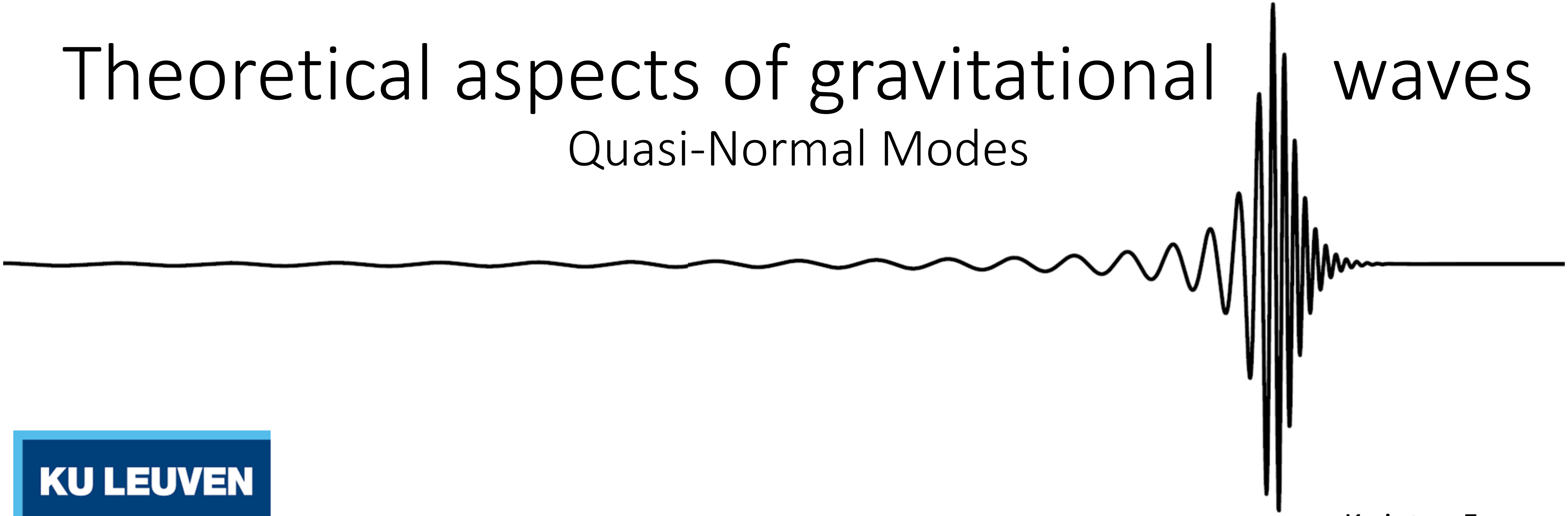
Theoretical aspects of gravitational waves

Quasi-Normal Modes



KU LEUVEN

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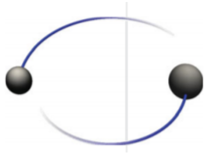
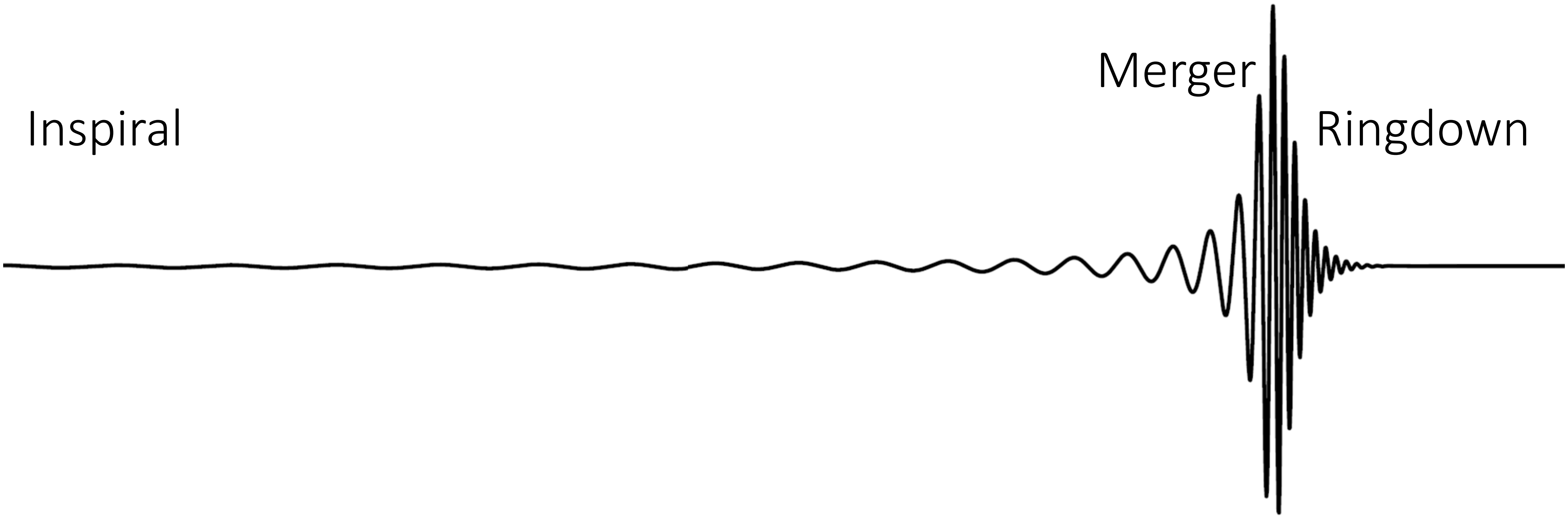
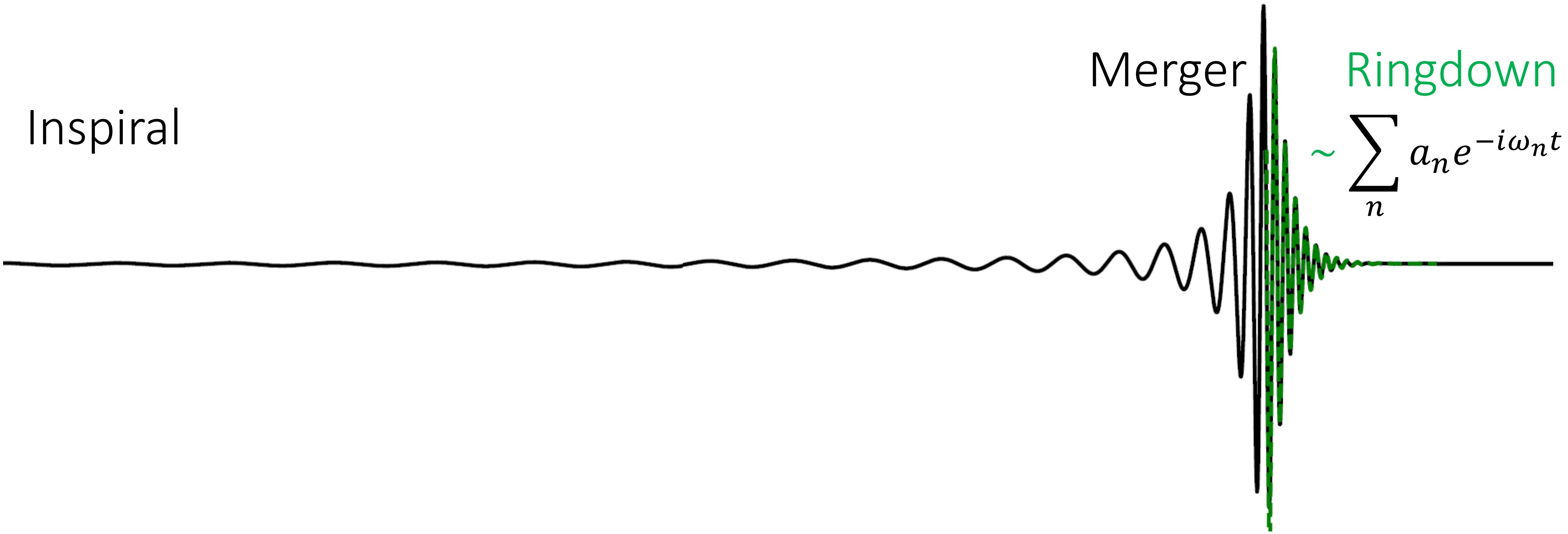
Kwinten Fransen
be.HEP
22/06/2020




Inspiral
Merger
Ringdown



Quasi-Normal Modes (QNM) of the remnant: $\omega_n = \Re(\omega_n) + i\,\Im(\omega_n)$

Plan

arXiv: 2005.03671
(Cano, KF, Hertog)

arXiv: 2005.12286
(KF, Koekoek, Tielemans, Vercnocke)

arXiv: 1910.02081
(Compère, KF, Hertog,, Liu)

Asteroseismology

from stars to black holes

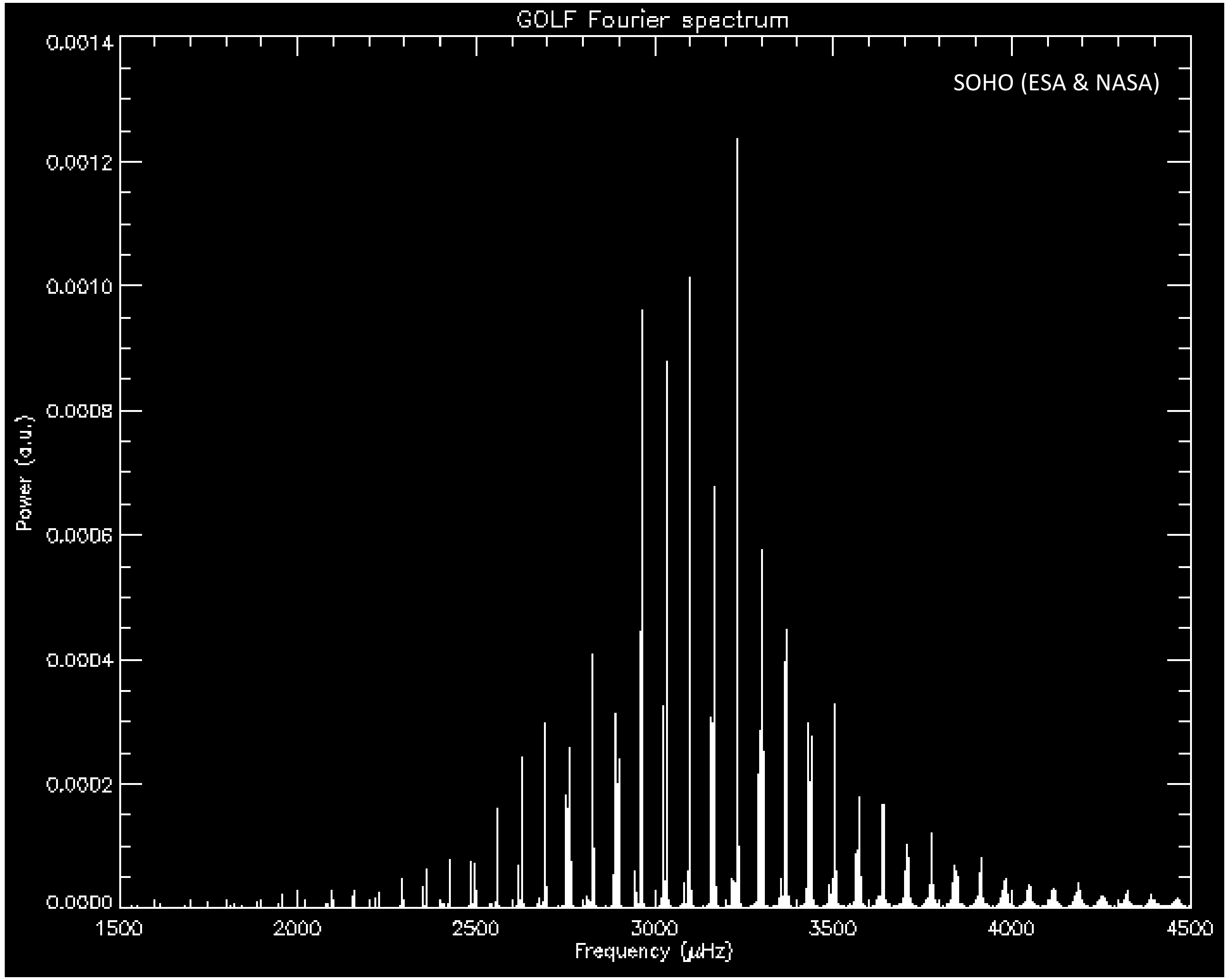
Unexpected Quasi-Normal Modes in Gravitational Waves

