

PRECISE DARK MATTER RELIC ABUNDANCE IN DECOUPLED SECTORS

based on

[2007.03696]

by

Torsten Bringmann, Paul Frederik Depta,
[Marco Hufnagel](#), Kai Schmidt-Hoberg

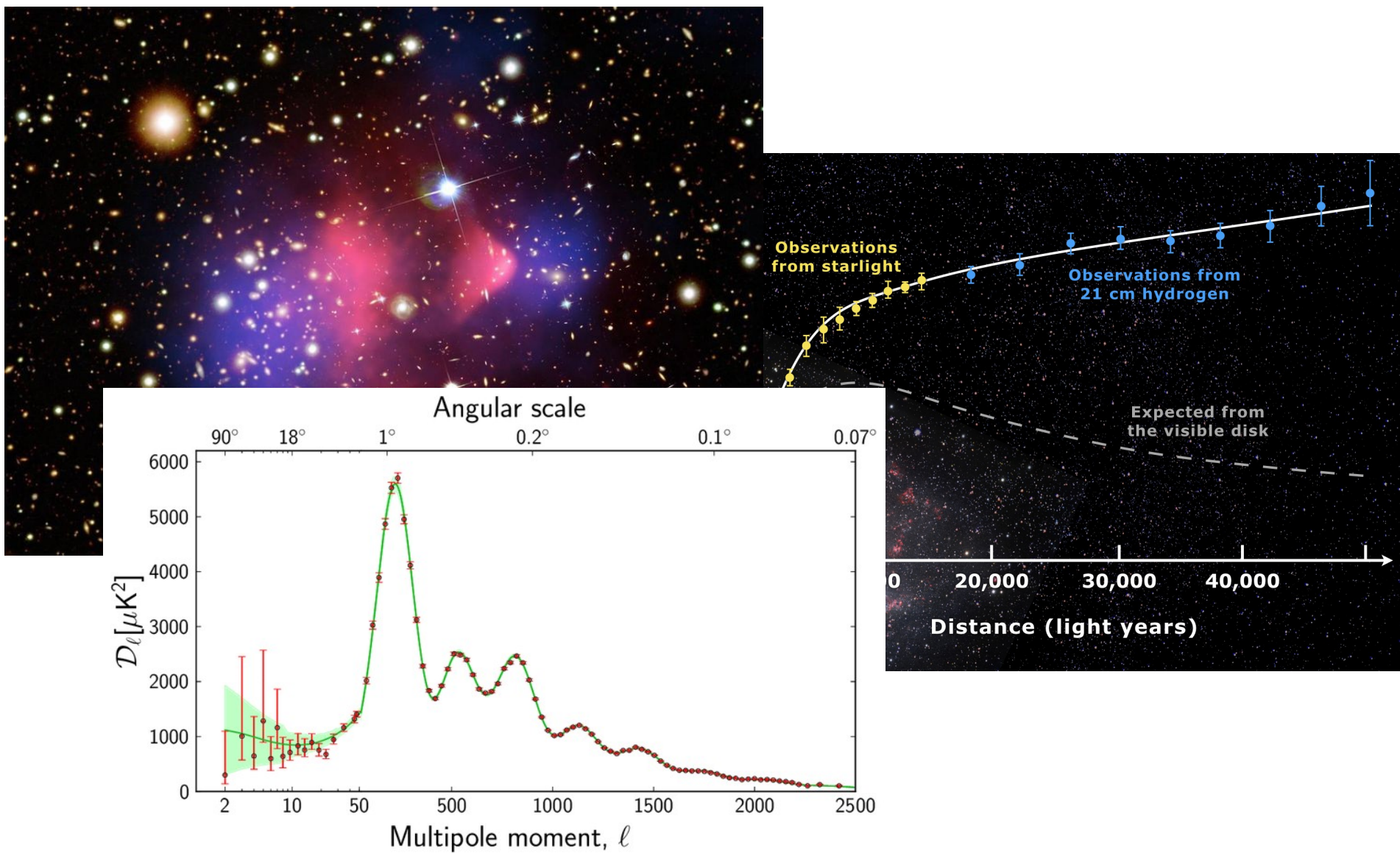
Wednesday, December 16, 2020



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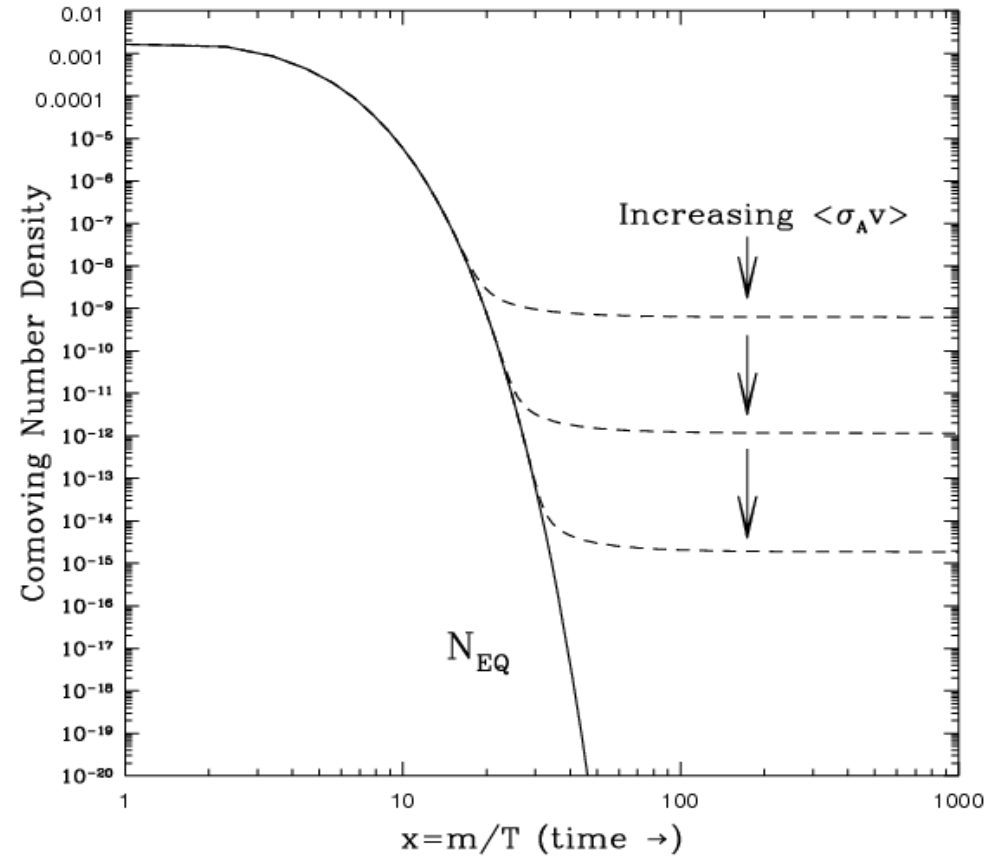
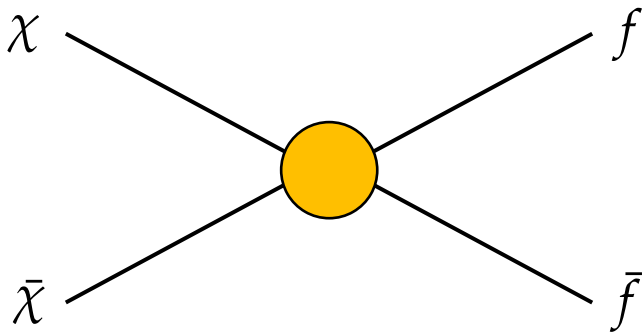


