# Entanglement and complexity of islands

be.HEP meeting

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Brussels, Dec 22 2021

# Outline

# What is a black hole?

Astrophysicist:

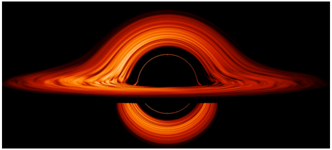
- Massive object in space
- Collapse from gravitational force

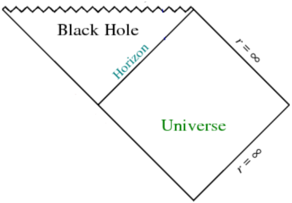
Theorist:

- Solution to Einstein's equations
- Singularity
- Event horizon

Information theorist:

- System with (nearly) maximal entropy
- Fast scrambling





# Black holes

Astrophysics

- Active galactic nuclei
- Gravitational waves
- EH telescope

Theorists

- Black hole thermodynamics
- Playground for QG
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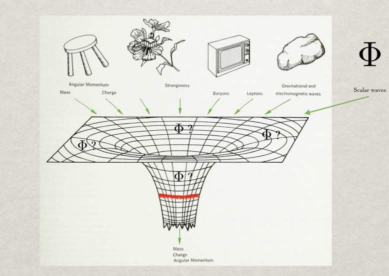

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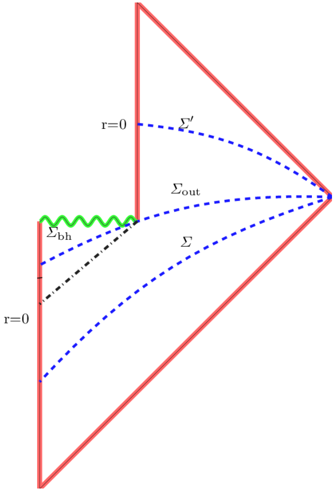

# Outline

# Entropy

Entanglement entropy

- $\rho_A = \mathrm{Tr}_{\bar{A}} |\Psi\rangle\langle\Psi|$
- $S(\rho_A) = -\mathrm{Tr}_A \left(\rho_A \mathrm{log} \rho_A\right)$
- Quantifies the amount of entanglement of $\rho_A$

Thermodynamic entropy

- $N = \#$ of states compatible with observables $\lambda_i$
- $S(\lambda_i) = \log N$
- For BH: $S(M, Q, J) = \frac{A}{4G}$

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In a quantum theory, entanglement entropy $\leq$ thermodynamic entropy

# Information paradox

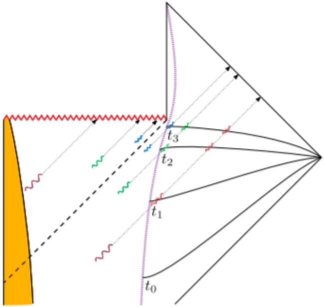
## Entropy curve

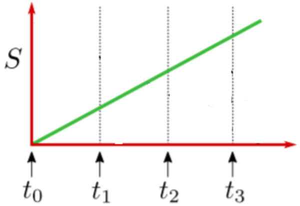
Hawking radiation

- Particle pair creation
- Near horizon
- Fall/escape of partners

Entanglement of black hole

- Pairs are entangled
- Constant Hawking radiation
- Linear increase in entropy



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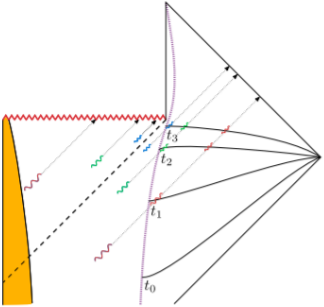