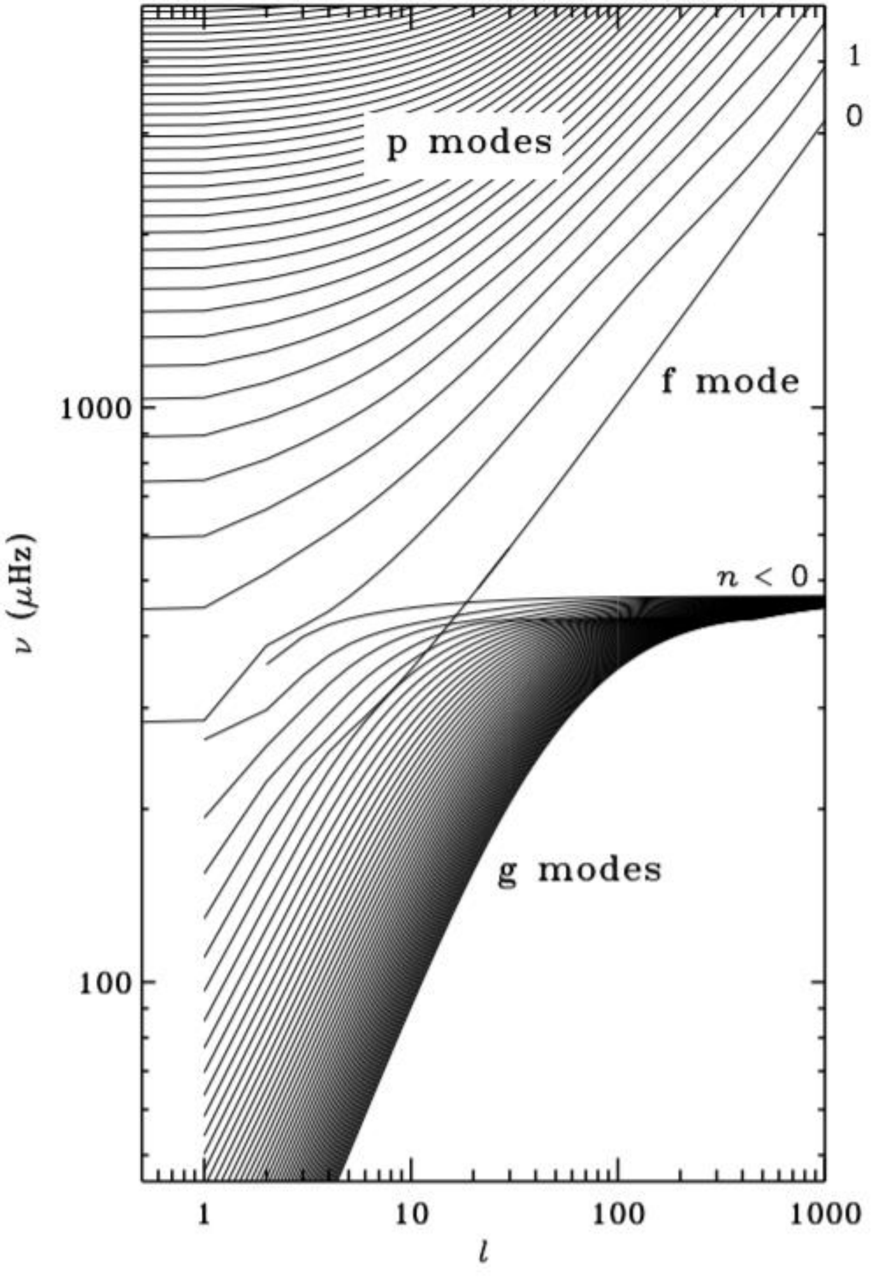
the perturbative and the non-perturbative

Symmetry at the other end

Spin me up, Scotty

Helioseismology

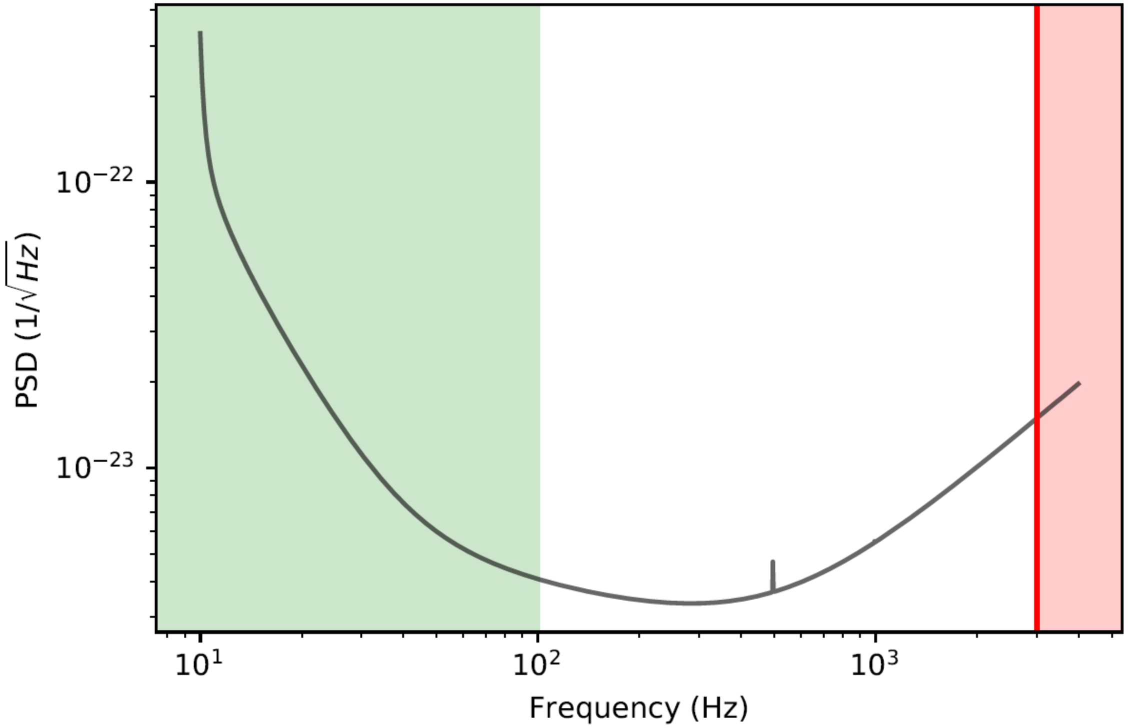




Christensen-Dalsgaard (2003)

Asteroseismology: Neutron stars

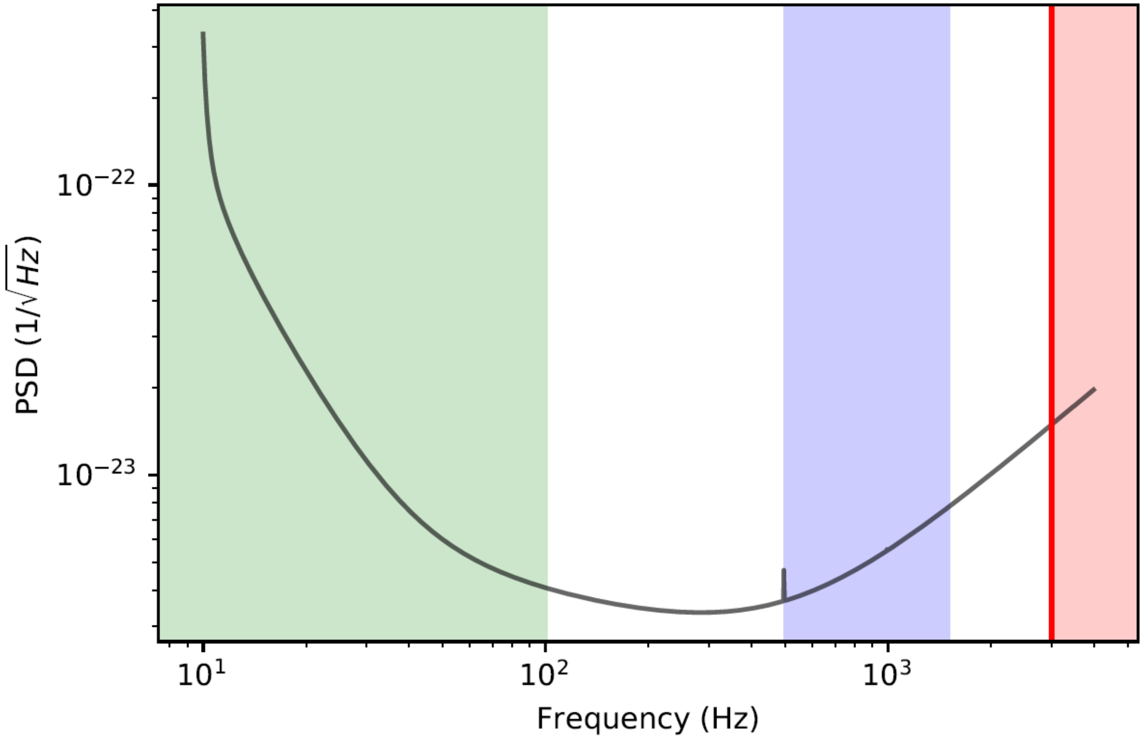
g-modes, f-mode, p-modes, …



Asteroseismology: Neutron stars

g-modes, f-mode, p-modes, …

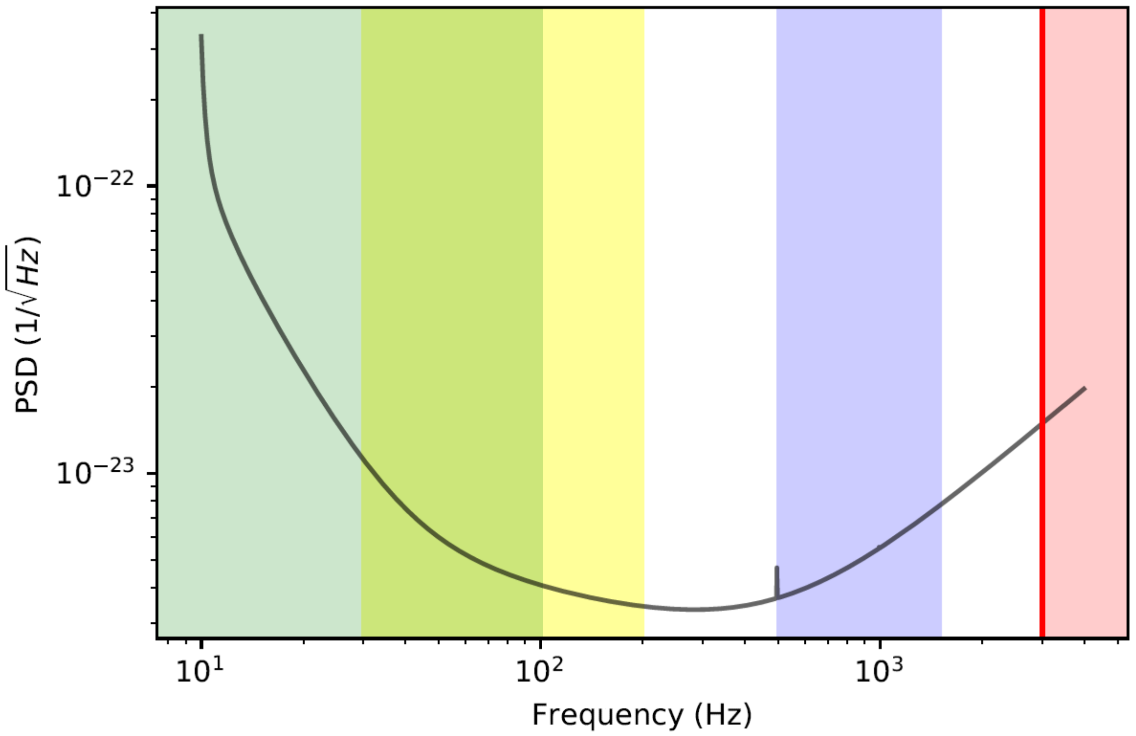
r-modes: see previous talk!



Asteroseismology: Neutron stars

g-modes, **r-modes**, f-mode, p-modes, …

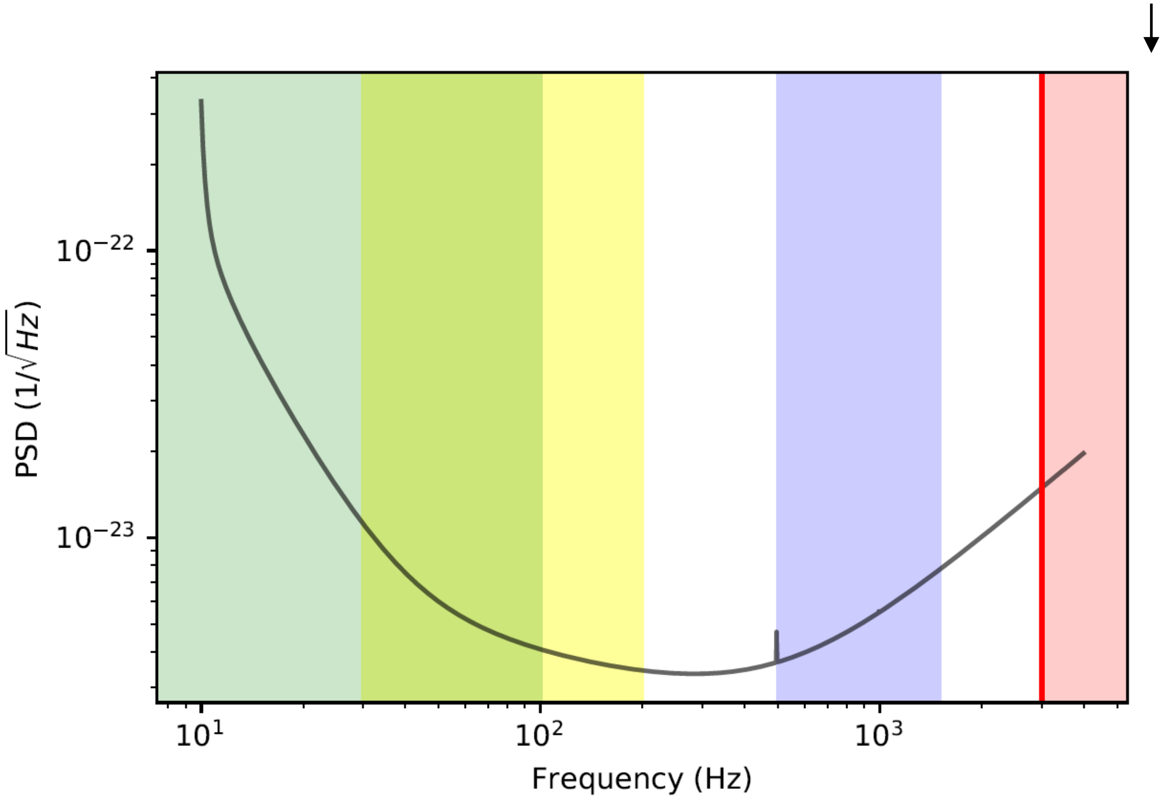
i-modes: crust-core interface



Asteroseismology: Neutron stars

g-modes, **i-modes**, **r-modes**, f-mode, p-modes, …

(g)w-modes: gravitational degrees of freedom



[insert name]-seismology: Black holes

Melasmaseismology?

Bothroseismology?

Nigricoseismology?

