From LHC to GWs: Heavy Scattering Amplitudes

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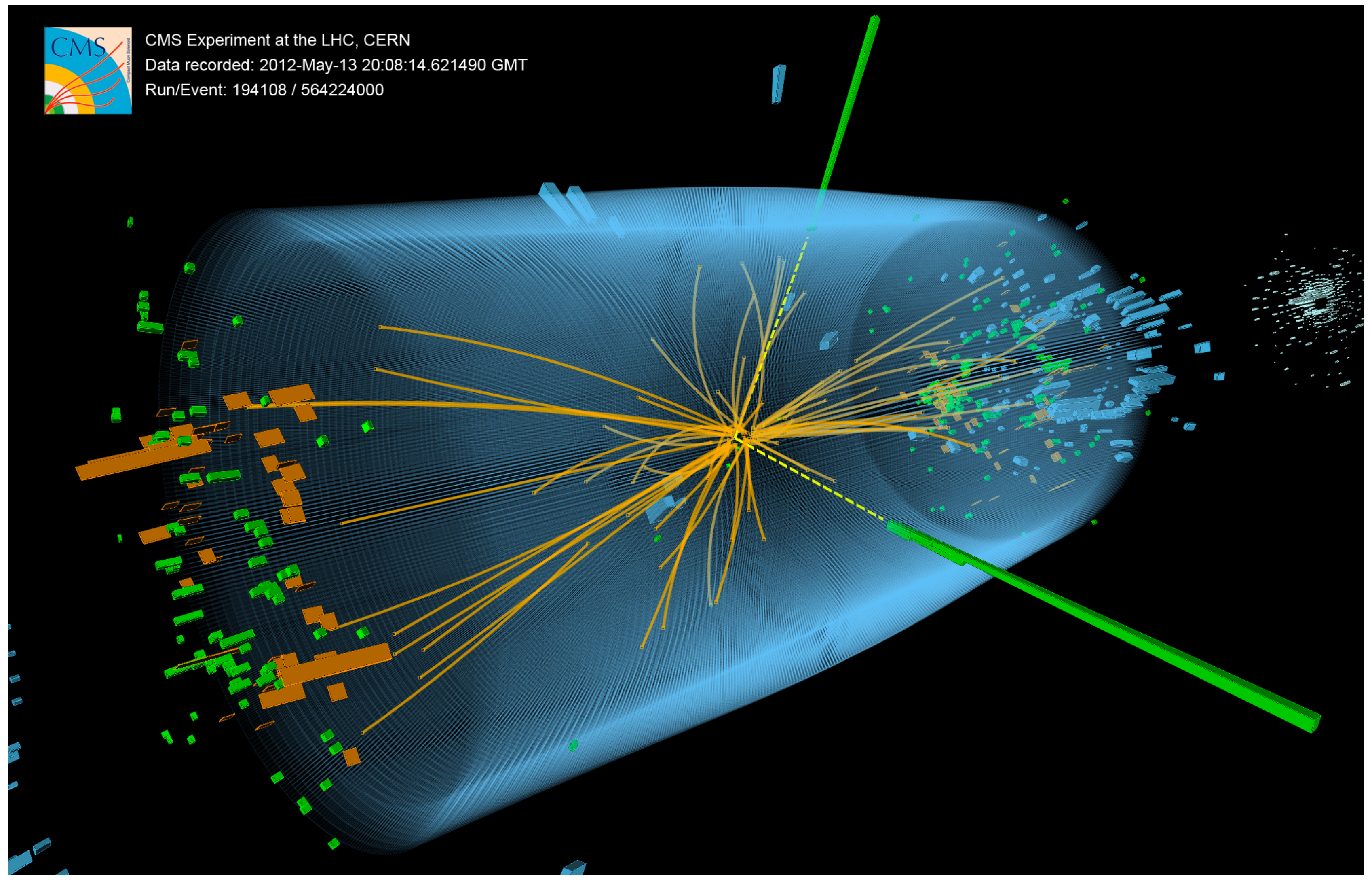
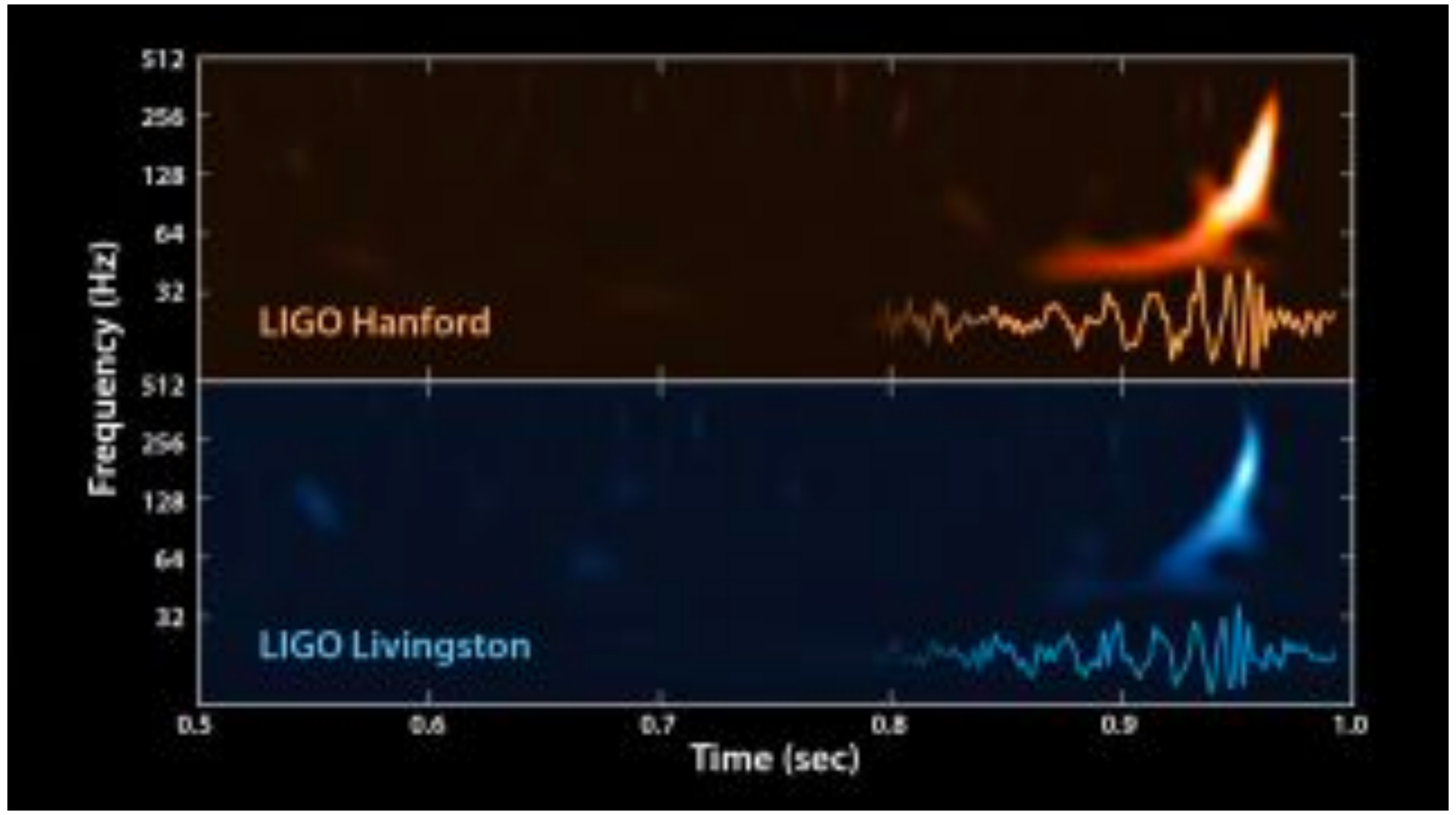
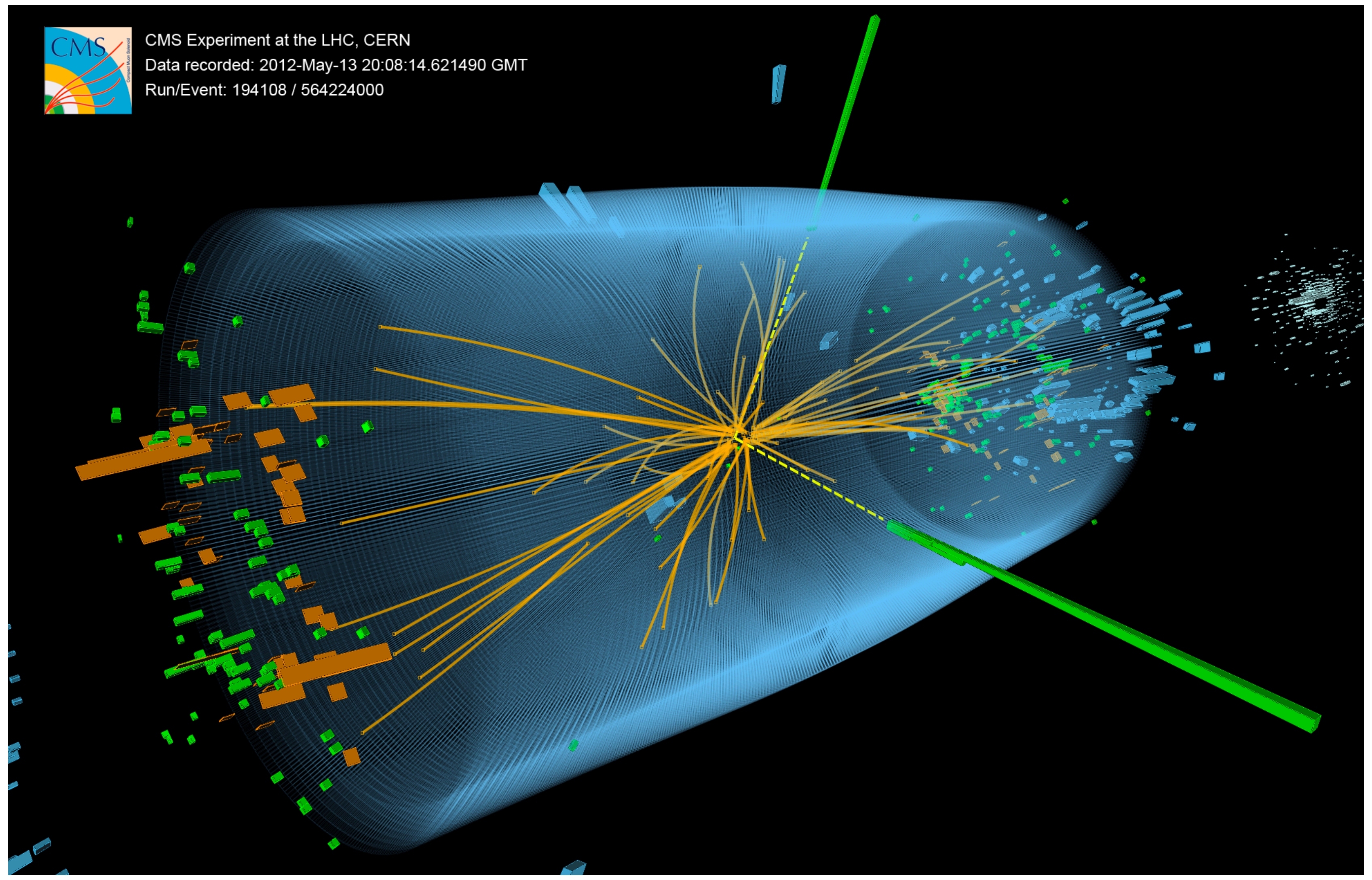
After amazing discoveries…

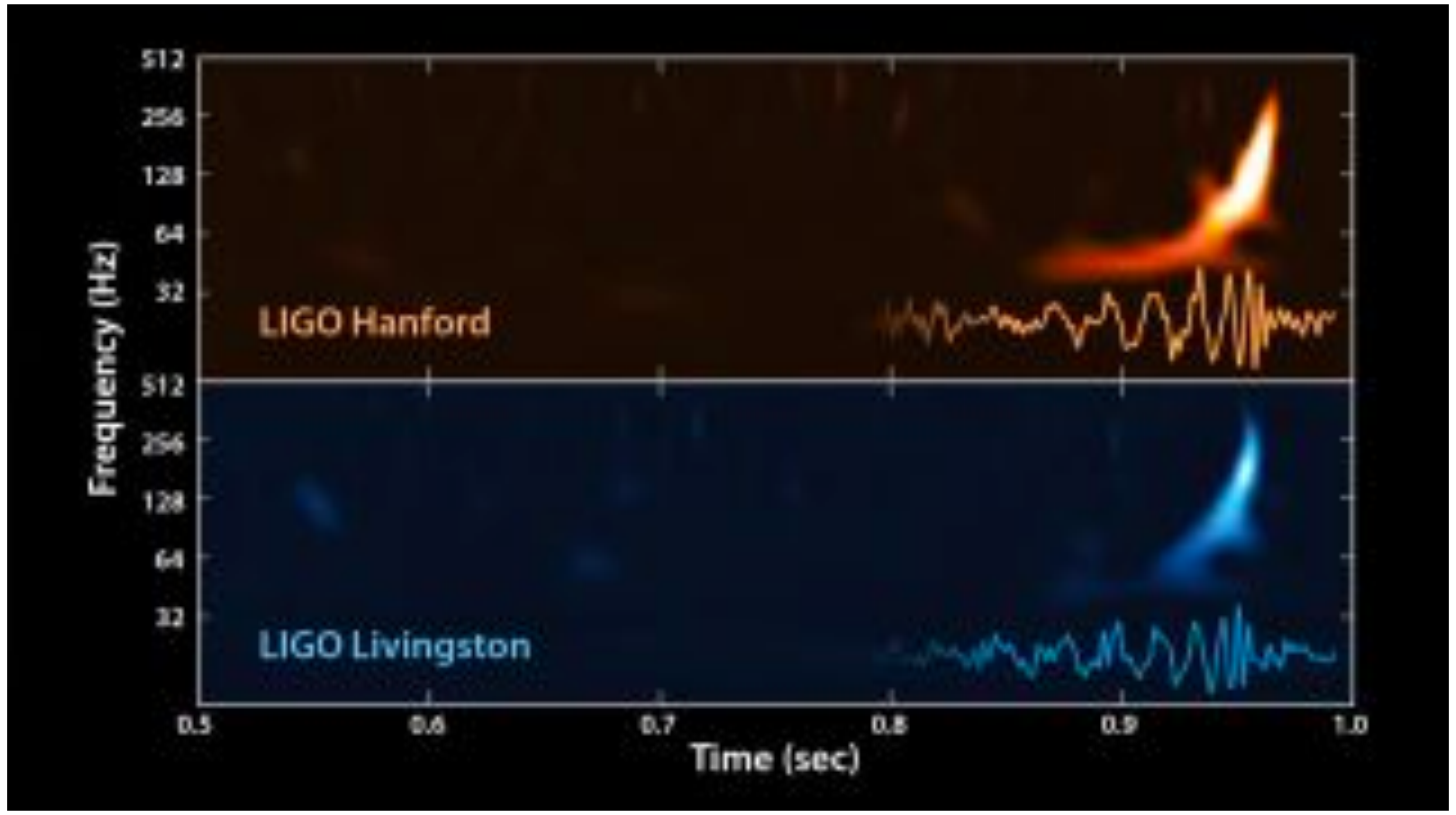




… we need precise theoretical predictions.

