Introduction to transverse beam dynamics I

Restricted to the LINEAR BEAM OPTICS ➔ THE IDEAL WORLD

Content of the course

TBD 1

- Charge particle motion in a magnetic field
- Equations of motion → derivation and assumptions
- Type of magnets

TUTO 1

- Rigidity formula
- Relativistic equations
- Create a storage ring with the Earth Magnetic field

TBD 2

- Particle trajectory
- Transfer Matrices
- Thin lens approximation
- Betatron oscillations
- Betatron tune
- Dispersion

TUTO 2

- Application of transfer matrices
- Thin lens
- FoDo cell

TBD 3

- Phase space ellipse
- Emittance
- Beam size
- Aperture
- Beta function evolution
- Periodic lattices

TUTO 3

- Beam size and aperture calculations

TBD 4

- Effect field errors
- Resonances
- Coupling
- Chromaticity

TUTO 4

- How the tune changes from a quadrupole defect
- Optimize beta beating
- Orbit bumps

Charge particle motion in a magnetic field

- When we build an accelerator or storage ring, the NOMINAL TRAJECTORY of the particle BEAM is FIXED by design

- The trajectory is a straight line if it is a linear accelerator, or very complicated shape if it is a circular accelerator

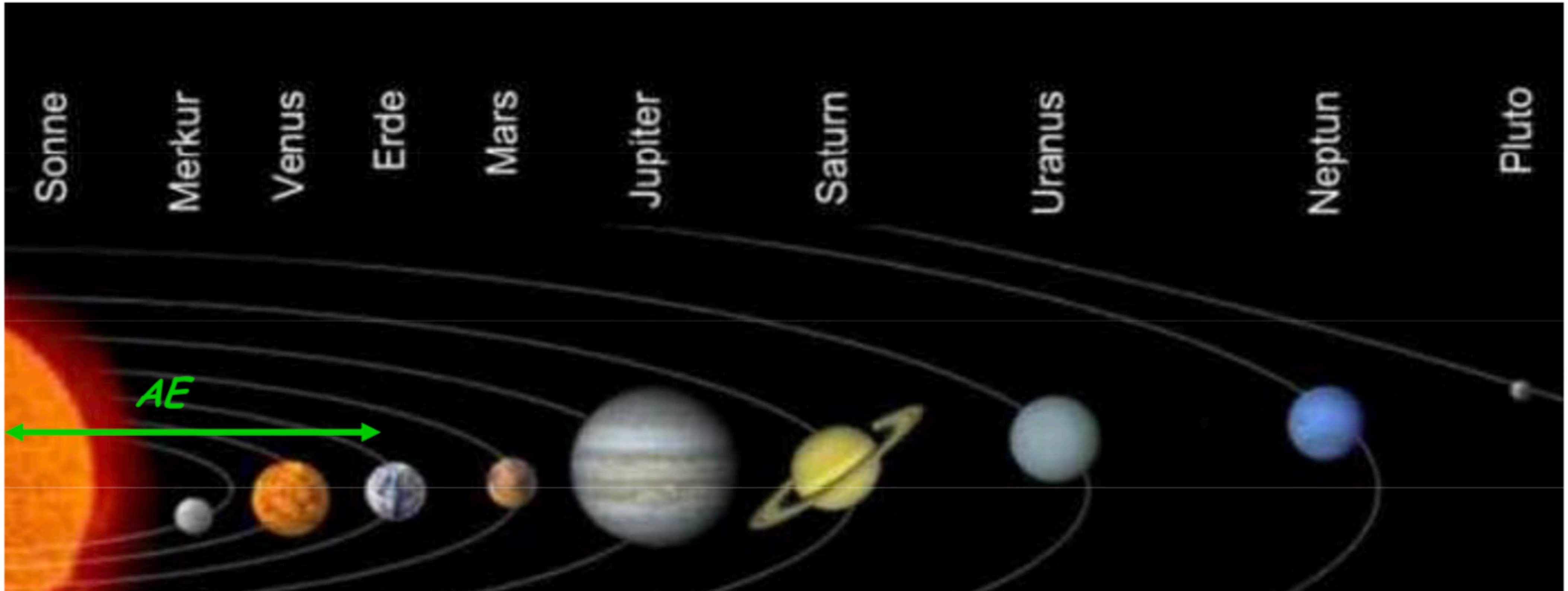
- The BEAM follows the resulting CLOSED PATH over and over again

- In LHC, 27 km circumference, the beams travel close to the speed of light for ~ 12 hours ➔ Do you have an idea of how many kilometers the beams travel?????

astronomical unit: *average distance earth-sun*

*1AE ≈ 150 *10⁶ km*

Distance Pluto-Sun ≈ 40 AE



Velocity of protons in LHC ≈ 300000 km/s

12 hours circulating = 43200 s

L= v·t ≈ 1.2 10^{10} km ≈ 86 AE

- The TRAJECTORIES of INDIVIDUAL PARTICLES in the beam always have a certain angular divergence, and without further measures, they would finally hit the wall of the vacuum chamber

- Therefore, the steps to follow are:
  1. Fix the NOMINAL trajectory
  2. Steer the divergent particles back to the NOMINAL trajectory

- To steer the particles we usually use MAGNETS ➔ why?

Lorentz Force $$F = q(\vec{E} + \vec{v} x \vec{B})$$

$$v \approx c \approx 3x10^8 m/s$$

Typical velocity in high energy machines

At relativistic energies, the electric field and the magnetic field have the same effect if:

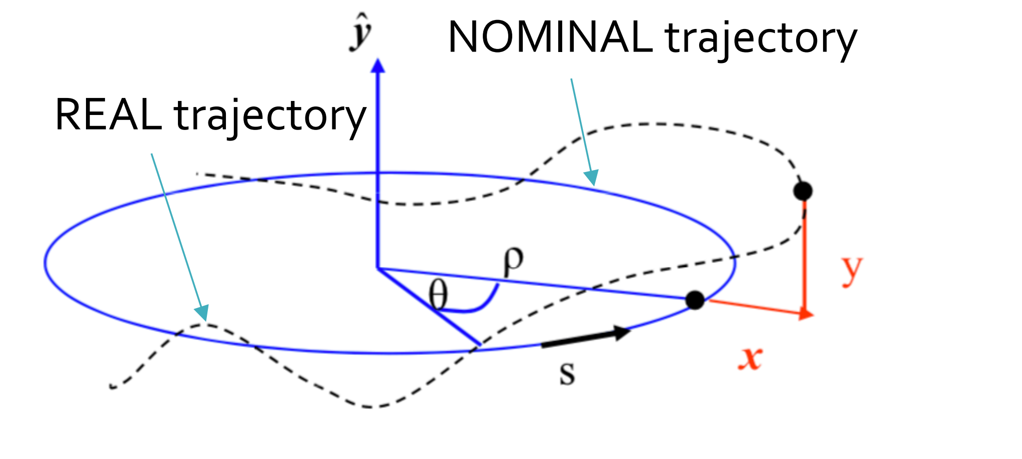
$$E = cB$$

For B = 1 T = 1 Vs/m² and c = 3x10^8 m/s ➔ E = 3x10^8 V/m = 300 MV/m

The maximum achievable electric gradients with RF cavities technologies are ~ 100 MV/m

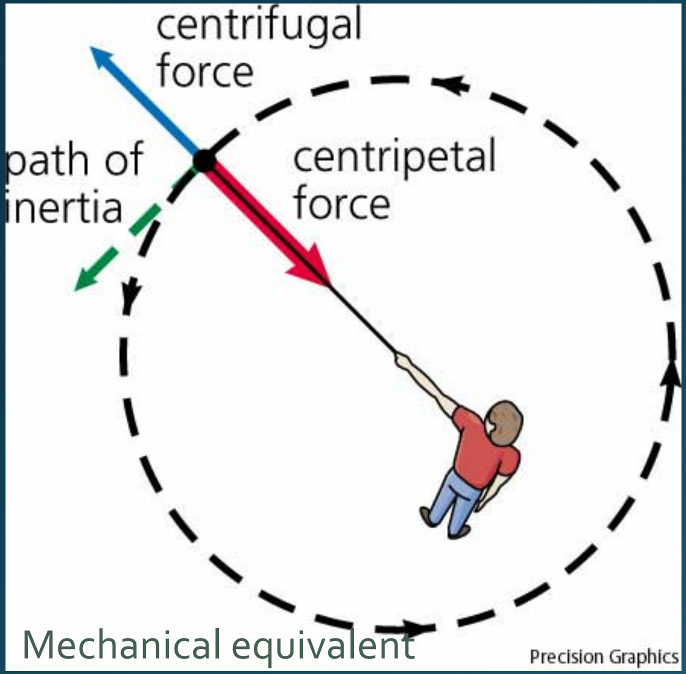
Seminar: Plasma Wake Accelerators ➔ achievable electric gradients > 100 000 MV/m

- To describe the motion of a particle in the vicinity of the NOMINAL trajectory we use a Cartesian coordinate system K = (x,y,s) whose origin moves along the trajectory of the beam



- For simplicity we assume:
  1. particles only move along the s direction: $v = (0, 0, v_s)$
  2. The magnetic field only has transverse components: $B = (B_x, B_y, 0)$

