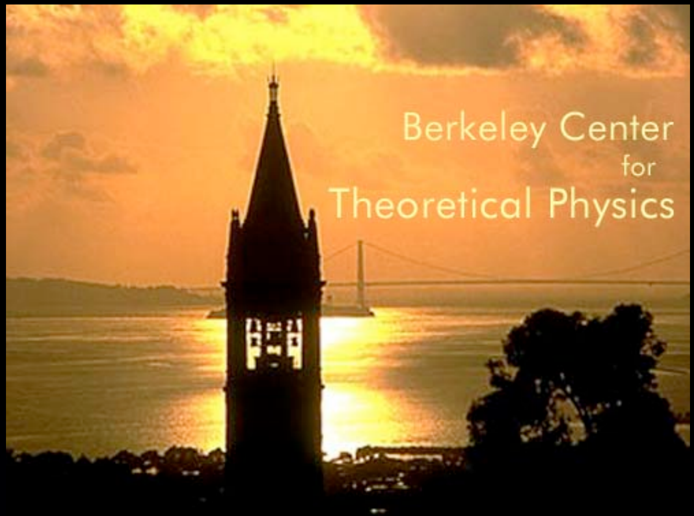


Probing the Structure of Jets with Effective Field Theory

Christopher Lee
Berkeley Center for Theoretical Physics
and Lawrence Berkeley National Laboratory

Catholic University of Louvain
Center for Particle Physics and Phenomenology
Theory Seminar
January 29, 2009

In collaboration with...

- Andrew Hornig, Grigory Ovanesyan (Berkeley)

  arXiv: 0901.3780, 0901.1879

- Christian Bauer (Berkeley), Sean Fleming (Arizona), George Sterman (Stony Brook)

  PRD **78**, 034027 (2008)
  PRD **75**, 014022 (2007)

QCD in e^+e^- annihilation

- At LHC, we are searching for new physics in large QCD dominated Standard Model backgrounds

- From studying e^+e^- annihilation first, learn about:
  - Effects of nonperturbative soft radiation in the final state
  - Useful observables to study jet-like structure in final state or substructure of individual jets
  - How to develop better methods in perturbation theory and models of nonperturbative physics in a simpler environment

Asymptotic Freedom

- At large energies, the strong coupling $\alpha_s(Q)$ is small, allowing reliable calculations in perturbation theory (**asymptotic freedom**)
    - e.g. total cross-section of e^+e^- to hadrons

- At low energies, $\alpha_s(\Lambda)$ grows large, inducing larger nonperturbative corrections
    - e.g. exclusive cross-sections

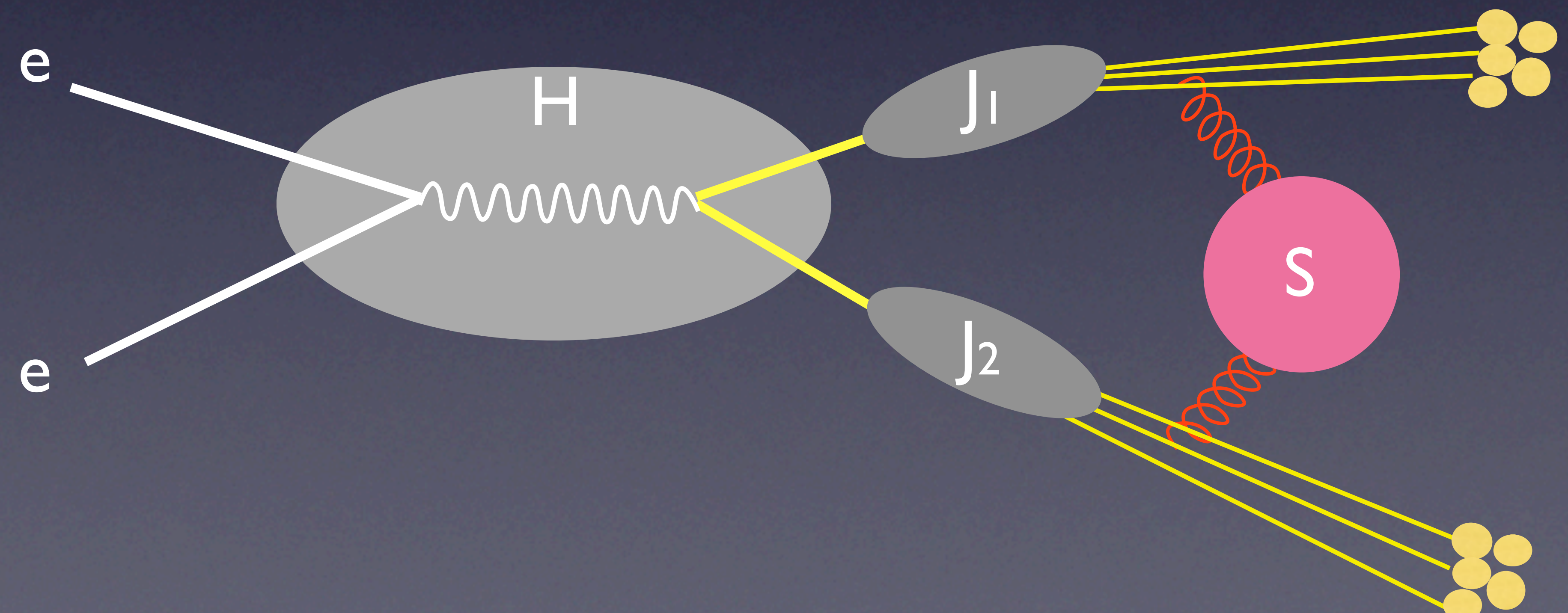
Factorization

Collins, Soper, Sterman (1980s)

- Separate full cross-sections into *calculable* partonic functions and *nonperturbative (noncalculable)* but (hopefully) universal functions

$$\sigma(e^+e^- \to n \text{ jets}) \sim \sigma(e^+e^- \to \text{ partons})\sigma(\text{partons} \to \text{ hadrons})$$

$$H \otimes J_1 \otimes J_2 \otimes S$$



Factorization

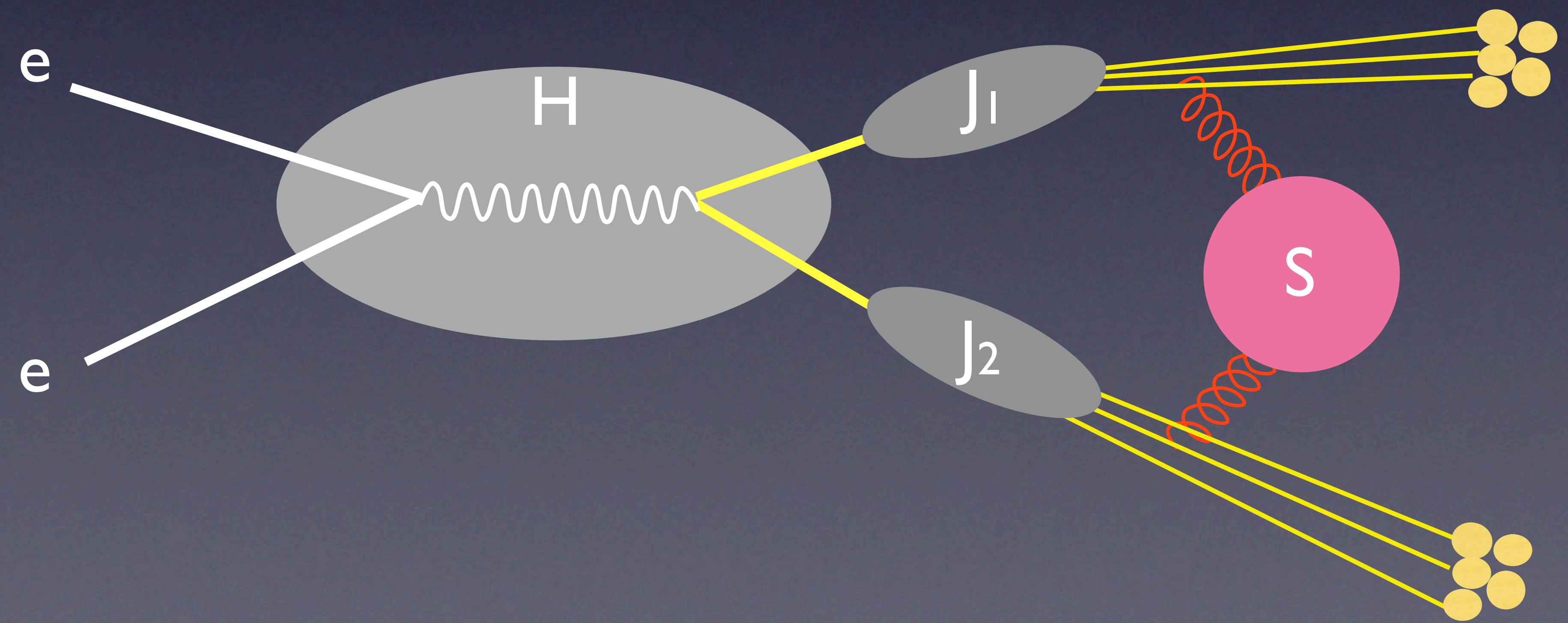
Collins, Soper, Sterman (1980s)

- Separate full cross-sections into *calculable* partonic functions and *nonperturbative (noncalculable)* but (hopefully) universal functions

$$\sigma(e^+e^- \to n \text{ jets}) \sim \sigma(e^+e^- \to \text{ partons})\sigma(\text{partons} \to \text{ hadrons})$$

$$H \otimes J_1 \otimes J_2 \otimes S$$

Does the observable factorize?



Factorization

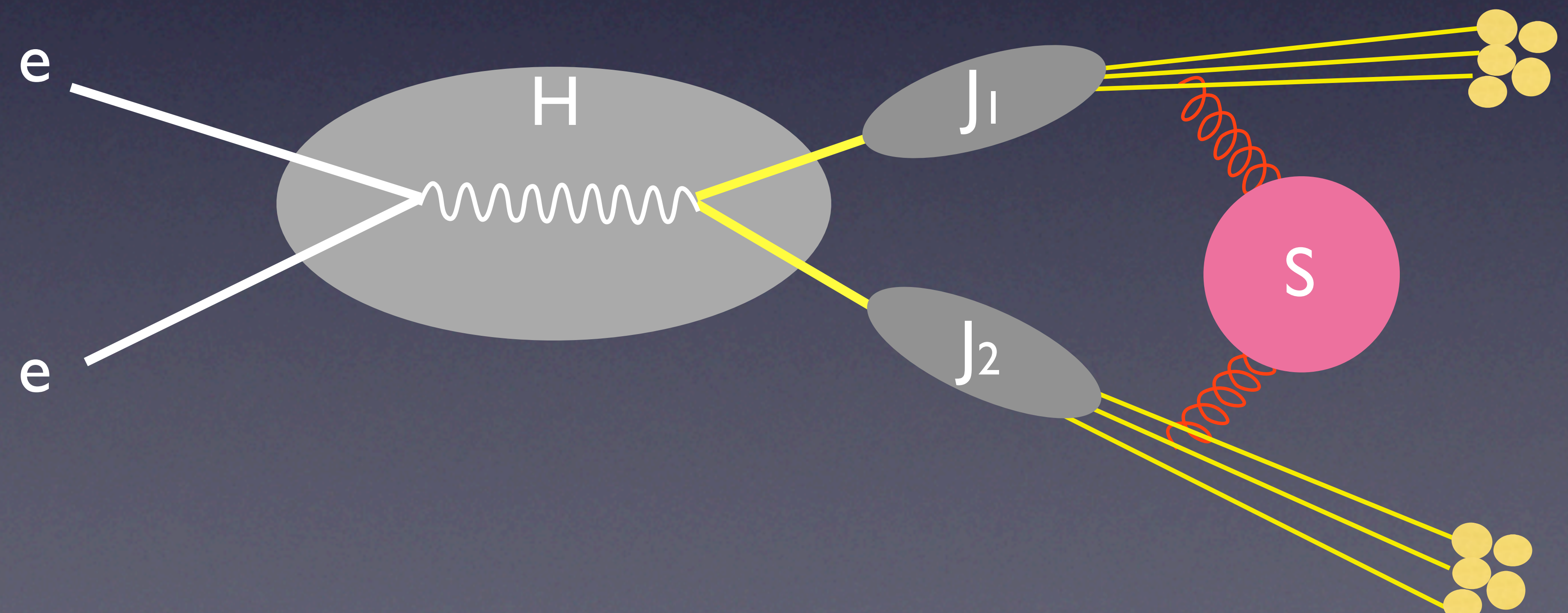
Collins, Soper, Sterman (1980s)

- Separate full cross-sections into *calculable* partonic functions and *nonperturbative (noncalculable)* but (hopefully) universal functions

$$\sigma(e^+e^- \to n \text{ jets}) \sim \sigma(e^+e^- \to \text{ partons})\sigma(\text{partons} \to \text{ hadrons})$$

$$H \otimes J_1 \otimes J_2 \otimes S$$

Does the observable factorize?



Are the noncalculable functions universal?

Plan of the talk

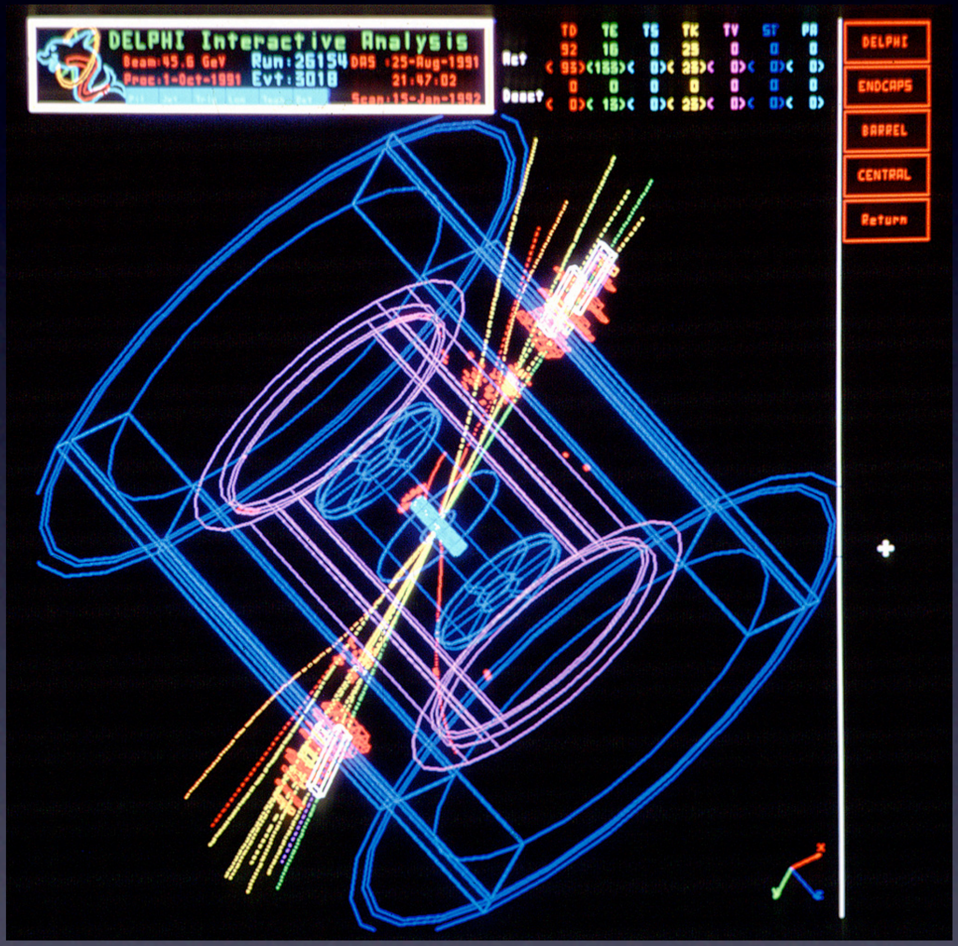
- General Introduction
- Event Shapes in e^+e^- annihilation
- Soft-Collinear Effective Theory (SCET)
- Energy Flow and Factorization in SCET
- RG Evolution and Resummation in SCET
- Nonperturbative Models of Hadronization Effects
- Final Predictions for Angularity Distributions
- Conclusions

Plan of the talk

- General Introduction
- Event Shapes in e^+e^- annihilation
- Soft-Collinear Effective Theory (SCET)
- Energy Flow and Factorization in SCET
- RG Evolution and Resummation in SCET
- Nonperturbative Models of Hadronization Effects
- Final Predictions for Angularity Distributions
- Conclusions

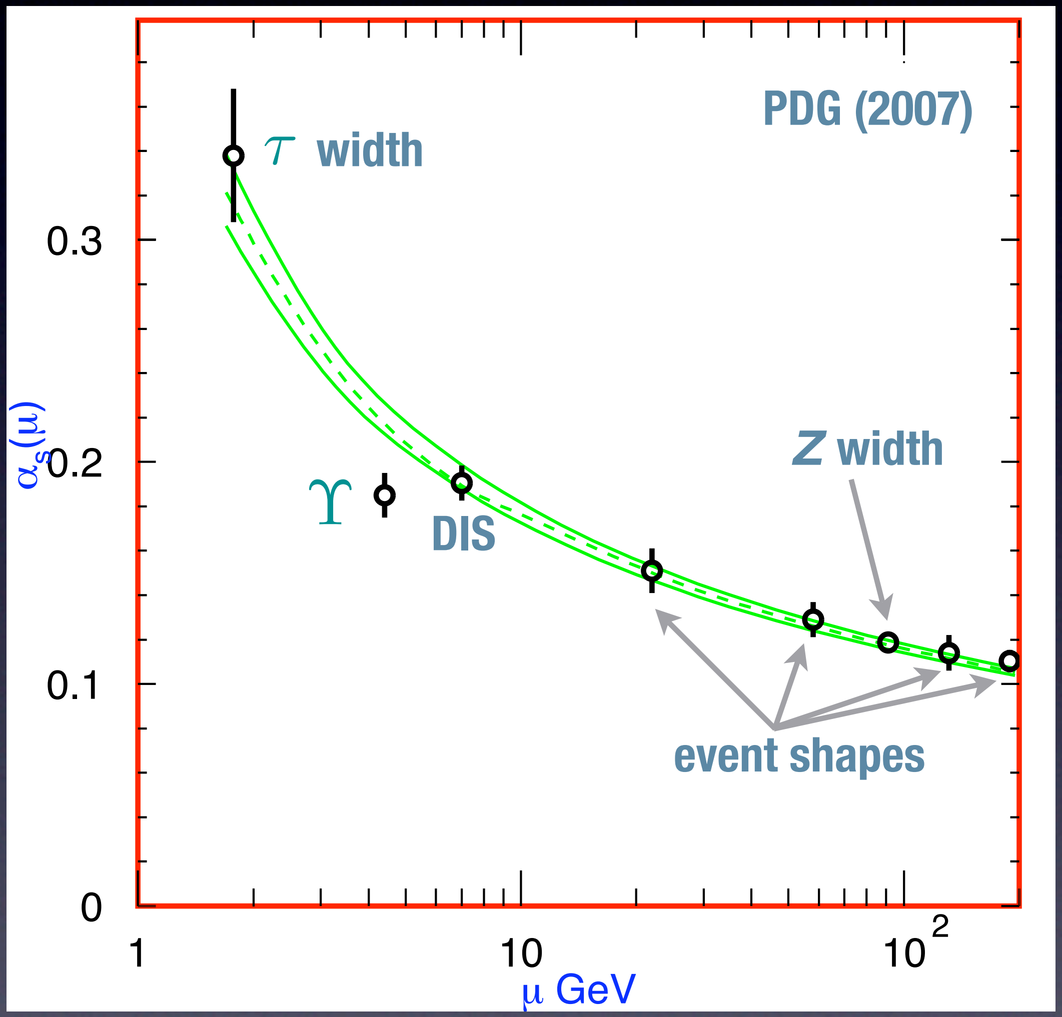
Event Shapes

- Hadrons produced in high-energy collisions usually produced as jets
- Two-jet events are easily distinguished using *event shapes*
- **Event shapes** e are number-valued observables, functions of all the final state momenta, such that two-jet events have $e \approx 0$



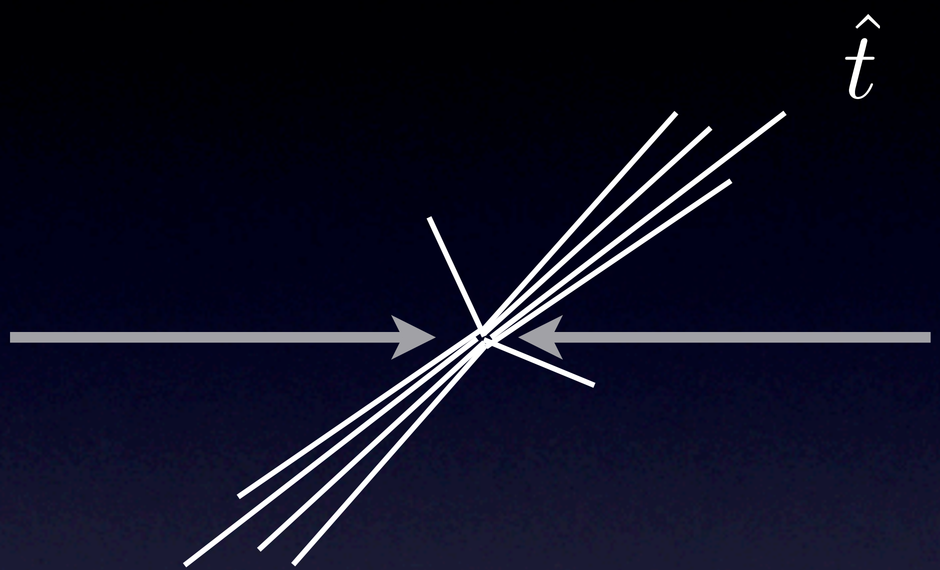
Event Shapes

- Hadrons produced in high-energy collisions usually produced as jets
- Two-jet events are easily distinguished using *event shapes*
- **Event shapes** e are number-valued observables, functions of all the final state momenta, such that two-jet events have $e \approx 0$



Some event shapes

review:
Dasgupta, Salam (2003)

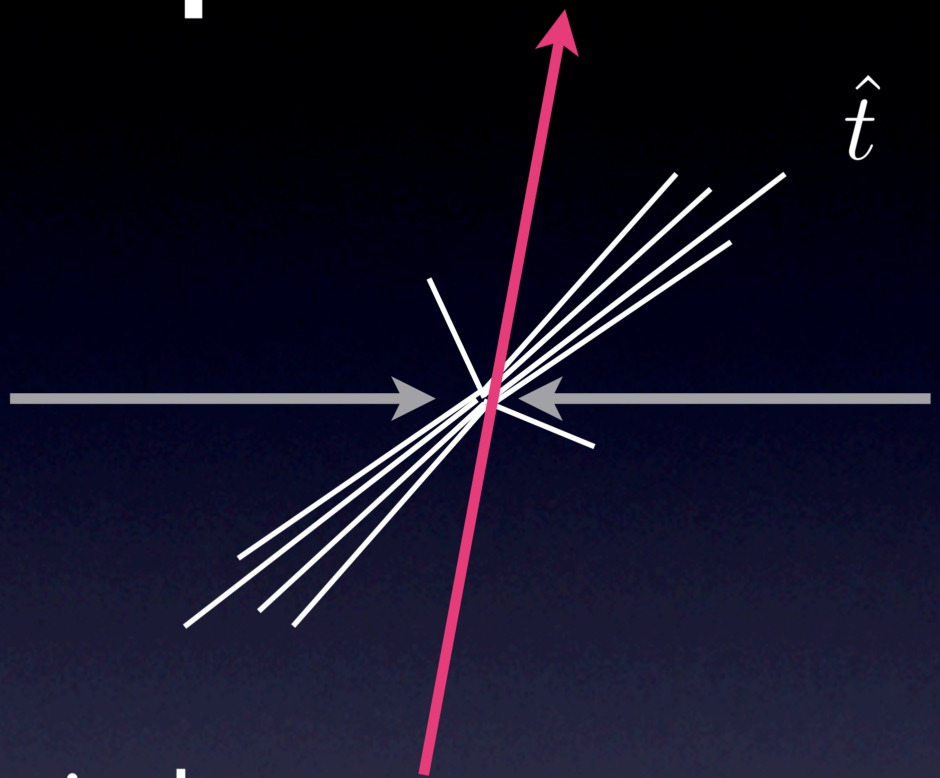


- **Thrust** $T = \frac{1}{Q} \max_{\hat{t}} \sum_i |\hat{t} \cdot \mathbf{p}_i|$
    - use $\tau = 1 - T$
    - Thrust axis is in the direction of the total hemisphere three-momentum $\hat{t} = \hat{\mathbf{p}}_A$
- **Jet broadening** $B = \frac{1}{Q} \sum_i |\hat{t} \times \mathbf{p}_i|$
- **Jet invariant masses** M_A^2, M_B^2

Some event shapes

review:
Dasgupta, Salam (2003)

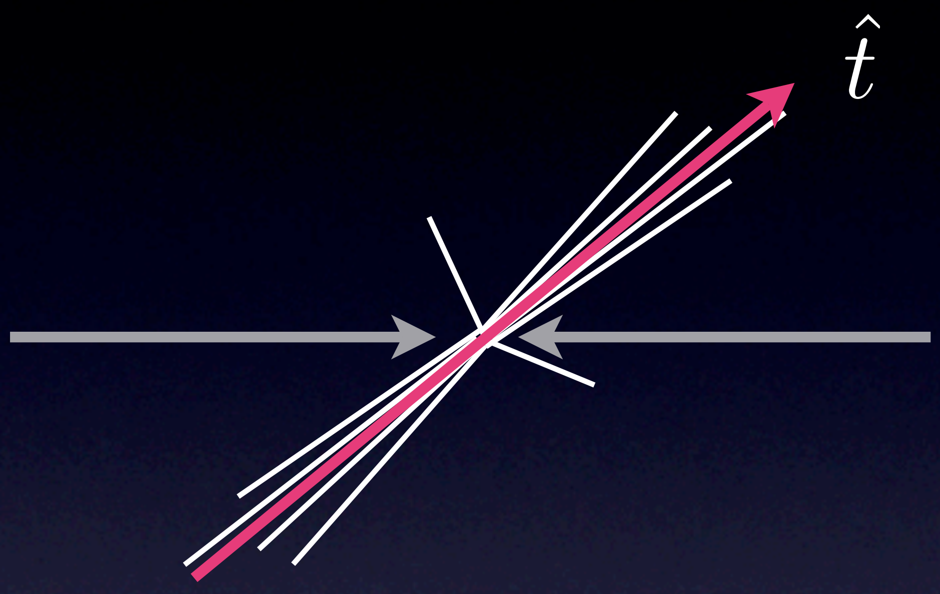
- **Thrust** $T = \frac{1}{Q} \max_{\hat{t}} \sum_i |\hat{t} \cdot \mathbf{p}_i|$
  - use $\tau = 1 - T$
  - Thrust axis is in the direction of the total hemisphere three-momentum $\hat{t} = \hat{\mathbf{p}}_A$