After amazing discoveries…





Classical limit ?

$$i\mathcal{M}(p_1, \ldots, p_n)$$

From the LHC side, future experimental analysis will require precise theoretical predictions

Higher loops

QFT : $i\mathcal{M}(p_1, ..., p_n)$

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From the GWs side, LIGO/Virgo require analytical predictions for the GW templates used.

↓

Two-body problem in GR: Knowledge of the interaction Hamiltonian to high-accuracy.

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Two-body problem in GR: Knowledge of the interaction Hamiltonian to high-accuracy.

Higher loops

loops ?

QFT : $i\mathcal{M}(p_1, \ldots, p_n)$

In order to improve precision, we need to sharpen our tools

Tradicional Feynman diagrammatic methods are still good and well-suited

But can be streamlined/helped with modern methods/on-shell methods

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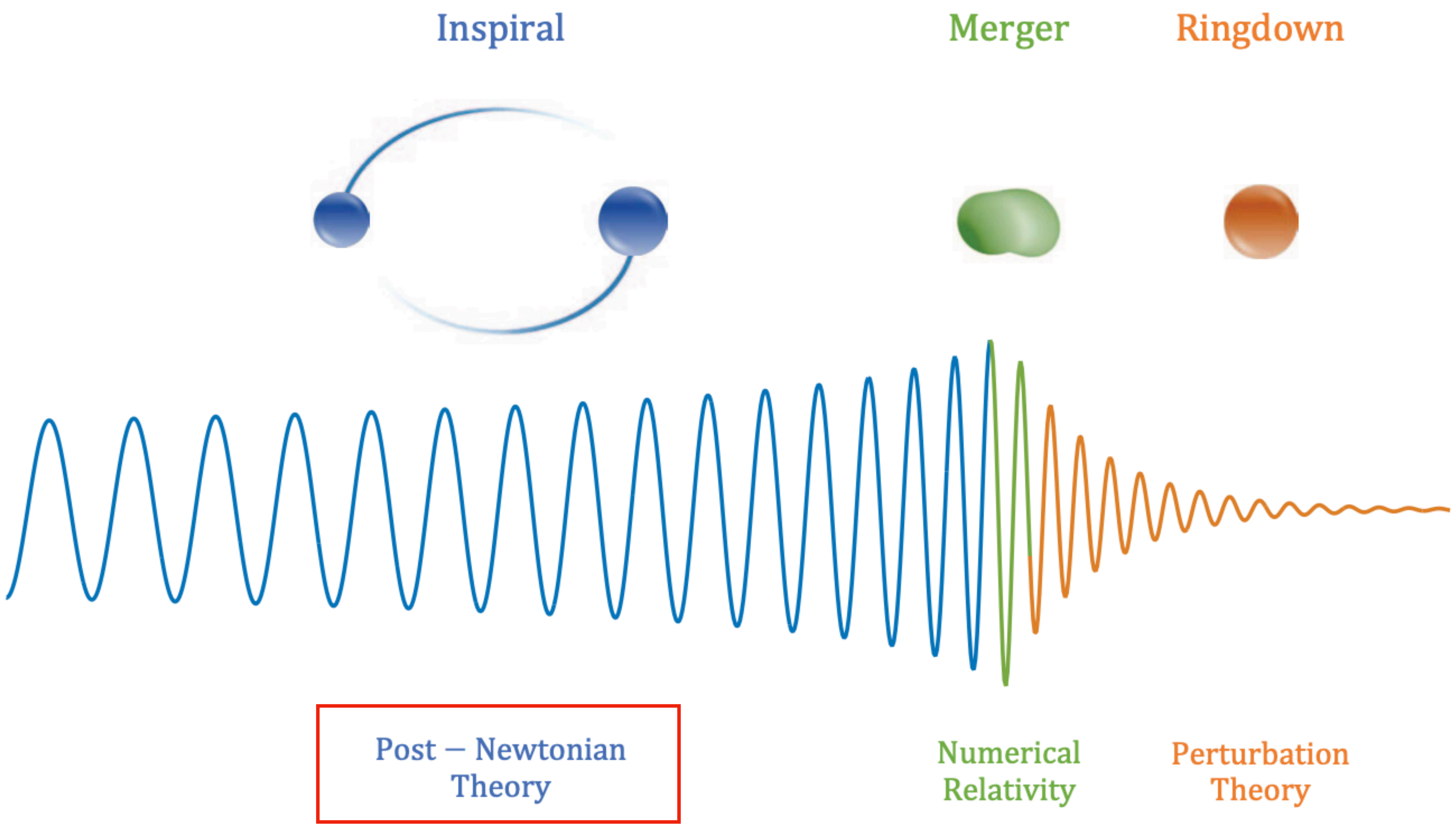
But can be streamlined/helped with modern methods/on-shell methods

Here, I will give some examples on how traditional and new methods could help for SM(EFT) and GW templates predictions

Where can we use for GW? Improve PN theory



Small deviations accumulate over time

PN: double expansion in v and G

PM: expansion in G: QFT methods

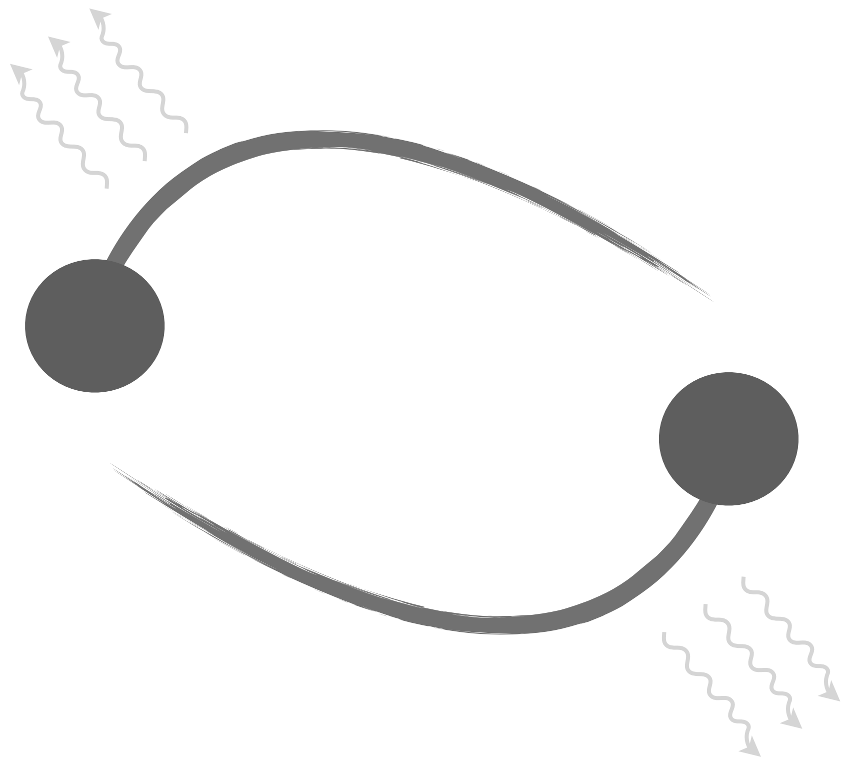
[Figure from Antelis and Moreno, 1610.03567]

[Cheung, Rothstein, Solon,19’]

From Amplitudes to Hamiltonians (or potentials)

(LIGO/Virgo is interested in potentials)

Two-body bounded problem



Effective theory

$$V(p,q)$$

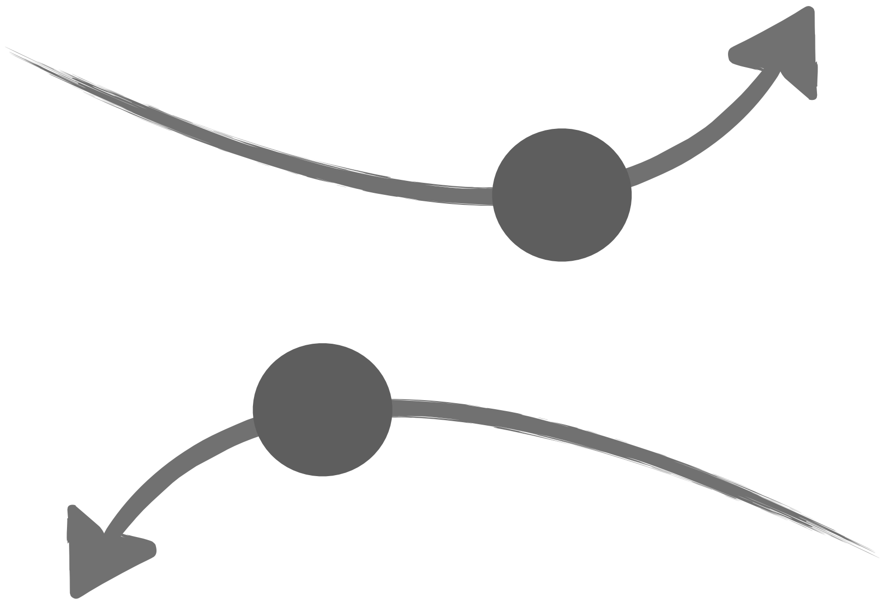
$$\downarrow$$

$$A_{\mathrm{EFT}}(p,q)$$

Matching

$$=$$

Scattering problem



Full theory

$$A_{\mathrm{full}}$$

$$\downarrow \quad \hbar \to 0$$

$$A(p,q)$$

Restoring $\hbar$ to obtain classical physics

We are used to set $\hbar = c = 1$ **hiding the classical limit:** $\hbar \to 0$

Rule of thumb to restore $\hbar$

[Kosower, Maybee, O'Connal, 19']

$$\text{gravity, QED/QCD couplings} : \hbar^{-1/2}$$

$$\text{massless momenta} : p^\mu = \hbar \bar{p}^\mu$$

wavenumber

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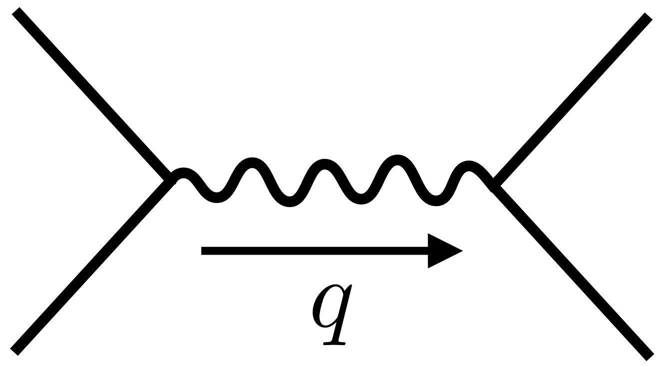
wavenumber

Loops can have classical information!

Newtonian Potential from scalar scattering

At tree-level (1PM) $\quad i\mathcal{M}(p_1, p_2 \to p_1 - \hbar\bar{q}, p_2 + \hbar\bar{q}).$



$$\mathcal{M}^{(1)} \approx -\frac{4\pi G m_1 m_2}{\hbar^3 \bar{q}^2}$$

Newtonian Potential

$$V = -\int \frac{d^3q}{(2\pi)^3} e^{-\frac{i}{\hbar}\vec{q}\cdot\vec{r}}\mathcal{M} = -\hbar^3 \int \frac{d^3\bar{q}}{(2\pi)^3} e^{-i\vec{\bar{q}}\cdot\vec{r}}\mathcal{M}, \longrightarrow V(r) = -\frac{G m_1 m_2}{r}$$

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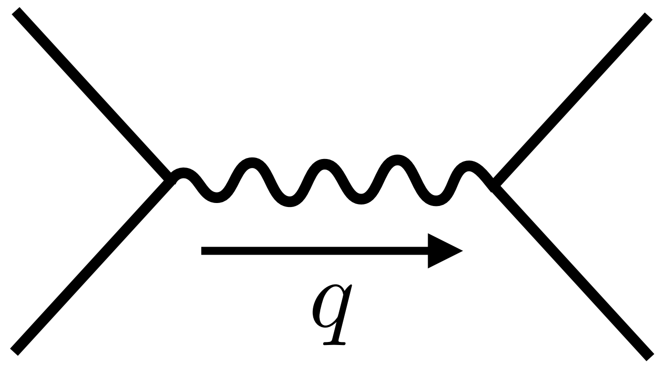

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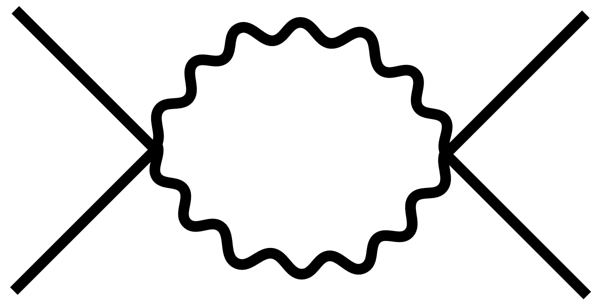
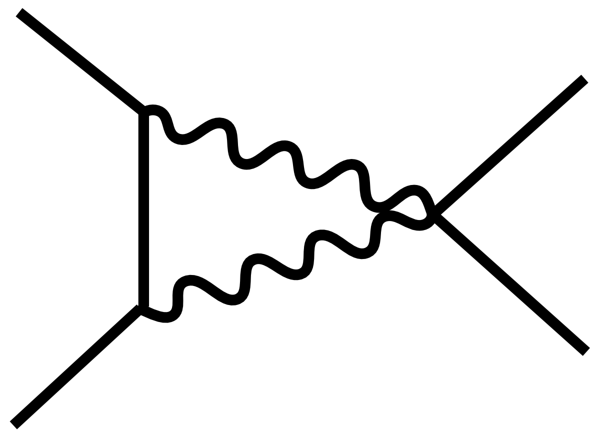
$\mathcal{M} \sim \hbar^{-3}$ $\longrightarrow$ classical contributions

