

Pascal Vanlaer (Université Libre de Bruxelles / IIHE) Belgium-Nederland-Deutschland doctoral school of particle physics - Spa, 2-13 September 2019

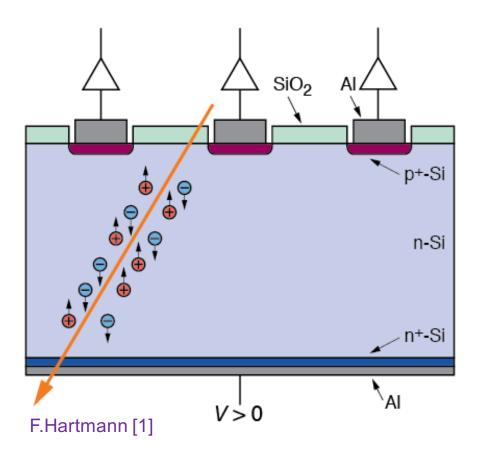




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Working principle



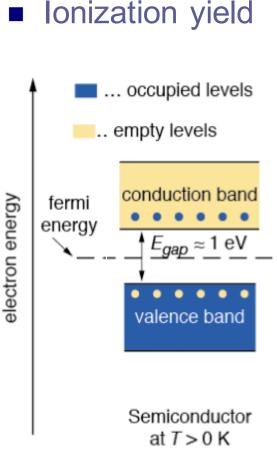
- Silicon wafer patterned with strip or pixel implants
 - Readout electronics connected to implants
- Each p-n junction is reversepolarized
 - Space charge region is depleted from mobile charge carriers and acts as an ionization chamber
- Ionizing particle produces electron-hole pairs
- Electrons and holes drift and induce signals in readout electronics

Attractive features

- About 3.10⁴ electron-hole pairs created by Minimum-Ionizing Particles (MIP) in 300µm thick wafer; detectable with low-noise electronics
- □ Fast signal (<10ns) thanks to high e- and hole mobility
- □ Implant pattern sizes allow excellent hit position resolution (5~10µm)
- □ Many different implant geometries are possible
- Front-end electronics can, in principle, be implemented in the same wafer as the sensor
- □ Can be made radiation-resistant

Less nice features...

- Sensors, mechanical structures, readout cables, cooling tubes inside tracking volume -> high x/X₀ "material budget"
- Cost per channel is high... but the driving cost is often not the sensors, it's the number of channels
 - it's the price to pay for high granularity



F.Hartmann [1]

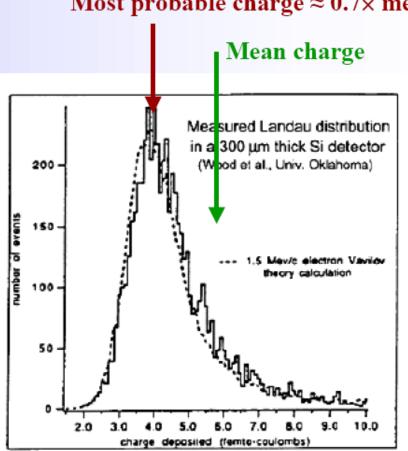
- Signal in silicon benefits from the small energy difference between the valence and conduction bands, also called the band gap, E_{gap}
- For diamond/silicon/germanium E_{gap} = 5.5 eV/1.12 eV/0.66 eV
- The average energy loss required from a charged particle to produce an electron-hole pair is measured to be: diamond/silicon/germanium: 13 eV/3.6 eV/2.9 eV
 - ~10 times smaller energy to release an electron than in a gas (~30eV)
- ~10³ times denser material: $\frac{dE}{dx}\Big|_{Si} = 3.87 MeV/cm$
- → ~10⁴ more e-hole pairs created per cm (Si) than in gas

Energy loss distribution for MIPs

 Mean energy loss dE/dx (Si) = 3.88 MeV/cm \Rightarrow 116 keV for 300µm thickness

- Most probable energy loss ≈ 0.7 ×mean \Rightarrow 81 keV
- 3.6 eV to create an e-h pair \Rightarrow 72 e-h / μ m (mean) \Rightarrow 108 e-h / μ m (most probable)
- Most probable charge (300 μm)

≈ 3.6 fC ≈ 22500 e



Most probable charge $\approx 0.7 \times$ mean

M.Moll [2]

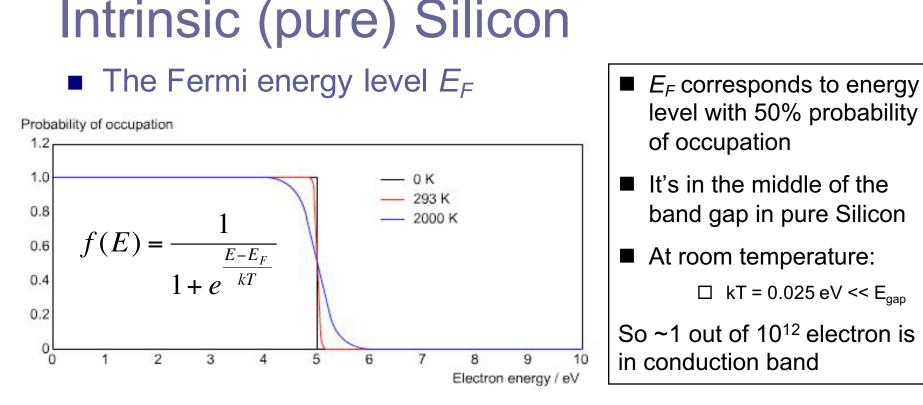
C. D'Ambrosio, T. Gys, C. Joram, M. Moll and L. Ropelewski

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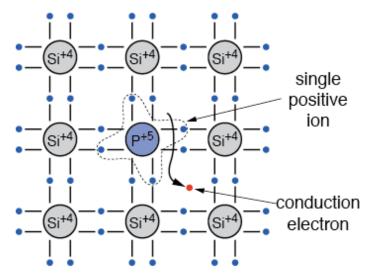
Concentration of electrons (and holes): $n_i \sim 1.45 \ 10^{10} \text{ cm}^{-3}$ at T = 300 K

In a detection cell of 300µm thickness x 10cm strip length x 100µm strip pitch: \sim 4.5 10⁷ free charge carriers >< \sim 3.3 10⁴ e-hole pairs from a MIP

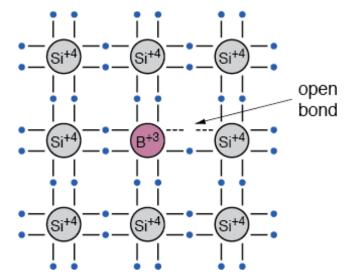
To avoid recombination of the signal with the free carriers, **reverse-polarized** (depleted) *p-n* junctions are used

Doping

A semi-conductor is called extrinsic when the equilibrium between negative and positive free charges is broken. There are thus majority charge carriers and minority charge carriers. This is realized by doping, i.e. introducing a well-chosen impurity in the crystal in small parts (doping fraction 10⁻⁸ – 10⁻¹¹). For Silicon:



Dopant of the Vth group (e- donor) 5th electron weakly bound, easily freed majority carriers: electrons **n-type**



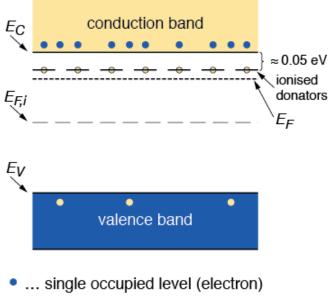
Dopant of the Illrd group (e- acceptor) Si electron easily captured majority carriers: holes **p-type**

Energy levels after doping

n-type

The energy level of the donor is just below the edge of the conduction band
At room temperature most donors are ionized, putting electrons in the conduction band

• The Fermi level E_F moves up



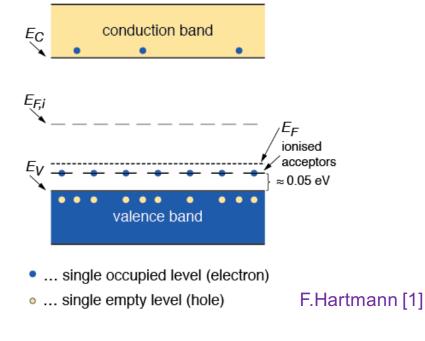
... single empty level (hole)