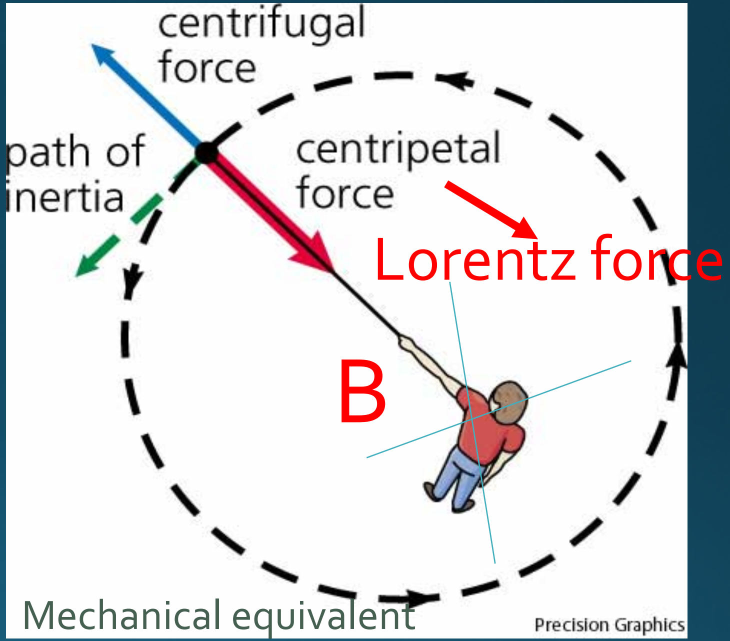
- For a particle moving in the horizontal plane through the magnetic field there is a balance between the Lorentz force and the centrifugal force



$$q \cdot v_s \cdot B_y = \frac{m \cdot v_s^2}{\rho}$$

$$p = m \cdot v$$

$$\boxed{B_y \rho = \frac{p_s}{q}}$$



Beam rigidity formula

- Since the beam dimensions and beam displacement around the NOMINAL trajectory are usually much smaller than the curvature radius (e.g. LHC beam size ~ 1 mm to few μm, and the beam excursions around the NOMINAL orbit are of ~ 2 mm, while LHC curvature radius ~ 2.8 km) we can expand the magnetic field in the vicinity of the NOMINAL trajectory:

$$B_y(s) = B_{y0} + \frac{dB_y}{dx}x + \frac{1}{2!}\frac{d^2B_y}{dx^2}x^2 + \frac{1}{3!}\frac{d^3B_y}{dx^3}x^3 + \cdots$$

- Multiplying by q/p:

$$\frac{q}{p}B_y(s) = \frac{q}{p}B_{y0} + \frac{q}{p}\frac{dB_y}{dx}x + \frac{1}{2!}\frac{q}{p}\frac{d^2B_y}{dx^2}x^2 + \frac{1}{3!}\frac{q}{p}\frac{d^3B_y}{dx^3}x^3 + \cdots$$

$$\frac{q}{p}B_y(s) = \frac{1}{\rho} + kx + \frac{1}{2!}mx^2 + \frac{1}{3!}ox^3 + \cdots$$

QUADRUPOLE

OCTUPOLE

DIPOLE

SEXTUPOLE

$$\frac{q}{p}B_y(s) = \frac{1}{\rho} + kx + \frac{1}{2!}mx^2 + \frac{1}{3!}ox^3 + \cdots$$

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The magnetic field around the beam is a sum of multipoles; each one has a different effect on the particles

Beam steering

Beam focusing

LINEAR BEAM OPTICS

ONLY LINEAR TERMS ARE TAKEN INTO ACCOUNT

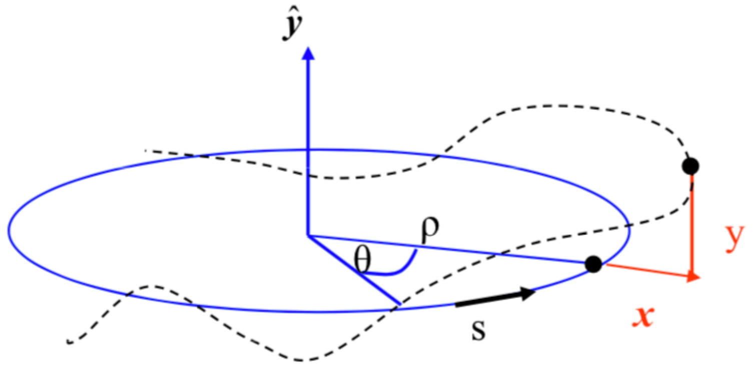
Chromaticity compensation

High amplitude oscillation damping and field errors compensation

Equations of motion

Equation of Motion:

Consider local segment of a particle trajectory



radial acceleration:

$$a_r = \frac{d^2\rho}{dt^2} - \rho\left(\frac{d\theta}{dt}\right)^2$$

Ideal orbit: $\rho = const, \quad \frac{d\rho}{dt} = 0$

Force: $F = m\rho\left(\frac{d\theta}{dt}\right)^2 = m\rho\omega^2$

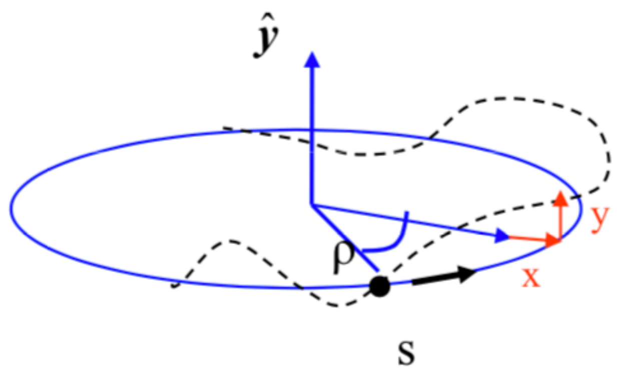
$$F = mv^2/\rho$$

general trajectory: $\rho \rightarrow \rho + x$

$$\boldsymbol{F = m\frac{d^2}{dt^2}(x+\rho) - \frac{mv^2}{x+\rho} = e\,B_y\,v}$$

$$F = m\frac{d^2}{dt^2}(x+\rho) - \frac{mv^2}{x+\rho} = e\,B_y\,v$$

1 2



1 $$\frac{d^2}{dt^2}(x+\rho) = \frac{d^2}{dt^2}x$$ *… as $\rho = const$*

2 ***remember:** $x \approx mm$, $\rho \approx m$ … → **develop for small x***

$$\frac{1}{x+\rho} \approx \frac{1}{\rho}\left(1-\frac{x}{\rho}\right)$$

Taylor Expansion

$$f(x) = f(x_0) + \frac{(x-x_0)}{1!}f'(x_0) + \frac{(x-x_0)^2}{2!}f''(x_0) + \;.$$

$$m\frac{d^2x}{dt^2} - \frac{mv^2}{\rho}\left(1-\frac{x}{\rho}\right) = eB_y v$$

guide field in linear approx.

$$B_y = B_0 + x\frac{\partial B_y}{\partial x} \qquad m\frac{d^2x}{dt^2} - \frac{mv^2}{\rho}(1-\frac{x}{\rho}) = ev\left\{B_0 + x\frac{\partial B_y}{\partial x}\right\} \quad \Big| \;: m$$

$$\frac{d^2x}{dt^2} - \frac{v^2}{\rho}(1-\frac{x}{\rho}) = \frac{e\,v\,B_0}{m} + \frac{e\,v\,x\,g}{m}$$

independent variable: $t \to s$

$$\frac{dx}{dt} = \frac{dx}{ds}\frac{ds}{dt}$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left(\frac{dx}{ds}\frac{ds}{dt}\right) = \frac{d}{ds}\left(\underbrace{\frac{dx}{ds}}_{x'}\underbrace{\frac{ds}{dt}}_{v}\right)\frac{ds}{dt}$$

$$\frac{d^2x}{dt^2} = x''v^2 + \cancel{\frac{dx}{ds}\frac{dv}{ds}v}$$

$$x''v^2 - \frac{v^2}{\rho}(1-\frac{x}{\rho}) = \frac{e\,v\,B_0}{m} + \frac{e\,v\,x\,g}{m} \quad \Big| \;: v^2$$

$$x'' - \frac{1}{\rho}\left(1 - \frac{x}{\rho}\right) = \frac{e\ B_0}{mv} + \frac{e\ x\ g}{mv}$$

$m\,v = p$

$$x'' - \frac{1}{\rho} + \frac{x}{\rho^2} = \frac{B_0}{p/e} + \frac{x\ g}{p/e}$$

normalize to momentum of particle

$$\frac{B_0}{p/e} = -\frac{1}{\rho}$$

$$\frac{g}{p/e} = k$$

$$x'' - \cancel{\frac{1}{\rho}} + \frac{x}{\rho^2} = -\cancel{\frac{1}{\rho}} + k\ x$$

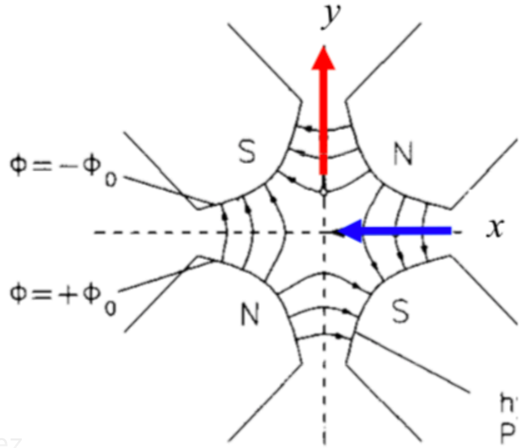
$$\boxed{x'' + x\left(\frac{1}{\rho^2} - k\right) = 0}$$