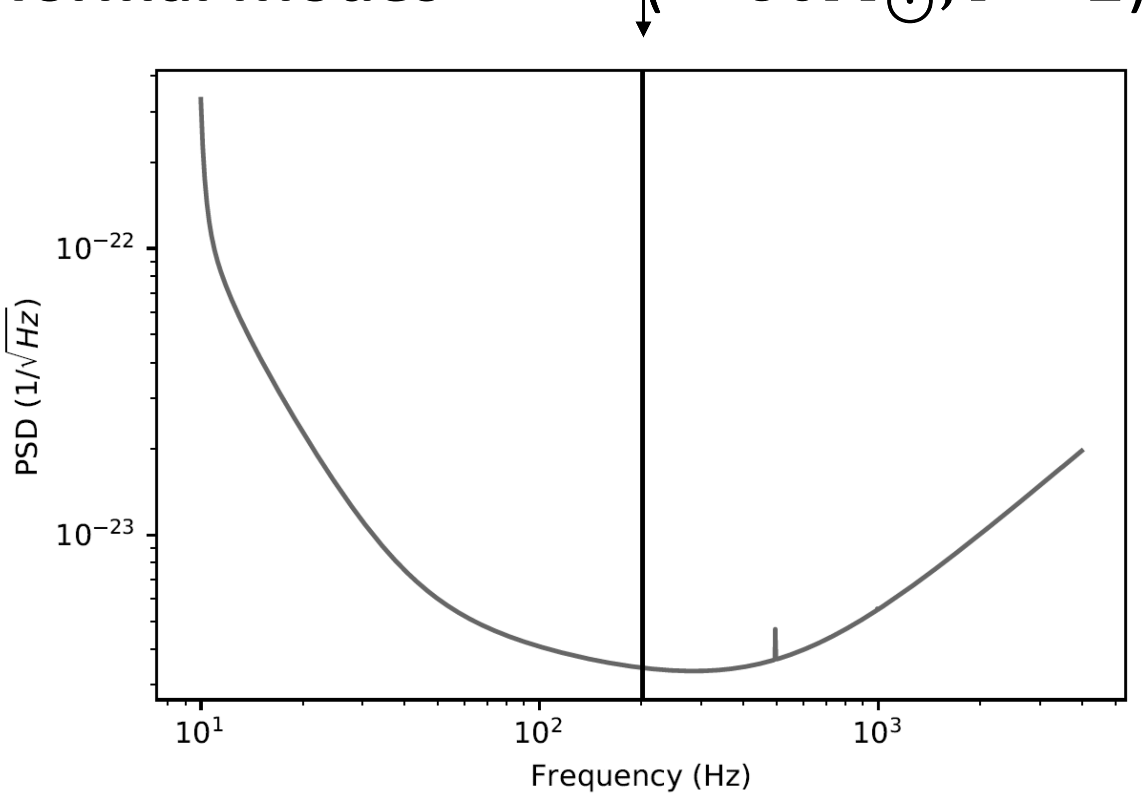
…

Black hole spectroscopy

~~f-mode, p-modes, g-modes, r-modes, i-modes + …~~

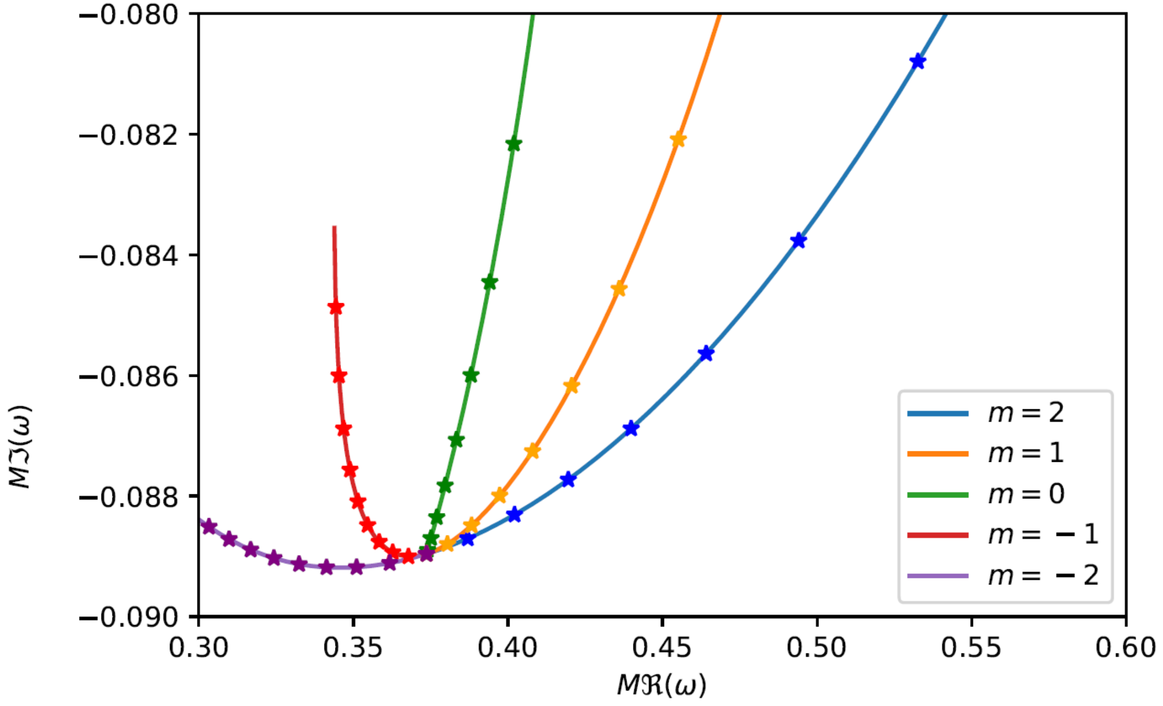
Only (g)w-modes aka 'the BH Quasi-Normal Modes'