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p-type

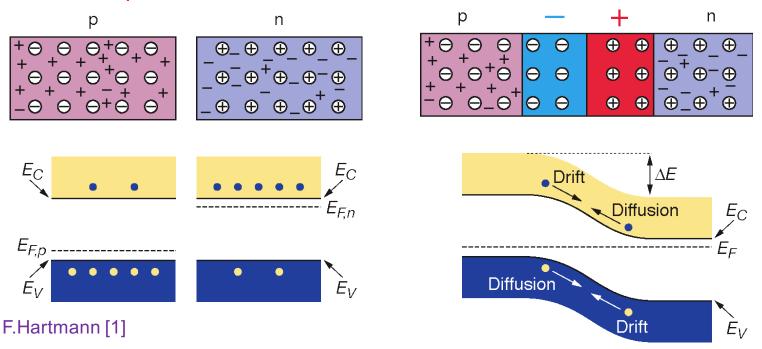
• The energy level of the acceptor is just above the edge of the valence band

- At room temperature most levels are occupied by electrons leaving holes in the valence band
- The Fermi level E_F moves down

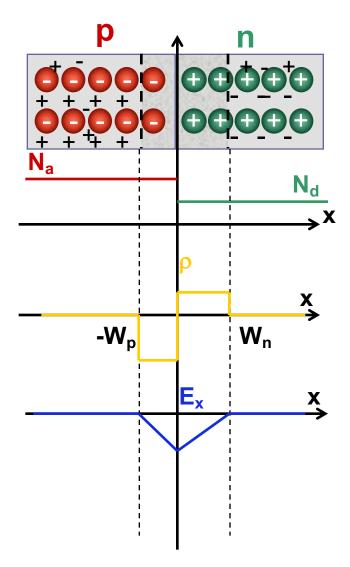


The p-n junction

- At the interface between an n-type and a p-type semiconductor, the difference in carrier concentrations cause diffusion of majority carries to the other material until thermal equilibrium is reached. At this point the Fermi levels are equal.
- The remaining fixed ions create a space charge and an electric field stopping further diffusion.
- The stable space charge region is emptied from mobile charge carries and is called the depletion zone.



The abrupt junction model



 N_a , N_d concentrations of acceptors and donors in the p- et n-type silicon parts

In the depleted zone, the charge density writes: $\rho(x) = -eN_a$, for x<0; $\rho(x) = eN_d$, for x>0

It is essentially zero elsewhere (electric neutrality)

By charge conservation:

 $N_aW_p = N_dW_{n,with} W_p$ and W_n the depleted depths in each silicon type

Integrating Gauss's law

$$\nabla . \vec{E} = \frac{\rho}{\varepsilon}$$

Leads to the expression of the electric field:

$$\mathbf{E}_{\mathbf{x}} = -\frac{\mathbf{d}\mathbf{V}(\mathbf{x})}{\mathbf{d}\mathbf{x}} = \mathbf{e}\frac{\mathbf{N}_{\mathbf{d}}}{\varepsilon}(\mathbf{x} - \mathbf{W}_{\mathbf{n}}), \text{ for } \mathbf{0} < \mathbf{x} < \mathbf{W}_{\mathbf{n}}$$
$$-\mathbf{e}\frac{\mathbf{N}_{\mathbf{a}}}{\varepsilon}(\mathbf{x} + \mathbf{W}_{\mathbf{p}}), \text{ for } - \mathbf{W}_{\mathbf{p}} < \mathbf{x} < \mathbf{0}$$

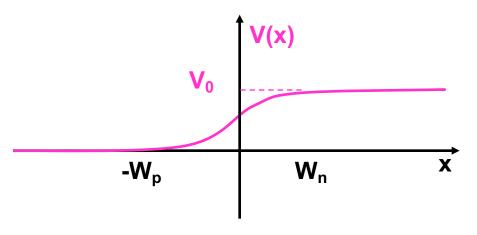
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The abrupt junction model

Integrating the expression of $E_x(x)$ leads to a parabolic expression for the potential V(x); the integration constants are fixed by imposing continuity at x=0, and considering:

- electric neutrality: $N_aW_p = N_dW_n$
- The diode potential barrier $V_0 \approx E_{gap}/e$



Unbiased junction:

diode potential barrier $V_0 \approx 0.6V$ Width of depletion region \approx a few tens of micrometers for N_a , N_d ~ $10^{12}/cm^3$

Reverse-biased junction:

Positive voltage $V_0 >> 0.6V$ on n-side increases potential barrier

Small current due to minority carriers thermally generated in depletion region

so-called leakage current

Reverse-biasing the junction

Imposing an external potential V₀ >> 0.6 V one finds: $V_0 = \frac{e}{2\epsilon} (N_d W_n^2 + N_a W_p^2)$

And using
$$N_a W_p = N_d W_n$$
 one gets: $V_0 = \frac{e}{2\varepsilon} W_n^2 N_d \left(1 + \frac{N_d}{N_a}\right)$

In practice, a p-n junction is produced from a homogeneous substrate (the bulk), for instance n-type. The p-type region is added by diffusing the appropriate dopant (the implant) in the n-type substrate. The depth of the diffusion is small compared to the thickness of the bulk. If one wishes a thick depleted region, it should extend in the bulk, implying in this example $N_a >> N_d$

• typical values: $N_a \sim 10^{15}/cm^3$, $N_d \sim 10^{12}/cm^3$

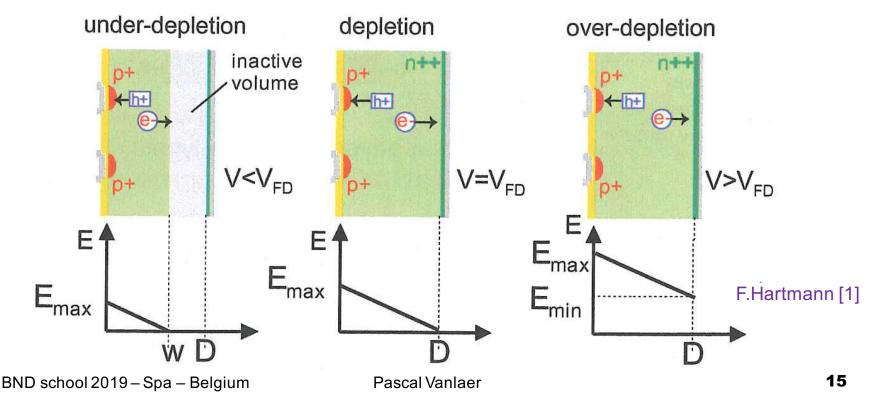
In this case:

$$\mathbf{W} \sim \mathbf{W}_{n} = \sqrt{\frac{2\varepsilon \mathbf{V}_{0}}{\mathbf{e} \mathbf{N}_{d}}}$$

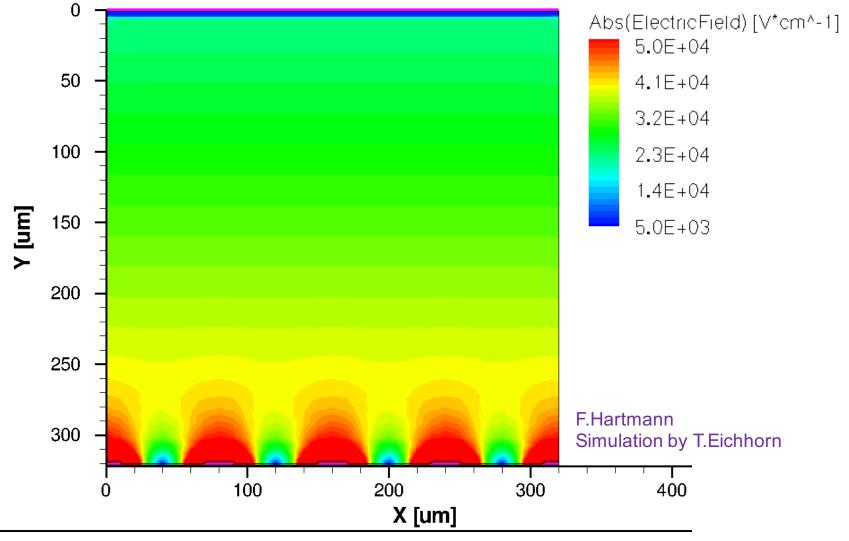
At 300µm, the voltage for full depletion $V_{FD} \approx 150V$ for an n-type bulk of ~2 k Ω .cm resistivity, i.e. with a doping concentration of 2.2 10^{12} /cm³ (exercice: check it)