* ***Equation for the vertical motion:***

$\frac{1}{\rho^2} = 0$ ***no dipoles … in general …***

$k \leftrightarrow -k$ ***quadrupole field changes sign***

$$y'' + k\ y = 0$$



Equation of motion in the linear approximation

$$x'' + \left(\frac{1}{\rho^2} - k\right)x = 0$$

$$y'' + ky = 0$$

Simple Harmonic Motion

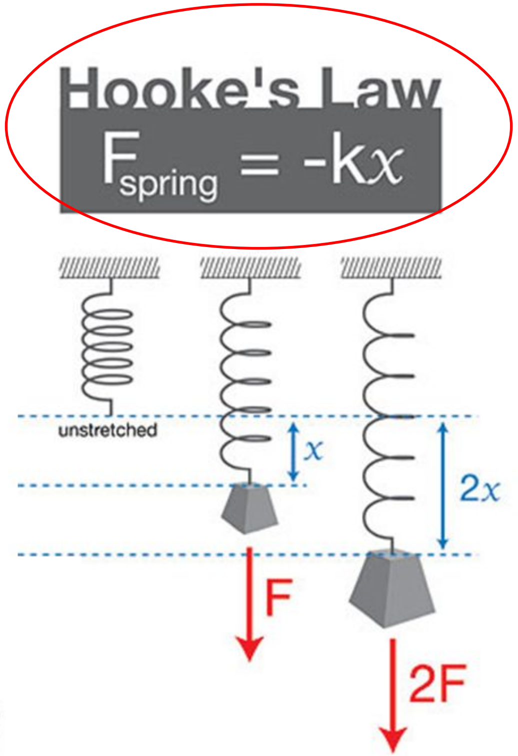
Spring Mass Systems

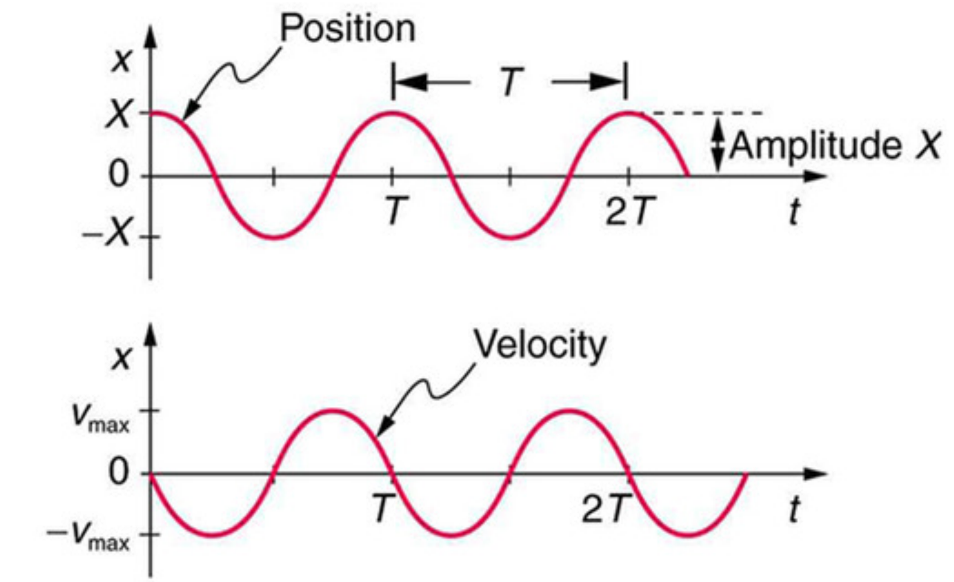
K = spring constant

X = displacement from relaxed position

F = restoring force

*If a spring is stretched or compressed, it oscillates in SHM when it is released.





Type of magnets

$$\frac{q}{p}B_y(s) = \frac{q}{p}B_{y0} + \frac{q}{p}\frac{dB_y}{dx}x + \frac{1}{2!}\frac{q}{p}\frac{d^2B_y}{dx^2}x^2 + \frac{1}{3!}\frac{q}{p}\frac{d^3B_y}{dx^3}x^3 + \cdots$$

$$\frac{q}{p}B_y(s) = \frac{1}{\rho} + kx + \frac{1}{2!}mx^2 + \frac{1}{3!}ox^3 + \cdots$$

QUADRUPOLE

OCTUPOLE

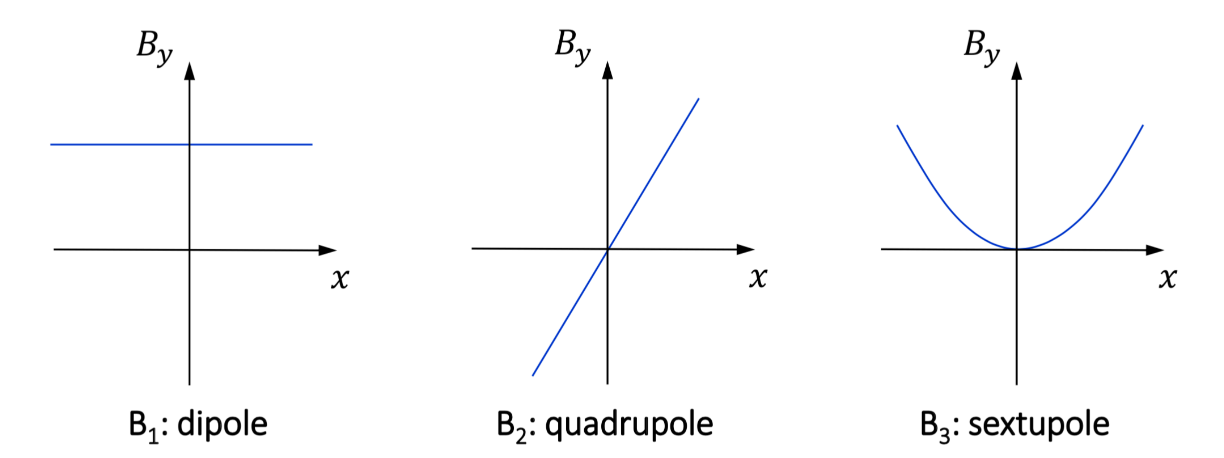
DIPOLE

SEXTUPOLE

$$k_0 = \frac{1}{\rho} = \frac{B}{B\rho}\left(\frac{1}{m}\right)$$

$$k_1 = \frac{q}{p}\frac{dB_y}{dx} = \frac{1}{B\rho}\frac{dB_y}{dx} = \frac{1}{B\rho}g\left(\frac{1}{m^2}\right)$$

$$k_2 = \frac{q}{p}\frac{d^2B_y}{dx^2} = \frac{1}{B\rho}\frac{d^2B_y}{dx^2}\left(\frac{1}{m^3}\right)$$



MAGNETS' TYPE

CONVENTIONAL FERROMAGNETS

Poles are made of iron

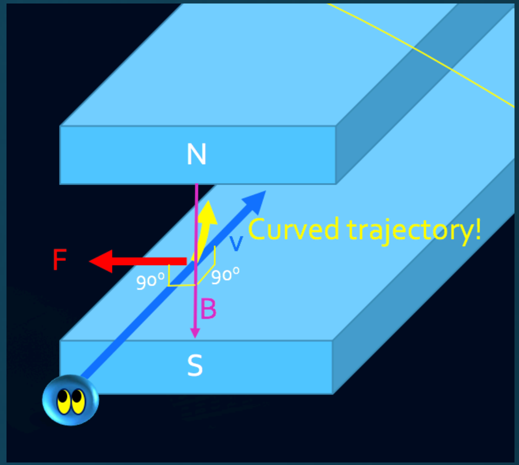
Field is generated by current flowing through windings

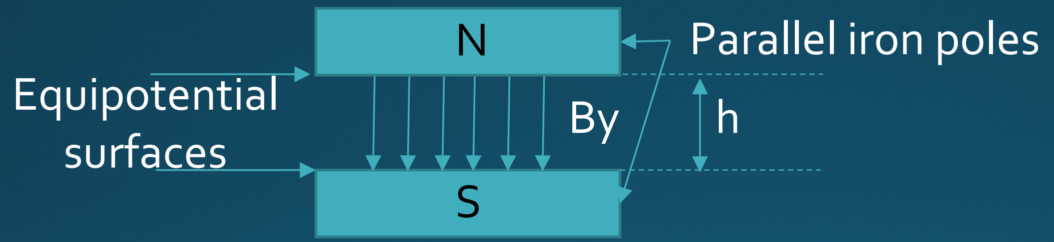
SUPERCONDUCTING MAGNETS

Conventional ferromagnets

DIPOLES

- Dipole magnets bend charged particles around a circular path
- Have a constant field along a given axis: if along the x(y)-axis ➔ horizontal(vertical) bending



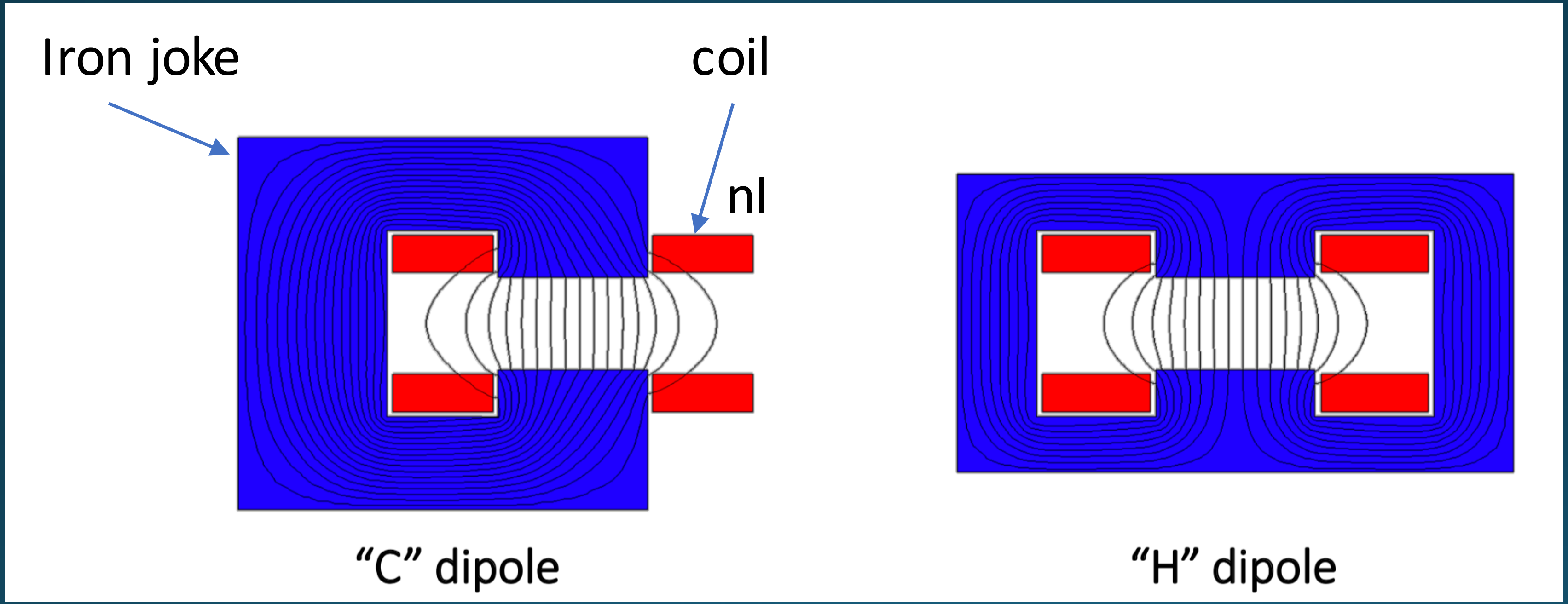


$$k_0 = \frac{1}{\rho} = \frac{B}{B\rho}\left(\frac{1}{m}\right) \approx 0.3\,\frac{B[T]}{p[\frac{GeV}{c}]}$$