Dark Matter



Standard Freeze-out

Boltzmann equation $\chi\bar{\chi} \leftrightarrow f\bar{f}$



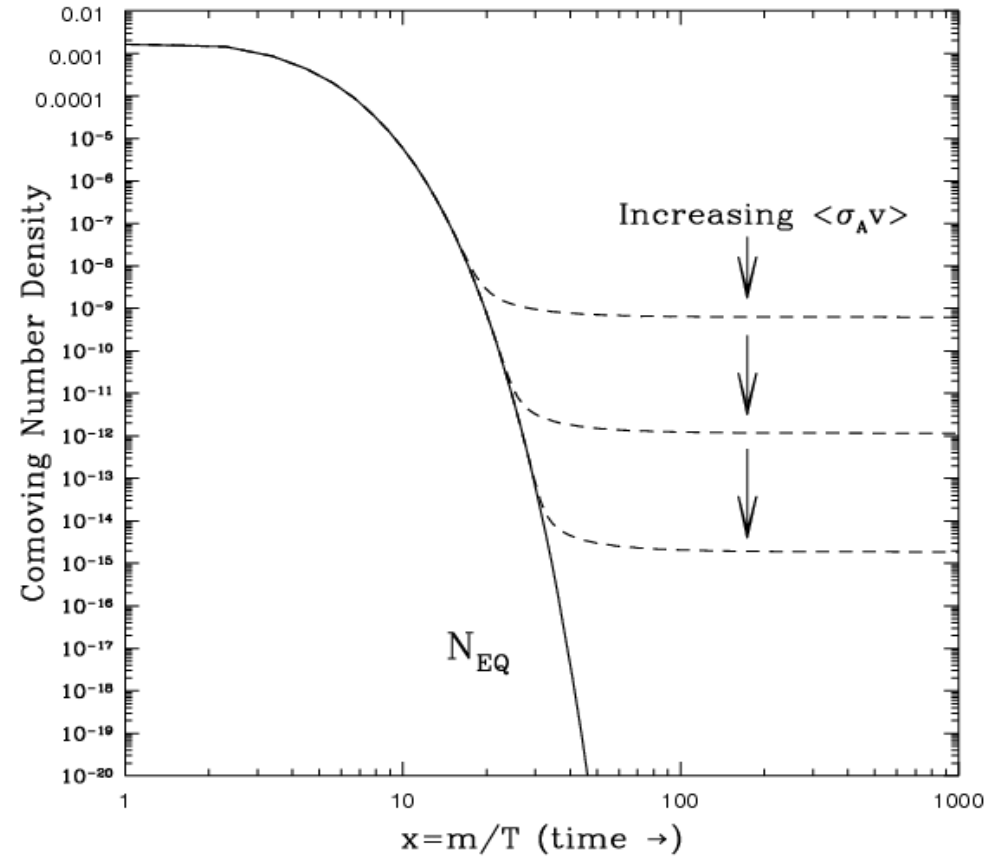
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$$\frac{dn_i}{dt} + 3Hn_i = \mathfrak{C}/N_\chi$$

dilution term

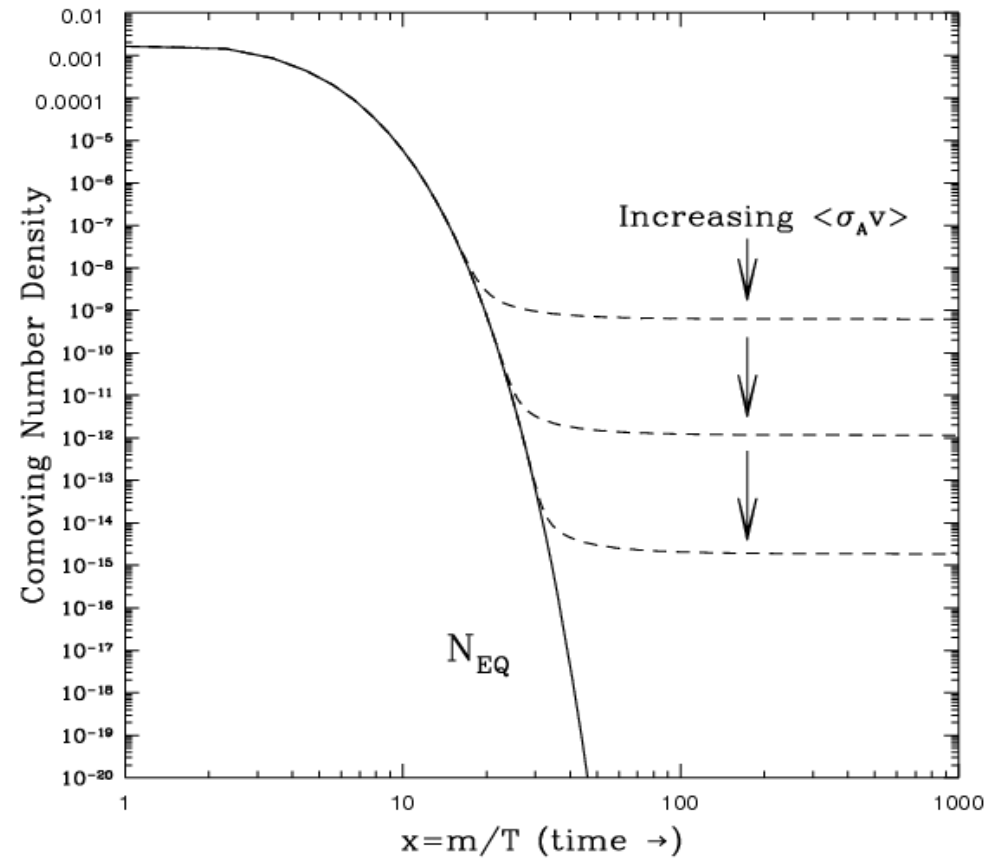
collision operator



Standard Freeze-out

Boltzmann equation $\chi\bar{\chi} \leftrightarrow f\bar{f}$

$$\frac{dn_i}{dt} + 3Hn_i = \langle\sigma v\rangle (n_{\chi,\text{eq}}n_{\bar{\chi},\text{eq}} - n_{\chi}n_{\bar{\chi}})$$