- **Jet broadening** $B = \frac{1}{Q} \sum_i |\hat{t} \times \mathbf{p}_i|$
- **Jet invariant masses** M_A^2, M_B^2

Some event shapes

review:
Dasgupta, Salam (2003)

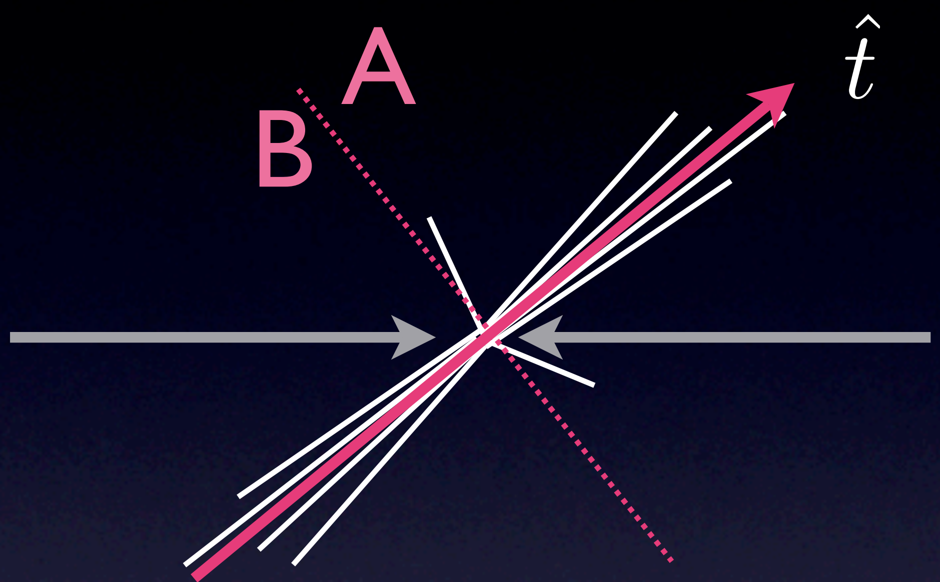


- **Thrust** $T = \frac{1}{Q} \max_{\hat{t}} \sum_i |\hat{t} \cdot \mathbf{p}_i|$
  - use $\tau = 1 - T$
  - Thrust axis is in the direction of the total hemisphere three-momentum $\hat{t} = \hat{\mathbf{p}}_A$
- **Jet broadening** $B = \frac{1}{Q} \sum_i |\hat{t} \times \mathbf{p}_i|$
- **Jet invariant masses** M_A^2, M_B^2

Some event shapes

review:
Dasgupta, Salam (2003)

- **Thrust** $T = \frac{1}{Q} \max_{\hat{t}} \sum_i |\hat{t} \cdot \mathbf{p}_i|$
  - use $\tau = 1 - T$
  - Thrust axis is in the direction of the total hemisphere three-momentum $\hat{t} = \hat{\mathbf{p}}_A$



- **Jet broadening** $B = \frac{1}{Q} \sum_i |\hat{t} \times \mathbf{p}_i|$
- **Jet invariant masses** M_A^2, M_B^2

Angularities

Berger, Kucs, Sterman (2003)

- Generalization of thrust:

$$\tau_a = \frac{1}{Q}\sum_i E_i(\sin\theta_i)^a(1-|\cos\theta_i|)^{1-a} \quad = \frac{1}{Q}\sum_i |\mathbf{p}_i^T| e^{-|\eta_i|(1-a)}$$

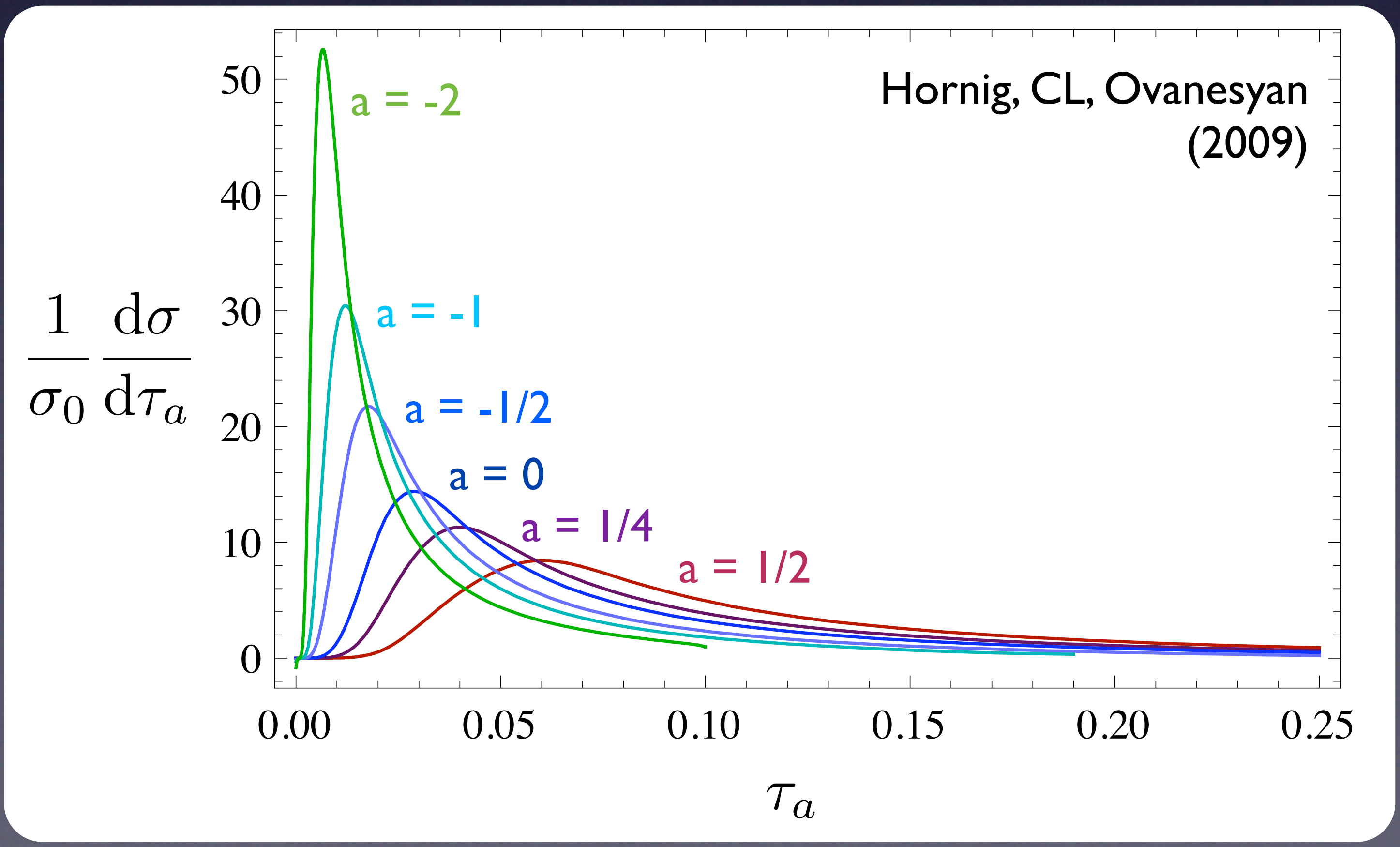
a = 0 thrust

a = 1 broadening

infrared safety: $-\infty < a < 2$

factorizability: $-\infty < a < 1$

Peak region dominated by narrower jets for large *a*, wider jets for small *a*



Angularities

Berger, Kucs, Sterman (2003)

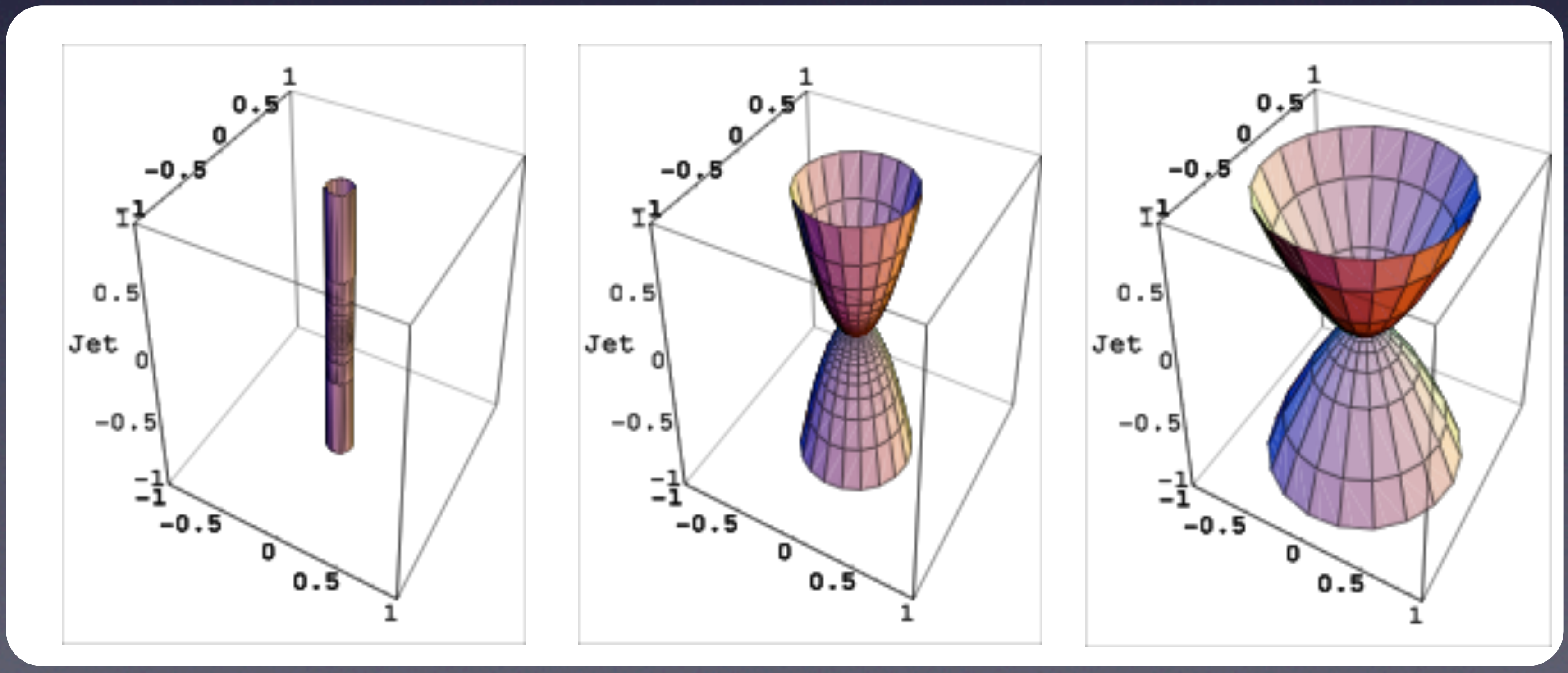
- Generalization of thrust:

$$\tau_a = \frac{1}{Q}\sum_i E_i (\sin\theta_i)^a (1-|\cos\theta_i|)^{1-a} \quad = \frac{1}{Q}\sum_i |\mathbf{p}_i^T| e^{-|\eta_i|(1-a)}$$

a = 0 thrust
a = 1 broadening

infrared safety: $-\infty < a < 2$
factorizability: $-\infty < a < 1$

Peak region dominated by narrower jets for large *a*, wider jets for small *a*



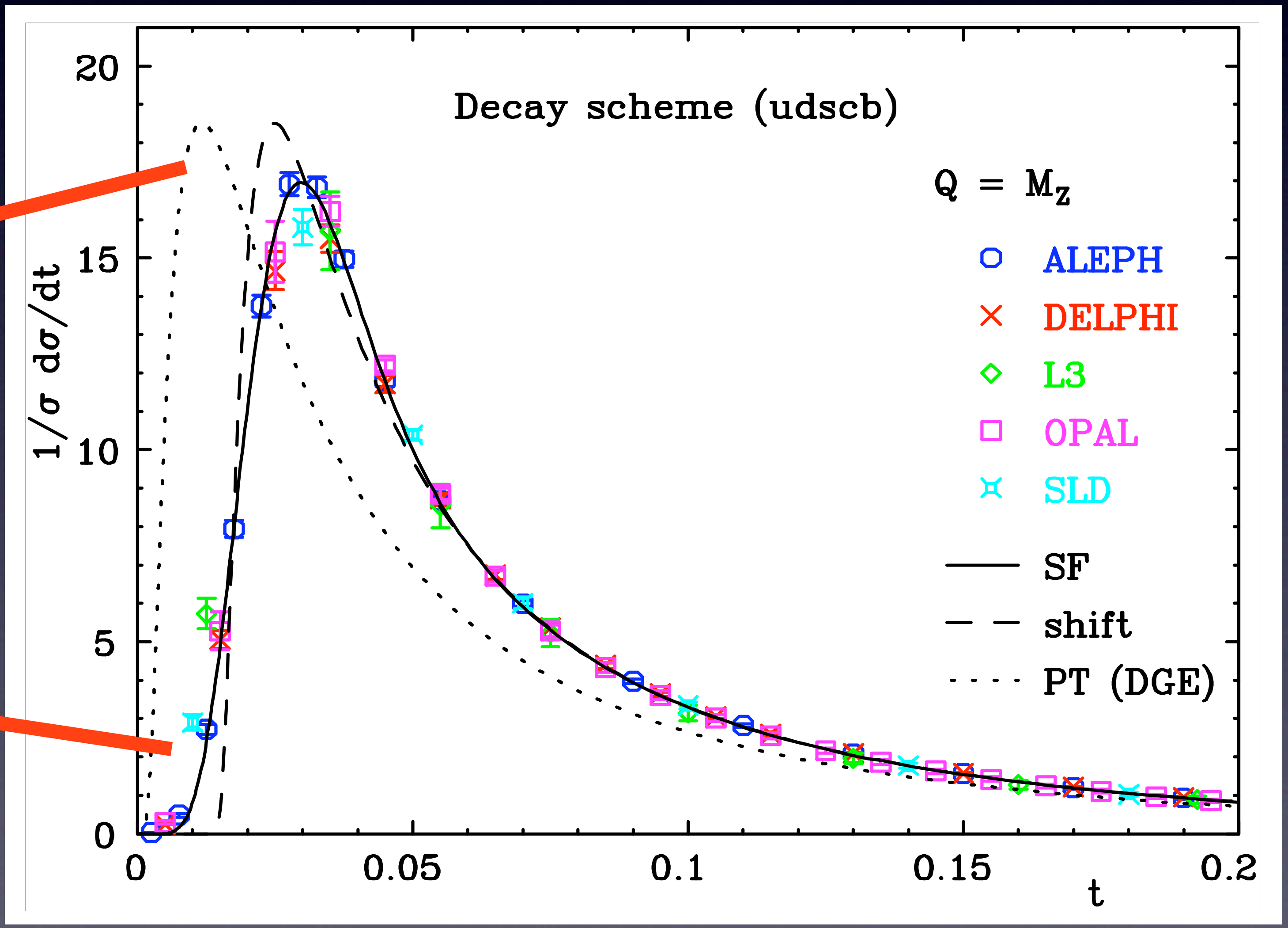
a = 1 a = 0 a = -1

Challenges in Predicting Distributions

- Distribution of events in $t = 1\text{-}T$:

$\frac{1}{Q}$ power corrections due to nonperturbative soft radiation

Large logarithms $\frac{\ln^n e}{e}$ must be resummed



Gardi, Rathsman (2002)

Plan of the talk

- General Introduction
- Event Shapes in e^+e^- annihilation
- Effective Field Theories and SCET
- Energy Flow and Factorization in SCET
- RG Evolution and Resummation in SCET
- Nonperturbative Models of Hadronization Effects
- Final Predictions for Angularity Distributions
- Conclusions

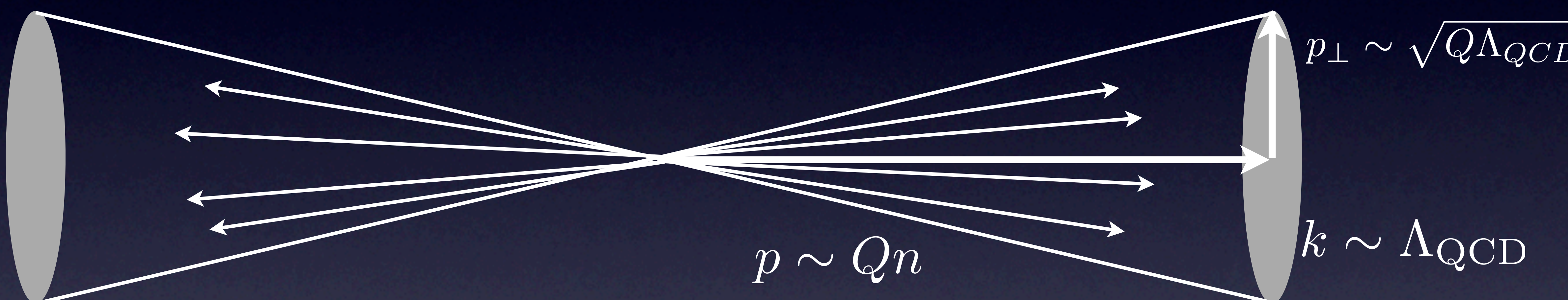
Effective Field Theory

- Integrate out degrees of freedom not relevant to the kinematics of the processes in question
- Expand full theory quantities in powers of small parameters, ratios of disparate energy scales
- Effective theory often possesses enhanced symmetries or simplifications not immediately evident in full theory

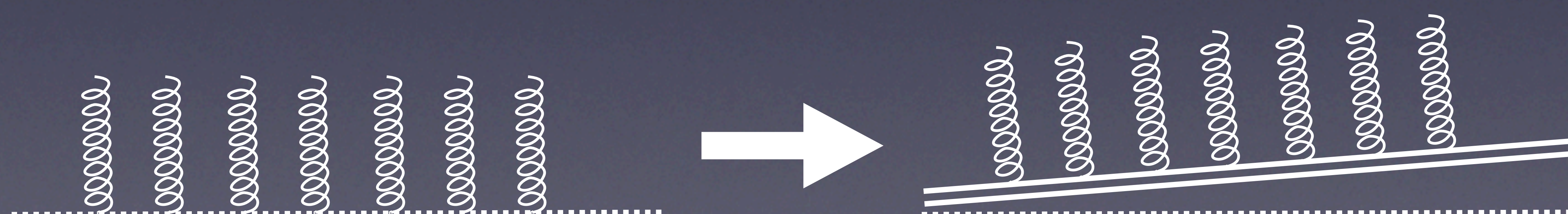
Soft-Collinear Effective Theory

Bauer, Fleming, Luke, Pirjol, Stewart (2000, 2001)

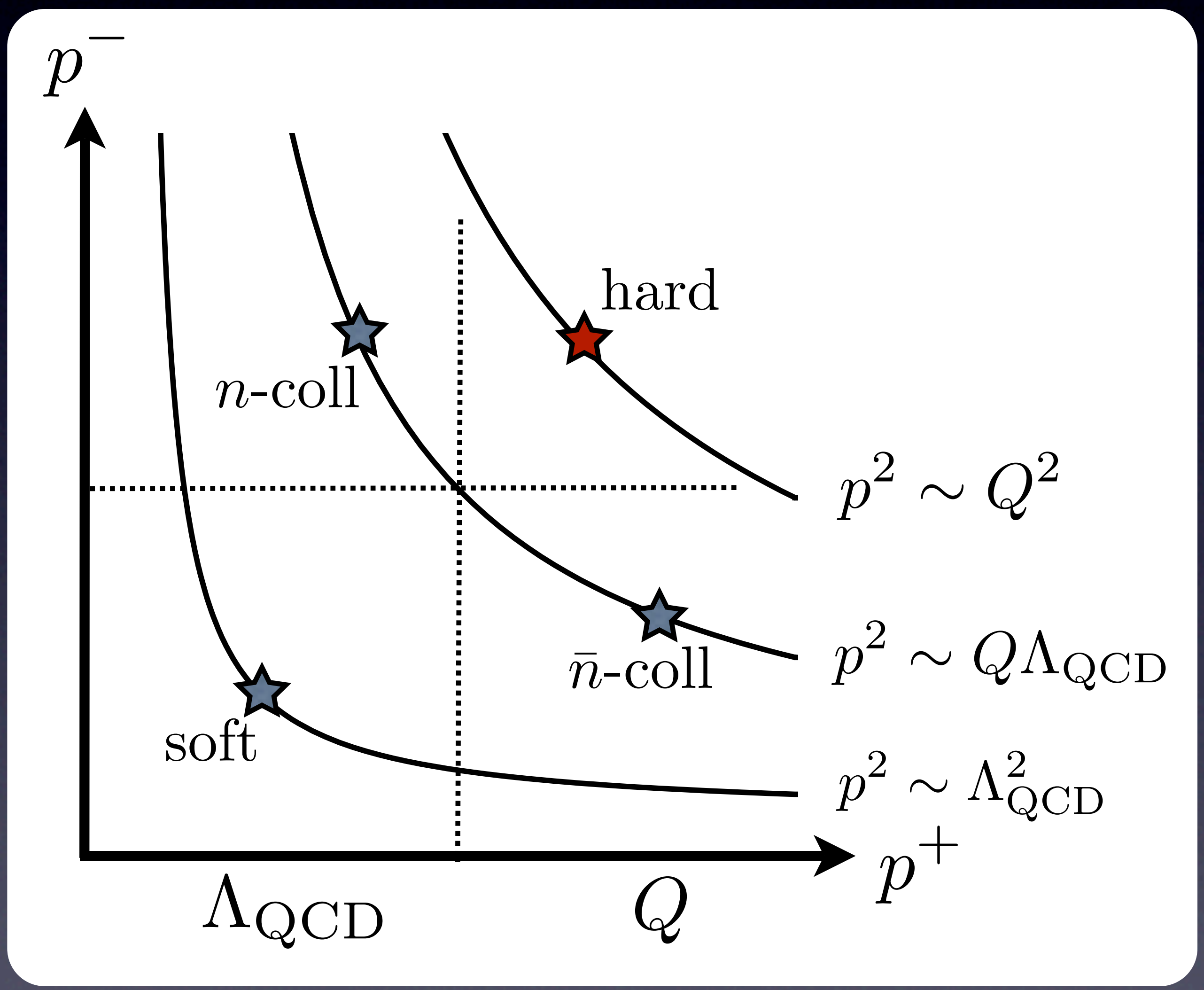
- Effective field theory for Λ_{QCD} fluctuations about lightlike trajectory *n* = (1,0,0,1)

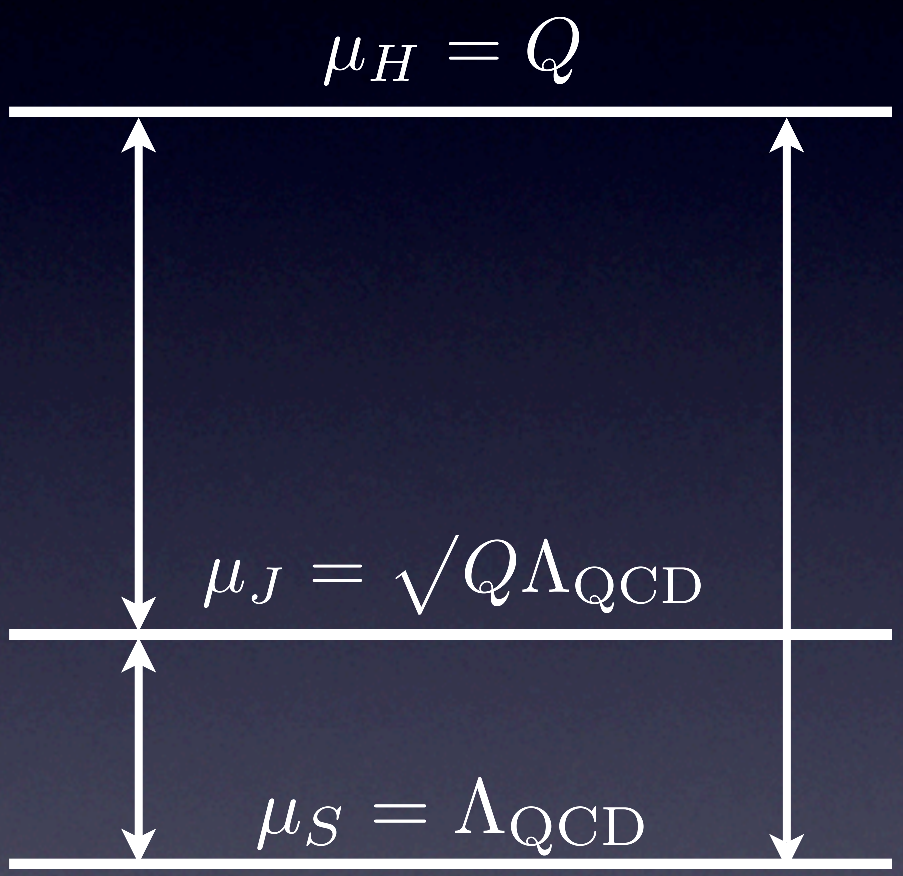


- Simplifies demonstration of *soft-collinear decoupling*



Collinear and soft modes

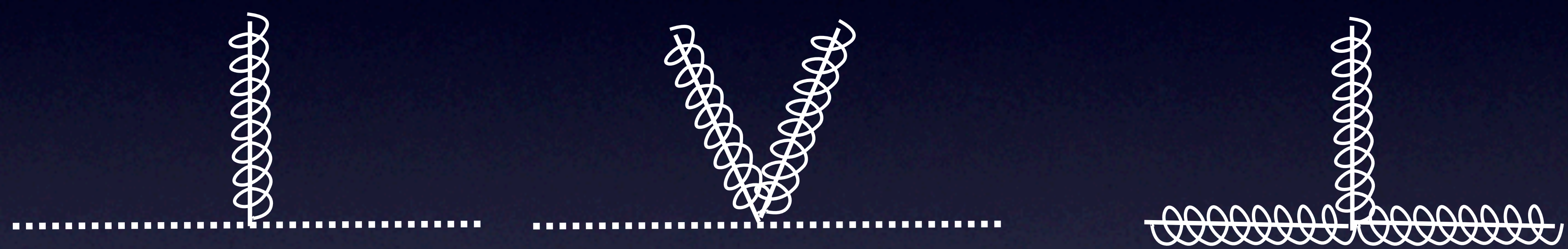




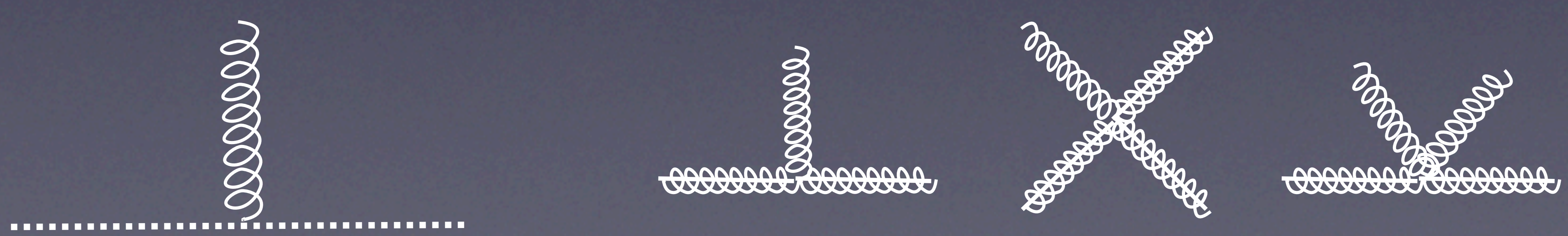
SCET Lagrangian is expansion of QCD Lagrangian in powers of $\lambda \sim \sqrt{\frac{\Lambda_{\rm QCD}}{Q}}$