Normalized bending strength

$$B_y = \mu_0 \frac{nI}{h}$$

Conventional ferromagnets



Iron ➔ $\mu_r >> 1$
Air ➔ $\mu_r = 1$

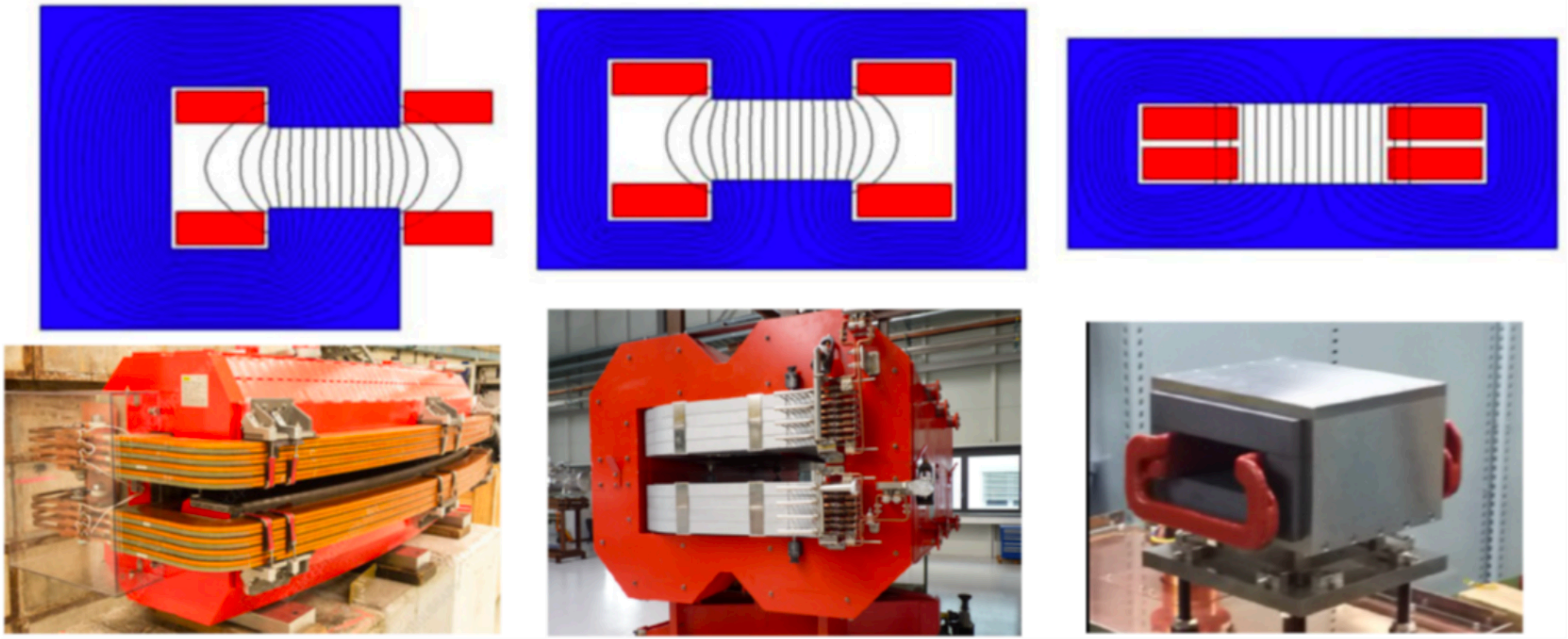
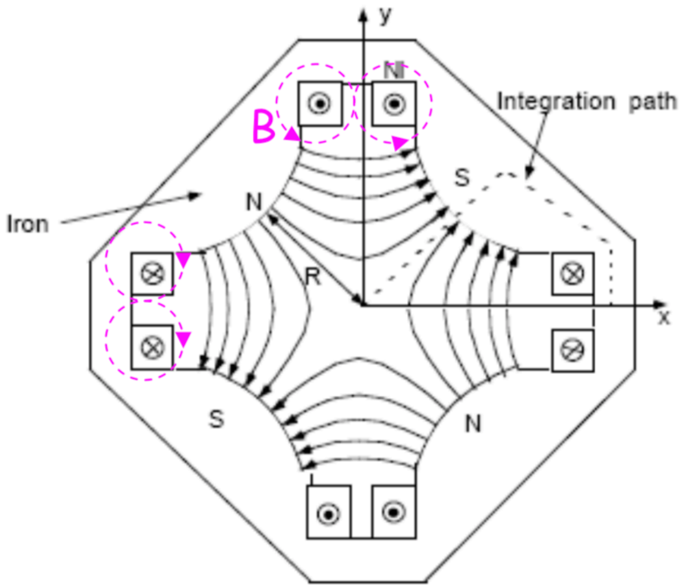


Fig. 7: Different types of iron dominated dipole magnets. Left: C dipole magnet; the picture shows a bending magnet of the SESAME storage ring. Middle: H dipole magnet; the picture shows a prototype of the normal conducting 11 Super-FRS dipole magnet. Right: window-frame dipole magnet; the picture shows the injection kicker for the KEK photon factor advanced ring.

QUADRUPOLES



The field lines are denser near the edges of the magnet, meaning the field is stronger there.

The strength of B_y is a function of x, and visa-versa. The field at the center is zero!

Gradient:

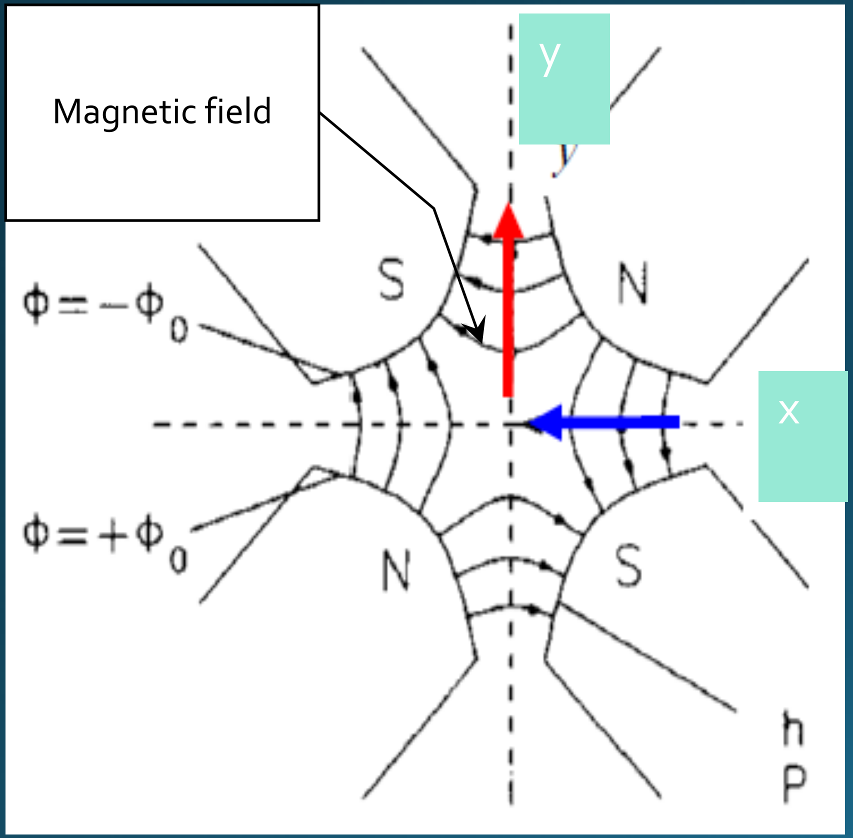
$$g = \frac{2\mu_o nI}{r^2}$$

Normalized gradient strength:

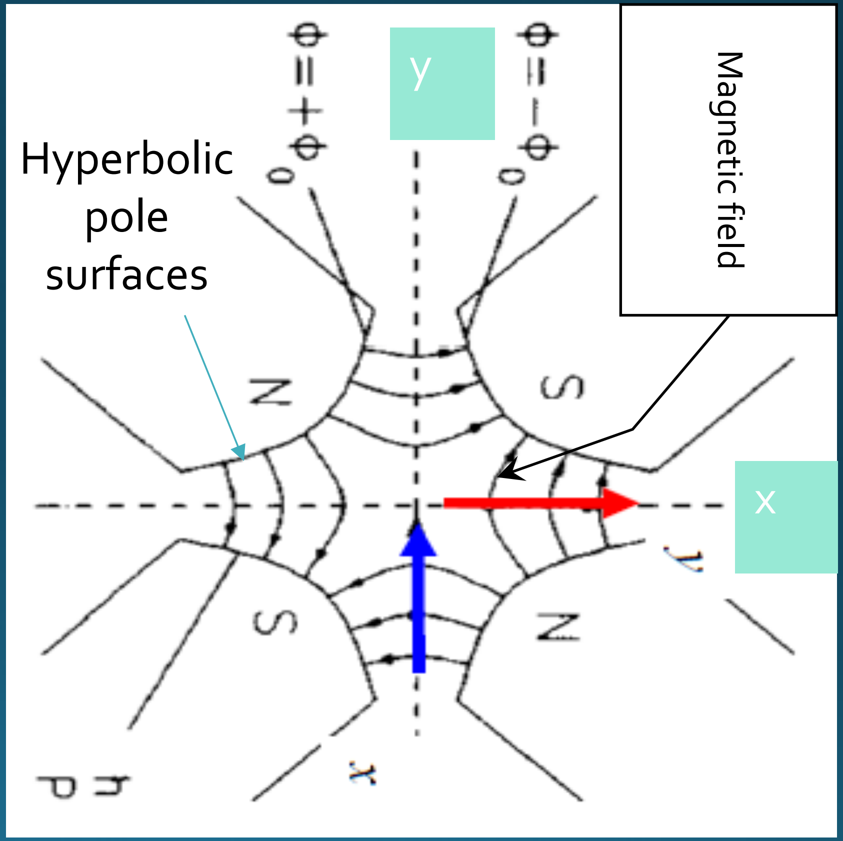
$$k_1 = \frac{q}{p}\frac{dB_y}{dx} = \frac{1}{B\rho}\frac{dB_y}{dx} = \frac{1}{B\rho}g\left(\frac{1}{m^2}\right) \approx 0.3\,\frac{g[\frac{T}{m}]}{p[\frac{GeV}{c}]}$$

