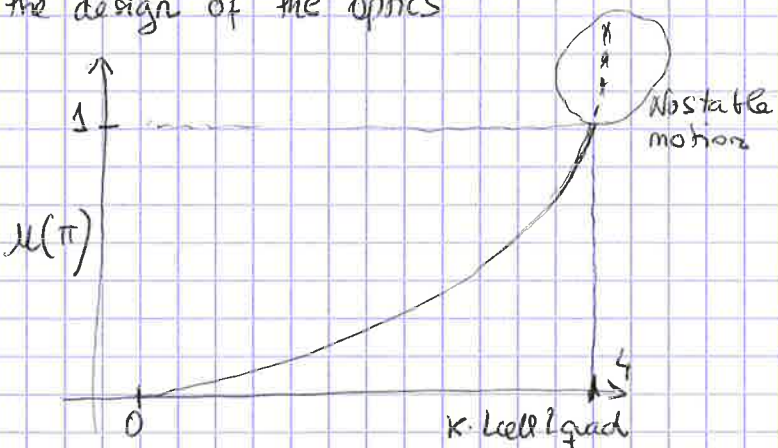
The phase advance of a periodic cell can be plotted as a function of the Lcell, Lquad, K to help the design of the optics

$$\mu = \psi_{cell} = 2 \arcsin \frac{K \cdot L_{cell} \cdot L_{quad}}{4}$$



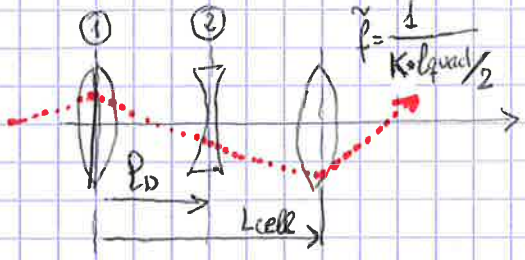
The phase advance and the β function are related by the following formula:

The maximum β function is: $\beta_{max} = \dfrac{\left(1 + \sin\frac{\psi_{cell}}{2}\right) L_{cell}}{\sin\psi_{cell}}$

The minimum β function is: $\beta_{min} = \dfrac{\left(1 - \sin\frac{\psi_{cell}}{2}\right) L_{cell}}{\sin\psi_{cell}}$

The demonstration of the two formulas above can be done in the following way:

① We start from the transfer matrix of a half FODO cell



$$M^{THIN}_{FODO-HALF} = \begin{pmatrix} 1 - \frac{l_D}{\tilde{f}} & l_D \\ -\frac{l_D}{\tilde{f}^2} & 1 + \frac{l_D}{\tilde{f}} \end{pmatrix}$$

② the matrix in Twiss parameters that transform the position in ① to a position in ② is:

$$M_{1\to 2} = \begin{pmatrix} \sqrt{\frac{\beta_2}{\beta_1}}\left(\cos\psi_{12} + \alpha_1 \sin\psi_{12}\right) & \sqrt{\beta_1\beta_2}\sin\psi_{12} \\ (\alpha_1 - \alpha_2)\cos\psi_{12} - (1 + \alpha_1\alpha_2)\sin\psi_{12} & \sqrt{\frac{\beta_1}{\beta_2}}\left(\cos\psi_{12} - \alpha_2 \sin\psi_{12}\right) \end{pmatrix}$$

③ The matrix above in the middle of a focusing (defocusing) quadrupole simplifies to $\Rightarrow \alpha = 0$