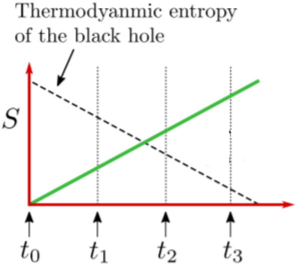

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# Holography

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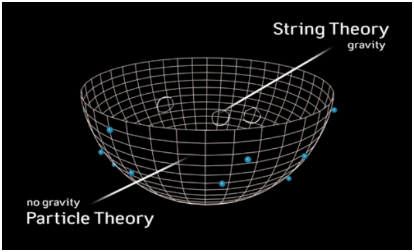
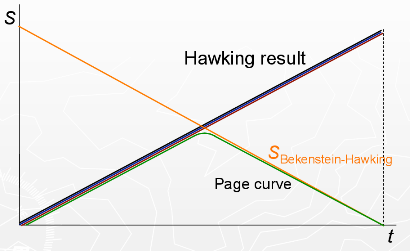
- Equivalence between two theories
- $d+1$ dimensional quantum gravity
- $d$ dimensional quantum field theory



# Holography

Information paradox

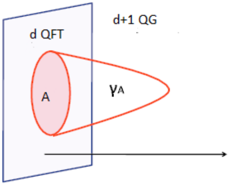
- Black hole in $d+1$ dimensional quantum gravity
- Think in terms of $d$ dimensional quantum field theory
- Entanglement entropy must somehow decrease



# Holographic entanglement entropy

Holographic dictionary

- Entanglement entropy of $\rho_A$
- Area of surface $\gamma_A$
- Geometric computation




$$S(\rho_A) = \frac{A(\gamma_A)}{4G}$$

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# Generalized entropy and island rule

For a theory with gravity, generalized entropy

$$S_{gen}(R) = S_{EE}(R) + \frac{A(\partial R)}{4G}$$

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$\frac{A(\partial I)}{4G}$ is very big. Islands only relevant when there is a lot of entanglement between $R$ and $I$

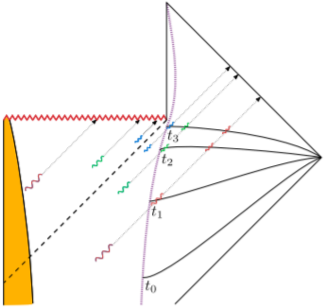
# The island rule

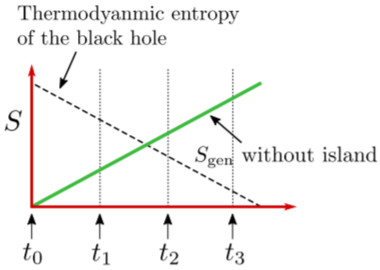
Island rule

- Entanglement entropy
- When gravity is included
- Allow for "entanglement islands"

Entanglement curve

- Early: no islands
- Entanglement increases
- Late: island configuration
- Entropy decreases



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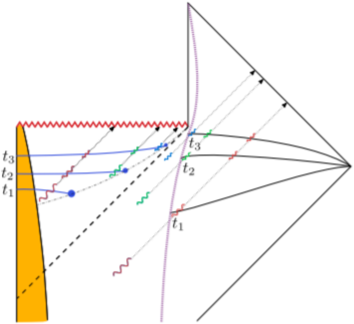


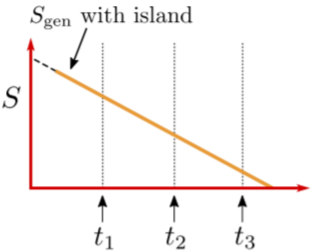

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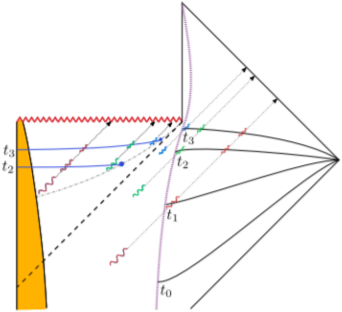
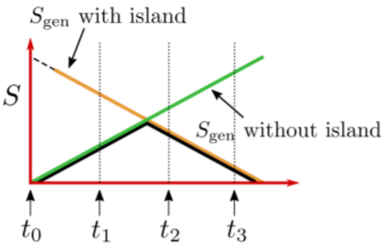
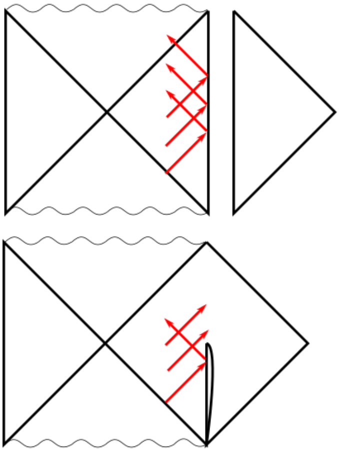