Field configurations

- There is a linearly-increasing E field across the sensor bulk
- The maximum is on the implants side
 - The maximum grows with bias voltage
- At V<V_{FD} there is an undepleted zone on the backside
- To collect e- and holes from the entire thickness, the E field has to be non-zero across the whole bulk -> apply V>V_{FD} (over-depletion)



Field configurations



Measuring the depletion voltage

□ By measuring the leakage current (not reliable):

Ideally, the leakage current due to e-hole pairs thermally created in the depletion region is proportional to the depleted volume:

$$I_{leak} {\sim} W {\sim} \sqrt{V_0}$$

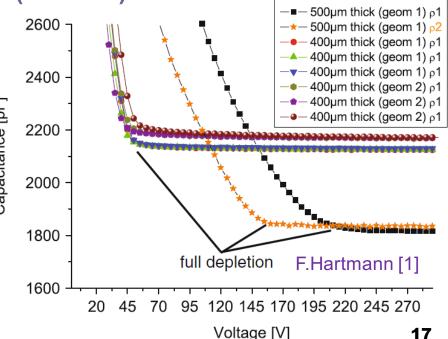
In practice, trapping of charge carriers at low voltage, and charge amplification at high voltage, make this method unreliable

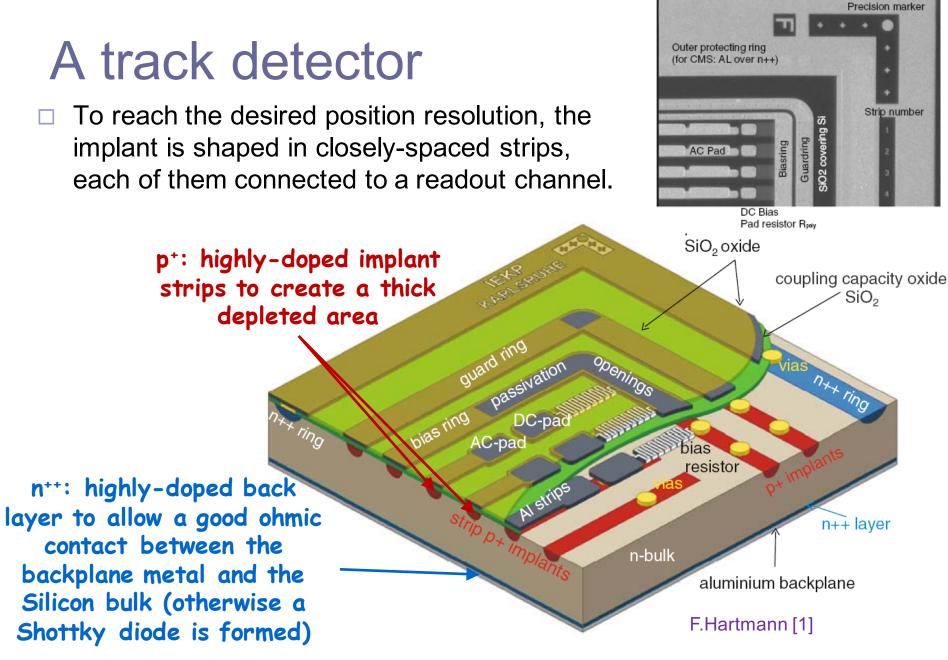
□ By measuring the capacitance (reliable):

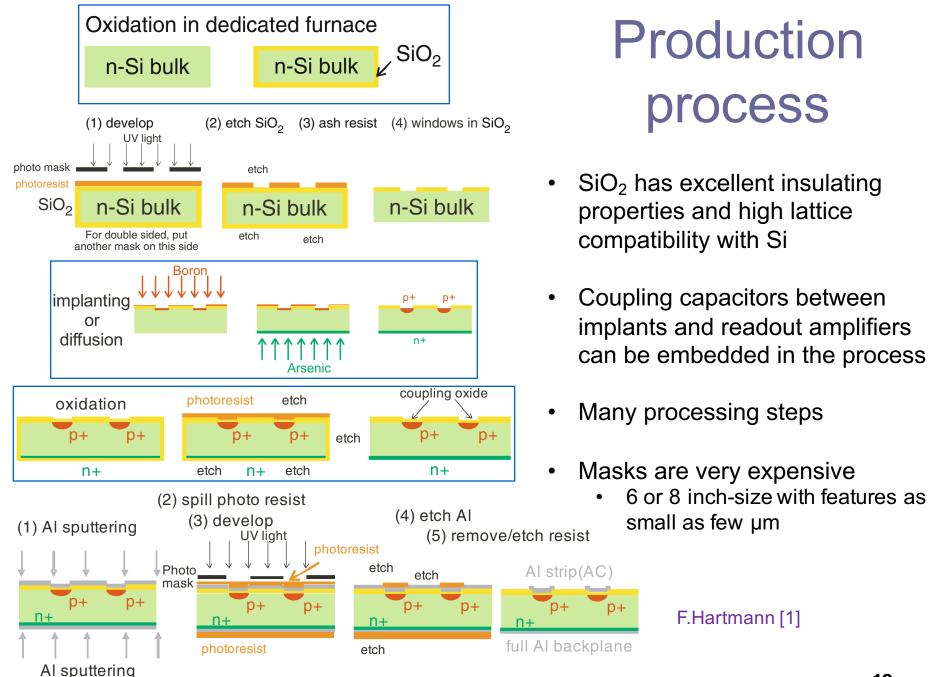
The depleted sensor acts as a //-plate capacitor of variable thickness *W*:

A ... detector surface

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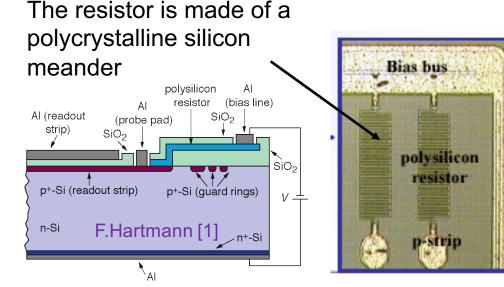


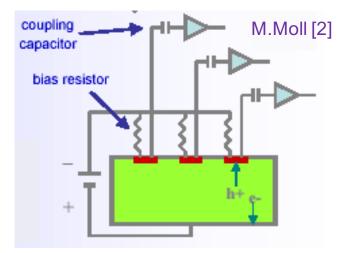




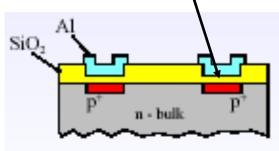
Coupling of the readout amplifiers

- To read the charge induced on each strip, the strips must be insulated from each other by a large resistance (10 MΩ), and the amplifier is coupled through a capacitor that filters the leakage current
- These elements are integrated to the substrate.

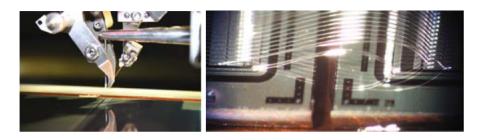


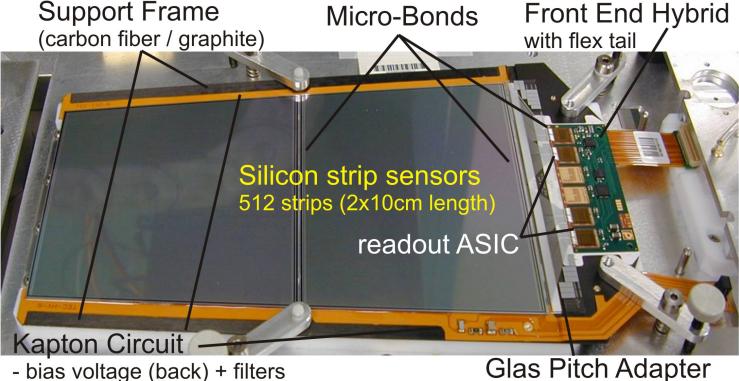


The capacitor is made by a SiO_2 dielectric layer between the p⁺ strip and the readout contact χ