Interactions in SCET

- Collinear sector interactions:



- Soft parton interactions with each other same as in QCD
- Soft gluon interactions with collinear sector:



Soft-collinear decoupling

Bauer, Pirjol, Stewart (2001)

- Collinear quark-soft gluon interaction:

$$\mathcal{L} \ni \bar{\xi}_n in \cdot D_s \frac{\bar{\not{n}}}{2} \xi_n = \bar{\xi}_n in \cdot (\partial_s + igA_s) \frac{\bar{\not{n}}}{2} \xi_n$$

- Soft Wilson line: $Y_n(z) = P \exp \left[ig \int_0^\infty ds\, n \cdot A_s(ns + z) \right]$
- Field redefinition:

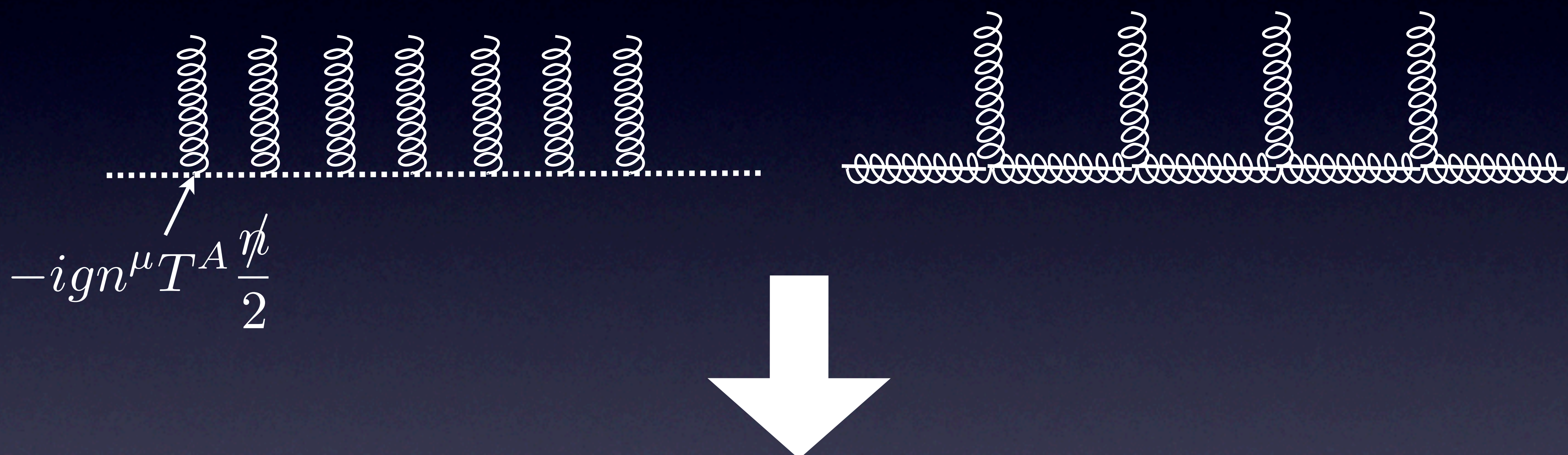
$$\xi_n = Y_n \xi_n^{(0)}$$

$$A_n = Y_n A_n^{(0)} Y_n^\dagger$$

- Removes coupling in Lagrangian:

$$n \cdot D_s Y_n(z) = 0 \Longrightarrow \mathcal{L}^{(0)} \ni \bar{\xi}_n^{(0)} in \cdot \partial_s \frac{\bar{n}}{2} \xi_n^{(0)}$$

Soft-collinear decoupling



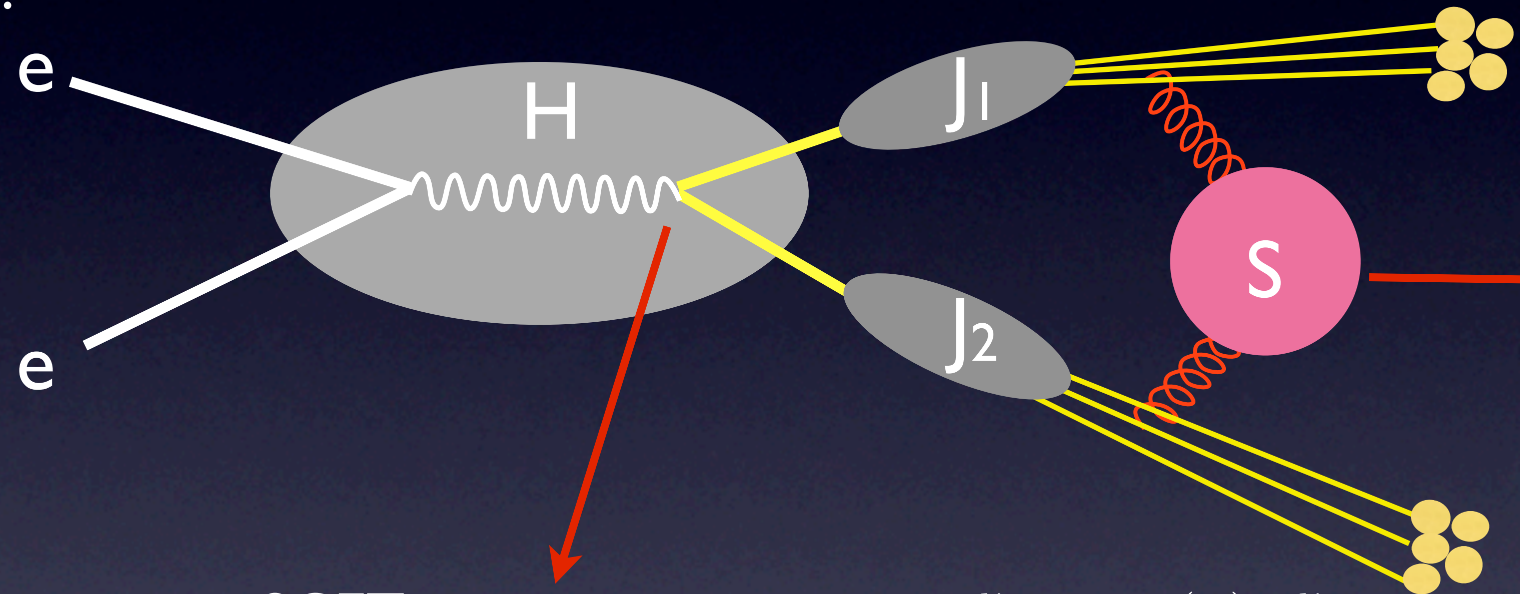
$$Y_n(z) = P \exp\left[ig \int_0^\infty ds\, n \cdot A_s(ns + z)\right]$$

Plan of the talk

- General Introduction
- Event Shapes in e^+e^- annihilation
- Effective Field Theories and SCET
- Energy Flow and Factorization in SCET
- RG Evolution and Resummation in SCET
- Nonperturbative Models of Hadronization Effects
- Final Predictions for Angularity Distributions
- Conclusions

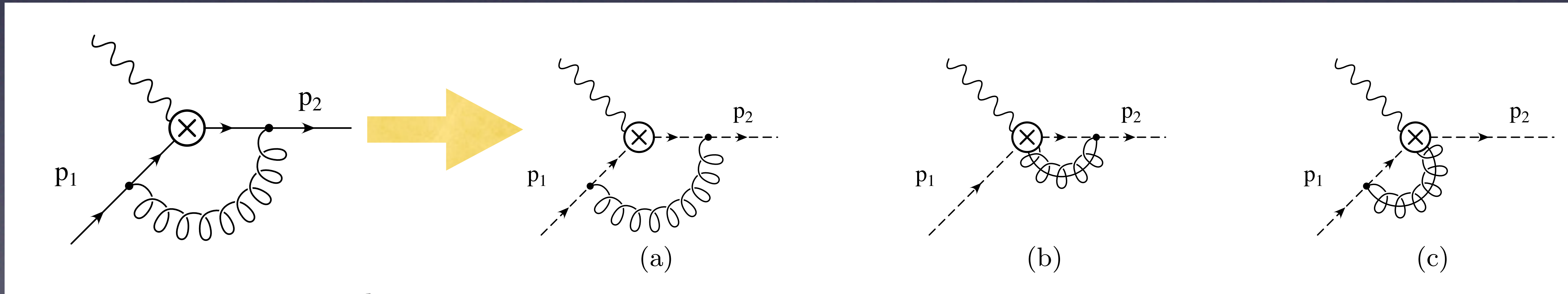
Event shape distribution

- Differential cross-section for e^+e^- annihilation to hadrons with respect to event shape *e:*



Redefine collinear fields in current to decouple soft gluon interactions

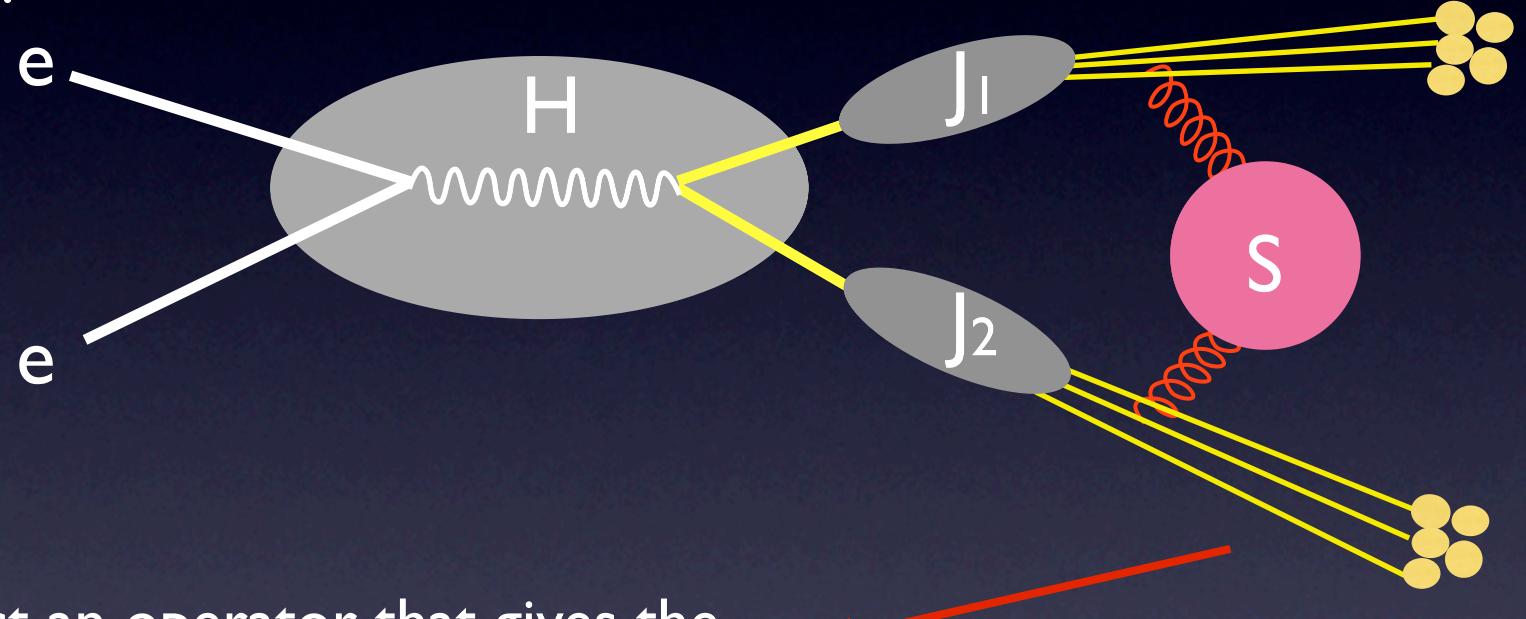
Match QCD current onto SCET two-jet operators $\bar{q}\gamma^{\nu}q = C_2(\mu)O_2^{\nu}$



IR divergences same as in QCD, UV behavior differs

Event shape distribution

- Differential cross-section for e^+e^- annihilation to hadrons with respect to event shape *e*:



Construct an operator that gives the value of the event shape of the final state

$$\hat{e}|N\rangle \equiv e(N)|N\rangle$$

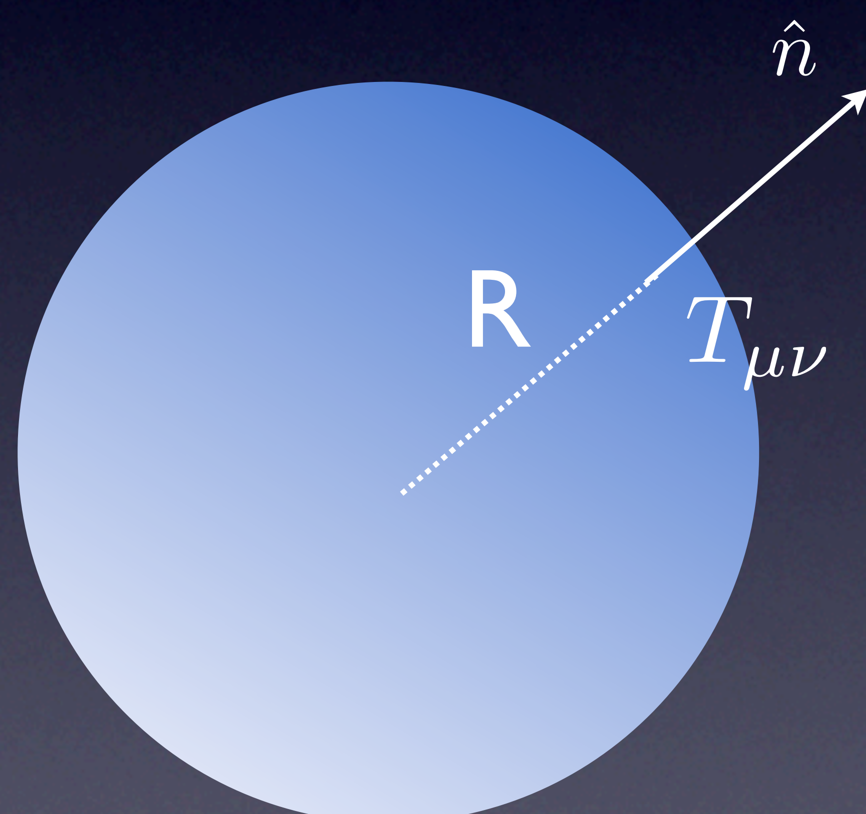
CL, Sterman (2007)

$$\mathcal{E}_T(\eta)|N\rangle = \sum_{i\in N} |\mathbf{p}_i^{\perp}|\delta(\eta - \eta_i)|N\rangle \Rightarrow \hat{e} = \frac{1}{Q}\int_{-\infty}^{\infty} d\eta\, f_e(\eta)\mathcal{E}_T(\eta)$$

Energy flow

- These operators can be constructed from the operator measuring energy flow in a given direction:

$$\mathcal{E}(\hat{n})|N\rangle = \sum_{i\in N} E_i \delta^2(\hat{n}-\hat{n}_i)|N\rangle$$



$$\mathcal{E}(\hat{n}) = \lim_{R\to\infty} \int_0^\infty dt\, \hat{n}_i T_{0i}(t, R\hat{n})$$

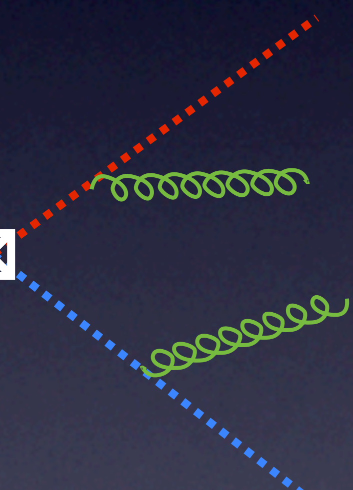
Korchemsky, Oderda, Sterman (1997); cf. Sveshnikov, Tkachov (1996)

Factorization

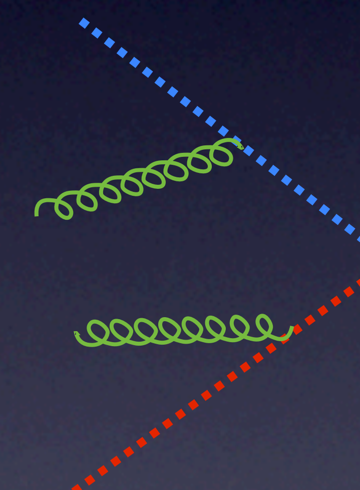
Bauer, Fleming, CL,
Sterman (2008)

- Matrix elements in event shape distribution factorize due to collinear-soft decoupling:

$$\frac{d\sigma}{de} \sim \sum_N \langle 0 | \boxtimes$$



$$\delta(e - e(N)) | N \rangle \langle N |$$



$$\boxtimes | 0 \rangle$$

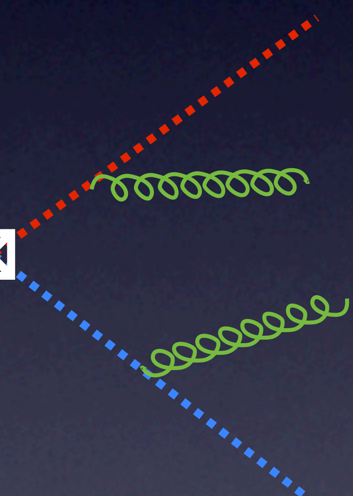
Factorization

Bauer, Fleming, CL, Sterman (2008)

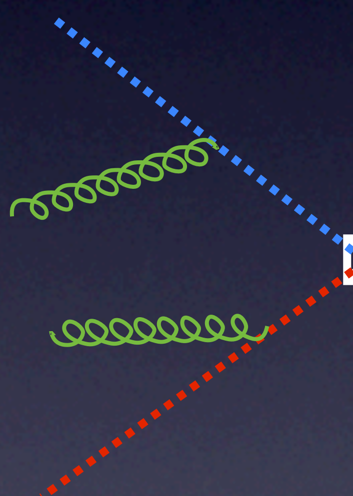
- Matrix elements in event shape distribution factorize due to collinear-soft decoupling:

Express event shape as operator:

$$\frac{d\sigma}{de} \sim \sum_N \langle 0| \boxtimes$$



$$\delta(e - \hat{e}(T_{\mu\nu}))|N\rangle\langle N|$$



$$\boxtimes\, |0\rangle$$

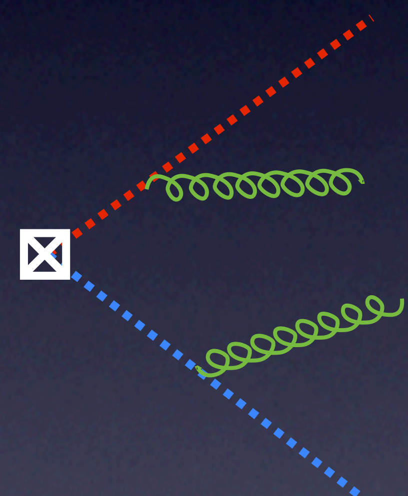
Factorization

Bauer, Fleming, CL, Sterman (2008)

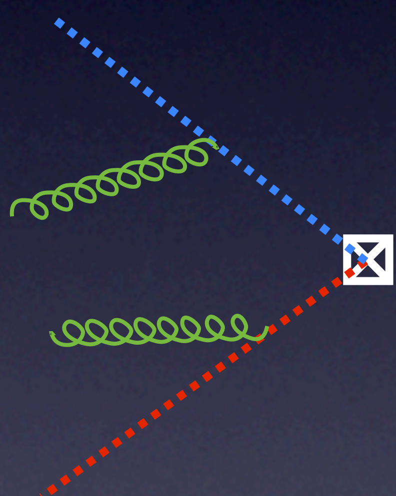
- Matrix elements in event shape distribution factorize due to collinear-soft decoupling:

Sum over complete set of hadronic states:

$$\frac{d\sigma}{de} \sim \langle 0|$$



$$\delta(e - \hat{e}(T_{\mu\nu}))$$



$$|0\rangle$$

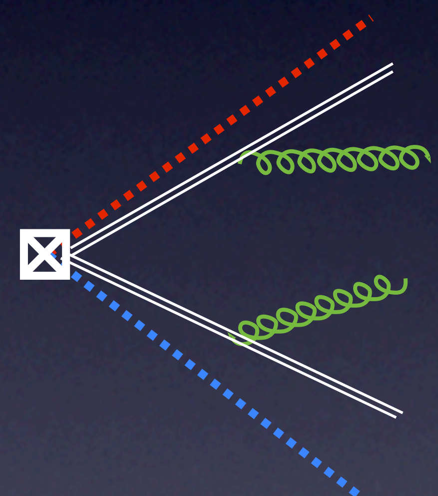
Factorization

Bauer, Fleming, CL, Sterman (2008)

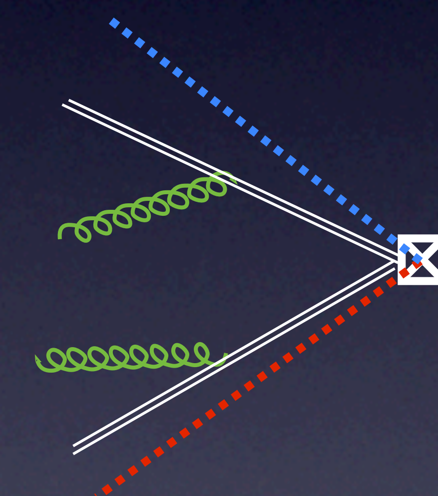
- Matrix elements in event shape distribution factorize due to collinear-soft decoupling:

Decouple soft gluons through collinear field redefinition:

$$\frac{d\sigma}{de} \sim \langle 0|$$



$$\delta(e - \hat{e}(T_{\mu\nu}))$$



$$|0\rangle$$

$$\boxed{T_{\mu\nu} = T^{n}_{\mu\nu} + T^{\bar{n}}_{\mu\nu} + T^{s}_{\mu\nu}}$$

Factorization

Bauer, Fleming, CL, Sterman (2008)

- Matrix elements in event shape distribution factorize due to collinear-soft decoupling:

Factorize matrix elements:

$$\frac{d\sigma}{de} \sim \langle 0| \boxtimes \cdots \delta(e_n - \hat{e}_n(T^n_{\mu\nu})) \cdots \boxtimes |0\rangle$$
$$\times \langle 0| \boxtimes \cdots \delta(e_{\bar{n}} - \hat{e}_{\bar{n}}(T^{\bar{n}}_{\mu\nu})) \cdots \boxtimes |0\rangle$$
$$\times \langle 0| \boxtimes \quad \delta(e_s - \hat{e}_s(T^s_{\mu\nu})) \quad \boxtimes |0\rangle$$

- Convolution of hard, jet, and soft functions:

$$\frac{d\sigma}{de} = H(Q,\mu) \int de_n de_{\bar{n}} J_n(e_n;\mu) J_{\bar{n}}(e_{\bar{n}};\mu) S(e - e_n - e_{\bar{n}};\mu)$$

Plan of the talk

- General Introduction
- Event Shapes in e^+e^- annihilation
- Effective Field Theories and SCET
- Energy Flow and Factorization in SCET
- RG Evolution and Resummation in SCET
- Nonperturbative Models of Hadronization Effects
- Final Predictions for Angularity Distributions
- Conclusions

Counting Logs

- Distribution contains divergent terms $\frac{\ln^n \tau_a}{\tau_a}$
- or, in integrated distribution $\alpha_s^n \ln^m \tau_a$

$$R(\tau_a) = \int d\tau_a \frac{1}{\sigma_0} \frac{d\sigma}{d\tau_a}$$

- Resum all terms up to a given order using $\ln \tau_a \sim \frac{1}{\alpha_s}$

$$\begin{aligned} R(\tau_a) \sim 1 \quad &+ \quad \alpha_s(\ln^2 \tau_a + \ln \tau_a + 1) \\ &+ \quad \alpha_s^2(\ln^4 \tau_a + \ln^3 \tau_a + \ln^2 \tau_a + \cdots) \\ &+ \quad \cdots \end{aligned}$$

