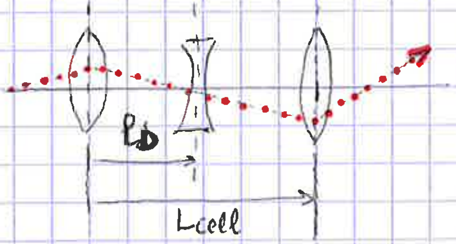
The formula $\overset{\Psi_{cell}}{\mu} = 2 \arcsin \frac{K L_{cell} \cdot L_{quad}}{4}$ $\mu = \Psi_{cell}$ = phase advance of the cell

can be obtained in the following way:

① We start from the matrix of a thin FODO:

$$M^{THIN}_{FODO} = \begin{pmatrix} 1 - \frac{2 l_D^2}{\tilde{f}^2} & 2 l_D \left(1 + \frac{l_D}{\tilde{f}}\right) \\ 2\left(\frac{l_D^2}{\tilde{f}^3} - \frac{l_D}{\tilde{f}^2}\right) & 1 - 2\frac{l_D^2}{\tilde{f}^2} \end{pmatrix} \qquad l_D = \frac{L_{cell}}{2} \qquad \tilde{f} = 2f$$

② The motion is stable if Trace(M) < 2



③ Let's calculate the trace and impose < 2:

$$\text{Trace}(M) = 2\left(1 - \frac{2 l_D^2}{\tilde{f}^2}\right) \qquad \text{Eq. 1}$$

④ The matrix of a periodic cell in full lens approximation is:

$$M_{FODO} = \begin{pmatrix} \cos\varphi_{cel} + \alpha_s \sin\Psi_{cell} & \beta_s \sin\Psi_{cell} \\ -\gamma_s \sin\Psi_{cell} & \cos\Psi_{cell} - \alpha_s \sin\Psi_{cell} \end{pmatrix}$$

⑤ For this matrix the motion is stable if Trace(M) < 2:

$$\text{Trace}(M) = \cos\Psi_{cell} < 2 \qquad \text{Eq. 2}$$

⑥ Since Eq. 1 = Eq 2:

$$\frac{1}{2}\text{Trace}(M) = \cos\Psi_{cell} = 1 - \frac{2 l_D^2}{\tilde{f}^2}$$

⑦ We can simplify the calculus by remembering that

$$\begin{cases} \cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \\ = 1 - 2 \cdot \sin^2\frac{x}{2} \end{cases}$$

⑧ $$\frac{1}{2}\text{Trace}(M) = \cancel{1} - \cancel{2} \cdot \sin^2\frac{\Psi_{cell}}{2} = \cancel{1} - \frac{\cancel{2} l_D^2}{\tilde{f}^2}$$

$$\boxed{\sin\frac{\Psi_{cell}}{2} = \frac{L_{cell}}{2f}} \Longleftarrow \sin\frac{\Psi_{cell}}{2} = \frac{l_D}{\tilde{f}} \Rightarrow \arcsin\frac{l_D}{\tilde{f}} = \frac{\Psi_{cell}}{2}$$

Eq. 3

$$\boxed{\Psi_{cell} = \mu = 2 \arcsin \frac{L_{cell} \cdot L_{quad} \cdot K}{4}}$$