QUADRUPOLES

Horizontal focusing quadrupole



Vertical focusing quadrupole



Useful field region due to edge field

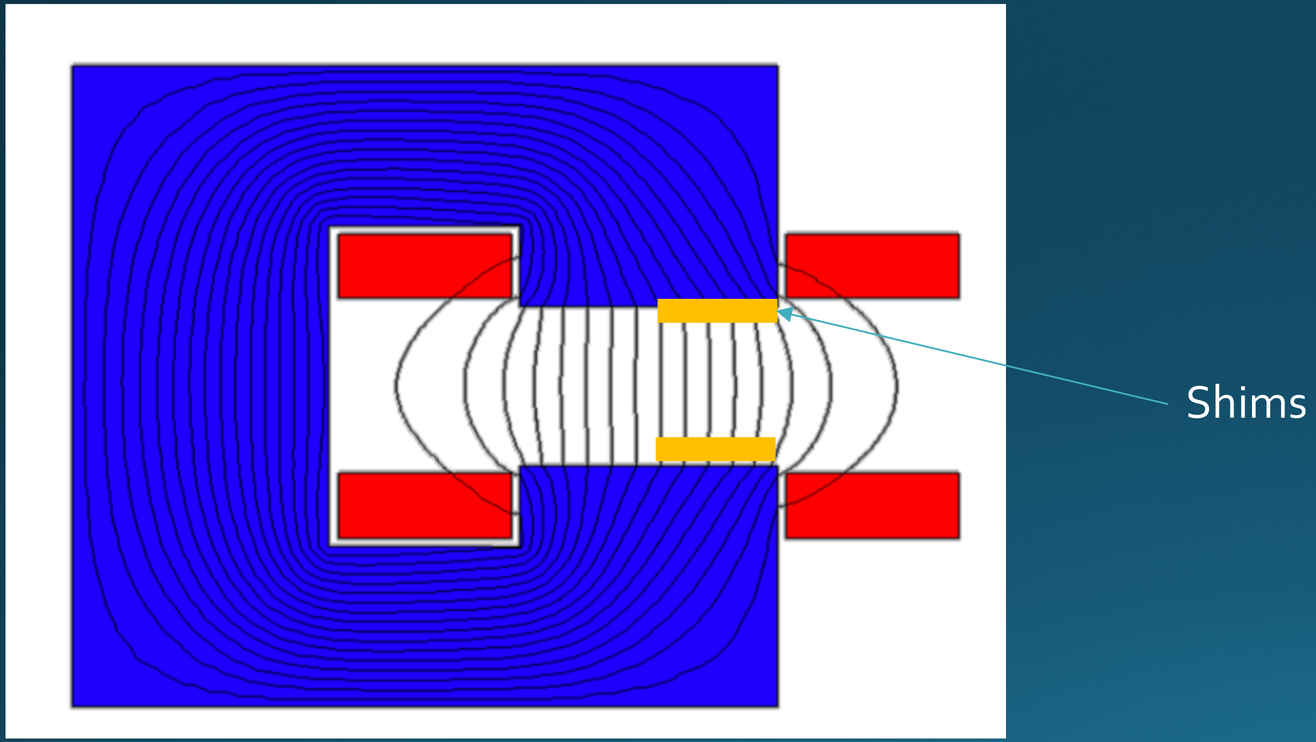
$$x'' + \left\{ \frac{1}{\rho^2} - k \right\} x = 0$$

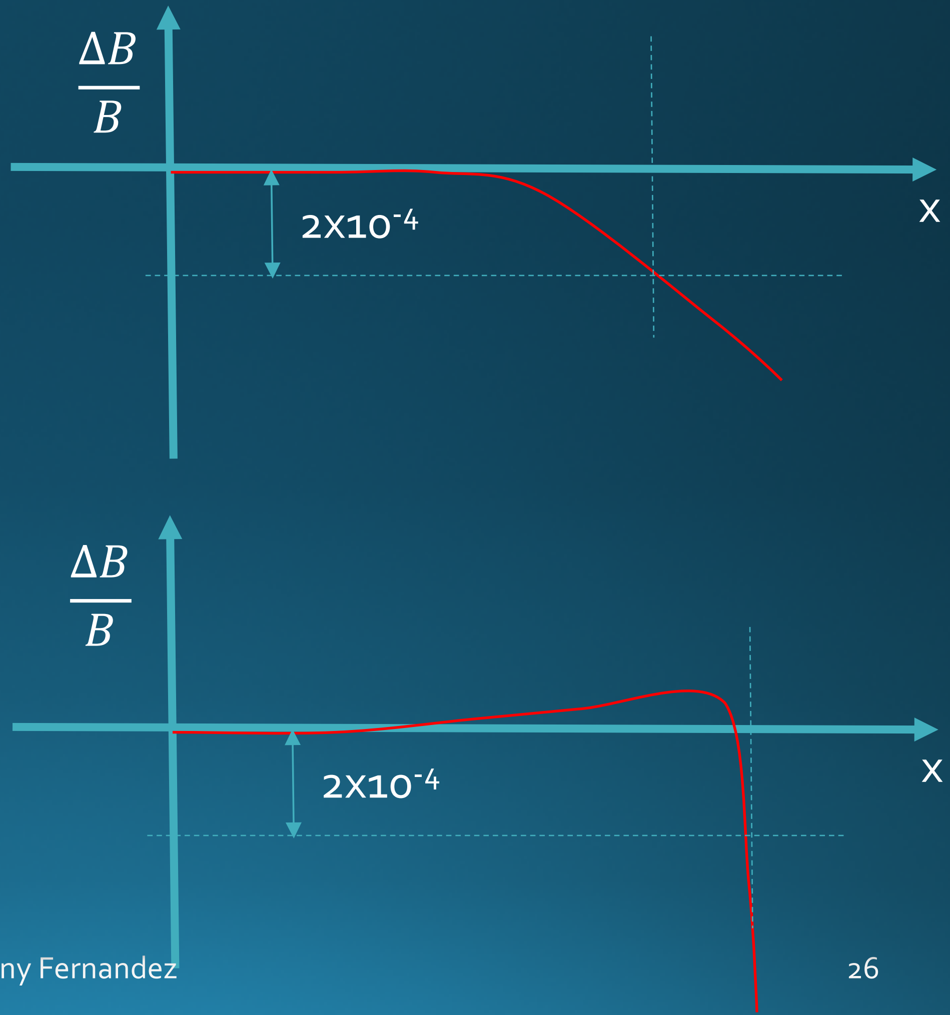
… this equation is not correct !!!

$$x''(s) + \left\{ \frac{1}{\rho^2(s)} - k(s) \right\} x(s) = 0$$

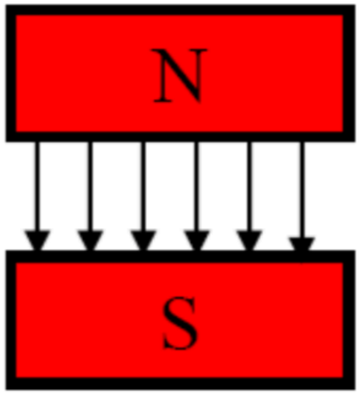
bending and focusing fields … are functi
of the independent variable „s“

However, we normally assume, for fast calculations, that the field inside the magnet is constant



