One-Loop Effective Actions

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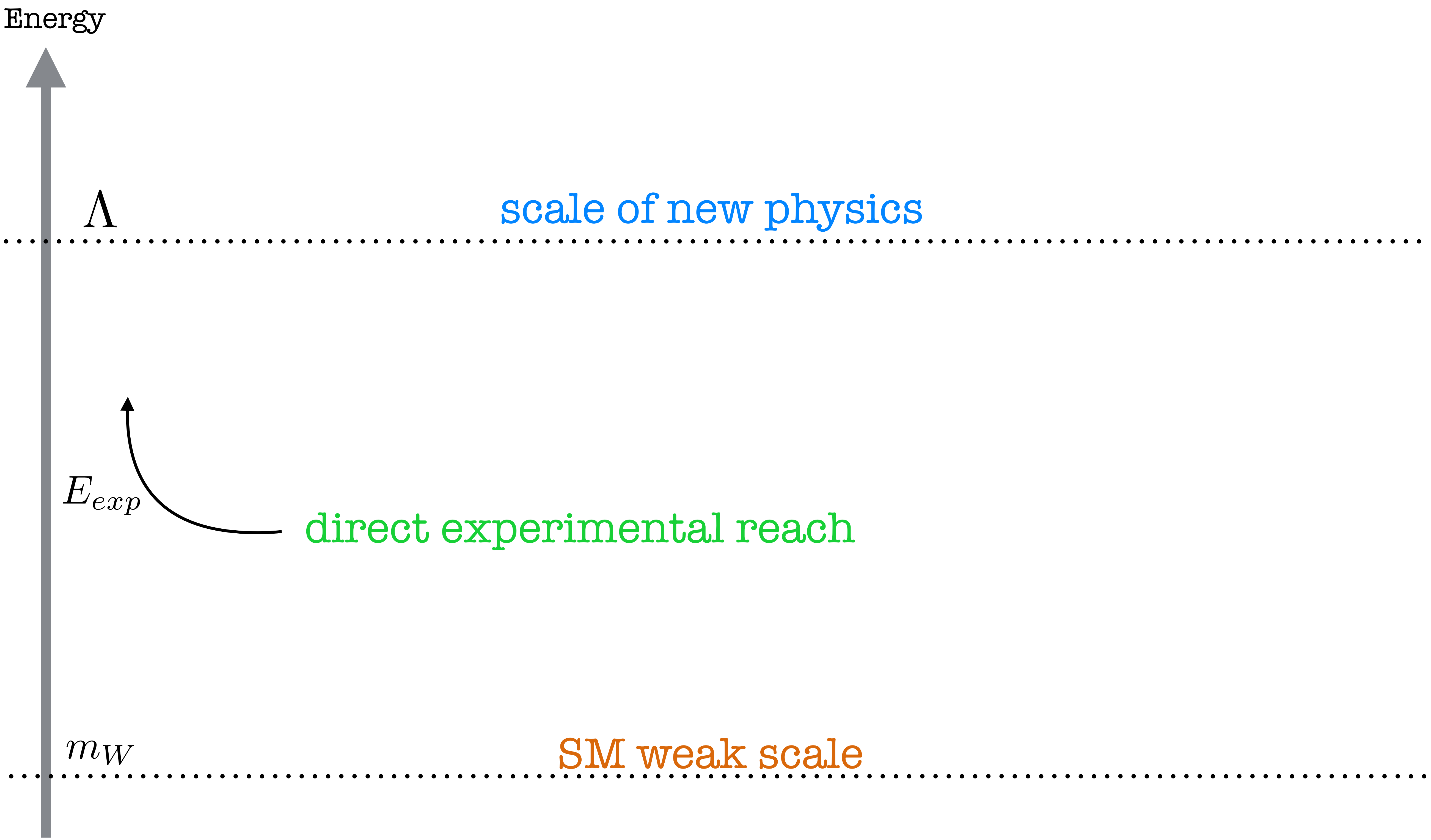
Overview

1. Universal One-Loop Effective Action

2. Effective action for gauge bosons

3. Effective action and anomalies (axion)

How far is new physics from the weak scale?



Program: Matching-Running-Mapping

Energy

$$\mathcal{L}_{UV}[\psi_{BSM}, \psi_{SM}]$$

BSM heavy states

Λ

matching

to do for each model

$$\mathcal{L}_{EFT}[\psi_{SM}] = \mathcal{L}_{SM} + \sum_i c_i(\Lambda)\mathcal{O}_i$$

EFT without heavy state

ex: SM EFT

RG running

(resum large logs)

done once for all

$$\mathcal{L}_{EFT}[\psi_{SM}] = \mathcal{L}_{SM} + \sum_i c_i(m_W)\mathcal{O}_i$$

m_W

mapping

done once for all

electro-weak
precision tests $\mathcal{O}(0.1\%) - \mathcal{O}(1\%)$

Program: Matching-Running-Mapping

Energy

$$\mathcal{L}_{UV}[\psi_{BSM}, \psi_{SM}]$$

Λ

Feynman diagrams

Fonctional path integral *

matching

to do for each model

$$\mathcal{L}_{EFT}[\psi_{SM}] = \mathcal{L}_{SM} + \sum_i c_i(\Lambda)\mathcal{O}_i$$

EFT without heavy state

ex: SM EFT

(*)Schwinger proper time,
Covariant Derivative Expansion,
Various log expansions,
heavy-light subtraction procedures,
integration by regions,
covariant diagrams, ...

RG running

(resum large logs)

done once for all

$$\mathcal{L}_{EFT}[\psi_{SM}] = \mathcal{L}_{SM} + \sum_i c_i(m_W)\mathcal{O}_i$$

m_W

mapping

done once for all

electro-weak
precision tests $\mathcal{O}(0.1\%) - \mathcal{O}(1\%)$

Effective Action by the path integral method

$$e^{iS_{\text{eff}}[\phi_{\text{SM}}](\mu)} = \int \mathcal{D}\Phi_{\text{heavy}} e^{iS[\phi_{\text{SM}},\Phi_{\text{heavy}}](\mu)}$$

Eq. of Motion : $$\frac{\delta S[\phi_{\text{SM}},\Phi]}{\delta\Phi} = 0 \Rightarrow \Phi_c(\phi_{\text{SM}})$$

Expand action around minimum :

Taylor exp. : $$S[\Phi] = S[\Phi_c + \eta] = S[\Phi_c] + \frac{1}{2}\frac{\delta^2 S}{\delta\Phi^2}(\Phi_c)\eta^2 + \mathcal{O}(\eta^3)$$

Write Gaussian integral as determinant :

$$\Rightarrow \quad e^{iS_{\text{eff}}[\phi_{\text{SM}}]} = e^{iS[\Phi_c]}\left[\det\left(-\frac{\delta^2 S}{\delta\Phi^2}(\Phi_c\right)\right]^{-1/2}$$

Write determinant as trace of log in exponent :

$$S_{\text{eff}} = S[\Phi_c] + \frac{i}{2}\text{Tr }\ln\left[-\frac{\delta^2 S}{\delta\Phi^2}(\Phi_c)\right]$$

tree level 1-loop level

One-loop Effective Action

$$\mathcal{L}_{UV} = \mathcal{L}_{SM} + \left(\Phi^\dagger F(\phi_{\rm SM}) + \text{h.c.}\right) + \Phi^\dagger\left(-D^2 - m_\Phi^2 - U(\phi_{\rm SM})\right)\Phi + \mathcal{O}(\Phi^3)$$

Heavy fields can be bosons or fermions

$$P_\mu \equiv iD_\mu$$

$$S_{\rm eff}^{\text{1-loop}} = i c_s \,\text{Tr}\,\ln\left[-\frac{\delta^2 S}{\delta\Phi^2}(\Phi_c)\right] = i c_s\,\text{Tr}\,\ln\left[-P^2 + m_\Phi^2 + U\right]$$

$$= i c_s \int d^4x \int d^4q\ \text{tr}\,\ln\left[e^{Op}(-P_\mu^2 + m_\Phi^2 + U)e^{-Op}\right]$$

1. $Op = q.x$
2. $Op = P_\mu \dfrac{\partial}{\partial q_\mu}$

Gaillard & Cheyette's trick : momentum shift

$$= i c_s \int d^4x \int d^4q\ \text{tr}\,\ln\left[-\left(\tilde{G}_{\nu\mu}\frac{\partial}{\partial q_\mu} + q_\mu\right)^2 + m_\Phi^2 + \tilde{U}\right]$$

So covariant derivatives are explicitly in commutators from beginning :

gauge invariance manifest through the computation

$$\tilde{G}_{\nu\mu} \equiv \sum_{n=0}^{\infty}\frac{n+1}{(n+2)!}[P_{\alpha_1},[\ldots[P_{\alpha_n},G'_{\nu\mu}]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}} \qquad \tilde{U} \equiv \sum_{n=0}^{\infty}\frac{1}{n!}[P_{\alpha_1},[\ldots[P_{\alpha_n},U]]]\frac{\partial^n}{\partial q_{\alpha_1}\ldots q_{\alpha_n}}$$

contain dim-6 operators & independent of momentum q!

integration on q can be done once for all!!!

- q-integrals factorize give usual & simple Feynman Integrals
- traces give Higher Dimensional Operators

M.K. Gaillard, Nucl. Phys. B 268 669 (1986)
O. Cheyette, Nucl. Phys. B 297 183 (1988)
B. Henning, X. Lu and H. Murayama arXiv:1412.1837

One-Loop Effective Action

assuming degenerate mass matrix

B. Henning, X. Lu and H. Murayama arXiv:1412.1837

$$\mathcal{L}_{\text{1-loop}} = \Phi^\dagger \left(-D^2 - m^2 - U\right) \Phi$$

$$\begin{aligned}
\Delta\mathcal{L}_{\text{eff,1-loop}} = \frac{c_s}{(4\pi)^2}\,\text{tr}\Bigg\{ &\\
&+ m^4\left[-\frac{1}{2}\left(\log\frac{m^2}{\mu^2} - \frac{3}{2}\right)\right] \\
&+ m^2\left[-\left(\log\frac{m^2}{\mu^2} - 1\right)U\right] \\
&+ m^0\left[-\frac{1}{12}\left(\log\frac{m^2}{\mu^2} - 1\right)G'^2_{\mu\nu} - \frac{1}{2}\log\frac{m^2}{\mu^2}\,U^2\right] \\
&+ \frac{1}{m^2}\left[-\frac{1}{60}\left(P_\mu G'_{\mu\nu}\right)^2 - \frac{1}{90}G'_{\mu\nu}G'_{\nu\sigma}G'_{\sigma\mu} - \frac{1}{12}\left(P_\mu U\right)^2 - \frac{1}{6}U^3 - \frac{1}{12}UG'_{\mu\nu}G'_{\mu\nu}\right] \\
&+ \frac{1}{m^4}\left[\frac{1}{24}U^4 + \frac{1}{12}U\left(P_\mu U\right)^2 + \frac{1}{120}\left(P^2U\right)^2 + \frac{1}{24}\left(U^2G'_{\mu\nu}G'_{\mu\nu}\right)\right. \\
&\qquad\left. - \frac{1}{120}\left[(P_\mu U),(P_\nu U)\right]G'_{\mu\nu} - \frac{1}{120}\left[U[U,G'_{\mu\nu}]\right]G'_{\mu\nu}\right] \\
&+ \frac{1}{m^6}\left[-\frac{1}{60}U^5 - \frac{1}{20}U^2\left(P_\mu U\right)^2 - \frac{1}{30}\left(UP_\mu U\right)^2\right] \\
&+ \frac{1}{m^8}\left[\frac{1}{120}U^6\right]\Bigg\}.
\end{aligned} \qquad (2.54)$$

dim-6 Operators

Universality of the One-Loop Effective Action

- No need to reinvent the wheel, every slide up to here can be ignored

- Universality of CDE expansion results first noticed in the **simplified** case of **degenerate mass** for heavy fields B. Henning, X. Lu and H. Murayama arXiv:1412.1837

- The general **Universal One-Loop Effective Action** (UOLEA) subsequently derived without such assumption A. Drozd, J. Ellis, JQ and T. You arXiv:1504.02409

- However, extra structures (**heavy-light terms**, « open »covariant derivatives, momentum-shifted-gamma matrices) in CDE expansion not included in initial UOLEA S.A.R. Ellis, JQ, T. You, Z. Zhang arXiv:1604.02445

- Universal **heavy-light** terms now done S.A.R. Ellis, JQ, T. You, Z. Zhang arXiv:1706.07765

- A complete UOLEA, including all possible CDE structures, is in sight…

Universal One-Loop Effective Action

for non degenerate mass heavy fields

A. Drozd, J. Ellis, JQ and T. You arXiv:1512.03003

$$\mathcal{L}_{\text{1-loop}} = \Phi^\dagger \left(-D^2 - m^2 - U\right) \Phi$$

f3 universal term calculated by 't Gooft '73

$$\mathcal{L}^{\text{eff}}_{\text{1-loop}}[\phi] \supset -ic_s \Big\{ f_1^i + f_2^i U_{ii} + f_3^i G'^2_{\mu\nu,ij} + f_4^{ij} U_{ij}^2$$
$$+ f_5^{ij} (P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij} (G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^{ij} [P_\mu, U_{ij}]^2$$
$$+ f_8^{ijk} (U_{ij} U_{jk} U_{ki}) + f_9^{ij} (U_{ij} G'_{\mu\nu,jk} G'_{\mu\nu,ki})$$
$$+ f_{10}^{ijkl} (U_{ij} U_{jk} U_{kl} U_{li}) + f_{11}^{ijk} U_{ij} [P_\mu, U_{jk}][P_\mu, U_{ki}]$$
$$+ f_{12,a}^{ij} [P_\mu, [P_\nu, U_{ij}]] [P_\mu, [P_\nu, U_{ji}]] + f_{12,b}^{ij} [P_\mu, [P_\nu, U_{ij}]] [P_\nu, [P_\mu, U_{ji}]]$$
$$+ f_{12,c}^{ij} [P_\mu, [P_\mu, U_{ij}]] [P_\nu, [P_\nu, U_{ji}]]$$
$$+ f_{13}^{ijk} U_{ij} U_{jk} G'_{\mu\nu,kl} G'_{\mu\nu,li} + f_{14}^{ijk} [P_\mu, U_{ij}] [P_\nu, U_{jk}] G'_{\nu\mu,ki}$$
$$+ \left(f_{15a}^{ijk} U_{i,j} [P_\mu, U_{j,k}] - f_{15b}^{ijk} [P_\mu, U_{i,j}] U_{j,k} \right) [P_\nu, G'_{\nu\mu,ki}]$$
$$+ f_{16}^{ijklm} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mi}) + f_{17}^{ijkl} U_{ij} U_{jk} [P_\mu, U_{kl}][P_\mu, U_{li}]$$
$$+ f_{18}^{ijkl} U_{ij} [P_\mu, U_{jk}] U_{kl} [P_\mu, U_{li}] + f_{19}^{ijklmn} (U_{ij} U_{jk} U_{kl} U_{lm} U_{mn} U_{ni}) \Big\} .$$

dim-6 Operators

Universal coefficients f encapsulate dependence on combinations of momentum master integrals

Application of the UOLEA: MSSM stops

- Write UV Lagrangian for heavy multiplet in appropriate form to extract U matrix, mass matrix and covariant derivative

$$\mathcal{L}_{UV} = \mathcal{L}_{SM} + (\Phi^\dagger F + \text{h.c.}) + \Phi^\dagger(-D^2 - M^2 - U)\Phi + \mathcal{O}(\Phi^3)$$

R-parity

$$\Phi = \begin{pmatrix} \tilde{Q} \\ \tilde{t}_R^* \end{pmatrix}$$

$$G'_{\mu\nu} = \begin{pmatrix} W'^a_{\mu\nu}\tau^a + Y_{\tilde{Q}} B'_{\mu\nu}\mathbb{1} & 0 \\ 0 & -Y_{\tilde{t}_R} B'_{\mu\nu} \end{pmatrix}$$

$$M^2 = \begin{pmatrix} m^2_{\tilde{Q}} & 0 \\ 0 & m^2_{\tilde{t}_R} \end{pmatrix}$$

$$U = \begin{pmatrix} (h_t^2 + \frac{1}{2}g_2^2 c_\beta^2)\tilde{H}\tilde{H}^\dagger + \frac{1}{2}g_2^2 s_\beta^2 H H^\dagger - \frac{1}{2}(g_1^2 Y_{\tilde{Q}} c_{2\beta} + \frac{1}{2}g_2^2)|H|^2 & h_t X_t \tilde{H} \\ h_t X_t \tilde{H}^\dagger & (h_t^2 - \frac{1}{2}g_1^2 Y_{\tilde{t}_R} c_{2\beta})|H|^2 \end{pmatrix}$$

- Example: $\mathcal{O}_{GG} \quad = \quad g_s^2 \left|H\right|^2 G^a_{\mu\nu} G^{a,\mu\nu}$

$$\begin{aligned}
\mathcal{L}^{\text{eff,}}_{\text{1-loop}} = -ic_s \Big\{ & f_1^i + f_2^i U_{ii} + f_3^{ij} G'^2_{\mu\nu,ij} + f_4^{ij} U_{ij}^2 \\
& + f_5^{ij}(P_\mu G'_{\mu\nu,ij})^2 + f_6^{ij}(G'_{\mu\nu,ij})(G'_{\nu\sigma,jk})(G'_{\sigma\mu,ki}) + f_7^i (P_\mu U_{ij})^2 + f_8^{ijk}(U_{ij}U_{jk}U_{ki}) \\
& \quad + f_9^{ij}(U_{ij} G'_{\mu\nu,jk} G'_{\mu\nu,ki}) \\
& + f_{10}^{ijkl}(U_{ij}U_{jk}U_{kl}U_{li}) + f_{11}^{ijk} U_{ij}[P_\mu, U_{jk}][P_\mu, U_{ki}] \\
& \quad + f_{12,a}^{ij}\left(P_\mu[P_\nu, U_{ij}]\right)\left(P_\mu[P_\nu, U_{ji}]\right) + f_{12,b}^{ij}\left(P_\mu[P_\nu, U_{ij}]\right)\left(P_\nu[P_\mu, U_{ji}]\right) \\
& \quad + f_{12,c}^{ij}\left(P_\mu[P_\mu, U_{ij}]\right)\left(P_\nu[P_\nu, U_{ji}]\right) \\
& \quad + f_{13}^{ijk} U_{ij}U_{jk} G'_{\mu\nu,kl} G'_{\mu\nu,li} + f_{14}^{ij}[P_\mu, U_{ij}][P_\nu, U_{jk}] G'_{\nu\mu,ki} \\
& \quad + (f_{15a}^{ijk} U_{i,j}[P_\mu, U_{j,k}] - f_{15b}^{ijk}[P_\mu, U_{i,j}]U_{j,k})[P_\nu, G'_{\nu\mu,ki}] \\
& + f_{16}^i (U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}) + f_{17}^{ijkl} U_{ij}U_{jk}[P_\mu, U_{kl}][P_\mu, U_{li}] + f_{18}^{ijkl} U_{ij}[P_\mu, U_{jk}]U_{kl}[P_\mu, U_{li}] \\
& + f_{19}^{ijklmn}(U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}) \Big\},
\end{aligned} \qquad (3.1)$$

$$\bar{c}_g = \frac{m_W^2}{(4\pi)^2}\frac{1}{24}\left(\frac{h_t^2 - \frac{1}{6}g_1^2 c_{2\beta}}{m^2_{\tilde{Q}}} + \frac{h_t^2 + \frac{1}{3}g_1^2 c_{2\beta}}{m^2_{\tilde{t}_R}} - \frac{h_t^2 X_t^2}{m^2_{\tilde{Q}} m^2_{\tilde{t}_R}}\right)$$