Scalar scattering with loops

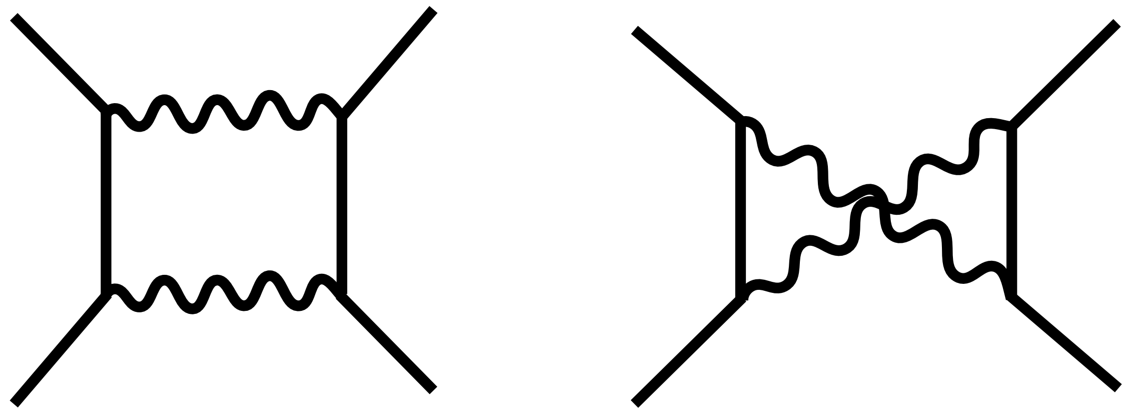
At one-loop level (2PM) : box, triangles and bubbles



$i\mathcal{M}_{\text{bubble}} \sim \mathcal{O}(\hbar^{-2})$



$i\mathcal{M}_{\text{triangle}} \sim \mathcal{O}(\hbar^{-3})$



$i\mathcal{M}_{\text{box, crossed-box}} \sim \mathcal{O}(\hbar^{-4})$

Scalar scattering with loops

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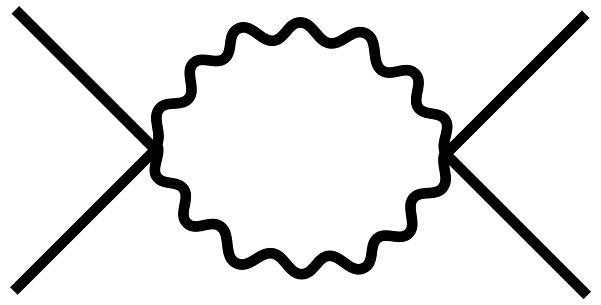
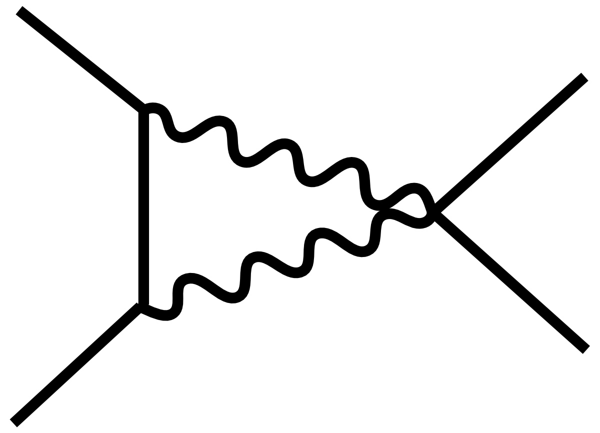


$$i\mathcal{M}_{\text{bubble}} \sim \mathcal{O}(\hbar^{-2})$$

quantum

Do not contribute at classical level



$$i\mathcal{M}_{\text{triangle}} \sim \mathcal{O}(\hbar^{-3})$$

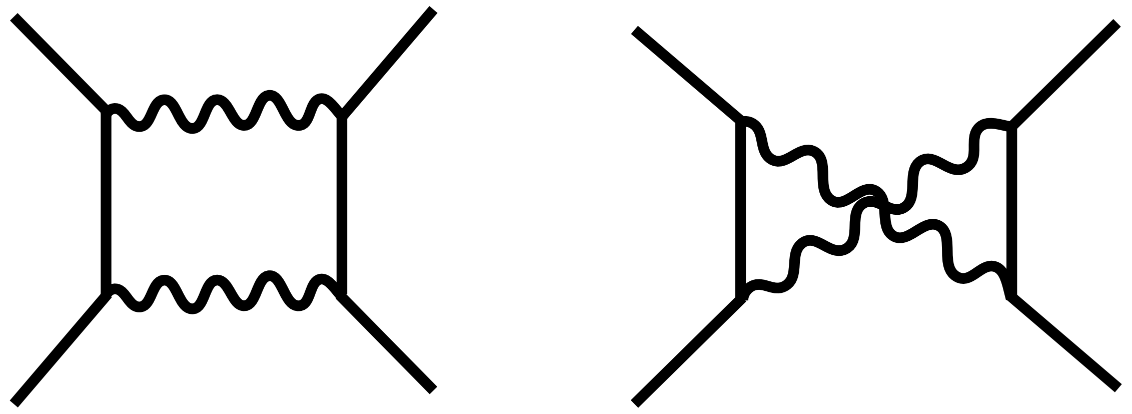
classical

$$i\mathcal{M}_{\text{triangle}} \sim \frac{G^2}{\hbar^3}\frac{\pi^2}{|q|}m_1 m_2(5\omega^2 - 1)$$

$$\downarrow$$

$$V(r) \sim G^2(5\omega^2 - 1)\frac{1}{r^2}$$

Corrections to the Newtonian Potential



$$i\mathcal{M}_{\text{box, crossed-box}} \sim \mathcal{O}(\hbar^{-4})$$

super-classical !

Box and crossed-box cancel + Born subtractions

[Kosower, Maybee, O'Connal, 19']

[Damgaard,Haddad,Helset, 19']

Classical observables

Linear and angular impulse, radiation, scattering angle

$\langle \Delta p_1^\mu \rangle$ ΔS_1^μ $R^\mu \equiv \langle k^\mu \rangle$

(Scattering observables)

$V(p,q)$ **Corrections to the Potential**

Some problems:

- **Higher orders in perturbation theory**
- **Spin**
- **Finite-size/tidal effects**
- **Radiation**

State of the art from amplitudes = 3PM 0-spin

[BCRSSZ 19']

Classical observables

Linear and angular impulse, radiation, scattering angle

$\langle \Delta p_1^\mu \rangle \qquad \Delta S_1^\mu \qquad R^\mu \equiv \langle k^\mu \rangle$

(Scattering observables)

$V(p,q)$ **Corrections to the Potential**

Some problems:

- **Higher orders in perturbation theory**
- **Spin** ←
- **Finite-size/tidal effects** ←
- **Radiation**

State of the art from amplitudes = 3PM 0-spin

[BCRSSZ 19']

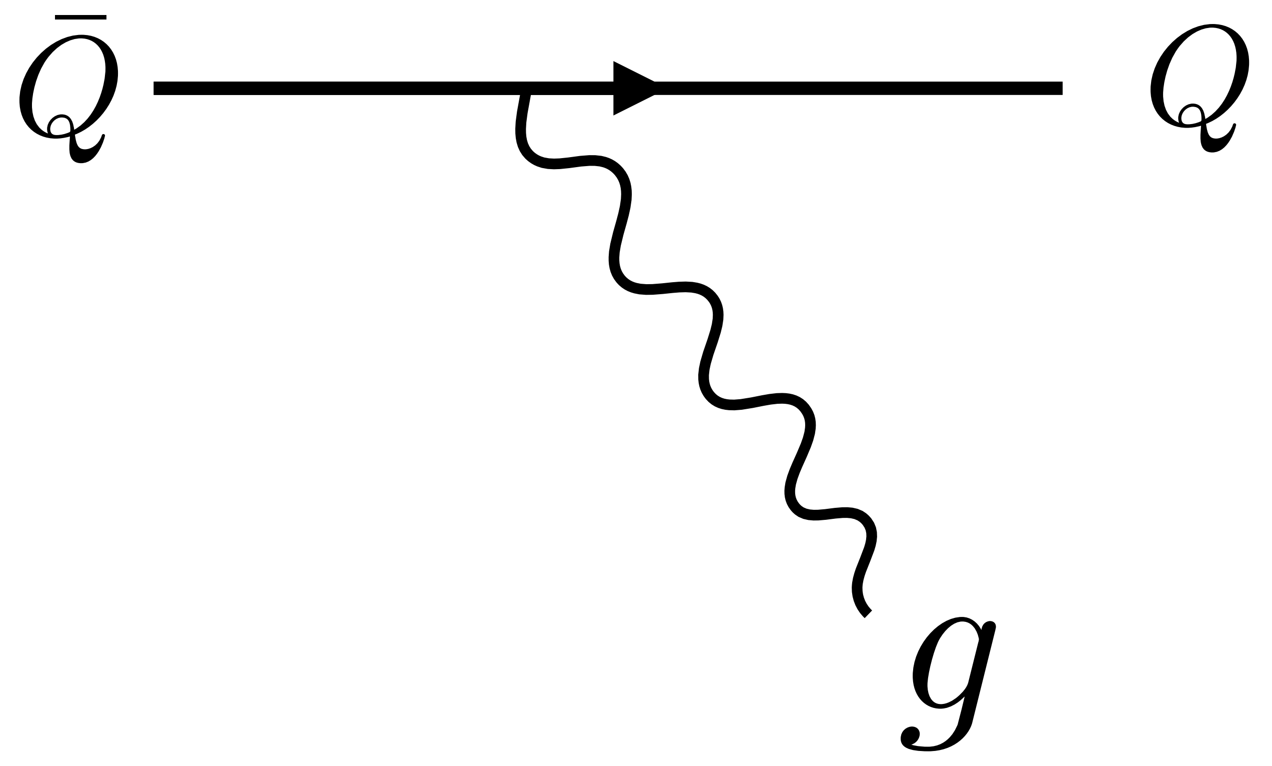
How can we describe such effects?

Can we use EFT methods to separate classical/quantum loop contributions?

Introduce On-shell Heavy Particle Effective Theory (HPET)

[Damgaard, Haddad, Helset, 19']
[Aoude, Haddad, Helset, Jan 20']

Take a Heavy meson decay at quark-level …



$$p^\mu = m v^\mu + k^\mu$$

$$|k^\mu| \sim \mathcal{O}(\Lambda_{\mathrm{QCD}}) \text{ where } \Lambda_{\mathrm{QCD}} \ll m_Q.$$

Description of B-decays
Spin-flavour symmetry

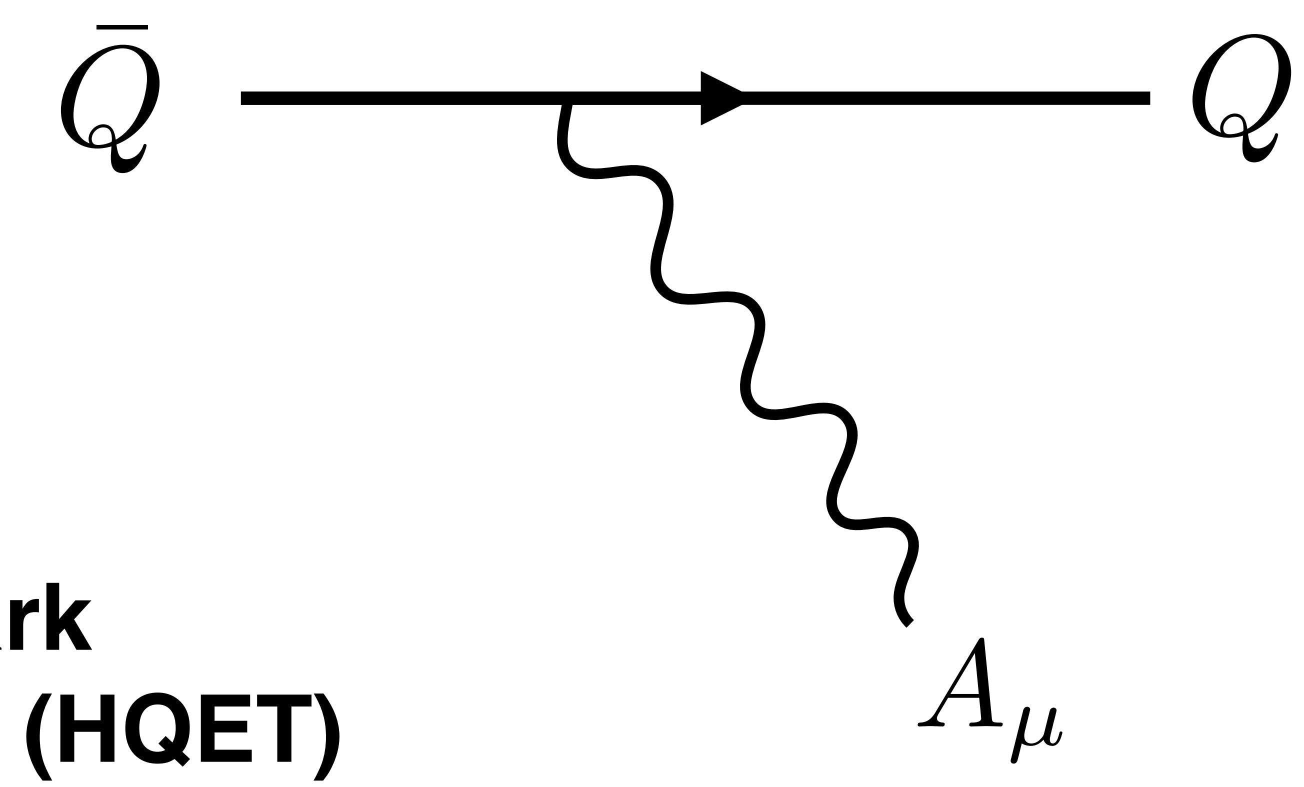
$$\psi(x) \to e^{-i m_Q v \cdot x} \frac{1+\not{v}}{2} Q_v(x)$$

Interaction

$$\bar{\psi}(i\not{D} - m_Q)\psi \to i\bar{Q}_v v \cdot D \frac{1+\not{v}}{2} Q_v$$

[Damgaard, Haddad, Helset, 19']

Take a Heavy meson decay at quark-level …



$$p^\mu = m_Q v^\mu + \hbar \bar{k}^\mu.$$

Heavy Quark Effective Theory (HQET)