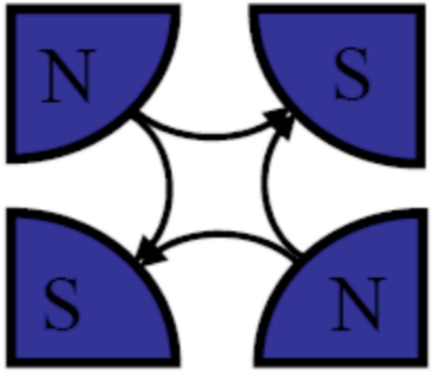


n=1: Dipole



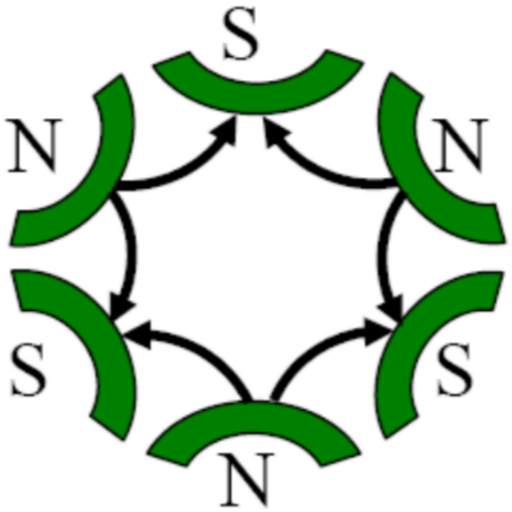
180° between poles

n=2: Quadrupole



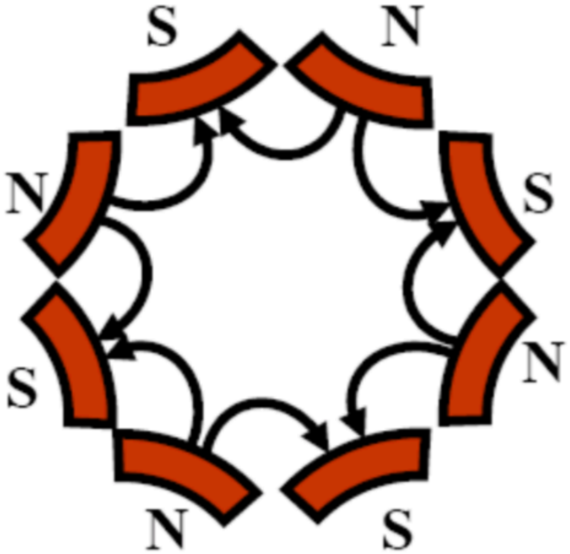
90° between poles

n=3: Sextupole



60° between poles

n=4: Octupole



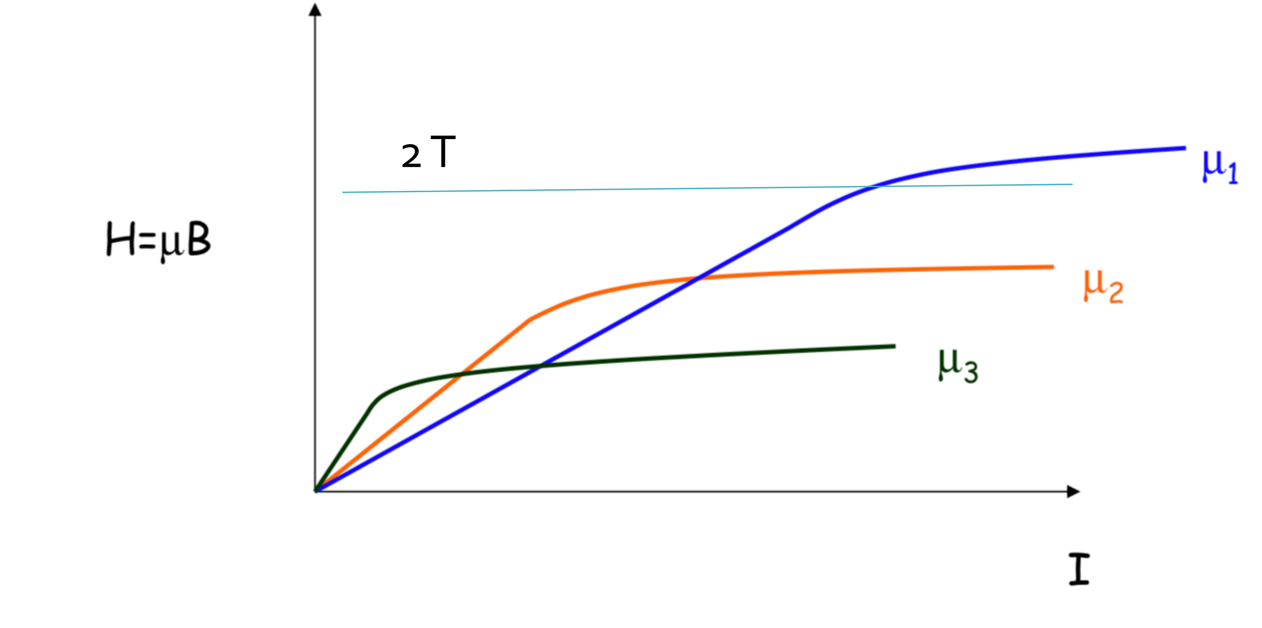
45° between poles

- In general, poles are 360°/2n apart.
- The skew version of the magnet is obtained by rotating the upright magnet by 180°/2n.

Saturation of magnetic materials

In a non-saturated field, the relationship between field strength, B, and driving current, I, is linear.

Above saturation, an increase in current does not generate a corresponding increase in field



Different materials saturate at different levels.

Saturation is a problem ➔ our control system needs to know the beam transfer function curves for each magnet to calculate as precise as possible the current to be injected into the magnets as a function of the requested magnetic field.

SUPERCONDUCTING MAGNETS

- The maximum magnetic field attainable with conventional iron magnets is ~ 2 T
- Above this value the magnets enters in saturation
- Another issue is that for higher magnetic fields, the current density transported by the filaments can be extremely high
- For example, if we need a field of 5 T at a distance of 5 cm from a conductor, a current of $I=1.25\times10^{6}$ A is needed. If the conductor has a thickness of 3 cm this gives dI/da = 1700 A mm^{-2}.
- Even with very effective water cooling, the maximum tolerable current density in cooper is ~ 100 A mm^{-2}.
- Such a requirement cannot be achieved by normal conductors, the ohmic resistance of the material will heat enormously the conductor and it will eventually melt.

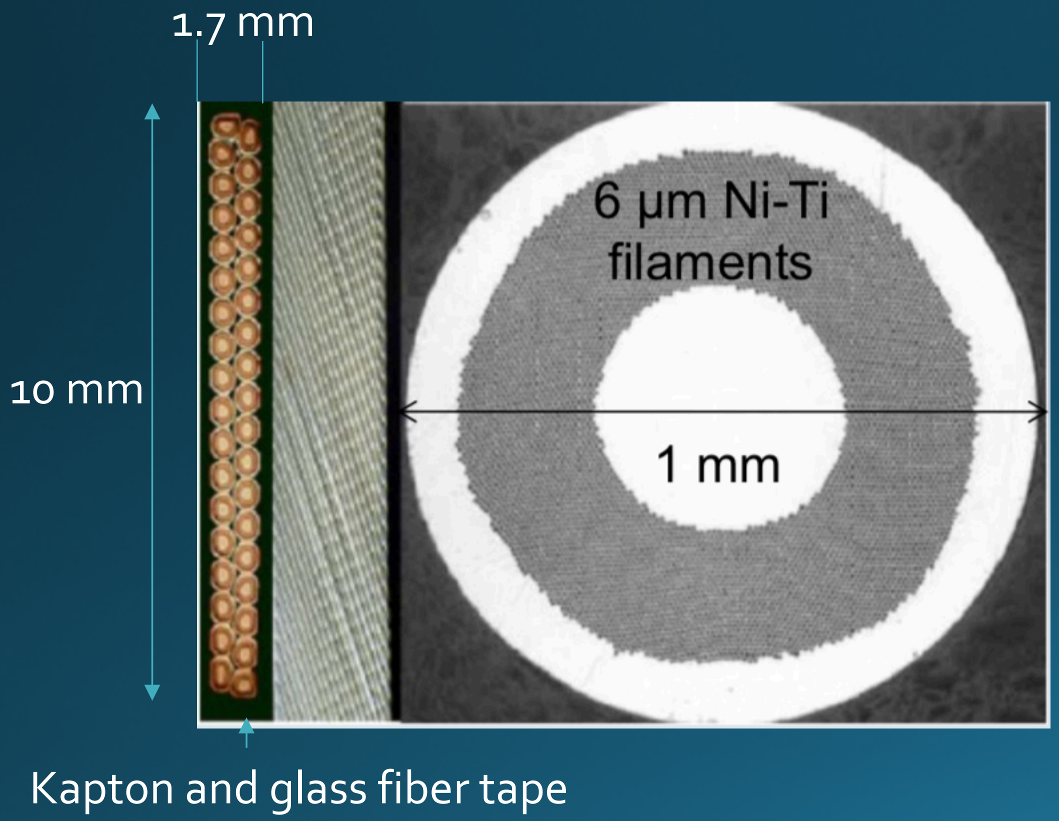
- The solution to this problem was found in 1911 by Dutchman H. Kamerling Onnes → SUPERCONDUCTORS

- He observed that when mercury is cooled to very low temperatures its ohmic resistance suddenly disappears below a critical temperature T_c=4.2 K → current flows without any losses.

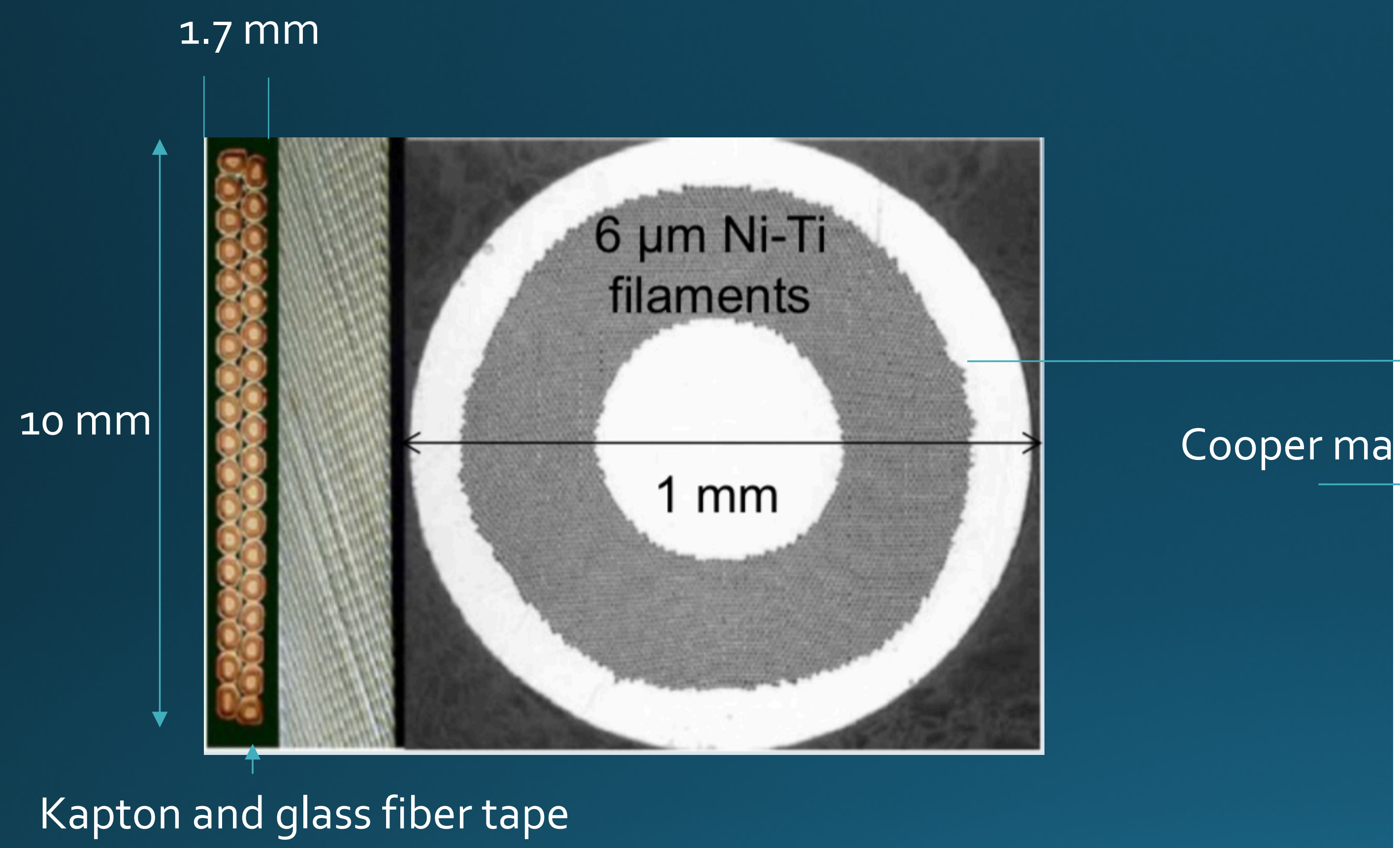
- Many other materials were discovered later to have the same property
- Striking is, though, that the best normal conductors, cooper and silver, do not have this property

- Today a widely used superconducting material is Niobium-Titanium → LHC

- Superconductivity is also influenced by external magnetic field, and there is a so called critical B field, above which superconductivity cannot be reached even at 0 K
- When superconducting regime is reached, B is pushed out of the conductor, therefore, the current only flows near the conductor surface



- This implies that large conductor areas are needed when building a cable
- This is achieved by placing thousands of filaments within the cooper matrix



