Mysterious neutron stars: dense-matter interiors and gravitational-wave searches

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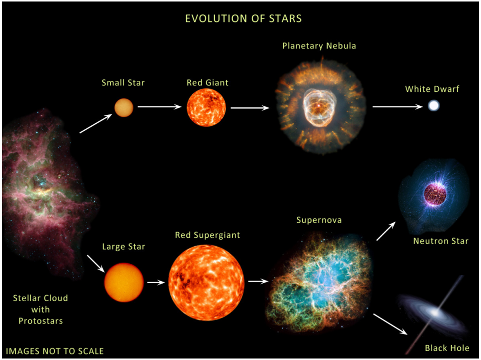
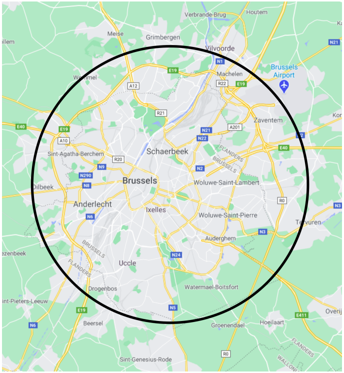
Outline

- Introduction:
  - Neutron stars
  - Gravitational wave sources
- Coalescencing binaries
- Continuous gravitational wave sources
  - Isolated neutron stars
  - Distance estimation error
- Gravitational wave parallax

Neutron stars - extreme objects

Neutron stars are mysterious and extraordinary remains of a cruel and unusual fate of massive stars (8-20 $M_{\odot}$).

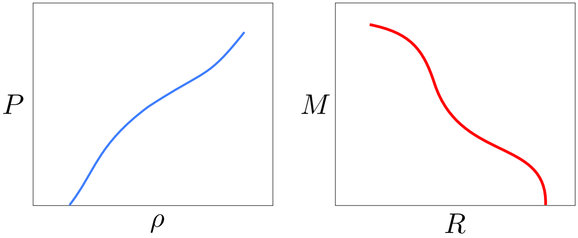




They have $M = 1 - 2$ $M_{\odot}$ and $R \approx 10 - 20$ km. Because of that they are the only known 'laboratories' that allow for testing theories of the densest, cold matter in extreme conditions unattainable at Earth.

From equation of state to $M(R)$ realation

By measurements of the neutron stars masses M and radii R one can, in principle, determine the properties of matter inside the neutron star: relation between pressure P and density ρ - so called equation of state (EOS).



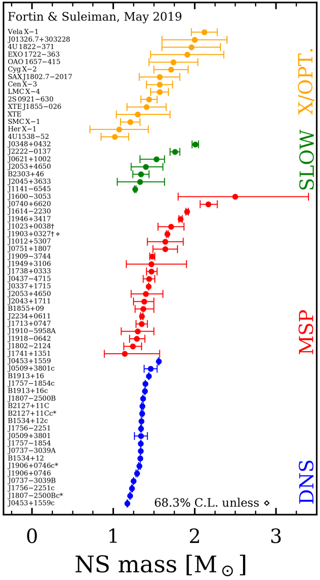
$M(R)$ of non-rotating stars are produced by solving Tolman–Oppenheimer–Volkoff (TOV) equations of the hydrostatic equilibrium (Oppenheimer & Volkoff 1939; Tolman 1939):

$$\frac{dP(r)}{dr} = -\frac{G}{r^2}\left[\rho(r) + \frac{P(r)}{c^2}\right]\left[M(r) + 4\pi r^3 \frac{P(r)}{c^2}\right]\left[1 - \frac{2GM(r)}{c^2 r}\right]^{-1}$$

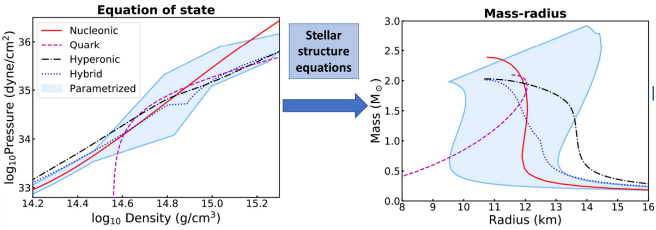
Neutron stars - mass and radius observations

Challenge

Some objects have precise measurements of their masses, but there is always a problem with radii measurements...