$(\approx 60M_{\odot}, \mathrm{l} = 2)$

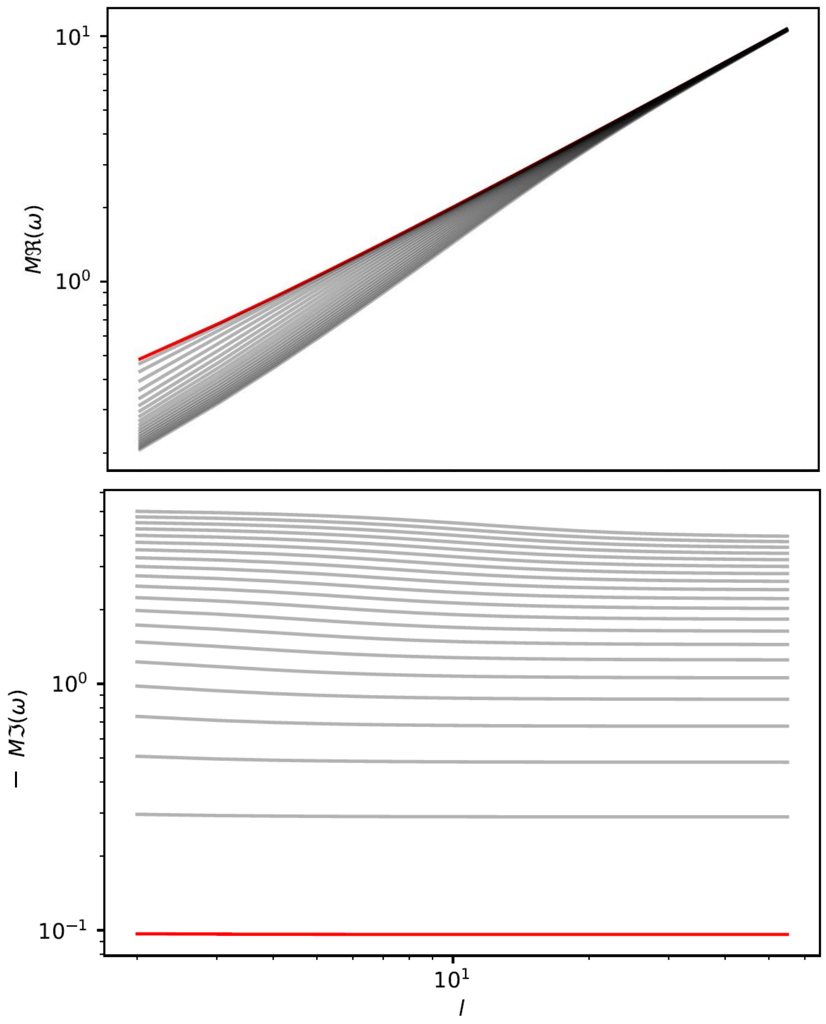


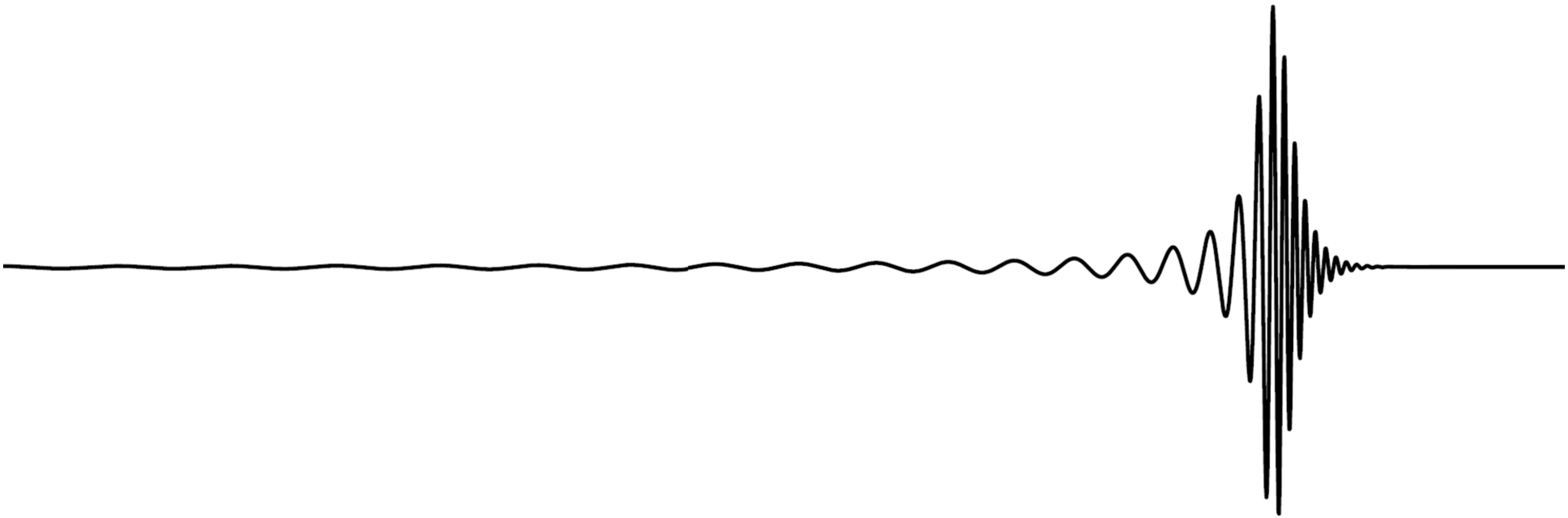
Quasi-normal modes

Rotating, l=2



Static

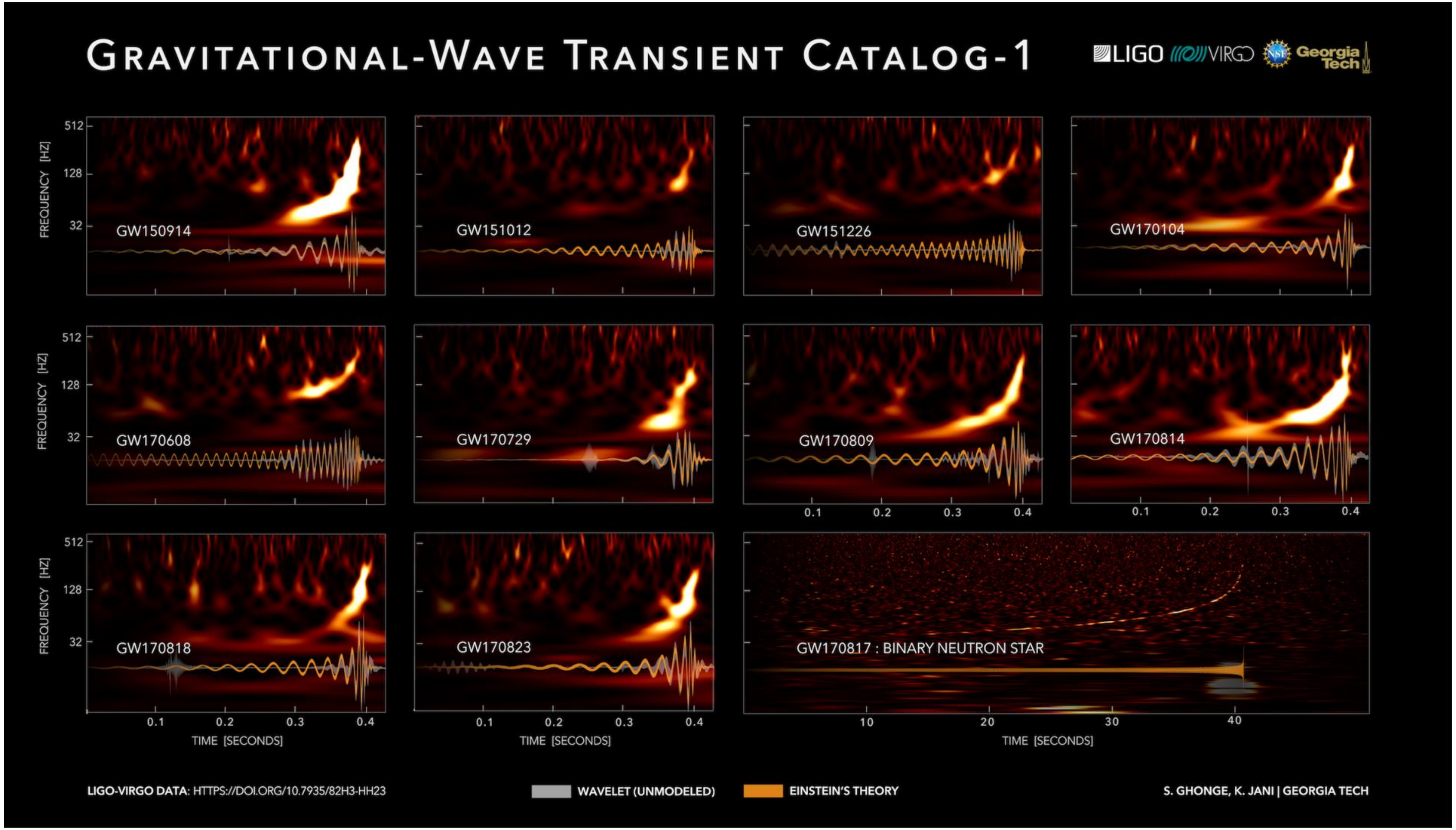




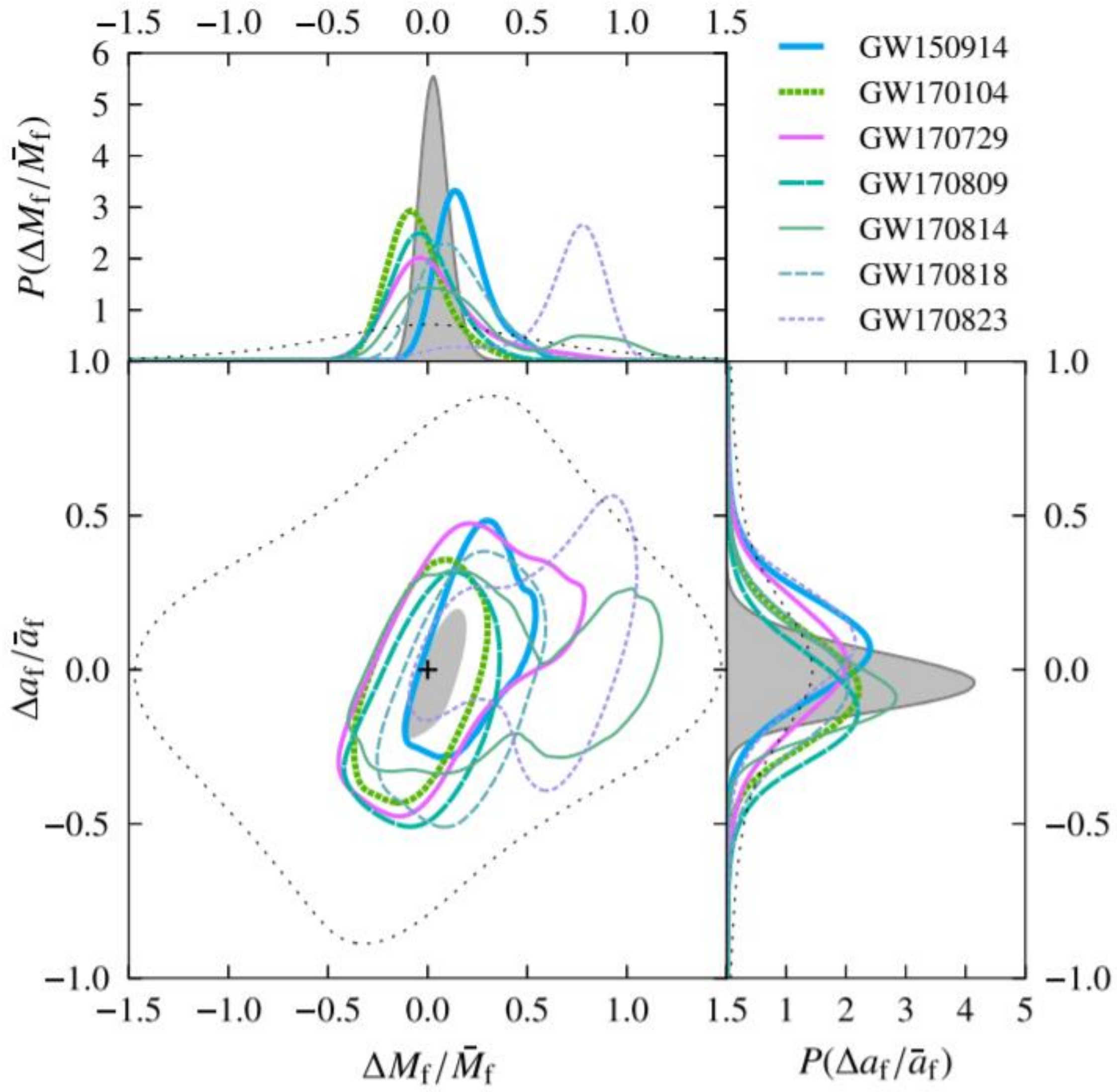
Cross-consistency checks: compare inspiral ↔ 1 QNM

Internal consistency checks: compare multiple QNMs

Theoretical?



Theoretical? O1/O2 Inspiral-Merger-Ringdown checks



LIGO/VIRGO collaboration: arXiv: 1903.04467

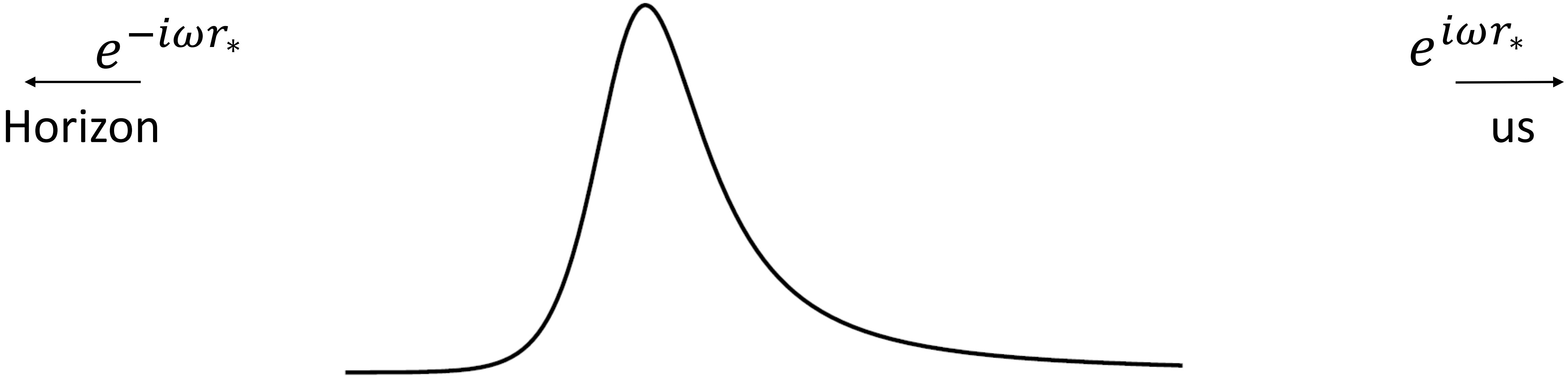
Further theoretical input?

- How would deviations vary with final spin? O1/O2 $\frac{cJ_f}{GM_f^2} \in [0.5 - 0.9]$

- Look for QNMs in the inspiral or 'post'-ringdown?

Core of the problem in GR (even rotating BH!)

$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}R$$



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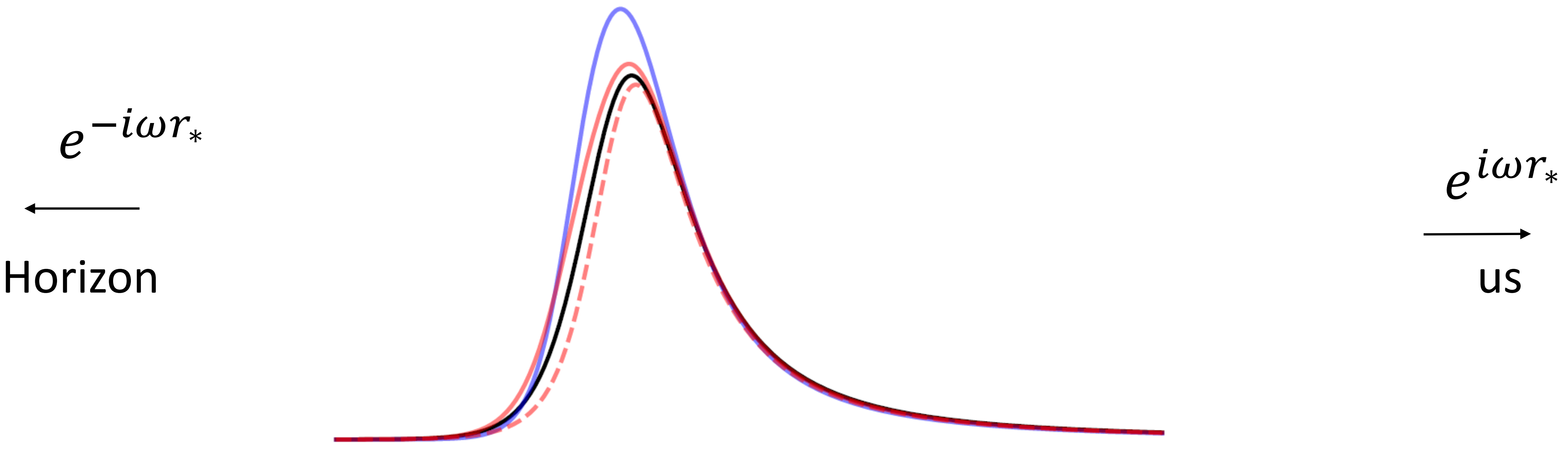
Spin me up, Scotty

Generically not true, but perturbatively …

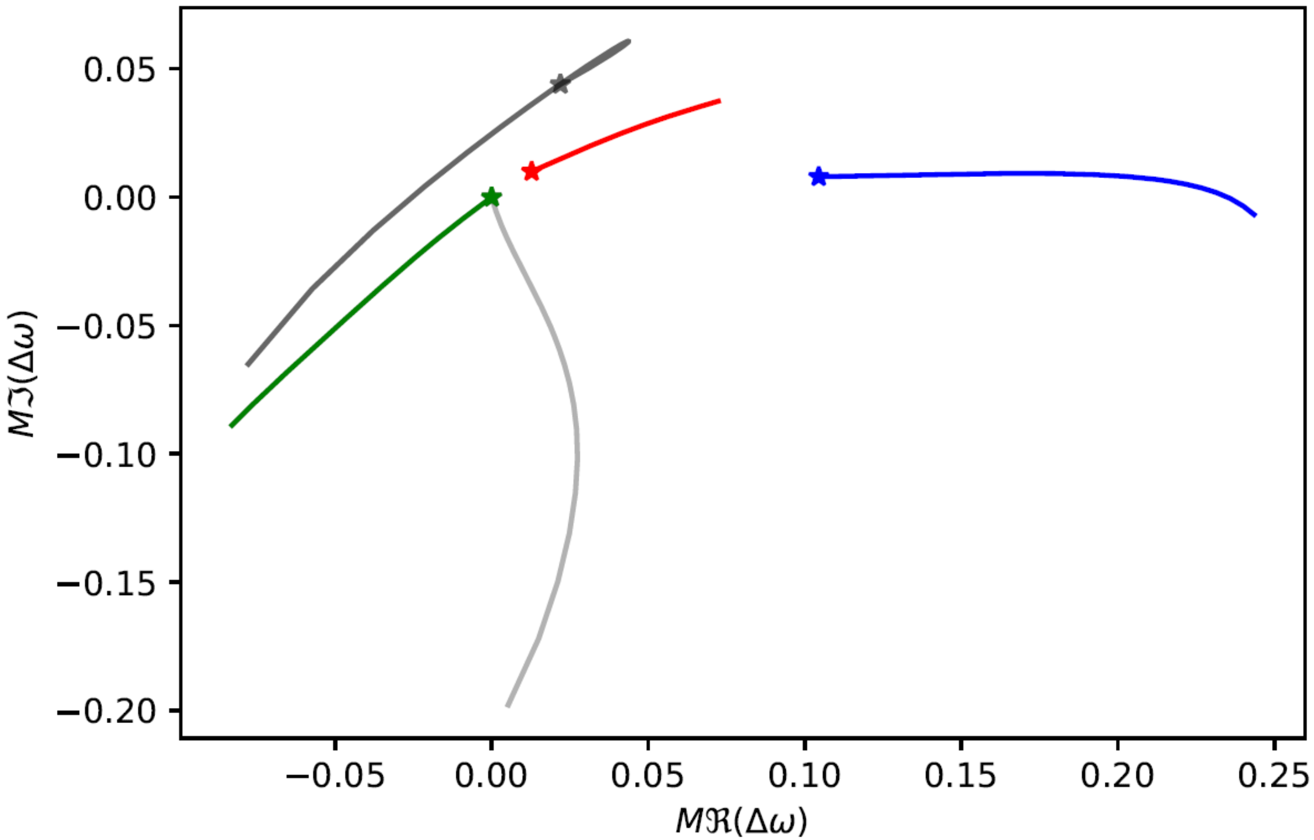
$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,\Big(R + \alpha_1 \ell^2 \phi_1\,\mathcal{X} + \alpha_2(\phi_2\cos\theta_m + \phi_1\sin\theta_m)\ell^2 R_{\mu\nu\rho\sigma}\,\tilde{R}^{\mu\nu\rho\sigma}$$
$$+\lambda_{ev}\ell^4 R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}R_{\delta\gamma}{}^{\mu\nu} + \lambda_{odd}\,\ell^4 R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\delta\gamma}\tilde{R}_{\delta\gamma}{}^{\mu\nu} + \epsilon_1\,\ell^6\big(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\big)^2 + \cdots$$



$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,\left(R + \alpha_1 \ell^2 \phi_1\, \mathcal{X} + \alpha_2(\phi_2 \cos\theta_m + \phi_1 \sin\theta_m)\ell^2 R_{\mu\nu\rho\sigma}\,\tilde{R}^{\mu\nu\rho\sigma}\right.$$

$$+\lambda_{ev}\ell^4 R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\delta\gamma} R_{\delta\gamma}{}^{\mu\nu} + \lambda_{odd}\,\ell^4 R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\delta\gamma} \tilde{R}_{\delta\gamma}{}^{\mu\nu} + \epsilon_1\,\ell^6 \left(R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}\right)^2$$