Counting Logs

- Distribution contains divergent terms $\dfrac{\ln^n \tau_a}{\tau_a}$
- or, in integrated distribution $\alpha_s^n \ln^m \tau_a$

$$R(\tau_a) = \int d\tau_a \frac{1}{\sigma_0} \frac{d\sigma}{d\tau_a}$$

- Resum all terms up to a given order using $\ln \tau_a \sim \dfrac{1}{\alpha_s}$

$$\begin{aligned} R(\tau_a) \sim 1 \quad &+ \quad \alpha_s(\boxed{\ln^2 \tau_a} + \ln \tau_a + 1) \\ &+ \quad \alpha_s^2(\boxed{\ln^4 \tau_a} + \ln^3 \tau_a + \ln^2 \tau_a + \cdots) \\ &+ \quad \cdots \end{aligned}$$

LL

Counting Logs

- Distribution contains divergent terms $\frac{\ln^n \tau_a}{\tau_a}$
- or, in integrated distribution $\alpha_s^n \ln^m \tau_a$

$$R(\tau_a) = \int d\tau_a \frac{1}{\sigma_0} \frac{d\sigma}{d\tau_a}$$

- Resum all terms up to a given order using $\ln \tau_a \sim \frac{1}{\alpha_s}$

$$\begin{aligned} R(\tau_a) \sim 1 \quad & + \quad \alpha_s(\ln^2 \tau_a + \ln \tau_a + 1) \\ & + \quad \alpha_s^2(\ln^4 \tau_a + \ln^3 \tau_a + \ln^2 \tau_a + \cdots) \\ & + \quad \cdots \end{aligned}$$

LL NLL

Counting Logs

- Distribution contains divergent terms $\frac{\ln^n \tau_a}{\tau_a}$
- or, in integrated distribution $\alpha_s^n \ln^m \tau_a$

$$R(\tau_a) = \int d\tau_a \frac{1}{\sigma_0} \frac{d\sigma}{d\tau_a}$$

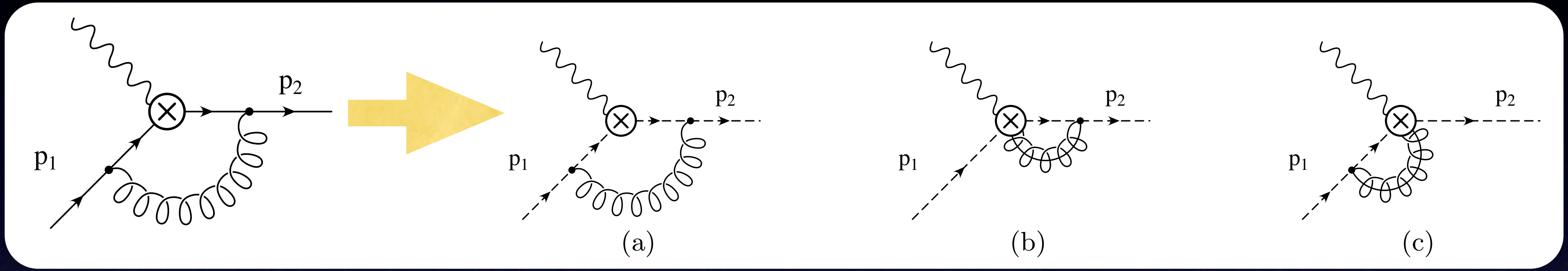
- Resum all terms up to a given order using $\ln \tau_a \sim \frac{1}{\alpha_s}$

$$\begin{aligned} R(\tau_a) \sim 1 \quad &+ \quad \alpha_s(\ln^2 \tau_a + \ln \tau_a + 1) \\ &+ \quad \alpha_s^2(\ln^4 \tau_a + \ln^3 \tau_a + \ln^2 \tau_a + \cdots) \\ &+ \quad \cdots \end{aligned}$$

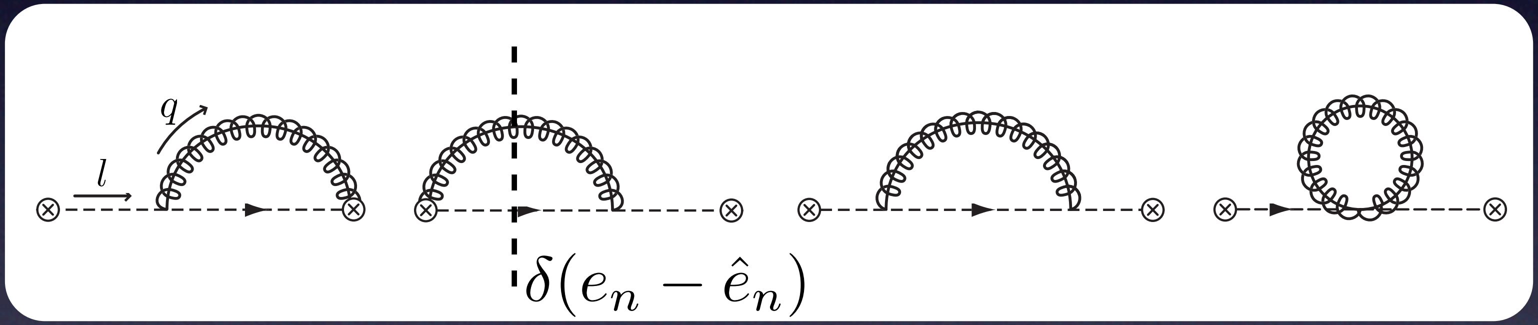
LL **NLL** **NNLL**

Factorized Functions at One Loop

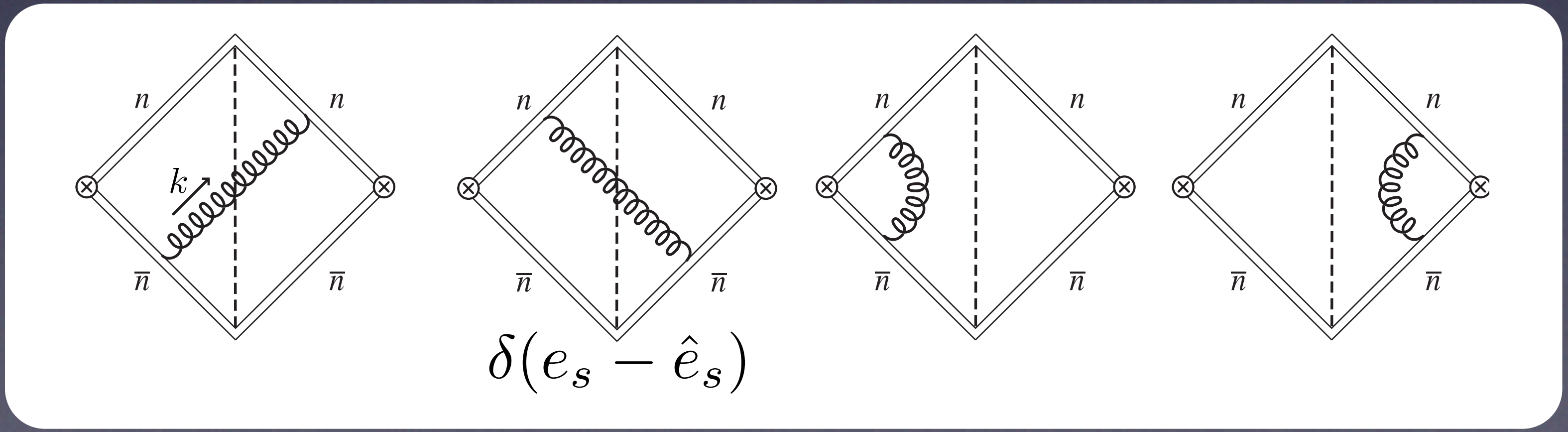
- Hard Function



- Jet Function



- Soft Function



Factorized Functions at One Loop

- Hard Function

$$H(Q;\mu) = 1 - \frac{\alpha_s(\mu)C_F}{2\pi}\left(8 - \frac{7\pi^2}{6} + \ln^2\frac{\mu^2}{Q^2} + 3\ln\frac{\mu^2}{Q^2}\right)$$

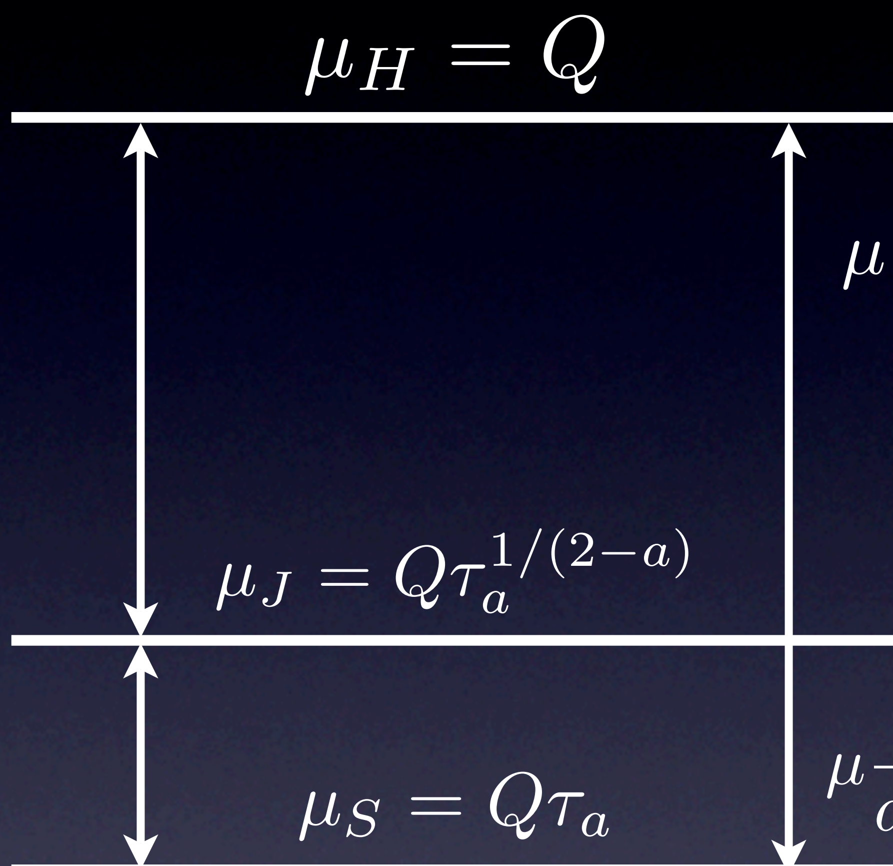
- Jet Function

$$\begin{aligned} J_a^n(\tau_a^n;\mu) &= \delta(\tau_a^n)\left\{1 + \frac{\alpha_s C_F}{\pi}\left[\frac{1-a/2}{2(1-a)}\ln^2\frac{\mu^2}{Q^2} + \frac{3}{4}\ln\frac{\mu^2}{Q^2} + f(a)\right]\right\} \\ &\quad - \frac{\alpha_s C_F}{\pi}\left[\left(\frac{3}{4}\frac{1}{1-a/2} + \frac{2}{1-a}\ln\frac{\mu}{Q(\tau_a^n)^{1/(2-a)}}\right)\left(\frac{\theta(\tau_a^n)}{\tau_a^n}\right)\right]_+ \end{aligned}$$

- Soft Function

$$S_a^{\mathrm{PT}}(\tau_a^s;\mu) = \delta(\tau_a^s)\left[1 - \frac{\alpha_s C_F}{\pi(1-a)}\left(\frac{1}{2}\ln^2\frac{\mu^2}{Q^2} - \frac{\pi^2}{12}\right)\right] + \frac{2\alpha_s C_F}{\pi(1-a)}\left[\frac{\theta(\tau_a^s)}{\tau_a^s}\ln\frac{\mu^2}{(Q\tau_a^s)^2}\right]_+$$

Running Among Scales



$$\mu \frac{d}{d\mu} H(Q;\mu) = \gamma_H(\mu) H(Q;\mu)$$

$$\gamma_H(\mu) = \Gamma_H[\alpha_s] \ln \frac{\mu^2}{Q^2} + \tilde{\gamma}_H[\alpha_s]$$

$$\mu \frac{d}{d\mu} F(\tau_a;\mu) = \int d\tau_a' \gamma_F(\tau_a - \tau_a';\mu) F(\tau_a';\mu)$$

$F = J, S$

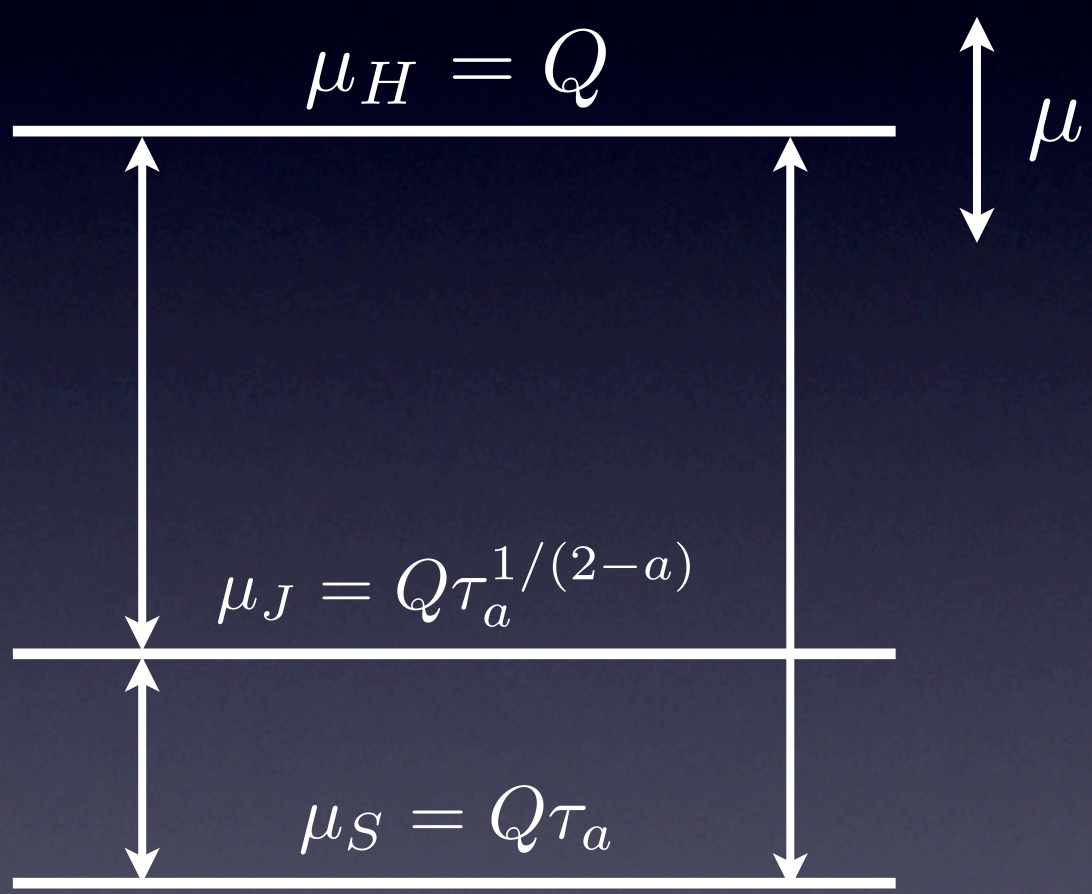
$$\gamma_F(\tau_a;\mu) = -\Gamma_F[\alpha_s] \left(\frac{2}{j_F} \left[\frac{\theta(\tau_a)}{\tau_a} \right]_+ - \ln \frac{\mu^2}{Q^2} \delta(\tau_a) \right) + \tilde{\gamma}_F[\alpha_s] \delta(\tau_a)$$

NLL: 2-loop 1-loop

Consistency condition:

$$-\gamma_H(\mu)\delta(\tau) = 2\gamma_J(\tau;\mu) + \gamma_S(\tau;\mu)$$

Orders in RG-improved PT



	Initial conditions	Running
	$H(Q;\mu_H)$	$\gamma_H(Q;\mu,\mu_H)$
	$J(\tau_a;\mu_J)$	$\gamma_J(\tau_a;\mu,\mu_J)$
	$S(\tau_a;\mu_S)$	$\gamma_S(\tau_a;\mu,\mu_s)$
Calculate to:	$\mathrm{N^nLO}$	$\mathrm{N^nLL}$

Plan of the talk

- General Introduction
- Event Shapes in e^+e^- annihilation
- Effective Field Theories and SCET
- Energy Flow and Factorization in SCET
- RG Evolution and Resummation in SCET
- Nonperturbative Models of Hadronization Effects
- Final Predictions for Angularity Distributions
- Conclusions

Hypothesis of Universal Soft Power Corrections

Dokshitzer, Webber (1995, 1997)

CL, Sterman (2007)

- Soft power corrections shift mean values of event shapes

$$\langle e \rangle = \langle e \rangle_{\mathrm{PT}} + c_e \frac{\mathcal{A}}{Q}$$

c_e observable dependent, calculable coefficient $\quad c_a = \frac{2}{1-a}, c_C = 3\pi$

$\mathcal{A}$ universal nonperturbative parameter

- Stronger version: simple shift of full distribution

$$\frac{d\sigma}{de}(e) = \frac{d\sigma}{de}\left(e - c_e \frac{\mathcal{A}}{Q}\right)\Big|_{\mathrm{PT}}$$

- Based on perturbative behavior of single soft gluon emission and model of "effective IR coupling"

Hypothesis of Universal Soft Power Corrections

Dokshitzer, Webber (1995, 1997)

CL, Sterman (2007)

- Soft power corrections shift mean values of event shapes

$$\langle e \rangle = \langle e \rangle_{\mathrm{PT}} + c_e \frac{\mathcal{A}}{Q}$$

Use factorization to prove true

c_e observable dependent, calculable coefficient $\quad c_a = \frac{2}{1-a}, c_C = 3\pi$

$\mathcal{A}$ universal nonperturbative parameter

- Stronger version: simple shift of full distribution

$$\frac{d\sigma}{de}(e) = \frac{d\sigma}{de}\left(e - c_e \frac{\mathcal{A}}{Q}\right)\bigg|_{\mathrm{PT}}$$

- Based on perturbative behavior of single soft gluon emission and model of "effective IR coupling"

Hypothesis of Universal Soft Power Corrections

Dokshitzer, Webber (1995, 1997)

CL, Sterman (2007)

- Soft power corrections shift mean values of event shapes

$$\langle e \rangle = \langle e \rangle_{\rm PT} + c_e \frac{\mathcal{A}}{Q}$$

Use factorization to prove true

c_e observable dependent, calculable coefficient $\quad c_a = \frac{2}{1-a}, c_C = 3\pi$

$\mathcal{A}$ universal nonperturbative parameter

- Stronger version: simple shift of full distribution

$$\frac{d\sigma}{de}(e) = \frac{d\sigma}{de}\left(e - c_e \frac{\mathcal{A}}{Q}\right)\bigg|_{\rm PT}$$

In general not true

- Based on perturbative behavior of single soft gluon emission and model of "effective IR coupling"

Boost Invariance in Soft Function

CL, Sterman (2007)

- Shift in mean value given by

$$\langle e \rangle = \langle e \rangle_J + \frac{1}{Q} \int d\eta \, f_e(\eta) \langle \mathcal{E}_T(\eta) \rangle$$

- Soft matrix element of transverse momentum flow is boost invariant:

$$\langle \mathcal{E}_T(\eta) \rangle = \frac{1}{N_C} \mathrm{Tr} \langle 0 | Y_{\bar{n}}^\dagger Y_n^\dagger \mathcal{E}_T(\eta) Y_n Y_{\bar{n}} | 0 \rangle = \langle \mathcal{E}_T(\eta + \eta') \rangle$$

for any η'

- Thus shift in mean value is calculable, in terms of one unknown nonperturbative parameter:

$$\langle e \rangle = \langle e \rangle_J + \frac{1}{Q} \int d\eta \, f_e(\eta) \langle \mathcal{E}_T(0) \rangle = \langle e \rangle_J + \frac{1}{Q} c_e \mathcal{A}$$

where $c_e = \int_{-\infty}^{\infty} d\eta \, f_e(\eta), \qquad \mathcal{A} = \langle \mathcal{E}_T(0) \rangle$

Boost Invariance in Soft Function

CL, Sterman (2007)

- Shift in mean value given by

$$\langle e \rangle = \langle e \rangle_J + \frac{1}{Q} \int d\eta\, f_e(\eta) \langle \mathcal{E}_T(\eta) \rangle$$

- Soft matrix element of transverse momentum flow is boost invariant:

$$\langle \mathcal{E}_T(\eta) \rangle = \frac{1}{N_C} \mathrm{Tr} \langle 0 | Y_{\bar{n}}^\dagger Y_n^\dagger \mathcal{E}_T(\eta) Y_n Y_{\bar{n}} | 0 \rangle = \langle \mathcal{E}_T(\eta + \eta') \rangle$$

for any η'

- Thus shift in mean value is calculable, in terms of one unknown nonperturbative parameter:

$$\langle e \rangle = \langle e \rangle_J + \frac{1}{Q} \int d\eta\, f_e(\eta) \langle \mathcal{E}_T(0) \rangle = \langle e \rangle_J + \frac{1}{Q} c_e \mathcal{A}$$

where $c_e = \int_{-\infty}^{\infty} d\eta\, f_e(\eta), \qquad \mathcal{A} = \langle \mathcal{E}_T(0) \rangle$

Field theory definition of DW parameter

Universal Soft Function Model

- Soft function interpolating between hadronic and partonic regimes:

$$S_a(\tau_a;\mu) = \int d\tau_a'\, S_a^{\rm PT}(\tau_a - \tau_a';\mu)\, f_a^{\rm exp}\left(\tau_a' - \frac{2\Delta_a}{Q}\right)$$

Hoang, Stewart (2007);
Ligeti, Stewart, Tackmann (2008)

- Model of nonpertubative hemisphere soft function, modified by a "gap parameter"

$$f^{\rm exp}(l^+, l^-) = \theta(l^+)\theta(l^-)\frac{\mathcal{N}(A,B)}{\Lambda^2}\left(\frac{l^+l^-}{\Lambda^2}\right)^{A-1} \exp\left(\frac{-(l^+)^2-(l^-)^2-2Bl^+l^-}{\Lambda^2}\right)$$

Korchemsky, Tafat (2000)

insert gap Δ_a for min. hadronic energy deposit

Hoang, Stewart (2007)

- Fit to jet mass distribution gives $A = 2.5,\ B = -0.4,\ \Lambda = 0.55\ \mathrm{GeV}$

Korchemsky, Tafat (2000)

Universal Soft Function Model

- Obtain thrust soft function from hemisphere soft function:

$$S_0(\tau_0;\mu) = \int dl^+ dl^- S_{\rm hemi}(l^+,l^-;\mu)\,\delta\left(\tau_0 - \frac{l^+ + l^-}{Q}\right)$$

- Minimal generalization: ensure first moment scaling is obeyed by defining

$$\Delta_a = \frac{\Delta}{1-a}, \quad \Lambda_a = \frac{\Lambda}{1-a}$$

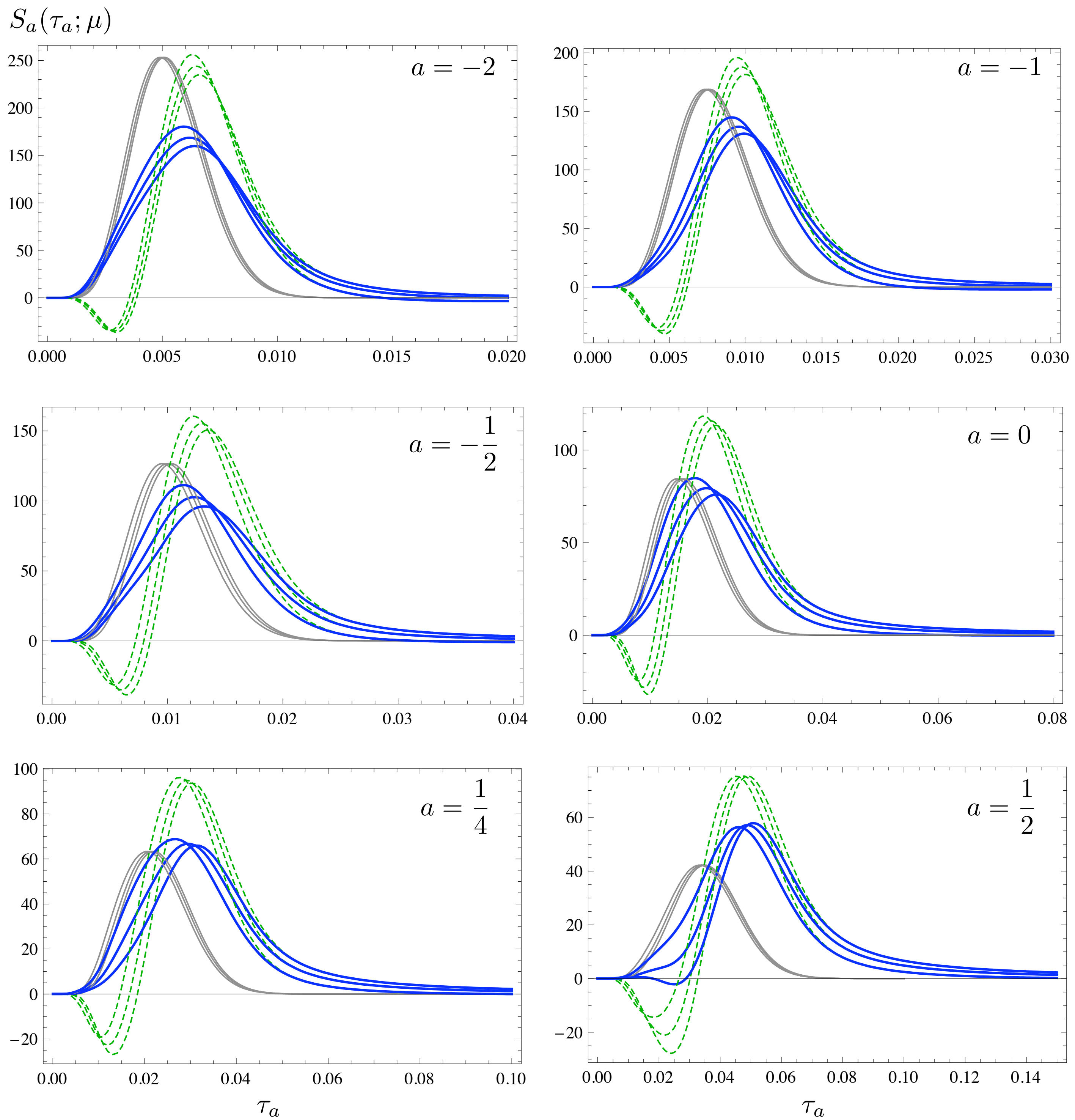
- Renormalon-free scheme for gap parameter cancels $1/Q$ renormalon ambiguity in partonic soft function