Effective action for gauge bosons (Euler-Heisenberg generalisation)

J.Q., C. Smith and S. Touati, Phys.Rev. D99 (2019) no.1, 013003

Photon effective interactions

$$Z_{QED}\left[J^\mu,\eta,\overline{\eta}\right]=\int DA^\mu D\psi D\overline{\psi}\ \exp i\int dx(\mathcal{L}_{QED}+\overline{\eta}\psi+\overline{\psi}\eta+J^\mu A_\mu)$$

$$\mathcal{L}_{QED}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\overline{\psi}(i\not{D}-m)\psi$$

- Construct EFT (integrate out fermion) :

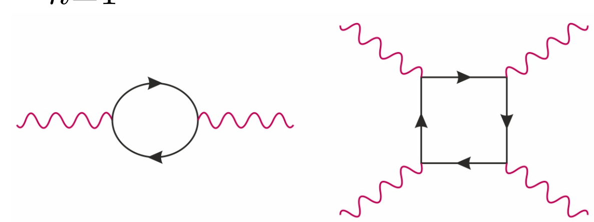
$$Z_{QED}\left[J^\mu,0,0\right]=\int DA^\mu\ \exp i\int dx\left\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+J^\mu A_\mu\right\}\times\det(i\not{D}-m)$$

$$\equiv\int DA^\mu\ \exp i\int dx(\mathcal{L}_{eff}+J^\mu A_\mu)\ .$$

$$\mathcal{L}_{eff}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-iTr\ \ln(i\not{D}-m)$$

all about perturbatively expand the « det »

$$\mathcal{L}_{eff}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+i\sum_{n=1}^{\infty}\frac{e^n}{n}Tr\left(\frac{1}{i\not{\partial}-m}\not{A}\right)^n$$



... : one-loop 1PI diagrams

- Match QED to EFT: light fields are not assumed on-shell!

$$\mathcal{L}_{eff}=-\frac{1}{4}\left\{1+\alpha_0\frac{e^2}{4!\pi^2}\right\}F_{\mu\nu}F^{\mu\nu}+\alpha_2\frac{e^2}{5!\pi^2m^2}\partial^\mu F_{\mu\nu}\partial_\rho F^{\rho\nu}+\alpha_4\frac{e^2}{6!\pi^2m^4}\partial^\mu F_{\mu\nu}\Box\partial_\rho F^{\rho\nu}$$

$$+\gamma_{4,1}\frac{e^4}{6!\pi^2m^4}(F_{\mu\nu}F^{\mu\nu})^2+\gamma_{4,2}\frac{e^4}{6!\pi^2m^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2+\mathcal{O}(m^{-6})\ .$$

	α_0	α_2	α_4	$\gamma_{4,1}$	$\gamma_{4,2}$
Fermion	$2D_\varepsilon Q^2$	$-Q^2$	$\frac{9}{14}Q^2$	$\frac{1}{2}Q^4$	$\frac{7}{8}Q^4$
Scalar	$\frac{1}{2}D_\varepsilon Q^2$	$-\frac{1}{8}Q^2$	$\frac{3}{56}Q^2$	$\frac{7}{32}Q^4$	$\frac{1}{32}Q^4$

E.H.

J.Q., C. Smith and S. Touati, Phys.Rev. D99 (2019) no.1, 013003

Photon effective interactions

integrate out vectors

- in 't Hooft-Feynman gauge: does not satisfy QED ward identities when photons are off-shell

⟶ 4 photons amplitude matches onto effective operators **only for on-shell photons**

usual prescription to construct effective action breaks down

- the problem: the gauge fixing procedure

$$\mathfrak{L}^{R_\xi,linear}_{gauge-fixing} = -\frac{1}{\xi}|\partial^\mu W^+_\mu + \xi M_W \phi^+|^2$$ this explicitly breaks $U(1)_{QED}$

unitary gauge?

$$\mathcal{L}_{U-gauge} = -\frac{1}{2}(D_\mu W^+_\nu - D_\nu W^+_\nu)(D^\mu W^{-\nu} - D^\nu W^{-\mu}) + ieF^{\mu\nu}W^+_\mu W^-_\nu + M^2_W W^+_\mu W^{-\mu}$$ ⟶ matching fails again!

- the non-linear gauge: $\partial^\mu W^\pm_\mu \to D^\mu W^\pm_\mu$

$$\mathfrak{L}^{non-linear}_{gauge-fixing} = -\frac{1}{\xi}|\partial^\mu W^+_\mu + i\kappa e A^\mu W^+_\mu + \xi M_W \phi^+|^2$$

interpolate between linear ($\kappa = 0$) and the U(1) gauge invariant non linear gauge ($\kappa = 1$)

Match QED to EFT:

$$\mathfrak{L}_{eff} = -\frac{1}{4}\left\{1 + \alpha_0 \frac{e^2}{4!\pi^2}\right\} F_{\mu\nu}F^{\mu\nu} + \alpha_2 \frac{e^2}{5!\pi^2 m^2}\partial^\mu F_{\mu\nu}\partial_\rho F^{\rho\nu} + \alpha_4 \frac{e^2}{6!\pi^2 m^4}\partial^\mu F_{\mu\nu}\Box\partial_\rho F^{\rho\nu}$$
$$+ \gamma_{4,1}\frac{e^4}{6!\pi^2 m^4}(F_{\mu\nu}F^{\mu\nu})^2 + \gamma_{4,2}\frac{e^4}{6!\pi^2 m^4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 + \mathcal{O}(m^{-6}) \ .$$

	α_0	α_2	α_4	$\gamma_{4,1}$	$\gamma_{4,2}$
Vector	$-\frac{21D_\varepsilon + 2}{2}Q^2$	$\frac{37}{8}Q^2$	$-\frac{159}{56}Q^2$	$\frac{261}{32}Q^4$	$\frac{243}{32}Q^4$

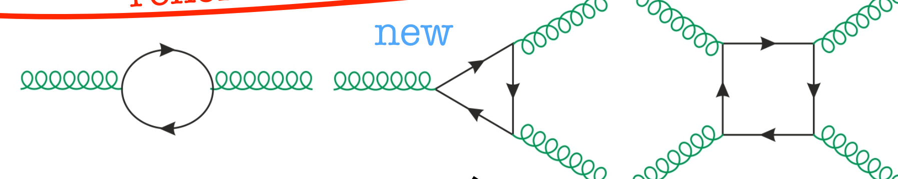
J.Q., C. Smith and S. Touati, Phys.Rev. D99 (2019) no.1, 013003

Gluon effective interactions

non-linear nature of field strength

Construct EFT i.e integrate out fermion :

$$\mathcal{L}_{eff} = -\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} - iTr\ \ln(i\not{D} - m)$$
$$= -\frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} + i\sum_{n=1}^{\infty}\frac{e^n}{n}Tr\left(\frac{1}{i\not\partial - m}\not{G}^a T^a\right)^n$$



- build the basis:

$$\mathfrak{L}^{(0+2)}_{eff} = -\frac{1}{4}\left\{1 + \alpha_0\frac{g_S^2}{4!\pi^2}\right\}G^a_{\mu\nu}G^{a,\mu\nu}$$
$$+ \alpha_2\frac{g_S^2}{5!\pi^2 m^2}D^\nu G^a_{\nu\mu}D_\rho G^{a,\rho\mu} + \alpha_4\frac{g_S^2}{6!\pi^2 m^4}D^\nu G^a_{\nu\mu}D^2 D_\rho G^{a,\rho\mu}$$
$$\mathfrak{L}^{(3)}_{eff} = \beta_2\frac{g_S^3}{5!\pi^2 m^2}f^{abc}G^{a\ \nu}_{\mu}G^{b\ \rho}_{\nu}G^{c\ \mu}_{\rho}$$
$$+ \beta_{4,1}\frac{g_S^3}{6!\pi^2 m^4}f^{abc}G^{a,\mu\nu}D^\alpha G^b_{\mu\nu}D^\beta G^c_{\alpha\beta} + \beta_{4,2}\frac{g_S^3}{6!\pi^2 m^4}f^{abc}G^{a,\mu\nu}D^\alpha G^b_{\alpha\mu}D^\beta G^c_{\beta\nu}$$
$$\mathfrak{L}^{(4)}_{eff} = \gamma_{4,1}\frac{g_S^4}{6!\pi^2 m^4}G^a_{\mu\nu}G^{a,\mu\nu}G^b_{\rho\sigma}G^{b,\rho\sigma} + \gamma_{4,2}\frac{g_S^4}{6!\pi^2 m^4}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}G^b_{\rho\sigma}\tilde{G}^{b,\rho\sigma}$$
$$+ \gamma_{4,3}\frac{g_S^4}{6!\pi^2 m^4}G^a_{\mu\nu}G^{b,\mu\nu}G^a_{\rho\sigma}G^{b,\rho\sigma} + \gamma_{4,4}\frac{g_S^4}{6!\pi^2 m^4}G^a_{\mu\nu}\tilde{G}^{b,\mu\nu}G^a_{\rho\sigma}\tilde{G}^{b,\rho\sigma}$$
$$+ \gamma_{4,5}\frac{g_S^4}{6!\pi^2 m^4}f^{abe}f^{cde}G^a_{\mu\nu}G^{c,\mu\nu}G^b_{\rho\sigma}G^{d,\rho\sigma} + \gamma_{4,6}\frac{g_S^4}{6!\pi^2 m^4}f^{abe}f^{cde}G^a_{\mu\nu}\tilde{G}^{c,\mu\nu}G^b_{\rho\sigma}\tilde{G}^{d,\rho\sigma}$$

no E.O.M. used

because op. contribute to several off-shell 1PI

- Match:

	α_0	α_2	α_4	β_2	$\beta_{4,1}$	$\beta_{4,2}$	$\gamma_{4,1}$	$\gamma_{4,2}$	$\gamma_{4,3}$	$\gamma_{4,4}$	$\gamma_{4,5}$	$\gamma_{4,6}$
Scalar	$\frac{1}{4}D_\varepsilon$	$-\frac{1}{16}$	$\frac{3}{112}$	$\frac{1}{48}$	$-\frac{1}{28}$	0	$\frac{7}{768}$	$\frac{1}{768}$	$\frac{7}{384}$	$\frac{1}{384}$	$\frac{1}{96}$	$\frac{1}{672}$
Fermion	D_ε	$-\frac{1}{2}$	$\frac{9}{28}$	$-\frac{1}{24}$	$\frac{1}{14}$	$-\frac{3}{4}$	$\frac{1}{48}$	$\frac{7}{192}$	$\frac{1}{24}$	$\frac{7}{96}$	$\frac{1}{96}$	$\frac{19}{672}$

J.Q., C. Smith and S. Touati, Phys.Rev. D99 (2019) no.1, 013

Gluon effective interactions

integrate out vectors

- calculation far more challenging!

- need to **generalize the non-linear gauge** to preserve QCD symmetry otherwise 1 PI off-shell amplitudes cannot **matched onto gauge invariant operators**

- non-linear gauge drastically reduce the number of diagrams to compute (4-gluon diagrams: **207** ⟶ **84**)

- R_ξ gauge: get rid of mixing term like $X_\mu^k \partial^\mu H_X^k$ (X charged under unbroken symmetries ⟶ $X_\mu^k D^\mu H_X^k$)

- non-linear gauge: no $X - V_{SM} - H_X$ couplings ⟶ all the mixed loops of vector with its WBG boson disappear (very welcome!)

- Match with vectors in the loops:

J.Q., C. Smith and S. Touati, Phys.Rev. D99 (2019) no.1, 013003

	α_0	α_2	α_4	β_2	$\beta_{4,1}$	$\beta_{4,2}$	$\gamma_{4,1}$	$\gamma_{4,2}$	$\gamma_{4,3}$	$\gamma_{4,4}$	$\gamma_{4,5}$	$\gamma_{4,6}$
Vector	$-\frac{21D_\varepsilon+2}{4}$	$\frac{37}{16}$	$-\frac{159}{112}$	$\frac{1}{16}$	$-\frac{3}{28}$	3	$\frac{87}{256}$	$\frac{81}{256}$	$\frac{87}{128}$	$\frac{81}{128}$	$-\frac{3}{32}$	$-\frac{27}{224}$

SU(N) effective interactions

- QCD case extended to arbitrary representations of other Lie groups :

Tr over the fundamental generators of SU(3) → Tr over generic rep. R of SU(N)

vacuum polarization : $\mathrm{Tr}(T^a_{\mathbf{R}}T^b_{\mathbf{R}}) = I_2(\mathbf{R})\delta^{ab}$

3 bosons amplitudes : $\mathrm{Tr}(T^a_{\mathbf{R}}[T^b_{\mathbf{R}},T^c_{\mathbf{R}}]) = iI_2(\mathbf{R})f^{abc}$

quadratic invariant ensures proper matching

4 bosons amplitudes :

reversing loop momentum

$$\mathcal{M}^{abcd} = C_1^{abcd}\mathcal{M}_1 + C_2^{abcd}\mathcal{M}_2 + C_3^{abcd}\mathcal{M}_3\ , \quad \begin{cases} C_1^{abcd} = \mathrm{Tr}(T^a_{\mathbf{R}}T^b_{\mathbf{R}}T^d_{\mathbf{R}}T^c_{\mathbf{R}}) + \mathrm{Tr}(T^a_{\mathbf{R}}T^c_{\mathbf{R}}T^d_{\mathbf{R}}T^b_{\mathbf{R}}) \\ C_2^{abcd} = \mathrm{Tr}(T^a_{\mathbf{R}}T^b_{\mathbf{R}}T^c_{\mathbf{R}}T^d_{\mathbf{R}}) + \mathrm{Tr}(T^a_{\mathbf{R}}T^d_{\mathbf{R}}T^c_{\mathbf{R}}T^b_{\mathbf{R}}) \\ C_3^{abcd} = \mathrm{Tr}(T^a_{\mathbf{R}}T^c_{\mathbf{R}}T^b_{\mathbf{R}}T^d_{\mathbf{R}}) + \mathrm{Tr}(T^a_{\mathbf{R}}T^d_{\mathbf{R}}T^b_{\mathbf{R}}T^c_{\mathbf{R}}) \end{cases}$$

$\mathcal{O}(m^0)$ and $\mathcal{O}(m^{-2})$: 2 independent combinations of Tr expressed in terms of $I_2(\mathbf{R})$

$$D_1^{abcd} = 2C_1^{abcd} - C_2^{abcd} - C_3^{abcd} = I_2(\mathbf{R})(2f^{ace}f^{bde} - f^{ade}f^{bce})\ ,$$
$$D_2^{abcd} = 2C_2^{abcd} - C_1^{abcd} - C_3^{abcd} = I_2(\mathbf{R})(2f^{ade}f^{bce} - f^{ace}f^{bde})\ ,$$
$$D_3^{abcd} = 2C_3^{abcd} - C_1^{abcd} - C_2^{abcd} = I_2(\mathbf{R})(-f^{ade}f^{bce} - f^{ace}f^{bde}) = -D_1^{abcd} - D_2^{abcd}$$

