Extracting bigravity from string amplitudes

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Overview

Gravity

1. field theory: Einstein's GR
   - Lorentz invariance + locality $\overset{\nexists \text{ ghost}}{\Longrightarrow}$ diffeos
   - unique kinematics of $g_{\mu\nu}$

     Gupta, Kraichnan, Weinberg, Boulware, Deser, … '50s – '80s

2. string theory: closed string spectra
   - massless level: $\alpha^i_{-1}\overline{\alpha}^j_{-1}|0\rangle \Rightarrow (g_{\mu\nu}, B_{\mu\nu}, \phi)$

     Scherk and Schwarz 1974
   - effective Lagrangian from three– and four–point amplitudes

     Green, Schwarz, Brink 1982
     Gross, Sloan 1987

Ghost–free bimetric theory

- two dynamical metrics:

$$\mathcal{L}_{\text{HR}} = m_g^2 \sqrt{g} R(g) + m_f^2 \sqrt{f} R(f) - 2\, m_g^2 m_f^2 \sqrt{g}\, V(S;\beta_n)$$

$$V(S;\beta_n) = \sum_{n=0}^{4} \beta_n e_n(S) \quad , \quad S^\mu{}_\nu = \left(\sqrt{g^{-1}f}\right)^\mu{}_\nu \, , \, \alpha \equiv m_f/m_g$$

Hassan, Rosen '11

- inspiration: ghost–free massive gravity

$$\mathcal{K}^\mu_\nu \equiv \delta^\mu_\nu - \left(\sqrt{g^{-1}f}\right)^\mu_\nu$$

de Rham, Gabadadze, Tolley '10, '11

7 propagating d.o.f.
no ghosts; else: Boulware, Deser 1972

String embedding

Idea: extract info on $V(S;\beta_n)$ from string scattering amplitudes

- Plan:
  1. expand: treat bimetric theory *peturbatively*
  2. identify: field $\leftrightarrow$ string state
  3. compute: string states as on–shell asymptotic states
  4. extract effective action and compare

1st non–trivial case: cubic interactions $\Rightarrow$ 3–point amplitudes

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Bimetric expansion, quadratic level

- expand around proportional bkgs (we choose equal Minkowski)

$$g_{\mu\nu} = \eta_{\mu\nu} + \delta g_{\mu\nu} \quad , \quad f_{\mu\nu} = \eta_{\mu\nu} + \delta f_{\mu\nu}$$

- mass eigenstates:

$$G_{\mu\nu} \equiv m_g \left(\delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}\right) \quad , \quad M_{\mu\nu} \equiv \alpha m_g \left(\delta f_{\mu\nu} - \delta g_{\mu\nu}\right)$$

- quadratic level:

$$\mathcal{L}^{(2)}(G) = \frac{1}{2} G^{\mu\nu} \hat{\mathcal{E}}^{\rho\sigma}_{\mu\nu} G_{\rho\sigma}$$

$$\mathcal{L}^{(2)}(M) = \frac{1}{2} M^{\mu\nu} \hat{\mathcal{E}}^{\rho\sigma}_{\mu\nu} M_{\rho\sigma} - \frac{m^2_{\mathrm{FP}}}{4} \left([M^2] - [M]^2\right)$$

Fierz, Pauli 1939

$$m^2_{\mathrm{FP}} \equiv m^2_g (1+\alpha^2) \left(\beta_1 + 2\beta_2 + \beta_3\right) \quad , \quad m^2_{\mathrm{Pl}} \equiv m^2_g (1+\alpha^2)$$

Hassan, Schmidt-May, von Strauss '12

- $M_{\mu\nu}$: viable DM candidate

Aoki, Mukohyama '16
Babichev, Marzola, Raidal, Schmidt-May, Urban, Veerm, von Strauss '16

Bimetric expansion, cubic order

Vertex	0–derivative	two–derivative
G^3	no	yes (Ricci)
M^3	yes	yes (Ricci)
GM^2	yes	yes

- no G^2M term

  Babichev, Marzola, Raidal, Schmidt-May, Urban, Veerm, von Strauss '16

- $G_{\mu\nu}$ *is* the graviton also @ cubic level
  graviton self–interactions: strictly GR