Jain, Scimemi, Stewart (2008); Hoang, Kluth (2008)

$$\Delta_a = \underbrace{\bar\Delta_a(\mu)}_{\text{renormalon-free}} + \underbrace{\delta_a(\mu)}_{\text{define by requiring } S_a^{\rm PT} \text{ become renormalon-free}}$$

- Analogous to switching between pole mass and renormalon-free mass schemes for heavy quark masses

Soft Functions for Angularities



Tree level with gap

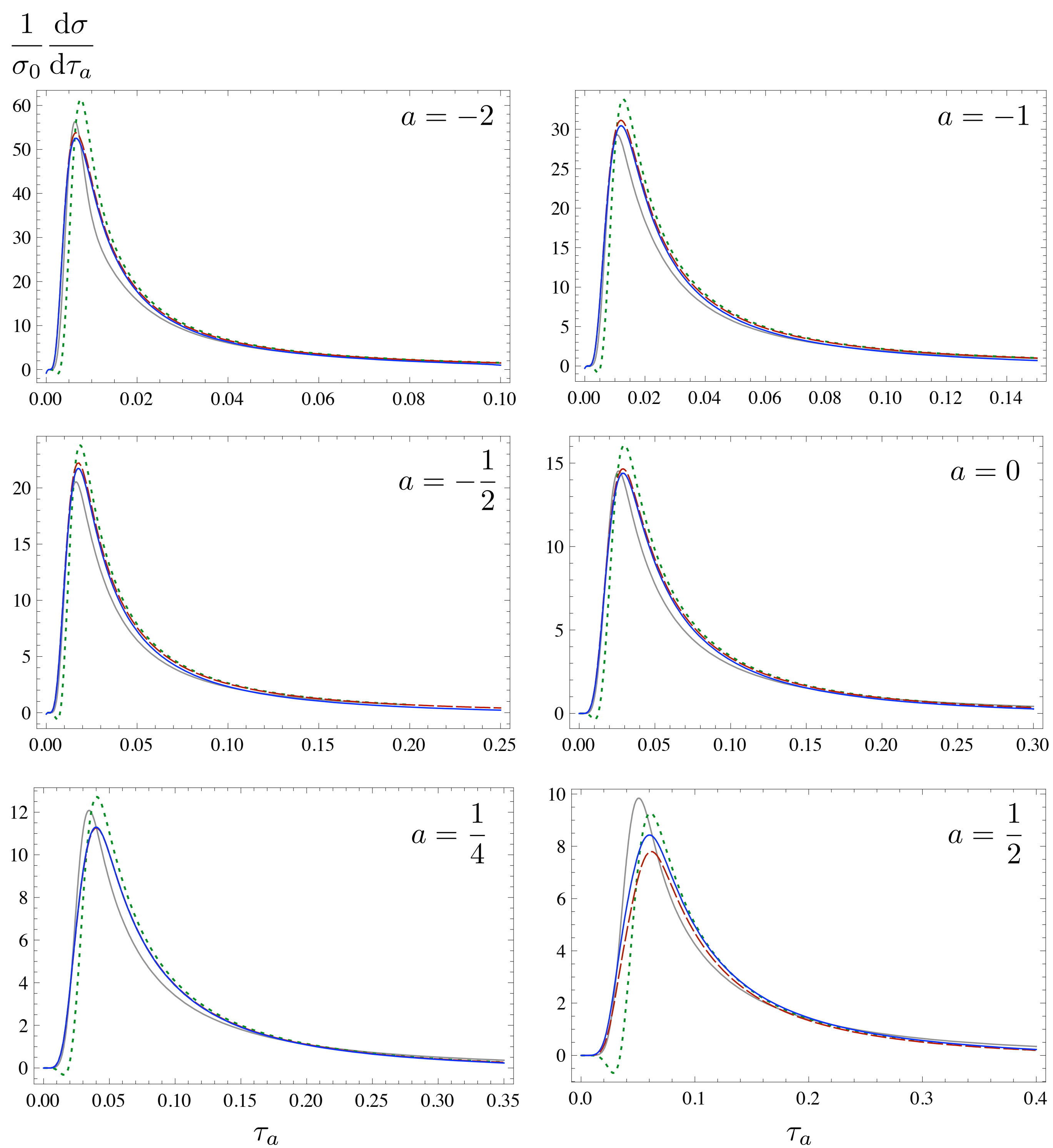
1-loop with gap

1-loop with renormalon-free gap

Plan of the talk

- General Introduction
- Event Shapes in e^+e^- annihilation
- Effective Field Theories and SCET
- Energy Flow and Factorization in SCET
- RG Evolution and Resummation in SCET
- Nonperturbative Models of Hadronization Effects
- Final Predictions for Angularity Distributions
- Conclusions

Factorized, Resummed, and Gapped Angularity Distributions



NLL/LO

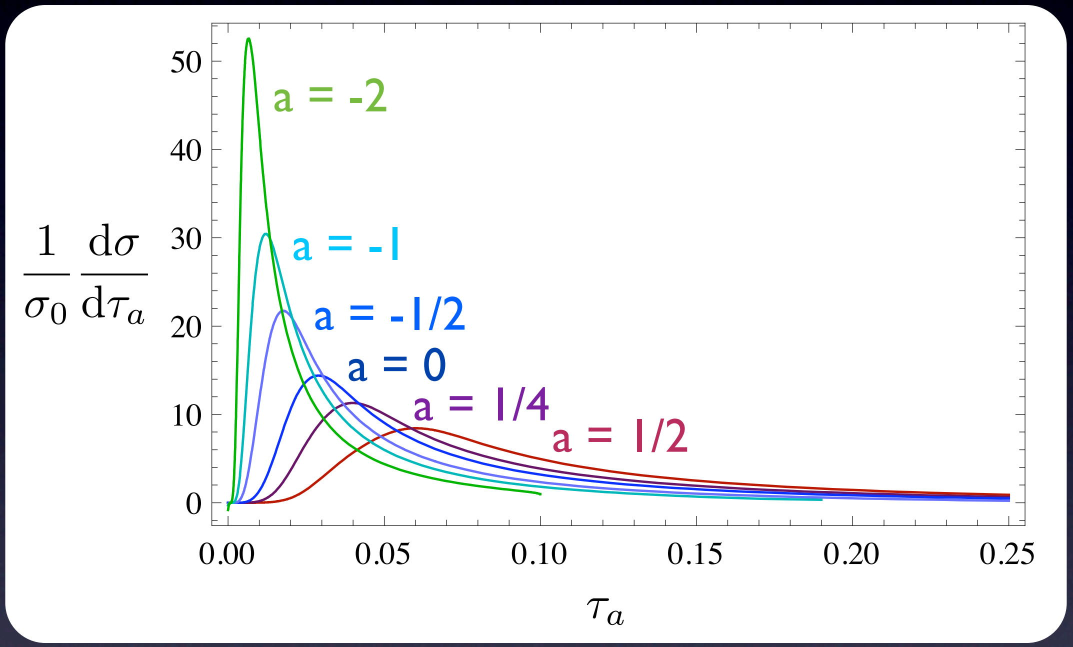
NLL/NLO

NLL/NLO w/ renormalon-free gap

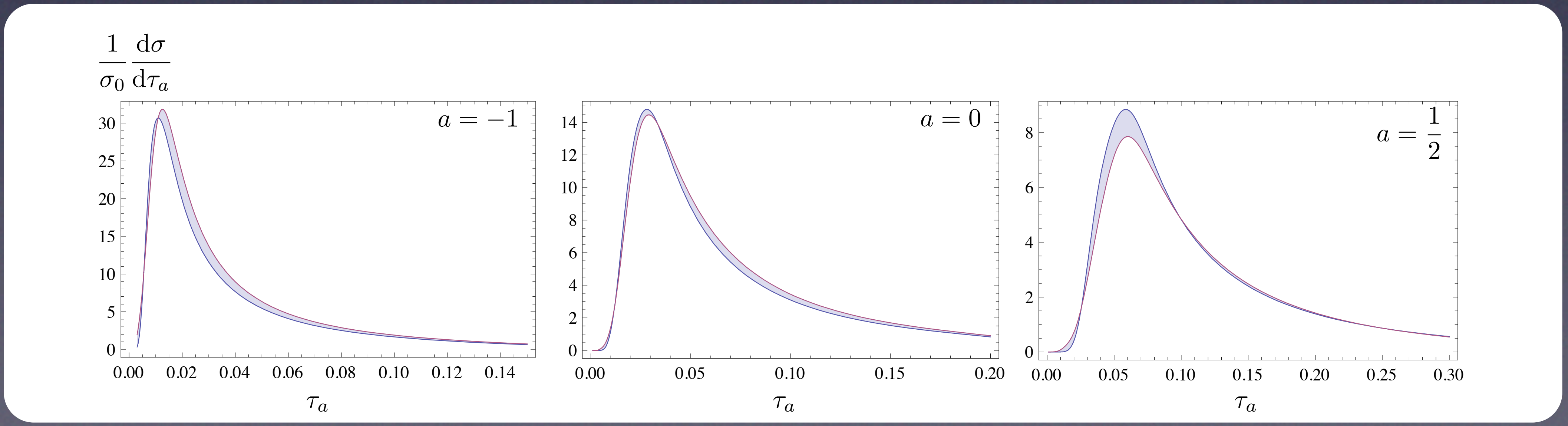
Renormalon-free NLL/NLO with NLO fixed-order matching

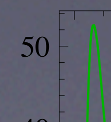
Factorized, Resummed, and Gapped Angularity Distributions

- Shape of distributions with varying *a*



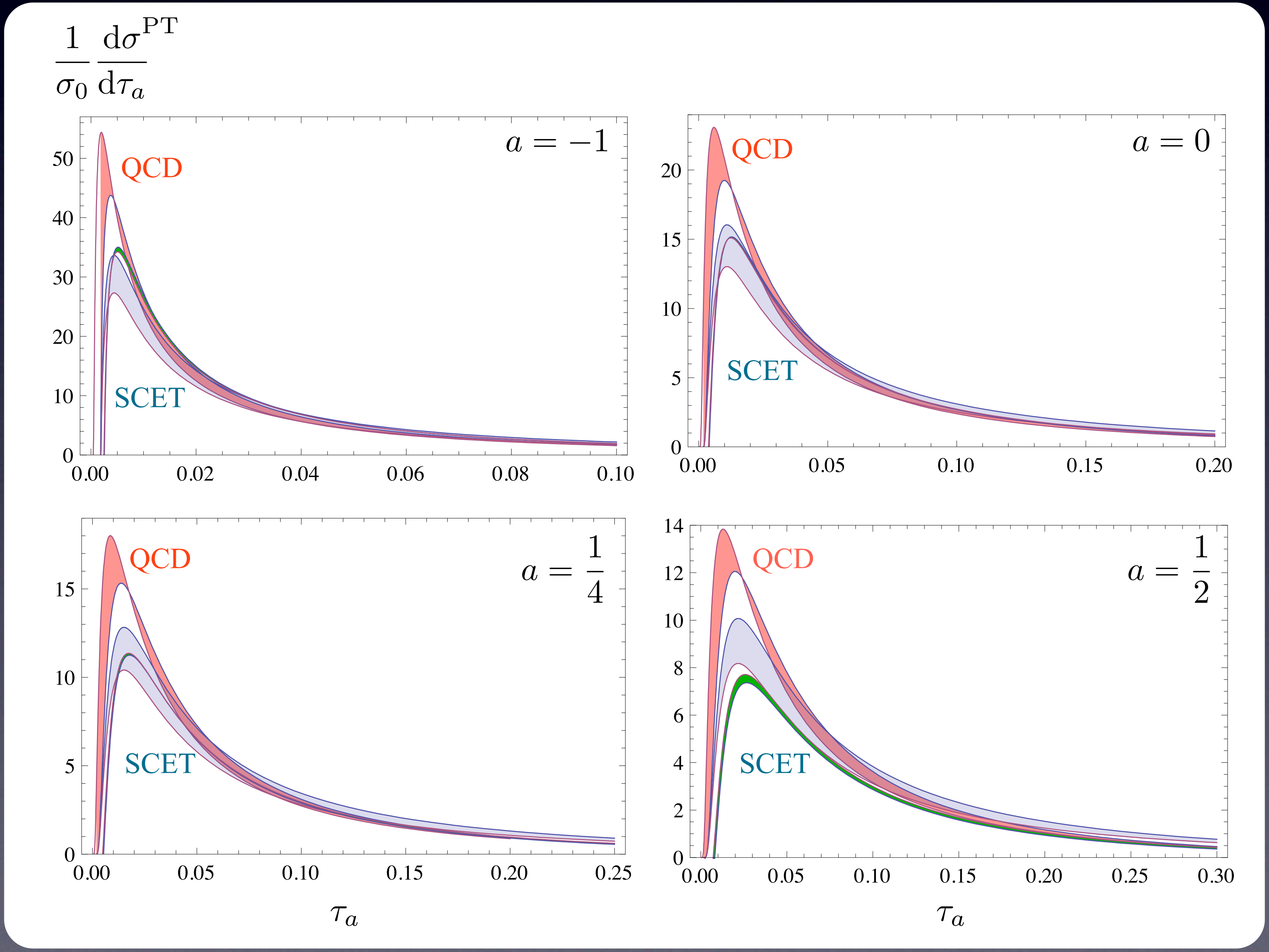
- Vary jet and soft scales together by factors of 1/2, 2





Hard or Factorization Scale Variation

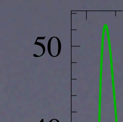
- Compare scale variation of traditional QCD NLL vs. SCET at NLL/LO and at NLL/NLO



In traditional QCD, $\mu = \mu_H$ and logs of $\mu/\mu_H, \mu_H/Q$ not resummed

SCET NLL/LO comparable variation to QCD

NLL/NLO significantly improved



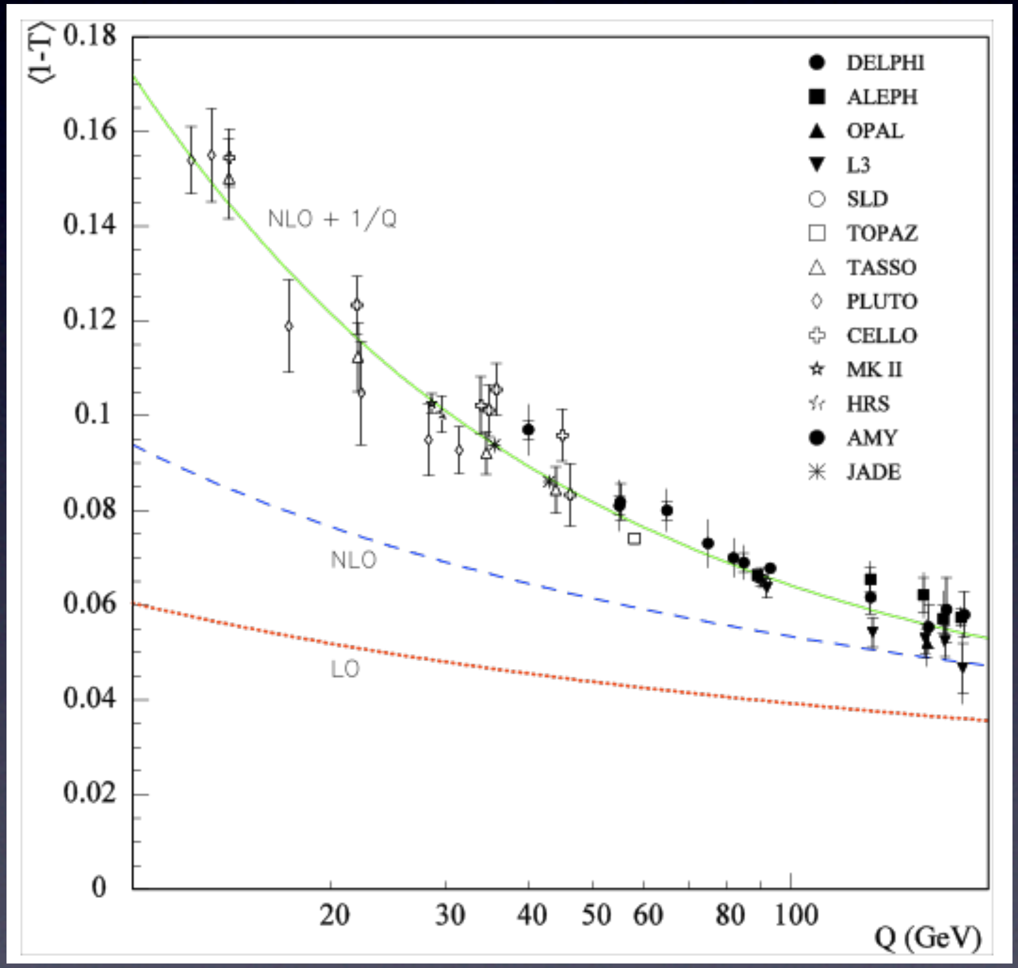
Conclusions

- **Utilized** a unified framework in EFT for factorization, resummation, and nonperturbative shape functions in effective field theory
- **Resummed** logs through RG evolution: NLL running of NLO initial conditions, can use identical framework to go to higher orders
- **Generalized** model of nonperturbative soft function for thrust to angularities based on universal shift in first moment
- **Applications** of precise predictions of angularity distributions include
  - Deducing jet origin from substructure (energy profile)
  - Improved extractions of strong coupling, nonperturbative model
- **Proceeding** from lessons learned in e^+e^- collisions to describing jets in hadron collisions at LHC.

Backup Slides

Shift in average thrust

- Shift scales with center-of-mass energy Q like $1/Q$:



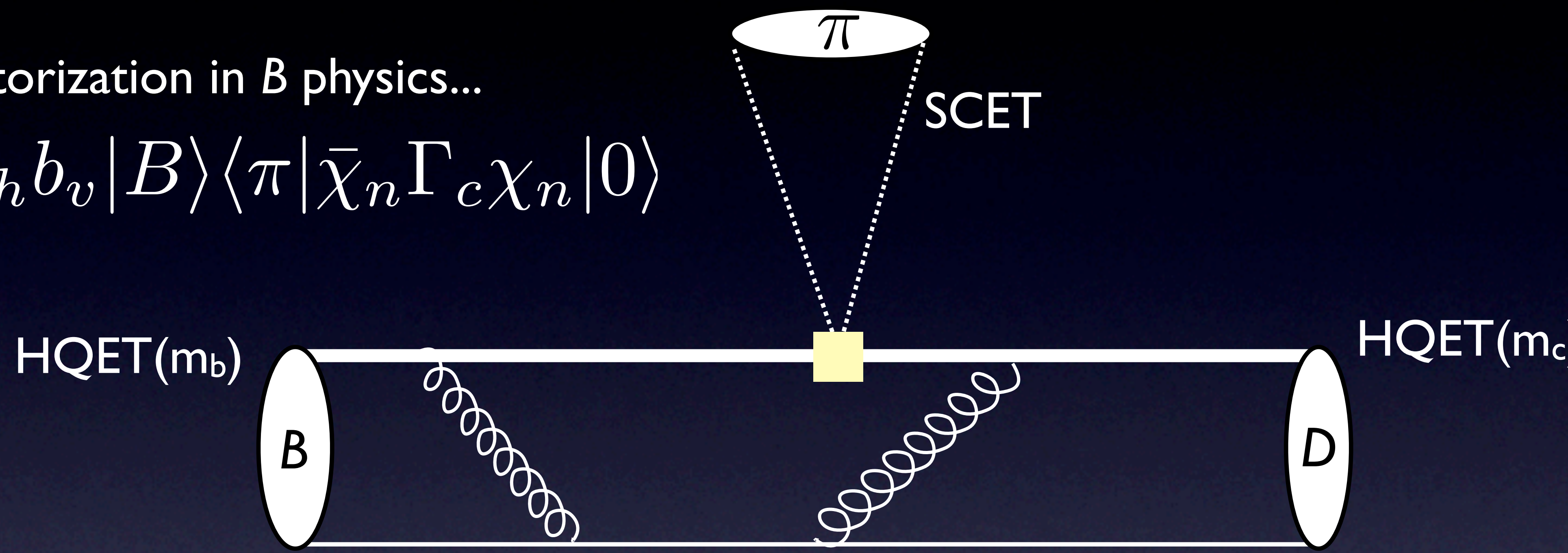
Dasgupta,
Salam (2003)

- These *power corrections* come from nonperturbative soft radiation from the jets

Collinear and soft states

- Contrast factorization in *B* physics...

$$\sim \langle D|\bar{c}'_v \Gamma_h b_v|B\rangle\langle\pi|\bar{\chi}_n \Gamma_c \chi_n|0\rangle$$



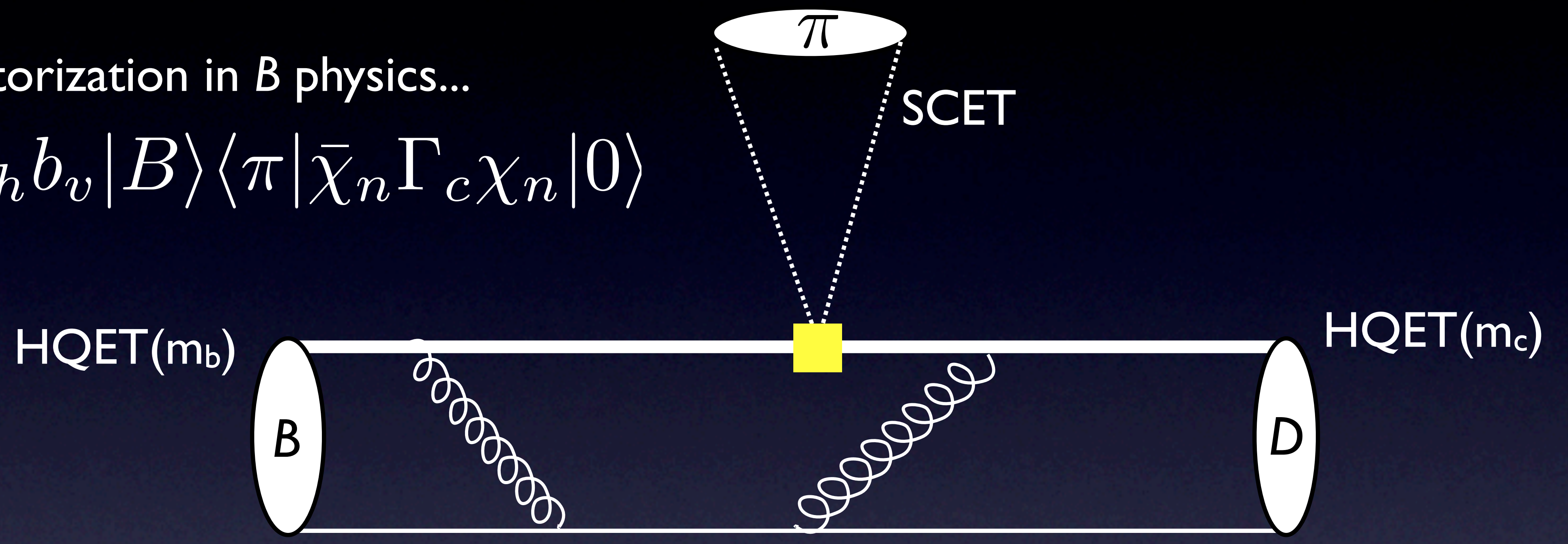
- ... with jet physics

$$\sim \langle J_1|\bar{\chi}_n|0\rangle\Gamma\langle J_2|\chi_{\bar{n}}|0\rangle\langle X_s|Y_n Y_{\bar{n}}|0\rangle$$

Collinear and soft states

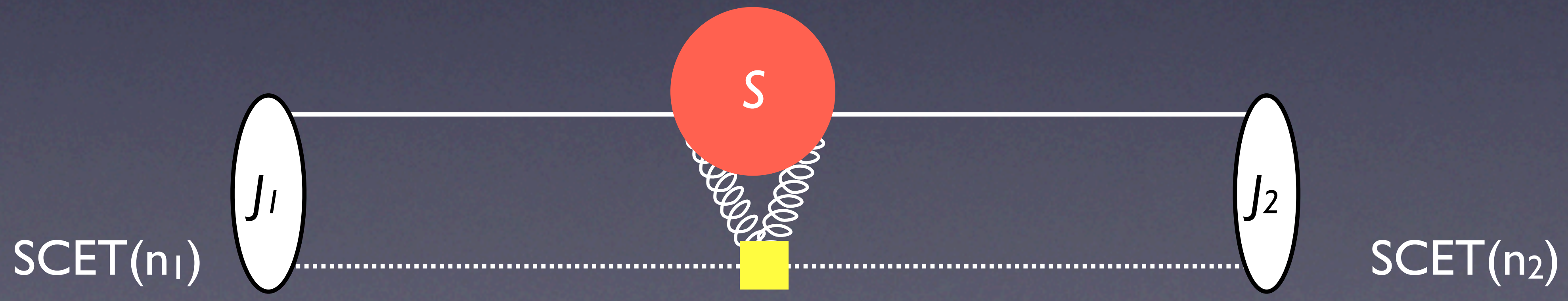
- Contrast factorization in *B* physics...

$$\sim \langle D|\bar{c}'_v \Gamma_h b_v|B\rangle\langle\pi|\bar{\chi}_n\Gamma_c\chi_n|0\rangle$$



- ... with jet physics

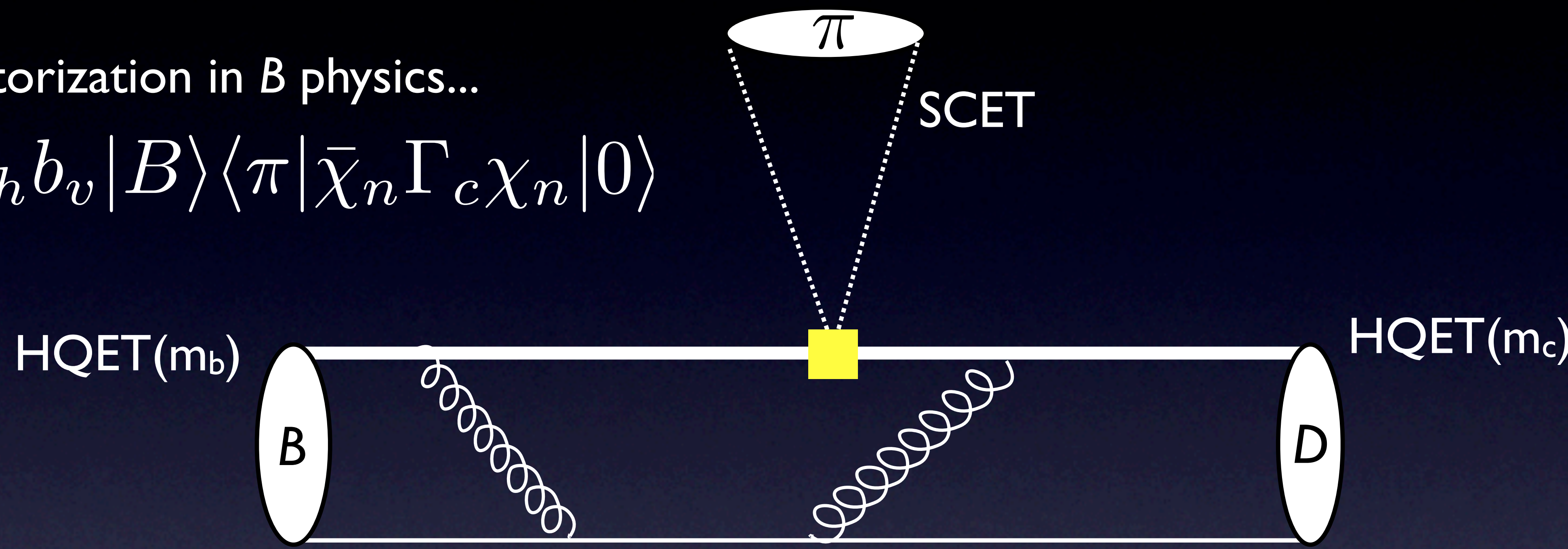
$$\sim \langle J_1|\bar{\chi}_n|0\rangle\Gamma\langle J_2|\chi_{\bar{n}}|0\rangle\langle X_s|Y_n Y_{\bar{n}}|0\rangle$$



Collinear and soft states

- Contrast factorization in *B* physics...