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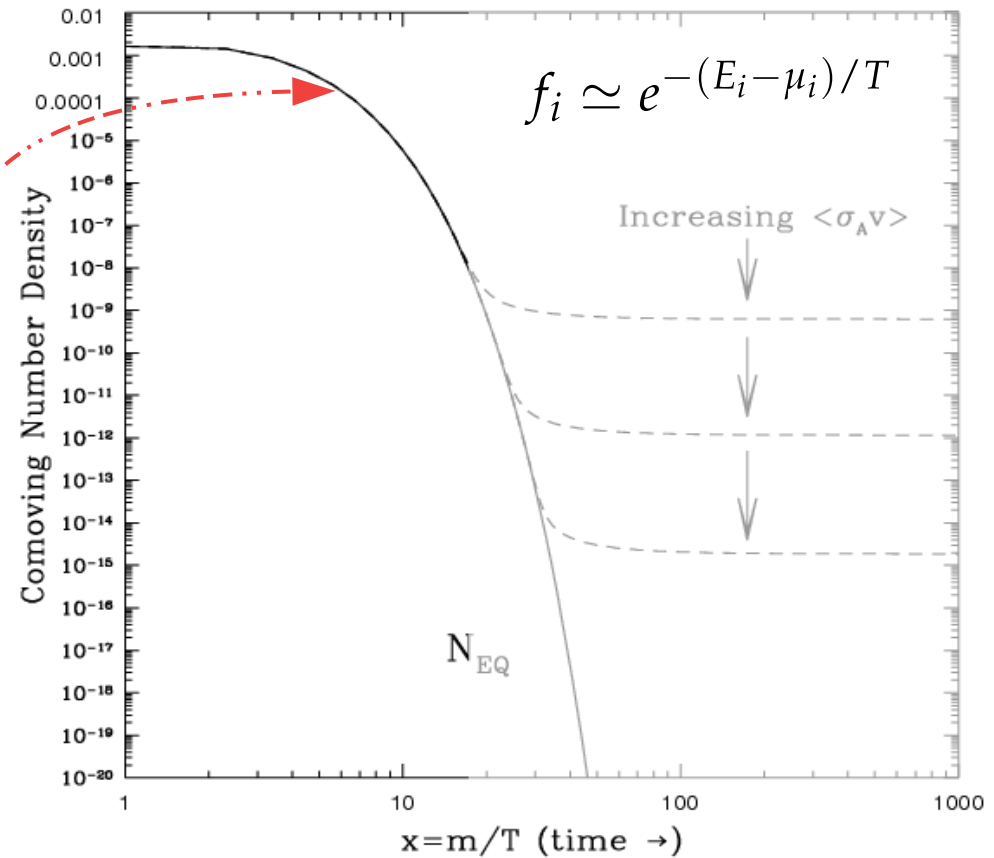
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chemical equilibrium

$$0 \simeq \mu_f + \mu_{\bar{f}} = \mu_{\chi} + \mu_{\bar{\chi}} \rightarrow \mu_{i,\text{eq}} = 0$$

$$\langle\sigma v\rangle n_{\chi} \gg H$$



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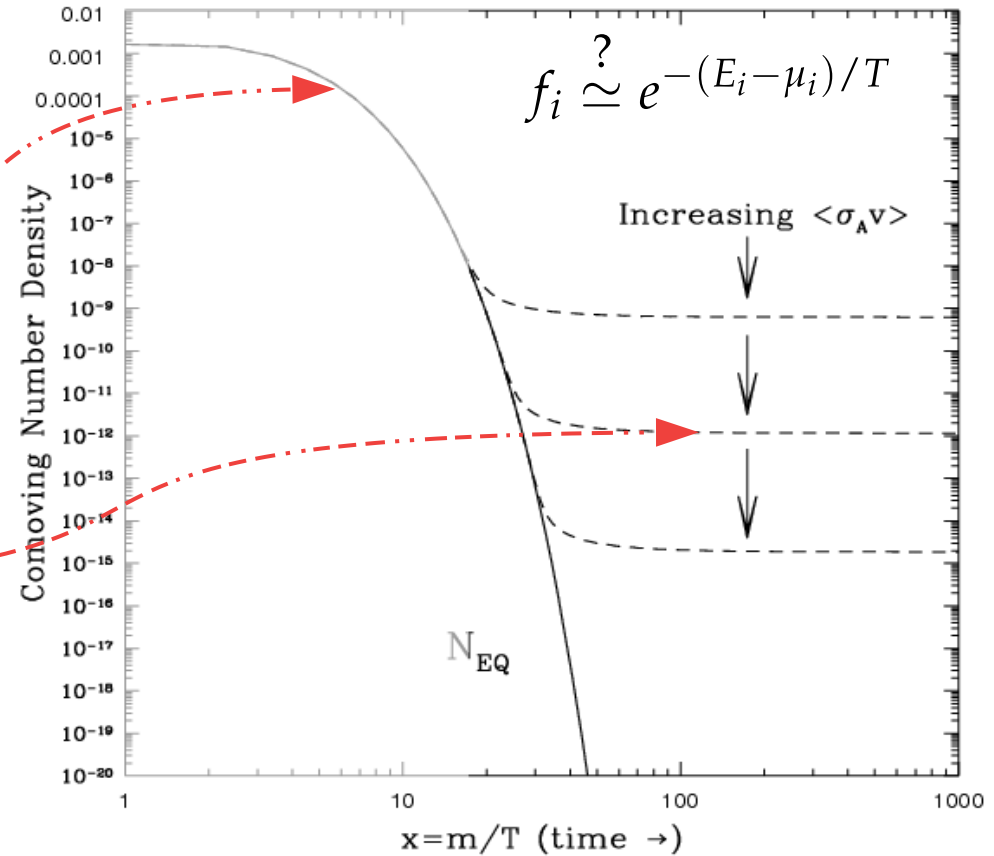
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chemical equilibrium
 $0 \simeq \mu_f + \mu_{\bar{f}} = \mu_{\chi} + \mu_{\bar{\chi}} \rightarrow \mu_{i,\text{eq}} = 0$

kinetic equilibrium
 $\mu_f + \mu_{\chi} = \mu_{\bar{f}} + \mu_{\bar{\chi}} \rightarrow \mu_{i,\text{eq}} \neq 0$