$\mathcal{O}(m^{-4})$: $D_{1,2,3}^{abcd}$ induce operators tuned by: $\gamma_{4,5}$ and $\gamma_{4,6}$

the rest is proportional to the **symmetrize Tr**: $D_0^{abcd} = C_1^{abcd} + C_2^{abcd} + C_3^{abcd} = \frac{1}{4}S\,\mathrm{Tr}(T^a_{\mathbf{R}}T^b_{\mathbf{R}}T^c_{\mathbf{R}}T^d_{\mathbf{R}})$

$$= 6I_4(\mathbf{R})d^{abcd} + 6\Lambda(\mathbf{R})(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc})$$

$$\Lambda(\mathbf{R}) = \left(\frac{N(\mathbf{A})I_2(\mathbf{R})}{N(\mathbf{R})} - \frac{I_2(\mathbf{A})}{6}\right)\frac{I_2(\mathbf{R})}{2+N(\mathbf{A})}$$

no matter the rep. or spin in the loop:

$$\gamma_{4,1} = \frac{1}{2}\gamma_{4,3} \qquad \gamma_{4,2} = \frac{1}{2}\gamma_{4,4}$$

$$\begin{aligned}\mathfrak{L}_{eff}^{(4)} = {} & \gamma_{4,1}\frac{g_S^4}{6!\pi^2m^4}G^a_{\mu\nu}G^{a,\mu\nu}G^b_{\rho\sigma}G^{b,\rho\sigma} + \gamma_{4,2}\frac{g_S^4}{6!\pi^2m^4}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}G^b_{\rho\sigma}\tilde{G}^{b,\rho\sigma} \\ & + \gamma_{4,3}\frac{g_S^4}{6!\pi^2m^4}G^a_{\mu\nu}G^{b,\mu\nu}G^a_{\rho\sigma}G^{b,\rho\sigma} + \gamma_{4,4}\frac{g_S^4}{6!\pi^2m^4}G^a_{\mu\nu}\tilde{G}^{b,\mu\nu}G^a_{\rho\sigma}\tilde{G}^{b,\rho\sigma} \\ & + \gamma_{4,5}\frac{g_S^4}{6!\pi^2m^4}f^{abe}f^{cde}G^a_{\mu\nu}G^{c,\mu\nu}G^b_{\rho\sigma}G^{d,\rho\sigma} + \gamma_{4,6}\frac{g_S^4}{6!\pi^2m^4}f^{abe}f^{cde}G^a_{\mu\nu}\tilde{G}^{c,\mu\nu}G^b_{\rho\sigma}\tilde{G}^{d,\rho\sigma} \\ & + \gamma_{4,7}\frac{g_S^4}{6!\pi^2m^4}d^{abcd}G^a_{\mu\nu}G^{b,\mu\nu}G^c_{\rho\sigma}G^{d,\rho\sigma} + \gamma_{4,8}\frac{g_S^4}{6!\pi^2m^4}d^{abcd}G^a_{\mu\nu}\tilde{G}^{b,\mu\nu}G^c_{\rho\sigma}\tilde{G}^{d,\rho\sigma}\end{aligned}$$

need to extend

SU(N), SO(N) effective interactions

$$\mathfrak{L}_{eff}^{(4)} = \gamma_{4,1}\frac{g_S^4}{6!\pi^2 m^4}G^a_{\mu\nu}G^{a,\mu\nu}G^b_{\rho\sigma}G^{b,\rho\sigma} + \gamma_{4,2}\frac{g_S^4}{6!\pi^2 m^4}G^a_{\mu\nu}\tilde{G}^{a,\mu\nu}G^b_{\rho\sigma}\tilde{G}^{b,\rho\sigma}$$
$$+\gamma_{4,3}\frac{g_S^4}{6!\pi^2 m^4}G^a_{\mu\nu}G^{b,\mu\nu}G^a_{\rho\sigma}G^{b,\rho\sigma} + \gamma_{4,4}\frac{g_S^4}{6!\pi^2 m^4}G^a_{\mu\nu}\tilde{G}^{b,\mu\nu}G^a_{\rho\sigma}\tilde{G}^{b,\rho\sigma}$$
$$+\gamma_{4,5}\frac{g_S^4}{6!\pi^2 m^4}f^{abe}f^{cde}G^a_{\mu\nu}G^{c,\mu\nu}G^b_{\rho\sigma}G^{d,\rho\sigma} + \gamma_{4,6}\frac{g_S^4}{6!\pi^2 m^4}f^{abe}f^{cde}G^a_{\mu\nu}\tilde{G}^{c,\mu\nu}G^b_{\rho\sigma}\tilde{G}^{d,\rho\sigma}$$
$$+\gamma_{4,7}\frac{g_S^4}{6!\pi^2 m^4}d^{abcd}G^a_{\mu\nu}G^{b,\mu\nu}G^c_{\rho\sigma}G^{d,\rho\sigma} + \gamma_{4,8}\frac{g_S^4}{6!\pi^2 m^4}d^{abcd}G^a_{\mu\nu}\tilde{G}^{b,\mu\nu}G^c_{\rho\sigma}\tilde{G}^{d,\rho\sigma}\ .$$

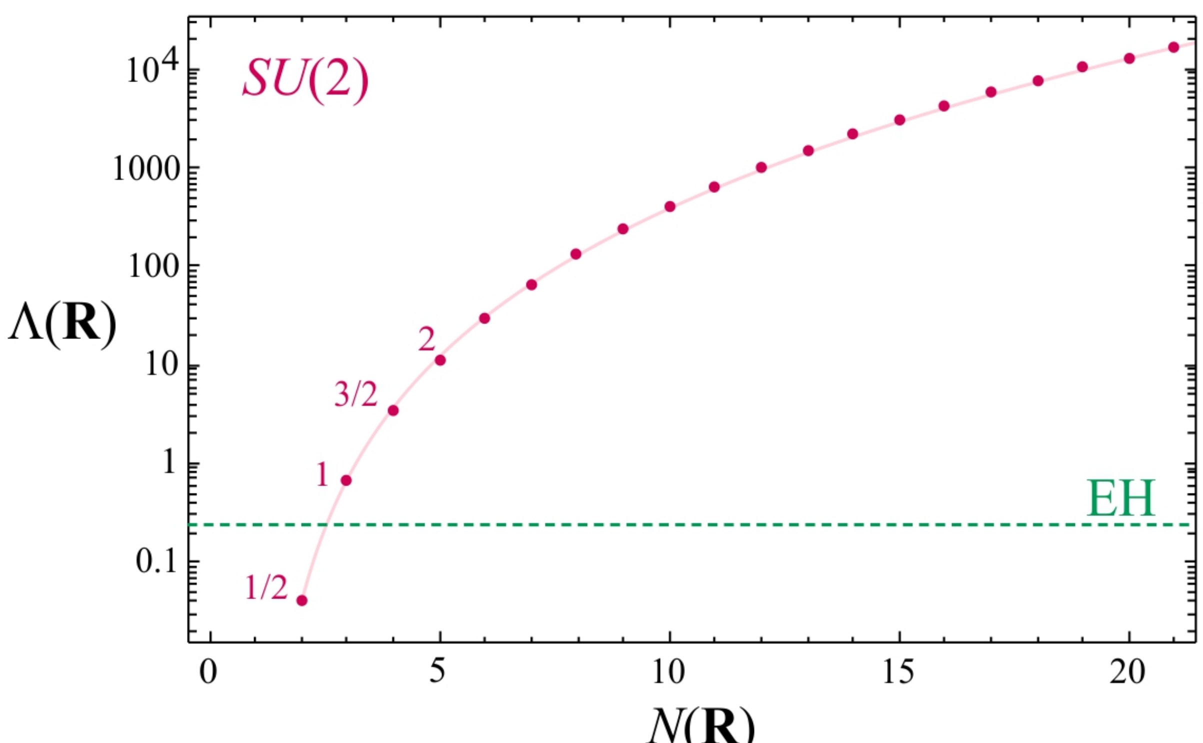
	α_0	α_2	α_4	β_2	$\beta_{4,1}$	$\beta_{4,2}$
Scalar	$\frac{1}{2}I_2(\mathbf{R})D_\varepsilon$	$-\frac{1}{8}I_2(\mathbf{R})$	$\frac{3}{56}I_2(\mathbf{R})$	$\frac{1}{24}I_2(\mathbf{R})$	$-\frac{1}{14}I_2(\mathbf{R})$	0
Fermion	$2I_2(\mathbf{R})D_\varepsilon$	$-I_2(\mathbf{R})$	$\frac{9}{14}I_2(\mathbf{R})$	$-\frac{1}{12}I_2(\mathbf{R})$	$\frac{1}{7}I_2(\mathbf{R})$	$-\frac{3}{2}I_2(\mathbf{R})$
Vector	$-\frac{21D_\varepsilon+2}{2}I_2(\mathbf{R})$	$\frac{37}{8}I_2(\mathbf{R})$	$-\frac{159}{56}I_2(\mathbf{R})$	$\frac{1}{8}I_2(\mathbf{R})$	$-\frac{3}{14}I_2(\mathbf{R})$	$6I_2(\mathbf{R})$
	$\gamma_{4,1}=\gamma_{4,3}/2$	$\gamma_{4,2}=\gamma_{4,4}/2$	$\gamma_{4,5}$	$\gamma_{4,6}$	$\gamma_{4,7}$	$\gamma_{4,8}$
Scalar	$\frac{7}{32}\Lambda(\mathbf{R})$	$\frac{1}{32}\Lambda(\mathbf{R})$	$\frac{1}{48}I_2(\mathbf{R})$	$\frac{1}{336}I_2(\mathbf{R})$	$\frac{7}{32}I_4(\mathbf{R})$	$\frac{1}{32}I_4(\mathbf{R})$
Fermion	$\frac{1}{2}\Lambda(\mathbf{R})$	$\frac{7}{8}\Lambda(\mathbf{R})$	$\frac{1}{48}I_2(\mathbf{R})$	$\frac{19}{336}I_2(\mathbf{R})$	$\frac{1}{2}I_4(\mathbf{R})$	$\frac{7}{8}I_4(\mathbf{R})$
Vector	$\frac{261}{32}\Lambda(\mathbf{R})$	$\frac{243}{32}\Lambda(\mathbf{R})$	$-\frac{3}{16}I_2(\mathbf{R})$	$-\frac{27}{112}I_2(\mathbf{R})$	$\frac{261}{32}I_4(\mathbf{R})$	$\frac{243}{32}I_4(\mathbf{R})$

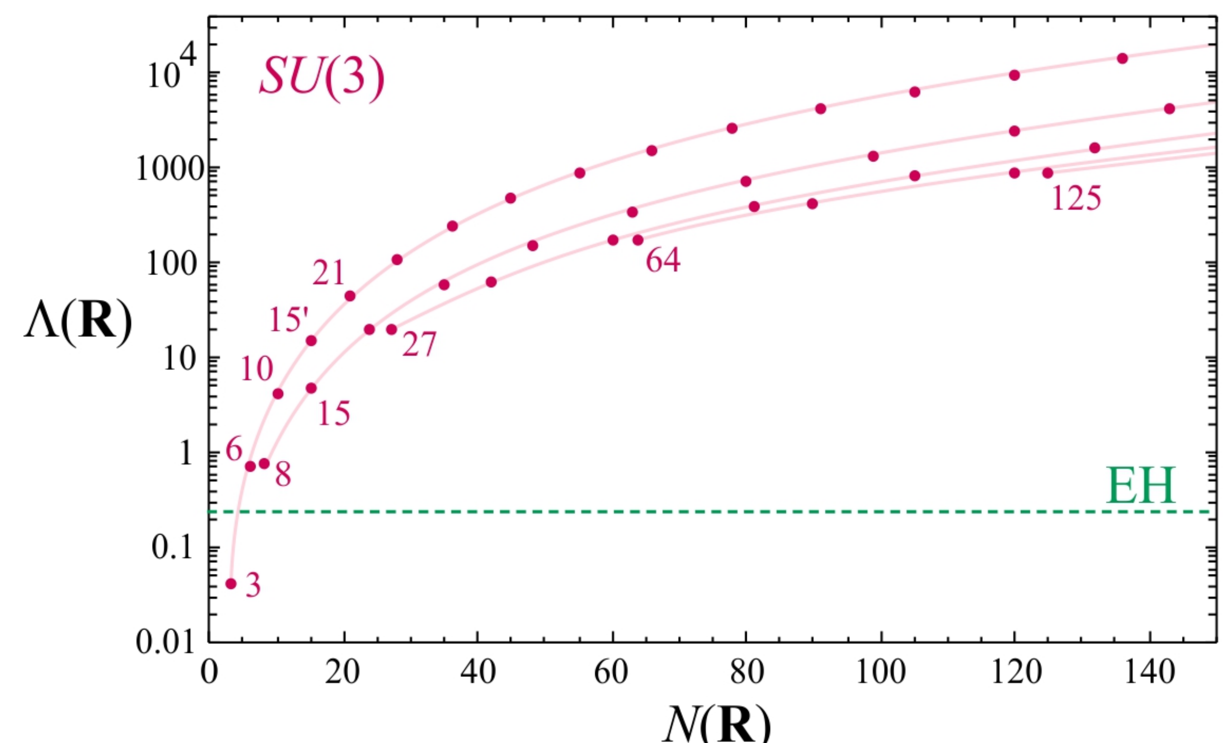
J.Q., C. Smith and S. Touati, Phys.Rev. D99 (2019) no.1, 013003

- reduction to U(1), SU(2) and SU(3) :

$$\mathfrak{L}_{eff,SU(2)_L}^{(4)} = \frac{(\gamma_{4,1}+\gamma_{4,3})g^4}{6!\pi^2 m^4}(W^3_{\mu\nu}W^{3,\mu\nu})^2 + \frac{(\gamma_{4,2}+\gamma_{4,4})g^4}{6!\pi^2 m^4}(W^3_{\mu\nu}\tilde{W}^{3,\mu\nu})^2$$
$$+\frac{4(\gamma_{4,1}+\gamma_{4,5})g^4}{6!\pi^2 m^4}W^3_{\mu\nu}W^{3,\mu\nu}W^+_{\rho\sigma}W^{-,\rho\sigma} + \frac{4(\gamma_{4,2}+\gamma_{4,6})g^4}{6!\pi^2 m^4}W^3_{\mu\nu}\tilde{W}^{3,\mu\nu}W^+_{\rho\sigma}\tilde{W}^{-,\rho\sigma}$$
$$+\frac{4(\gamma_{4,3}-\gamma_{4,5})g^4}{6!\pi^2 m^4}|W^3_{\mu\nu}W^{+,\mu\nu}|^2 + \frac{4(\gamma_{4,4}-\gamma_{4,6})g^4}{6!\pi^2 m^4}|W^3_{\mu\nu}\tilde{W}^{+,\mu\nu}|^2$$
$$+\frac{2(2\gamma_{4,1}+\gamma_{4,3}+\gamma_{4,5})g^4}{6!\pi^2 m^4}(W^+_{\mu\nu}W^{-,\mu\nu})^2 + \frac{2(\gamma_{4,4}-\gamma_{4,6})g^4}{6!\pi^2 m^4}|W^+_{\mu\nu}\tilde{W}^{+,\mu\nu}|^2$$
$$+\frac{2(\gamma_{4,3}-\gamma_{4,5})g^4}{6!\pi^2 m^4}|W^+_{\mu\nu}W^{+,\mu\nu}|^2 + \frac{2(2\gamma_{4,2}+\gamma_{4,4}+\gamma_{4,6})g^4}{6!\pi^2 m^4}(W^+_{\mu\nu}\tilde{W}^{-,\mu\nu})^2\ .$$

$$\mathfrak{L}_{eff}^{(4)}(U(1)_\alpha\subset SU(N)) = (\gamma_{4,1}+\gamma_{4,3}+d^{\alpha\alpha\alpha\alpha}\gamma_{4,7})\frac{g_S^4}{6!\pi^2 m^4}G^\alpha_{\mu\nu}G^{\alpha,\mu\nu}G^\alpha_{\rho\sigma}G^{\alpha,\rho\sigma}$$
$$+(\gamma_{4,2}+\gamma_{4,4}+d^{\alpha\alpha\alpha\alpha}\gamma_{4,8})\frac{g_S^4}{6!\pi^2 m^4}G^\alpha_{\mu\nu}\tilde{G}^{\alpha,\mu\nu}G^\alpha_{\rho\sigma}\tilde{G}^{\alpha,\rho\sigma}$$





Axions are blind to anomalies

J.Q. and C. Smith, arXiv:1903.12559

An axionic toy model: simple QED extension

- local $U(1)_{em}$, new scalar field ϕ :

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_L(iD)\psi_L + \bar{\psi}_R(iD)\psi_R + (y\phi\bar{\psi}_L\psi_R + h.c.) + \partial_\mu\phi^\dagger\partial^\mu\phi - V(\phi)$$

- global $U(1)_{PQ}$:

chiral anomaly

$$\phi \to \exp(-i\theta)\phi \ , \quad \psi_L \to \exp(i\alpha\theta)\psi_L \ , \ \psi_R \to \exp(i(\alpha+1)\theta)\psi_R$$

global fermion number

→ Goldstone boson (**axion**) remnant of $U(1)_{PQ}$ S.S.B.

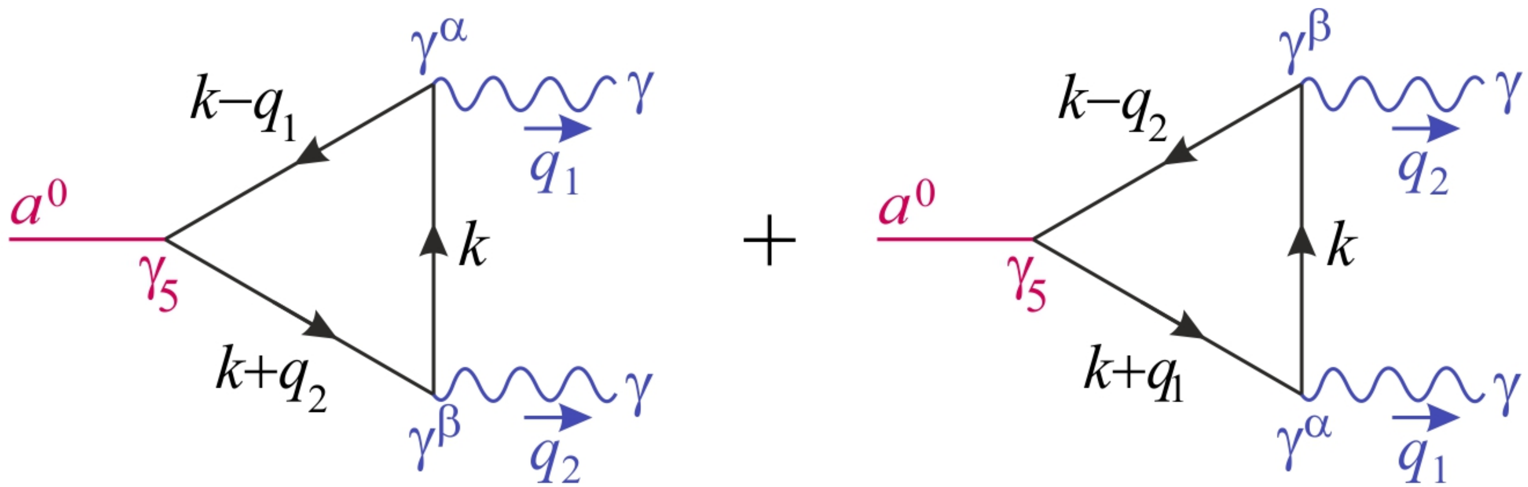
Linear representation: $\phi(x) = \sigma^0(x) + ia^0(x) + v$

Polar representation: $\phi(x) = \frac{1}{\sqrt{2}}(v + \sigma^0(x))e^{-ia^0(x)/v}$

Linear representation

$$\phi(x) = \sigma^0(x) + i a^0(x) + v$$

$$\mathcal{L}_{\text{Linear}} \supset \frac{1}{2}\partial_\mu a^0 \partial^\mu a^0 + \frac{m}{v} a^0 \bar{\psi} i \gamma_5 \psi$$



$$\mathcal{T}^{\alpha\beta}_{PVV} = \int \frac{d^4k}{(2\pi)^4}(-1)\,\mathrm{Tr}\left[\frac{i}{\not{k} - \not{q}_1 - m}\gamma^\alpha \frac{i}{\not{k} - m}\gamma^\beta \frac{i}{\not{k} + \not{q}_2 - m}\gamma_5\right] + (1, \alpha \leftrightarrow 2, \beta) + (m \to M)$$

$$= -i\frac{1}{2\pi^2}\varepsilon^{\alpha\beta\rho\sigma} q_{1,\rho} q_{2,\sigma}(m C_0(m^2) - M C_0(M^2)) \ .$$

Pauli-Villars regulator

$m \to \infty$ $$\mathcal{L}^{\text{eff}}_{\text{Linear}} = -\frac{e^2}{16\pi^2 v} a^0 F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Anomalies do no show up : loop amplitude can safely be computed using a naive regularization procedure even if $U(1)_{PQ}$ is anomalous