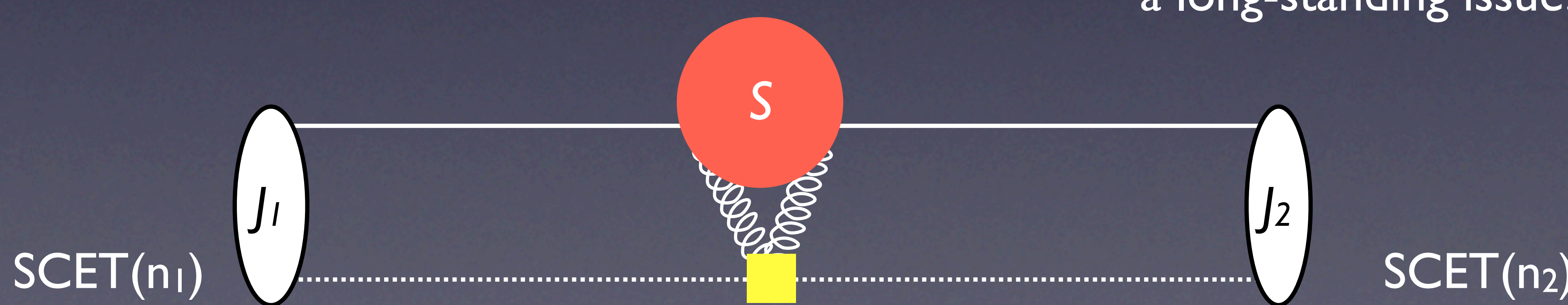
$$\sim \langle D|\bar{c}'_v \Gamma_h b_v|B\rangle\langle\pi|\bar{\chi}_n\Gamma_c\chi_n|0\rangle$$



- ... with jet physics

$$\sim \langle J_1|\bar{\chi}_n|0\rangle\Gamma\langle J_2|\chi_{\bar{n}}|0\rangle\langle X_s|Y_n Y_{\bar{n}}|0\rangle$$

Factorization of states justified?
a long-standing issue...



Energy flow

- Relate transverse energy flow to energy flow operator:
$$\mathcal{E}_T(\eta) = \frac{1}{\cosh^3 \eta} \int_0^{2\pi} d\phi \, \mathcal{E}(\hat{n})$$

$$\mathcal{E}(\hat{n})|N\rangle = \sum_{i \in N} E_i \delta^2(\hat{n} - \hat{n}_i)|N\rangle$$

- Energy flow from energy-momentum tensor:

$$\mathcal{E}(\hat{n}) = \lim_{R \to \infty} \int_0^\infty dt \, \hat{n}_i T_{0i}(t, R\hat{n})$$

Korchemsky, Oderda, Sterman (1997);
cf. Sveshnikov, Tkachov (1996)

Factorized Functions at One Loop

- Hard Function

$$H(Q;\mu) = 1 - \frac{\alpha_s(\mu) C_F}{2\pi}\left(8 - \frac{7\pi^2}{6} + \ln^2\frac{\mu^2}{Q^2} + 3\ln\frac{\mu^2}{Q^2}\right)$$

- Jet Function

$$\begin{aligned} J_a^n(\tau_a^n;\mu) &= \delta(\tau_a^n)\left\{1 + \frac{\alpha_s C_F}{\pi}\left[\frac{1-a/2}{2(1-a)}\ln^2\frac{\mu^2}{Q^2} + \frac{3}{4}\ln\frac{\mu^2}{Q^2} + f(a)\right]\right\} \\ &\quad - \frac{\alpha_s C_F}{\pi}\left[\left(\frac{3}{4}\frac{1}{1-a/2} + \frac{2}{1-a}\ln\frac{\mu}{Q(\tau_a^n)^{1/(2-a)}}\right)\left(\frac{\theta(\tau_a^n)}{\tau_a^n}\right)\right]_+ \end{aligned}$$

$$f(a) \equiv \frac{1}{1-a/2}\left(\frac{7-13a/2}{4} - \frac{\pi^2}{12}\frac{3-5a+9a^2/4}{1-a} - \int_0^1 dx\frac{1-x+x^2/2}{x}\ln[(1-x)^{1-a} + x^{1-a}]\right.$$

- Soft Function

$$S_a^{\mathrm{PT}}(\tau_a^s;\mu) = \delta(\tau_a^s)\left[1 - \frac{\alpha_s C_F}{\pi(1-a)}\left(\frac{1}{2}\ln^2\frac{\mu^2}{Q^2} - \frac{\pi^2}{12}\right)\right] + \frac{2\alpha_s C_F}{\pi(1-a)}\left[\frac{\theta(\tau_a^s)}{\tau_a^s}\ln\frac{\mu^2}{(Q\tau_a^s)^2}\right]_+$$

When does factorization break down?

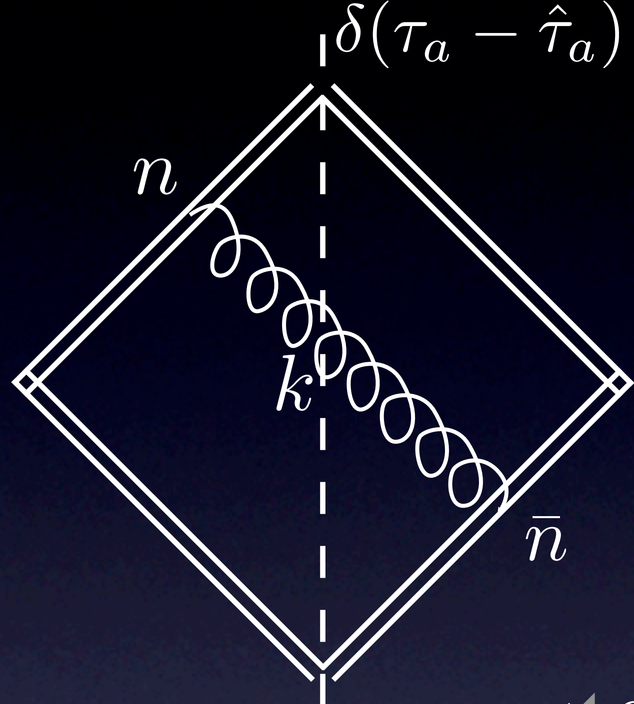
Hornig, CL, Ovanesyan
(2008, in progress)

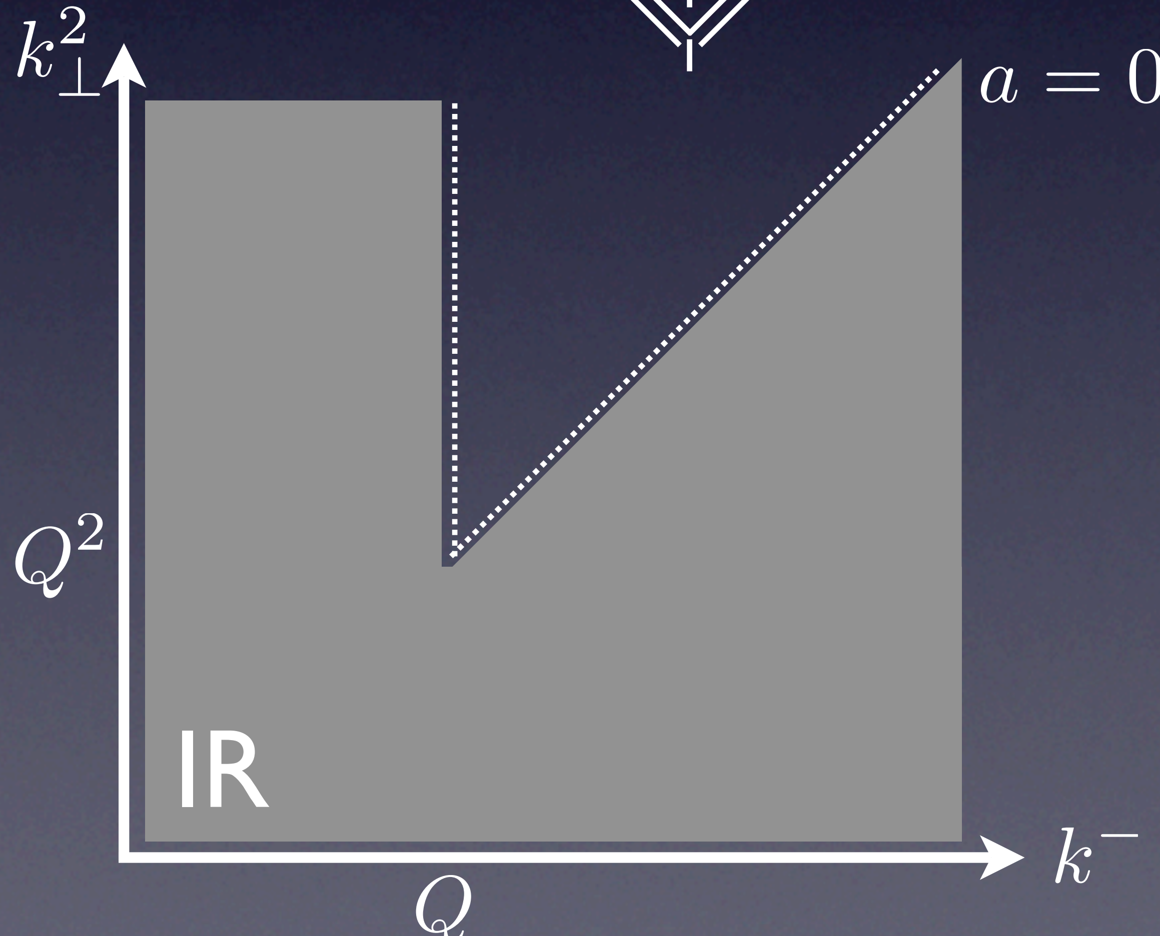
- Successful factorization means well-defined separation of soft and collinear contributions to cross-section
- Jet and soft functions must separately be infrared-finite
- For a >1, distribution dominated by narrow jets in which soft and collinear modes not completely distinguished by $\mathrm{SCET_I}$. Jet and soft functions IR divergent.
- Following example illustrates NLO calculation of soft function, jet function, integrated over kinematically-allowed region $0 < \tau_a < 1$

Calculation of soft function

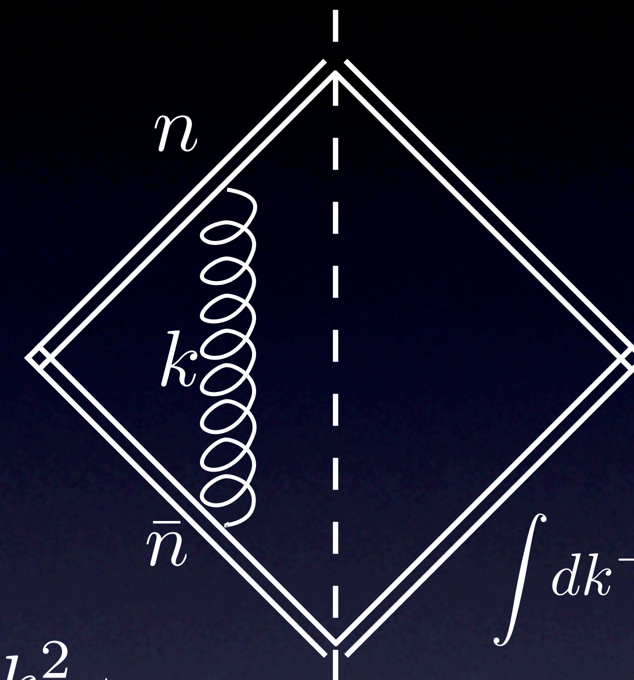
Hornig, CL, Ovanesyan (2009)

Real emission:

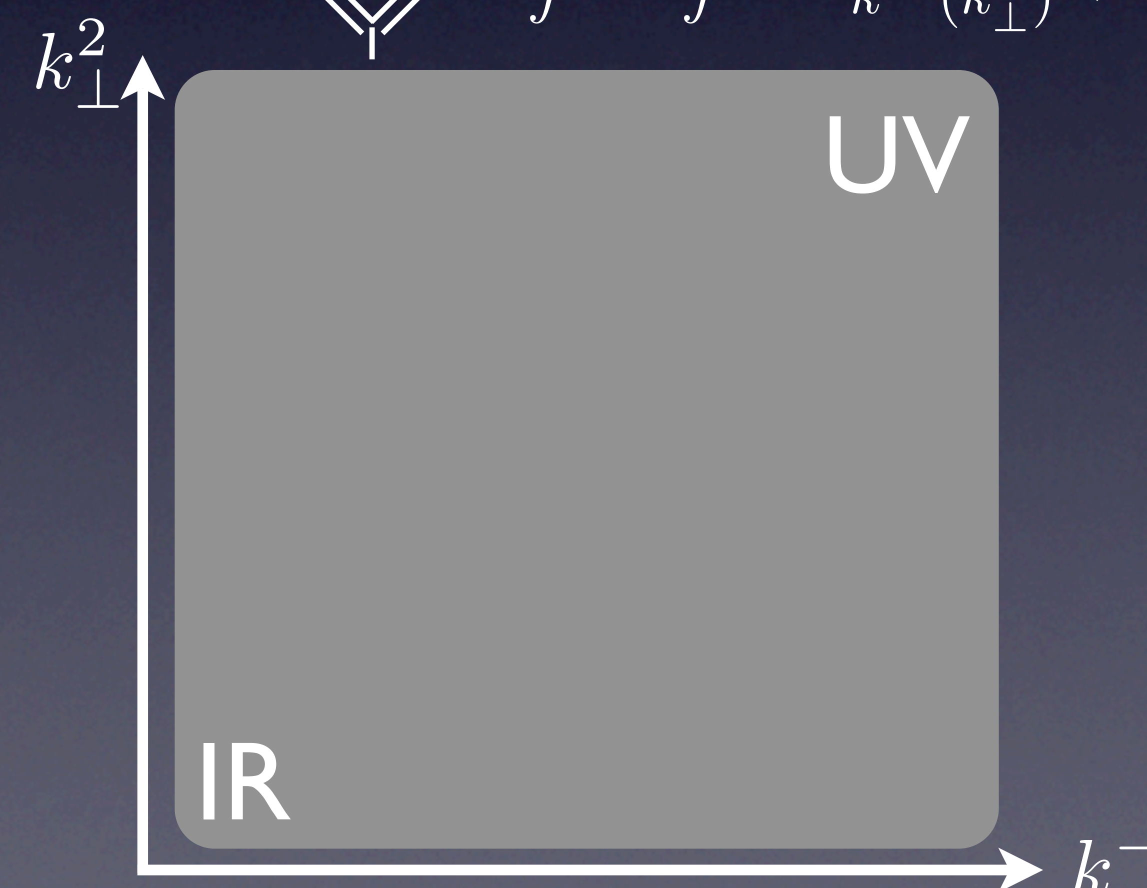




Virtual:



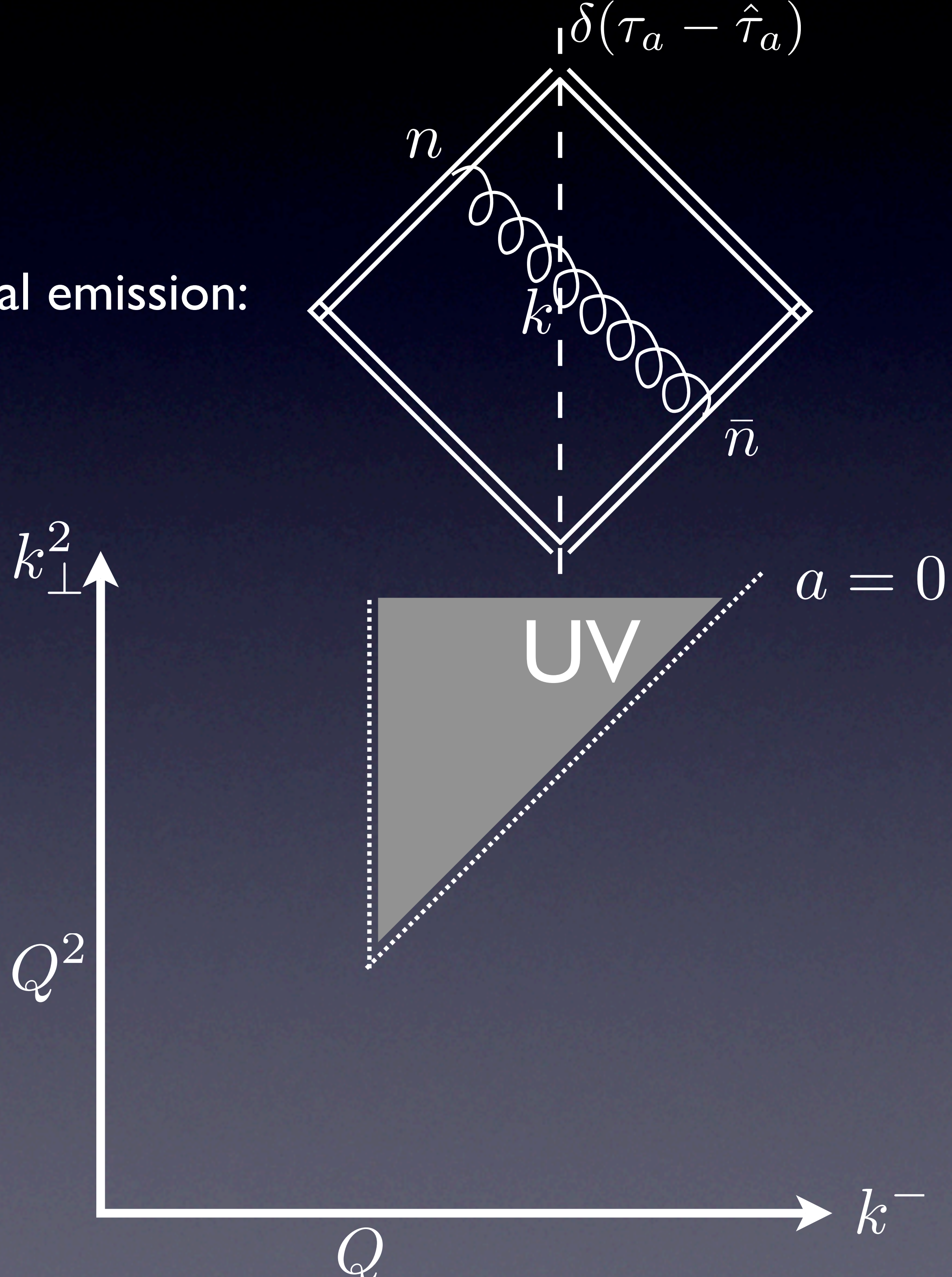
$$\int dk^- \int dk_\perp^2 \frac{1}{k^-} \frac{1}{(k_\perp^2)^{1+\epsilon}}$$



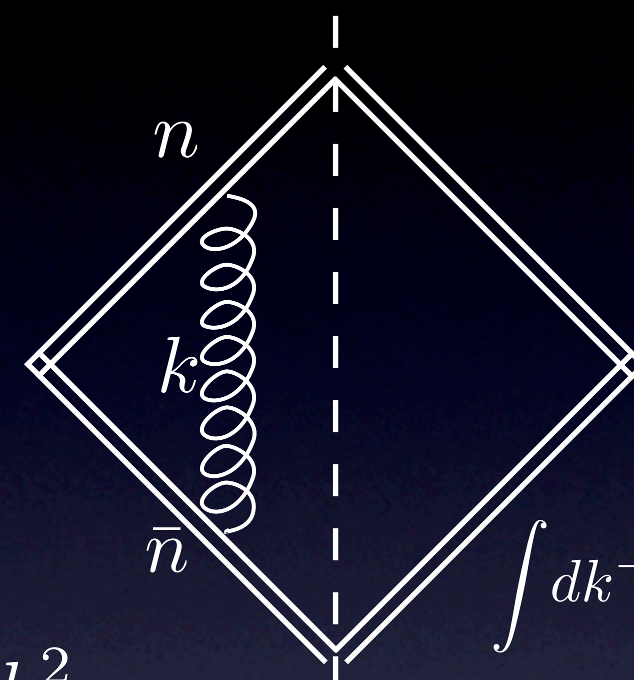
Calculation of soft function

Hornig, CL, Ovanesyan (2009)

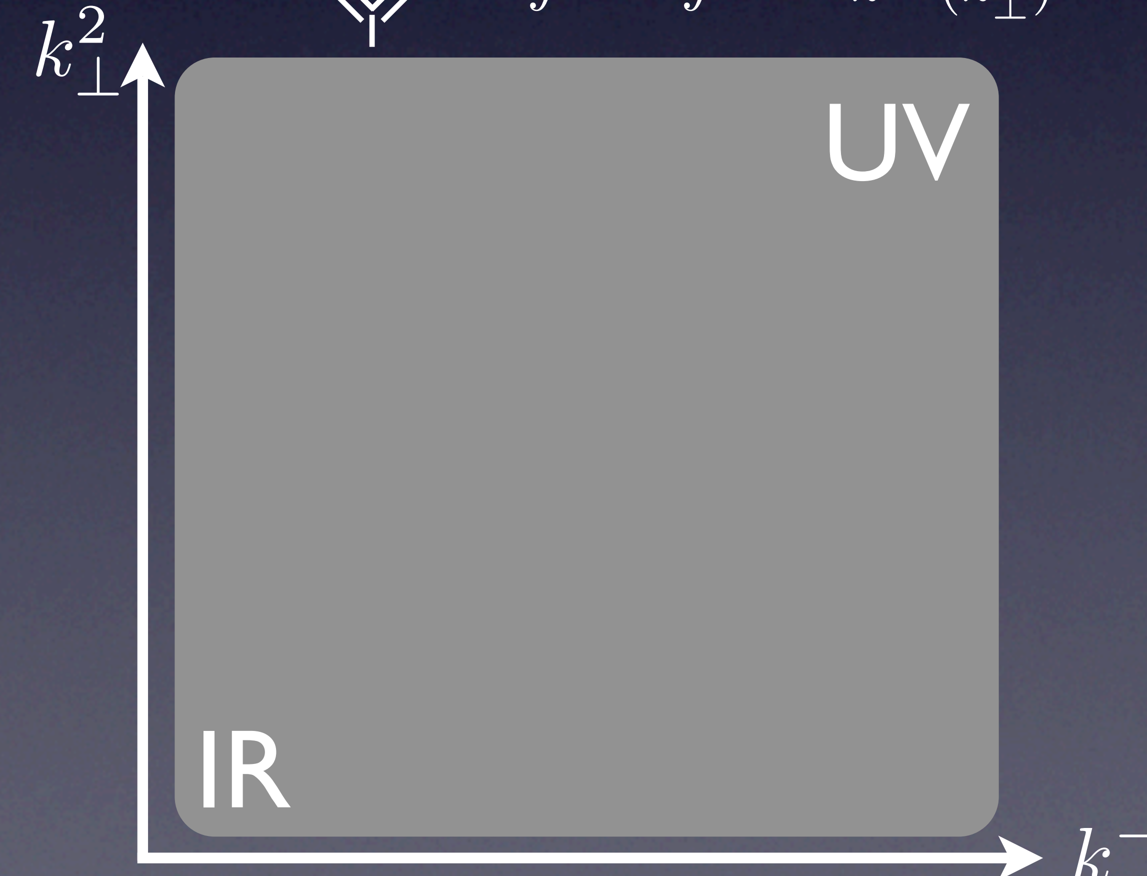
Real emission:



Virtual:



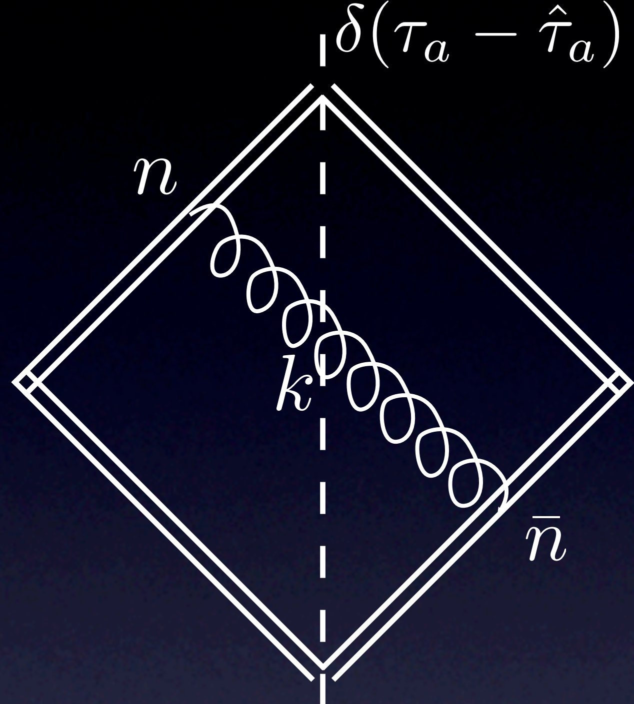
$$\int dk^- \int dk_\perp^2 \frac{1}{k^-} \frac{1}{(k_\perp^2)^{1+\epsilon}}$$



