{0,12} + d^2\sigma_{0,22}\right).$$

lq and d are known

a

c

b

Fitting $\sigma_{1,11}(k)$ to a parabola $y = ak^2 + bk + c$ will determine the whole beam matrix at P0

$$\sigma_{0,11} = \frac{a}{d^2\ell_q^2},$$

$$\sigma_{0,12} = \frac{-b - 2d\ell_q\sigma_{0,11}}{2d^2\ell_q},$$

$$\sigma_{0,22} = \frac{c - \sigma_{0,11} - 2d\sigma_{0,12}}{d^2}.$$

Geometrical emittance

$$\varepsilon^2 = \sigma_{11}\,\sigma_{22} - \sigma_{12}^2$$

Normalized emittance

$$\varepsilon_n = \gamma_{rel}\beta_{rel}\varepsilon$$

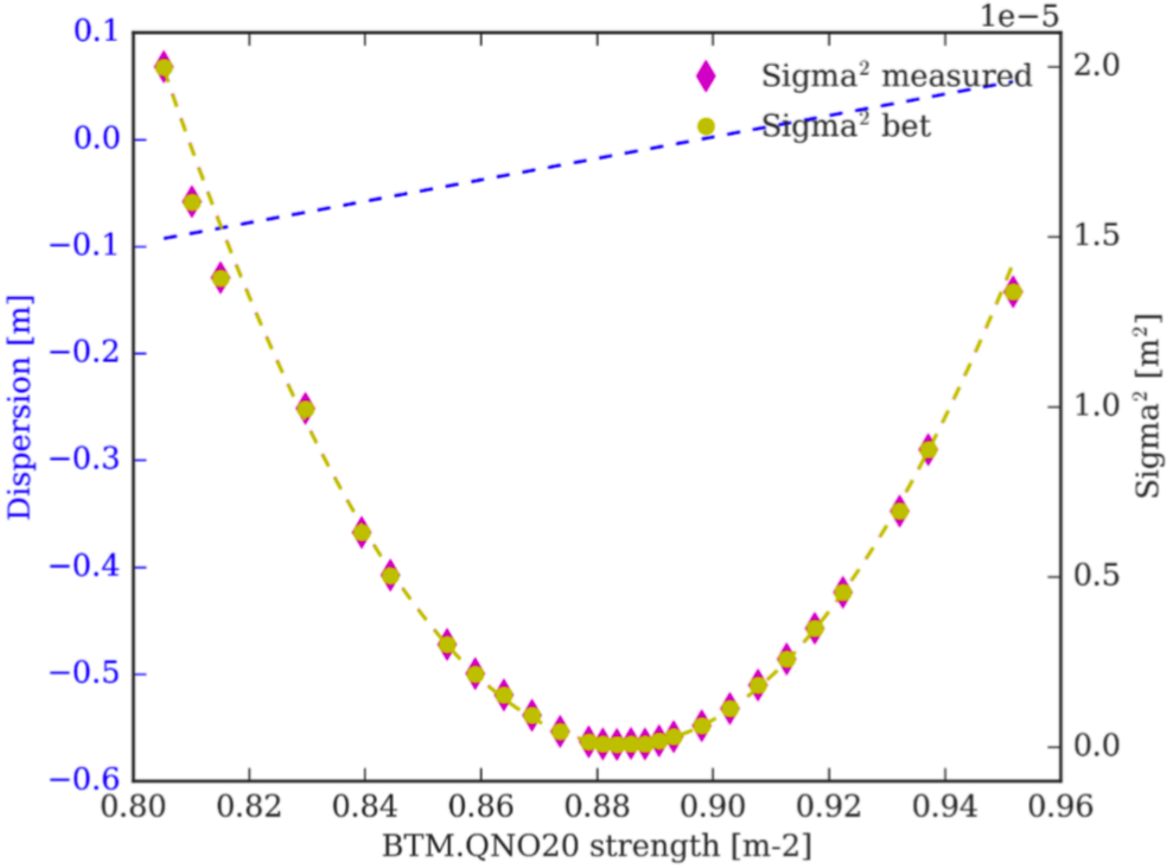
The beam matrix not only defines the beam emittance but also the betatron functions at the beginning of the quadrupole in this measurement. We gain with this measurement a full set of initial beam parameters $(\alpha_0, \beta_0, \gamma_0, \varepsilon)$ and may now calculate beam parameters at any point along the transport line.

$$\sigma_0 = \begin{pmatrix} \sigma_{0,11} & \sigma_{0,12} \\ \sigma_{0,21} & \sigma_{0,22} \end{pmatrix} = \varepsilon \begin{pmatrix} \beta_0 & -\alpha_0 \\ -\alpha_0 & \gamma_0 \end{pmatrix}$$