Equation of state is still unknown

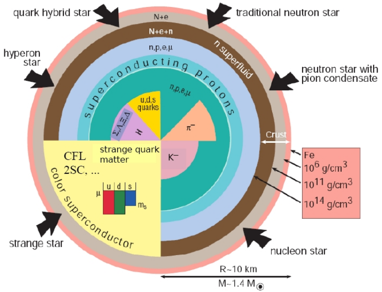


Watts (2019)

Equation of state status

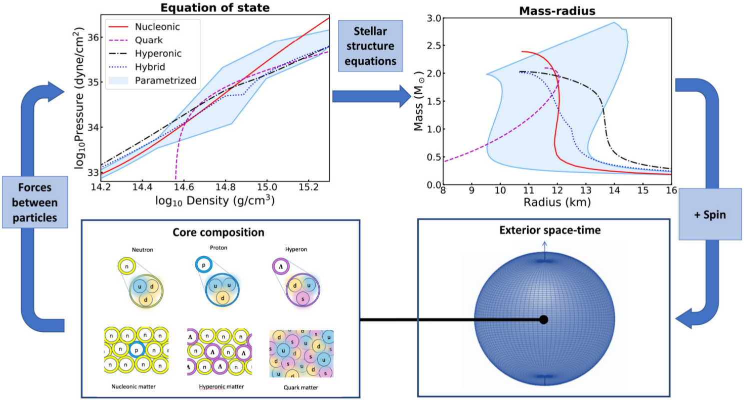
In reality it is (currently) impossible to determine equation of state due to the observational error.

Weber (2004) ⟶



Numerical relativity needed!

Sequences of rotating stars parametrized by the spin frequency and the equation of state parameter (e.g. the central pressure) can be obtained by using e.g. a multi-domain spectral methods: library LORENE (Gourgoulhon et al. 2016; Bonazzola et al. 1993; Gourgoulhon et al. 1999; Gourgoulhon 2010)



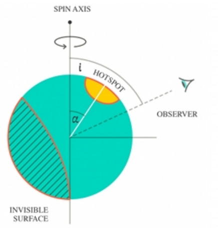
Watts (2019)

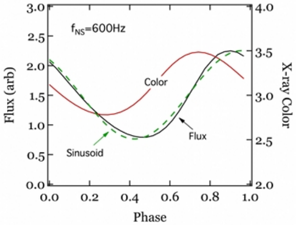
Observational challenges: NICER

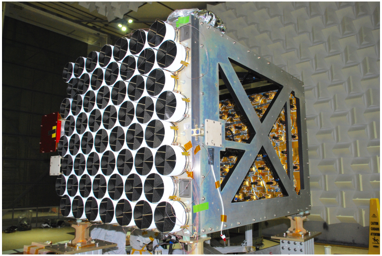
To test equation of state we need better estimation of the neutron stars global parameters.

Neutron star Interior Composition ExploreR (NICER) → Predicted accuracy of M and R measurements: few % by using pulse profile modelling

Psaltis, Özel & Chakrabarty (2014); Psaltis & Özel (2014); Lo, Miller, Bhattacharyya & Lamb (2013); Miller & Lamb (2016)