$$\langle\sigma v\rangle n_{\chi} \ll H$$

$$f_i \stackrel{?}{\simeq} e^{-(E_i - \mu_i)/T}$$



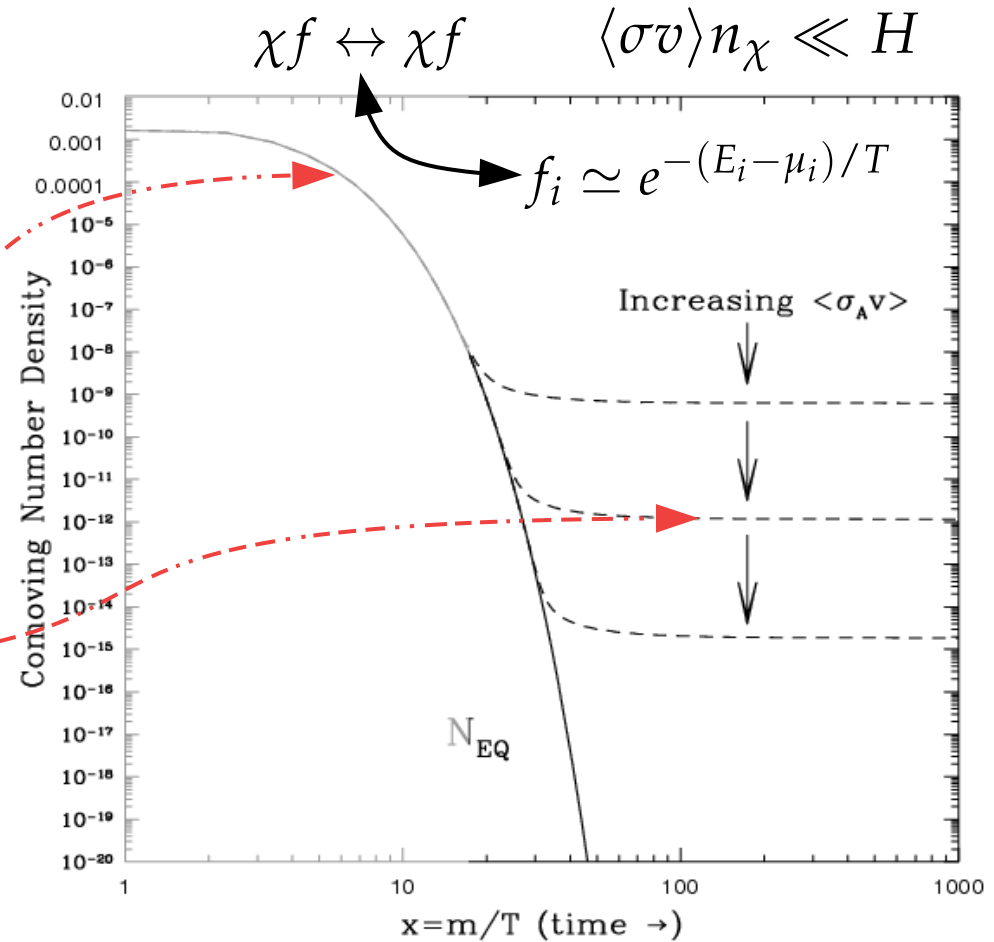
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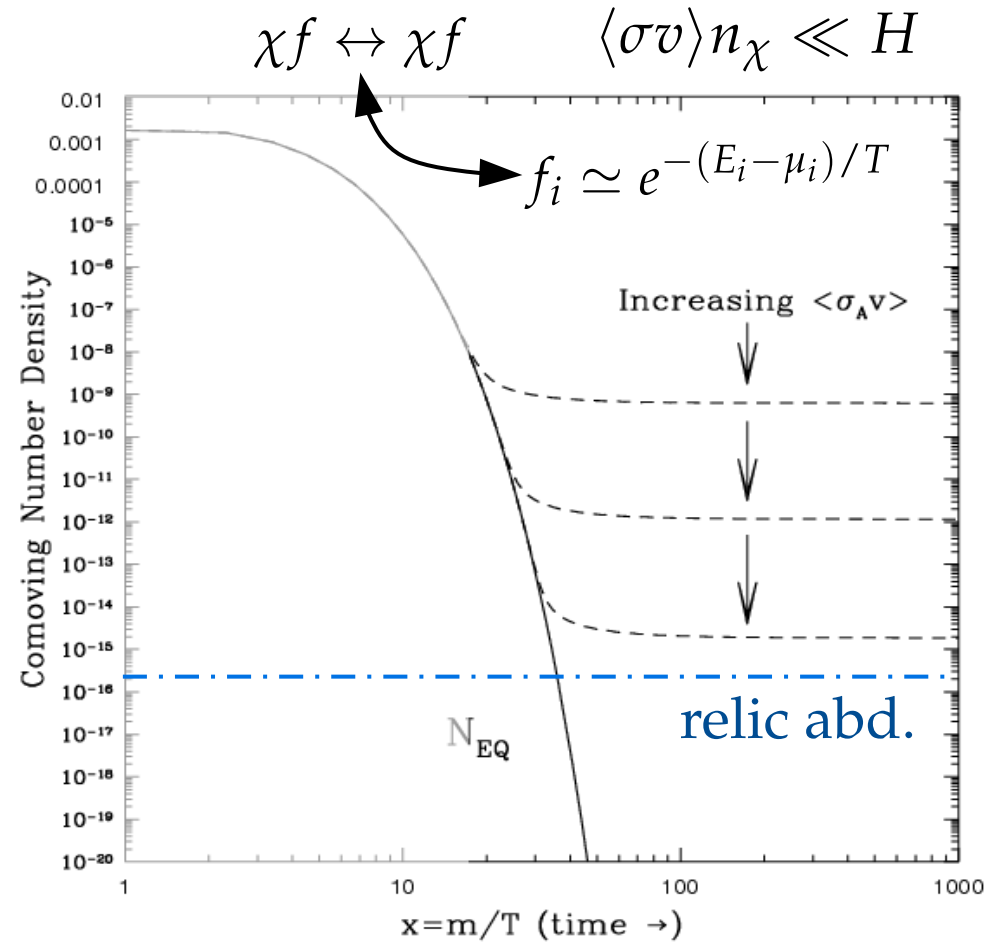
kinetic equilibrium
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