Polar representation

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \sigma^0(x))e^{-ia^0(x)/v}$$

reparametrize fermion fields (invariant under $U(1)_{PQ}$):

$$\psi_L(x) \to \exp(i\alpha a^0(x)/v)\psi_L(x) \ , \ \psi_R(x) \to \exp(i(\alpha+1)a^0(x)/v)\psi_R(x)$$

- a^0 disapears from the Yukawa $(y\phi\bar{\psi}_L\psi_R + h.c.)$
- Fermion kinetic term induce **derivative interactions** $\bar{\psi}_L(iD)\psi_L + \bar{\psi}_R(iD)\psi_R$

$$\delta\mathcal{L}_{\rm Der} = -\frac{\partial_\mu a^0}{v}(\alpha\bar{\psi}_L\gamma^\mu\psi_L + (\alpha+1)\bar{\psi}_R\gamma^\mu\psi_R) = -\frac{\partial_\mu a^0}{2v}((2\alpha+1)\bar{\psi}\gamma^\mu\psi + \bar{\psi}\gamma^\mu\gamma_5\psi)$$

- Fermionic path integral measure is not invariant:

new local interaction (Jacobian of the transformation)

$$\delta\mathcal{L}_{\rm Jac} = \frac{e^2}{16\pi^2 v}a^0(\alpha - (\alpha+1))F_{\mu\nu}\tilde{F}^{\mu\nu} = -\frac{e^2}{16\pi^2 v}a^0 F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\longrightarrow \mathcal{L}_{\rm Polar} \supset \frac{1}{2}\partial_\mu a^0 \partial^\mu a^0 + \delta\mathcal{L}_{\rm Der} + \delta\mathcal{L}_{\rm Jac}$$

Polar representation

$$\mathcal{T}^{\gamma\alpha\beta}_{AVV} = \int \frac{d^4k}{(2\pi)^4}(-1)\,\mathrm{Tr}\left[\frac{i}{\not k - \not q_1 - m}\gamma^\alpha \frac{i}{\not k - m}\gamma^\beta \frac{i}{\not k + \not q_2 - m}\gamma^\gamma\gamma_5\right] + (1,\alpha \leftrightarrow 2,\beta)$$

$$\mathcal{M}(a^0 \to \gamma\gamma)_{\mathrm{Der}} = \frac{-e^2}{2v} i(q_1+q_2)_\gamma \mathcal{T}^{\gamma\alpha\beta}_{AVV}\varepsilon(q_1)^*_\alpha \varepsilon(q_2)^*_\beta$$

no « VVV » diagram (Furry theorem)

$$= \frac{-e^2}{4\pi^2 v}(2m^2 C_0(m^2) - 2M^2 C_0(M^2))\varepsilon^{\alpha\beta\rho\sigma}\varepsilon(q_1)^*_\alpha \varepsilon(q_2)^*_\beta q_{1,\rho} q_{2,\sigma}$$

$\xrightarrow{M\to\infty}$ the regulator term precisely cancels the local term from $\delta\mathcal{L}_{\mathrm{Jac}}$

$$\mathcal{M}(a^0 \to \gamma\gamma)_{\mathrm{Polar}} = \mathcal{M}(a^0 \to \gamma\gamma)_{\mathrm{Der}} + \mathcal{M}(a^0 \to \gamma\gamma)_{\mathrm{Jac}}$$

$$= -\frac{e^2}{2\pi^2 v} m^2 C_0(m^2)\varepsilon^{\alpha\beta\rho\sigma}\varepsilon(q_1)^*_\alpha \varepsilon(q_2)^*_\beta q_{1,\rho} q_{2,\sigma} = \underline{\mathcal{M}(a^0 \to \gamma\gamma)_{\mathrm{Linear}}}$$

- in polar rep. : the local term $a^0 F_{\mu\nu}\tilde{F}^{\mu\nu}$ is **spurious**

  it only serves to cancel out the anomalous term arising from $\partial_\mu a^0 \bar\psi\gamma^\mu\gamma_5\psi$

  nothing but axial Ward identity: $\partial_\mu A^\mu - \frac{e^2}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} = 2imP$ $\quad A^\mu = \bar\psi\gamma^\mu\gamma_5\psi$, $\quad P = \bar\psi\gamma_5\psi$

- $a^0 \to \gamma\gamma$ often **misinterpreted** as induced by the **anomaly**

$$\mathcal{M}(a^0 \to \gamma\gamma)_{\mathrm{Der}} \overset{m\to\infty}{=} 0 \;(\neq 0) \quad + \mathcal{M}(a^0 \to \gamma\gamma)_{\mathrm{Jac}}$$

$$(\;A \;\; -A\;) \;+\; A \;=\; A$$ ← no more than a book-keeping device

$$\underset{B}{A} + (-A \;+\; A\;) \;=\; \underset{B}{A}$$ ← **physical identification!**

Consistent use of anomalies

Jacobian of the transformation: triangle graph with left handed currents

$$\mathcal{T}^{\alpha\beta\gamma}_{LLL} = \int \frac{d^4k}{(2\pi)^4}(-1)\,\mathrm{Tr}\left[\frac{i}{\not k - \not q_1}\boxed{\gamma^\beta P_L}\frac{i}{\not k}\boxed{\gamma^\gamma P_L}\frac{i}{\not k + \not q_2}\boxed{\gamma^\alpha P_L}\right] + (1,\beta \leftrightarrow 2,\gamma)$$

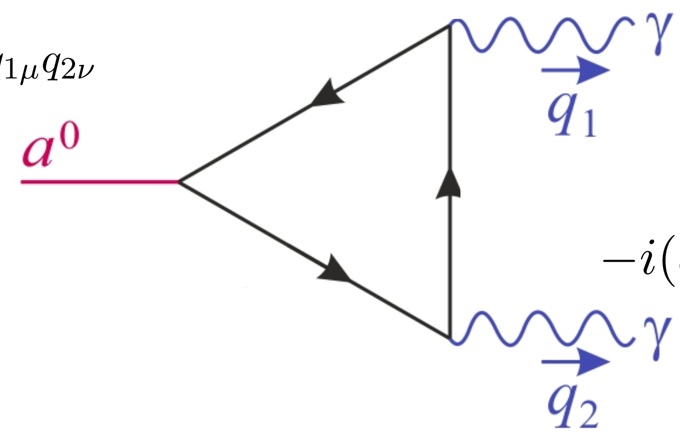
$\partial_\mu a^0$

$$-i(q_1)_\beta \mathcal{T}^{\alpha\beta\gamma}_{LLL} = \frac{1}{8\pi^2}\boxed{(1+b)}\,\varepsilon^{\gamma\alpha\mu\nu}q_{1\mu}q_{2\nu}$$

$U(1)_{em}$ non anomalous $\Rightarrow b = -1$

$$i(q_1+q_2)_\alpha \mathcal{T}^{\alpha\beta\gamma}_{LLL} = \frac{1}{8\pi^2}(a-b)\,\varepsilon^{\beta\gamma\mu\nu}q_{1\mu}q_{2\nu}$$

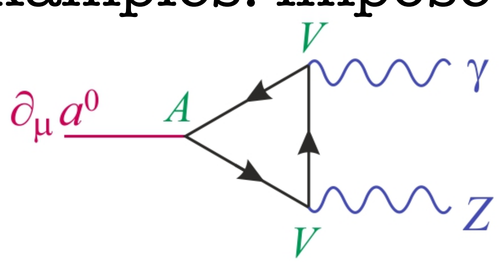
$= \underline{0}$ since $a = -b = 1$



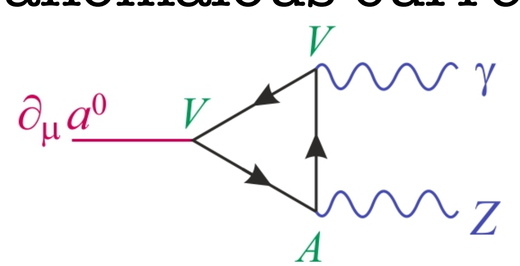
$$-i(q_2)_\gamma \mathcal{T}^{\alpha\beta\gamma}_{LLL} = \frac{1}{8\pi^2}\boxed{(1-a)}\,\varepsilon^{\alpha\beta\mu\nu}q_{1\mu}q_{2\nu}$$

$U(1)_{em}$ non anomalous $\Rightarrow a = 1$

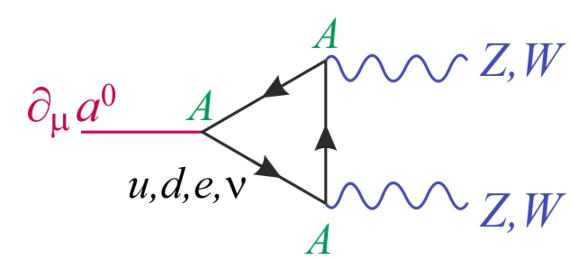
- a and b are free parameters :needed to **keep track of the anomalies**
- anomaly equally distributed (Bose symmetry) with $a = -b = 1/3$
- In terms of **vector** and **axial** currents only « **AVV**s » and « **AAA** » do not cancel (Furry)
- careful to the regularization procedure: Pauli-Villars or dim. reg. enforce automatically a and b (AVV: $a = -b = 1$; AAA: $a = -b = 1/3$) *we do not want that! *
- examples: impose which are the anomalous currents



$\mathcal{M}(a^0 \to \gamma Z)^{AVV}\big|_{m\to\infty} = \underline{0}$



$\mathcal{M}(a^0 \to \gamma Z)^{VAV}\big|_{m\to\infty}\boxed{\neq 0}$

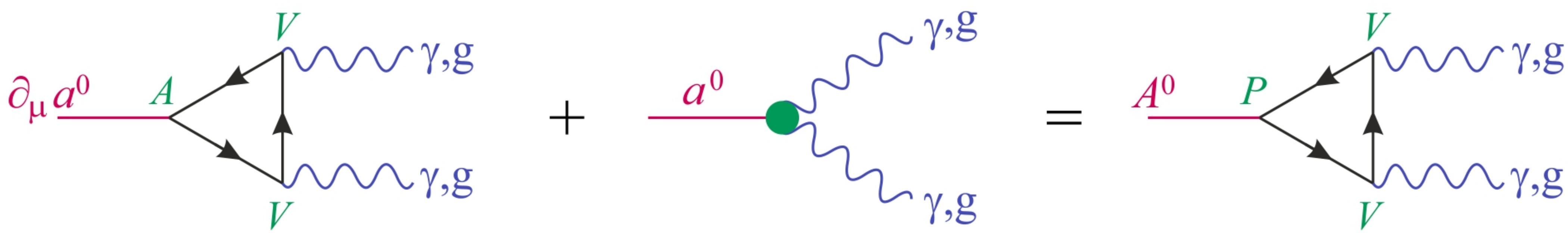


$\mathcal{M}(a^0 \to \gamma Z)^{AAA}\big|_{m\to\infty}\boxed{\neq 0}$

Couplings of the PQ axion: Matching the polar and linear rep.

- $a^0 \to \gamma\gamma, gg$:

simple usual 2HDM



$$\mathcal{M}(a^0 \to \gamma\gamma, gg)_{\text{Polar}} = \mathcal{M}(a^0 \to \gamma\gamma, gg)^{AVV}_{\text{Der}} + \mathcal{M}(a^0 \to \gamma\gamma, gg)_{\text{Jac}}$$

$$\mathcal{M}(a^0 \to \gamma\gamma, gg)^{AVV}_{\text{Der}} = -\mathcal{M}(a^0 \to \gamma\gamma, gg)_{\text{Jac}} + \mathcal{M}(A^0 \to \gamma\gamma, gg)_{\text{Linear}}$$

$$\mathcal{M}(a^0 \to \gamma\gamma, gg)_{\text{Polar}} = \mathcal{M}(A^0 \to \gamma\gamma, gg)_{\text{Linear}}$$

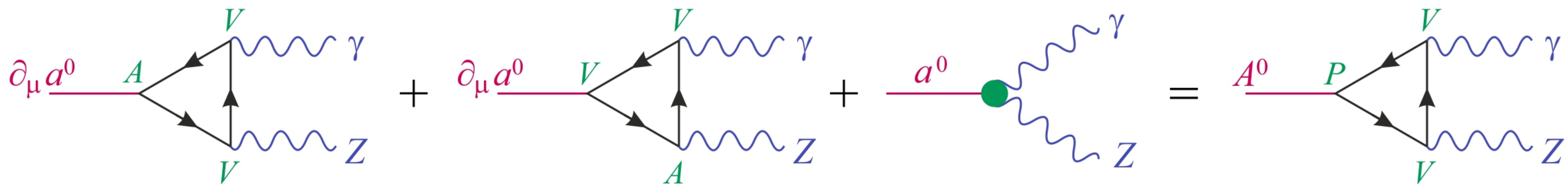
The anomalous contact int. do cancel out systematically with the anomalous part to the triangle graphs

$$\mathcal{M}(a^0 \to \gamma\gamma, gg)^{AVV}_{\text{Der}} \overset{m\to\infty}{=} 0 \implies \mathcal{M}(A^0 \to \gamma\gamma, gg)_{\text{Linear}} \overset{m\to\infty}{=} \mathcal{M}(a^0 \to \gamma\gamma, gg)_{\text{Jac}}$$

though interpreting the axion coupling as induced by the anomaly is incorrect!

Couplings of the PQ axion: Matching the polar and linear representations

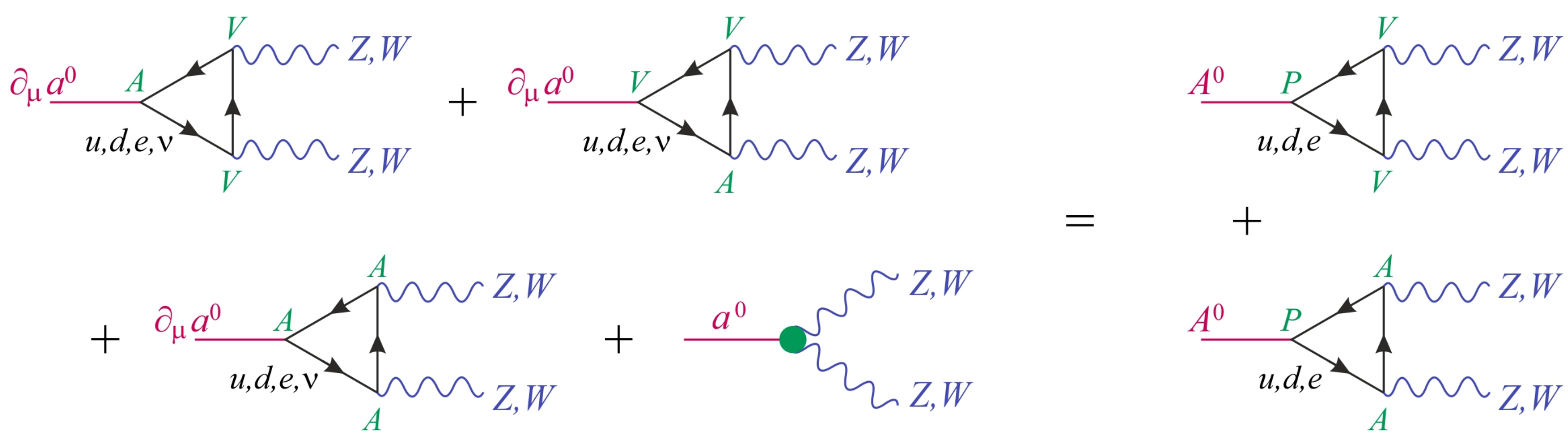
- $a^0 \to \gamma Z$:



$$\mathcal{M}(a^0 \to \gamma Z)_{\rm Polar} = \mathcal{M}(a^0 \to \gamma Z)^{AVV}_{\rm Der} + \underbrace{\mathcal{M}(a^0 \to \gamma Z)^{VAV}_{\rm Der}}_{\neq 0} + \mathcal{M}(a^0 \to \gamma Z)_{\rm Jac} \quad = \mathcal{M}(A^0 \to \gamma Z)_{\rm Linear}$$

For **chiral gauge theory**: the local terms from $\delta\mathcal{L}_{\rm Jac}$ are no longer reliable book-keeping of the effect of heavy fermions because part of the anomaly is hidden in the VAV triangle

- $a^0 \to ZZ, W^+W^-$:



same conclusion: all the local anomalous contributions cancel exactly

$$\mathcal{M}(a^0 \to VV)_{\rm Polar} = \mathcal{M}(a^0 \to VV)^{AVV}_{\rm Der} + \underline{\mathcal{M}(a^0 \to VV)^{VAV}_{\rm Der}}$$
$$\underbrace{+\,\mathcal{M}(a^0 \to VV)^{AAA}_{\rm Der}}_{\neq 0} + \mathcal{M}(a^0 \to VV)_{\rm Jac} = \mathcal{M}(A^0 \to VV)_{\rm Linear}$$

Conclusion

- All decoupled new physics is a non zero Wilson coefficient:

The One-Loop Universal Action is a simplified way to express collider constraints on realistic BSM theories

- Construct EFTs for gauge bosons up to dim. 8 interactions (loop of spin 0, 1/2, 1)
- Spin 1: usual diagrammatic procedure to build effective action breaks down
  ↳ quantized the SM in the non-linear gauge: matching consistent off-shell (closely parallels the CDE path integral method)
- Generalization to QCD gluon and SU(N), $U(1) \otimes SU(N)$, $SU(N) \otimes SU(M)$ boson EFTs
- At one-loop some operators are redundant! no matter the rep. or spin of particle circulating in the loops

- Match the axion decay modes computed using either a linear or a polar representation for the scalar field breaking the $U(1)_{PQ}$ symmetry
- we derived the couplings of axions to gauge bosons, they are not induced by the anomaly
- Could have consequences for ALP searches

spare slides

Euler-Heisenberg Effective Action

« Consequences of Dirac's Theory of the Positron », W. Heisenberg & H. Euler (1936)

- Classical field theory: Lagrangian encapsulates the relevant E.O.M and the symmetries of the system
- Dirac's theory: an E.M. field create pairs of particles which change Maxwell's equations in the vacuum
- QFT: effective Lagrangian encodes quantum corrections to the classical Lagrangian (ex: vacuum polarization)
- E.H. (1936): compute nonperturbative, renormalized, one-loop effective (no e+,e-) action for QED in a classical E.M. background of constant field strength