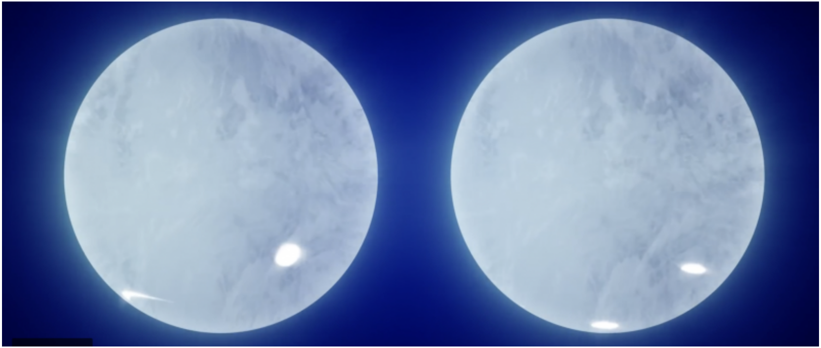






Observational results: NICER

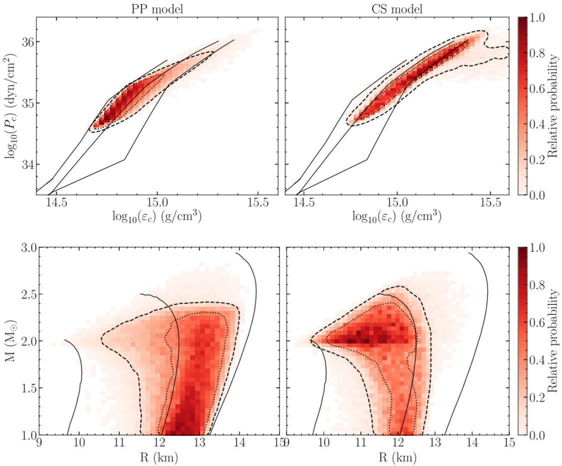
To achieve $\sim 5\%$ accuracy in M and R measurements many assumption have to be fulfilled...



... so far 10% was achieved for PSR J0030+0451 ($f \approx 200$ Hz).

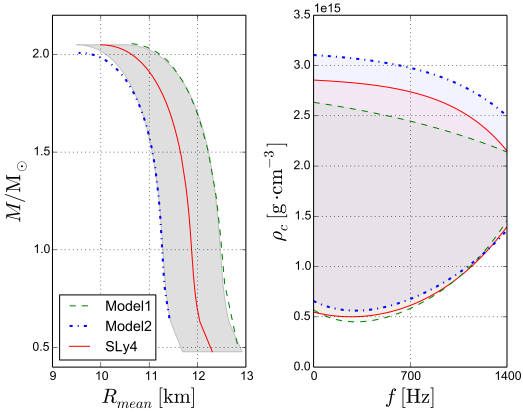
Riley et al. (2019); Raaijmakers et al. (2019); Bilous et al. (2019); Miller et al. (2019); Bogdanov et al. (2019); Guillot et al. (2019)

A NICER view of PSR J0030+0451: implications for the dense-matter equation of state.



Raaijmakers et al. (2019)

Sieniawska, Bejger & Haskell (2018) $\mathcal{A}\&\mathcal{A}$, 616, A105 arXiv:1803.08813



Reference model: SLy4 model: crust + liquid core with *npeν* composition (Douchin & Haensel 2001)

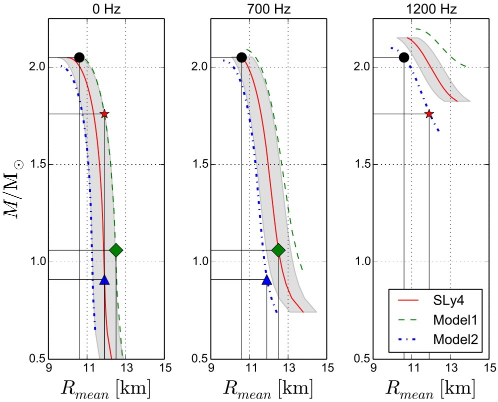
Model1 and Model2: our polytropic equations of state: SLy4 crust + three piecewise relativistic polytropes:

$$P(n) = \kappa_i n^{\gamma_i},$$

$$\epsilon(n) = \rho c^2 = \frac{P}{\gamma_i - 1} + n m_{b_i} c^2$$

$P(n)$ - pressure as function of the baryon density n
$\epsilon(n)$ - mass-energy density
κ_i - pressure coefficient for i-th polytrope ($i = 1, \ldots, 3$)
γ_i - polytropic index
m_{b_i} - baryon mass

Sieniawska, Bejger & Haskell (2018) *A&A*, 616, A105 arXiv:1803.08813



PSR J1748-2446ad: $\nu = 716$ Hz
Hessels et al. (2006)

XTE J1739-285: $\nu = 1122$ Hz
Kaaret et al. (2007)
not confirmed

5% accuracy in R measurements leads to errors:

- $8 - 10\%$ for the oblateness and area
- up to 10% for n_c, P_c and ρ_c for $1M_{\odot}$
- $20 - 40\%$ for n_c, P_c and ρ_c for $2M_{\odot}$

With rotation one can distinguish between equation of states!

A non-negligible time-varying quadrupole moment is needed to produce GWs!