  Boulanger, Damour, Gualtieri, Henneaux 2000

On-shell conditions

- string theory: operator–state correspondence
  1. on–shell mass
  2. transversality and tracelessness

     on–shell asymptotic states ⇒ *on–shell* string amplitudes

- field theory:
  1. $\Box G_{\mu\nu} = 0 \quad , \quad \partial^\mu G_{\mu\nu} = 0 \quad , \quad [G] = 0 \qquad \Rightarrow 2$ d.o.f.
  2. $(\Box - m_{\text{FP}}^2) M_{\mu\nu} = 0 \, , \, \partial^\mu M_{\mu\nu} = 0 \, , \, [M] = 0 \qquad \Rightarrow 5$ d.o.f.

     we have to impose these on the cubic vertices

On-shell bimetric cubic

$$\mathcal{L}_{\mathrm{G}^3} = \frac{1}{m_g\sqrt{1+\alpha^2}}\, G^{\mu\nu}\left(\partial_\mu G_{\rho\sigma}\partial_\nu G^{\rho\sigma} - 2\partial_\nu G_{\rho\sigma}\partial^\sigma G^\rho_\mu\right)$$

$$\mathcal{L}_{\mathrm{GM}^2} = \frac{1}{m_g\sqrt{1+\alpha^2}}\Big[G^{\mu\nu}\left(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 4\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\right) + 2M^{\mu\nu}\left(\partial_\mu G_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\rho G_{\mu\sigma}\partial_\nu M^{\rho\sigma}\right)\Big]$$

$$\mathcal{L}_{\mathrm{M}^3} = \frac{(-\beta_1+\beta_3)\,(1+\alpha^2)^{3/2} m_g}{6\,\alpha}\,[M^3] + \frac{(1-\alpha^2)}{m_g\alpha\sqrt{1+\alpha^2}}\, M^{\mu\nu}\left(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 2\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\right)$$

these terms are not unique to ghost–free bimetric theory, but the particular coefficients and couplings are [excluding $G^2 M$]

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Identification of states

$M_{\mu\nu}$: closed or open string state?

- open string choice: $G_{\mu\nu}$ is a bulk state, $M_{\mu\nu}$ is a brane state

[motivation from Ferrara, Kehagias, Lüst '19]

$\Rightarrow$ we *choose* the first massive spin–2 state

$$b^i_{-3/2}|0\rangle \quad , \quad \alpha^i_{-1} b^j_{-1/2}|0\rangle$$

$$m_g^2(1+\alpha^2)\left(\beta_1 + 2\beta_2 + \beta_3\right) \stackrel{!}{=} \frac{1}{\alpha'}$$

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- closed string choice: both $G_{\mu\nu}$ and $M_{\mu\nu}$ are bulk states

Lüst, CM, Mazloumi, Stieberger *to appear*

Scattering of open and closed strings

- tree–level disk amplitude:

$$\mathcal{A}(N_o, N_c) = \sum_\sigma \left(\int_{\mathcal{I}_\sigma} \prod_{j=1}^{N_o} \mathrm{d}x_j \prod_{i=1}^{N_c} \int_{\mathcal{H}_+} \mathrm{d}^2 z_i \right) V_{\mathrm{CKG}}^{-1}$$

$$\langle \prod_{j=1}^{N_o} : V_o(x_j) : \prod_{i=1}^{N_c} : V_c(z_i, \overline{z}_i) : \rangle_{\mathbb{D}_2}$$

Stieberger '09, Stieberger, Taylor '15

- massless external states: late '90s

Klebanov, Thorlacius
Gubser, Hashimoto, Klebanov, Maldacena
Garousi, Myers
Hashimoto, Klebanov

- 1 massive and other massless

Feng, Lüst, Schlotterer, Stieberger, Taylor '10

- our novelty:
  1. *all* external states are either helicity–2 or spin–2
  2. at least *two* external states are massive