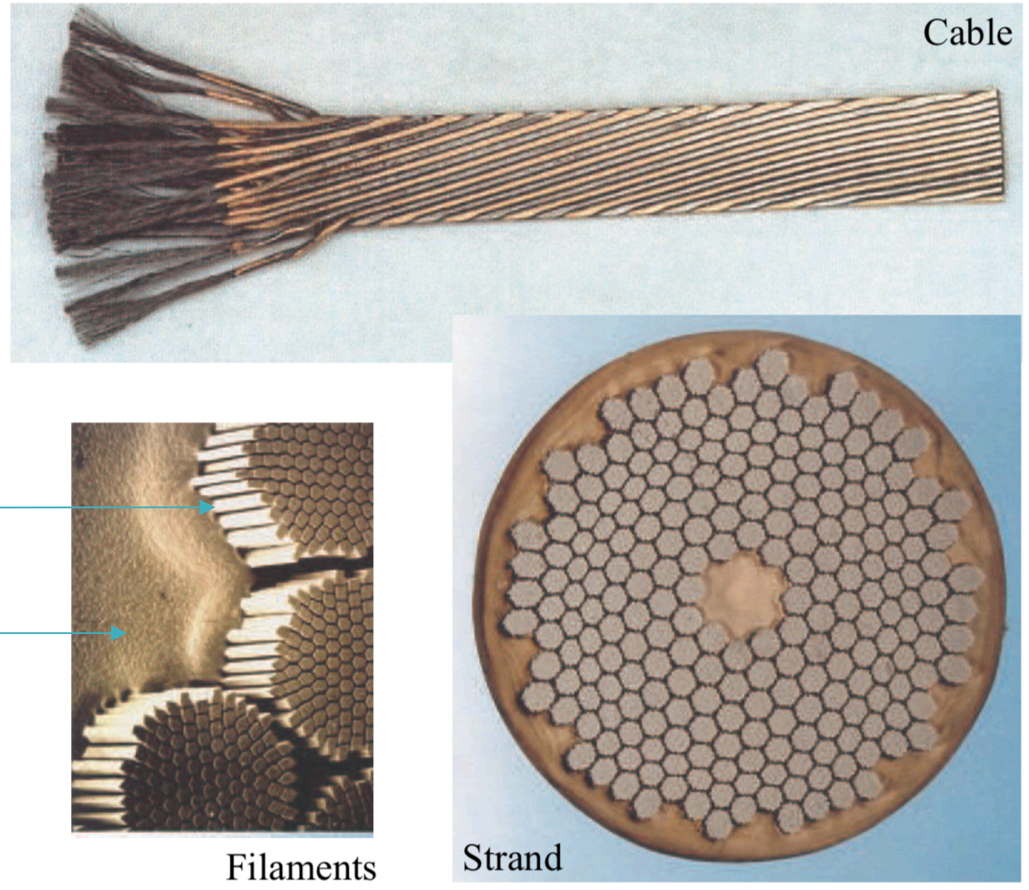
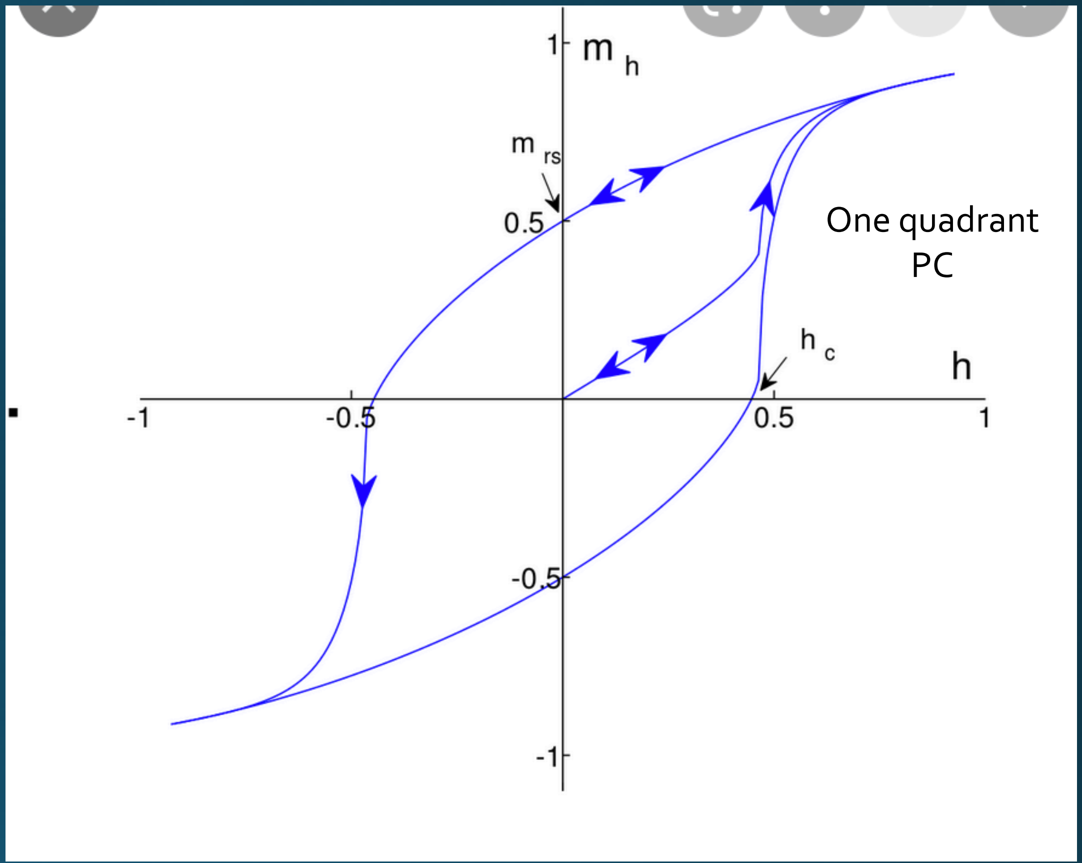


Figure 1.11 Cable for the inner layer of the LHC main dipole coils; microphotograph of the strand cross section and the superconducting filaments. Shown is a prototype wire produced with a double-stacking process.

Effects that perturb the field quality

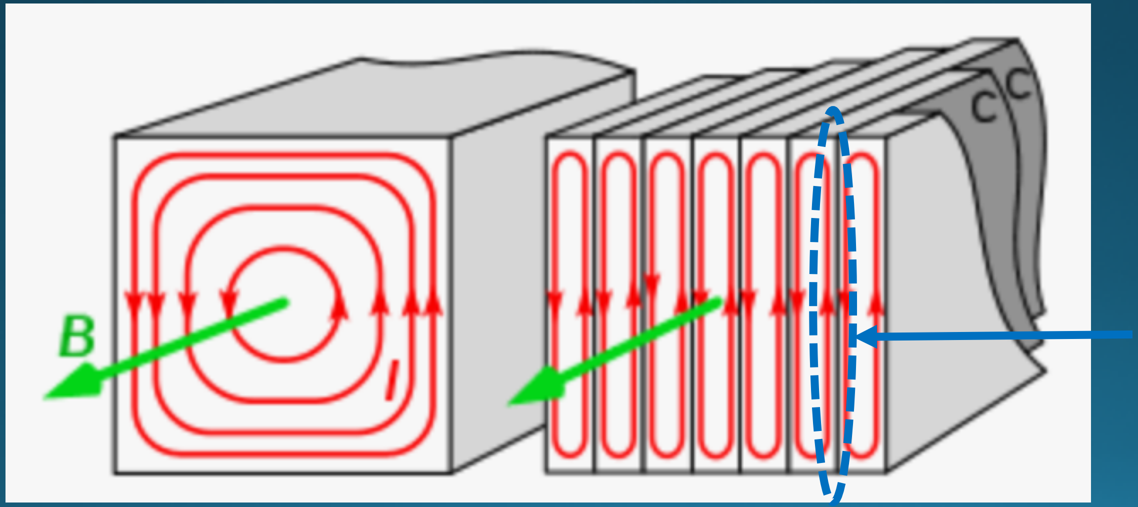
Magnet Hysteresis

- Magnetic hysteresis occurs when an external magnetic field is applied to a magnet such as iron and the atomic dipoles align themselves with it
- Even when the field is removed, part of the alignment will be retained: the material has become magnetized
- Once magnetized, the magnet will stay magnetized indefinitely
- To demagnetize it requires heat or a given magnetic field



Eddy currents

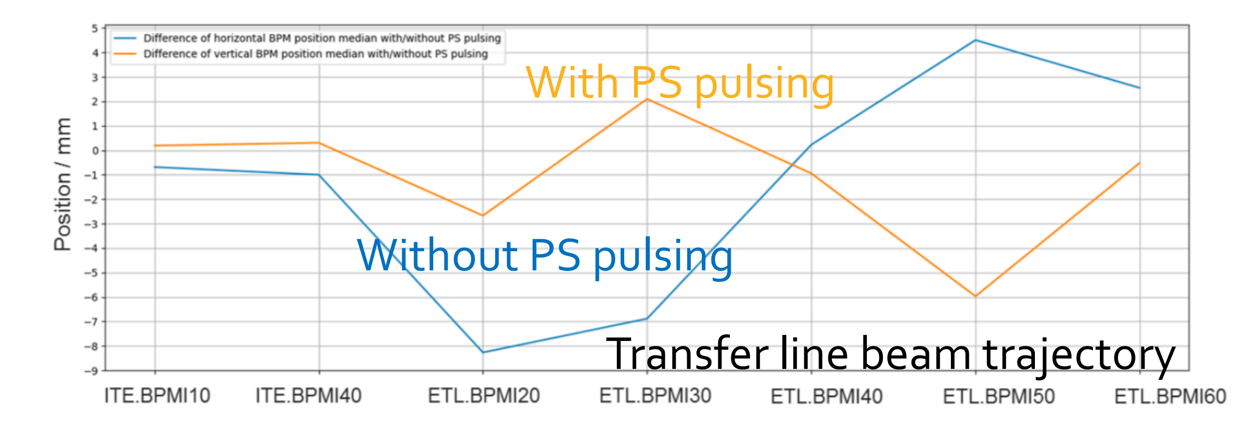
- Eddy currents (also called Foucault's currents) are loops of electrical current induced within conductors **by a changing magnetic field** in the conductor according to Faraday's law of induction
- Eddy currents flow in closed loops within conductors, in planes perpendicular to the magnetic field
- An **eddy current** creates a magnetic field that **opposes the change** in the magnetic field that created it, and thus eddy currents react back on the source of the magnetic field
- Eddy currents therefore also modify the magnetic field for a given requested current ➔ we get less field
- To prevent this effect as much as possible, pulsing magnets are machined in "laminated" and not in solid cores