GW amplitude strain tensor h_{ij} at position r (Einstein 1916, 1918):

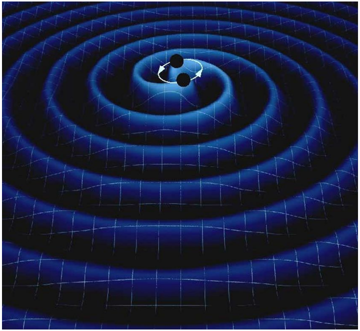
$$h_{ij} = \frac{2G}{c^4 r}\ddot{Q}_{ij}(t - \frac{r}{c}),$$

where the mass-quadrupole moment:

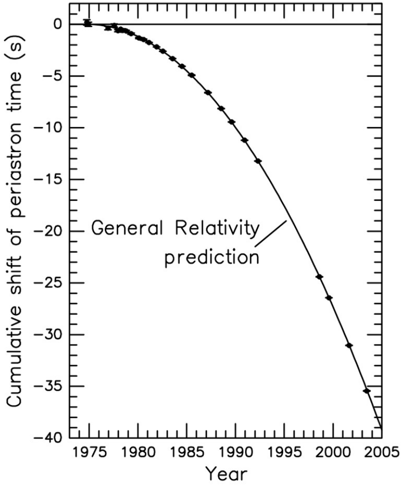
$$Q_{ij}(x) = \int \rho(x_i x_j - \frac{1}{3}\delta_{ij} r^2) d^3 x.$$

Propagation of the GWs in vacuum is governed by a standard wave equation:

$$(\frac{\partial^2}{\partial t^2} - \nabla^2) h_{ij} = \Box h_{ij} = 0.$$



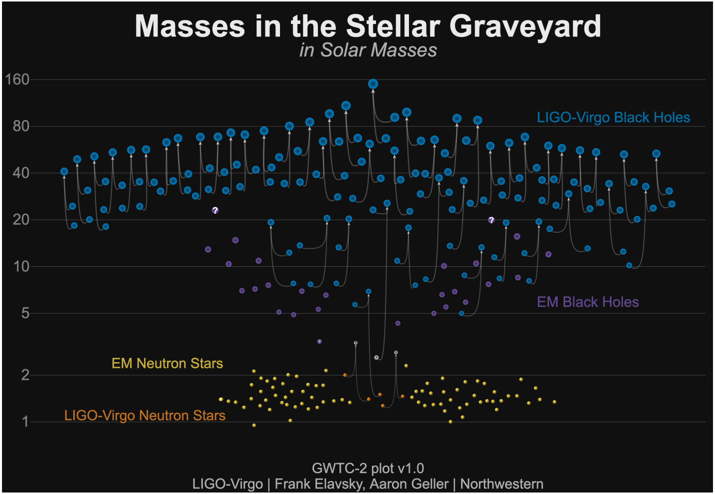
First evidence



- Hulse–Taylor binary (PSR B1913+16)
- Discovered in 1974; Nobel Prize in Physics 1993
- A binary star system composed of a neutron star and a pulsar $\rightarrow$ precise measurements
- Great agreement with the loss of energy due to gravitational waves

New era: gravitational waves astronomy (O1/O2/O3)

90 events confirmed! (GW150914,GW170817)



→ Detections catalog: https://www.gw-openscience.org

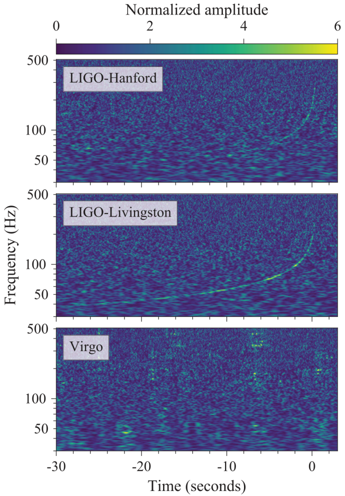
Coalescencing binaries

Gravitational waves from coalescencing binary systems are standard sirens (Schutz 1986) - the GW analog of an astronomical standard candle - as determination of their luminosity distance depends only on measurable quantities like amplitude, frequency and frequency derivative of the signal.

$$h_{0,\mathrm{bin}} = \frac{4\pi^{2/3}G^{5/3}}{c^4}(f_{\mathrm{GW}}\mathcal{M})^{5/3}\frac{1}{f_{\mathrm{GW}}}\frac{1}{d}$$

