BND19 Electroweak exercises

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Ex. 1 Show that the Fermi Lagrangian lead to

$$\frac{\Gamma_{\uparrow}\left(\mu \to e^{-}\nu_{\mu}\bar{\nu}_{e}\right)}{\Gamma\left(\mu \to e^{-}\nu_{\mu}\bar{\nu}_{e}\right)} = \frac{1+\cos(\theta)}{2} \tag{1}$$

and

$$\frac{\Gamma_{\downarrow}\left(\mu \to e^{-}\nu_{\mu}\bar{\nu}_{e}\right)}{\Gamma\left(\mu \to e^{-}\nu_{\mu}\bar{\nu}_{e}\right)} = \frac{1-\cos(\theta)}{2} \tag{2}$$

where θ is the angle between the muon spin direction and the momentum of the electron.

Ex. 2 Show that the cross-section for lepton scattering grows like $s = (p_1 + p_2)^2$ at high energy where $p_{1,2}$ are the initial momenta.

Ex. 3 Show that the Dirac mass term only contains term mixing the left and right chiralities of the fermions.

Ex. 4 Show that the scattering amplitude of the longitudinal gauge bosons is

$$A \sim \frac{1}{m_W^2}(s+t) \tag{3}$$

at high energy in the $SU(2)_L \times U(1)_Y$ Lagrangian with a symmetry breaking mass term and no scalar doublet. Then add the contribution from the Higgs and show that it cancels this term.

Ex. 5 Do the symmetry breaking of $SU(2)_L \times U(1)_Y$ with a scalar triplet and derive the relation between the masses of bosons and the mixing Z-A angle.

Ex. 6 Compute the decay width of the Z in a pair of neutrino in the SM.

Ex. 7 Compute the coefficients of the Flavour Changing Neutral Current (FCNC) if there is only one quark doublet,

$$Q_L = \left(u_L, \cos\theta_c d_L + \sin\theta_c s_L\right)^T, \tag{4}$$

and show that they disappear when adding a second doublet,

$$Q_L = \left(c_L, -\sin\theta_c d_L + \cos\theta_c s_L\right)^T, \tag{5}$$

Ex 8 Show that the operators

$$iD_\mu \Phi^\dagger W_a^{\mu\nu} T^a D_\nu \Phi \quad and \quad iD_\mu \Phi^\dagger B^{\mu\nu} D_\nu \Phi \tag{6}$$

can be rewritten as two linearly independent combinations of operators of the Warsaw basis and therefore exchanged with the operators

$$\Phi^\dagger W_a^{\mu\nu} T^a B_{\mu\nu} \Phi \quad and \quad \left(\Phi^\dagger D_\mu \Phi\right)^* \Phi^\dagger D^\mu \Phi. \tag{7}$$

Compute the modification of the W boson mass and of the couplings of the photon and the Z boson due to the introduction of those two last operators on top of the SM Lagrangian.