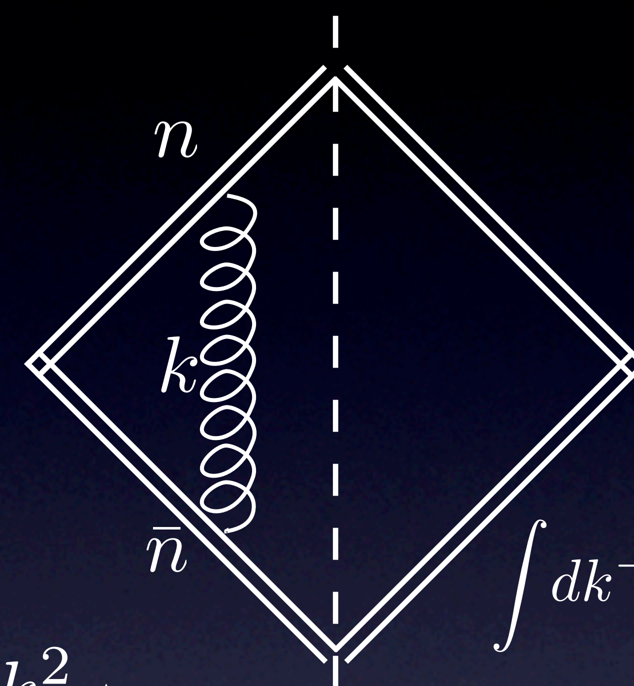
Calculation of soft function

Hornig, CL, Ovanesyan (2009)

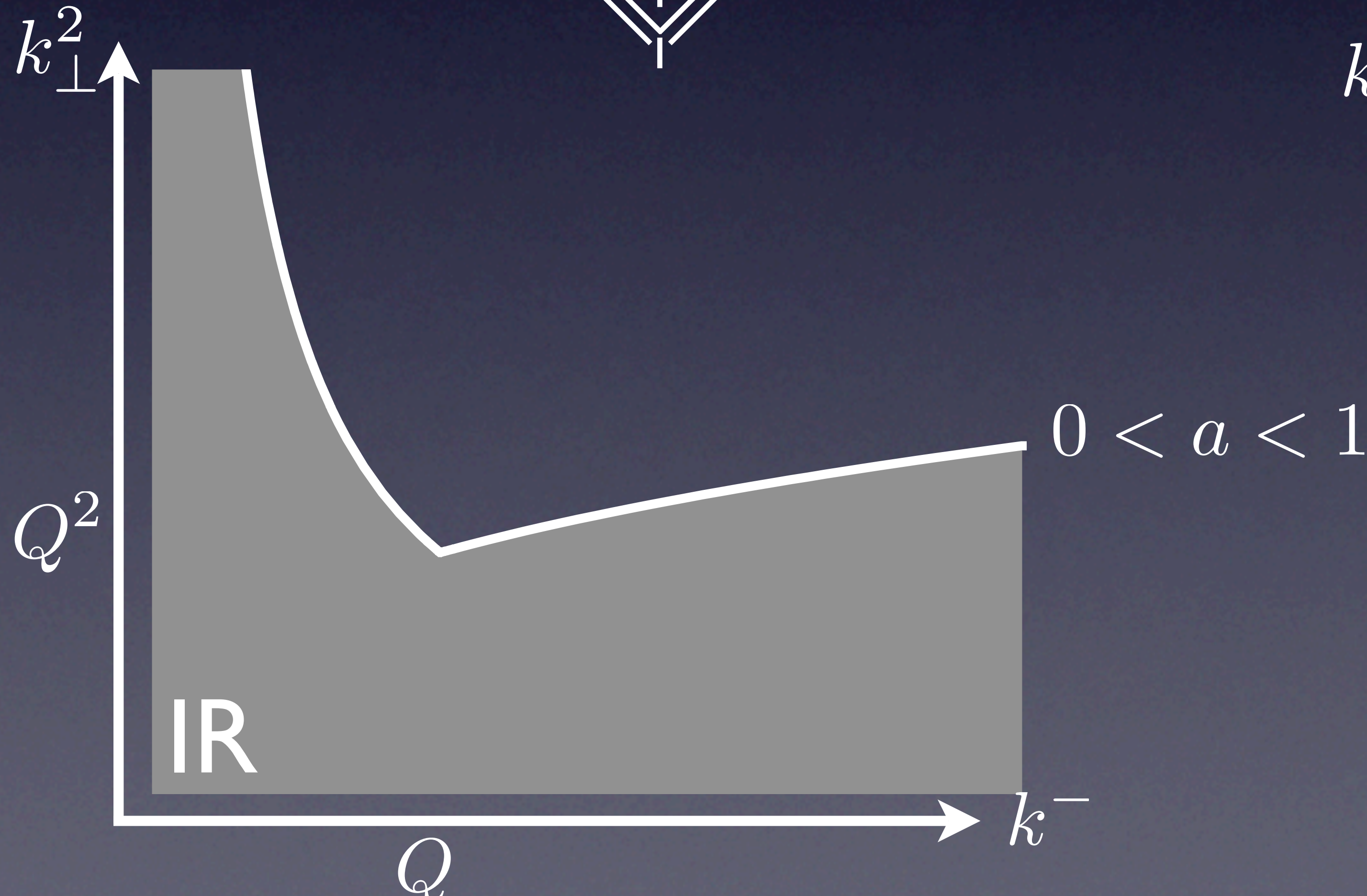
Real emission:

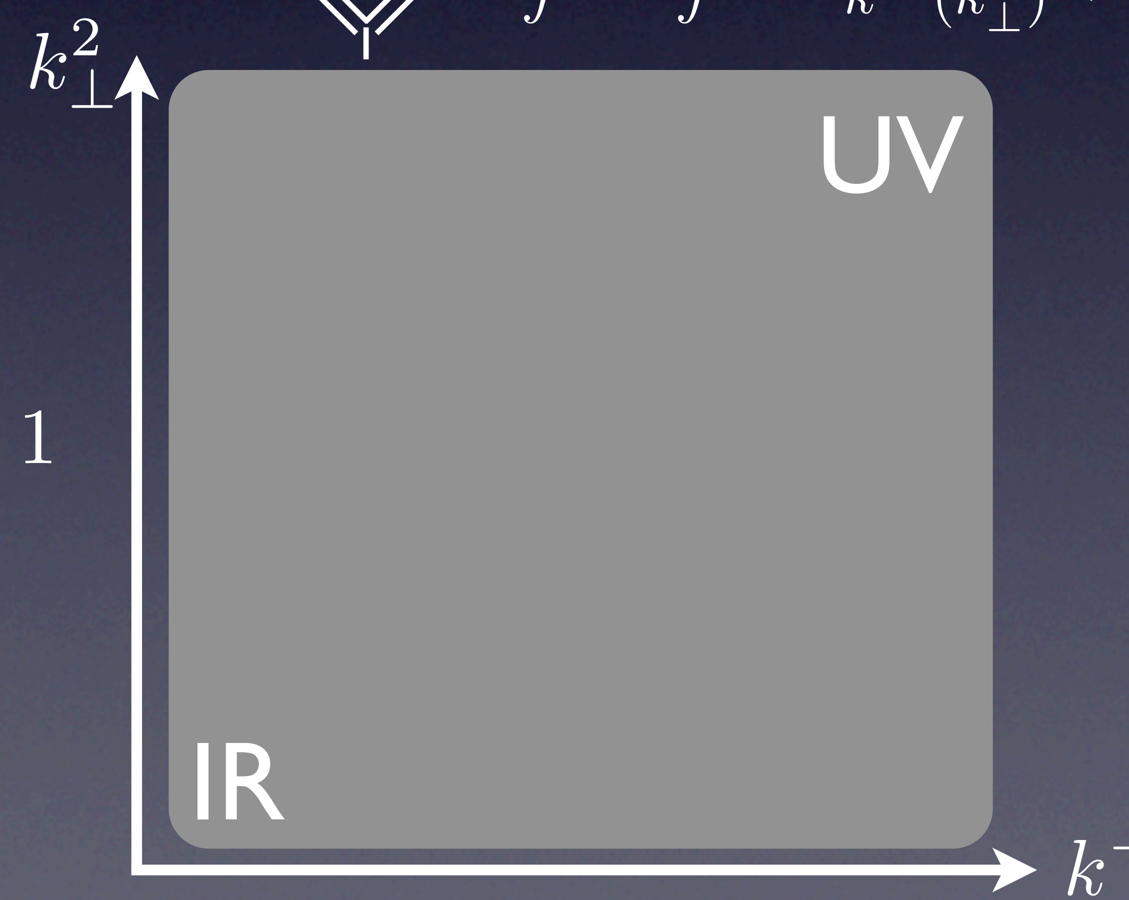


Virtual:



$$\int dk^- \int dk_\perp^2 \frac{1}{k^-} \frac{1}{(k_\perp^2)^{1+\epsilon}}$$

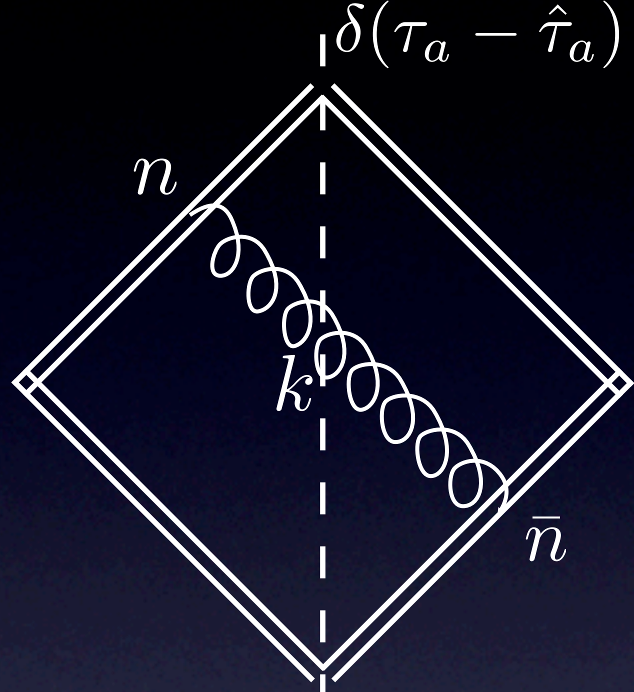


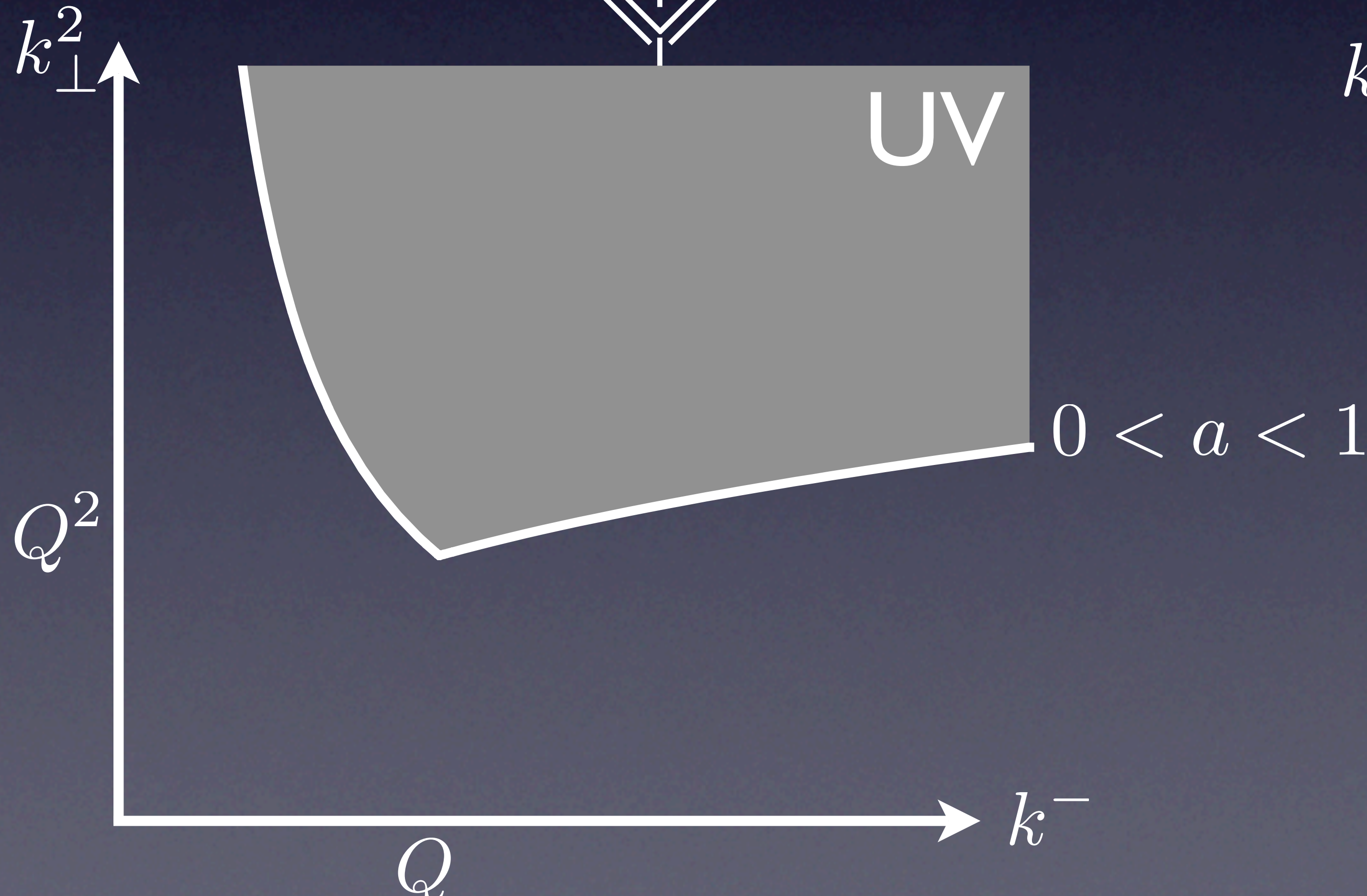


Calculation of soft function

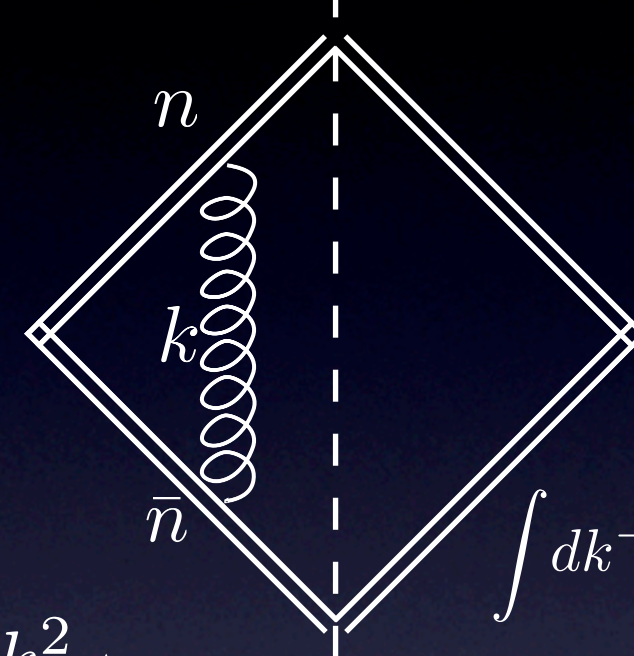
Hornig, CL, Ovanesyan (2009)

Real emission:

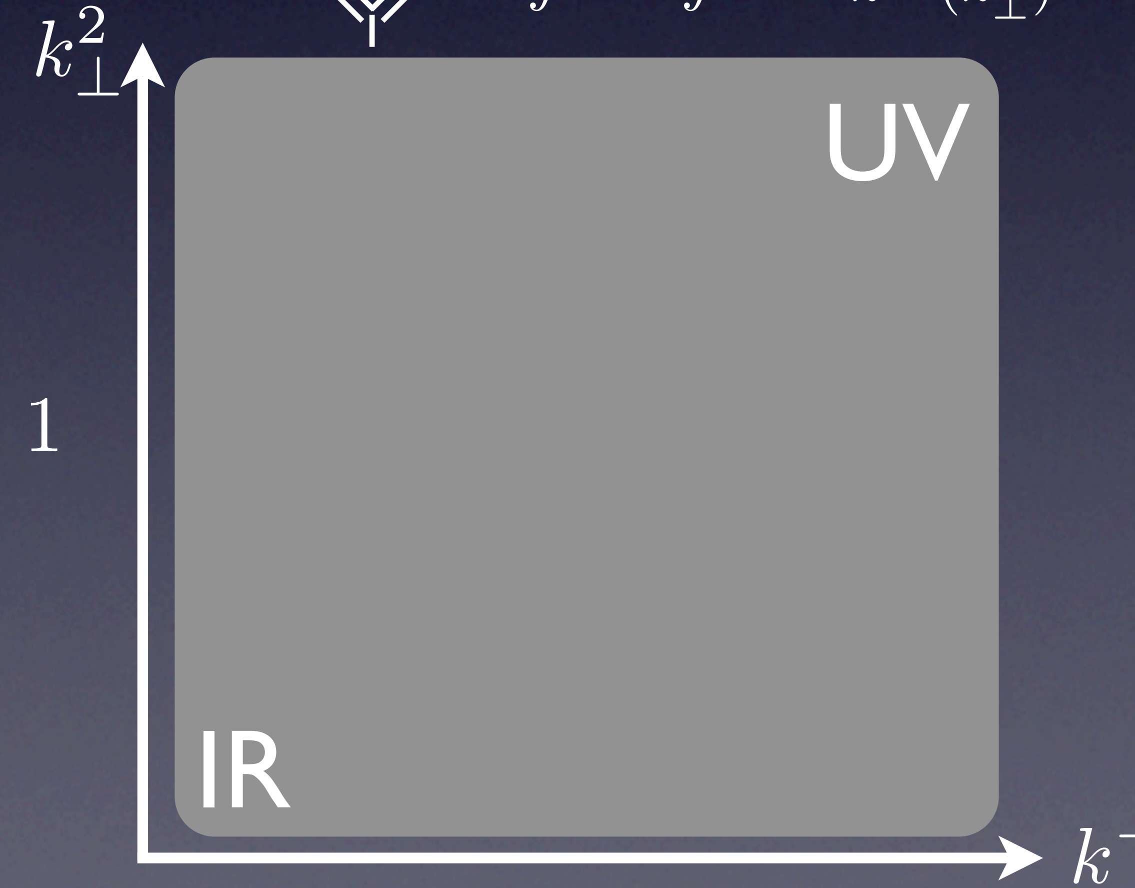




Virtual:



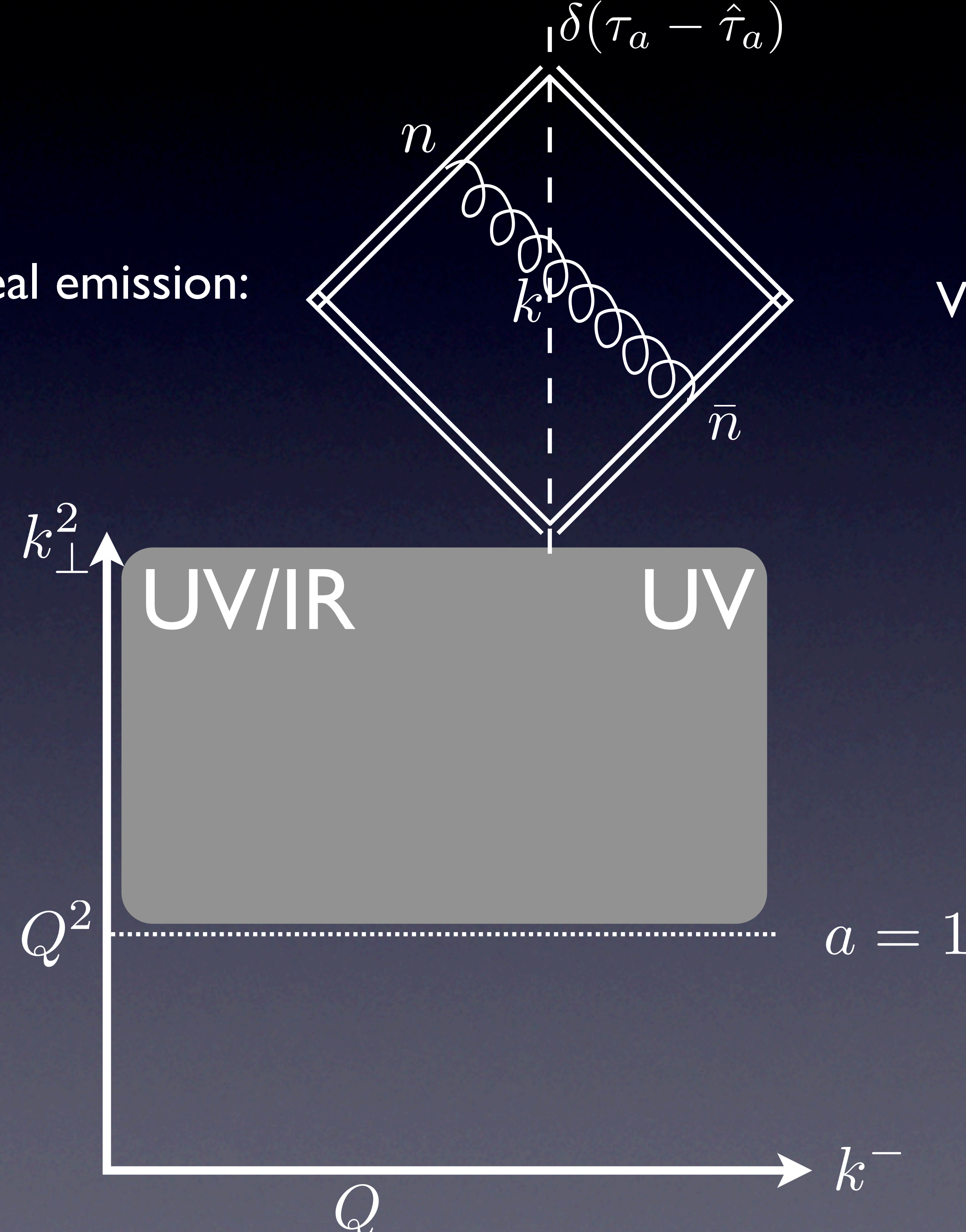
$$\int dk^- \int dk_\perp^2 \frac{1}{k^-} \frac{1}{(k_\perp^2)^{1+\epsilon}}$$



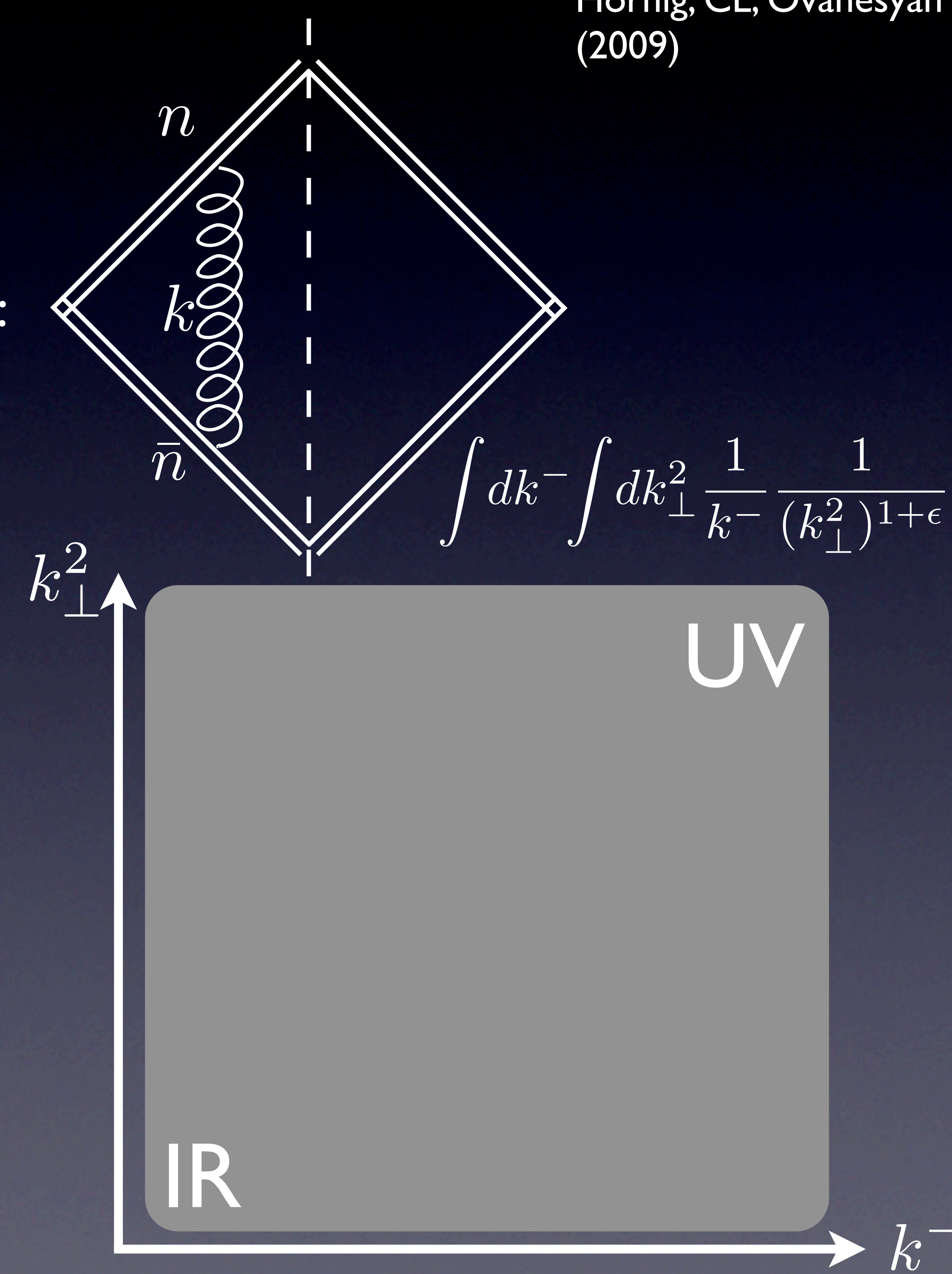
Calculation of soft function

Hornig, CL, Ovanesyan (2009)