Makes explicit the hbar dependence

$$i\mathcal{M}^{(2)}_{\text{bubble}} \sim \frac{G^2}{\hbar^2}\hbar^4 \int d^4\bar{l} \frac{1}{\hbar^2\bar{l}^2}\frac{1}{\hbar^2(\bar{l}+\bar{q})^2} + \mathcal{O}(\hbar^{-1}) = \mathcal{O}(\hbar^{-2}).$$

$$i\mathcal{M}^{(2)}_{\text{triangle}} \sim \frac{G^2}{\hbar^2}\hbar^4 \int d^4\bar{l} \frac{1}{\hbar^2\bar{l}^2}\frac{1}{\hbar^2(\bar{l}+\bar{q})^2}\frac{1}{\hbar v\cdot(\bar{l}+\bar{k})} + \mathcal{O}(\hbar^{-2}) = \mathcal{O}(\hbar^{-3}).$$

Propagator $$D_v^{s=\frac{1}{2}}(k) = \frac{i}{\hbar v\cdot k}\frac{1+\not{v}}{2}.$$

$$iM^{(2)}_{\text{(crossed}-\text{)box}} \sim \frac{G^2}{\hbar^2}\hbar^4 \int d^4\bar{l} \frac{1}{\hbar^2\bar{l}^2}\frac{1}{\hbar^2(\bar{l}+\bar{q})^2}\frac{1}{\hbar v\cdot(\bar{l}+\bar{k}_1)}\frac{1}{\hbar v\cdot(\bar{l}+\bar{k}_2)} + \mathcal{O}(\hbar^{-3}) = \mathcal{O}(\hbar^{-4}).$$

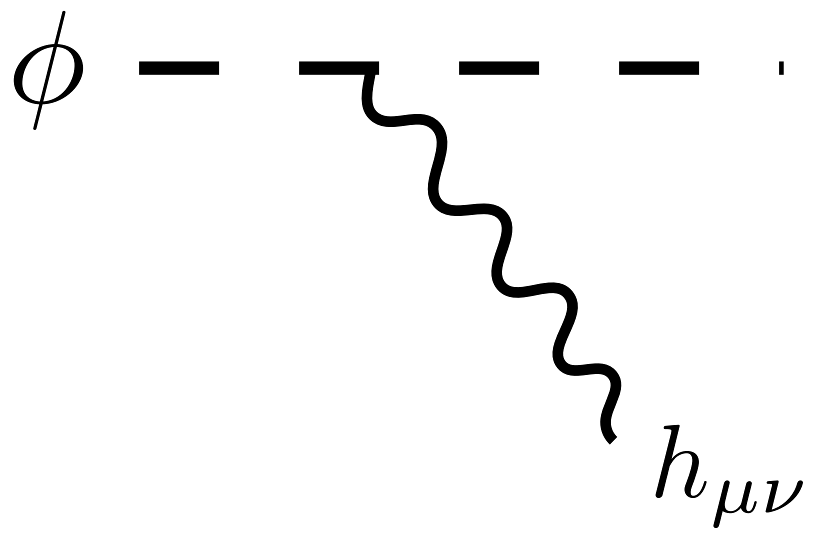
Explicit $\hbar$ power-counting

[Damgaard, Haddad, Helset, 19’]

Heavy Black-Hole Effective Theory (HBET)

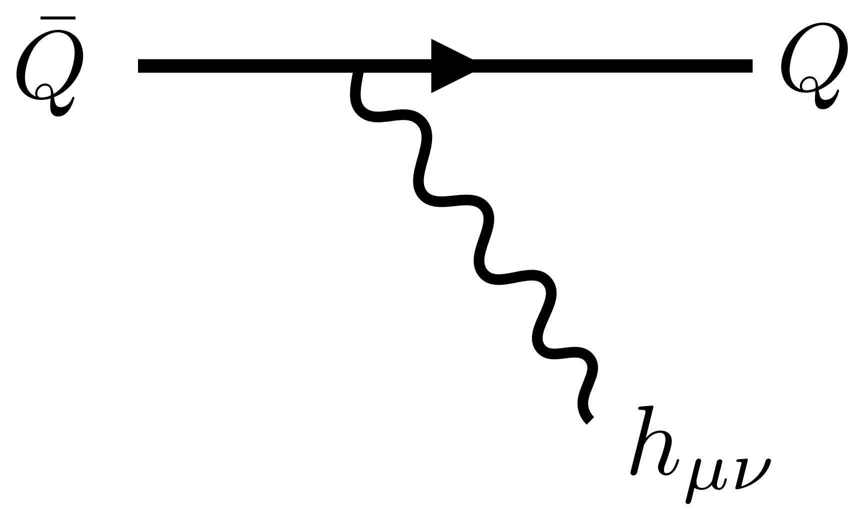
$$p^\mu = m_Q v^\mu + \hbar \bar{k}^\mu .$$

scalar-graviton



$$D_v^{s=0}(k) = \frac{i}{\hbar v \cdot k}$$

fermion-graviton



$$D_v^{s=\frac{1}{2}}(k) = \frac{i}{\hbar v \cdot k} \frac{1 + \not{v}}{2} .$$

Lagrangian description for scalars and fermions coupled to gravity

Hard for higher-spins

In order to describe a classical spinning particle…

Upgrading the interaction for any spin … need better tools!

Modern Methods for scattering amplitudes

(gauge invariant building blocks, bypass Lagrangians)

Spinor-helicity formalism

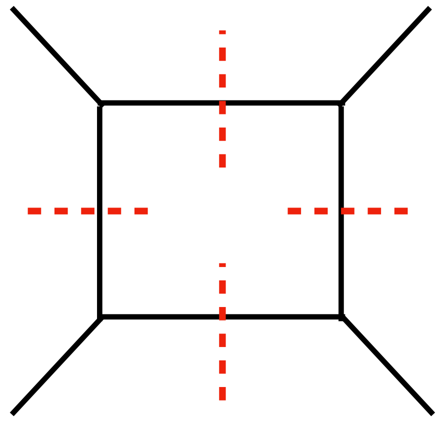
(instead of momenta/pol. vectors)

$$\langle ij \rangle = \lambda_{i\alpha}\lambda_{j\beta}\epsilon^{\alpha\beta}$$
$$[ij] = \tilde{\lambda}_{i\dot{\alpha}}\tilde{\lambda}_{j\dot{\beta}}\epsilon^{\dot{\alpha}\dot{\beta}}$$

- Massless
- Massive
- Heavy

Unitarity methods

(loops from trees)

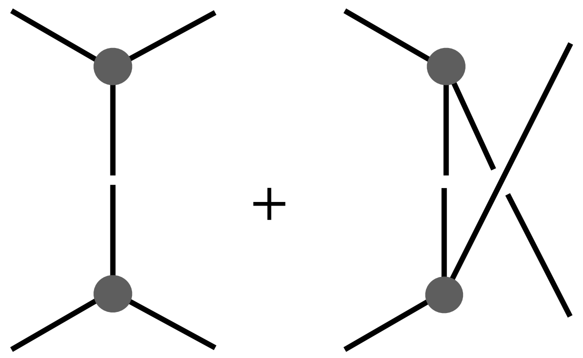


$$\sim A_1 A_2 A_3 A_4$$

Recursion relations

(higher-points from lower-points)

$$\mathcal{M} = -\sum_K \frac{\hat{\mathcal{M}}_L(z_K)\hat{\mathcal{M}}_R(z_K)}{p_K^2 - m^2} + B_\infty$$



Double-copy

(uses YM to calculate GR)

$$\mathrm{GR} \sim (\mathrm{YM})^2$$

and more…

Sharping our calculation tools: *massless* on-shell methods

$$p_{\alpha\dot{\alpha}} = p_\mu \sigma^\mu_{\alpha\dot{\alpha}}$$

$$\det(p) = 0 \to p_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$$

Rank 1 matrix !

Spinor-helicity building blocks

$$\langle ij \rangle = \lambda_{i\alpha} \lambda_{j\beta} \epsilon^{\alpha\beta}$$

$$[ij] = \tilde{\lambda}_{i\dot{\alpha}} \tilde{\lambda}_{j\dot{\beta}} \epsilon^{\dot{\alpha}\dot{\beta}}$$

A photon has 2 polarizations, why using polarization tensor with 4 entries?

polarization vectors are represented by the same obj.

$$\epsilon^\mu_+ = \frac{\langle \zeta | \sigma^\mu | \lambda]}{\sqrt{2} \langle \lambda \zeta \rangle}, \qquad \epsilon^\mu_- = \frac{\langle \lambda | \sigma_\mu | \zeta]}{\sqrt{2} [\lambda \zeta]},$$

Special kinematics and little-group uniquely …

$$\mathcal{M}_3(1^{h_1} 2^{h_2} 3^{h_3}) = g \begin{cases} \langle 12 \rangle^{-h_1-h_2+h_3} \langle 23 \rangle^{h_1-h_2-h_3} \langle 31 \rangle^{-h_1+h_2-h_3}, & h < 0 \ \text{(H)}, \\ [12]^{h_1+h_2-h_3} \ [23]^{-h_1+h_2+h_3} \ [31]^{h_1-h_2+h_3}, & h > 0 \ \text{(AH)}, \end{cases}$$

Sharping our calculation tools: massive on-shell methods

[Arkani-Hamed, Huang, Huang. 17’]

[Ochirov 18’]

$$\det(p_{\alpha\dot{\beta}}) = m^2 \to p_{\alpha\beta} = \lambda^a_\alpha \tilde{\lambda}_{\beta a} = \lambda^a_\alpha \epsilon_{ab} \tilde{\lambda}^b_\beta$$

Rank 2 matrix   Similar variables, extra index

$$\lambda^a_\alpha \leftrightarrow |p^a\rangle_\alpha$$

$$\tilde{\lambda}_{\dot{\beta}a} \leftrightarrow [p_a|_{\dot{\beta}}$$

Dirac spinors and polarization tensors

$$u_p^{Aa} = \begin{pmatrix} \lambda_{p\alpha}^{\ \ a} \\ \tilde{\lambda}_p^{\dot{\alpha}a} \end{pmatrix}, \qquad \bar{u}^a_{pA} = \begin{pmatrix} -\lambda_p^{\alpha a} \\ \tilde{\lambda}_{p\dot{\alpha}}^{\ \ a} \end{pmatrix} \qquad \varepsilon^{ab}_\mu(p) = \frac{i\langle p^{(a}|\sigma_\mu|p^{b)}]}{\sqrt{2}m}$$

Recover the massless one in the high-energy limit

$$\lambda_{p\alpha}^{\ \ a} \xrightarrow[m\to 0]{} \lambda_{p\alpha}\zeta^a_-, \qquad \tilde{\lambda}_p^{\dot{\alpha}a} \xrightarrow[m\to 0]{} \tilde{\lambda}_p^{\dot{\alpha}}\zeta^a_+,$$

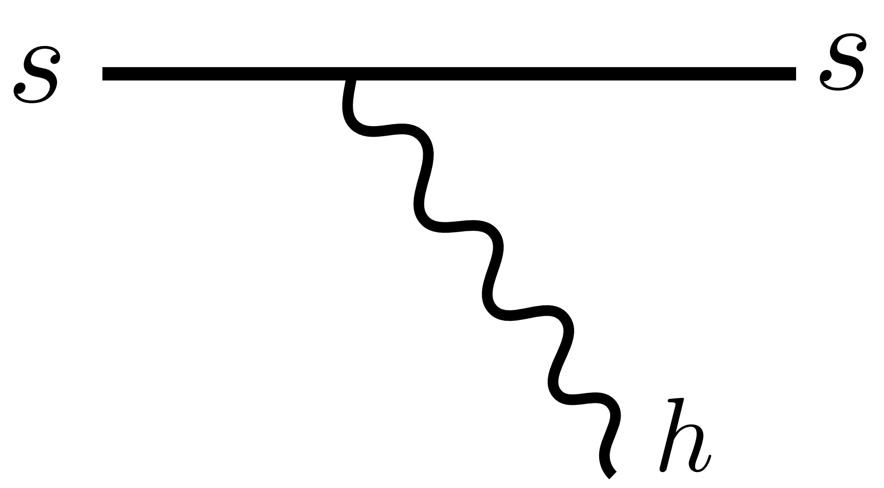
e.g: 1 scalar + 2 vectors, all massive (bold notation, a,b symmetrized)

$$\mathcal{M}(\mathbf{1}_h\, \mathbf{2}_V^{a_1, a_2}\, \mathbf{3}_{\bar{V}}^{b_1 b_2}) = g_0\langle\mathbf{23}\rangle[\mathbf{23}] + g_1\langle\mathbf{23}\rangle^2 + g_2[\mathbf{23}]^2$$

Sharping our calculation tools: massive on-shell methods

[Arkani-Hamed, Huang, Huang. 17']

General 3-point amplitudes (any spin-s, helicity-h):



$$\mathcal{M}^{+|h|,s} = (-1)^{2s+h} \frac{x^{|h|}}{m^{2s}} \left[g_0 \langle \mathbf{21} \rangle^{2s} + g_1 \langle \mathbf{21} \rangle^{2s-1} \frac{x \langle \mathbf{2} q \rangle \langle q \mathbf{1} \rangle}{m} + \cdots + g_{2s} \frac{(x \langle \mathbf{2} q \rangle \langle q \mathbf{1} \rangle)^{2s}}{m^{2s}} \right]$$

(similar for negative helicity)

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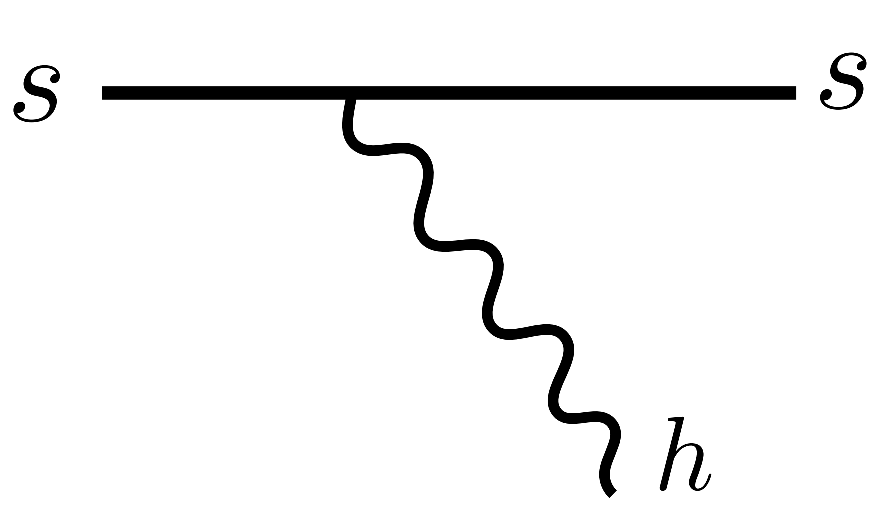