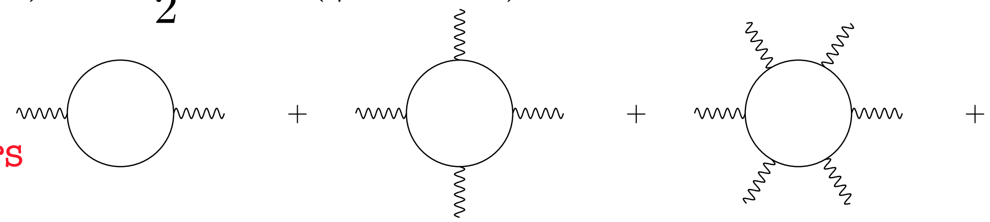
→ leads to several insights and applications

integrate out electron from path integral: $\int \mathcal{D}A \exp\left(i\Gamma[A_\mu]\right) \equiv \int \mathcal{D}A\mathcal{D}\bar{\psi}\mathcal{D}\psi \exp\left[i\int d^4x\left(-\frac{1}{4}F_{\mu\nu}^{\,2} + \bar{\psi}\,(i\not{D} - m)\psi\right)\right]$

one-loop effective action of QED: $S^{(1)} = -i\ln\det(i\not{D} - m) = -\frac{i}{2}\ln\det(\not{D}^2 + m^2)$

Perturbative expansion in powers of the external photon field:

at low energies: effective action in terms of local operators



low energy limit: closed form which generates **all** the perturbative diagrams for the effective action

$$S^{(1)} = -\frac{1}{hc}\int_0^\infty \frac{d\eta}{\eta^3}\, e^{-\eta\, e\mathcal{E}_c}\left\{\frac{e^2 a\, b\, \eta^2}{\tanh(eb\eta)\tan(ea\eta)} - 1 - \frac{e^2\eta^2}{3}(b^2 - a^2)\right\}$$

$$a^2 - b^2 = \vec{E}^2 - \vec{B}^2 = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} \equiv -2\mathcal{F}\ ,$$

$$a\, b = \vec{E}\cdot\vec{B} = -\frac{1}{4}F_{\mu\nu}\tilde{F}^{\mu\nu} \equiv -\mathcal{G}\ .$$

$$a = \sqrt{\sqrt{\mathcal{F}^2 + \mathcal{G}^2} - \mathcal{F}}\ , \quad b = \sqrt{\sqrt{\mathcal{F}^2 + \mathcal{G}^2} + \mathcal{F}}\ .$$

Euler-Heisenberg Effective Action

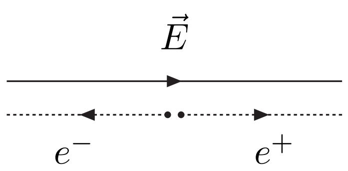
- nonlinear QED processes:

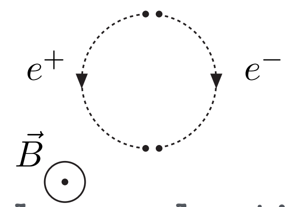
expanding E.H. to quartic order: $S^{(1)} = \frac{e^4}{360\pi^2 m^4} \int d^4x \left[(\vec{E}^2 - \vec{B}^2)^2 + 7(\vec{E}\cdot\vec{B})^2 \right] + \dots$

low energy limit of
light by light scattering
full QED process solved in 1951

nonlinearities $\sim$ dielectric effects, the quantum vacuum behaving as a polarizable medium

- pair-production from vacuum in **E**-field:





E field accelerates and splits virtual vacuum dipole pairs, leading to e+e- particle production

$$\Gamma = 2\, Im\mathcal{L}$$

rate of vacuum non persistence due to pair production

$$\Gamma \sim \frac{e^2 E^2}{4\pi^3} \exp\left[-\frac{m^2\pi}{eE} \right]$$

- charge renormalization, β-function:

E.H.'s result correctly anticipated charge renormalization

subtraction of a log divergent term

$$S^{(1)} = -\frac{1}{hc} \int_0^\infty \frac{d\eta}{\eta^3} e^{-\eta\, e\mathcal{E}_c} \left\{ \frac{e^2 a\, b\, \eta^2}{\tanh(eb\eta)\tan(ea\eta)} - 1 - \frac{e^2\eta^2}{3}(b^2 - a^2) \right\}$$

bare result

$$\frac{\mathcal{L}^{(1)}_{\text{spinor}}}{\mathcal{L}_{\text{Maxwell}}} \sim -\frac{e^2}{12\pi^2} \log\left(\frac{eB}{m^2} \right), \qquad B \to \infty$$

: one-loop QED β-function

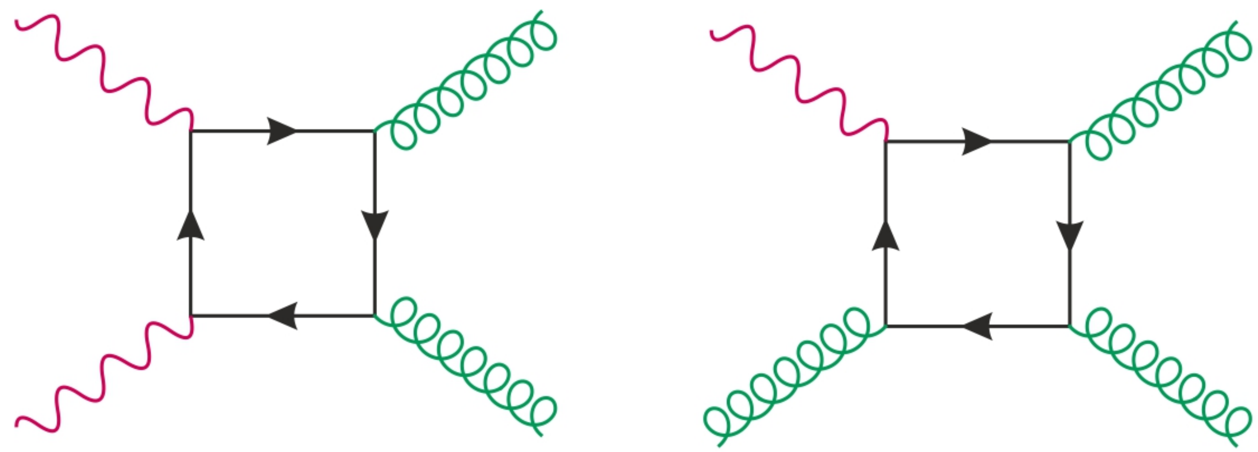
- paradigm of what is now called « low energy EFT »:

describes the physics of light d.o.f at energies much lower than some energy scale (heavy d.o.f. are integrated out)
Lagrangian expanded in terms of gauge and Lorentz invariant operators for the light fields

$$\mathcal{L}_{\text{eff}} = m^4 \sum_n a_n \frac{O^{(n)}}{m^n}$$

at mass dim. 8 : $(F_{\mu\nu}F^{\mu\nu})^2$ or $(F_{\mu\nu}\tilde{F}^{\mu\nu})^2$

Mixed effective interactions



$$\mathfrak{L}^{(4)}_{eff}(\underline{U(1)} \otimes \underline{SU(N)}) = \alpha_1 \frac{g_1^2 g_n^2}{6!\pi^2 m^4} F_{\mu\nu} F^{\mu\nu} G^a_{\rho\sigma} G^{a,\rho\sigma} + \alpha_2 \frac{g_1^2 g_n^2}{6!\pi^2 m^4} F_{\mu\nu} \tilde{F}^{\mu\nu} G^a_{\rho\sigma} \tilde{G}^{a,\rho\sigma}$$
$$+ \alpha_3 \frac{g_1^2 g_n^2}{6!\pi^2 m^4} F_{\mu\nu} G^{a,\mu\nu} F_{\rho\sigma} G^{a,\rho\sigma} + \alpha_4 \frac{g_1^2 g_n^2}{6!\pi^2 m^4} F_{\mu\nu} \tilde{G}^{a,\mu\nu} F_{\rho\sigma} \tilde{G}^{a,\rho\sigma}$$
$$+ \beta_1 \frac{g_1 g_n^3}{6!\pi^2 m^4} d^{abc} F_{\mu\nu} G^{a,\mu\nu} G^b_{\rho\sigma} G^{c,\rho\sigma} + \beta_2 \frac{g_1 g_n^3}{6!\pi^2 m^4} d^{abc} F_{\mu\nu} \tilde{G}^{a,\mu\nu} G^b_{\rho\sigma} \tilde{G}^{c,\rho\sigma}$$

$$\mathfrak{L}^{(4)}_{eff}(\underline{SU(M)} \otimes \underline{SU(N)}) = \alpha_1 \frac{g_m^2 g_n^2}{6!\pi^2 m^4} W^i_{\mu\nu} W^{i,\mu\nu} G^a_{\rho\sigma} G^{a,\rho\sigma} + \alpha_2 \frac{g_m^2 g_n^2}{6!\pi^2 m^4} W^i_{\mu\nu} \tilde{W}^{i,\mu\nu} G^a_{\rho\sigma} \tilde{G}^{a,\rho\sigma}$$
$$+ \alpha_3 \frac{g_m^2 g_n^2}{6!\pi^2 m^4} W^i_{\mu\nu} G^{a,\mu\nu} W^i_{\rho\sigma} G^{a,\rho\sigma} + \alpha_4 \frac{g_m^2 g_n^2}{6!\pi^2 m^4} W^i_{\mu\nu} \tilde{G}^{a,\mu\nu} W^i_{\rho\sigma} \tilde{G}^{a,\rho\sigma}$$

	$\alpha_1 = \alpha_3/2$	$\alpha_2 = \alpha_4/2$	β_1	β_2
Scalar	$\frac{7}{16} Q(\mathbf{R})^2 I_2(\mathbf{R})$	$\frac{1}{16} Q(\mathbf{R})^2 I_2(\mathbf{R})$	$\frac{7}{32} Q(\mathbf{R}) I_3(\mathbf{R})$	$\frac{1}{32} Q(\mathbf{R}) I_3(\mathbf{R})$
Fermion	$Q(\mathbf{R})^2 I_2(\mathbf{R})$	$\frac{7}{4} Q(\mathbf{R})^2 I_2(\mathbf{R})$	$\frac{1}{2} Q(\mathbf{R}) I_3(\mathbf{R})$	$\frac{7}{8} Q(\mathbf{R}) I_3(\mathbf{R})$
Vector	$\frac{261}{16} Q(\mathbf{R})^2 I_2(\mathbf{R})$	$\frac{243}{16} Q(\mathbf{R})^2 I_2(\mathbf{R})$	$\frac{261}{32} Q(\mathbf{R}) I_3(\mathbf{R})$	$\frac{243}{32} Q(\mathbf{R}) I_3(\mathbf{R})$

J.Q., C. Smith and S. Touati, Phys.Rev. D99 (2019) no.1, 013003

Universal coefficients in terms of standard master integrals

$$\int \frac{d^dq}{(2\pi)^d} \frac{q^{\mu_1}\cdots q^{\mu_{2n_c}}}{(q^2-M_i^2)^{n_i}(q^2-M_j^2)^{n_j}\cdots(q^2)^{n_L}} \equiv g^{\mu_1\ldots\mu_{2n_c}}\,\mathcal{I}[q^{2n_c}]^{n_in_j\ldots n_L}_{ij\ldots 0}$$

Universal coefficient	Operator
$f_2^i = \mathcal{I}_i^1$	U_{ii}
$f_3^i = 2\,\mathcal{I}[q^4]_i^4$	$G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$	$U_{ij}U_{ji}$
$f_5^i = 16\,\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu,i}][P_\rho, G_i'^{\rho\nu}]$
$f_6^i = \frac{32}{3}\mathcal{I}[q^6]_i^6$	$G'^{\mu}{}_{\nu,i}G'^{\nu}{}_{\rho,i}G'^{\rho}{}_{\mu,i}$
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{ij}][P_\mu, U_{ji}]$
$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$	$U_{ij}U_{jk}U_{ki}$
$f_9^i = 8\,\mathcal{I}[q^4]_i^5$	$U_{ii}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{ij}U_{jk}U_{kl}U_{li}$
$f_{11}^{ijk} = 2\big(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\big)$	$U_{ij}[P^\mu, U_{jk}][P_\mu, U_{ki}]$
$f_{12}^{ij} = 4\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu,[P_\mu,U_{ij}]]\,[P^\nu,[P_\nu,U_{ji}]]$
$f_{13}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + 2\,\mathcal{I}[q^4]_{ij}^{42} + 2\,\mathcal{I}[q^4]_{ij}^{51}\big)$	$U_{ij}U_{ji}G_i'^{\mu\nu}G'_{\mu\nu,i}$
$f_{14}^{ij} = -8\,\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{ij}][P^\nu, U_{ji}]G'_{\nu\mu,i}$
$f_{15}^{ij} = 4\big(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\big)$	$(U_{ij}[P^\mu, U_{ji}] - [P^\mu, U_{ij}]U_{ji})[P^\nu, G'_{\nu\mu,i}]$
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}_{ijklm}^{11111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mi}$
$f_{17}^{ijkl} = 2\big(\mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\big)$	$U_{ij}U_{jk}[P^\mu, U_{kl}][P_\mu, U_{li}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{ij}[P^\mu, U_{jk}]U_{kl}[P_\mu, U_{li}]$
$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{I}_{ijklmn}^{111111}$	$U_{ij}U_{jk}U_{kl}U_{lm}U_{mn}U_{ni}$

A. Drozd, J. Ellis, JQ and T. You arXiv:1512.03003

for degenerate mass heavy fields

B. Henning, X. Lu and H. Murayama arXiv:1412.1837

$$f_5 = -\frac{i}{(4\pi)^2 60m^2},\quad f_{11} = \frac{i}{(4\pi)^2 12m^4},\quad f_{15a} = \frac{i}{(4\pi)^2 60m^4},$$

$$f_6 = -\frac{i}{(4\pi)^2 90m^2},\quad f_{12,a} = 0,\quad f_{15b} = \frac{i}{(4\pi)^2 60m^4},$$

$$f_7 = -\frac{i}{(4\pi)^2 12m^2},\quad f_{12,b} = 0,\quad f_{16} = -\frac{i}{(4\pi)^2 60m^6},$$

$$f_8 = -\frac{i}{(4\pi)^2 6m^2},\quad f_{12,c} = \frac{i}{(4\pi)^2 120m^4},\quad f_{17} = -\frac{i}{(4\pi)^2 20m^6},$$

$$f_9 = -\frac{i}{(4\pi)^2 12m^2},\quad f_{13} = \frac{i}{(4\pi)^2 24m^4},\quad f_{18} = -\frac{i}{(4\pi)^2 30m^6},$$

$$f_{10} = \frac{i}{(4\pi)^2 24m^4},\quad f_{14} = \frac{-i}{(4\pi)^2 60m^4},\quad f_{19} = \frac{i}{(4\pi)^2 120m^8}.$$

Functional methods: Heavy-Light loops?

- Linear coupling = tree-level, quadratic coupling = *heavy-only* one-loop

$$\mathcal{L}_{\rm UV} = \mathcal{L}_{\rm SM} + (\Phi^\dagger F(x) + \text{h.c.}) + \Phi^\dagger (P^2 - M^2 - U(x))\Phi + \mathcal{O}(\Phi^3)\,,$$

- What about loops involving both heavy and light fields?
- Naively not accounted for in functional method

See e.g. Bilenky & Santamaria, hep-ph/9310302; Del Aguila, Kunszt, Santiago, 1602.00126.

- Solution: apply background field method to both **heavy** *and* **light** fields

$$\phi \to \phi_c + \phi' \quad , \quad \Phi \to \Phi_c + \Phi'$$

$$\mathcal{L}_{\rm quad} = \frac{1}{2}(\Phi', \phi') \begin{pmatrix} P^2 - M^2 - U_{\Phi\Phi} & -U_{\Phi\phi} \\ -U_{\phi\Phi} & P^2 - m^2 - U_{\phi\phi} \end{pmatrix} \begin{pmatrix} \Phi' \\ \phi' \end{pmatrix}$$

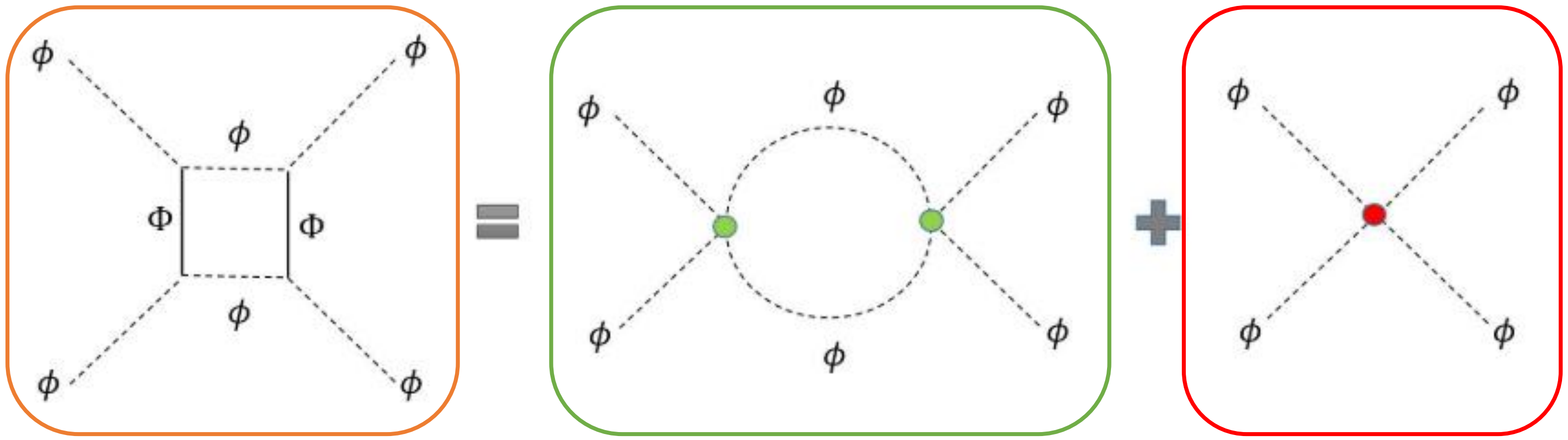
Functional methods: Heavy-Light loops?

