Effective theory amplitudes: on-shell & effectively

Yael Shadmi
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w/ Yaniv Weiss 1809.09644
+ Gauthier Durieux, Teppei Kitahara 190?.????
Effective field theory:

parametrizing our ignorance
UV
full MSSM/composite Higgs/RS..
IR (~ -100m)

experiment must guide the way
EFT: a model independent & well-defined framework
Effective field theory:
parametrizing our \textit{(considerable)} ignorance

\[ V(\Phi) = -\frac{1}{2}m^2\Phi^2 + \lambda\Phi^4 \]

?? origin of this potential
Why mass-squared < 0? (dynamical mechanism?)
What sets its value?
What protects it from quantum corrections? Naturalness?
   particularly promising: Higgs, W, Z, top couplings
   ?clues on the mechanism of \textit{electroweak gauge symmetry breaking}
On-shell methods for calculating scattering amplitudes:

a vast (and growing) toolbox
simplify the calculation of amplitudes
in some cases: guess the form of the amplitude
starting from three-point amplitudes: bootstrap to higher-point, higher-loop amplitudes

HERE: apply to EFT amplitudes
particularly useful:

• 1\textsuperscript{st} step in EFT calculation: EFT Lagrangian
  (modulo field redefinitions; equations of motion, integration by parts)

• Higher-dim: many fields/derivatives: complicated Feynman rules
OUTLINE

On-shell methods (lightening review)

Putting these together in simple EFT examples
- SM + new (massive) scalar: scalar+gluon amplitudes
- SM + new (massive) vector: vector+gluon amplitudes
Effective field theory:

parametrizing our ignorance

\[ L = L_{\text{renormalizable}} + \sum \frac{c_i}{\Lambda^{d-4}} O_i^{(d)} \]

with no assumptions about UV theory:

a complete set of independent operators

compare to experiment \(\rightarrow\) measure \(c_i\)
finding a complete set of independent operators:

polynomials of operators subject to a set of constraints
equations of motion (EOM), integration by parts (IBP)
→ mathematical problem: Hilbert series
   lots of progress in recent years using sophisticated methods

Alonso Jenkins Manohar Trott
Henning Lu Melia Murayama
Lehman Martin

Yael Shadmi, Technion HEFT19
source of problem:

no one has ever seen a Lagrangian in the lab

(or anywhere else apart from physicists writings)

experiments see particles ≠ fields

so let’s abandon Lagrangians and turn to amplitudes..
on-shell methods for calculating scattering amplitudes:

structure of amplitudes constrained by
1. Lorentz (little group)
2. unitarity: particularly simple for tree level amplitudes

output: gauge symmetry, full structure of amplitude etc
1) Lorentz symmetry:
• spinor helicity variables
• little group scaling
• complex momenta
**writing amplitudes:**

scalar amplitude:

\[ A = A(p_1, p_2, p_3, p_4) \]

+ Lorentz: \( A = A(p_i \cdot p_j) \)

**go to basic building blocks:**

(lightlike) 4-momenta (spin 1) \( \rightarrow \) **spin-1/2 massless spinors**

\( \rightarrow \) \( |p\rangle = u_+(p) \) (dotted) \hspace{1cm} |p\rangle = u_-(p) \) (undotted)
writing amplitudes:

2 massless spinors $|p\rangle$, $|p\rangle$

$$p^{\alpha\dot{\alpha}} = |p\rangle\langle p|, \quad p_{\dot{\alpha}\alpha} = |p\rangle\langle p|$$

$$p^{\alpha\dot{\alpha}} \equiv \sigma_\mu^{\alpha\dot{\alpha}} p^\mu, \quad p_{\dot{\alpha}\alpha} \equiv \bar{\sigma}_\mu^{\dot{\alpha}\alpha} p_\mu,$$

or translated to Dirac spinor notation: $u_+(p) \bar{u}_+(p) + u_-(p) \bar{u}_-(p)$
writing amplitudes:

a few more necessities:

\[ |p> = |p|^* \quad \text{for real momenta (not for long..)} \]

\[ <qp> \equiv <q|p> \quad [qp] \equiv [qp] \quad \text{(both antisymmetric under } q \leftrightarrow p) \]

\[ <qp> [pq] = 2p \cdot q = s_{pq} \]
so: express amplitudes using massless spinor products

\[ \langle p_i p_j \rangle \quad [p_i p_j] \]

the spinor-helicity basis

what is this good for?
Spinors carry *more* symmetry information:

\[ p^{\alpha\dot{\alpha}} = |p\rangle\langle p|, \quad p^{\dot{\alpha}\alpha} = |p\rangle\langle p| \]