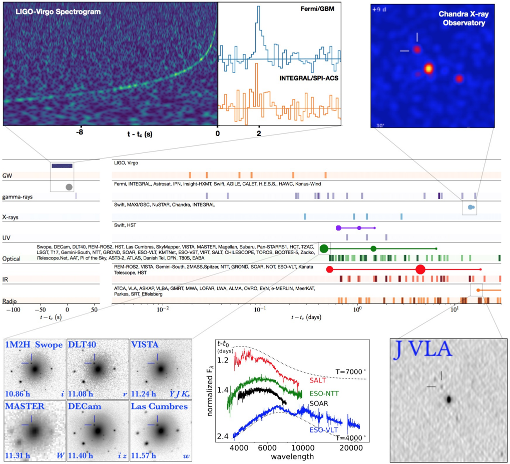
$$\dot{f}_{\mathrm{GW}} = \frac{96}{5}\pi^{8/3}\left(\frac{G\mathcal{M}}{c^3}\right)^{5/3} f_{\mathrm{GW}}^{11/3}$$

Chirp mass: $\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1+M_2)^{1/5}}$



GW170817 (Abbott et al. 2017)

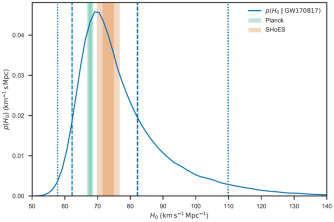
Gravitational wave cosmology



Cosmological corrections

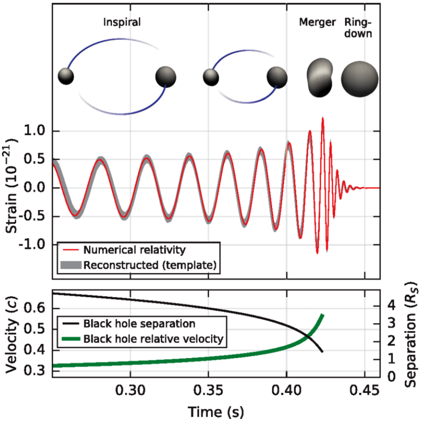
$$h_{0,\mathrm{bin}} = \frac{4\pi^{2/3}G^{2/3}}{c^4}(f_{\mathrm{d,GW}}\mathcal{M}_{\mathrm{d}})^{5/3}\frac{1}{f_{\mathrm{d,GW}}}\frac{1}{d_l}$$

$$f_{s,GW} = f_{d,GW}(1+z);$$
$$\dot{f}_{\mathrm{d,GW}} = \dot{f}_{s,GW}/(1+z)^2;$$
$$\mathcal{M}_{\mathrm{d}} = (1+z)\mathcal{M}$$



Electromagnetic counterpart needed!

Tidal deformability: GW170817 (Abbott et al. 2018)



Reaction of the star on the external tidal field (lowest-order approximation) Love (1911):

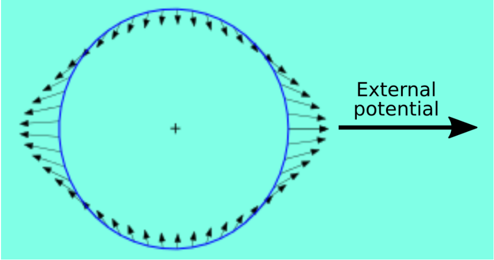
$$\lambda_{td} = \frac{2}{3} R^5 k_2,$$ k_2 depends on M and EOS

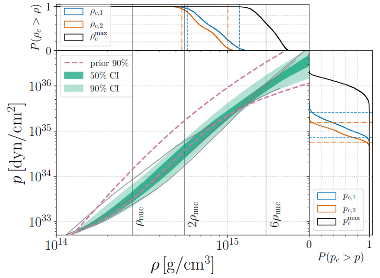
Normalised value

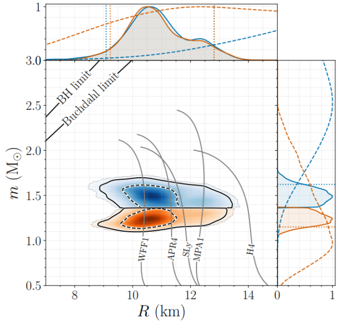
$$\Lambda = \lambda_{td} \left(GM/c^2 \right)^{-5} \in (100, 1000)$$

Effective tidal deformability

$$\tilde{\Lambda} = \frac{16}{13} \frac{(M_1 + 12M_2)M_1^4 \Lambda_1 + (M_2 + 12M_1)M_2^4 \Lambda_2}{(M_1 + M_2)^5}$$