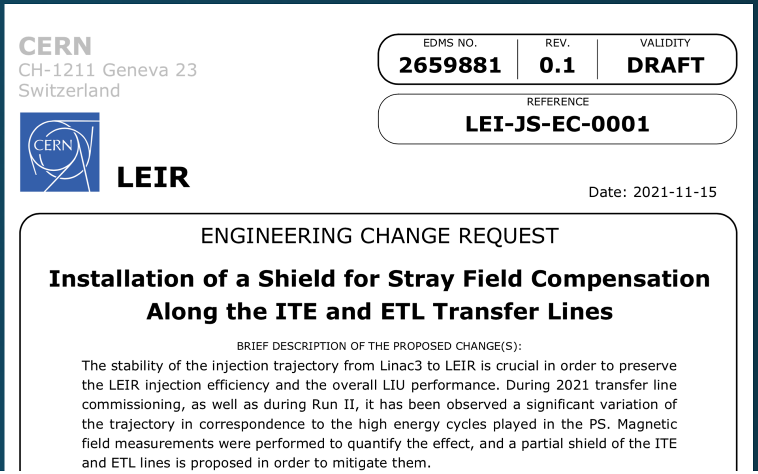


Cancel each other

Stray fields from other magnets

- The magnetic field leaking out from other surrounding magnets, can reach our magnets providing an extra B field which effect superimposes to the one requested by the operational applied current
- The transfer line of LEIR suffers from the PS stray fields in such a way that the particle trajectory is different from the ideal one





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LEIR

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2659881	0.1	DRAFT

REFERENCE
LEI-JS-EC-0001

Date: 2021-11-15

ENGINEERING CHANGE REQUEST

Installation of a Shield for Stray Field Compensation Along the ITE and ETL Transfer Lines

BRIEF DESCRIPTION OF THE PROPOSED CHANGE(S):

The stability of the injection trajectory from Linac3 to LEIR is crucial in order to preserve the LEIR injection efficiency and the overall LIU performance. During 2021 transfer line commissioning, as well as during Run II, it has been observed a significant variation of the trajectory in correspondence to the high energy cycles played in the PS. Magnetic field measurements were performed to quantify the effect, and a partial shield of the ITE and ETL lines is proposed in order to mitigate them.

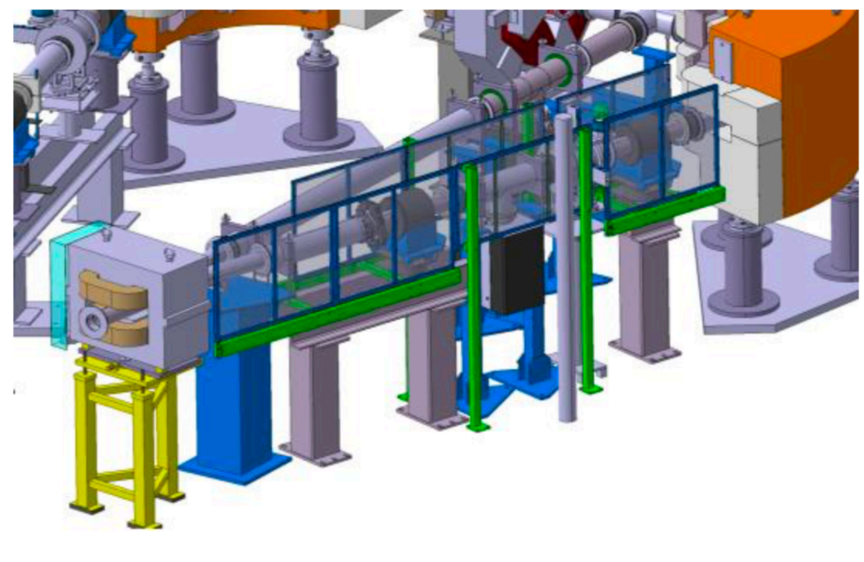


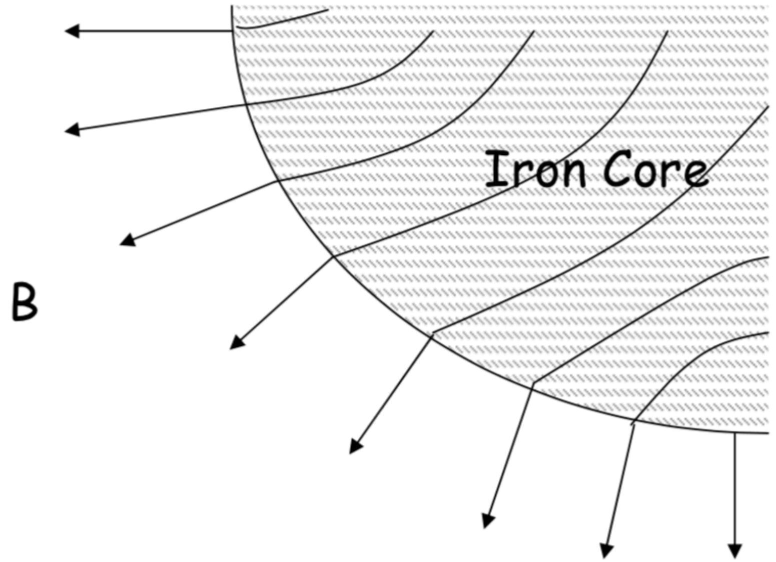
Figure 5: ITE/ETL lines with stray field shields

Spare slides

B-Fields at the Pole Tips

Now we need to add a B field to the material.

Below saturation of iron or similar material, the field lines on the vacuum side are always perpendicular to the pole tip surface:

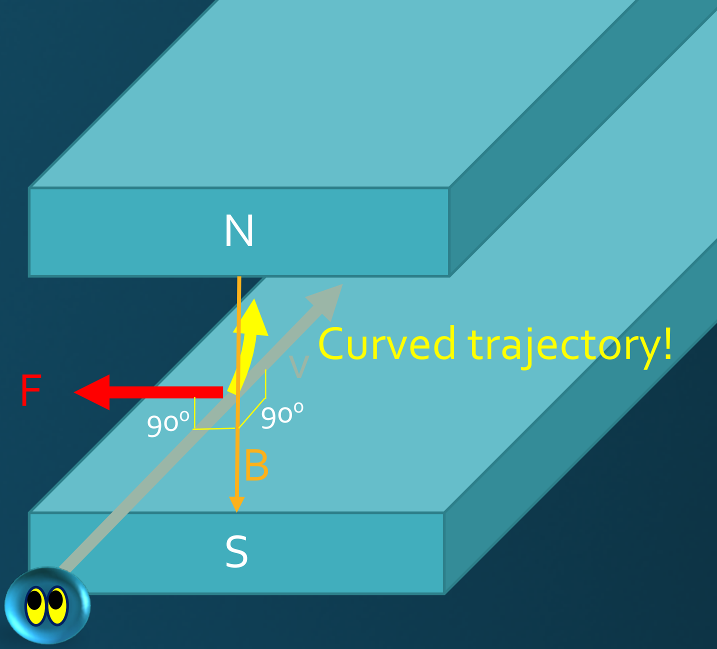


Magnetic lines may have both || and ⊥ path inside the material, but outside, only the field ⊥ to the surface survives.

Below saturation, we can add the B field any way we want inside the material. By setting the pole tip geometry to the magnetic equipotential for a multipole, we get B fields of the desired multipoles.

We need dipole magnets

- A dipole with a uniform field deviates a particle by an angle θ
- The bending angle θ depends on:
  - the length L
  - the magnetic field B
  - the particle momentum



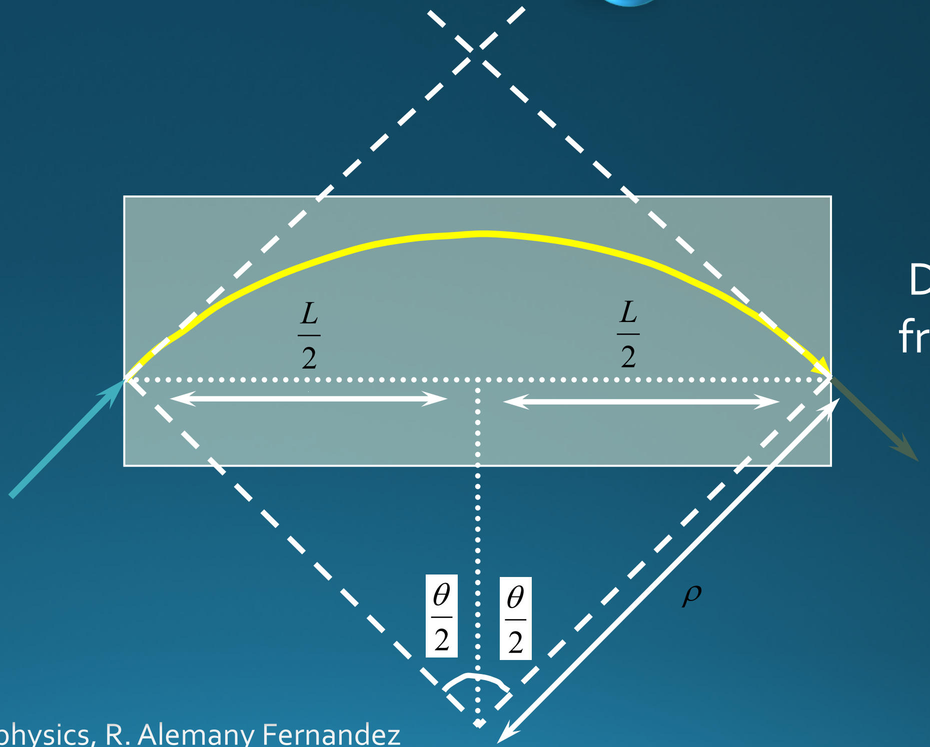
$$arc \approx angle \cdot radius$$

$$arc = L \qquad angle = \theta \qquad radius = \rho$$

$$L = \theta \cdot \rho$$

$$\theta = \frac{L}{\rho} \cdot \frac{B}{B} \qquad \theta = \frac{LB}{B\rho} = \frac{LB}{\frac{p}{q}}$$

$$LB = calibration\ factor(\frac{Tm}{A}) * I$$



Dipole seen from the top