- Just apply background field method to both heavy and light fields?

$$\phi \to \phi_c + \phi' \quad , \quad \Phi \to \Phi_c + \Phi'$$

- Actually, this gives the one-loop 1PI effective action and **not** $\mathcal{L}_{\text{eff}}$

  - Feynman diagram intuition: Heavy-light loops in UV theory match onto both **tree-level-generated** EFT operators *inserted at one-loop*, and **one-loop-generated** EFT operators *inserted at tree-level*



(soft part) (hard part)

  - The **former** is not part of $\mathcal{L}_{\text{eff}}$, must be subtracted to keep only the **latter**

Functional methods: Heavy-Light subtractions

- Various subtraction procedures proposed

See e.g. Boggia, Gomez-Ambrisio, Passarino arXiv:1603.03660
B. Henning, X. Lu and H. Murayama arXiv:1604.01019
S.A.R. Ellis, JQ, T. You, Z. Zhang arXiv:1604.02445 → Universality properties also applies to heavy-light case

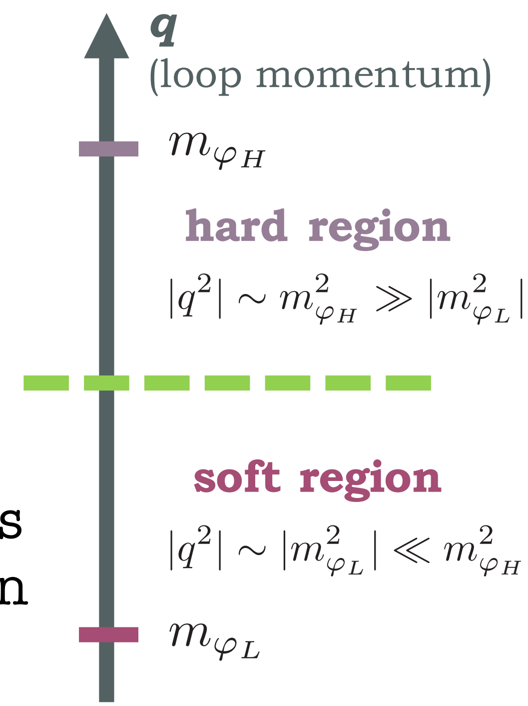
Fuentes-Martin, Portoles, Ruiz-Femenia arXiv:1607.02142

$$\int d^d x \, \mathcal{L}^{\text{1-loop}}_{\text{EFT}}[\varphi_L] \neq \Gamma^{\text{1-loop}}_{\text{L,UV}}[\varphi_L]$$

- **1PI effective actions** include quantum fluctuations at **all scales**

$$\int d^d x \, \mathcal{L}^{\text{1-loop}}_{\text{EFT}}[\varphi_L] = \Gamma^{\text{1-loop}}_{\text{L,UV}}[\varphi_L]\Big|_{\text{hard}}$$

- Extract **short-distance** fluctuations ->**local** operators in EFT Lagrangian



Integration by regions method avoids subtraction, separates **hard** and **soft** part in integral, greatly simplifies heavy-light treatment

- Simplification of evaluating CDE from these developments lead to a **Covariant Diagram** formulation

- But **Universality** of CDE results means evaluation via all these different methods gives same model-independent expression

B. Henning, X. Lu and H. Murayama arXiv:1412.1837
A. Drozd, J. Ellis, JQ and T. You arXiv:1504.02409
S.A.R. Ellis, JQ, T. You, Z. Zhang arXiv:1706.07765

Universal One-Loop Effective Action

- with **Heavy-Light** extension S.A.R. Ellis, JQ, T. You, Z. Zhang arXiv:1706.07765

$\mathcal{O}(U_H^4 P^2)$ terms	
$f_{17}^{ijkl} = 2\left(\mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1212} + \mathcal{I}[q^2]_{ijkl}^{1122}\right)$	$U_{Hij}U_{Hjk}[P^\mu, U_{Hkl}][P_\mu, U_{Hli}]$
$f_{18}^{ijkl} = \mathcal{I}[q^2]_{ijkl}^{2121} + \mathcal{I}[q^2]_{ijkl}^{2112} + \mathcal{I}[q^2]_{ijkl}^{1221} + \mathcal{I}[q^2]_{ijkl}^{1212}$	$U_{Hij}[P^\mu, U_{Hjk}]U_{Hkl}[P_\mu, U_{Hli}]$
$\mathcal{O}(U_H^2 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{17A}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk0}^{1122} + \mathcal{I}[q^2]_{ijk0}^{1221} + \mathcal{I}[q^2]_{ijk0}^{2121}\right)$	$U_{Hij}U_{HLji'}[P^\mu, U_{LHi'k}][P_\mu, U_{Hki}]$ $+U_{LHi'i}U_{Hij}[P^\mu, U_{Hjk}][P_\mu, U_{HLki'}]$
$f_{17B}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk0}^{1122} + \mathcal{I}[q^2]_{ijk0}^{1212} + \mathcal{I}[q^2]_{ijk0}^{2112}\right)$	$U_{Hij}U_{Hjk}[P^\mu, U_{HLki'}][P_\mu, U_{LHi'i}]$
$f_{17C}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk0}^{1122} + \mathcal{I}[q^2]_{ijk0}^{2121} + \mathcal{I}[q^2]_{ijk0}^{1221}\right)$	$U_{HLii'}U_{LHi'j}[P^\mu, U_{Hjk}], [P_\mu, U_{Hki}]$
$f_{18A}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk0}^{1221} + \mathcal{I}[q^2]_{ijk0}^{2121} + \mathcal{I}[q^2]_{ijk0}^{1212} + \mathcal{I}[q^2]_{ijk0}^{2112}\right)$	$U_{Hij}[P^\mu, U_{HLji'}]U_{LHi'k}[P_\mu, U_{Hki}]$ $+U_{Hij}[P^\mu, U_{Hjk}]U_{HLki'}[P_\mu, U_{LHi'i}]$
$\mathcal{O}(U_H^1 U_L^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{17D}^{ij} = 2\left(2\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{222}\right)$	$U_{HLii'}U_{Li'j'}[P^\mu, U_{LHj'j}][P_\mu, U_{Hji}]$ $+U_{Li'j'}U_{LHj'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}]$
$f_{17E}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{114} + \mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{213}\right)$	$U_{Hij}U_{HLji'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}]$ $+U_{LHi'i}U_{Hij}[P^\mu, U_{HLjj'}][P_\mu, U_{Lj'i'}]$
$f_{18B}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{222} + \mathcal{I}[q^2]_{ij0}^{114} + \mathcal{I}[q^2]_{ij0}^{213}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}]U_{LHj'j}[P_\mu, U_{Hji}]$
$f_{18C}^{ij} = 4\left(\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{213}\right)$	$U_{Hij}[P^\mu, U_{HLji'}]U_{Li'j'}[P_\mu, U_{LHj'i}]$
$\mathcal{O}(U_L^2 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{17F}^{i} = 2\left(2\mathcal{I}[q^2]_{i0}^{15} + \mathcal{I}[q^2]_{i0}^{24}\right)$	$U_{HLii'}U_{Li'j'}[P^\mu, U_{Lj'k'}][P_\mu, U_{LHk'i}]$ $+U_{Li'j'}U_{LHj'i}[P^\mu, U_{HLik'}][P_\mu, U_{Lk'i'}]$
$f_{17G}^{i} = 2\left(2\mathcal{I}[q^2]_{i0}^{15} + \mathcal{I}[q^2]_{i0}^{24}\right)$	$U_{LHi'i}U_{HLij'}[P^\mu, U_{Lj'k'}][P_\mu, U_{Lk'i'}]$
$f_{17H}^{i} = 6\mathcal{I}[q^2]_{i0}^{24}$	$U_{Li'j'}U_{Lj'k'}[P^\mu, U_{LHk'i}][P_\mu, U_{HLii'}]$
$f_{18D}^{i} = 4\left(\mathcal{I}[q^2]_{i0}^{15} + \mathcal{I}[q^2]_{i0}^{24}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}]U_{Lj'k'}[P_\mu, U_{LHk'i}]$ $+U_{LHi'i}[P^\mu, U_{HLij'}]U_{Lj'k'}[P_\mu, U_{Lk'i}]$
$\mathcal{O}(U_{HL}^2 U_{LH}^2 P^2)$ terms	
$f_{17I}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{114} + \mathcal{I}[q^2]_{ij0}^{213} + \mathcal{I}[q^2]_{ij0}^{123}\right)$	$U_{HLii'}U_{LHi'j}[P^\mu, U_{HLjj'}][P_\mu, U_{LHj'i}]$
$f_{17J}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{222} + 2\mathcal{I}[q^2]_{ij0}^{123}\right)$	$U_{LHi'i}U_{HLij'}[P^\mu, U_{LHj'j}][P_\mu, U_{HLji'}]$
$f_{18E}^{ij} = \mathcal{I}[q^2]_{ij0}^{114} + 2\mathcal{I}[q^2]_{ij0}^{123} + \mathcal{I}[q^2]_{ij0}^{222}$	$U_{HLii'}[P^\mu, U_{LHi'j}]U_{HLjj'}[P_\mu, U_{LHj'i}]$ $+U_{LHi'i}[P^\mu, U_{HLij'}]U_{LHj'j}[P_\mu, U_{HLji'}]$

$\mathcal{O}(U_H^3 P^2)$ terms	
$f_{11}^{ijk} = 2\left(\mathcal{I}[q^2]_{ijk}^{122} + \mathcal{I}[q^2]_{ijk}^{212}\right)$	$U_{Hij}[P^\mu, U_{Hjk}][P_\mu, U_{Hki}]$
$\mathcal{O}(U_H^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{11A}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{122} + \mathcal{I}[q^2]_{ij0}^{212}\right)$	$U_{Hij}[P^\mu, U_{HLji'}][P_\mu, U_{LHi'i}$
$f_{11B}^{ij} = 2\left(\mathcal{I}[q^2]_{ij0}^{221} + \mathcal{I}[q^2]_{ij0}^{122}\right)$	$U_{LHi'i}[P^\mu, U_{Hij}][P_\mu, U_{HLji'}] + U_{HLii'}[P^\mu, U_{LHi'j}][P_\mu, U_{Hji}]$
$\mathcal{O}(U_L^1 U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{11C}^{ij} = 4\mathcal{I}[q^2]_{i0}^{23}$	$U_{Li'j'}[P^\mu, U_{LHj'i}][P_\mu, U_{HLii'}]$
$f_{11D}^{ij} = 2\left(\mathcal{I}[q^2]_{i0}^{14} + \mathcal{I}[q^2]_{i0}^{23}\right)$	$U_{HLii'}[P^\mu, U_{Li'j'}][P_\mu, U_{LHj'i}] + U_{LHi'i}[P^\mu, U_{HLij'}][P_\mu, U_{Lj'i'}]$

$\mathcal{O}(U_H^2 P^4)$ terms	
$f_{12}^{ij} = 4\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, [P_\mu, U_{Hij}]][P^\nu, [P_\nu, U_{Hji}]]$
$f_{13}^{ij} = 4\left(\mathcal{I}[q^4]_{ij}^{33} + 2\mathcal{I}[q^4]_{ij}^{33} + 2\mathcal{I}[q^4]_{ij}^{51}\right)$	$U_{Hij}U_{Hji}G'^{\mu\nu}_i G'_{\mu\nu_i}$
$f_{14}^{ij} = -8\mathcal{I}[q^4]_{ij}^{33}$	$[P^\mu, U_{Hij}][P^\nu, U_{Hji}]G'_{\nu\mu_i}$
$f_{15}^{ij} = 4\left(\mathcal{I}[q^4]_{ij}^{33} + \mathcal{I}[q^4]_{ij}^{42}\right)$	$\left(U_{Hij}[P^\mu, U_{Hji}] - [P^\mu, U_{Hij}]U_{Hji}\right)[P^\nu, G'_{\nu\mu_i}]$
$\mathcal{O}(U_{HL}^1 U_{LH}^1 P^4)$ terms	
$f_{12A}^{i} = 8\mathcal{I}[q^4]_{i0}^{33}$	$[P^\mu, [P_\mu, U_{HLii'}]][P^\nu, [P_\nu, U_{LHi'i}]]$
$f_{13A}^{i} = 2\left(\mathcal{I}[q^4]_{i0}^{24} + 2\mathcal{I}[q^4]_{i0}^{33} + 3\mathcal{I}[q^4]_{i0}^{42} + 4\mathcal{I}[q^4]_{i0}^{51}\right)$	$U_{HLii'}U_{LHi'i}G'^{\mu\nu}_i G'_{\mu\nu_i}$
$f_{13B}^{i} = 2\left(4\mathcal{I}[q^4]_{i0}^{15} + 3\mathcal{I}[q^4]_{i0}^{24} + 2\mathcal{I}[q^4]_{i0}^{33} + \mathcal{I}[q^4]_{i0}^{42}\right)$	$U_{LHi'i}U_{HLii'}G'^{\mu\nu}_{i'} G'_{\mu\nu_{i'}}$
$f_{14A}^{i} = 4\left(-\mathcal{I}[q^4]_{i0}^{24} - 2\mathcal{I}[q^4]_{i0}^{33} + \mathcal{I}[q^4]_{i0}^{42}\right)$	$[P^\mu, U_{HLii'}][P^\nu, U_{LHi'i}]G'_{\nu\mu_i}$
$f_{14B}^{i} = 4\left(\mathcal{I}[q^4]_{i0}^{24} - 2\mathcal{I}[q^4]_{i0}^{33} - \mathcal{I}[q^4]_{i0}^{42}\right)$	$[P^\mu, U_{LHi'i}][P^\nu, U_{HLii'}]G'_{\nu\mu_{i'}}$
$f_{15A}^{i} = 2\left(\mathcal{I}[q^4]_{i0}^{24} + 2\mathcal{I}[q^4]_{i0}^{33} + \mathcal{I}[q^4]_{i0}^{42}\right)$	$\left(U_{HLii'}[P^\mu, U_{LHi'i}] - [P^\mu, U_{HLii'}]U_{LHi'i}\right)[P^\nu, G'_{\nu\mu_i}]$ $+\left(U_{LHi'i}[P^\mu, U_{HLii'}] - [P^\mu, U_{LHi'i}]U_{HLii'}\right)[P^\nu, G'_{\nu\mu_{i'}}]$

$\mathcal{O}(U)$ term		$\mathcal{O}(U^3)$ terms	
$f_2^i = \mathcal{I}_i^1$	U_{Hii}	$f_8^{ijk} = \frac{1}{3}\mathcal{I}_{ijk}^{111}$	$U_{Hij}U_{Hjk}U_{Hki}$
$\mathcal{O}(U^2)$ terms		$f_{8A}^{ij} = \mathcal{I}_{ij0}^{111}$	$U_{Hij}U_{HLji'}U_{LHi'i}$
$f_4^{ij} = \frac{1}{2}\mathcal{I}_{ij}^{11}$	$U_{Hij}U_{Hji}$	$f_{8B}^{i} = \mathcal{I}_{i0}^{12}$	$U_{HLii'}U_{Li'j'}U_{LHj'i}$
$f_{4A}^{ij} = \mathcal{I}_{i0}^{11}$	$U_{HLii'}U_{LHi'i}$	$\mathcal{O}(U^6)$ terms	
$\mathcal{O}(U^4)$ terms		$f_{19}^{ijklmn} = \frac{1}{6}\mathcal{I}_{ijklmn}^{111111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmn}U_{Hni}$
$f_{10}^{ijkl} = \frac{1}{4}\mathcal{I}_{ijkl}^{1111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hli}$	$f_{19A}^{ijklm} = \mathcal{I}_{ijklm0}^{111111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{HLmi'}U_{LHi'i}$
$f_{10A}^{ijk} = \mathcal{I}_{ijk0}^{1111}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'i}$	$f_{19B}^{ijkl} = \mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{Li'j'}U_{LHj'i}$
$f_{10B}^{ij} = \mathcal{I}_{ij0}^{112}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'i}$	$f_{19C}^{ijkl} = \mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{LHi'l}U_{HLlj'}U_{LHj'i}$
$f_{10C}^{ij} = \frac{1}{2}\mathcal{I}_{ij0}^{112}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'i}$	$f_{19D}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$
$f_{10D}^{i} = \mathcal{I}_{i0}^{13}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f_{19E}^{ijkl} = \frac{1}{2}\mathcal{I}_{ijkl0}^{11112}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{Hkl}U_{HLlj'}U_{LHj'i}$
$\mathcal{O}(U^5)$ terms		$f_{19F}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{Lj'k'}U_{LHk'i}$
$f_{16}^{ijklm} = \frac{1}{5}\mathcal{I}_{ijklm}^{11111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{Hlm}U_{Hmi}$	$f_{19G}^{ijk} = \mathcal{I}_{ijk0}^{1113}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{16A}^{ijkl} = \mathcal{I}_{ijkl0}^{11111}$	$U_{Hij}U_{Hjk}U_{Hkl}U_{HLli'}U_{LHi'i}$	$f_{19H}^{ij} = \mathcal{I}_{ij0}^{114}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{16B}^{ijk} = \mathcal{I}_{ijk0}^{1112}$	$U_{Hij}U_{Hjk}U_{HLki'}U_{Li'j'}U_{LHj'i}$	$f_{19I}^{ijk} = \frac{1}{3}\mathcal{I}_{ijk0}^{1113}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{LHj'k}U_{HLkk'}U_{LHk'i}$
$f_{16C}^{ijk} = \mathcal{I}_{ijk0}^{1112}$	$U_{Hij}U_{HLji'}U_{LHi'k}U_{HLkj'}U_{LHj'i}$	$f_{19J}^{ij} = \mathcal{I}_{ij0}^{114}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$
$f_{16D}^{ij} = \mathcal{I}_{ij0}^{113}$	$U_{Hij}U_{HLji'}U_{Li'j'}U_{Lj'k'}U_{LHk'i}$	$f_{19K}^{ij} = \frac{1}{2}\mathcal{I}_{ij0}^{114}$	$U_{HLii'}U_{Li'j'}U_{LHj'j}U_{HLjk'}U_{Lk'l'}U_{LHl'i}$
$f_{16E}^{ij} = \mathcal{I}_{ij0}^{113}$	$U_{HLii'}U_{LHi'j}U_{HLjj'}U_{Lj'k'}U_{LHk'i}$	$f_{19L}^{i} = \mathcal{I}_{i0}^{15}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{Ll'm'}U_{LHm'i}$
$f_{16F}^{i} = \mathcal{I}_{i0}^{14}$	$U_{HLii'}U_{Li'j'}U_{Lj'k'}U_{Lk'l'}U_{LHl'i}$		

P-only terms	
$f_3^i = 2\mathcal{I}[q^4]_i^4$	$G'^{\mu\nu}_i G'_{\mu\nu_i}$
$f_5^i = 16\mathcal{I}[q^6]_i^6$	$[P^\mu, G'_{\mu\nu_i}][P_\rho, G'^{\rho\nu}_i]$
$f_6^i = (32/3)\mathcal{I}[q^6]_i^6$	$G'^\mu_{\nu i} G'^\nu_{\rho i} G'^\rho_{\mu i}$

$\mathcal{O}(U_H^2 P^2)$ terms	
$f_7^{ij} = \mathcal{I}[q^2]_{ij}^{22}$	$[P^\mu, U_{Hij}][P_\mu, U_{Hji}]$
$\mathcal{O}(U_{HL}^1 U_{LH}^1 P^2)$ terms	
$f_{7A}^{ij} = 2\mathcal{I}[q^2]_{i0}^{22}$	$[P^\mu, U_{HLii'}][P_\mu, U_{LHi'i}]$

$\mathcal{O}(U_H^1 P^4)$ terms	
$f_9^i = 8\mathcal{I}[q^4]_i^5$	$U_{Hij}G'^{\mu\nu}_i G'_{\mu\nu_i}$

Application: matching SM-EFT vs UV model

susy first...