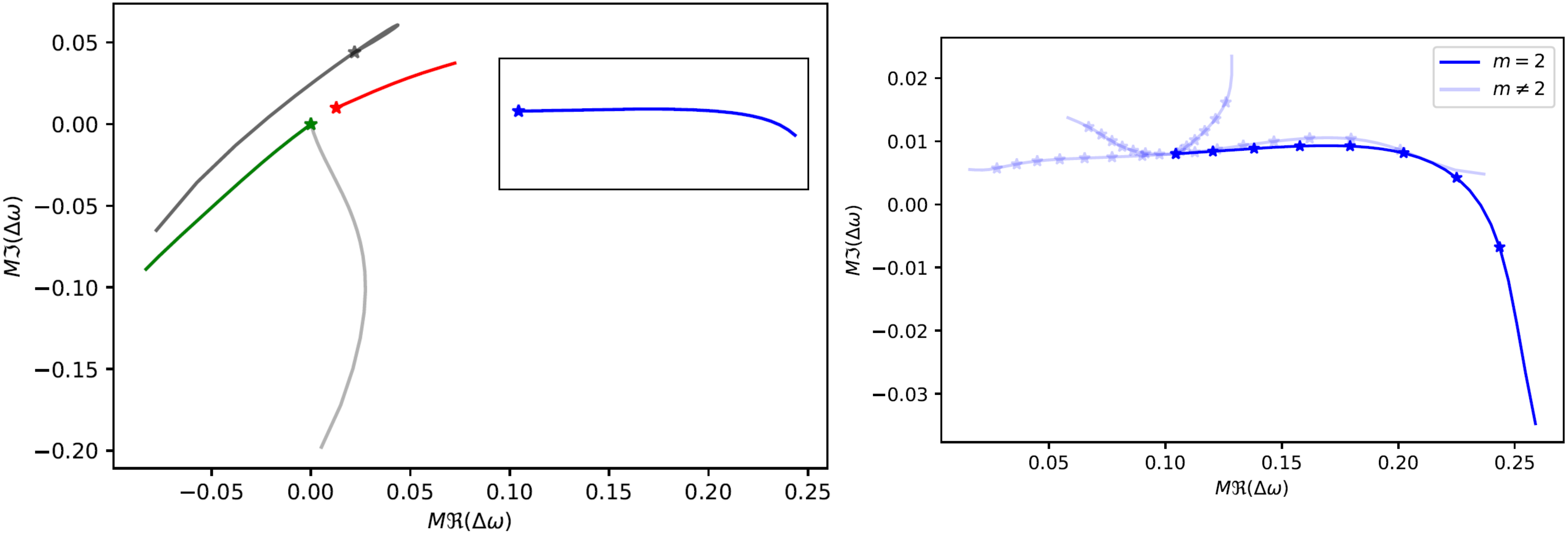
$$+\epsilon_2\,\ell^6\left(R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}\right)^2 + \epsilon_3 \ell^6 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} R_{\alpha\beta\gamma\kappa}\tilde{R}^{\alpha\beta\gamma\kappa} + \cdots$$



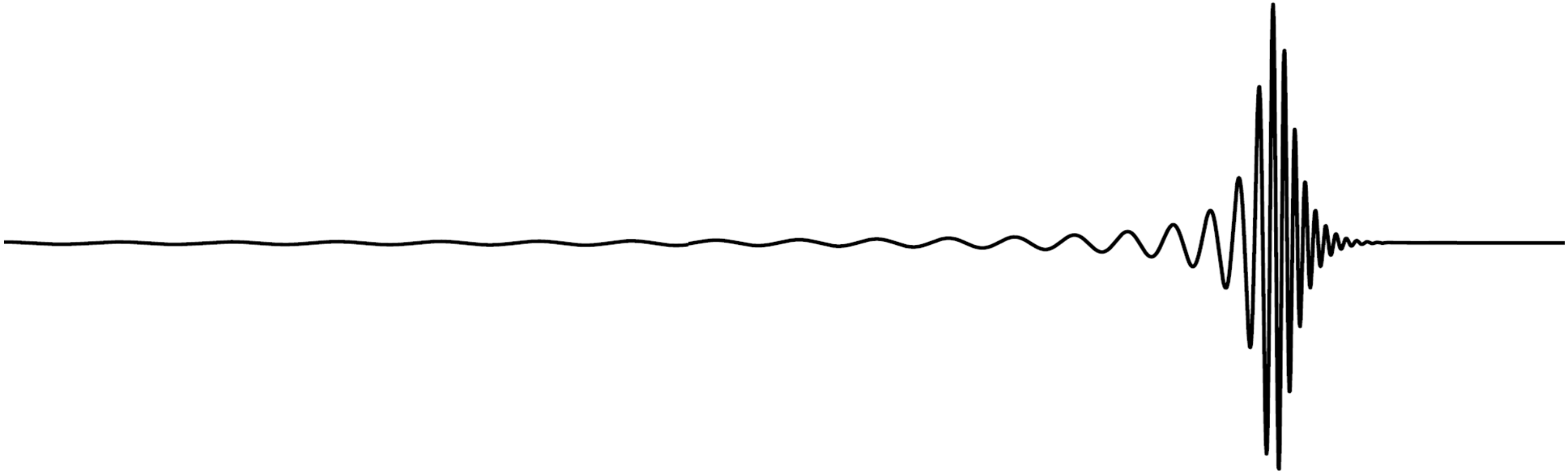
$$S = \frac{1}{16\pi G}\int d^4x\,\sqrt{-g}\,\left(R + \alpha_1 \ell^2 \phi_1\, \mathcal{X} + \alpha_2(\phi_2 \cos\theta_m + \phi_1 \sin\theta_m)\ell^2 R_{\mu\nu\rho\sigma}\,\tilde{R}^{\mu\nu\rho\sigma}\right.$$

$$+\lambda_{ev}\ell^4 R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\delta\gamma} R_{\delta\gamma}{}^{\mu\nu} + \lambda_{odd}\,\ell^4 R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\delta\gamma} \tilde{R}_{\delta\gamma}{}^{\mu\nu} + \epsilon_1\,\ell^6\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\right)^2$$

$$+\epsilon_2\,\ell^6\left(R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}\right)^2 + \epsilon_3 \ell^6 R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}R_{\alpha\beta\gamma\kappa}\tilde{R}^{\alpha\beta\gamma\kappa} + \cdots$$



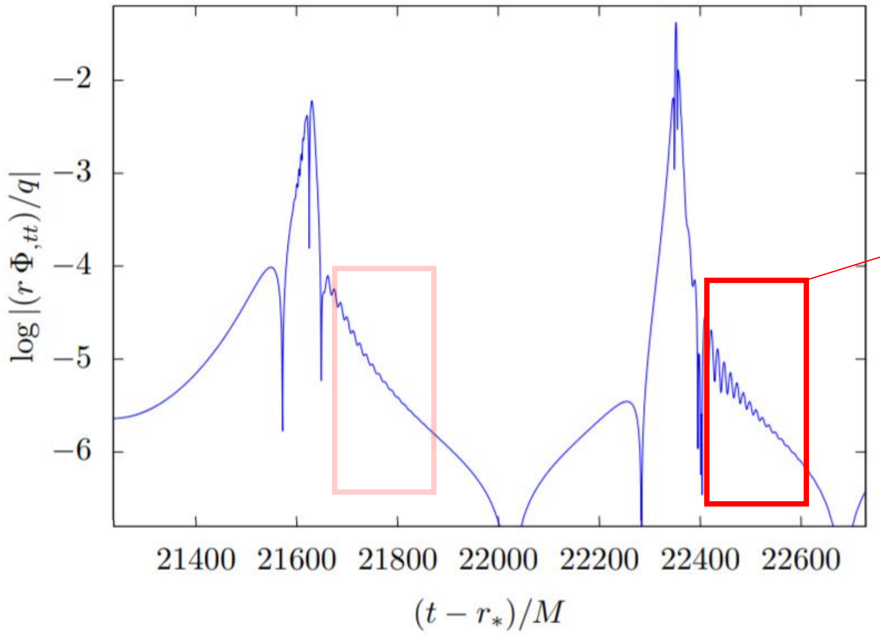
Look for QNMs in the inspiral?

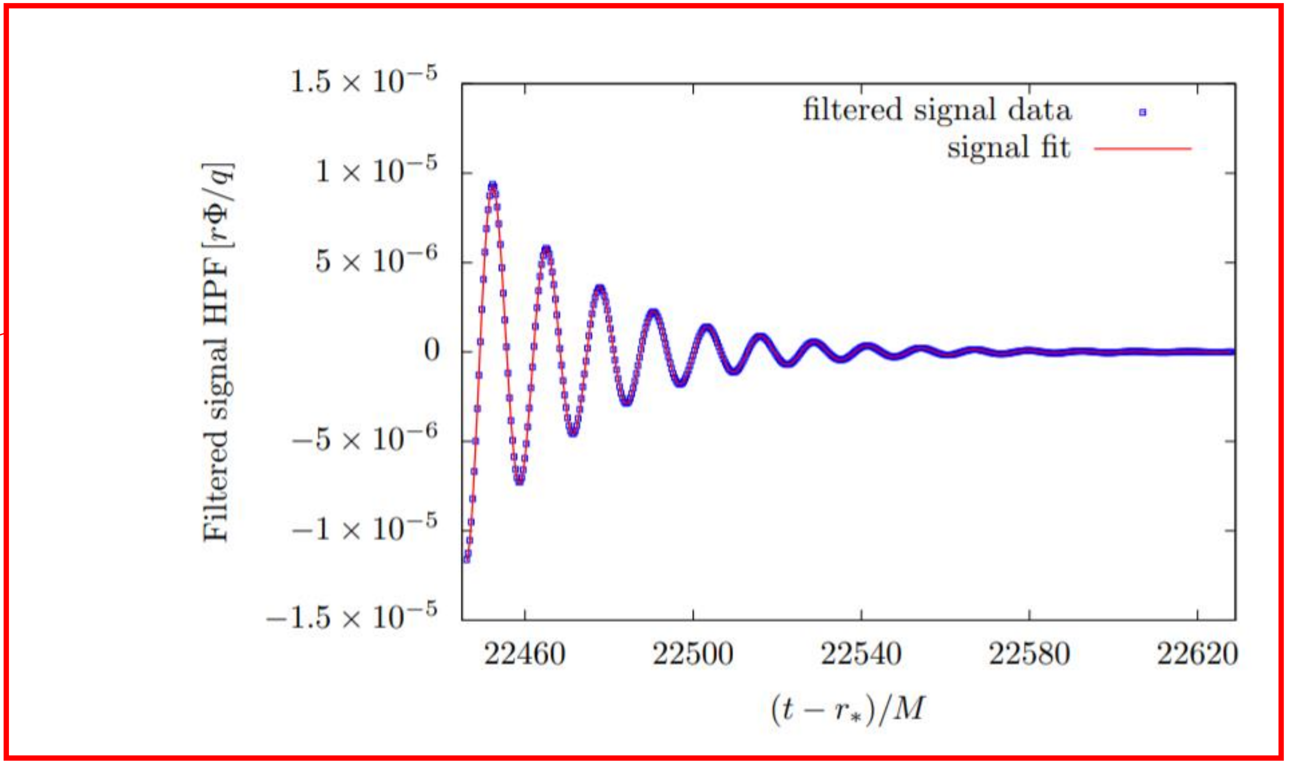


Possible for GR BHs but generically too high frequency

QNM bursts at periastron passage

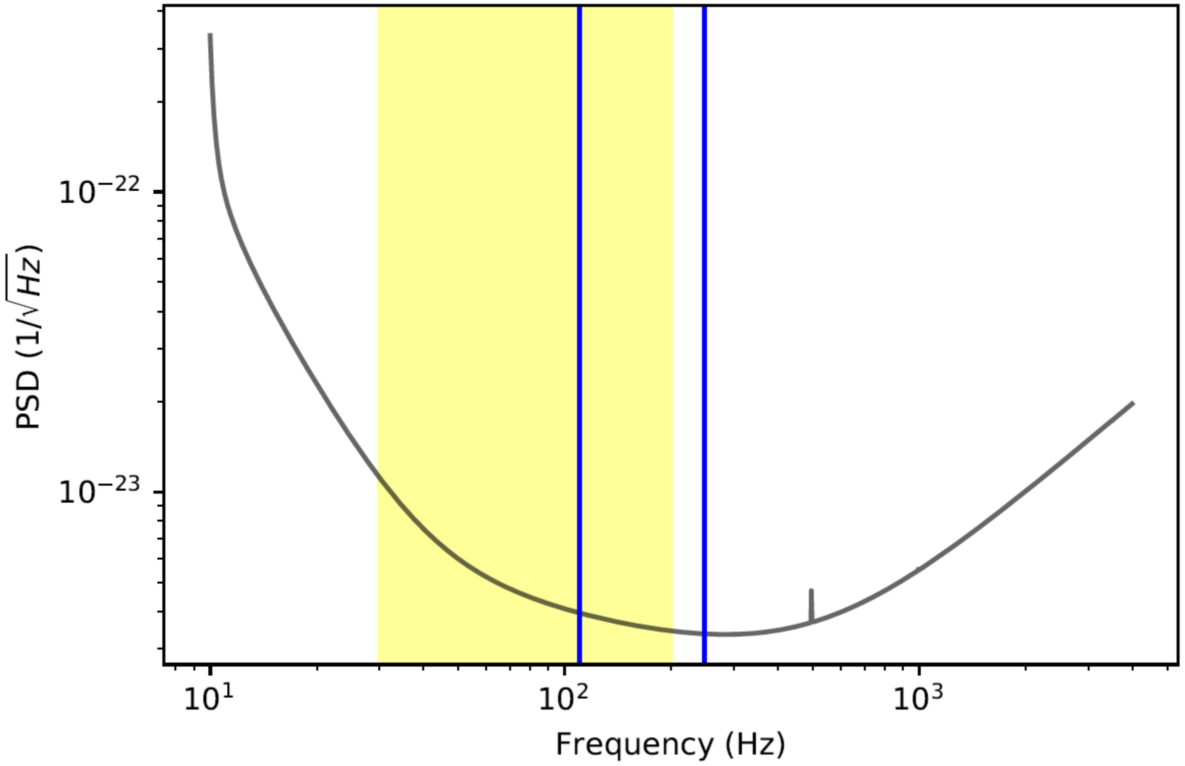
(highly eccentric, rotating)





Nasipak, Osburn, Evans: arXiv: 1905.13237

Look for QNMs in the inspiral?