From sensor to module

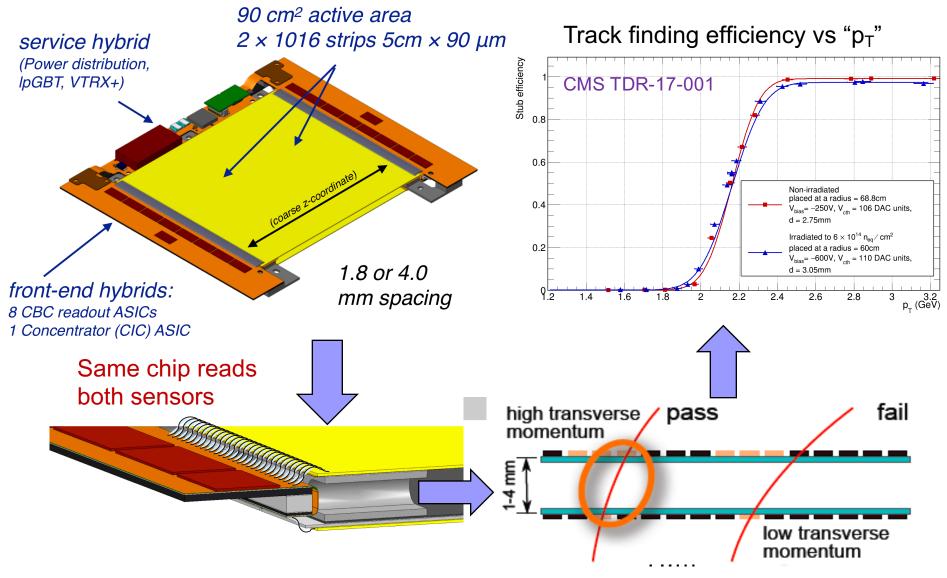


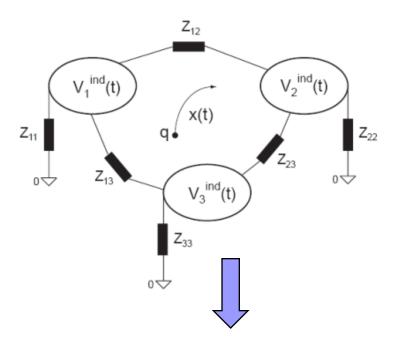


- bias voltage (back) + filters
- temperature sensor

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CMS upgrade trigger-enabled tracking module

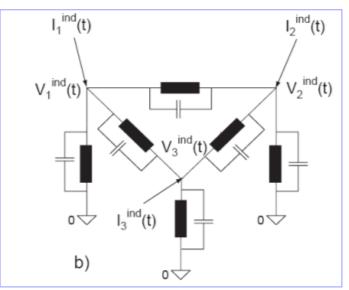




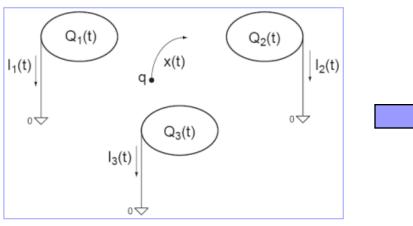
Signal induction (from slides by W.Riegler [3])

What are the currents induced by the drift of eand holes on a network of electrodes (ovals) connected to each other by interstrip and backplane capacitances and resistances (Z_{ij}) and front-end electronics (triangles) ?

These act as current sources into the impedance network

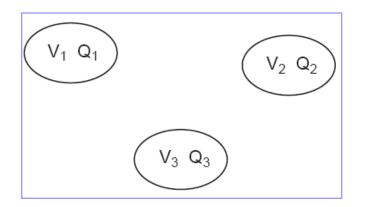


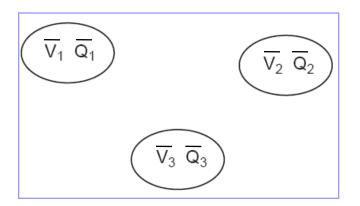
Drift of e- and holes (q with position x(t)) induce currents $I_i(t)$ on the electrodes



Signal induction

Placing charges on metal electrodes results in certain potentials of these electrodes. A different set of charges results in a different set of potentials.





The reciprocity theorem states that:

$$\sum_{n=1}^{N} Q_n \overline{V}_n = \sum_{n=1}^{N} \overline{Q}_n V_n$$

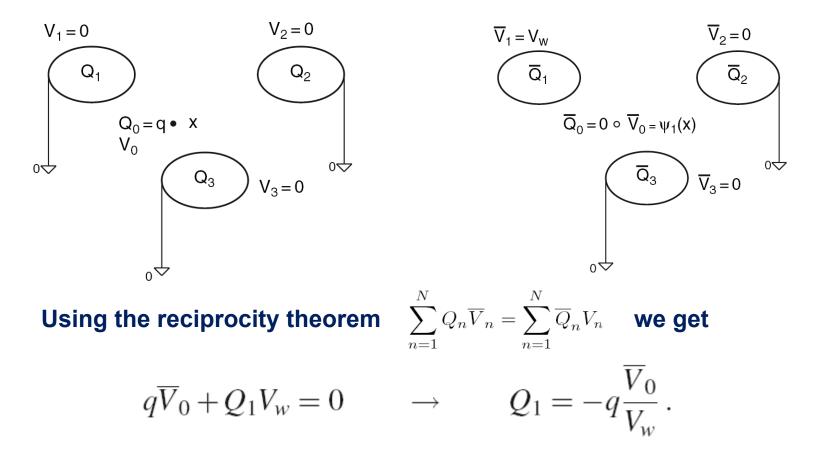
This can be shown using one of Green's identities (see backup).

By choosing $\overline{V_n}$ and $\overline{Q_n}$ wisely, we can get the charge Q_i on the electrode of interest

Signal induction

We assume three grounded electrodes and a point charge in between. We want to know the charges induced on the grounded electrodes. We assume the point charge to be an very small metal electrode with charge q, so we have a system of 4 electrodes with $V_1=0$, $V_2=0$, $V_3=0$, $Q_0=q$.

We can now assume another set of voltages and charges where we remove the charge from electrode zero, we put electrode 1 to voltage V_w and keep electrodes 2 and 3 grounded.



Signal induction

The voltage $\overline{V_0}$ is the voltage at the place x of the moving point charge q, in the second electrostatic state, where the only electrode with non-zero voltage is the electrode of interest (electrode 1).

Let's call the potential field in this configuration $\psi(x)$, the weighting potential of electrode 1. We have:

$$Q = -\frac{q}{V_w} \,\psi(\vec{x})$$

Since the charge is moving, \vec{x} depends on time t: $Q(t) = -\frac{q}{V_w} \psi(\vec{x}(t))$

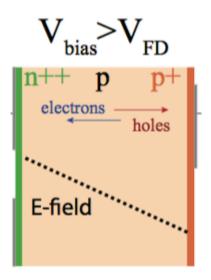
Inducent current, *I(t)* taken positive when exiting the electrode:

$$I(t) = -\frac{dQ}{dt} = \frac{q}{V_W} \vec{\nabla} \psi(\vec{x}(t)) \frac{d\vec{x}(t)}{dt} = -\frac{q}{V_W} \cdot \vec{E_W}(\vec{x}(t)) \cdot \vec{v}(t)$$

where $\vec{v}(t) = \frac{d\vec{x}(t)}{dt}$ is the speed of the moving charge, and $\overrightarrow{E_W}(\vec{x}(t))$ is the weighting field of electrode 1. This expression of I(t) is called Ramo's theorem.