Let's match dim6-EFT and the MSSM :

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \sum_i c_i \frac{\mathcal{O}_i}{\Lambda^2}$$

$m_{\rm stop}$

$$\begin{aligned}
\mathcal{O}_{GG} &= g_s^2 \, |H|^2 G^a_{\mu\nu} G^{a,\mu\nu} & \mathcal{O}_H &= \tfrac{1}{2}(\partial_\mu |H|^2)^2 \\
\mathcal{O}_{WW} &= g^2 \, |H|^2 W^a_{\mu\nu} W^{a,\mu\nu} & \mathcal{O}_T &= \tfrac{1}{2}(H^\dagger \overleftrightarrow{D}_\mu H)^2 \\
\mathcal{O}_{BB} &= g'^2 \, |H|^2 B_{\mu\nu} B^{\mu\nu} & \mathcal{O}_R &= |H|^2 |D_\mu H|^2 \\
\mathcal{O}_{WB} &= 2 g g' H^\dagger t^a H W^a_{\mu\nu} B^{\mu\nu} & \mathcal{O}_D &= |D^2 H|^2 \\
\mathcal{O}_W &= ig(H^\dagger t^a \overleftrightarrow{D}^\mu H) D^\nu W^a_{\mu\nu} & \mathcal{O}_6 &= |H|^6 \\
\mathcal{O}_B &= ig' Y_H (H^\dagger \overleftrightarrow{D}^\mu H) \partial^\nu B_{\mu\nu} & \mathcal{O}_{2G} &= -\tfrac{1}{2}(D^\mu G^a_{\mu\nu})^2 \\
\mathcal{O}_{3G} &= \tfrac{1}{3!} g_s f^{abc} G^{a\mu}_\rho G^{b\nu}_\mu G^{c\rho}_\nu & \mathcal{O}_{2W} &= -\tfrac{1}{2}(D^\mu W^a_{\mu\nu})^2 \\
\mathcal{O}_{3W} &= \tfrac{1}{3!} g \epsilon^{abc} W^{a\mu}_\rho W^{b\nu}_\mu W^{c\rho}_\nu & \mathcal{O}_{2B} &= -\tfrac{1}{2}(\partial^\mu B_{\mu\nu})^2
\end{aligned}$$

$$c_{GG} = \frac{h_t^2}{(4\pi)^2}\frac{1}{12}\left[\left(1+\frac{1}{12}\frac{g'^2 c_{2\beta}}{h_t^2}\right) - \frac{1}{2}\frac{X_t^2}{m_{\tilde t}^2}\right] \qquad c_{WB} = -\frac{h_t^2}{(4\pi)^2}\frac{1}{24}\left[\left(1+\frac{1}{2}\frac{g^2 c_{2\beta}}{h_t^2}\right) - \frac{4}{5}\frac{X_t^2}{m_{\tilde t}^2}\right]$$

$$c_{WW} = \frac{h_t^2}{(4\pi)^2}\frac{1}{16}\left[\left(1-\frac{1}{6}\frac{g'^2 c_{2\beta}}{h_t^2}\right) - \frac{2}{5}\frac{X_t^2}{m_{\tilde t}^2}\right] \qquad c_W = \frac{h_t^2}{(4\pi)^2}\frac{1}{40}\frac{X_t^2}{m_{\tilde t}^2}$$

$$c_{BB} = \frac{h_t^2}{(4\pi)^2}\frac{17}{144}\left[\left(1+\frac{31}{102}\frac{g'^2 c_{2\beta}}{h_t^2}\right) - \frac{38}{85}\frac{X_t^2}{m_{\tilde t}^2}\right] \qquad c_B = \frac{h_t^2}{(4\pi)^2}\frac{1}{40}\frac{X_t^2}{m_{\tilde t}^2}$$

$$c_{3G} = \frac{g_s^2}{(4\pi)^2}\frac{1}{20} \qquad c_{3W} = \frac{g^2}{(4\pi)^2}\frac{1}{20} \qquad c_{2G} = \frac{g_s^2}{(4\pi)^2}\frac{1}{20} \qquad c_{2W} = \frac{g^2}{(4\pi)^2}\frac{1}{20} \qquad c_{2B} = \frac{g'^2}{(4\pi)^2}\frac{1}{20}$$

$$c_H = \frac{h_t^4}{(4\pi)^2}\frac{3}{4}\left[\left(1+\frac{1}{3}\frac{g'^2 c_{2\beta}}{h_t^2}+\frac{1}{12}\frac{g'^4 c_{2\beta}^2}{h_t^4}\right) - \frac{7}{6}\frac{X_t^2}{m_{\tilde t}^2}\left(1+\frac{1}{14}\frac{(g^2+2g'^2)c_{2\beta}}{h_t^2}\right) + \frac{7}{30}\frac{X_t^4}{m_{\tilde t}^2}\right]$$

$$c_T = \frac{h_t^4}{(4\pi)^2}\frac{1}{4}\left[\left(1+\frac{1}{2}\frac{g^2 c_{2\beta}}{h_t^2}\right)^2 - \frac{1}{2}\frac{X_t^2}{m_{\tilde t}^2}\left(1+\frac{1}{2}\frac{g^2 c_{2\beta}}{h_t^2}\right) + \frac{1}{10}\frac{X_t^4}{m_{\tilde t}^4}\right]$$

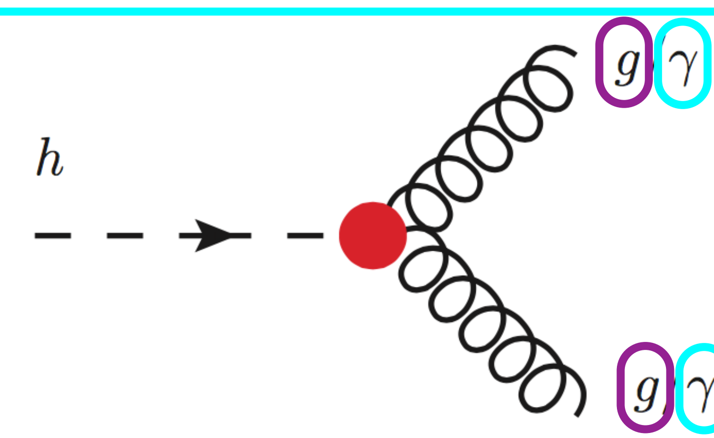
$$c_R = \frac{h_t^4}{(4\pi)^2}\frac{1}{2}\left[\left(1+\frac{1}{2}\frac{g^2 c_{2\beta}}{h_t^2}\right)^2 - \frac{3}{2}\frac{X_t^2}{m_{\tilde t}^2}\left(1+\frac{1}{12}\frac{(3g^2+g'^2)c_{2\beta}}{h_t^2}\right) + \frac{3}{10}\frac{X_t^4}{m_{\tilde t}^4}\right]$$

$$c_D = \frac{h_t^2}{(4\pi)^2}\frac{1}{20}\frac{X_t^2}{m_{\tilde t}^2}$$

$$c_6 = -\frac{h_t^6}{(4\pi)^2}\frac{1}{2}\left\{\begin{array}{l}\left[1+\frac{1}{12}\frac{(3g^2-g'^2)c_{2\beta}}{h_t^2}\right]^3 + \left[-\frac{1}{12}\frac{(3g^2+g'^2)c_{2\beta}}{h_t^2}\right]^3 + \left(1+\frac{1}{3}\frac{g'^2 c_{2\beta}}{h_t^2}\right)^3 \\ -\frac{X_t^2}{m_{\tilde t}^2}\left[2\left(1+\frac{1}{12}\frac{(3g^2-g'^2)c_{2\beta}}{h_t^2}\right)\left(1+\frac{1}{8}\frac{(g^2+g'^2)c_{2\beta}}{h_t^2}\right) + \left(1+\frac{1}{3}\frac{g'^2 c_{2\beta}}{h_t^2}\right)^2\right] \\ +\frac{X_t^4}{m_{\tilde t}^4}\left[1+\frac{1}{8}\frac{(g^2+g'^2)c_{2\beta}}{h_t^2}\right] - \frac{X_t^6}{m_{\tilde t}^6}\frac{1}{10}\end{array}\right\}$$

B. Henning, X. Lu and H. Murayama
arXiv:1412.1837
Wilson coef. for degenerate stops

$$\mathcal{O}_\gamma = \mathcal{O}_{BB} + \mathcal{O}_{WW} - \mathcal{O}_{WB}$$

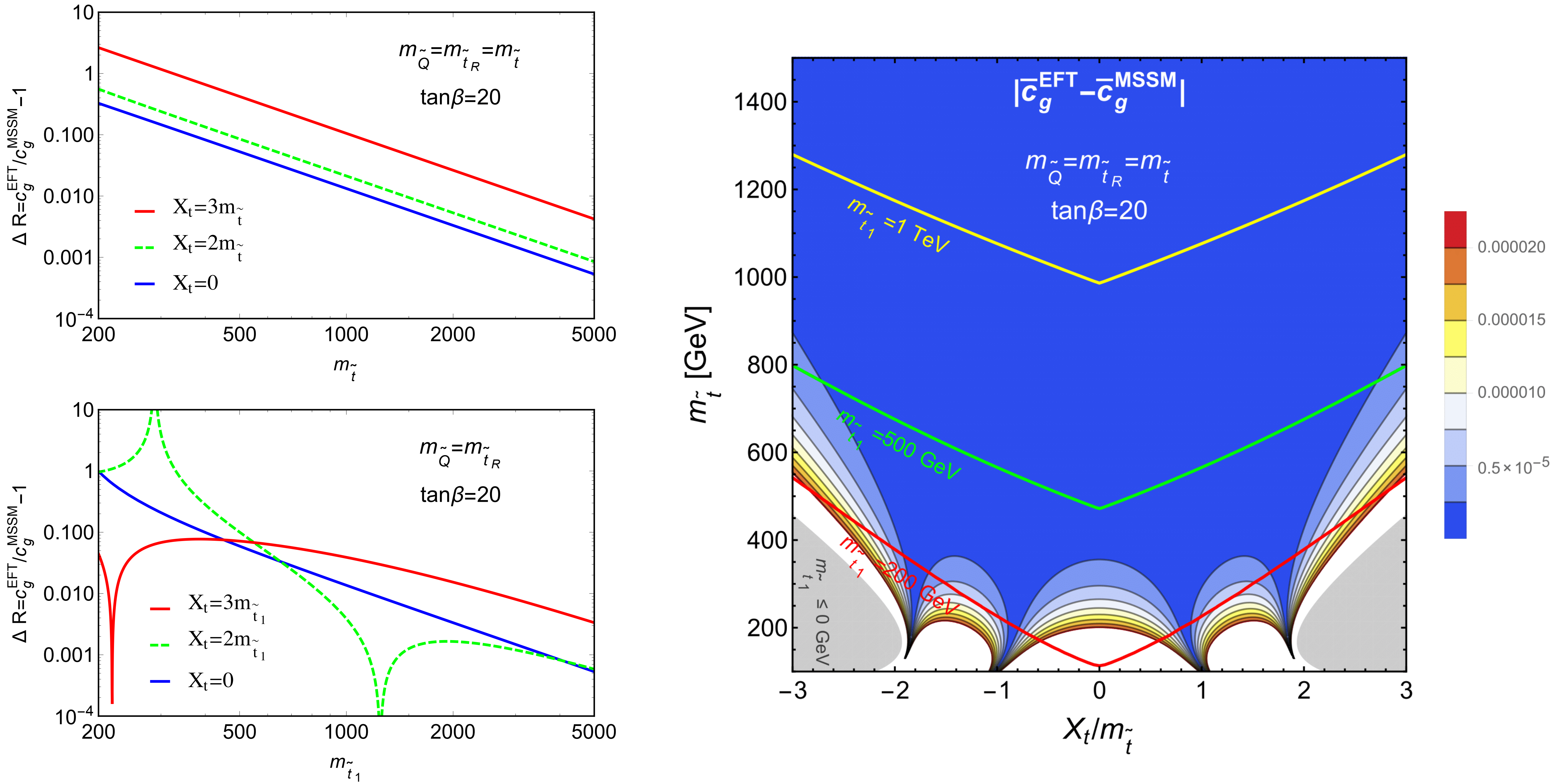


A. Drozd, J. Ellis, JQ and T. You
arXiv:1504.02409
Wilson coef. for non-degenerate stops

EFT vs Loop Calculation

+ EFT vs full MSSM calculation agrees well (non trivial check!)

+ Operators > dim-6 become important when EFT cut-off (stop mass) is too low

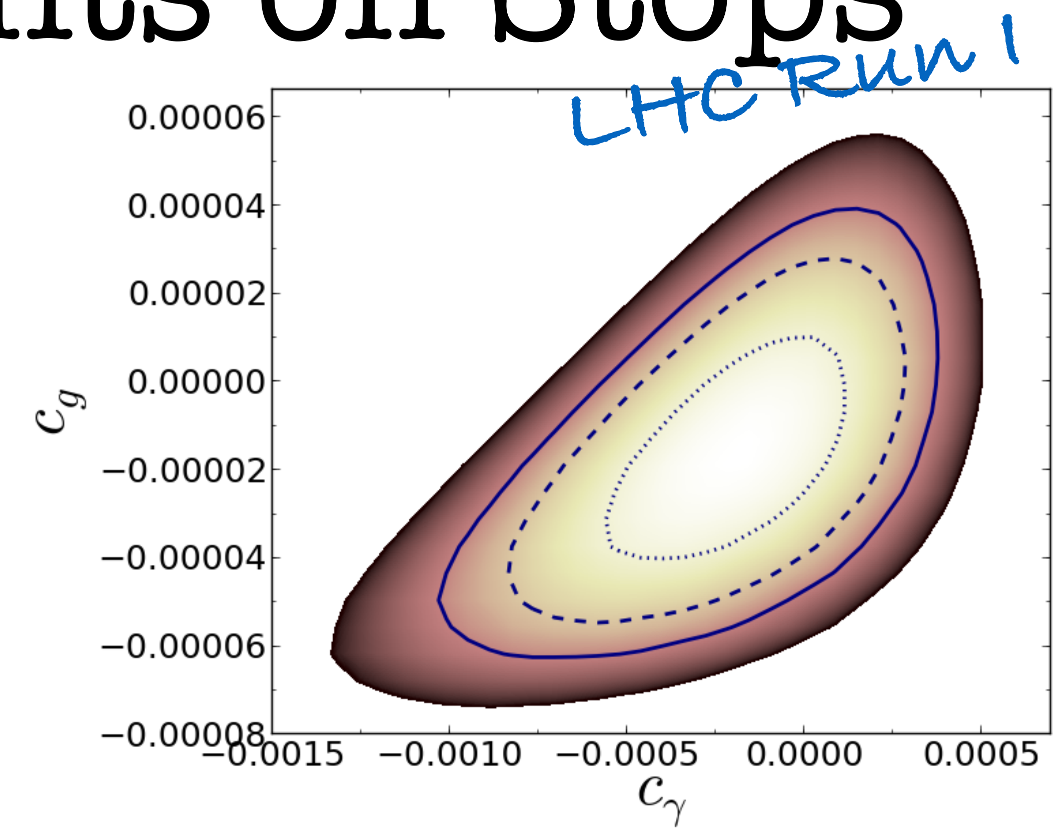


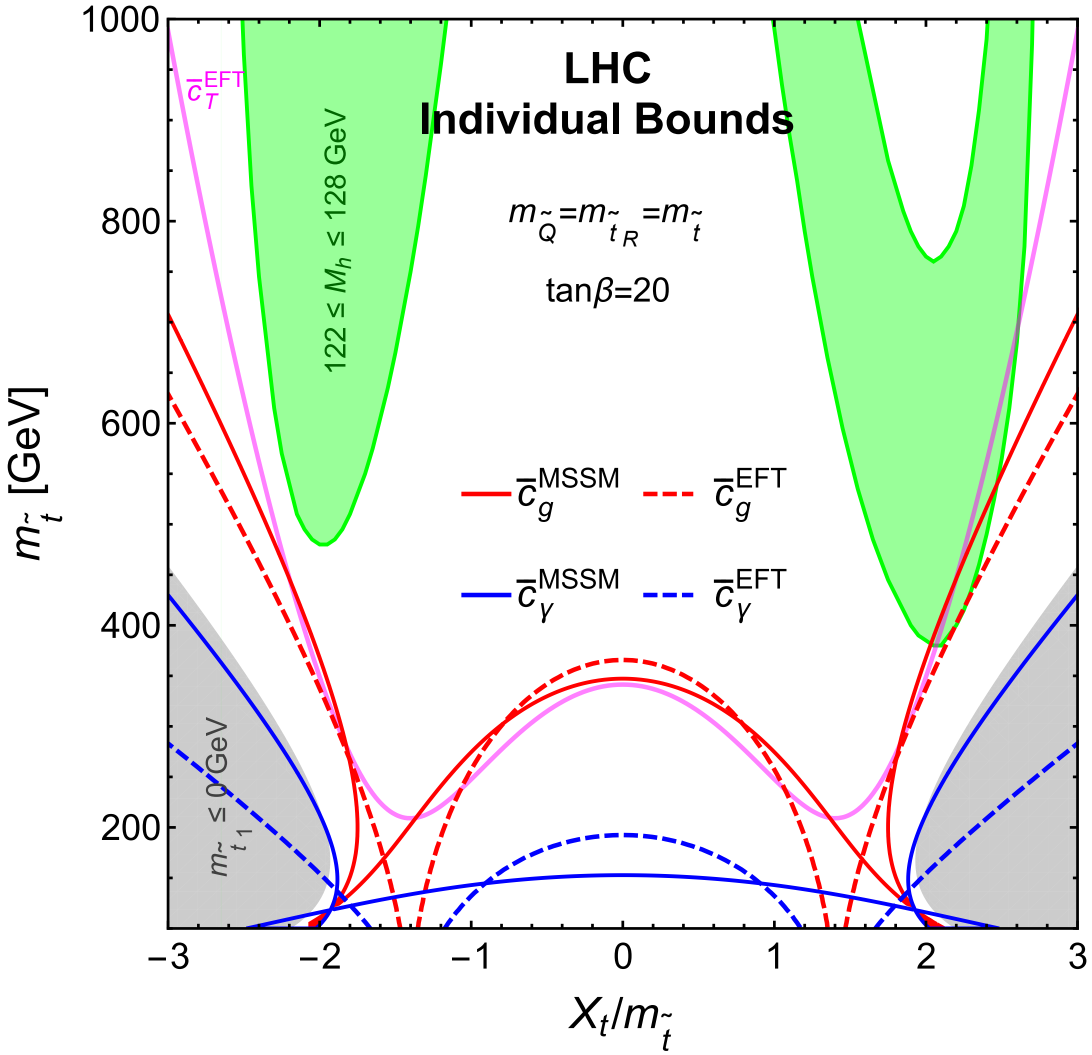
A. Drozd, J. Ellis, JQ and T. You arXiv:1504.02409