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(similar for negative helicity)

Minimal coupling:

$$\mathcal{M}^{+|h|,s}_{\text{min}} = (-1)^{2s+h} \frac{g_0 x^{+|h|}}{m^{2s}} \langle \mathbf{21} \rangle^{2s},$$

(similar for negative helicity)

SMEFT applications

[Aoude, Machado 19']

[Duriex, Kitahara Shadmi, Weiss 19']

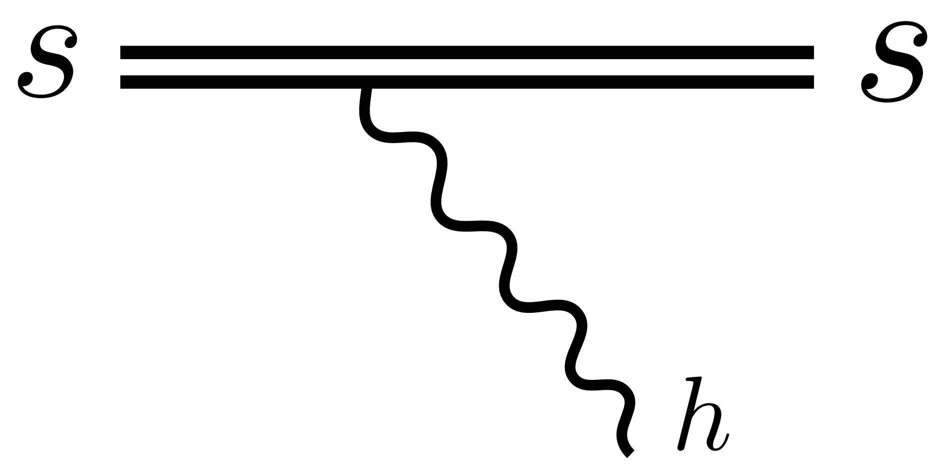
$\tilde{g}_{k\geq 0}$ **Non-minimal coupling**

$g_{k\geq 0}$ **Mapping to the SMEFT Wilson coefficients**

Lessons from the high-energy (massless) limit of the amplitudes

[Aoude, Haddad, Helset, Jan 20']

Sharping our calculation tools: heavy spinors



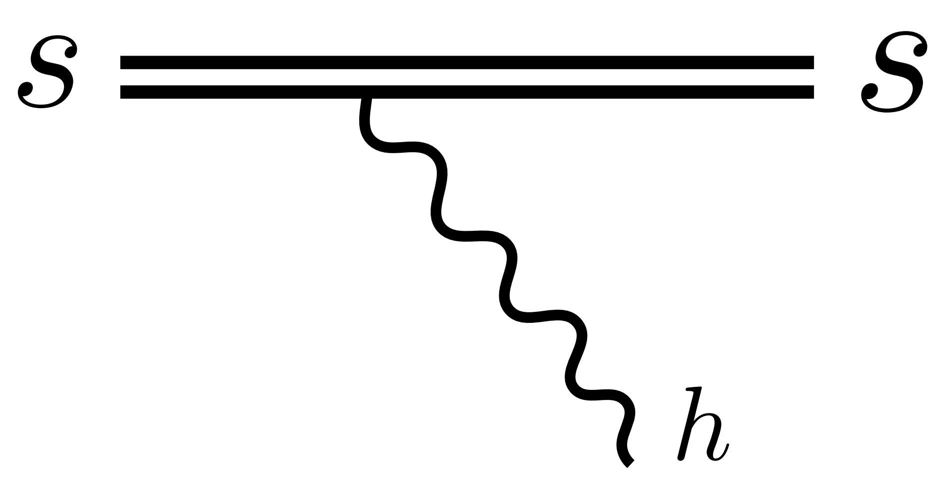
$$p^\mu = m v^\mu + k^\mu,$$

$$u_v^I(p) = \left(\frac{\mathbb{I} + \not{v}}{2}\right) u^I(p) = \left(\mathbb{I} - \frac{\not{k}}{2m}\right) u^I(p),$$

$$\begin{pmatrix} |\mathbf{p}_v\rangle \\ |\mathbf{p}_v] \end{pmatrix} = \left(\mathbb{I} - \frac{\not{k}}{2m}\right) \begin{pmatrix} |\mathbf{p}\rangle \\ |\mathbf{p}] \end{pmatrix}.$$

[Aoude, Haddad, Helset, Jan 20']

Sharping our calculation tools: **heavy** spinors



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Same amp with heavy spinors

$$\mathcal{M}_3^{+|h|,s} = (-1)^{2s+h} \frac{x^{|h|}}{m^{2s}} \sum_{k=0}^{2s} g_{s,k}^{\mathrm{H}} \langle \mathbf{2}_v \mathbf{1}_v \rangle^{2s-k} \left(\frac{x}{2m} \langle \mathbf{2}_v q \rangle \langle q \mathbf{1}_v \rangle \right)^k$$

Relation between coefficients

$$g_{s,k}^{\mathrm{H}} = \sum_{i=0}^{k} g_i \binom{2s-i}{2s-k}$$

Minimal coupling:

$$\mathcal{M}^{+|h|,s} = (-1)^{2s+h} \frac{g_0 x^{|h|}}{m^{2s}} \langle \mathbf{2}_v |^{2s} \sum_{k=0}^{2s} \frac{\left(\frac{q \cdot S}{m}\right)^k}{k!} | \mathbf{1}_v \rangle^{2s}.$$

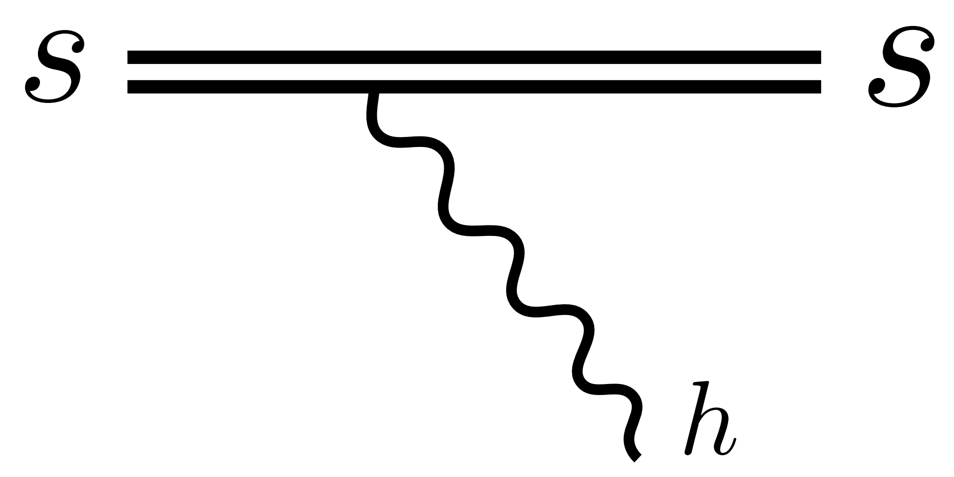
Infinity spin-limit (classical)

$$\lim_{s \to \infty} \mathcal{M}^{+|h|,s} = (-1)^h g_0 x^{|h|} e^{q \cdot S / m}.$$

Same exponentials as [Vines 18']

Why is that important? Any-spin generalization; and facilitates higher-PM orders

“Kerr Black Holes”



One particle effective action

[Goldberger Rothstein, 06’]
Porto 06’, Levi Steinhoff 15’]

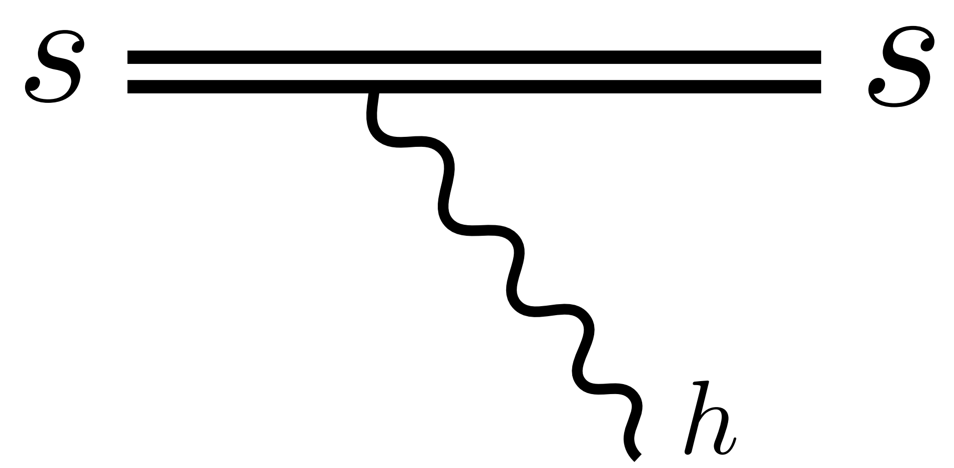
$$S = \int d\sigma \left\{ -m\sqrt{u^2} - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{\mathrm{SI}}[u^\mu, S_{\mu\nu}, g_{\mu\nu}(x^\mu)] \right\}$$

Non-minimal Spin-multipole expansion

$$L_{\mathrm{SI}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{S^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \dots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \dots S^{\mu_{2n}}$$

$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{S^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \dots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \dots S^{\mu_{2n+1}}.$$

“Kerr Black Holes”



One particle effective action

[Goldberger Rothstein, 06’]
Porto 06’, Levi Steinhoff 15’]

$$S = \int d\sigma \left\{ -m\sqrt{u^2} - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{\rm SI}[u^\mu, S_{\mu\nu}, g_{\mu\nu}(x^\mu)] \right\}$$

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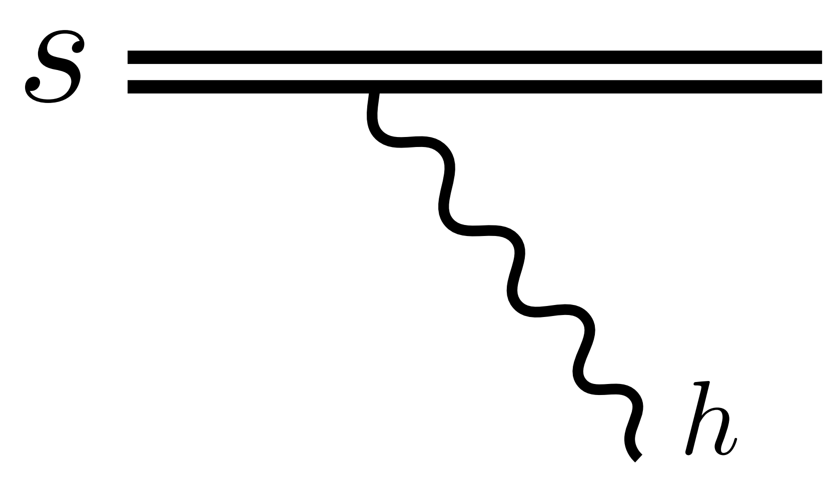
Wilson Coefficients

$$C_{S^k}^{\rm Kerr} = 1 \text{ for all } k$$

“Kerr Black Holes”

[Aoude, Haddad, Helset, Jan 20’]



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Wilson Coefficients

$$C^{\rm Kerr}_{S^k} = 1 \text{ for all } k$$

Direct matching to the HPET amplitudes

$$\mathcal{M}_3^{+|h|,s} = (-1)^{2s+h} \frac{x^{|h|}}{m^{2s}} \sum_{k=0}^{2s} g^{\rm H}_{s,k} \langle \mathbf{2}_v \mathbf{1}_v \rangle^{2s-k} \left(\frac{x}{2m} \langle \mathbf{2}_v q \rangle \langle q \mathbf{1}_v \rangle \right)^k$$

Classical limit →

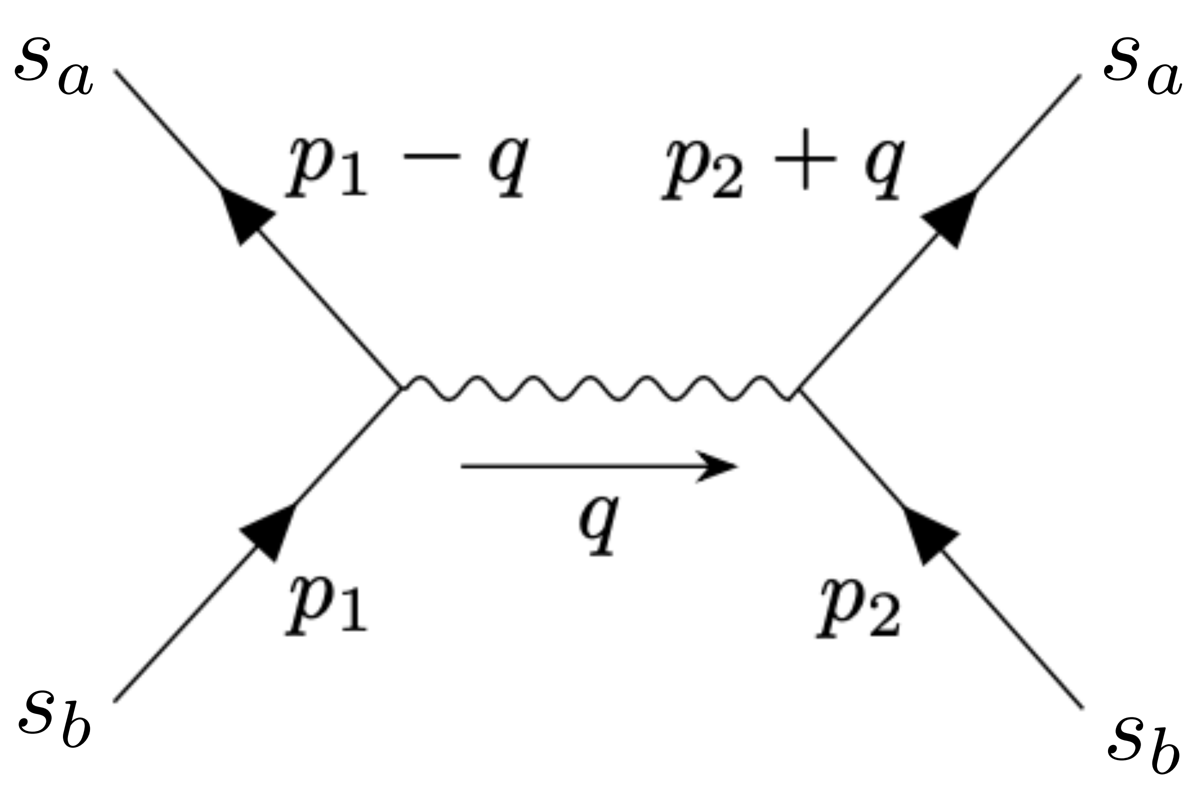
$$\lim_{s\to\infty} \mathcal{M}^{+|h|,s} = (-1)^h g_0 x^{|h|} e^{q\cdot S/m}$$