# Example: BH + bath

Black hole + bath models

- Black hole in gravitational theory
- Couple to non gravitational baths
- Black hole evaporates into baths



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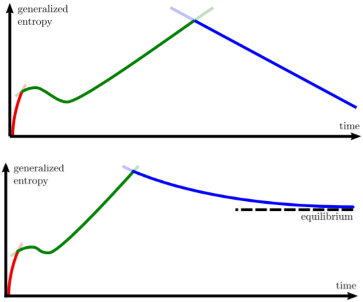


# Outline

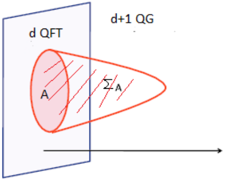
# Holographic complexity

Entangelment entropy is not enough

- Entanglement entropy equilibrates fast
- Quantifies entanglement between $A$ and $A^c$

Another measure of entanglement

- Keeps increasing for very long times
- Sensitive to entanglement within $A$




$$C(\rho_A) = \frac{V(\Sigma_A)}{4G\ell}$$

# Complexity

## Circuit complexity

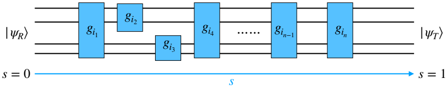
What is holographic complexity on the field theory side?

- Slow to thermalize
- Linear increase in time
- Reach very large values

Circuit complexity

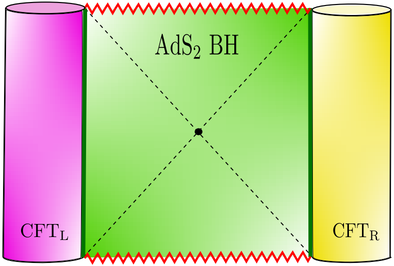
$$C_{\Psi_R}(\Psi_T) = \min_U D(U)$$

$$s.t. \quad U|\Psi_R\rangle = |\Psi_T\rangle$$



# Entanglement vs complexity

Example: black hole + circular baths




Three extremal surfaces

- Early time surface
- Thermal surface
- Island surface

Two phases

- Early
- Late

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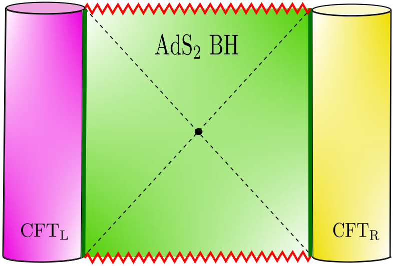
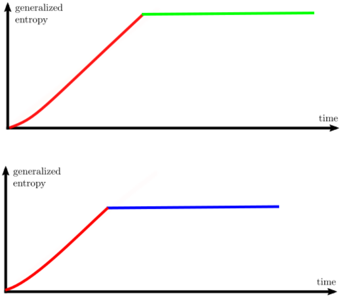



Entropy curve



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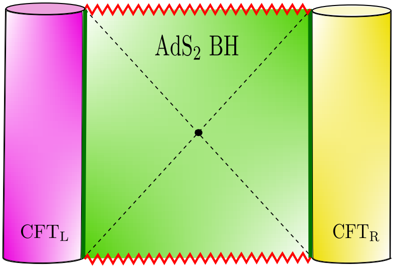
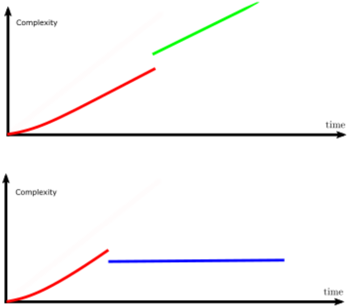



Complexity curve



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# Conclusion

Summary

- Black holes and information paradox
- Resolution: islands
- Two measures of entanglement
- Entanglement entropy
- Complexity