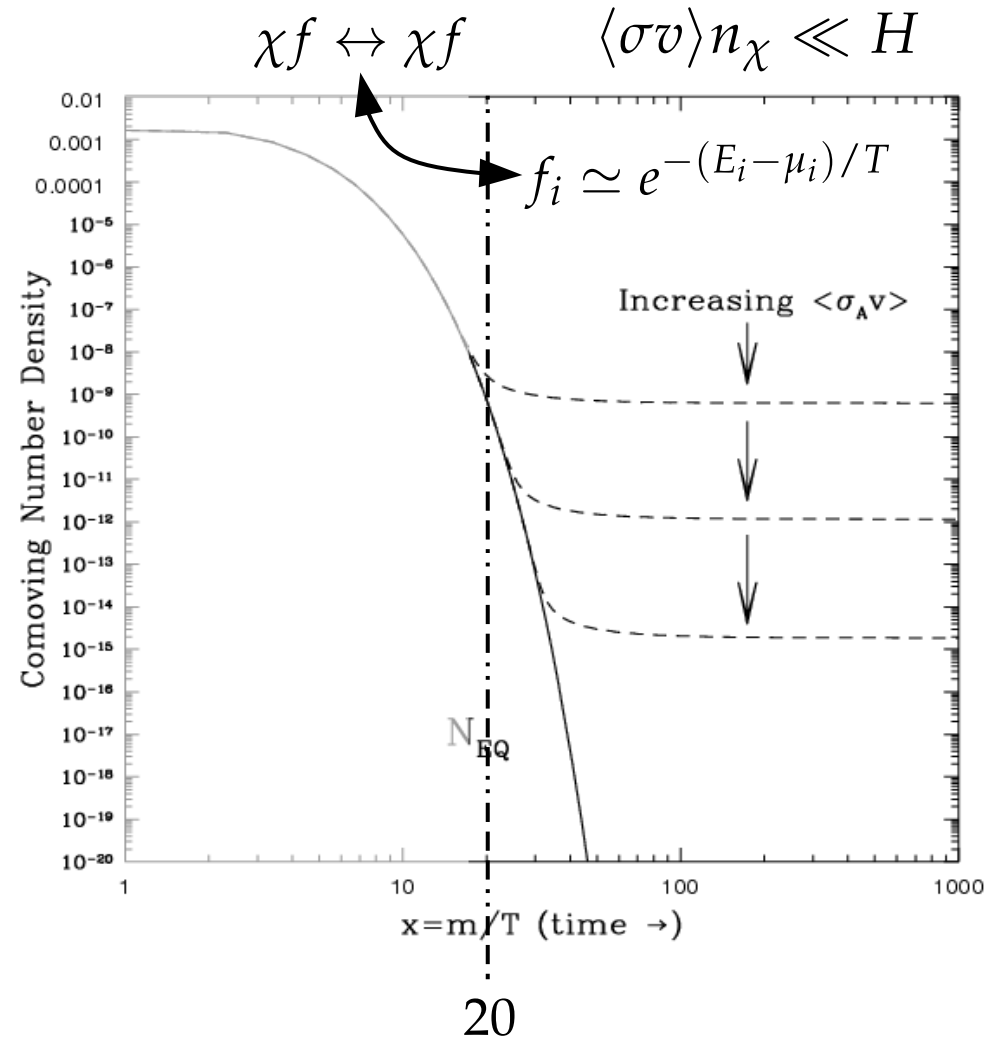
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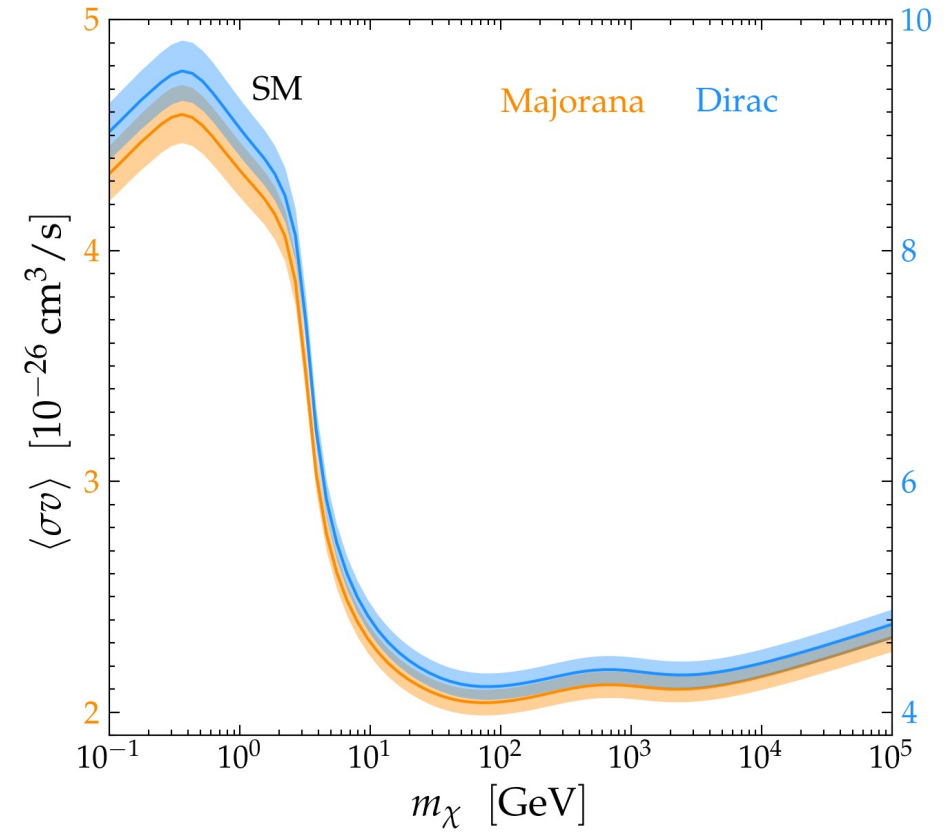
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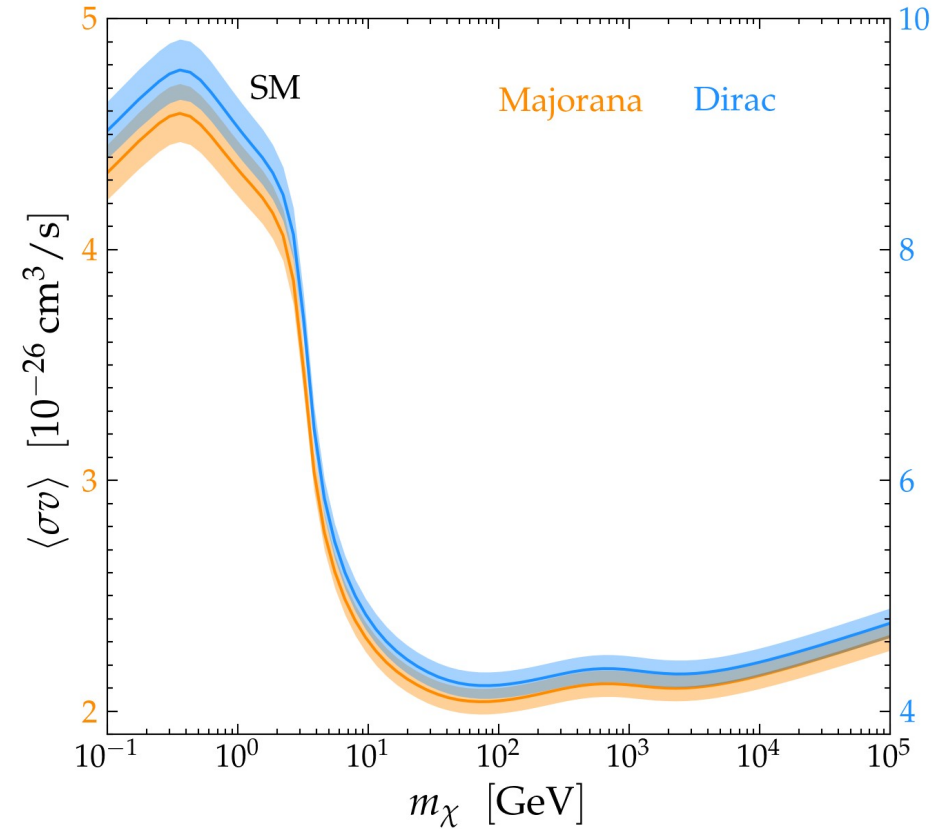
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Assumptions