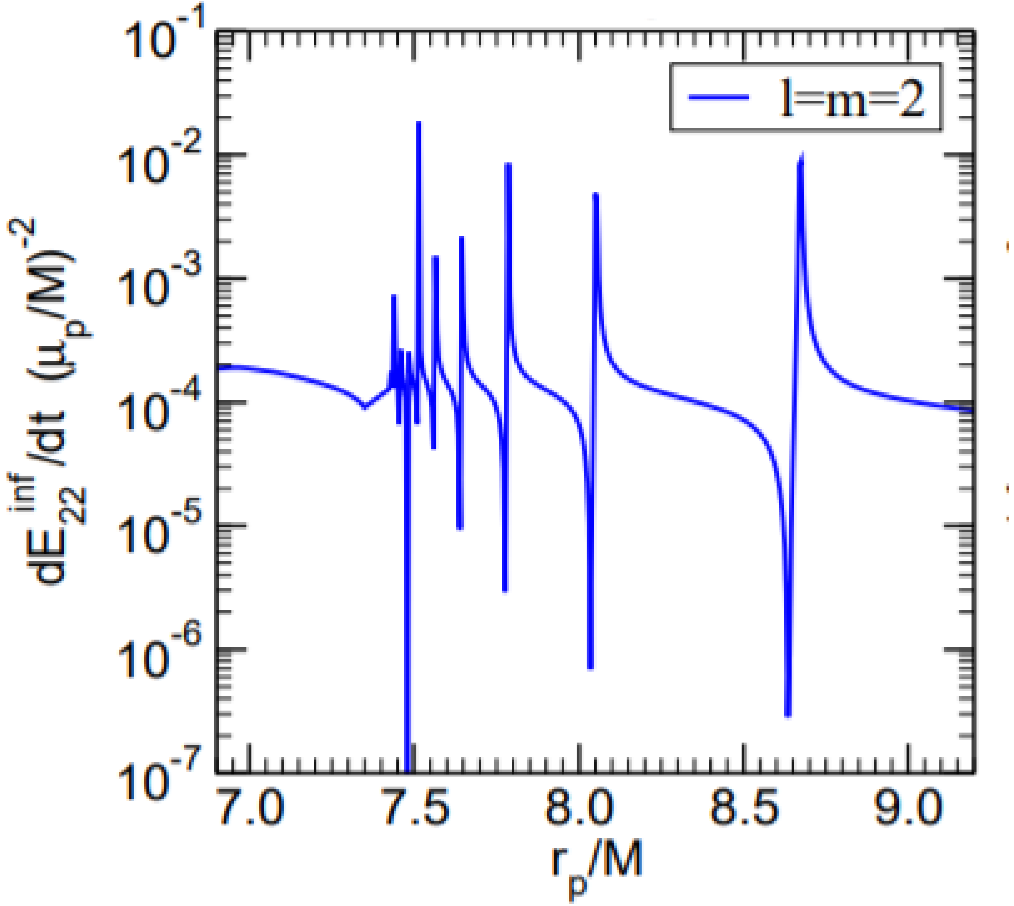


Like neutron star i-modes, …

But also excitations associated to light boson clouds: previous talk!

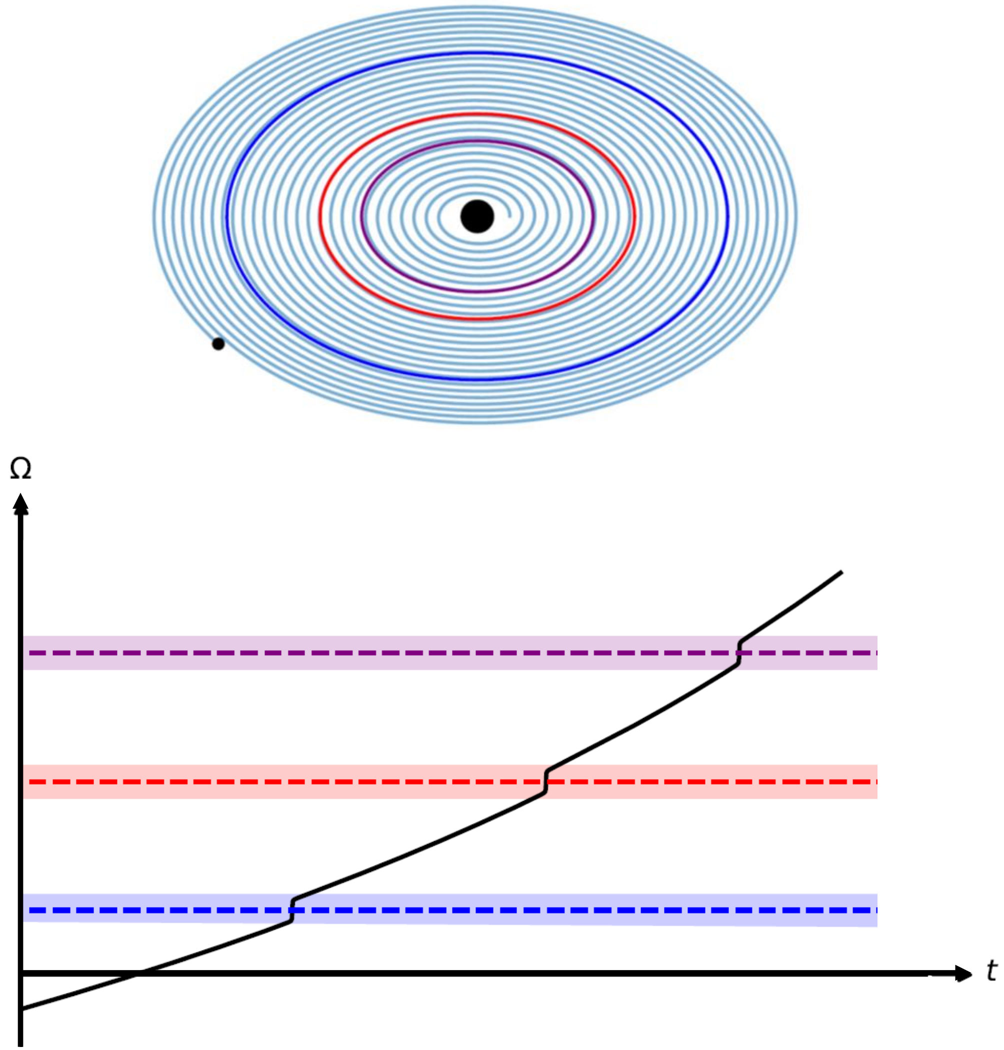
M$\approx 30 M_{\odot}, \mu \approx 3 \times 10^{-12} eV$