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Vertex operators

- Graviton $G_{\mu\nu}$:

$$V_G^{(0,0)}(z,\overline{z},\varepsilon,q) \;=\; -\tfrac{2g_c}{\alpha'}\,\varepsilon_{\mu\nu}\left[i\overline{\partial}X^\mu + \tfrac{\alpha'}{2}(q\widetilde{\psi})\widetilde{\psi}^\mu(\overline{z})\right] \times\left[i\partial X^\nu + \tfrac{\alpha'}{2}(q\psi)\psi^\nu(z)\right] e^{iqX(z,\overline{z})}$$

$$\varepsilon_{\mu\nu}q^\mu = \varepsilon_{\mu\nu}q^\nu = 0 \quad , \quad q^2 = 0 \quad , \quad \varepsilon_{\mu\nu} = \varepsilon_{\nu\mu} \quad , \quad \varepsilon_{\mu\nu}\eta^{\mu\nu} = 0$$

eg. Mayr, Stieberger '94

- massive spin–2 $M_{\mu\nu}$:

$$V_M^{(-1)}(x,\alpha,k) \;=\; \tfrac{g_o}{(2\alpha')^{1/2}}T^a\, e^{-\phi(x)}\,\alpha_{\mu\nu}\, i\partial X^\mu(x)\psi^\nu(x)\; e^{ikX(x)}$$

$$V_M^{(0)}(x,\alpha,k) \;=\; \tfrac{g_o}{(2\alpha')}T^a\,\alpha_{\mu\nu}\Big[i\partial X^\mu(x)\partial X^\nu(x) - 2i\alpha'\partial\psi^\mu(x)\psi^\nu(x) +2\alpha'\,(k\psi)(x)\,\psi^\nu(x)\partial X^\mu(x)\Big]e^{ikX(x)}$$

$$\alpha_{\mu\nu}k^\mu = 0 \quad , \quad k^2 = -\tfrac{1}{\alpha'} \quad , \quad \alpha_{\mu\nu} = \alpha_{\nu\mu} \quad , \quad \alpha_{\mu\nu}\eta^{\mu\nu} = 0$$

Koh, Troost, van Proeyen 1987
Feng, Lüst, Schlotterer, Stieberger, Taylor '10
Bianchi, Guerrieri '15

One graviton and two massive spin–2 states

$$\mathcal{A}(2,1) \;=\; \int_{\mathcal{R}}\int_{\mathcal{H}_+} \frac{\mathrm{d}x_1 \mathrm{d}x_2\, \mathrm{d}^2 z}{V_{\text{CKG}}}$$

$$\langle :V_M^{(-1)}(x_1,\alpha_1,k_1)::V_M^{(-1)}(x_2,\alpha_2,k_2)::V_G^{(0,0)}(z,\overline{z},\varepsilon,q):\rangle_{\mathbb{D}_2}$$

- heavy brane ⇒ momentum conservation along the brane

$$(k_1+k_2+q_{\|})^\mu = 0$$

$$q_{\|} = \tfrac{1}{2}(q + D\cdot q) \quad , \quad q_{\perp} = \tfrac{1}{2}(q - D\cdot q)$$

- can define Mandelstam (!)

$$s \equiv \alpha'(k_1+k_2)^2\,,\; t \equiv \alpha'(k_1+q/2)^2\,,\; u \equiv \alpha'(k_1+Dq/2)^2$$

⇒ ∃ a single kinematic invariant, eg. $s = -2 + 2\alpha' k_1 k_2$

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One graviton and two massive spin–2 states

- we compute all contractions using correlators on $\mathbb{D}_2$

- $PSL(2,\mathbb{R})$: fix *three* vertex op. positions + insert c–ghosts ⇒ integral over real line ⇒ evaluate

$$\mathcal{A}(2,1) = \frac{g_c}{\alpha'^2}\ \mathrm{Tr}\left(T^a T^b\right) \sum_{i=1}^{4} \mathbf{A}_i$$

- schematically: $\mathbf{A}_i = \mathcal{K}(k_1, k_2, q; \alpha') \times \mathcal{I}(s)$ e.g.