Heavy spin-s particle (minimally coupled to gravity) has same spin-multipole expansion

HPET facilitates the matching
No need for any boost

Boson exchange

[Aoude, Haddad, Helset, Jan 20']



Relevant amplitude for classical potential.

for spin-1/2

$$i\mathcal{A}_{\text{tree}}\left(-\mathbf{1}_a^{\frac{1}{2}}, \mathbf{2}_a^{\frac{1}{2}}, -\mathbf{3}_b^{\frac{1}{2}}, \mathbf{4}_b^{\frac{1}{2}}\right) = \sum_h \mathcal{A}_{\text{tree}}\left(-\mathbf{1}^{\frac{1}{2}}, \mathbf{2}^{\frac{1}{2}}, -q^h\right) \frac{i}{q^2} \mathcal{A}_{\text{tree}}\left(q^{-h}, -\mathbf{3}^{\frac{1}{2}}, \mathbf{4}^{\frac{1}{2}}\right)$$

… agrees with known results

Infinity spin case

$$\lim_{s_a, s_b \to \infty} \mathcal{A}_{\text{tree}}^{s_a, s_b} = -\frac{2e^2}{q^2} \sum_{\pm} (\omega \pm \sqrt{\omega^2 - 1}) \exp\left[\pm q \cdot \left(\frac{S_a}{m_a} + \frac{S_b}{m_b}\right)\right],$$

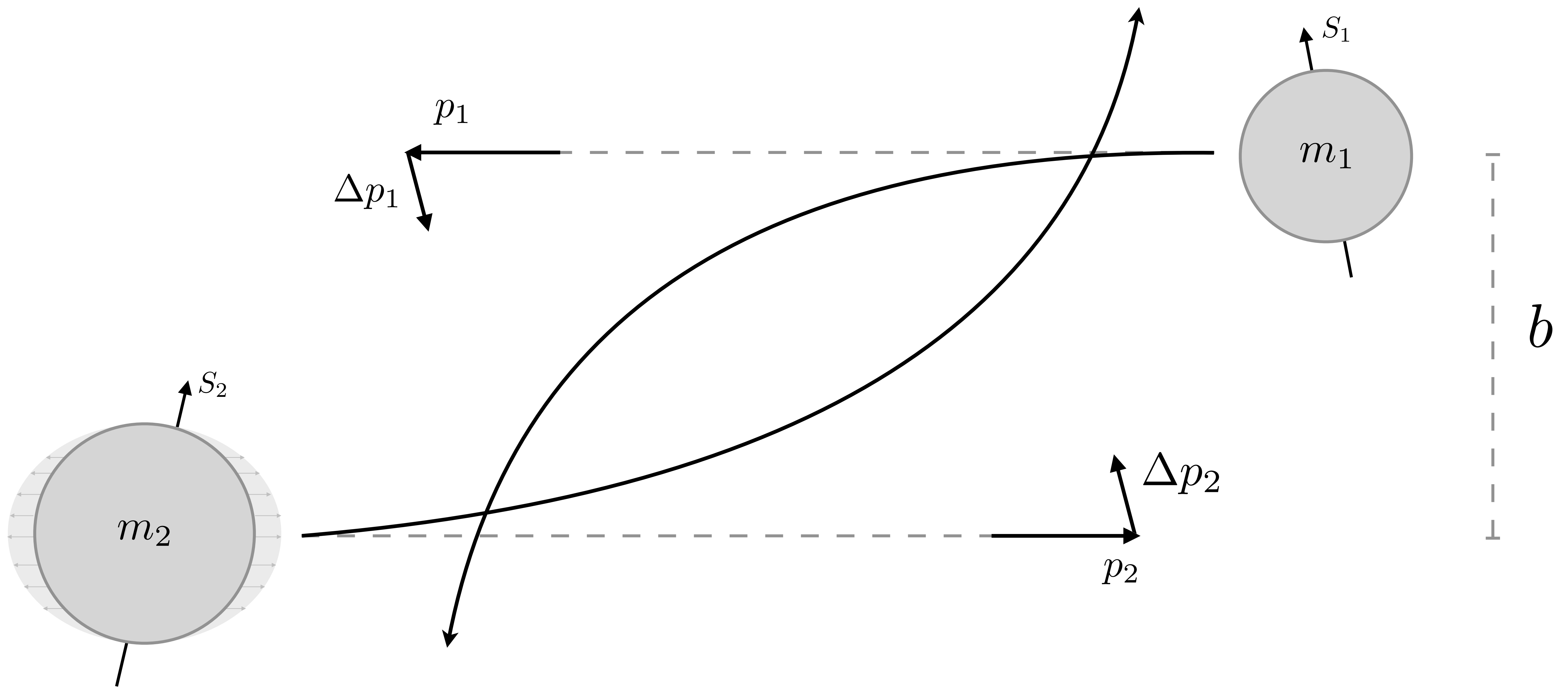
$$\lim_{s_a, s_b \to \infty} \mathcal{M}_{\text{tree}}^{s_a, s_b} = -\frac{\kappa^2 m_a m_b}{4q^2} \sum_{\pm} (\omega \pm \sqrt{\omega^2 - 1})^2 \exp\left[\pm q \cdot \left(\frac{S_a}{m_a} + \frac{S_b}{m_b}\right)\right].$$

Other processes

- Compton scattering
- n-boson emission

Tidal effects for spinning particles

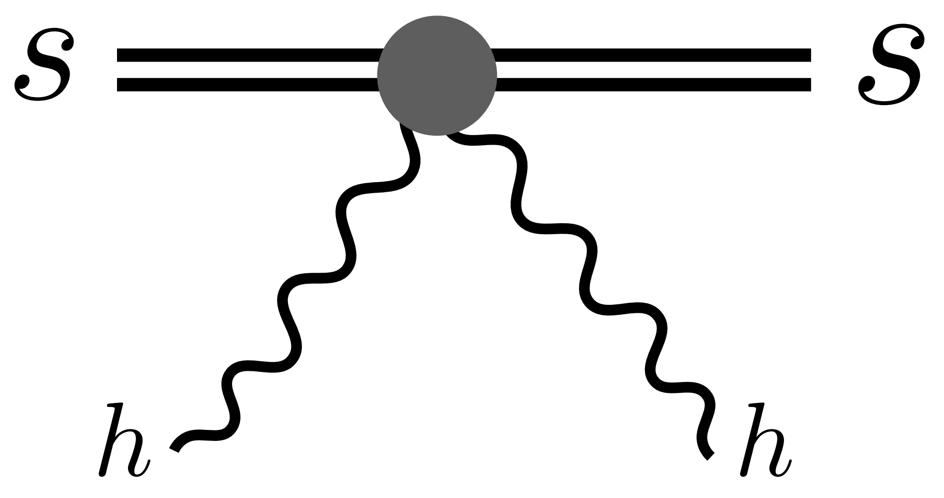
[Aoude, Haddad, Helset, Dez 20’]



The higher-dimension are tidal/finite-size effects.

The WCs are tidal love numbers

All contact interaction



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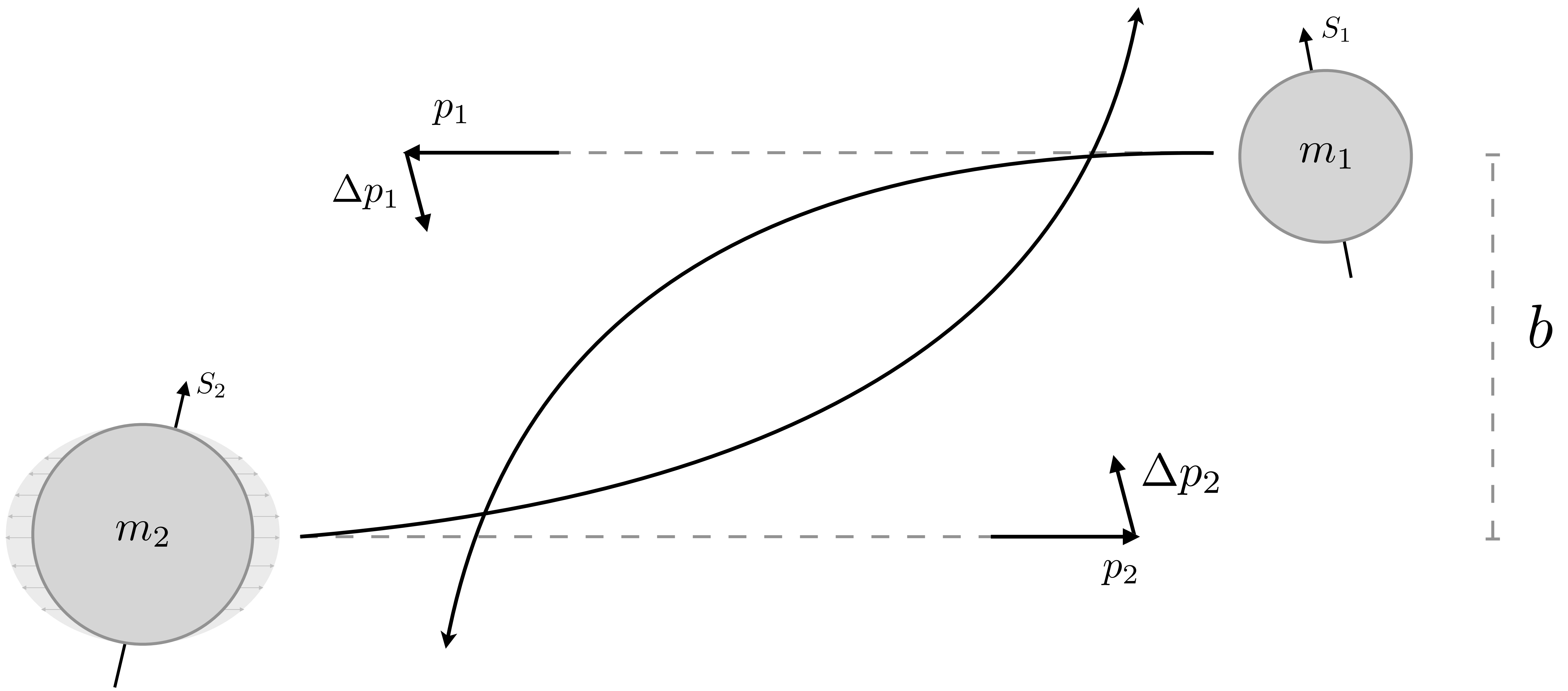
[Aoude, Haddad, Helset, Dez 20']



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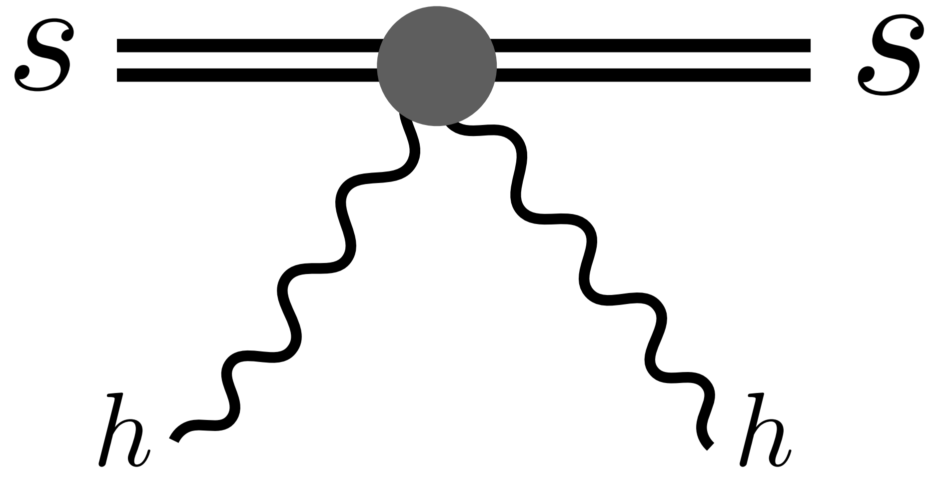
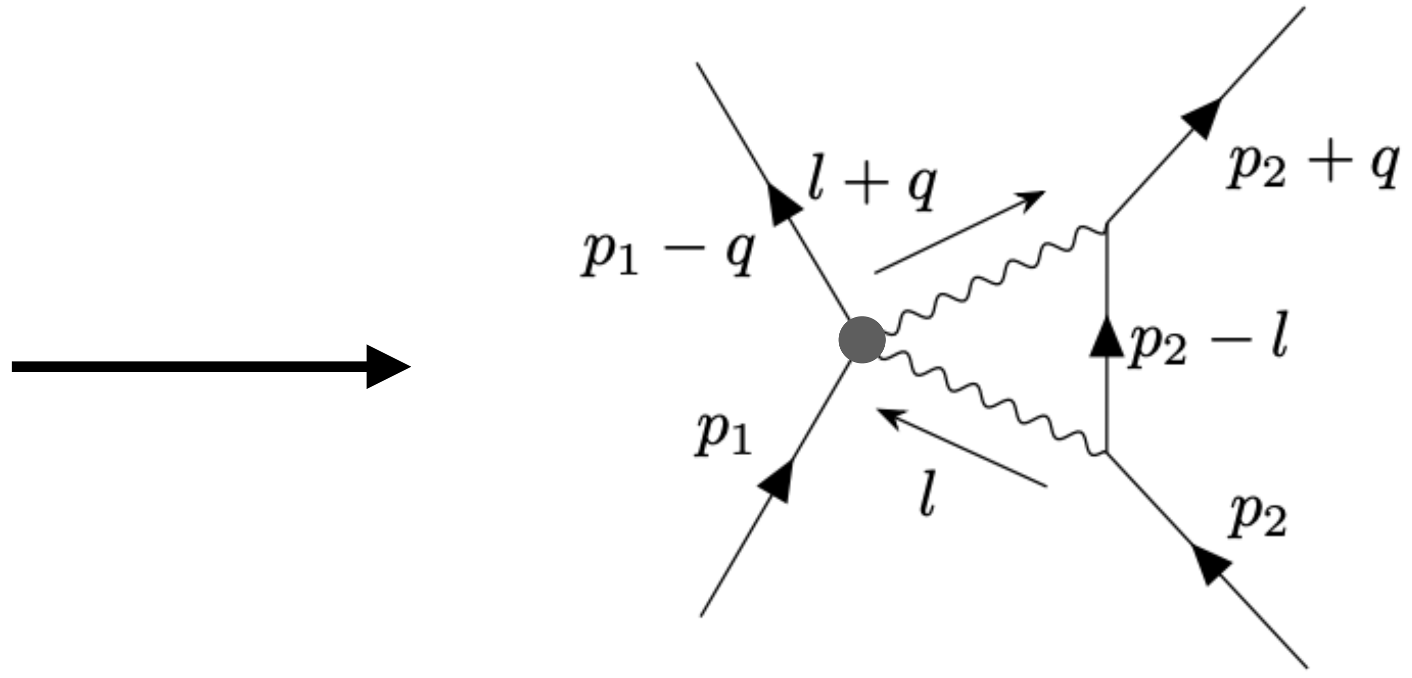