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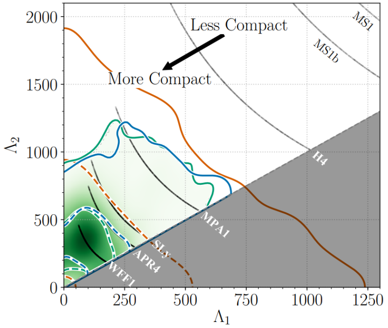
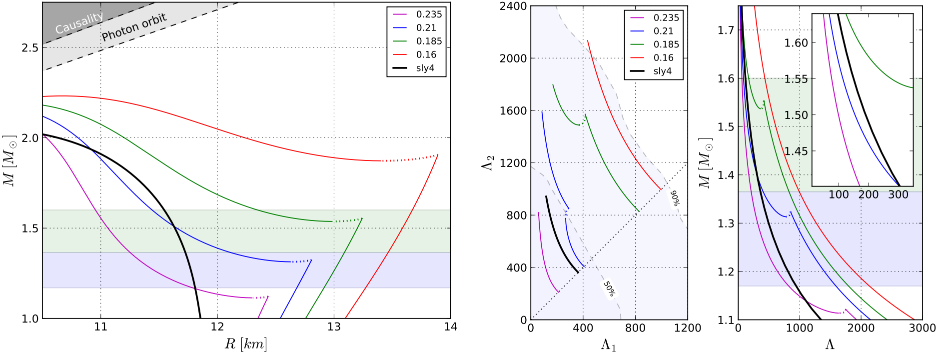


GW170817: can be hybrid (twin) stars

Sieniawska, Turczański, Bejger & Zdunik (2019) A&A 622, A174 arXiv:1807.11581

Tidal deformability and other global parameters of compact stars with strong phase transitions

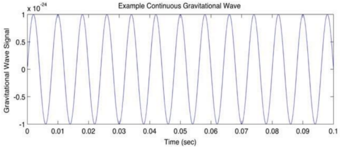
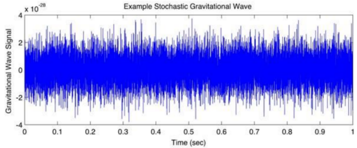
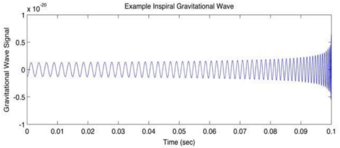
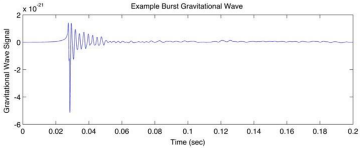
One of the possibilities of a very dense matter is the deconfinement of the quarks → existence of the phase transition between the normal matter and the quark matter.



Simulations of the hybrid stars are consistent with the GW170817 tidal deformability measurements.

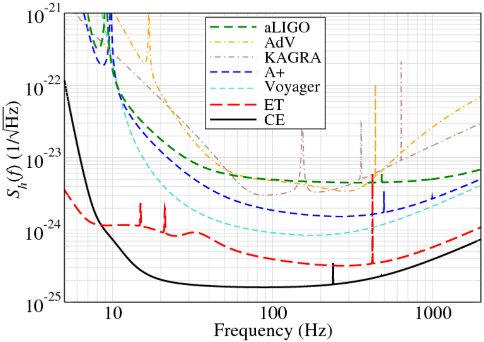
Gravitational waves - sources

So far only compact objects mergers were detected, but it's just a beginning!

	Waveform known / Barely known	Unknown waveform
'Long-lived' signals		
Transient Signals **Burst Signals**		

Upgrade of the existing detectors + new methods in data analysis + new detectors = detections of the more subtle signals

Signal-to-noise ratio (SNR)



Regimbau et al. (2017)

Signal-to-noise ratio

$$SNR \propto \frac{h_0}{\sqrt{S_n}}\sqrt{T}$$

S_n - strain noise
(aLIGO: $\sqrt{S_n} \sim 10^{-23}\text{Hz}^{-1/2}$)

T - observational time

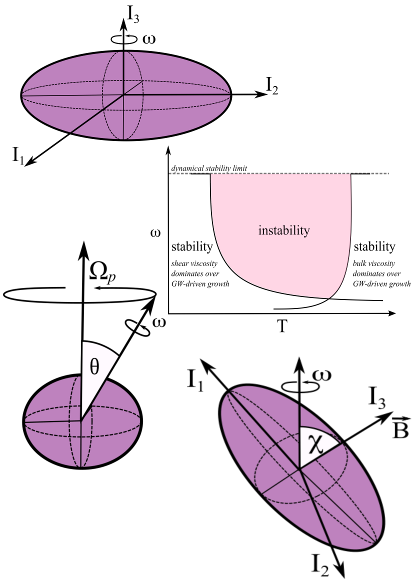
Network of the detectors

$$SNR \propto \sqrt{N}$$

N - number of detectors with comparable sensitivity

- GW150914: $h_0 \sim 10^{-21}$, $T \sim 0.2\text{s} \rightarrow SNR \sim 24$
- CGW: $h_0 \lesssim 10^{-25}$, $T \sim$ days, months, years...