$$\begin{aligned}\mathbf{A}_4 \quad &= \quad \frac{1}{16}\, 4^s \Bigg\{ 2A\, \frac{\sqrt{\pi} 2^{-s}\, s\, \Gamma\left(\frac{s-1}{2}\right)}{\Gamma\left(\frac{s}{2}+1\right)} - (C + \widetilde{\Delta})\, \frac{\sqrt{\pi} 2^{-s}\, \Gamma\left(\frac{s-1}{2}\right)}{\Gamma\left(\frac{s}{2}+1\right)} \\ &+ (E - F)\, \frac{(s-1)\left[\Gamma\left(\frac{s-1}{2}\right)\right]^2}{4\Gamma(s)} \Bigg\},\end{aligned}$$

we have computed the *full* amplitude, valid in both 10D and 4D and for any D–brane dimension

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Three massive spin–2 states

$$
\begin{aligned}
\mathcal{A}(3,0) \quad = \quad & \frac{g_o}{4\alpha'^3}\,\mathrm{Tr}(T^{a_1}\{T^{a_2},T^{a_3}\})\Big\{3\,(2\alpha')^2\,\mathrm{Tr}(\alpha^1\cdot\alpha^2\cdot\alpha^3)+(2\alpha')^3\times \\
& \Big[(k_1\cdot\alpha^2\cdot k_1)(\alpha^3\cdot\alpha^1)+(k_2\cdot\alpha^3\cdot k_2)(\alpha^2\cdot\alpha^1)+(k_3\cdot\alpha^1\cdot k_3)(\alpha^2\cdot\alpha^3) \\
& +3\,k_1\cdot\alpha^2\cdot\alpha^1\cdot\alpha^3\cdot k_2+3\,k_2\cdot\alpha^3\cdot\alpha^2\cdot\alpha^1\cdot k_3+3\,k_3\cdot\alpha^1\cdot\alpha^3\cdot\alpha^2\cdot k_1\Big] \\
& +(2\alpha')^4\Big[(k_1\cdot\alpha^2\cdot k_1)(k_2\cdot\alpha^3\cdot\alpha^1\cdot k_3)+(k_2\cdot\alpha^3\cdot k_2)(k_3\cdot\alpha^1\cdot\alpha^2\cdot k_1) \\
& +(k_3\cdot\alpha^1\cdot k_3)(k_1\cdot\alpha^2\cdot\alpha^3\cdot k_2)\Big]\Big\}
\end{aligned}
$$

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- no integrals ⇒ simpler extraction of Lagrangian
- compare with universal three–graviton amplitude !

Gross, Sloan 1987
Lüst, Theisen, Zoupanos 1988
Stieberger, Taylor ’14 – ’16

Effective Lagrangians

- Simplifications:
  1. $D^{\mu}_{\nu} = \delta^{\mu}_{\nu} \quad , \quad D^{\mu\nu} = D^{\mu}_{\lambda} g^{\lambda\nu} = g^{\mu\nu}$
  2. brane gauge group: $U(1)$
- Replacements

$$\varepsilon_{\mu\nu} \to G_{\mu\nu} \quad , \quad \alpha^{1,2}_{\mu\nu} \to M_{\mu\nu} \quad , \quad k_{\mu}\, , q_{\mu} \to i\partial_{\mu}$$

- How to truncate? How to expand in α'?

$$\alpha'\, k_1 \cdot k_2 \xrightarrow{\alpha' \to 0} 1 \quad , \quad \alpha'\, k_{1,2} \cdot q \xrightarrow{\alpha' \to 0} 0$$

$$s = -2 + 2\alpha' k_1 k_2 \xrightarrow{\alpha' \to 0} 0$$

- expansion in small s and substitution via $s = -2 + 2\alpha' k_1 k_2$

$\Rightarrow$ not meaningful truncation

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Effective Lagrangians

1. *instead*: expand $\mathcal{I}(s)$ in small $s = -2\alpha'\, k_1 \cdot q$

2. simplify $\mathcal{K}(k_1, k_2, q; \alpha')$ for spacetime–filling D–branes

3. keep terms up to order α'^2 in $\mathbf{A}_i = \mathcal{K}(k_1, k_2, q; \alpha') \times \mathcal{I}(s)$