- $f_i \simeq e^{-(E_i - \mu_i)/T} = e^{\mu_i/T} f_{i,\text{eq}}$
 → kinetic equilibrium | non-relativistic during freeze-out | $\mu_i/m_i \ll 1$
- $f_j \simeq e^{-E_j/T} = f_{j,\text{eq}}$
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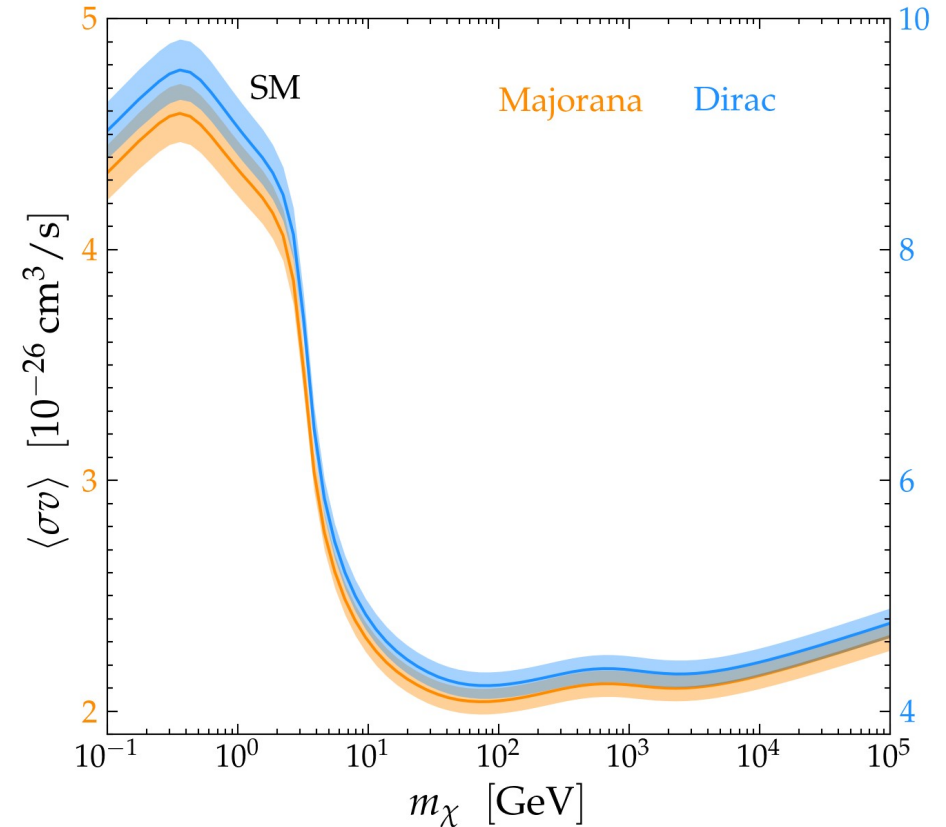
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Relativistic and quantum effects are small