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Exercise



Let's compute the collection time of electrons and the expression of the current induced by their motion, in the following situation:

- an overdepleted detector of p-type bulk
- d = 300µm active depth
- Electrons are injected at a single point at the p⁺-backside
- The readout electrode is on the n⁺⁺-side
- $E_{min} = A = 2.10^4 V.cm^{-1}$
- $E_{max} = 5.10^4 V. cm^{-1}$
- Electrons mobility: $\mu_e = \frac{v_e}{E} = 1400 cm^2 s^{-1} V^{-1}$

$$p^{+} = p^{+} = p^{++} = p^{++} = kx + A$$

$$|\vec{E}(x)| = kx + A$$

$$|\vec{E}_{w}| = dk$$

$$|\vec{w}| = dk$$

$$|\vec{w}| = \sqrt{k}(x) = p^{-} E(x) = p^{-} (kx + A)$$

$$x(t) = \int_{0}^{t} \sqrt{k}(x) dt = \int_{0}^{t} p^{-} (kx(t) + A)$$

$$dx(t) = p^{-} kx(t) + p^{-} A$$

$$dt$$

$$Selumm: \quad x(t) = \frac{A}{k} \begin{bmatrix} e^{-t} \\ -1 \end{bmatrix}; \quad x(0) = 0 \text{ cm}$$

$$t) \text{ collection hims: } t = \frac{A}{p^{-} k} \ln \left(\frac{kd}{A} + t\right)$$

$$k = \frac{5 \cdot b^{4} - 2 \cdot b^{4}}{300 \cdot 10^{-4}} \begin{bmatrix} v \cdot cm^{-4} \end{bmatrix} = 10^{6} \text{ V. cm}^{-2}$$

$$t = \frac{1}{1400 - 10^{6}} \ln \left(\frac{10^{6} \cdot 3 \cdot b^{-2}}{2 \cdot b^{4}} + 1\right) = 6, 5 \cdot 10^{-10} \text{ cm}$$

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b) Induced current:

$$V_W = 0$$
 F_W
 $V_W = 1$
 $V_W = 0$
 $V_W = 0$
 $V_W = 0$
 $V_W = 1$
 $V_W = 0$
 $V_W = 0$
 $V_W = 1$
 $V_W = 0$
 $V_W = 0$

Pulse shapes

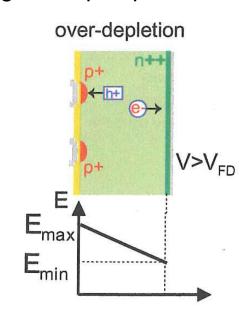
 $I(t) = -\frac{q}{V_W} \cdot \overrightarrow{E_W}(\vec{x}(t)) \cdot \vec{v}(t)$

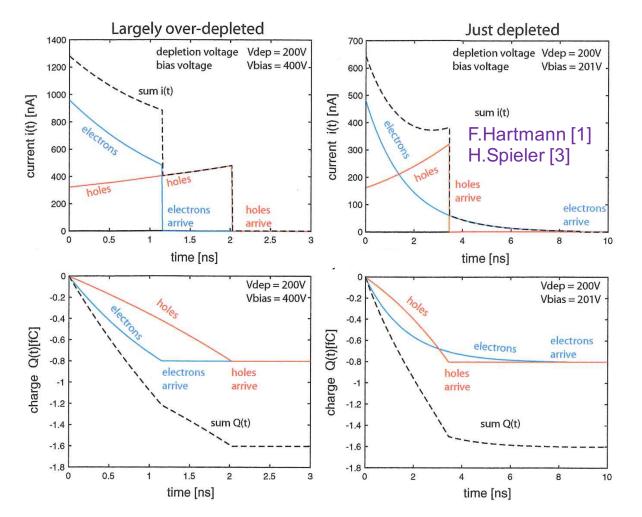
The current induced is large:

- 1. when the speed of the e- or holes is large, i.e. when the drift field is large
- 2. when the weighting field is large

Simulation:

300 µm diode; n-type bulk 10000 e-h pairs injected at 150 µm depth Signal on p-implant





Position resolution

The interstrip distance or "**pitch**", p, is typically in the range 20-200 μ m.

Binary (digital) readout

If the strips are read out in a binary way (signal above or below threshold) and if a single strip carries a signal, the hit position is the middle of that strip, and the position resolution is:

 $\sigma = \frac{\mathbf{p}}{\sqrt{12}} \simeq 5 - 50 \mu \text{m}$ (R.M.S. of a uniform distribution of width p)

Analog readout

- The amplitudes of the signals in neighbouring strips can be used to compute the center-of-gravity of the deposited ionization charge
- For tracks impinging at normal incidence, this requires specific design, as the diffusion of charges in silicon is small
 - A few tens of µm for 300 µm of drift

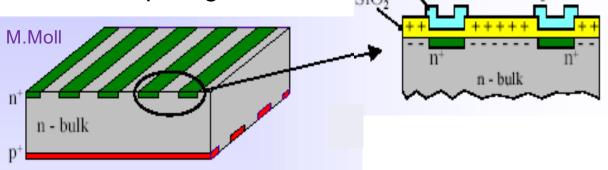
n-strips in a p-bulk

 \Box Patterning the back side gives 2nd coordinate (in principle)

n-in-p sensors are more radiation-hard

□ Caveat:

 typical good-quality oxide layer has fixed positive charges, creating a conducting electron channel under the oxide, shorting the strips together

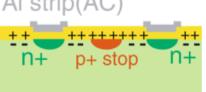


F.Hartmann [1] M.Moll [2] G.Lutz^[4]

\Box Solution:

Al strip(AC)

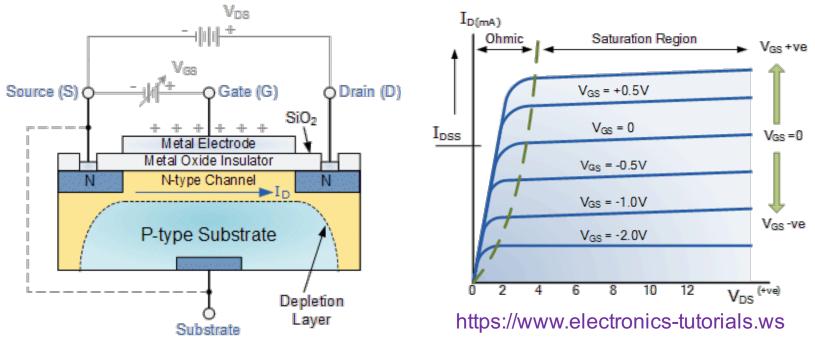
p⁺ implants (p-stops) create fixed negative charges repelling electrons, insulating the strips



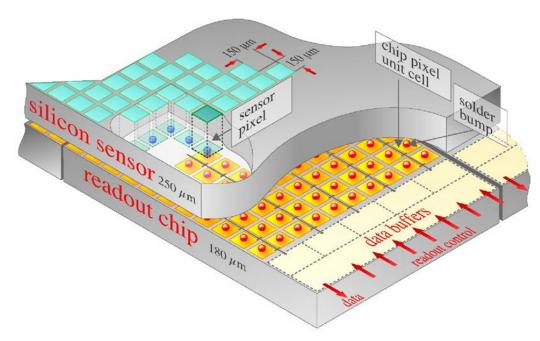
n-strips in a p-bulk

The structure is very similar to the widely-used Metal-Oxide-Silicon Field-Effect Transistor (MOSFET)

- Without p-stops, our n-in-p sensor is similar to a depletion-mode (normally-ON) N-channel MOSFET ⁽²⁾
- MOSFETs also appear in integrated readout amplifiers, and in monolithic detectors combining sensor and amplifiers in same Silicon wafer (see later)