$$\begin{aligned}\mathcal{A}(2,1) \quad = \quad & g_c \Big\{ -2\,\mathrm{Tr}(\alpha^1 \cdot \alpha^2)\, \varepsilon_{\mu\nu} k_1^\mu k_2^\nu + 2(\varepsilon \cdot \alpha^2 \cdot \alpha^1)_{\mu\nu} k_1^\mu k_2^\nu \\ & +2(\varepsilon \cdot \alpha^1 \cdot \alpha^2)_{\mu\nu} k_1^\nu k_2^\mu + 2(\varepsilon \cdot \alpha^2 \cdot \alpha^1)_{\mu\nu}\, k_2^\mu q^\nu + 2(\varepsilon \cdot \alpha^1 \cdot \alpha^2)_{\mu\nu}\, k_1^\mu q^\nu \\ & +2\,\mathrm{Tr}(\varepsilon \cdot \alpha^1 \cdot \alpha^2)\,(k_1 \cdot q) + \big[\,\mathrm{Tr}(\varepsilon \cdot \alpha^2)\alpha^1_{\mu\nu} - 2(\alpha^1 \cdot \varepsilon \cdot \alpha^2)_{\mu\nu} \\ & + \mathrm{Tr}(\varepsilon \cdot \alpha^1)\alpha^2_{\mu\nu}\big]\, q^\mu q^\nu \Big\} + \mathcal{O}\left(\alpha'^3\right)\,.\end{aligned}$$

Lüst, CM, Mazloumi, Stieberger ’21

Effective Lagrangians

- up to two–derivatives:

$$\mathcal{L}^{\text{eff}}_{\text{G}^3} = g_c\, G^{\mu\nu}\left[\partial_\mu G_{\rho\sigma}\partial_\nu G^{\rho\sigma} - 2\partial_\nu G_{\rho\sigma}\partial^\sigma G^\rho_\mu\right]$$

$$\mathcal{L}^{\text{eff}}_{\text{GM}^2} = g_c\left[G^{\mu\nu}\left(\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 4\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\right)\right.$$
$$\left. + M^{\mu\nu}\left(\partial_\mu G_{\rho\sigma}\partial_\nu M^{\rho\sigma} - \partial_\rho G_{\mu\sigma}\partial_\nu M^{\rho\sigma}\right)\right]$$

- can we set $\frac{1}{m_g\sqrt{1+\alpha^2}} \equiv g_c\,$?

$$\mathcal{L}^{\text{eff}}_{\text{M}^3} = \tfrac{g_o}{\alpha'}\left\{\left[M^3\right] + 2\alpha' M^{\mu\nu}\left[\partial_\mu M_{\rho\sigma}\partial_\nu M^{\rho\sigma} - 3\partial_\nu M_{\rho\sigma}\partial^\sigma M^\rho_\mu\right]\right.$$
$$\left. + 4\alpha'^2\,\partial^\mu\partial^\nu M_{\rho\sigma}\partial^\rho M^\kappa_\nu\partial^\sigma M_{\mu\kappa}\right\}$$

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Conclusions

- We have extracted the low–energy interactions of massive string states from string amplitudes for the first time

- GM^2 and M^3: the string and bimetric yield the *same* set of terms, but $\exists$ coefficient discrepancy $\Rightarrow M_{\mu\nu}$ kinematics not GR–like

  Lüst, CM, Mazloumi, Stieberger '21

- our 2 vs 3 discrepancy is strikingly reminiscent of but qualitatively different from the vDVZ discontinuity

$$\mathcal{D}^{M}_{\mu\nu,\kappa\lambda}(p) \xrightarrow{m_{\rm FP}\to 0} \frac{-i}{p^2}\left[\frac{1}{2}(\eta_{\mu\kappa}\eta_{\nu\lambda}+\eta_{\mu\lambda}\eta_{\nu\kappa})-\frac{1}{3}\eta_{\mu\nu}\eta_{\kappa\lambda}\right]+\text{singular terms}$$

  van Dam, Veltman, Zakharov 1970

- closed string case more promising

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