Hidden Freeze-out

Boltzmann equation $\chi\bar{\chi} \leftrightarrow SS$

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?

Possible scenarios

$m_S = 0$	$\Gamma_{\chi\bar{\chi} \leftrightarrow \chi\bar{\chi}S} \gg \Gamma_{\chi\bar{\chi} \leftrightarrow SS}$	$\mu_S = 0$
$m_S \neq 0$	$\Gamma_{N\bar{N} \leftrightarrow SS} \gg \Gamma_{\chi\bar{\chi} \leftrightarrow SS}$	$\mu_S = 0$
$m_S \neq 0$	$\Gamma_{\chi\bar{\chi} \leftrightarrow \chi\bar{\chi}S} \stackrel{?}{\sim} \Gamma_{\chi\bar{\chi} \leftrightarrow SS}$	$\mu_S = ?$

Assumptions

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✓

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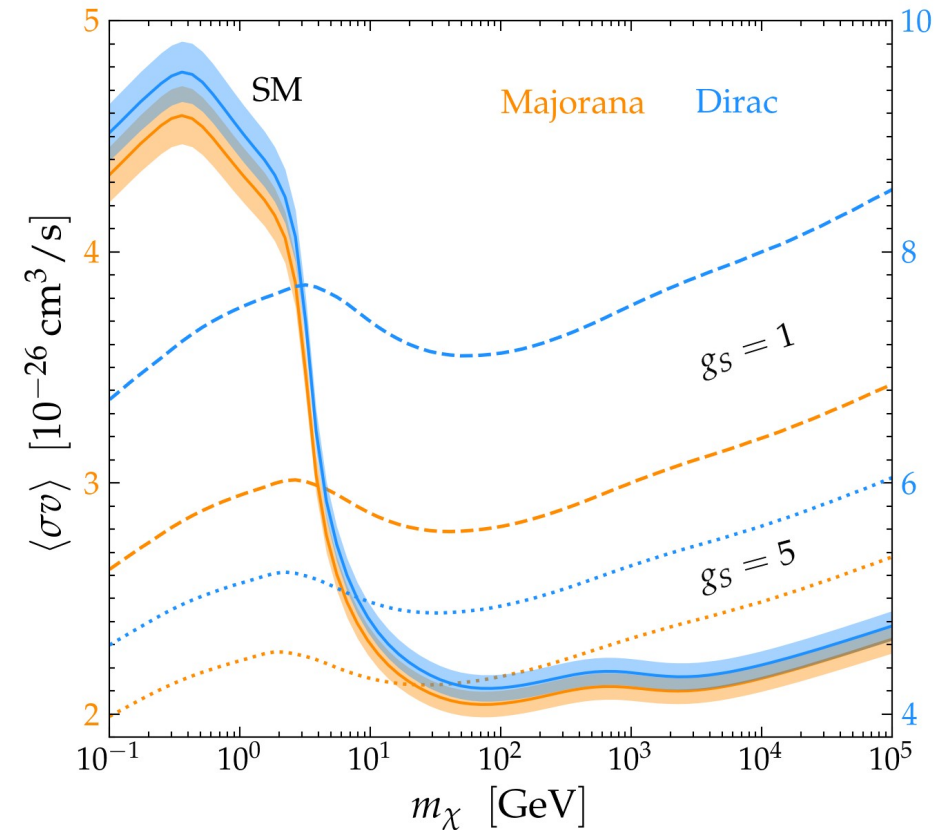
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DarkSUSY

Hidden Freeze-out

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Possible scenarios

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Assumptions

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For $m_S \sim m_{\chi}, \mu_S \sim m_S$