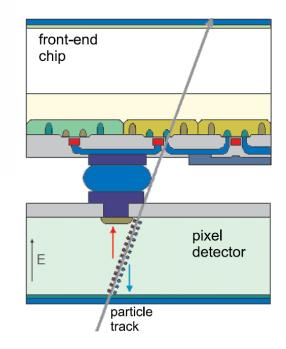


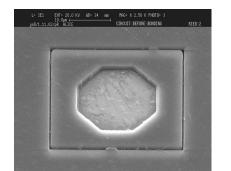
□ Pixelated sensor and readout chip



Difficulty: connect the electronics to each pixel

Solution: bump bonding





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15 µm



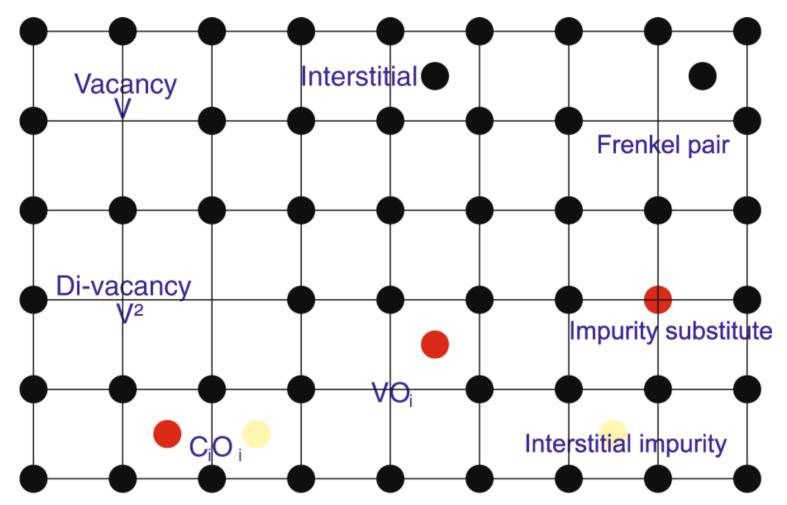
Radiation Jamage

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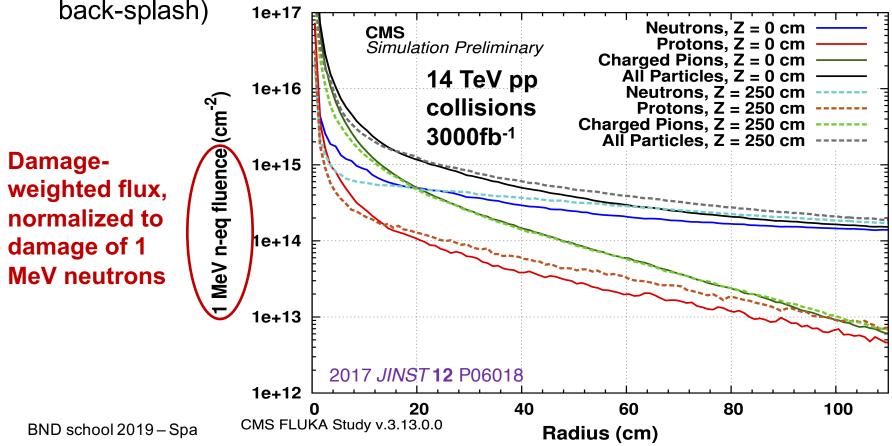
Radiation damage

□ Radiation produces defects in Silicon lattice

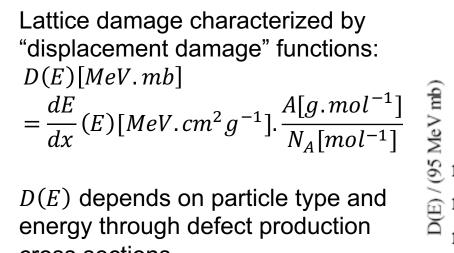


Radiation damage

- □ A concern at high-luminosity experiments, notably at HL-LHC
- Damage mostly due to neutrons, protons and charged pions, of very different energy spectra
- Particle fluxes are normalized to 1 MeV neutrons, characteristic of emission by detector activated by primary hadrons (e.g. calorimeter back-splash) = 10:17



Non-Ionizing Energy Loss (NIEL) model

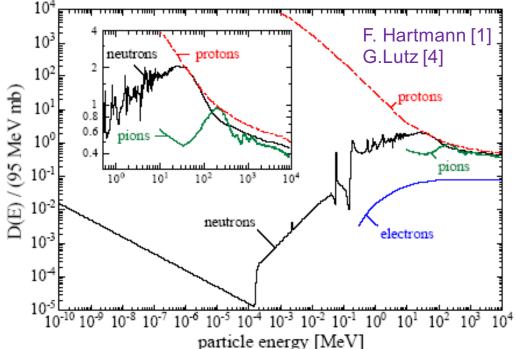


cross sections

 \rightarrow Neutrons of 1MeV as reference particles "equivalent fluence Φ_{eq} "

Irradiation "hardness" factor:

$$\kappa = \frac{\int D(E)\phi(E)dE}{95MeVmb\cdot\Phi} = \frac{\Phi_{eq}}{\Phi}$$

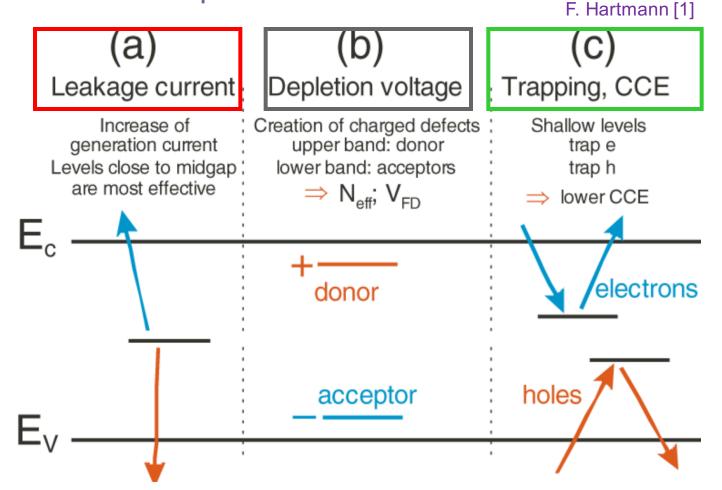


Note the huge difference: in Silicon:

- Ionization (MIP): $\frac{dE}{dx} = 3.87 MeV/cm$
- NIEL (1 MeV neutron): $\frac{dE}{dx} = 5.0 KeV/cm$

Radiation damage

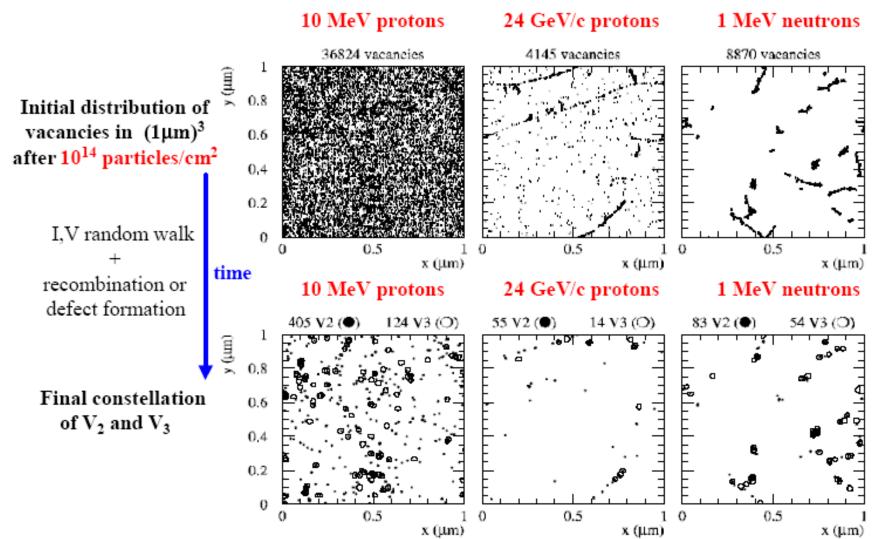
Defects introduce extra energy levels in band gap, with different consequences:



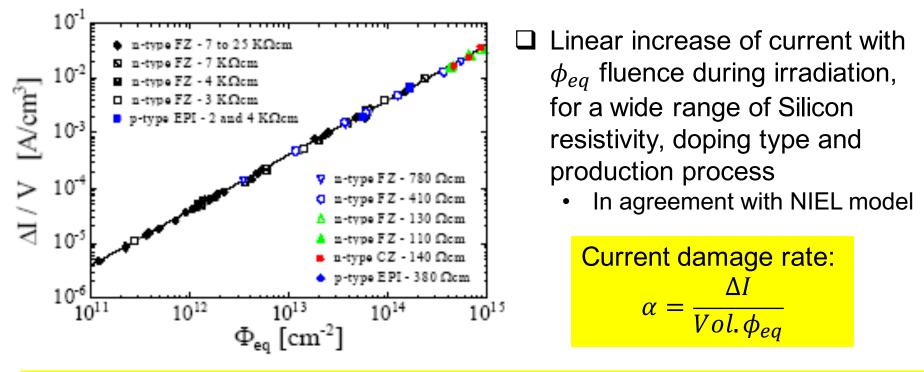
Annealing

Defects diffuse, migrate and combine: annealing

□ Temperature and time-dependent !!