~~Direct detection:~~

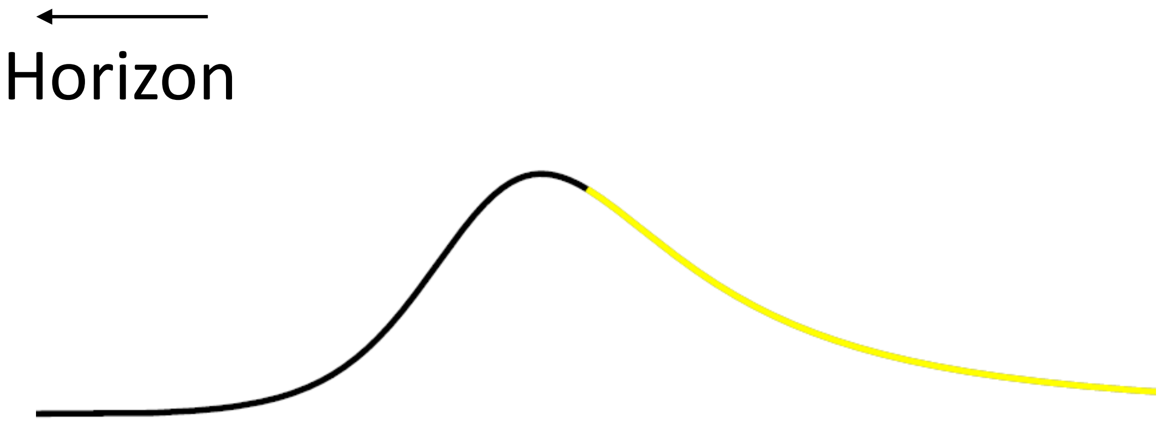


Macedo, Pani, Cardoso, Crispino: arXiv: 1307.4812

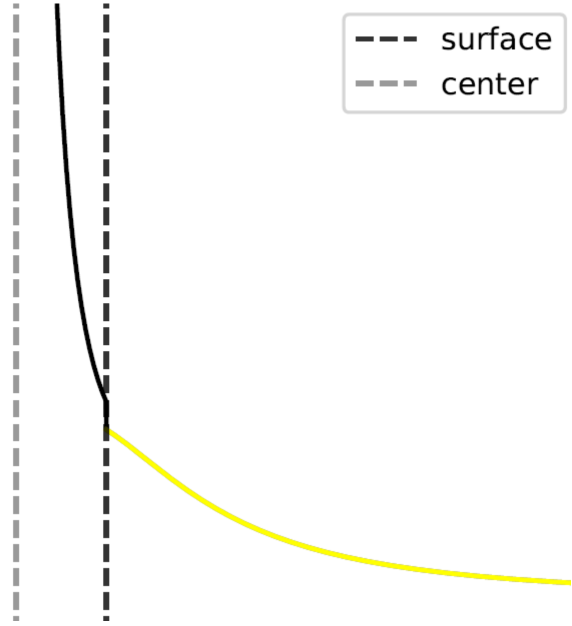
Backreaction on the orbit



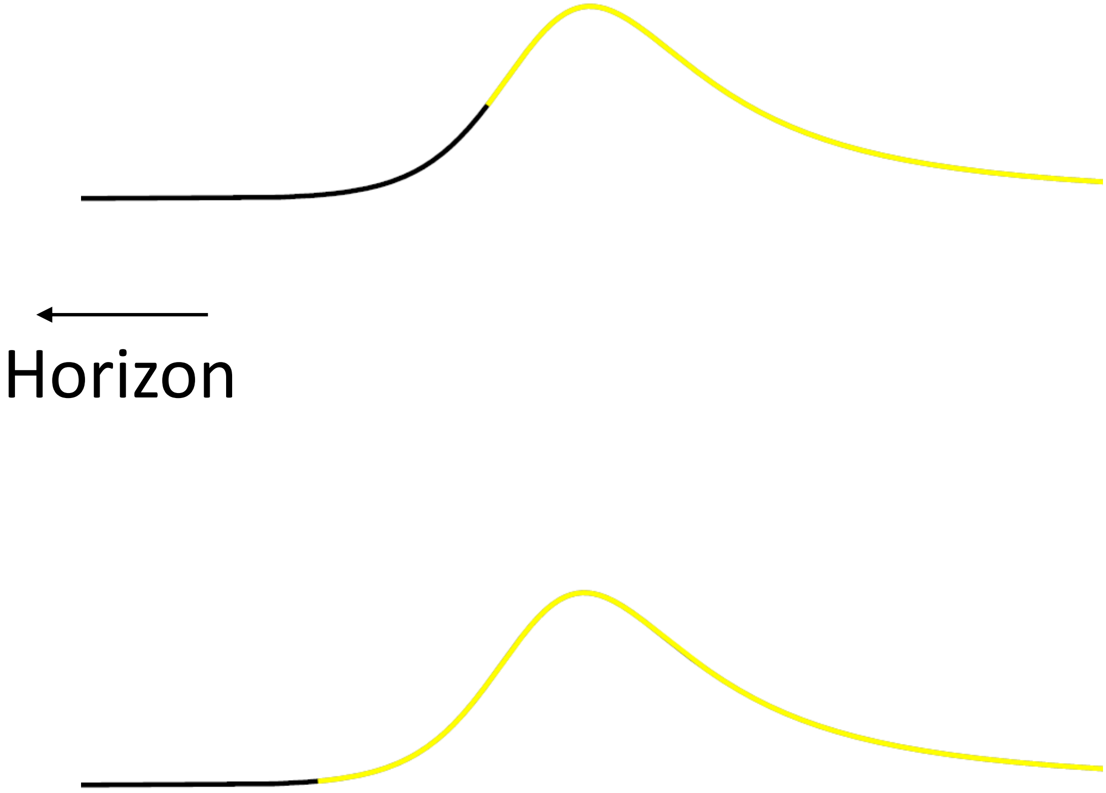
w-mode: Black holes

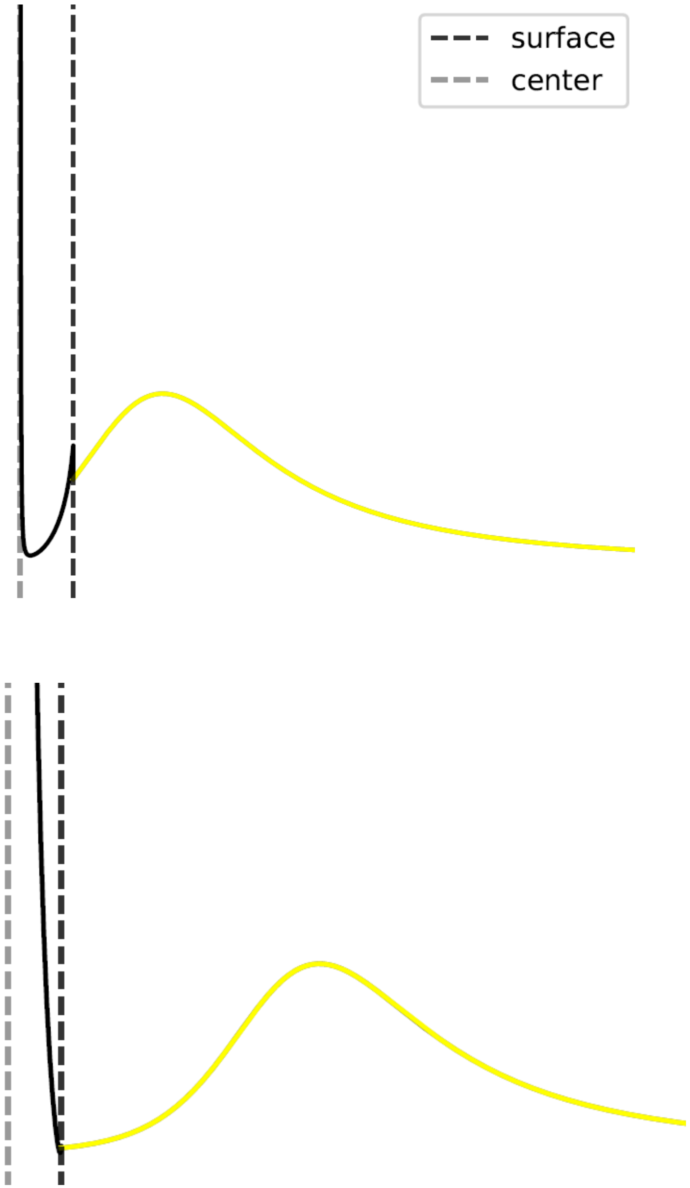


Neutron star



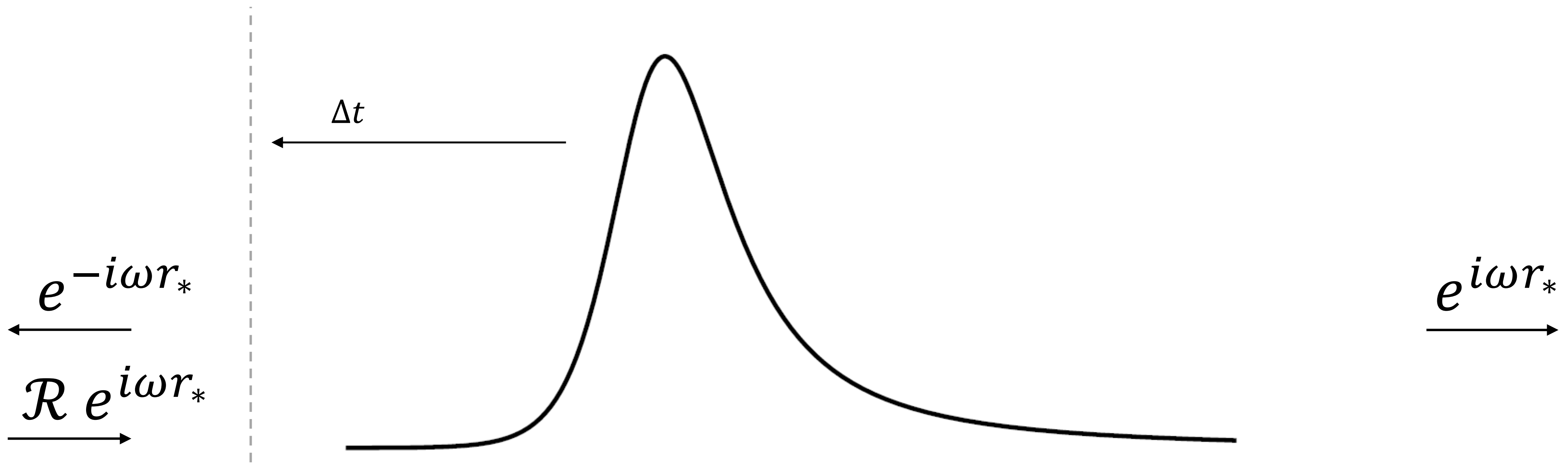
Trapped modes



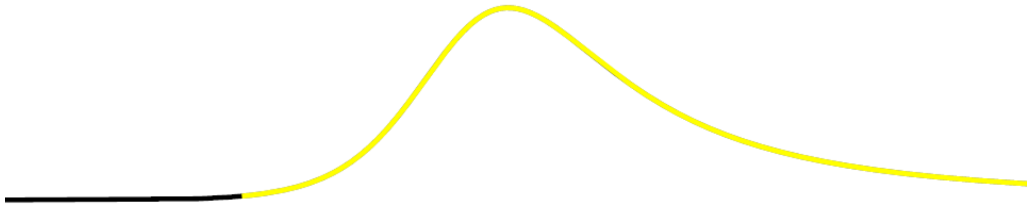


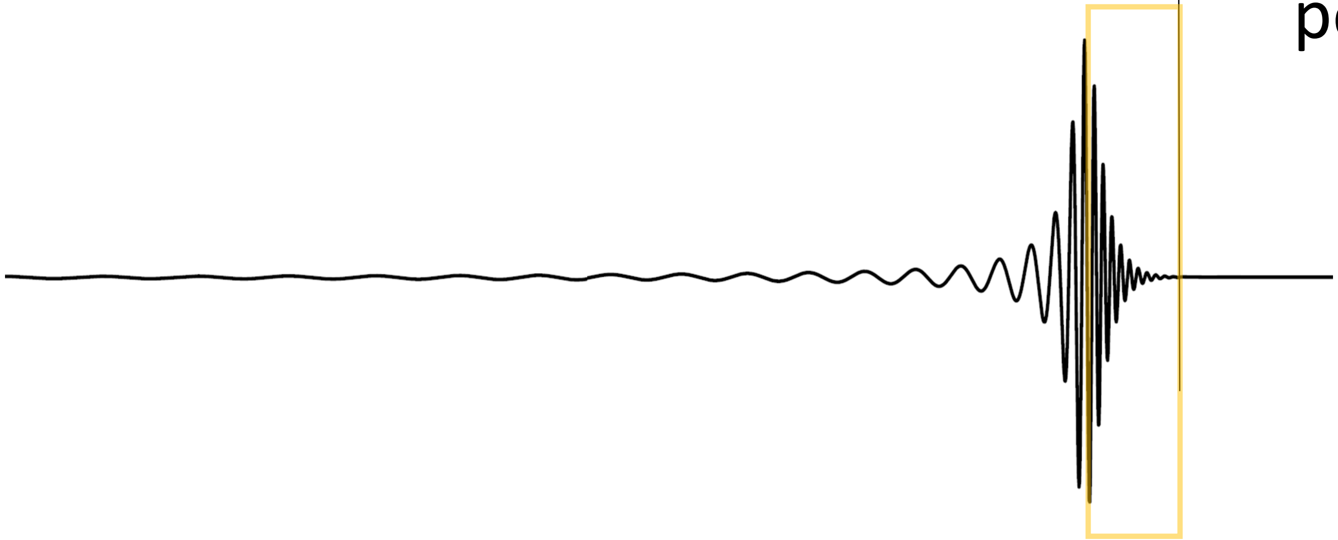
Trapped modes

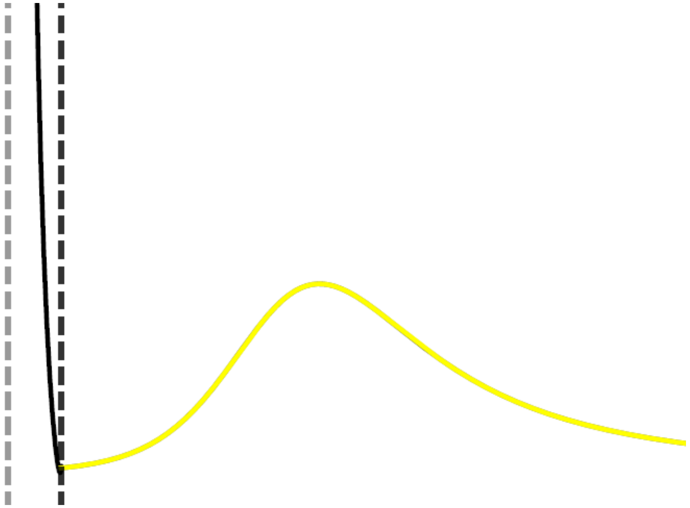
$$\Re(\omega_n) \approx \frac{n\pi}{\Delta t} \qquad \Re(f_n) \approx n \left(\frac{30\ M_\odot}{M}\right)\left(\frac{\log 10^{-12.8242712}}{\log \Delta s/r_s}\right) 100\text{Hz}$$



Trapped modes







particular complementarity with perturbed ringdown

Detectable?

In LIGO/VIRGO? (Asali, Pang, Samajdar, Van Den Broeck, arXiv: 2004.05128)

Keep in mind with ‘echo’ detection claims:

Claim Δt_{echo} (Abedi, Dykaar, Afshordi arXiv: 1612.00266) $\rightarrow f_{trapped} \approx 20$Hz

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Theoretical, optimistic order of magnitude:

need minimally 3G detectors

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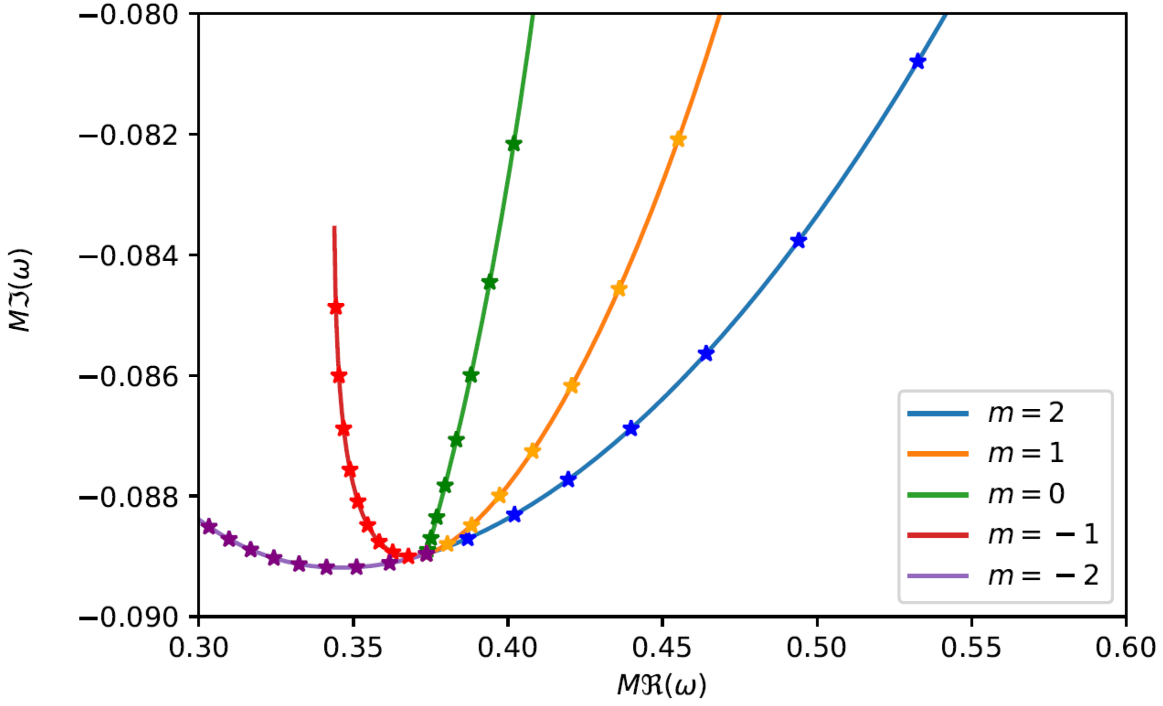
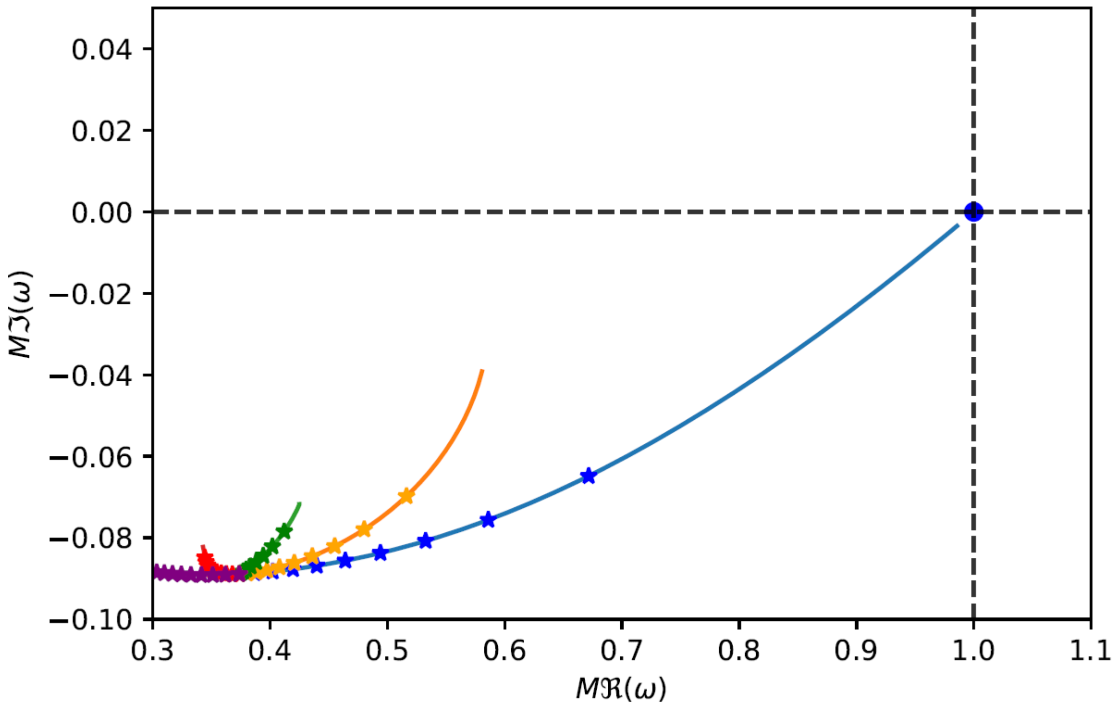
Spin me up, Scotty

Quasi-normal modes (again)