Continuous gravitational waves



Emission mechanisms (NS)

- Mountains (elastic, magnetic, viscosity stresses)
  $f_{GW} = 2 f_{rot}$
- Oscillations (r-modes)
  $f_{GW} = 4/3 f_{rot}$
- Free precession
- Magnetic field

Reviews

Sieniawska & Bejger (2019)
Bejger (2018)
Lasky (2015)

Deformed neutron stars

Commonly used model

Non-axisymmetric rotating NS (described as a triaxial ellipsoid) radiating purely quadrupolar CGW.

Strain amplitude

$$h_0 = 4 \times 10^{-25} \left(\frac{\epsilon}{10^{-6}}\right) \left(\frac{I_3}{10^{45}\ \mathrm{g\ cm^2}}\right) \left(\frac{f}{100\ \mathrm{Hz}}\right)^2 \left(\frac{100\ \mathrm{pc}}{d}\right)$$

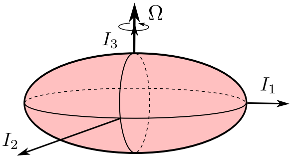
Compare GW 150914: $h_0 \sim 10^{-21}$ (Abbott et al. 2016)

$\epsilon = (I_1 - I_2)/I_3$

$I = I_3$

$f = \Omega/2\pi$

d - distance



Target:

rapidly spinning neutron stars in our Galaxy

$\sim$ 2600 known (http://www.atnf.csiro.au/people/pulsar/psrcat/)

potentially 10^8 objects

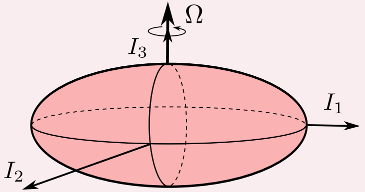
Can we also use CGWs as standard sirens?

Mountains

Rigid rotation of a triaxial star, whose triaxiality or 'mountain' is supported by elastic and/or magnetic strains.

$$h_{0,\mathrm{tr}} = \frac{4G}{c^4}\frac{1}{d} I_3 \epsilon \omega_{\mathrm{rot}}^2$$

$$\dot{\omega}_{\mathrm{rot}} = \frac{32G}{5c^5}\omega_{\mathrm{rot}}^5 \epsilon^2 I_3$$



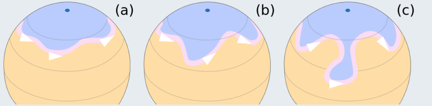
R-modes

Inertial waves, caused by the Coriolis force acting as restoring force (Rossby 1939).

$$h_{0,\mathrm{rm}} = \sqrt{\frac{8\pi}{5}}\frac{G}{c^5}(\alpha M R^3 \tilde{J})\frac{1}{d}\omega_{\mathrm{mode}}^3$$

$$\tilde{J} = \frac{1}{MR^4}\int_0^R \hat{\rho} r^6 dr$$

$$\dot{\omega}_{\mathrm{rot}} = -\frac{2^{18}\pi G}{3^8 5^2 c^7}(\alpha M R^3 \tilde{J})^2 \frac{1}{I_3}\omega_{\mathrm{rot}}^7$$