Leakage current



Strongly temperature dependent $\rightarrow \alpha(T)$ 20° C: $\alpha = 4,00 \cdot 10^{-17}$ A/cmCurrent: I $\propto \exp(-E_{eff}/2k_{B}T)$ -10° C: $\alpha = 1.86 \cdot 10^{-18}$ A/cm

Our concerns:

- Electronics noise due to current fluctuations at the input of the amplifiers
- Joule effect in sensor: $P_J = V.I$ increases exponentially with sensor temperature, while cooling capacity only proportional to $(T_{sensor}^o T_{coolant}^o)$

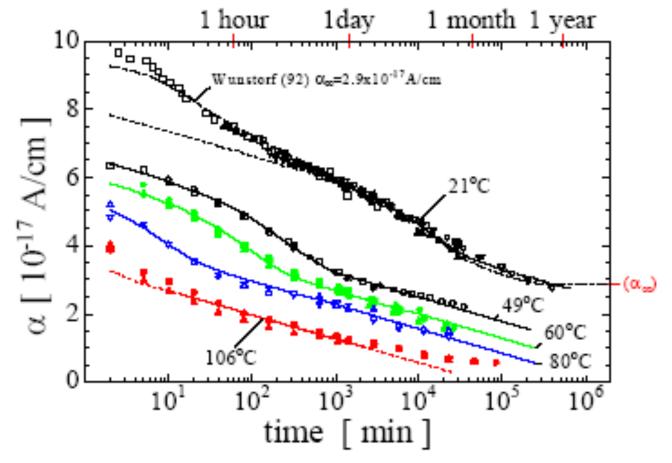
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Annealing of the leakage current

Leakage current decreases with time

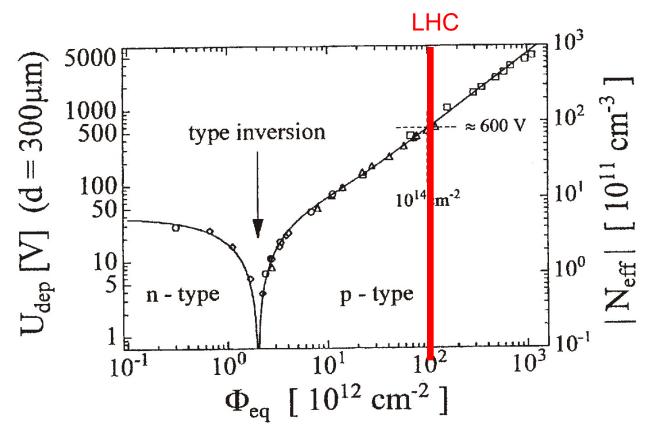
Decrease is faster at higher temperature

Can be used to some extent to mitigate radiation damage



Depletion voltage

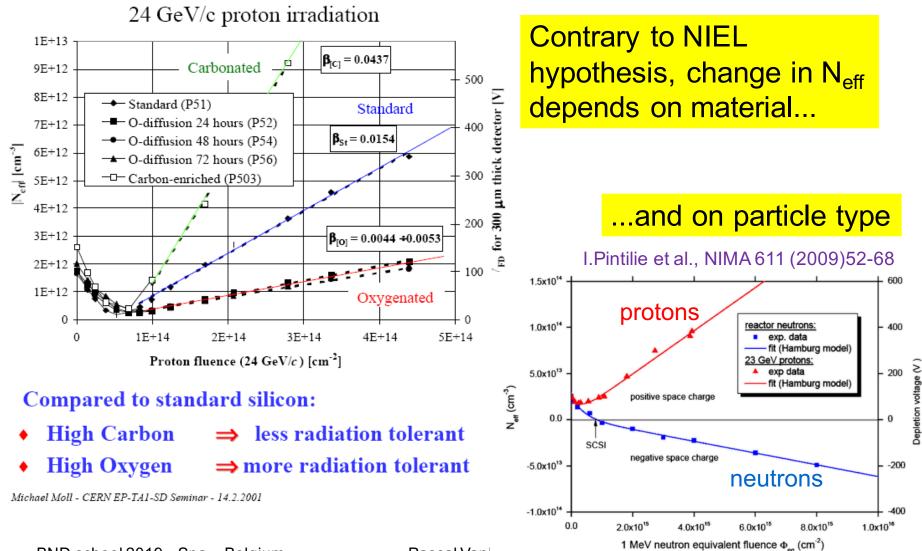
- □ Most irradiation damages in Silicon behave like acceptors
- ❑ Starting with n-type Si → p-type: "type inversion" also called Space Charge Sign Inversion (SCSI)



For LHC fluences (2.4·10¹⁴n_{1MeV}/cm²) high full depletion voltages are expected !

Effective doping concentration N_{eff}

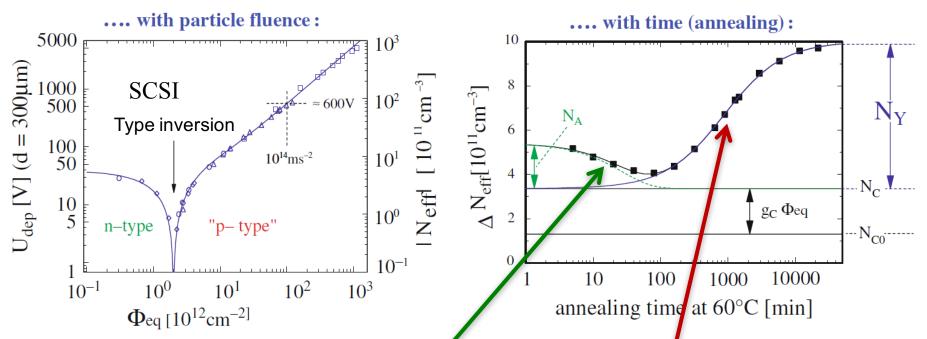
Influence of Carbon and Oxygen concentration



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N_{eff} and annealing



Different defects evolve differently and with different time constants

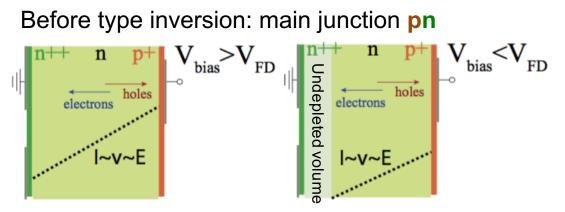
- Short-term beneficial annealing; long-term reverse annealing
- The evolution also depends on temperature
 - Annealing can be slowed down or even "frozen" at -10° C

Bottom line:

- 1) Make irradiation tests with both neutrons and protons
- 2) Characterize every new material (also wrt. annealing of N_{eff})
- 3) Beware of reverse annealing

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Where is the depletion zone ??



After type inversion: main junction **np** (remember n-bulk \rightarrow p-bulk)

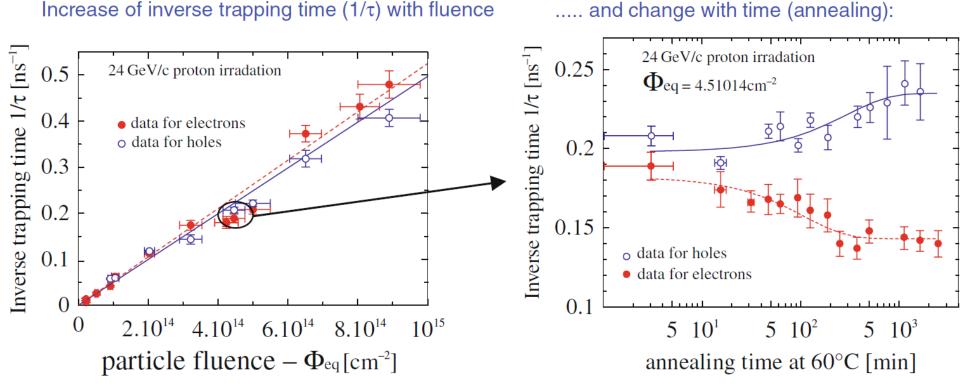
After type inversion:

- n-p is main pn-junction (not p-n anymore)
 - Depletion zone grows from other side
- On p-side: low field; and if underdepleted, the electrodes are screened (no signal induced)
 - P strips -in-n bulk sensors cannot be run under-depleted after type inversion
 - Present CMS strip sensors
- n-in-p or n-in-n can run partially depleted !
 - CMS pixel sensors (n-in-n)
 - HL-LHC pixel and strip sensors (n-in-p)