Assumptions

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X

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X

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Boltzmann equation $\chi\bar{\chi} \leftrightarrow SS$

$$\frac{dn_i}{dt} + 3Hn_i = \mathfrak{e}/N_\chi$$

$$\frac{dn_S}{dt} + 3Hn_S = -\mathfrak{e}$$

→ Solve for both μ_S and μ_χ
with **full spin-statistics**

$$f = \frac{1}{e^{(E-\mu)/T_\chi} \pm 1}$$

Possible scenarios

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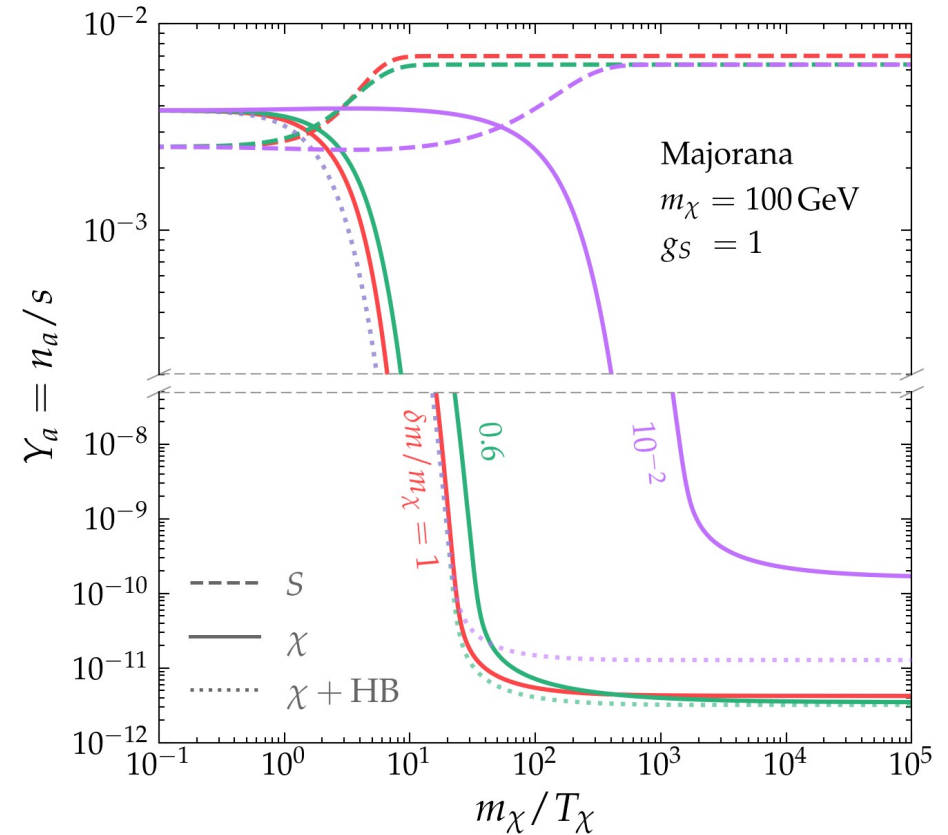
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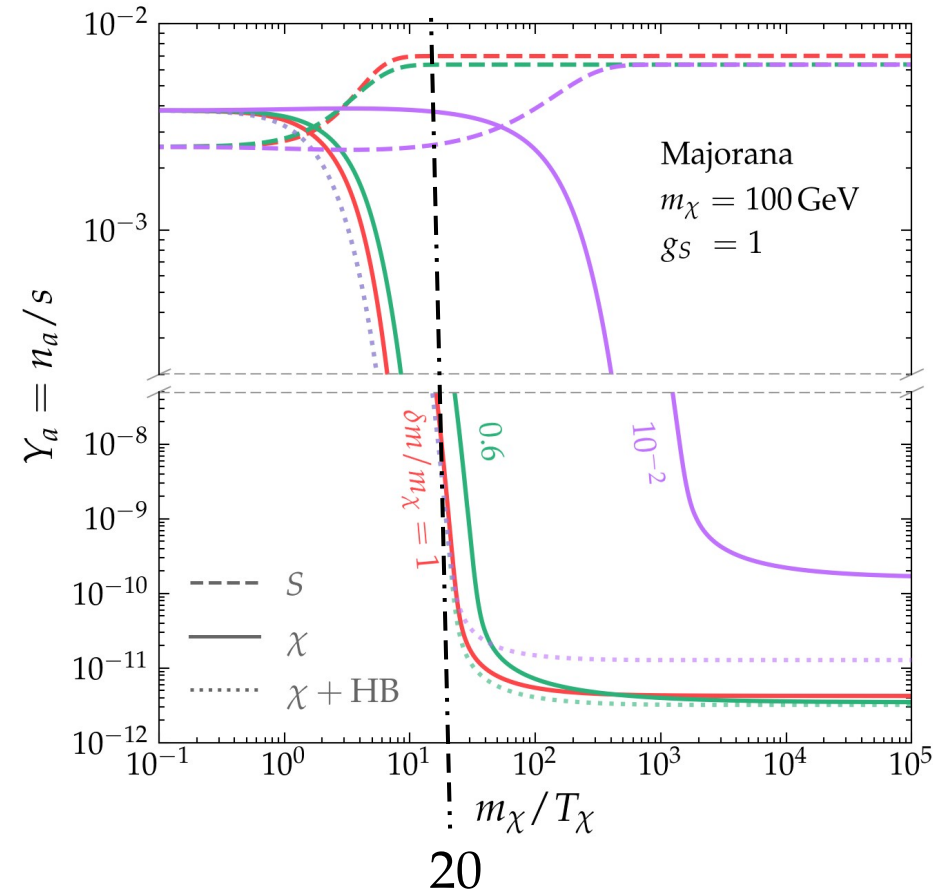
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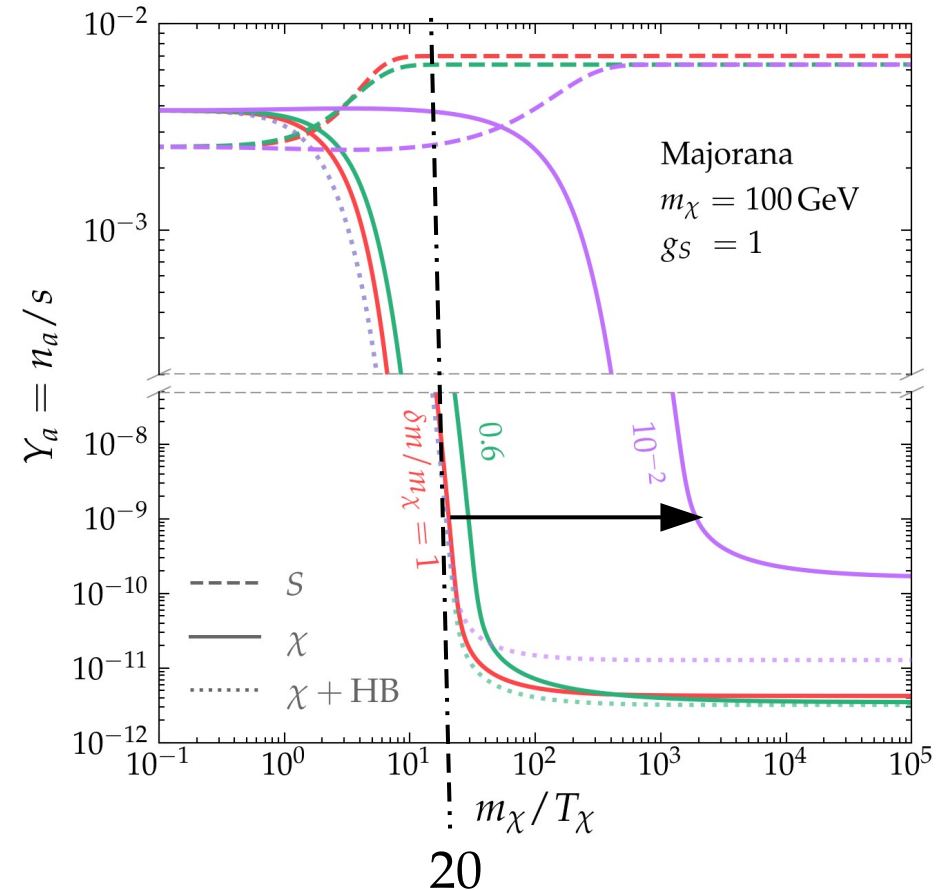
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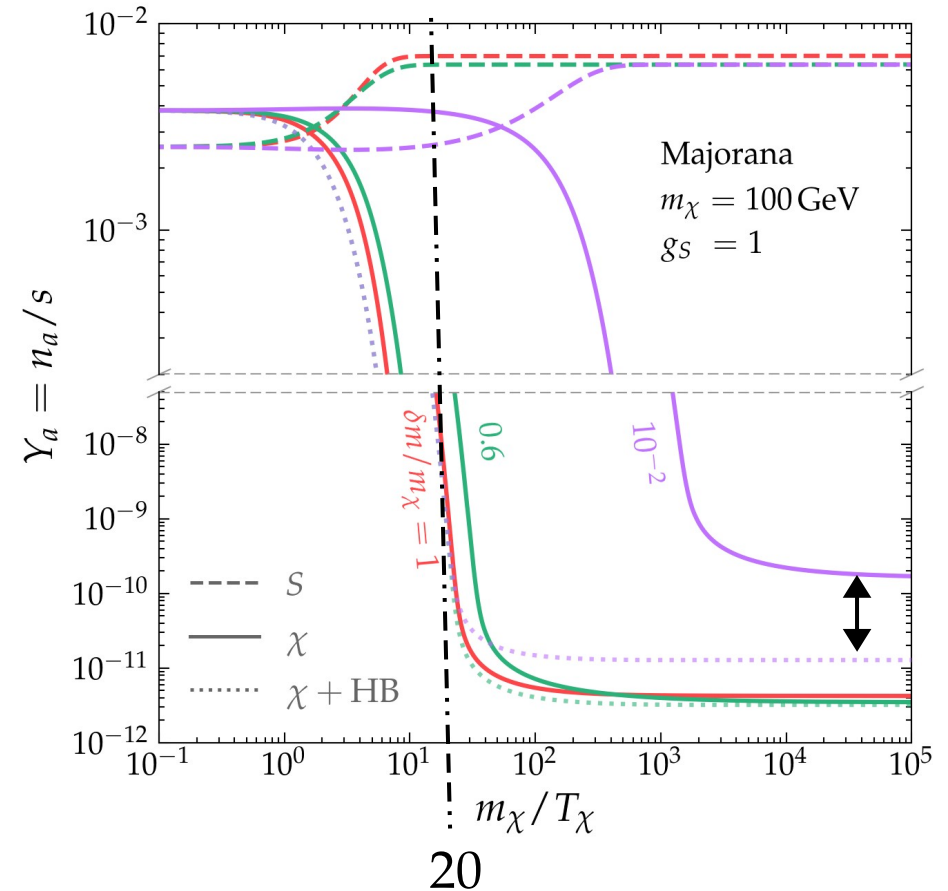
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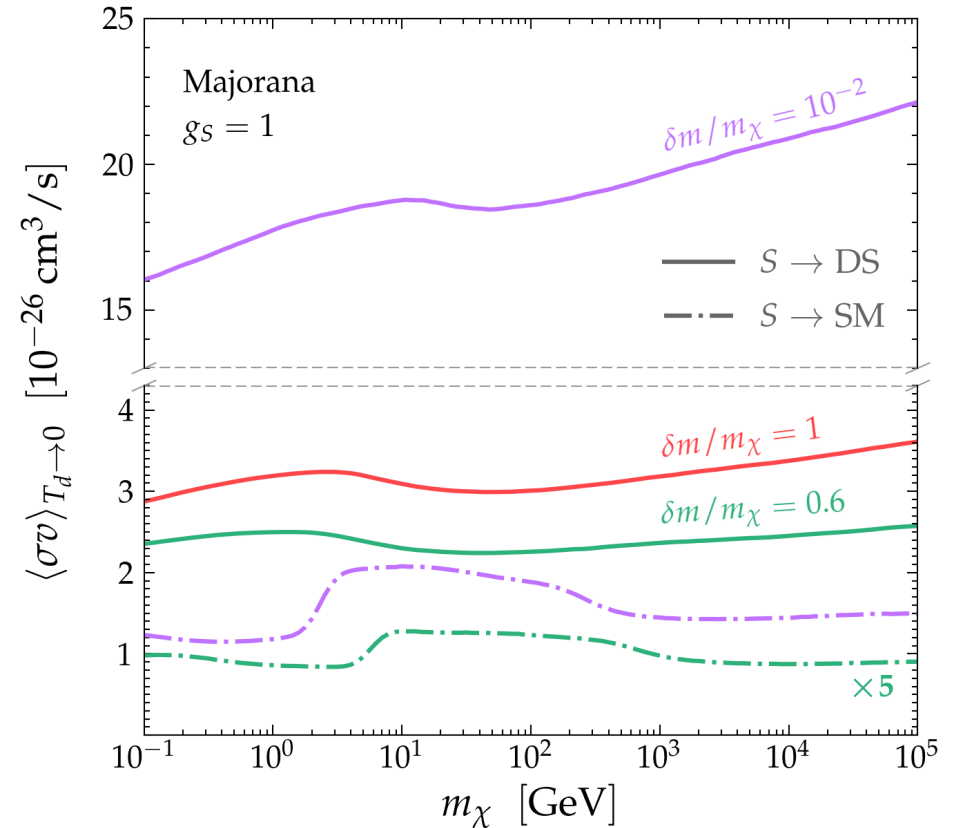
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→ Solve for both μ_S and μ_χ
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Conclusions

- We presented a formalism that can be used to make precision calculations for DM freeze-out in secluded dark sectors
- For dark-sector particles that are degenerate in mass, the standard treatment breaks down and **correction can be rather large**