At 2PM



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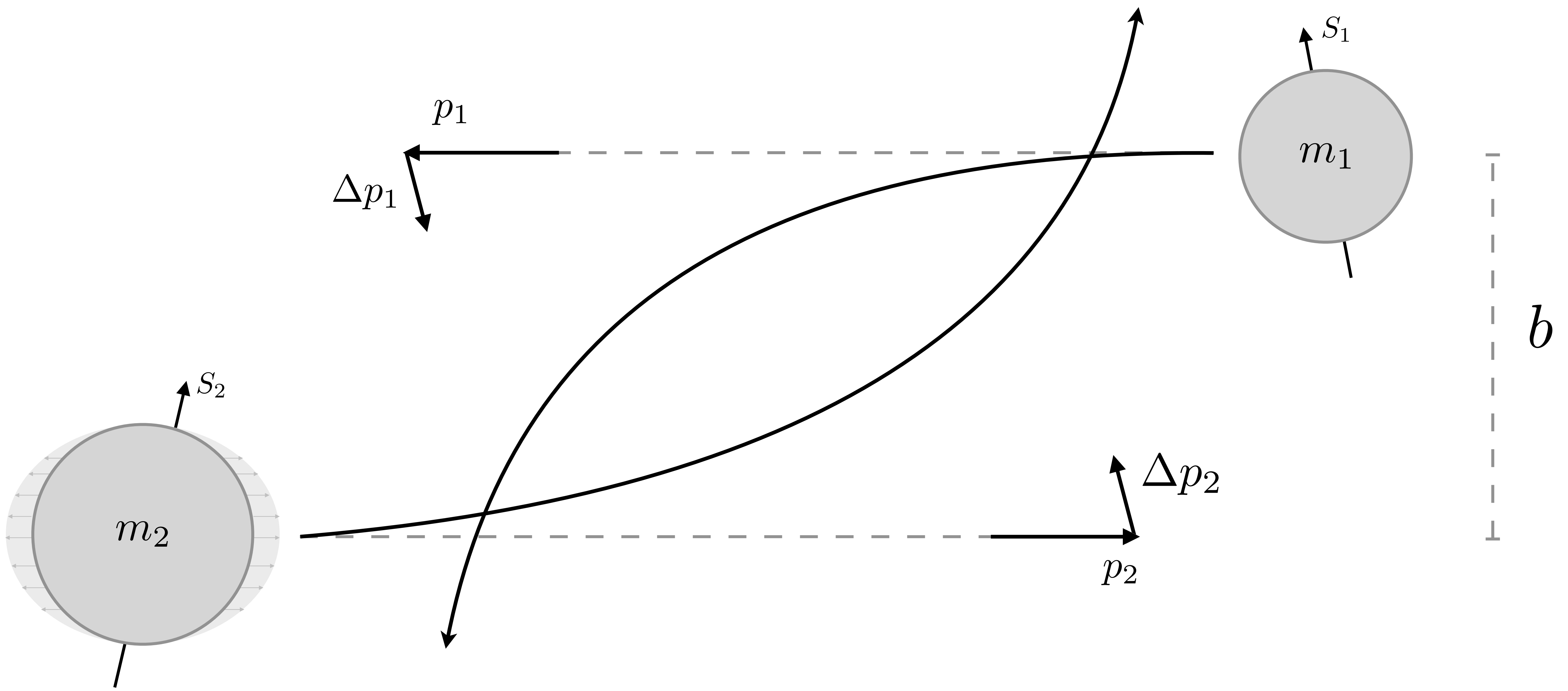

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Schwarzschild BHs do not tidally deform!

Still debate for Kerr BHs

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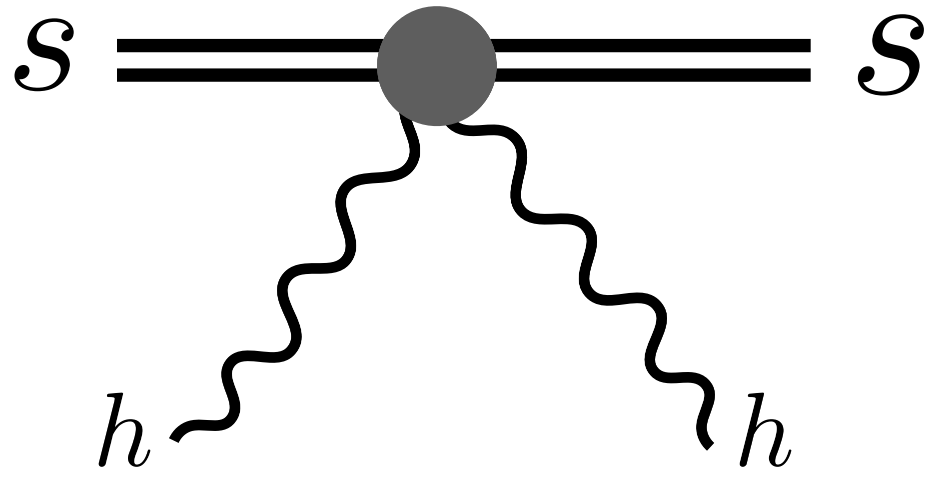

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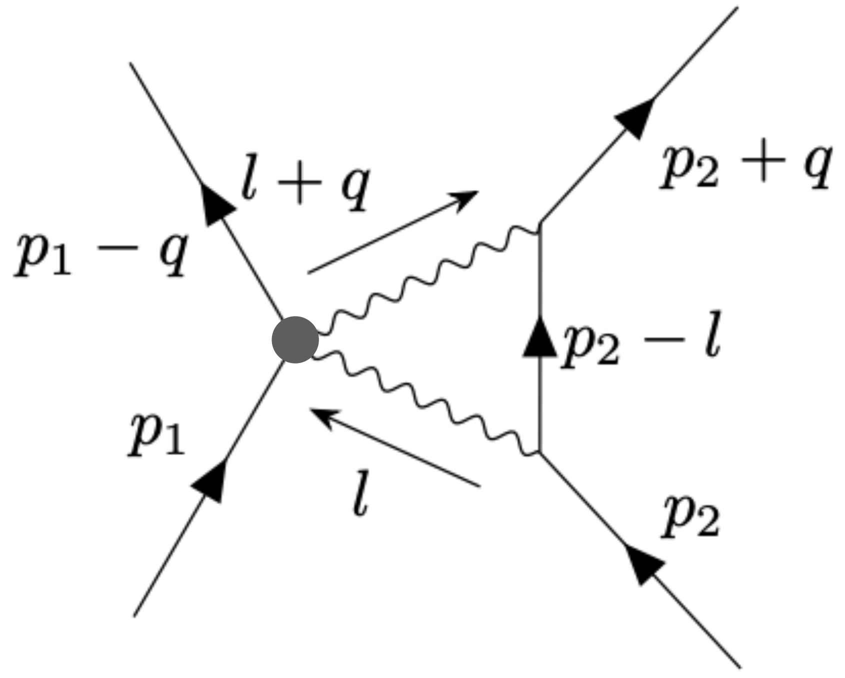

Classical Observables

$H(\boldsymbol{q},\boldsymbol{p})$ Δp_1^μ

ΔS_1^μ $\Delta\chi_2$

$\Delta\theta_2$ (aligned case)

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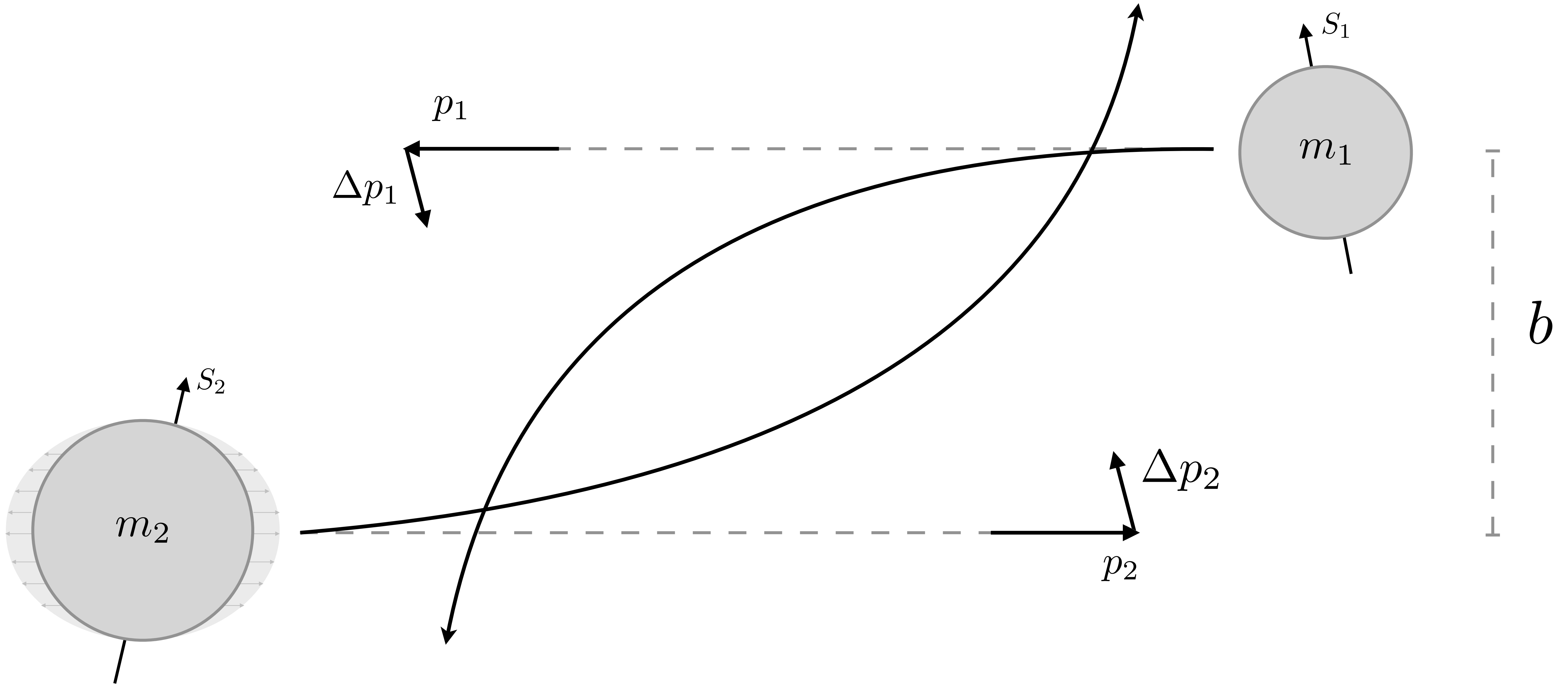
[Aoude, Haddad, Helset, Dez 20']



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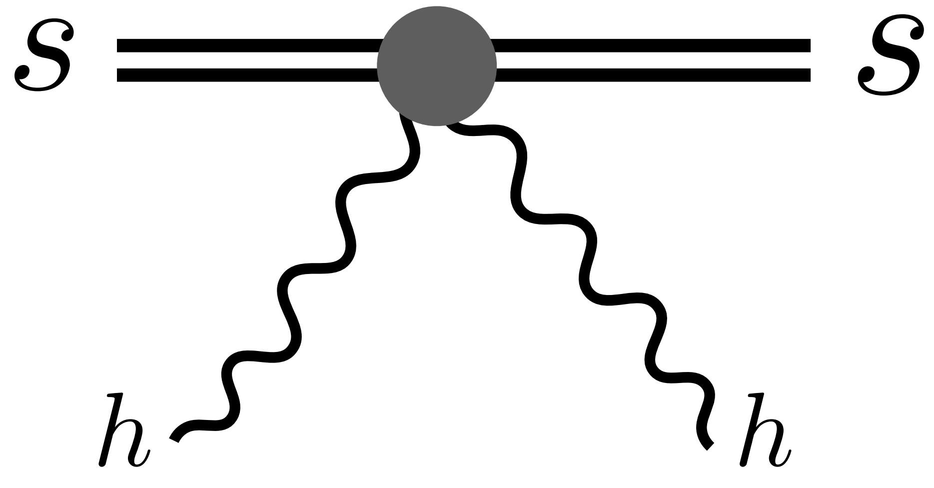
Still debate for Kerr BHs

Are the WCs = 0 ?