Trapping

- Charge carriers can be trapped by some type of defects (long-lived energy levels)
- Trapping time τ_{eff} changes with Φ_{eq} ; little annealing
- Different materials behave differently
- $\tau_{eff} (10^{15} n1 MeV/cm2) = 2 ns$: $x = (10^7 cm/s) \cdot 2 \cdot ns = 200 \mu m$
- $\tau_{eff} (10^{16} n1 MeV/cm2) = 0.2 ns$: $x = (10^7 cm/s) \cdot 0.2 \cdot ns = 20 \mu m$

Bottom line: at large Φ_{eq} , charge collection becomes a competition between drift and trapping \Rightarrow operate at high fields! (and with n-in-p sensors)



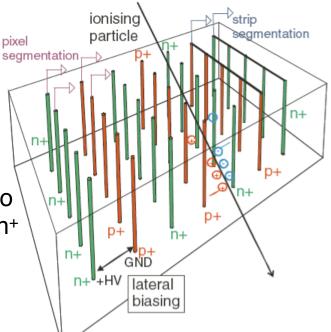


Sensor developments

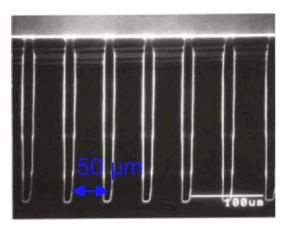
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3D sensors

- Non-planar implant structure
- Deep holes are etched into the Silicon and filled with n⁺ and p⁺ material
- Depletion is sideways.
 Distances between electrodes are small ⇒ depletion voltage can be much smaller and charge carriers travel much shorter distances
- Hence less vulnerable to trapping



DRIE – Deep Reactive Ion Etching



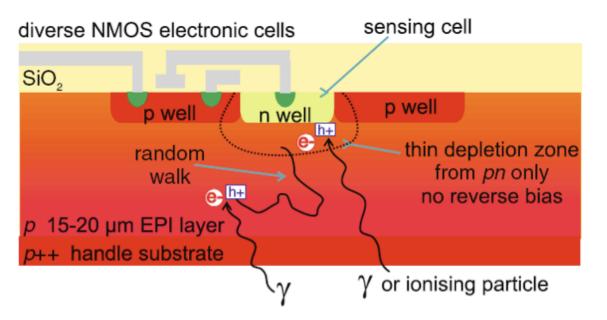


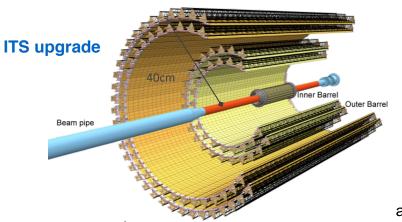
Very radiation tolerant detectors, first use in ATLAS Insertable B layer (IBL) 3.3 cm radius - improved impact parameter resolution

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Monolithic Active Pixel Sensors

• Electronics and sensor are integrated into the same Silicon wafer





Pros:

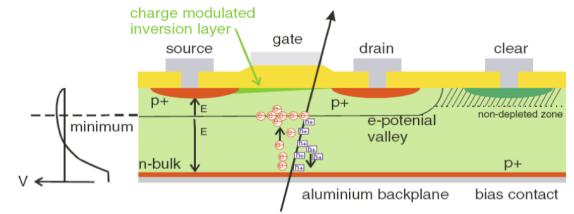
- No depletion voltage
- Very thin
- Very low noise
- In cell signal processing
- Very high resolution **Cons:**
- Small signal
- Slower
- Not radiation-hard

10m² in production for ALICE upgrade (LHC Run 3) 29µm x 27µm pixels 5µm x 5µm resolution

Y.Corrales Morales, EPS2019

DEPFET pixel detector

- Each pixel is a **p**-channel FET on top of an **n**-type bulk
- The bulk is fully depleted, with a potential minimum under the FET
- Electrons released by an ionizing particle accumulate in the potential minimum
 - "internal gate" effect: these electrons modulate the FET current
- Periodic clear needed



H.Ye, EPS2019



Belle II pixel detector

- 2 layers of DEPFET sensors @ r = 14(22) mm
- Sensitive area per module: L1: 12.5mm x
- 44.8mm, L2: 12.5mm x 61.44mm
- Sensor thickness: 75 μm, 0.21% X₀ per layer

Pascal Vanlaer

Pros:

• Large signal: Belle II:

$$g_q = \frac{\partial I}{\partial q} \approx 500 \, \frac{pA}{e^-}$$

• low power

Cons:

Slower; not radiation-hard

Summary

- □ Si detectors can provide fast (<10 ns) trackers of very high position resolution (down to a few µm) and of very high granularity (25µmx25µm or smaller)
- □ The main cost is often not the sensors
 - It's granularity (#electronics channels) that is expensive...
 - ...although, sensors beyond industry standards can be very costly
- □ Main disadvantages are:
 - a large amount of scattering material
 - sensitivity to radiation
- Radiation hardness is being improved for HL-LHC applications

Silicon track detectors

References:

[1] F.Hartmann, « Evolution of silicon sensor technology in particle physics », Springer, 2009.

[2] W.Riegler, "Particle detectors – part 4: Tracking with Gas and Solid State Detectors", 2008 CERN Summer student lectures,

http://indico.cern.ch/event/34199/material/slides/

[3] H.Spieler, "Semiconductor Detector Systems", Clarendon Press, Oxford, 2005.

[4] G.Lutz, "Semiconductor Radiation Detectors", Springer, 1999.

BACKUP

Reciprocity theorem $v_1 q_1$ $v_2 q_2$ $v_3 q_3$

Setting the three electrodes to potentials V_1 , V_2 , V_3 results in charges Q_1 , Q_2 , Q_3 . In order to find them we have to solve the Laplace equation

$$\Delta \varphi = 0$$

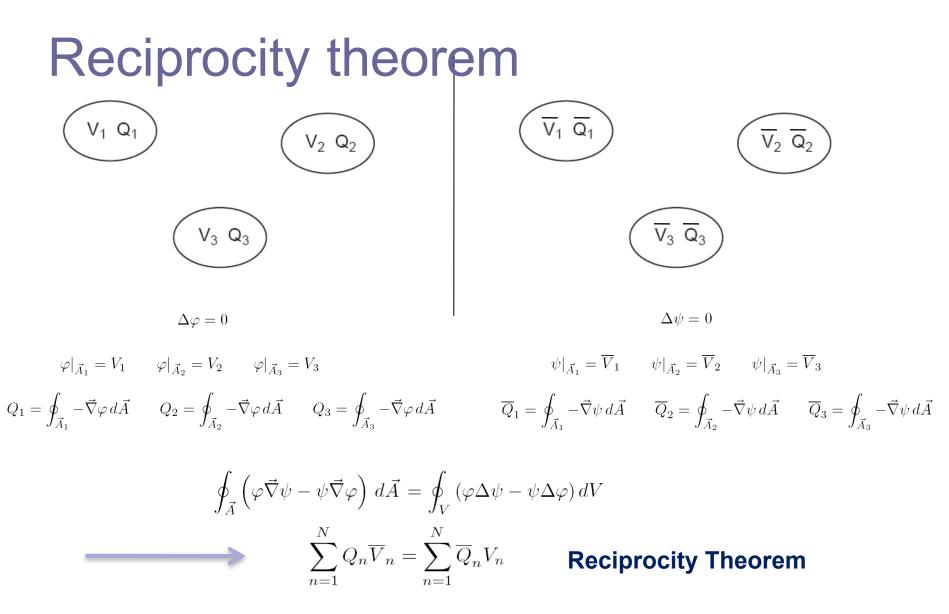
with boundary condition

$$\varphi|_{\vec{A}_1} = V_1 \qquad \varphi|_{\vec{A}_2} = V_2 \qquad \varphi|_{\vec{A}_3} = V_3$$

And the calculate

$$Q_1 = \oint_{\vec{A}_1} -\vec{\nabla}\varphi \, d\vec{A} \qquad Q_2 = \oint_{\vec{A}_2} -\vec{\nabla}\varphi \, d\vec{A} \qquad Q_3 = \oint_{\vec{A}_3} -\vec{\nabla}\varphi \, d\vec{A}$$

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It related two electrostatic states, i.e. two sets of voltages and charges

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