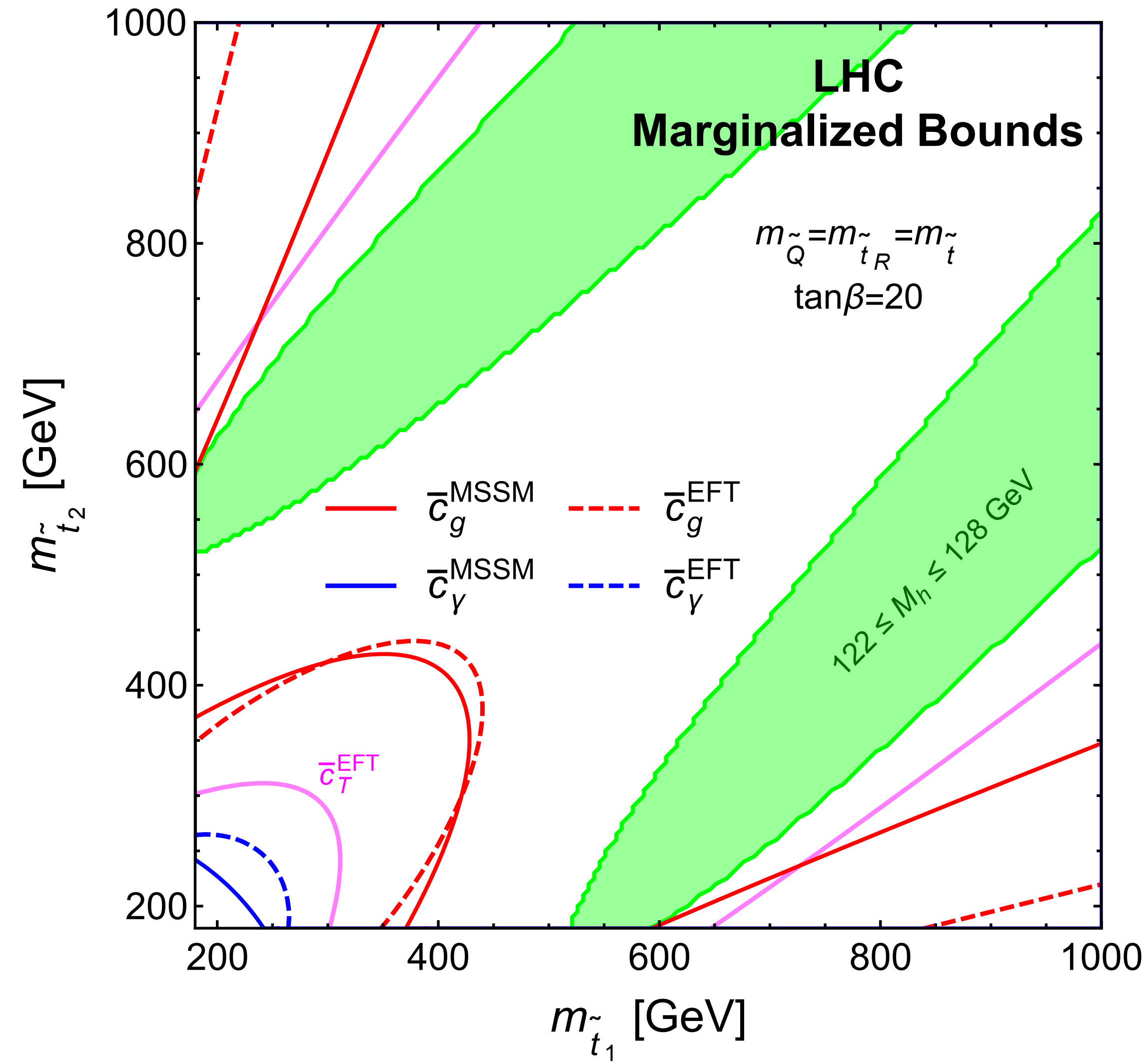
Indirect Constraints on Stops

A. Drozd, J. Ellis, JQ and T. You arXiv:1504.02409

Coeff.	Experimental constraints		95 % CL limit	deg. $m_{\tilde{t}_1}$, $X_t = 0$
$\bar{c}_g$	LHC	marginalized	$[-4.5, 2.2] \times 10^{-5}$	~ 410 GeV
		individual	$[-3.0, 2.5] \times 10^{-5}$	~ 390 GeV
$\bar{c}_\gamma$	LHC	marginalized	$[-6.5, 2.7] \times 10^{-4}$	~ 215 GeV
		individual	$[-4.0, 2.3] \times 10^{-4}$	~ 230 GeV
$\bar{c}_T$	LEP	marginalized	$[-10, 10] \times 10^{-4}$	~ 290 GeV
		individual	$[-5, 5] \times 10^{-4}$	~ 380 GeV
$\bar{c}_W + \bar{c}_B$	LEP	marginalized	$[-7, 7] \times 10^{-4}$	~ 185 GeV
		individual	$[-5, 5] \times 10^{-4}$	~ 195 GeV

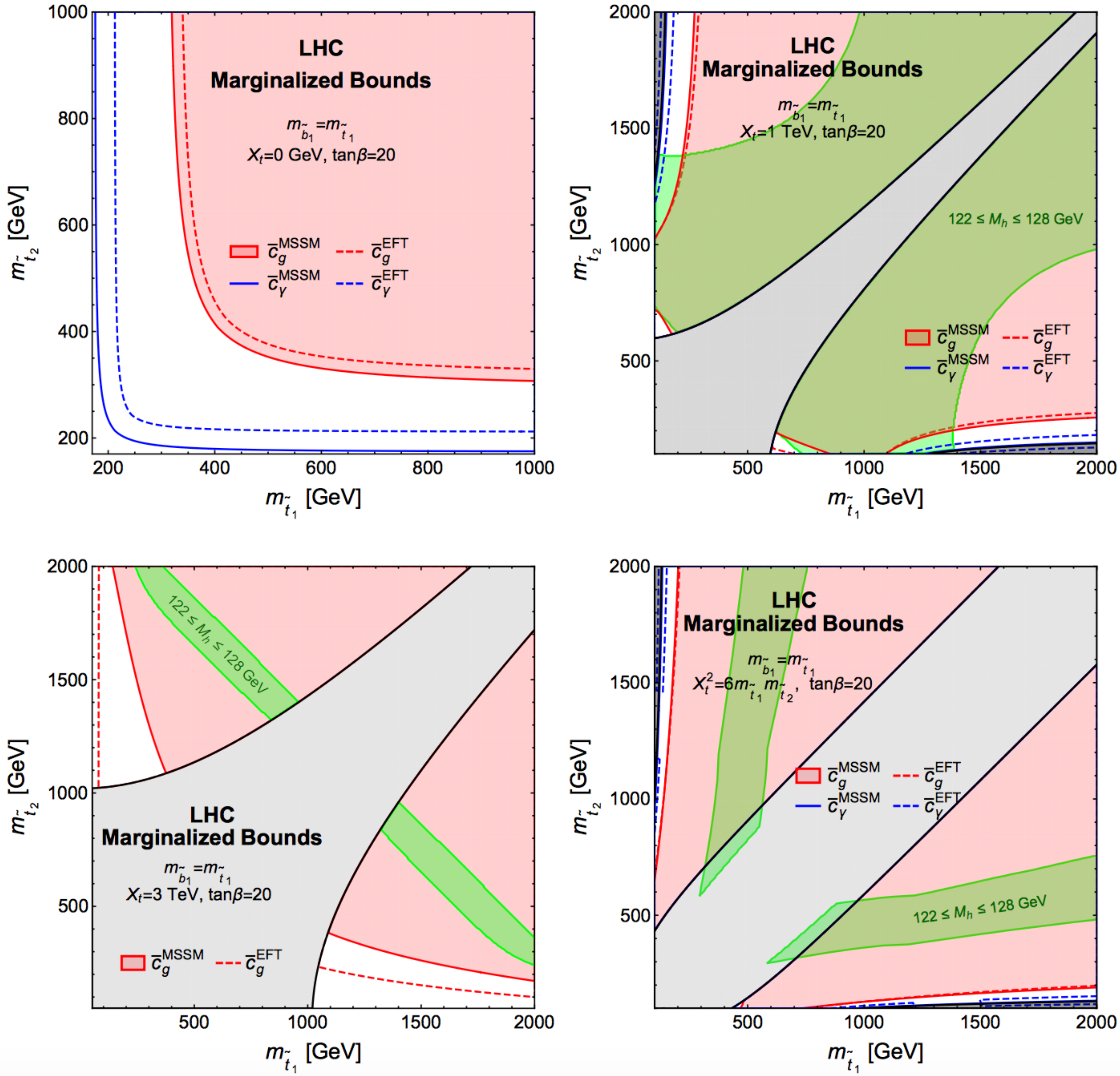






Indirect Constraints on Stops

LHC Run I



A. Drozd, J. Ellis, JQ and T. You arXiv:1504.02409

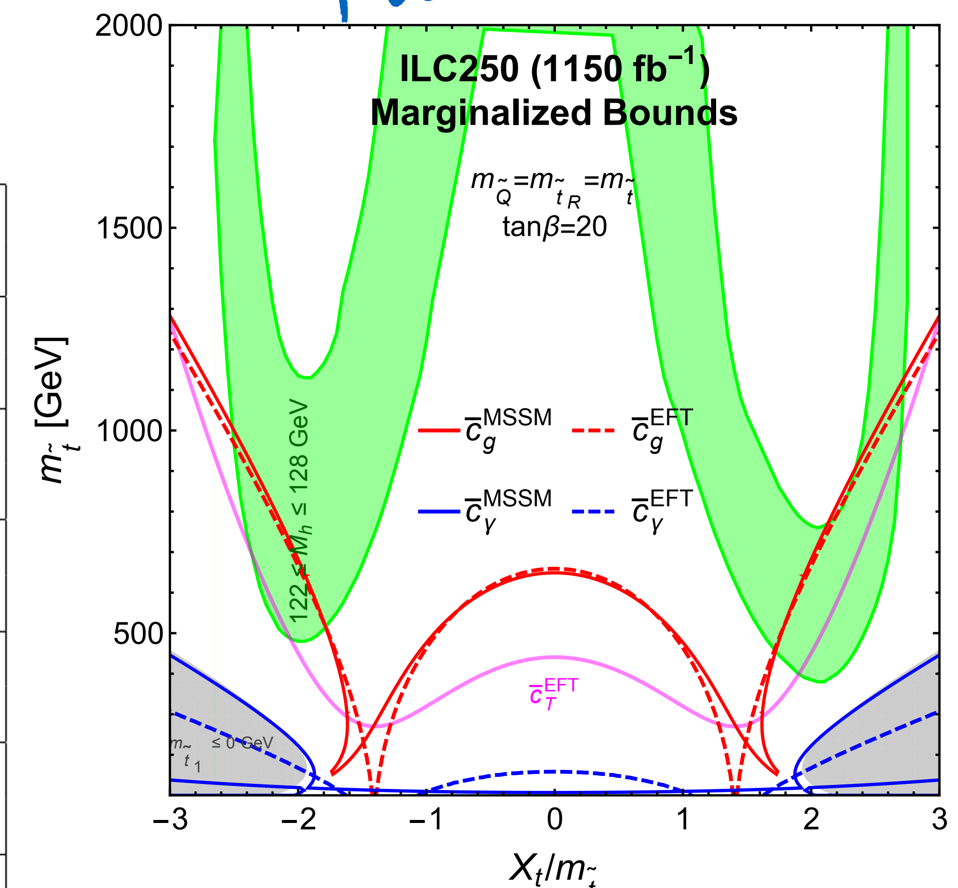
The current sensitivity is already comparable to that of direct LHC searches

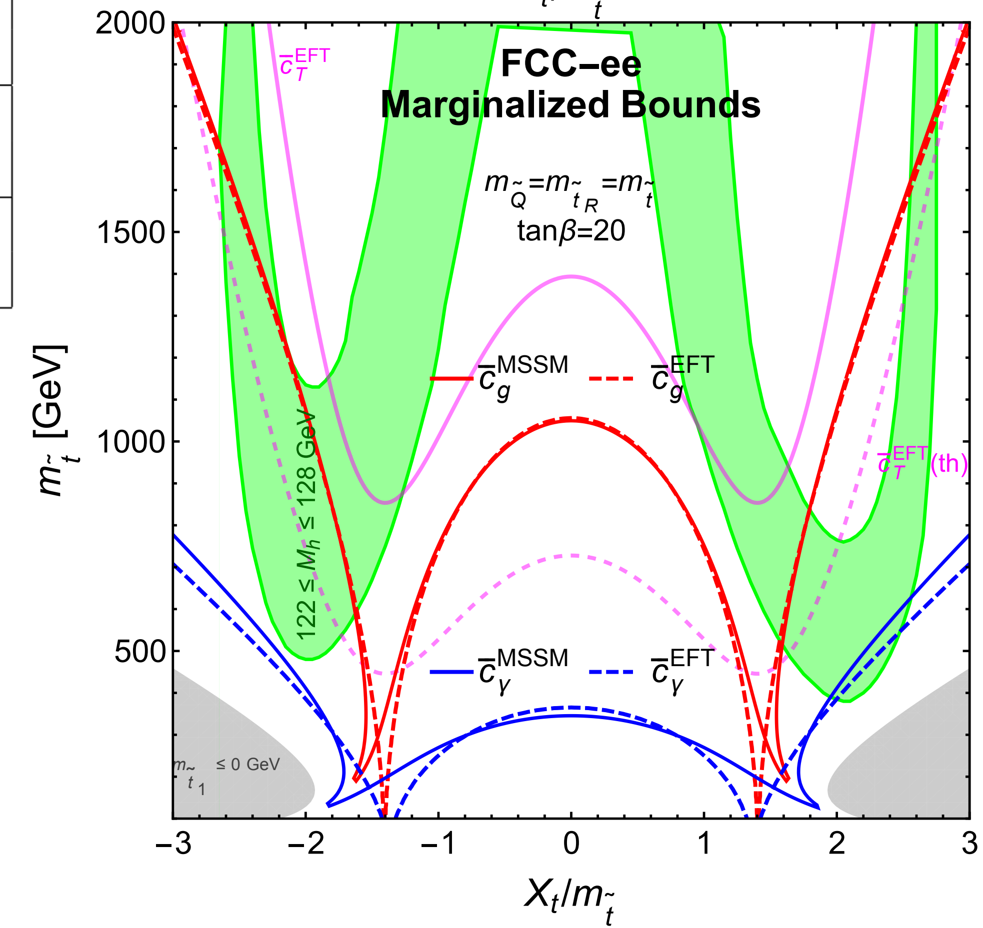
Indirect Constraints on Stops

Future colliders

A. Drozd, J. Ellis, JQ and T. You arXiv:1504.02409

Coeff.	Experimental constraints		95 % CL limit	deg. $m_{\tilde{t}_1}$ $X_t = 0$	deg. $m_{\tilde{t}_1}$ $X_t = m_{\tilde{t}}/2$
$\bar{c}_g$	$\text{ILC}^{1150\text{fb}^{-1}}_{250\text{GeV}}$	marginalized	$[-7.7, 7.7] \times 10^{-6}$	$\sim$ 675 GeV	$\sim$ 520 GeV
		individual	$[-7.5, 7.5] \times 10^{-6}$	$\sim$ 680 GeV	$\sim$ 545 GeV
	FCC-ee	marginalized	$[-3.0, 3.0] \times 10^{-6}$	$\sim$ 1065 GeV	$\sim$ 920 GeV
		individual	$[-3.0, 3.0] \times 10^{-6}$	$\sim$ 1065 GeV	$\sim$ 915 GeV
$\bar{c}_\gamma$	$\text{ILC}^{1150\text{fb}^{-1}}_{250\text{GeV}}$	marginalized	$[-3.4, 3.4] \times 10^{-4}$	$\sim$ 200 GeV	$\sim$ 40 GeV
		individual	$[-3.3, 3.3] \times 10^{-4}$	$\sim$ 200 GeV	$\sim$ 35 GeV
	FCC-ee	marginalized	$[-6.4, 6.4] \times 10^{-5}$	$\sim$ 385 GeV	$\sim$ 250 GeV
		individual	$[-6.3, 6.3] \times 10^{-5}$	$\sim$ 390 GeV	$\sim$ 260 GeV
$\bar{c}_T$	$\text{ILC}^{1150\text{fb}^{-1}}_{250\text{GeV}}$	marginalized	$[-3, 3] \times 10^{-4}$	$\sim$ 480 GeV	$\sim$ 285 GeV
		individual	$[-7, 7] \times 10^{-5}$	$\sim$ 930 GeV	$\sim$ 780 GeV
	FCC-ee	marginalized	$[-3, 3] \times 10^{-5}$	$\sim$ 1410 GeV	$\sim$ 1285 GeV
		individual	$[-0.9, 0.9] \times 10^{-5}$	$\sim$ 2555 GeV	$\sim$ 2460 GeV
$\bar{c}_W + \bar{c}_B$	$\text{ILC}^{1150\text{fb}^{-1}}_{250\text{GeV}}$	marginalized	$[-2, 2] \times 10^{-4}$	$\sim$ 230 GeV	$\sim$ 170 GeV
		individual	$[-6, 6] \times 10^{-5}$	$\sim$ 340 GeV	$\sim$ 470 GeV
	FCC-ee	marginalized	$[-2, 2] \times 10^{-5}$	$\sim$ 545 GeV	$\sim$ 960 GeV
		individual	$[-0.8, 0.8] \times 10^{-5}$	$\sim$ 830 GeV	$\sim$ 1590 GeV





+ Future FCC-ee measurements could be sensitive to stop masses above a TeV