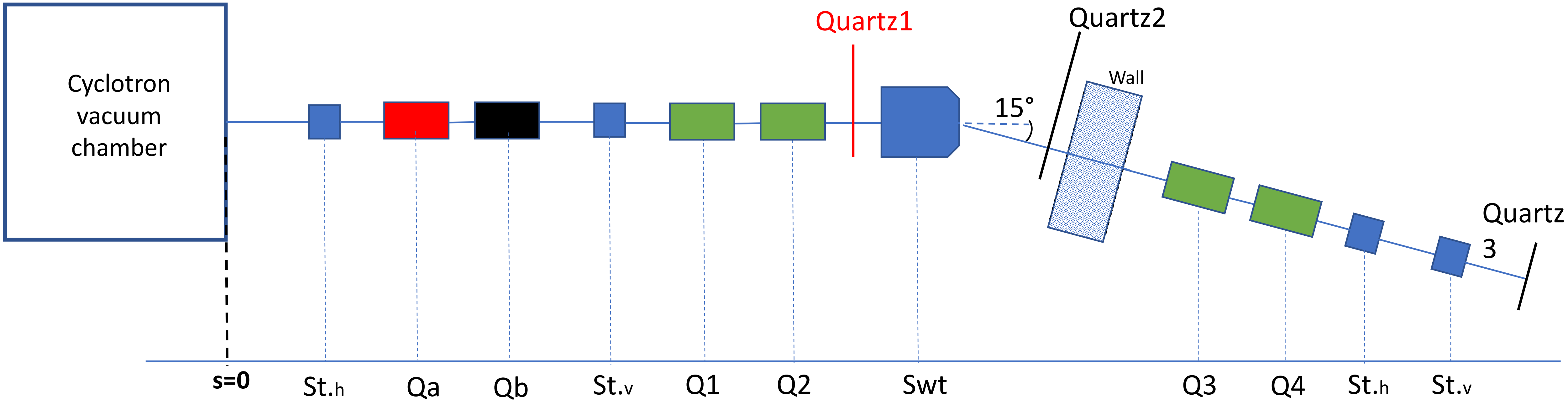
COMMENTS

- Chose setting with **focus closed to** the **SEM grid**
- **Careful at the focus** – beam very small and possible space charge effects
- Guarantee **large beam size variation with quadrupole strength**, to be able to accurately fit the 3 parameters.

Disp-free optics



Proposal: use the quadrupole scan method varying Qa and measuring the beam size in Quartz1 ➔ advantage: there is no dispersion. While doing the Qa scan you should keep Qb, Q1 and Q2 OFF.



Watch up! Once you obtain the Twiss parameters for Qa, you have to transport them back through the drift up to the entrance of the cyclotron, s_0.

Keep the steer magnets St.h/v OFF for all this exercise

Question 3: define the transfer line of the cyclotron in MADX

1. Define the cyclotron transfer line in MADX
2. Use the Twiss parameters at s_0 obtained via the quad scan method as initial conditions for the Twiss
3. Check the beta functions and the dispersion
4. Check the line trajectory with SURVEY command (plot x,y vs. z not s)
5. Plot the beam size along the transfer line. Assume a $\frac{\Delta p}{p_0} = 10^{-3}$ when there is dispersion.
6. Check that the demanded currents in the power converters are within the maximum and minimum allowed values.
7. If the beam size in some parts of the line is too large, you can constrain the beta function those regions, e.g.

```
# Several matching methods can be used: LMDIF, MIGRAD, SIMPLEX, JACOBIAN
myString='''
savebeta, label=betaEnd, place=#e;

MATCH, SEQUENCE=myCell, betx=10, bety=10;
constraint, betx=40, range=#e;
constraint, alfx=0, range=#e;
constraint, bety=40, range=#e;
constraint, alfy=0, range=#e;
constraint, betx<1000, range=#s/#e;
constraint, bety<1000, range=#s/#e;
VARY, NAME= k_Qa, STEP=0.00001;
VARY, NAME= k_Qb, STEP=0.00001;
VARY, NAME= k_Q1, STEP=0.00001;
VARY, NAME= k_Q2, STEP=0.00001;
VARY, NAME= k_Q3, STEP=0.00001;
VARY, NAME= k_Q4, STEP=0.00001;
JACOBIAN, CALLS=50, TOLERANCE=1e-20;//method adopted
ENDMATCH;

twiss,betx=10, bety=10;
'''
myMad.input(myString);
```