All contact interaction



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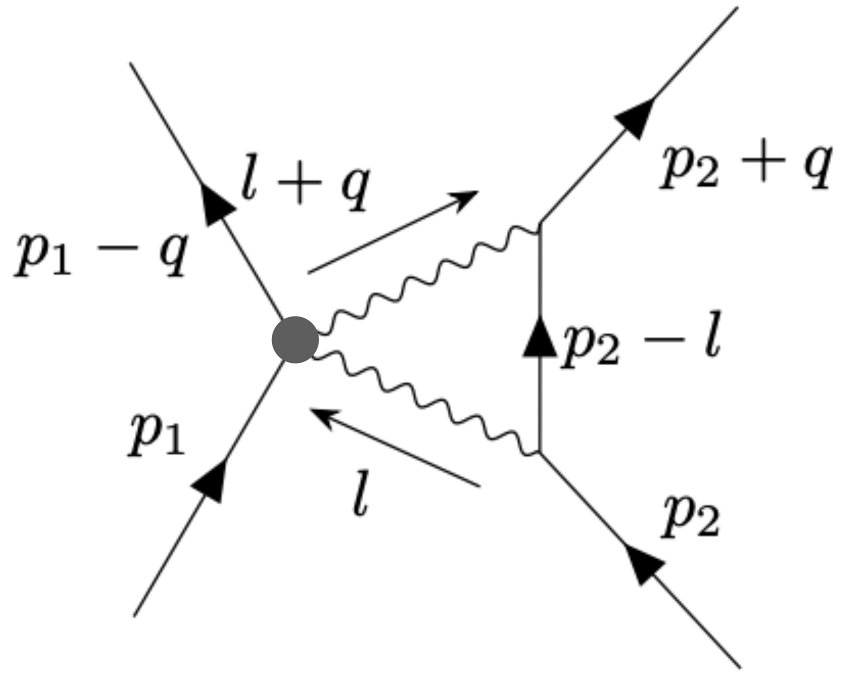

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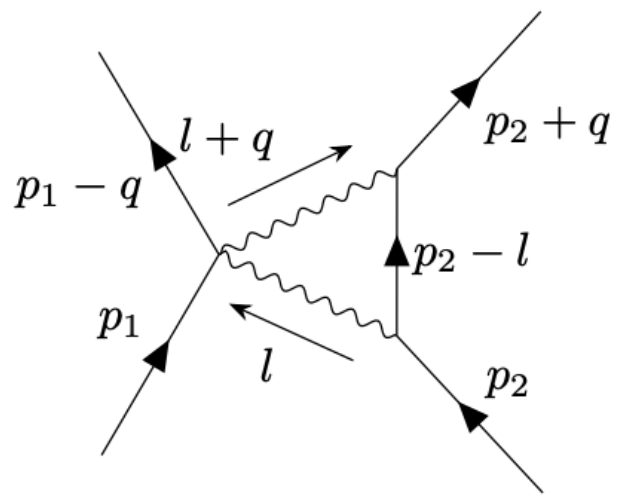
Conclusion

Similar questions from different problems



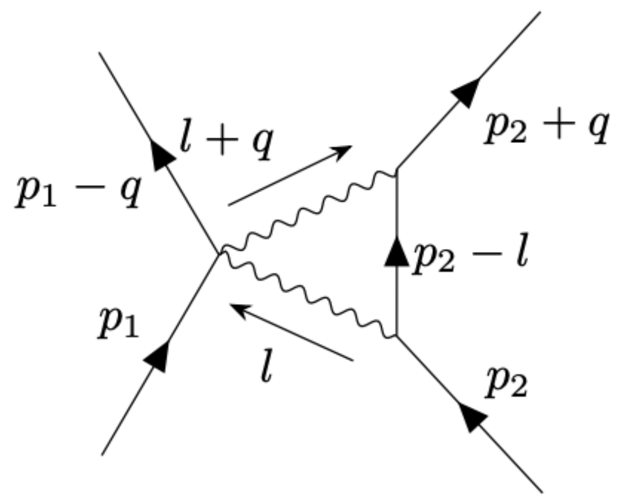
Amplitudes methods (and Feynman diagrammatic) are well-suited for LHC and GWs

Efficient loop evaluation allows precise theoretical predictions for the amplitudes

Conclusion

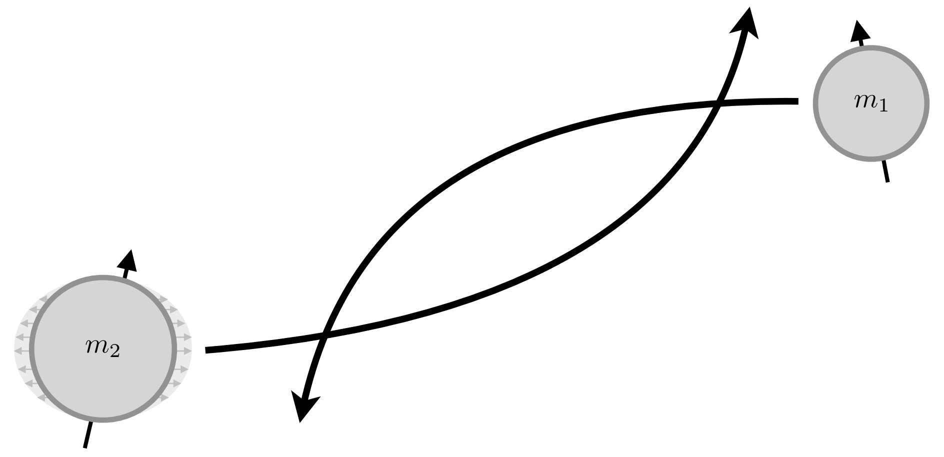
Similar questions from different problems

Amplitudes methods (and Feynman diagrammatic) are well-suited for LHC and GWs



Efficient loop evaluation allows precise theoretical predictions for the amplitudes

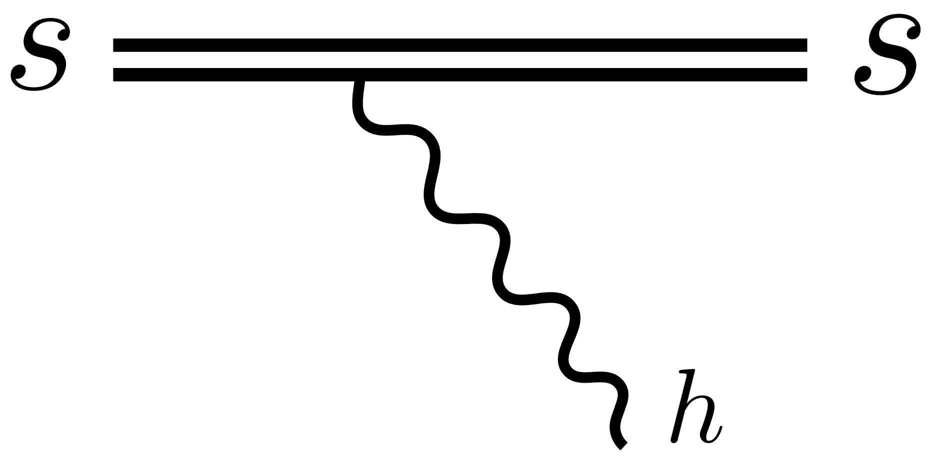
At LHC, SM and SMEFT high-accuracy cross-section required for next years.



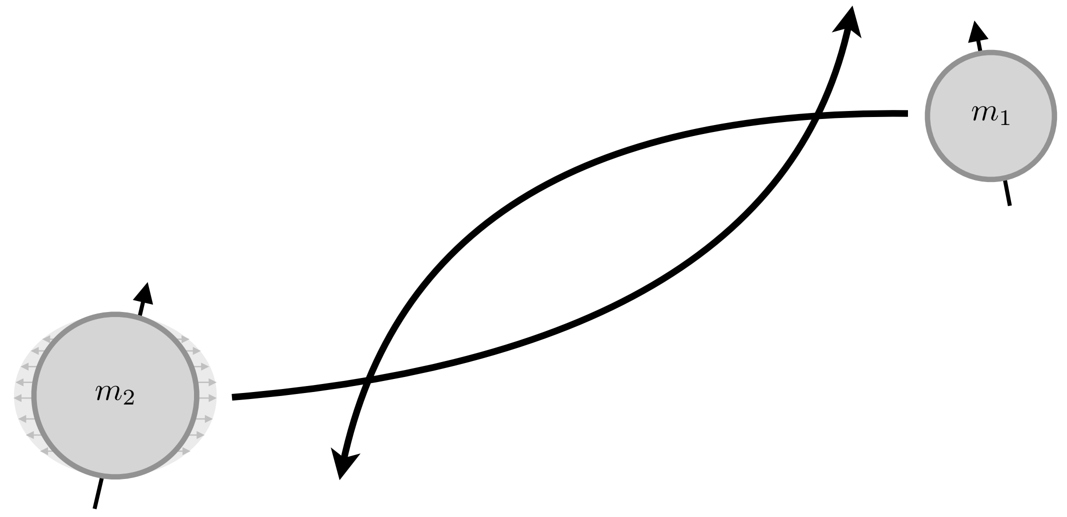
For GWs, precise description on the two-body hamiltonian.
Description of BHs and NS. Tidal effects.

New insights from looking at the same problem from a different perspective: Double copy, soft-theorems, …

A lot to learn! (and to calculate)



Thank you!



Obtaining the Potential from Scattering Amplitudes

[Cheung, Rothstein, Solon,19']

Full theory

amplitudes Methods

$$A_{\text{tree}}$$

generalized Unitarity

$$A = \sum_i d^{(i)} I^{(i)}$$

multi-loop Integration

$$A(p, q)$$

Effective theory

ansatz

$$V(p, q)$$

Feynman ciagrams

$$A_{\text{EFT}} = \sum_i d^{(i)}_{\text{EFT}} I^{(i)}$$

multi-loop Integration

$$A_{\text{EFT}}(p, q)$$

Matching

$$A(p, q) = A_{\text{EFT}}(p, q)$$

[slide based on Cheung

PM vs. PN

Viral theorem

$$v^2 \sim \frac{GM}{r} \ll 1$$

PN double expansion

1PN 2PN 3PN 4PN 5PN 5PN 6PN

1PM $\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + v^{14} + \ldots\right) G$

2PM $\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \ldots\right) G^2$

3PM $\left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \ldots\right) G^3$

4PM $\left(1 + v^2 + v^4 + v^6 + v^8 + \ldots\right) G^4$

5PM $\left(1 + v^2 + v^4 + v^6 + \ldots\right) G^5$

Kerr Black Holes as heavy particles

[Aoude, Haddad, Helset, Jan 20']

Effective action for spinning gravitating bodies

$$S = \int d\sigma \left\{ -m\sqrt{u^2} - \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu} + L_{\text{SI}}[u^\mu, S_{\mu\nu}, g_{\mu\nu}(x^\mu)] \right\}$$

worldline of the particle

spin monopole and spin-dipole

(universal for any spinning body)

$$C_{S^0} = C_{S^1} = 1$$

$u^\mu = \frac{dx^\mu}{d\sigma}$ coordinate velocity

$S_{\mu\nu}$ spin operator

$\Omega^{\mu\nu}$ angular velocity

higher spin multipoles

$$L_{\text{SI}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{S^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \dots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \dots S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{S^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \dots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \dots S^{\mu_{2n+1}}$$

The WCs contain info about the internal structure of the body

for Kerr BHs

$$C_{S^k}^{\text{Kerr}} = 1 \text{ for all } k.$$

Kerr Black Holes as heavy particles

[Aoude, Haddad, Helset, Jan 20']

The 3-point amplitude can be derived from this action.

[Chung, Huang, Kim, Lee, 19']

[Chung, Huang, Kim, 19']

In the HPET variables (all incoming) …

$$\mathcal{M}^{+2,s} = \sum_{a+b\leq s} \frac{\kappa m x^2}{2m^{2s}} C_{S^{a+b}} n^s_{a,b} \langle \mathbf{2}_{-v} \mathbf{1}_v \rangle^{s-a} \left(-x \frac{\langle \mathbf{2}_{-v} q\rangle \langle q \mathbf{1}_v \rangle}{2m} \right)^a [\mathbf{2}_{-v} \mathbf{1}_v]^{s-b} \left(x^{-1} \frac{[\mathbf{2}_{-v} q][q \mathbf{1}_v]}{2m} \right)^b, \qquad n^s_{a,b} \equiv \binom{s}{a}\binom{s}{b}$$

converting to the chiral basis

$$\mathcal{M}^{+2,s} = \frac{x^2}{m^{2s}} (-1)^{2s} \sum_{a+b\leq 2s} \frac{\kappa m}{2} C_{S^{a+b}} n^s_{a,b} \langle \mathbf{2}_v \mathbf{1}_v \rangle^{2s-a-b} \left(\frac{x}{2m} \langle \mathbf{2}_v q\rangle \langle q \mathbf{1}_v \rangle \right)^{a+b}$$

Comparing with the general 3-pt HPET

$$\mathcal{M}_3^{+|h|,s} = (-1)^{2s+h} \frac{x^{|h|}}{m^{2s}} \sum_{k=0}^{2s} g^{\mathrm{H}}_{s,k} \langle \mathbf{2}_v \mathbf{1}_v \rangle^{2s-k} \left(\frac{x}{2m} \langle \mathbf{2}_v q\rangle \langle q \mathbf{1}_v \rangle \right)^k$$

$$g^{\mathrm{H}}_{s,k} = \frac{\kappa m}{2} C_{S^k} \sum_{j=0}^{k} n^s_{k-j,j}.$$

Focusing on the minimal coupling, and normalizing

$$g^{\mathrm{H}}_{s,k} = g_0 \binom{2s}{k} \qquad g_0 = \kappa m/2. \longrightarrow \qquad C^{\min}_{S^k} = \binom{2s}{k} \left[\sum_{j=0}^{k} \binom{s}{k-j}\binom{s}{j} \right]^{-1} = 1.$$

Minimal coupling in HPET,
Precisely the multipoles of a Kerr BH

(way simpler, no need for boost)

Spin-1/2 coupled to a photon

$$\mathcal{A}\left(-\mathbf{1}^{\frac{1}{2}}, \mathbf{2}^{\frac{1}{2}}, q^h\right) = f(m,v,q) e v_\mu \epsilon_q^{h,\mu} \bar{u}_v(p_2) u_v(p_1) + g(m,v,q) e q^\mu \epsilon_q^{h,\nu} \bar{u}_v(p_2)\sigma_{\mu\nu} u_v(p_1).$$

In the HPET variables (all incoming) …

$$v_\mu \epsilon_q^{+,\mu} \bar{u}_v(p_2) u_v(p_1) = -\sqrt{2} x \langle \mathbf{2}_v \mathbf{1}_v \rangle = -\frac{x}{\sqrt{2}}\left(-\frac{x}{m}\langle \mathbf{2} q\rangle\langle q \mathbf{1}\rangle + 2\langle \mathbf{2}\mathbf{1}\rangle\right),$$

$$\bar{u}_v(p_2)\sigma_{\mu\nu} u_v(p_1) q^\mu \epsilon_q^{+,\nu} = \sqrt{2} i x^2 \langle \mathbf{2}_v q\rangle\langle q \mathbf{1}_v\rangle = \sqrt{2} i x^2 \langle \mathbf{2} q\rangle\langle q \mathbf{1}\rangle.$$

plugging back…

$$\mathcal{A}\left(-\mathbf{1}^{\frac{1}{2}}, \mathbf{2}^{\frac{1}{2}}, q^+\right) = \sqrt{2} x e \left(-f(m,v,q)\langle \mathbf{2}_v \mathbf{1}_v\rangle + g(m,v,q) i x \langle \mathbf{2}_v q\rangle\langle q \mathbf{1}_v\rangle\right).$$

from QED, we have

$$\mathcal{A}_{\text{QED}}\left(-\mathbf{1}^{\frac{1}{2}}, \mathbf{2}^{\frac{1}{2}}, q^+\right) = e\bar{u}(p_2)\gamma_\mu u(p_1)\epsilon_q^{+,\mu}$$
$$= \sqrt{2} e x \langle \mathbf{2}\mathbf{1}\rangle,$$

→ fixes f and g

$$f(m,v,q) = -1,$$

$$g(m,v,q) = \frac{i}{2m}.$$

due to on-shellness

$$v \cdot q \sim q^2 = 0$$