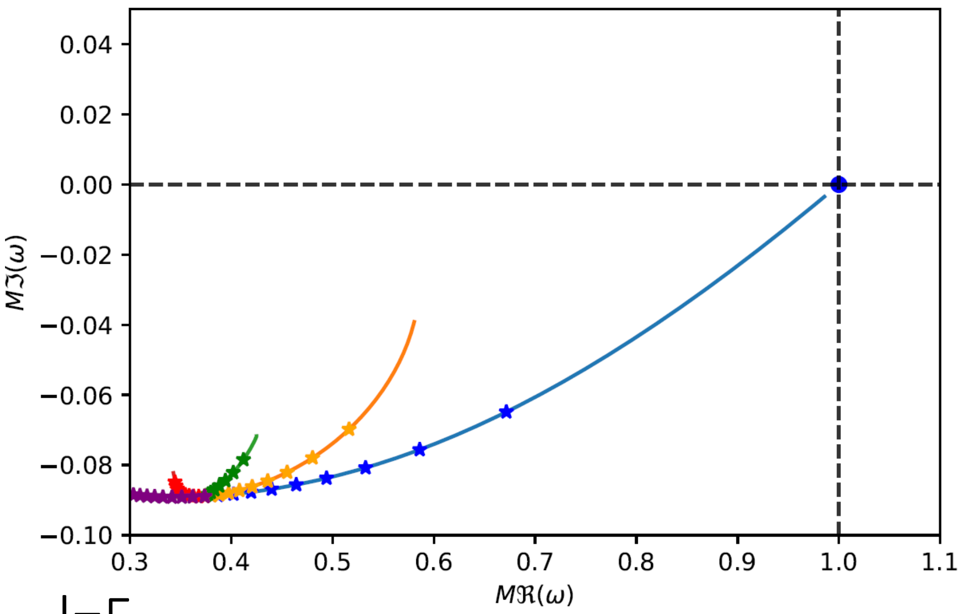
Rotating, l=2

$$J \to \frac{GM^2}{c}$$

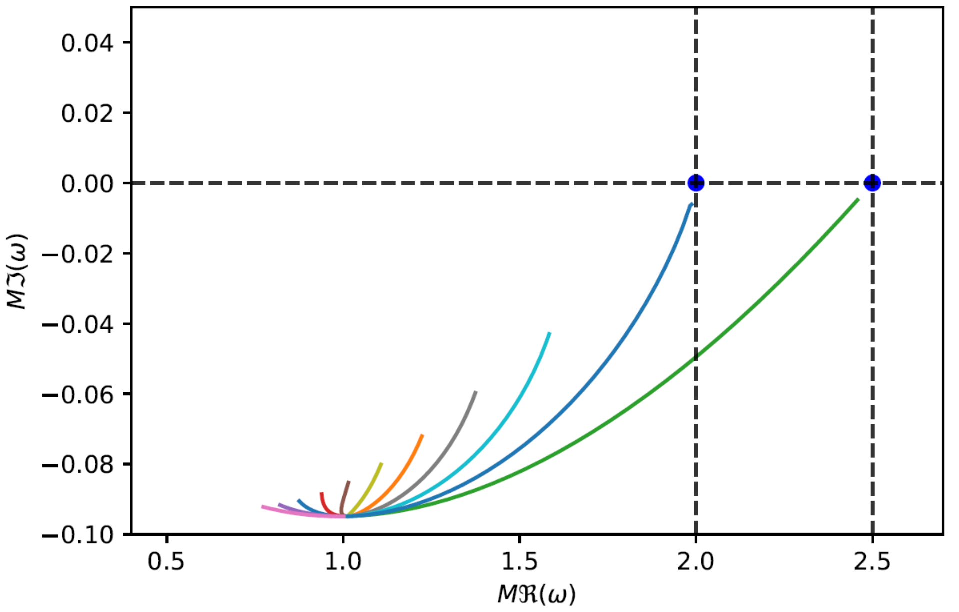




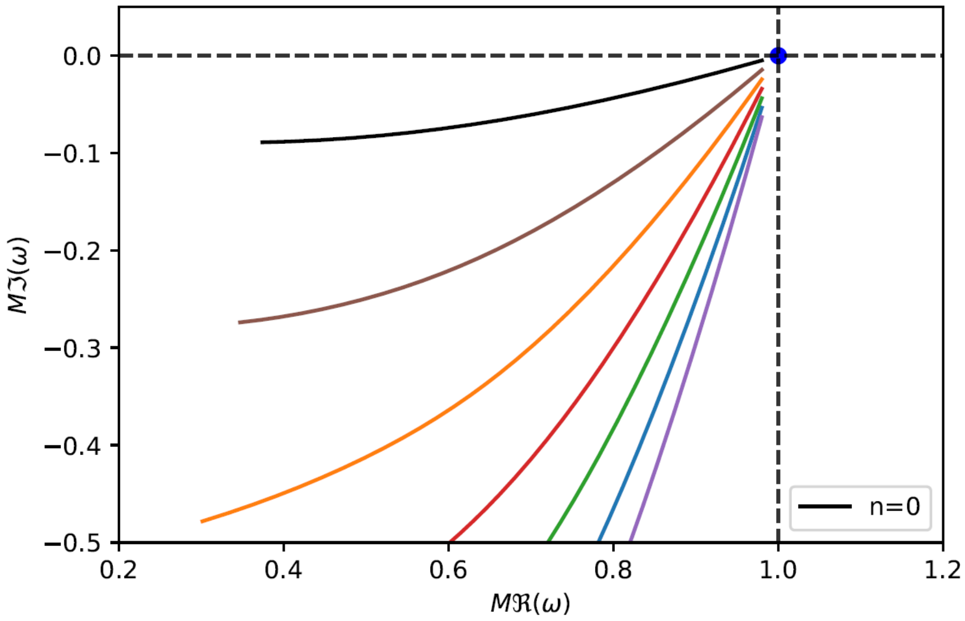
l=2



l=5



l=2, m=2 (+ overtones)

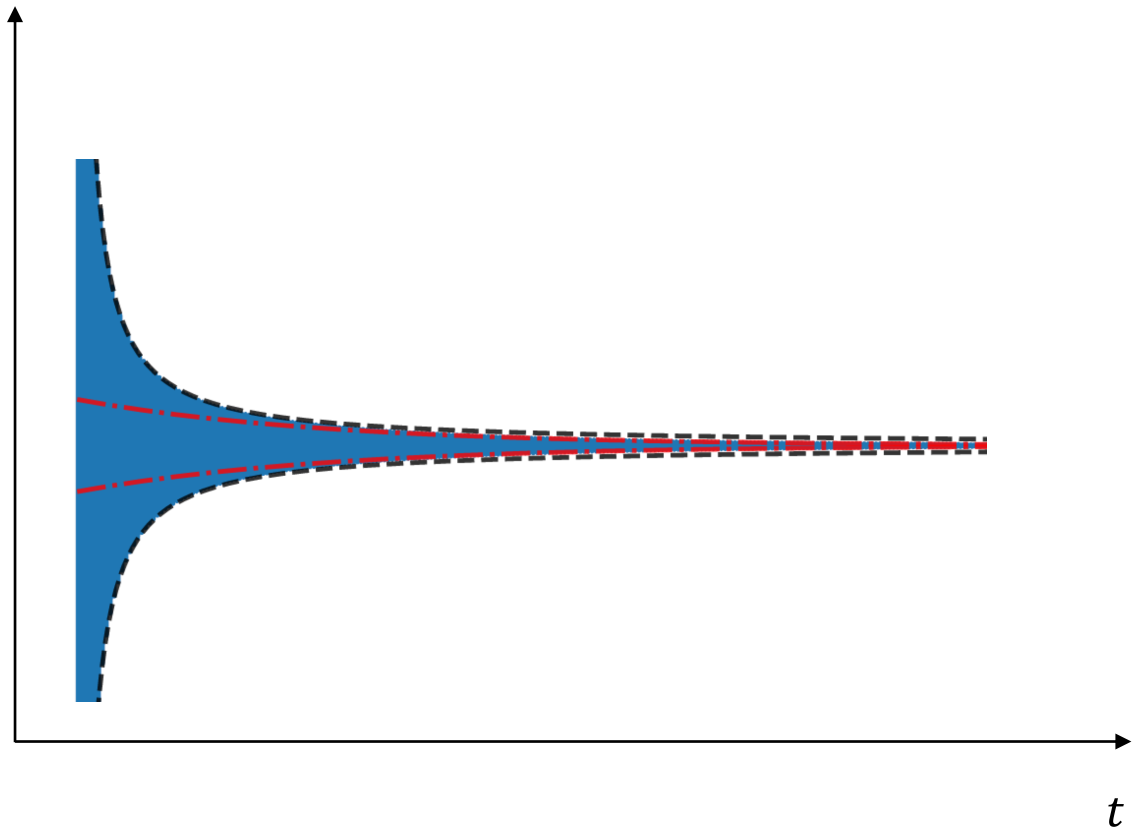


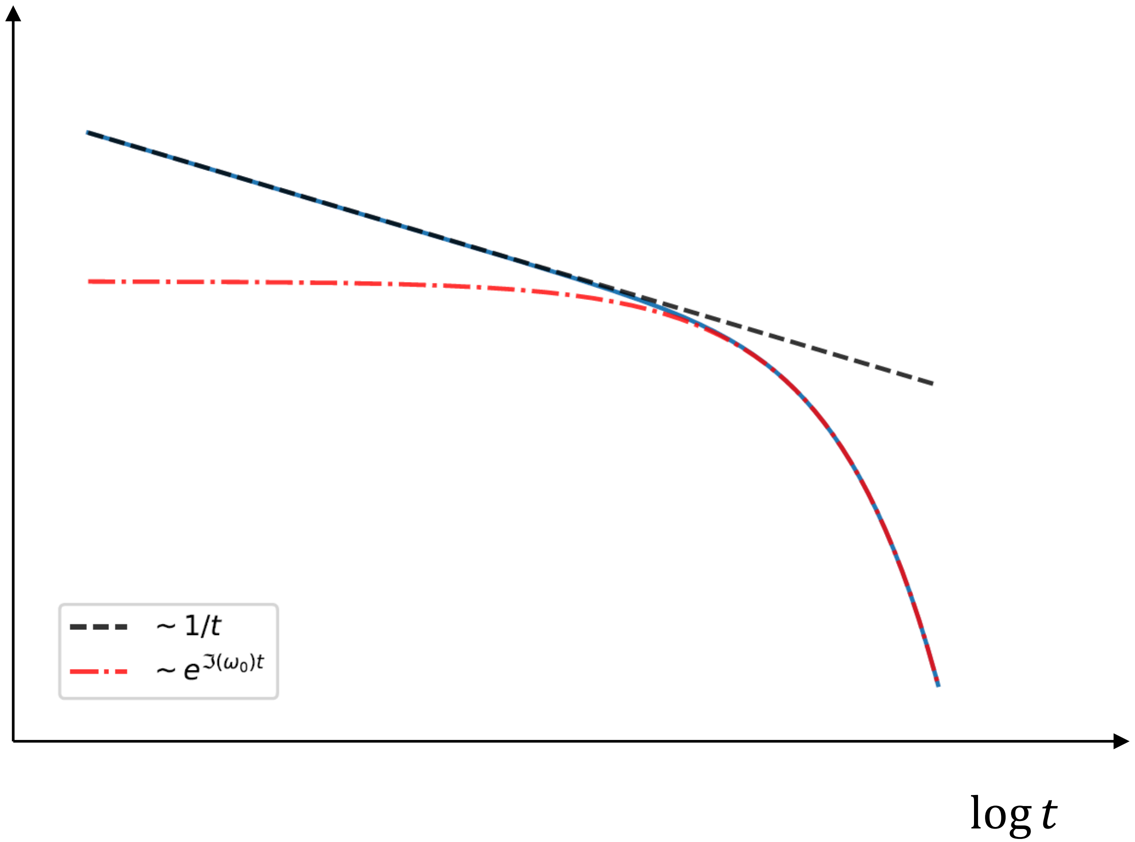
‘Zero-damped’ QNMs:

$$\omega_n \approx \frac{m}{2M} - i2\pi T(n+h)$$

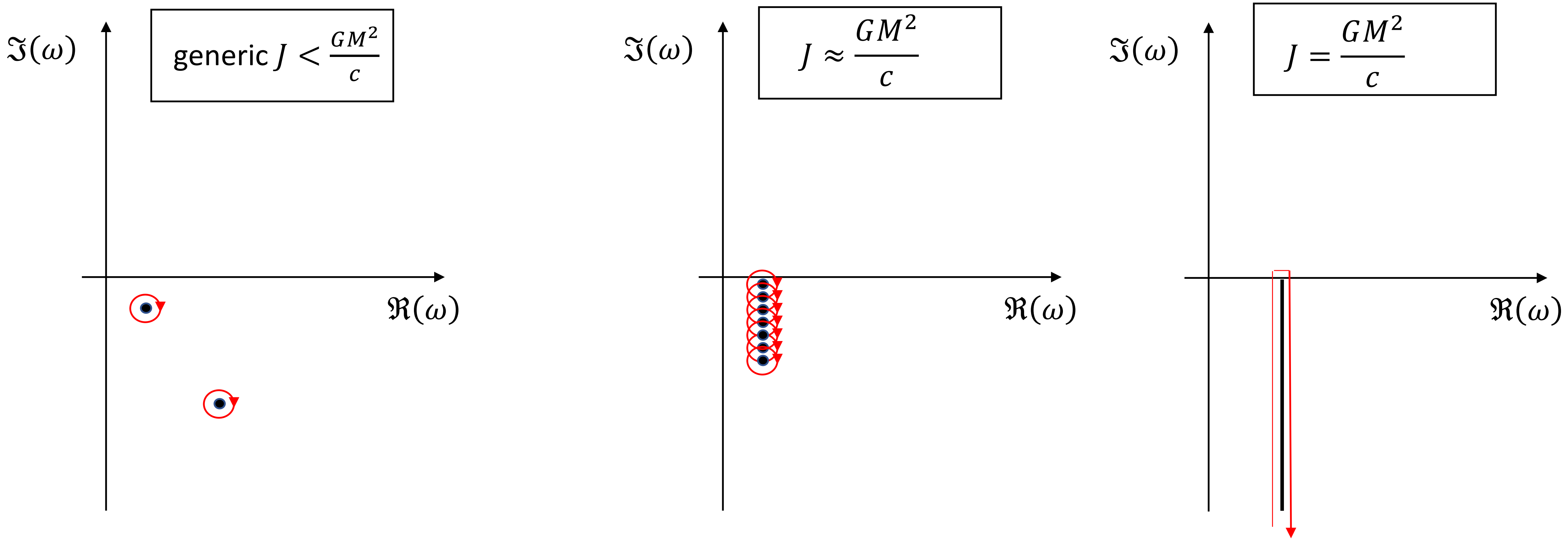
$$T \to 0, \text{ as } J \to \frac{GM^2}{c}$$

Powerlaw ringdown





$$\int_C A(\omega)e^{-i\omega t}d\omega \sim \sum_n a_n e^{-i\omega_n t} \quad \leftarrow \quad \int_C A(\omega)e^{-i\omega t}d\omega \sim \frac{a\, e^{-i\Re(\omega)t}}{t^{\alpha}}$$



Conclusion

Most astrophysical objects have a rich spectrum of characteristic modes, black holes are boring by comparison … ?

More concretely:

- Still work to be done computing QNMs at high spin beyond GR, BHs
- Exotic compact objects cannot (‘cheaply’) mimic BH ringdown with impunity (from 3G onwards?)
- Distinguish very high spin black holes from very, very, high spin black holes

The End