Real emission:

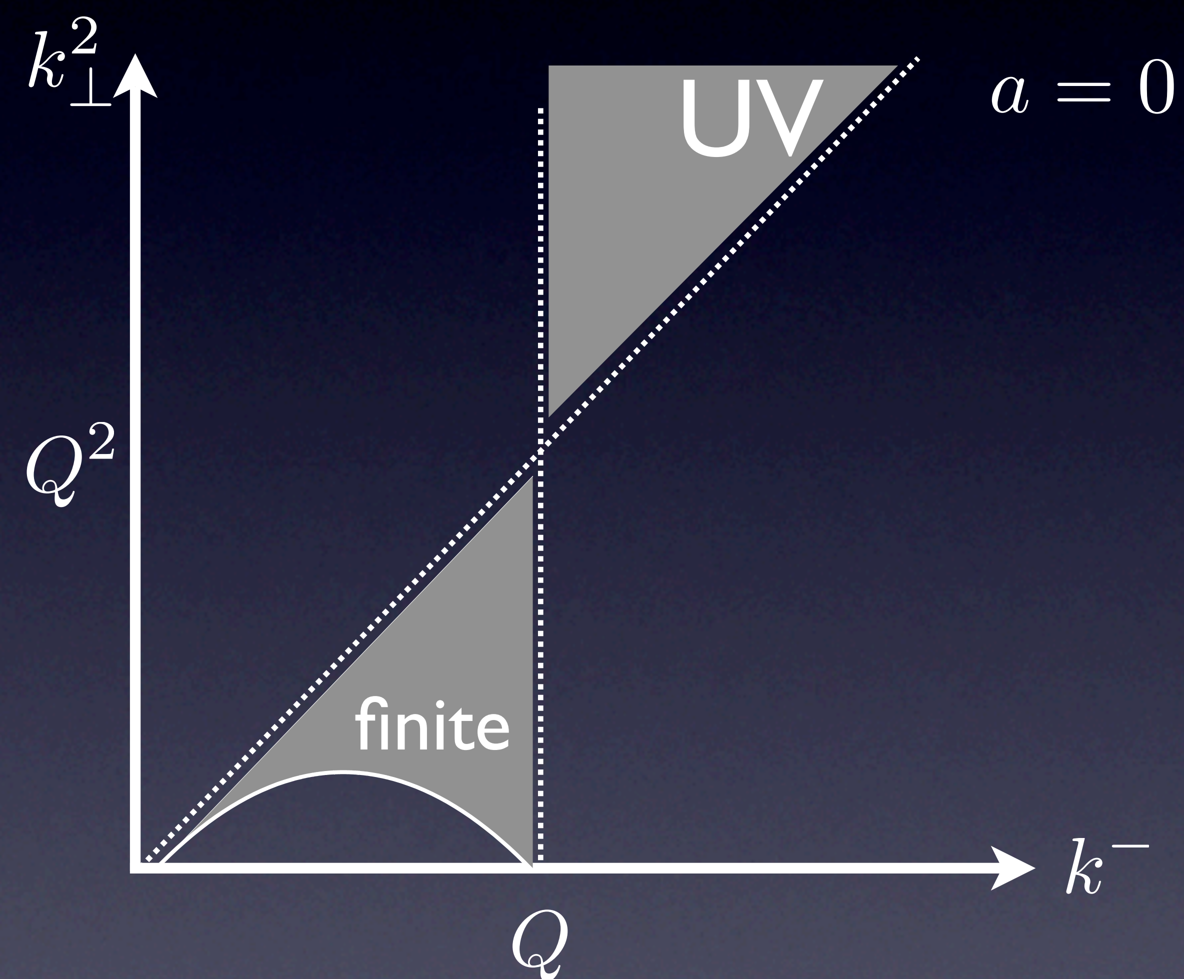


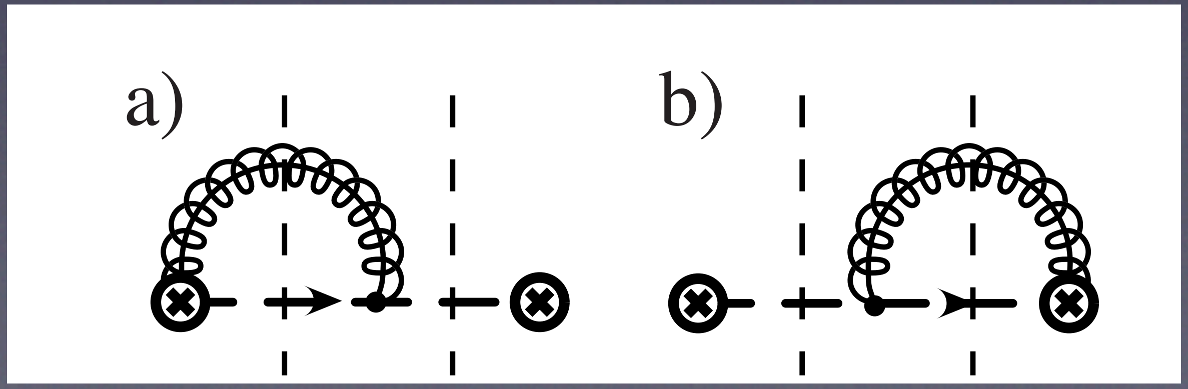
Virtual:



Calculation of jet function

Hornig, CL, Ovanesyan (2009)

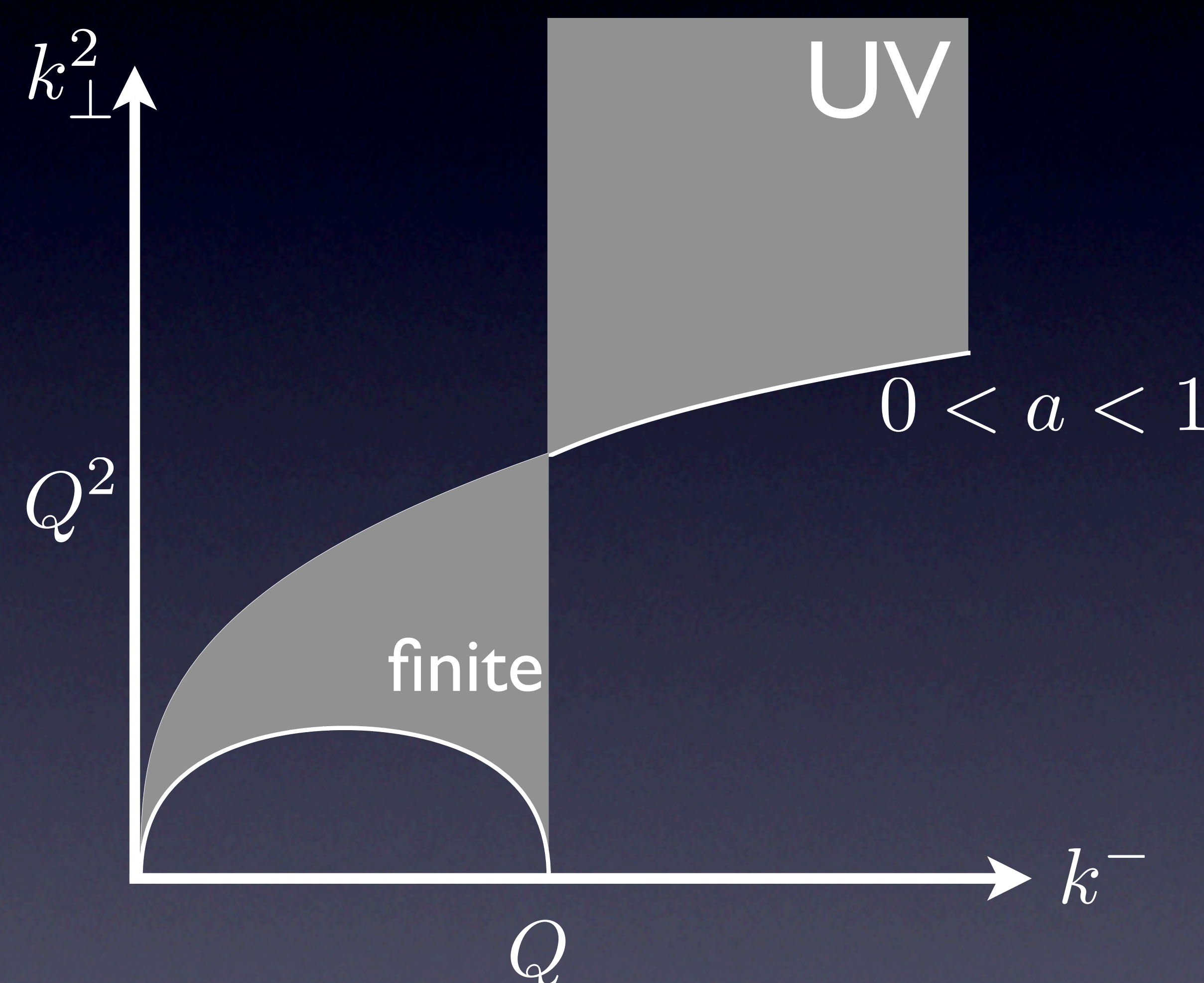


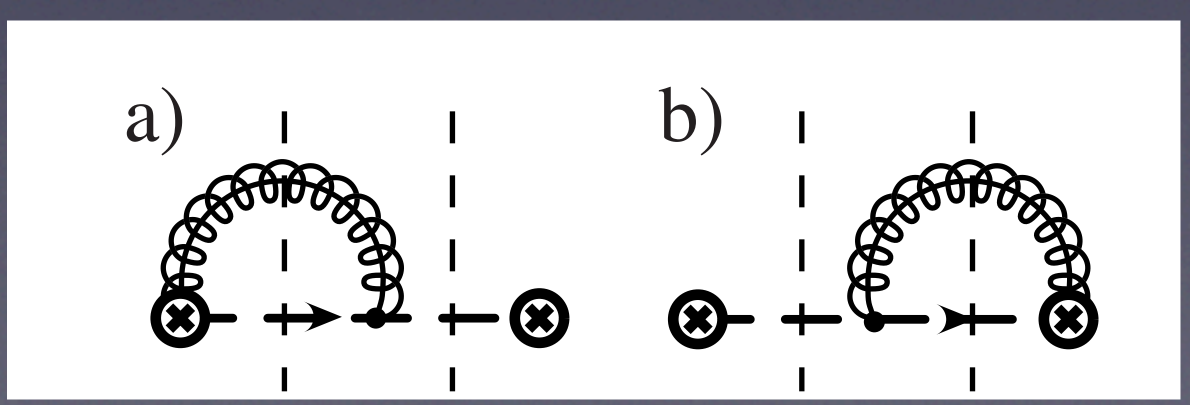


$$\int dk^- \int dk_\perp^2 \frac{1}{k^-} \frac{1}{(k_\perp^2)^{1+\epsilon}}$$

Calculation of jet function

Hornig, CL, Ovanesyan (2009)

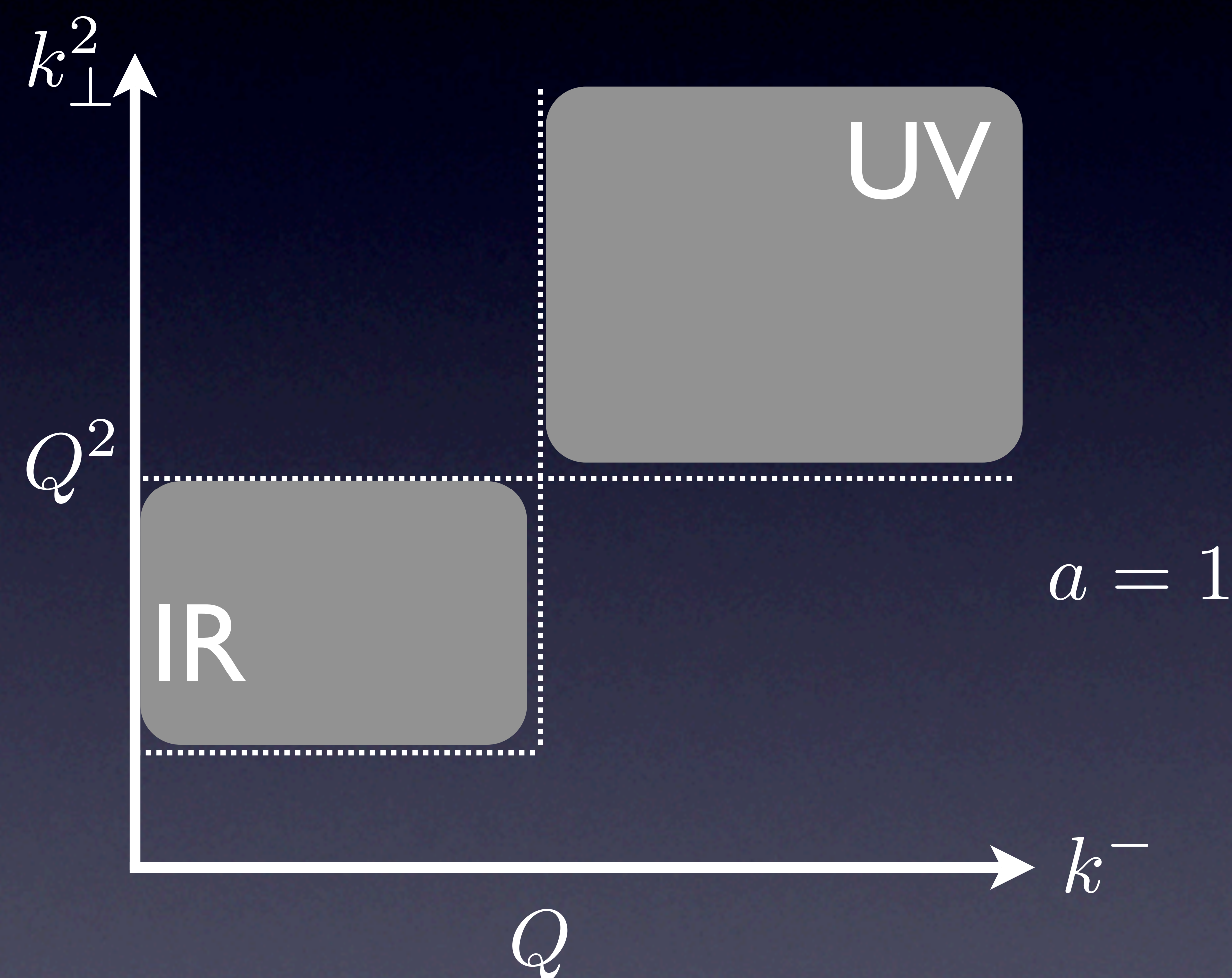


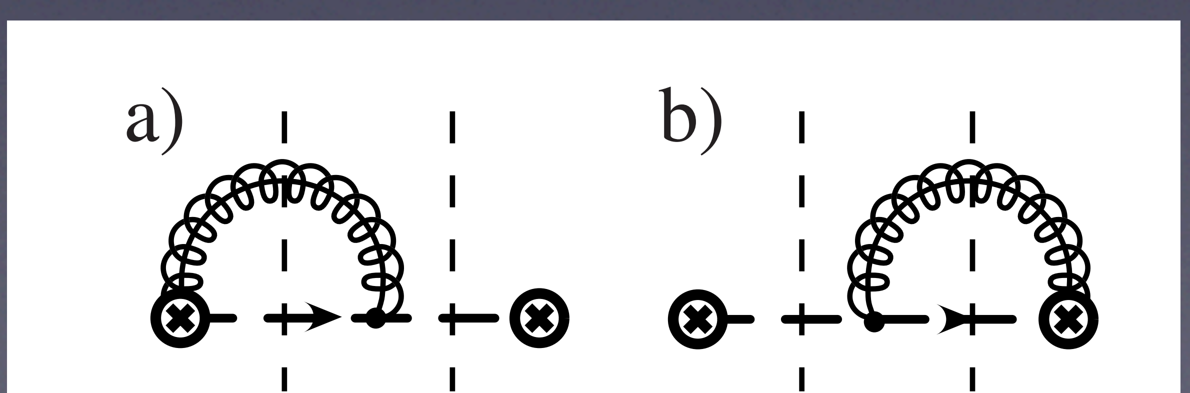


$$\int dk^- \int dk_\perp^2 \frac{1}{k^-} \frac{1}{(k_\perp^2)^{1+\epsilon}}$$

Calculation of jet function

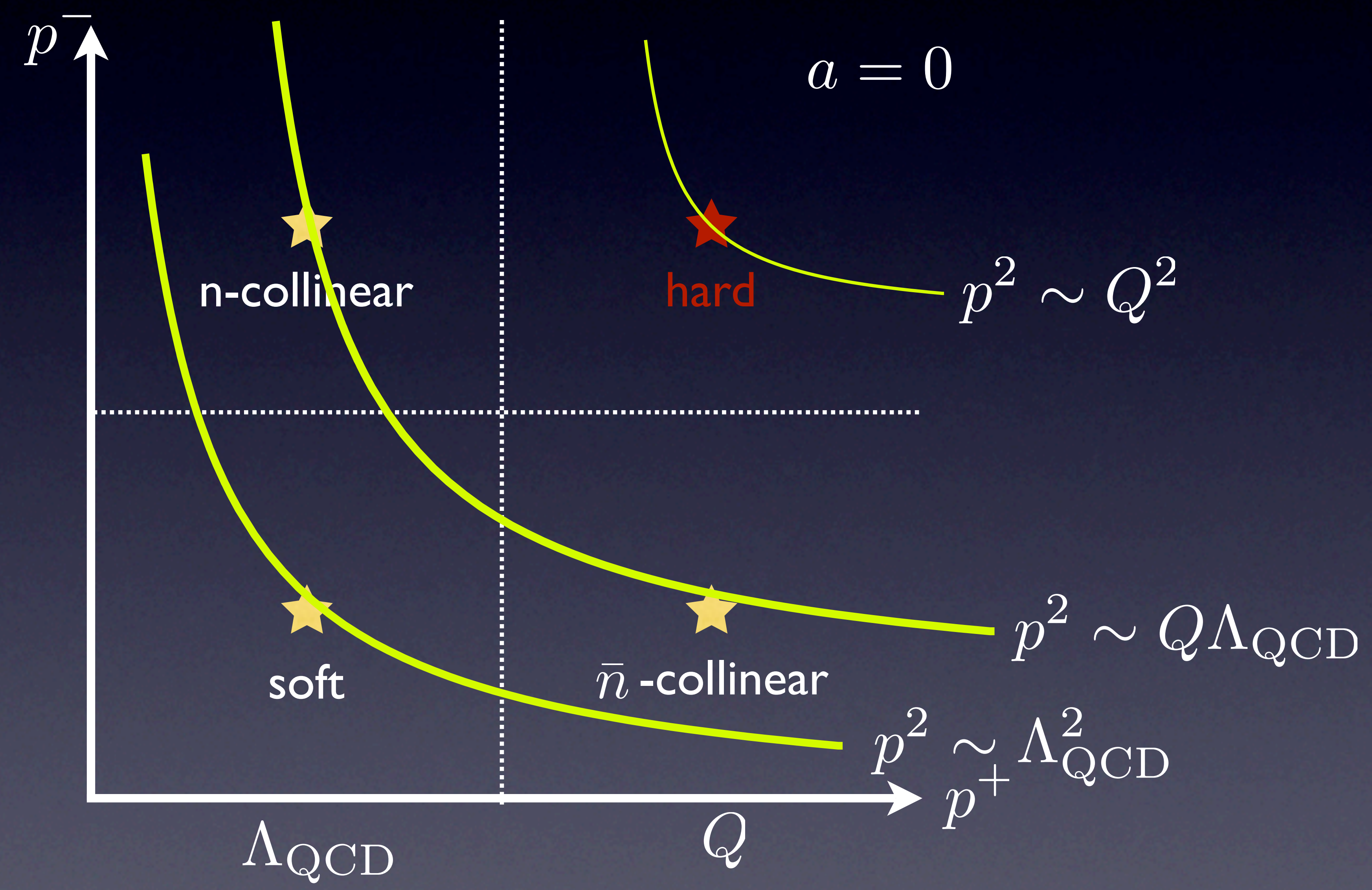
Hornig, CL, Ovanesyan (2009)



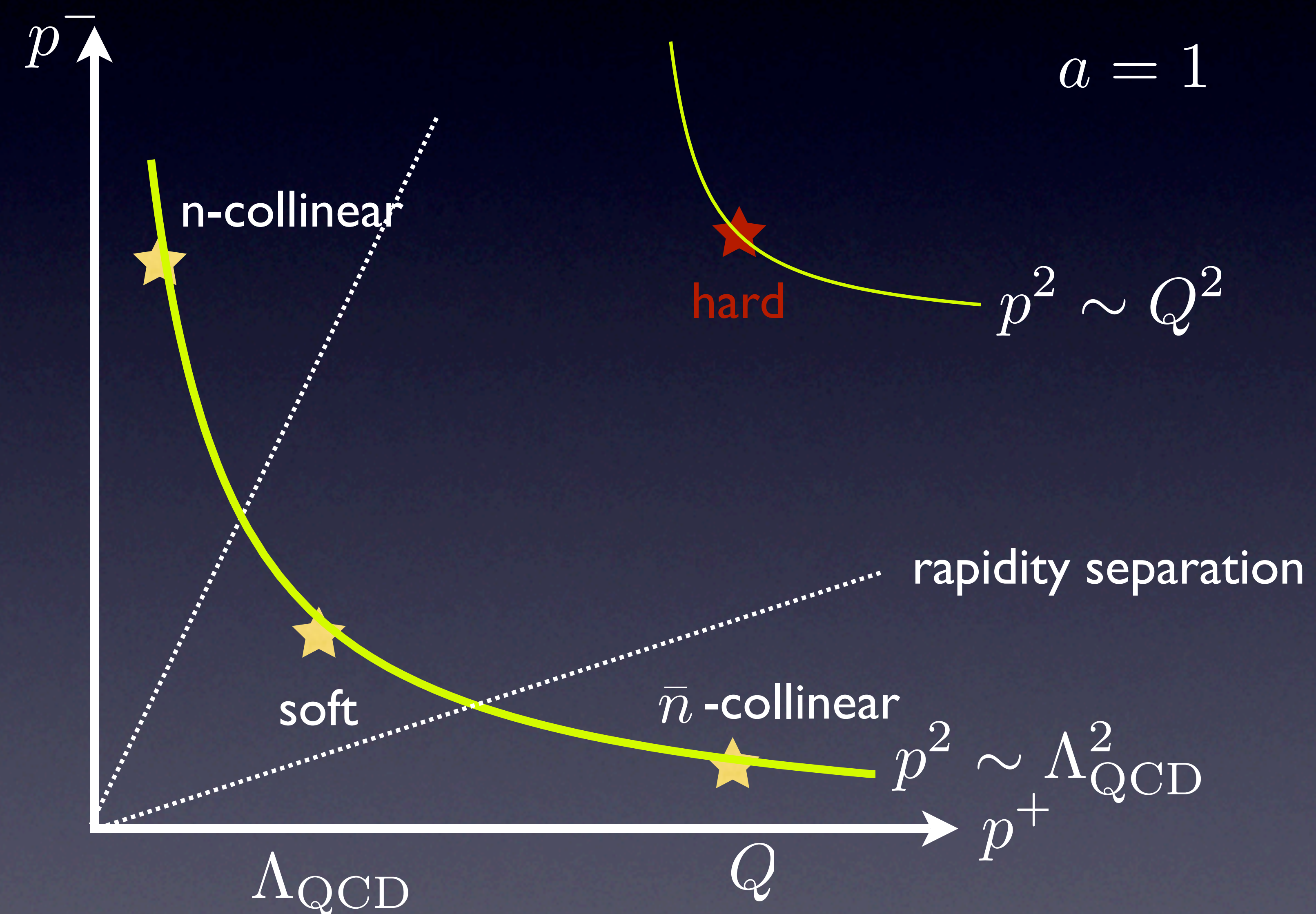


$$\int dk^- \int dk_\perp^2 \, \frac{1}{k^-} \frac{1}{(k_\perp^2)^{1+\epsilon}}$$

Collinear and soft modes



Collinear and soft modes



Factorization now requires distinguishing collinear and soft modes by rapidity, not virtuality (i.e. transverse momentum)

Manohar, Stewart (2006)

Towards Jets and Event Shapes at LHC

Factorization in pp → Jets

Bauer, Hornig, Tackmann (2008)

- Hadrons in both initial and final state
  - Initial state hadrons → PDFs, operator definitions of beam remnant and underlying event
  - Final state jets → jet functions
  - New soft functions connecting incoming and outgoing lines

- Factorization shown so far for subprocess $q\bar{q} \to q\bar{q}$, needs generalization to other partonic subprocesses.

pp Observables

Bauer, Hornig, Tackmann (2008)

- Any jet algorithm can be described as acting on a list of final state momenta, that is, the energy-momentum flow.
- Energy-momentum flow operator at heart of field theoretic treatment of jet algorithms.
- Parameters of jet algorithm choose typical size of jets, setting the scale of jet functions, and determining the factorizability of jet cross sections.
- Jet algorithms may be combined with event shapes to study internal structure of individual jets.

Angularities as measures of jet substructure

cf. Almeida, S.J. Lee, Perez, Sterman, Sung, Virzi (2008)

- *a* dependence of angularities distinguishes "diffuse" jets with energy spread out more evenly from jets whose constituents are sharply bunched
- Potential to distinguish light quark, heavy quark, and gluon jets; or QCD jets from W, Z jets
- Possible to construct other jet shapes, study factorization properties and distinguishing power

Angularities as measures of jet substructure

Almeida, S.J. Lee, Perez, Sterman, Sung, Virzi (2008)