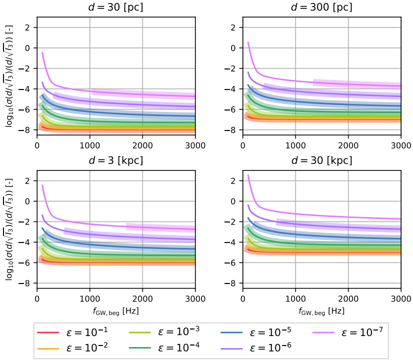


Sieniawska & Jones (2021), arXiv:2108.11710, accepted to MNRAS

CGWs as not-quite-standard sirens

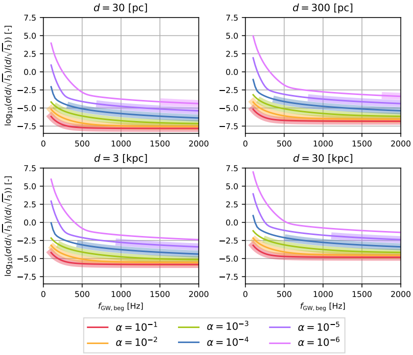
Mountains

$$\Rightarrow h_{0,\mathrm{tr}} = \sqrt{\frac{5G}{2c^3}}\sqrt{\frac{\dot{\omega}_{\mathrm{rot}}}{\omega_{\mathrm{rot}}}}\frac{\sqrt{I_3}}{d}$$



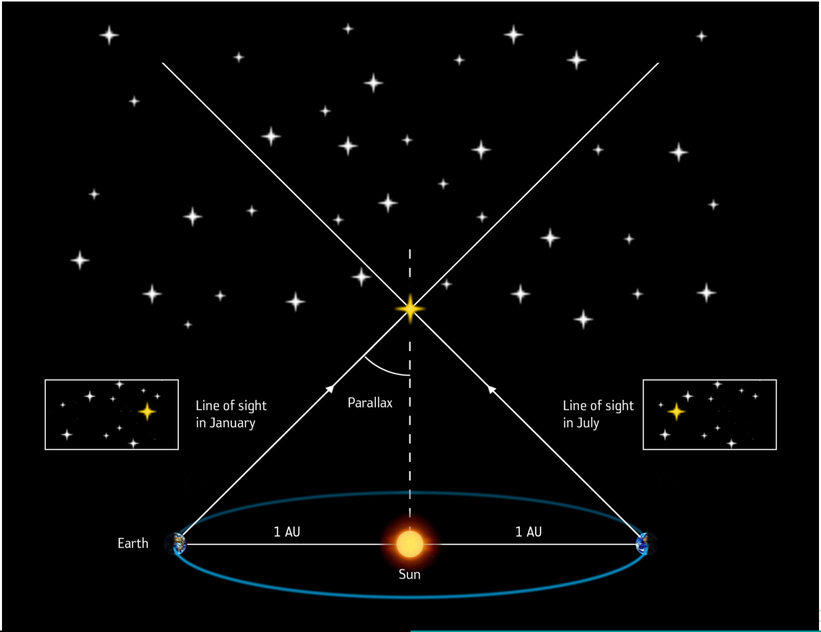
R-modes

$$\Rightarrow h_{0,\mathrm{rm}} = \sqrt{\frac{45G}{8c^3}}\sqrt{\frac{\dot{\omega}_{\mathrm{rot}}}{\omega_{\mathrm{rot}}}}\frac{\sqrt{I_3}}{d}$$



Detectable signals have relative errors $\frac{\sigma(d/\sqrt{I_3})}{d/\sqrt{I_3}} < 1\%$ for ET (10% for aLIGO).

Gravitational wave parallax

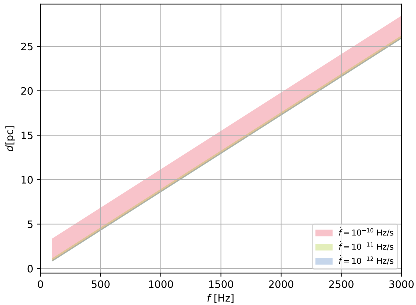


Gravitational wave parallax

Sieniawska & Miller - preliminary results

$$d = \sqrt{\frac{K_{sky}}{\pi}} R_{orb} T_{FFT} \left[\frac{\dot{f}}{\Omega_{orb} \cos(\beta)} + \frac{f_0 \Omega_{orb} R_{orb}}{c} \right]$$

$$\sigma(d) = \frac{1}{\rho} \sqrt{\frac{K_{sky}}{\pi^3} \left[\frac{48 R_{orb}^4 \Omega_{obs}^2}{c} + \frac{180 R_{orb}^2}{\Omega_{obs}^2 T_{obs}^2 \cos^2 \beta} - \frac{180 R_{orb}^3}{c T_{obs} \cos \beta} \right]}$$



- Suitable for the near sources (how near?)
- Long-lived signal (CGW)
- What search grid/sky resolution do we need?
- The ONLY work: Seto (2005)
- ... but contains (too) many approximations