Rescale by \( c \), \( c^{-1} \), \( c = e^{i\varphi} \), \( U(1) \)

And momentum stays the same

\[ p = (p, 0, 0, p): \quad LG = SO(2) \quad \checkmark \]

\( |p\rangle \) carries LG weight +1; \( |p\rangle \) carries LG weight -1
what’s this good for?
scalar amplitudes: not much?
  but for particles with spin: need external polarizations

fermion amplitudes:
	negative helicity:  \(|p> = u_-(p)\)
  positive helicity:  \(|p] = u_+(p)\)
what’s this good for?

scalar amplitudes: not much?
  but for particles with spin: need external polarizations

fermion amplitudes:

\[
\begin{array}{ll}
\text{LG weight} = -1 & \text{negative helicity: } |p\rangle = u_-(p) \\
\text{LG weight} = +1 & \text{positive helicity: } |p\rangle = u_+(p)
\end{array}
\]
vector amplitudes: need polarization vectors:

\[ \mathbf{\varepsilon}^\pm(p) \]

\[ p \]

can be written in terms of 2 spinors, e.g.

\[ \mathbf{\varepsilon}^-(p) = \sqrt{2} \frac{|p\rangle[r]}{|pr\rangle} \]

reference momentum:
choice doesn’t affect any physical amplitude by virtue of gauge invariance
vector amplitudes: need polarization vectors:

\[
\varepsilon^\pm(p) \quad p
\]

LG weight = +2
LG weight = −2

can be written in terms of 2 spinors, eg

\[
\varepsilon^-(p) = \sqrt{2} \frac{|p\rangle\langle r|}{[p]r}
\]

reference momentum:
choice doesn’t affect any physical amplitude by virtue of gauge invariance
assembling the pieces

\[ p_1 \quad p_2 \quad \ldots \quad p_n \]

function of:  \( <p_i p_j> \equiv <ij>, \ [ij], \ (p_i \cdot p_j) \)

helicity weights: under \( LG(p_i) \):
- \( i=\text{scalar} \): weight 0
- \( i=\text{fermion of helicity } h = \pm \frac{1}{2} \): weight \( h \)
- \( i=\text{vector of helicity } h = \pm 1 \): weight \( h \)
example: scalar decay to 2 (massless) vectors @ tree level:

\[ h ; p = p_1 + p_2 \]
$$h \; p = p_1 + p_2$$

$$[12] \quad \& \quad <12>$$

$$A = [12]^n \quad F(s_{12} = m^2)$$

\[ \text{LG: } \quad n = h_1 \quad n = h_2 \quad \Rightarrow \quad h_1 = h_2 \]

\[ h_1 = h_2 = + \quad \Rightarrow \quad n = 2 \]

\[ \dim(A) = 4 - 3 = 1 \quad \Rightarrow \quad \dim(F) = -1 \]

\[ A = \frac{[12]^2}{\Lambda} \]

nothing at the renormalizable level

angular momentum is conserved..
and indeed \( \mathcal{L} \ni \frac{c}{\Lambda} h F_{\mu \nu} F_{\mu \nu} \)

- our first EFT "calculation"
  (admittedly a very simple example anyway..)
- no general function of \( s_{ij} \) : either zero or \( m^2 \rightarrow \) constant
  general feature of 3-point amplitudes \( \rightarrow \) completely determined
• massless $h$ limit? 3-point amplitude unphysical
• too bad, because 3-point amplitudes are handy, massless limits are handy..
• so: complex momenta: $p_1 + p_2 + p_3 = 0$ & $p_i^2 = 0$
3-particle amplitudes (complex momenta)

the real power of the spinor formalism
all massless:

\[ \langle ij \rangle \cdot [ji] = s_{ij} = 0 \Rightarrow [ji] = 0 \quad or \quad \langle ij \rangle = 0 \]

say \( \langle 12 \rangle \neq 0 \quad [12] = 0 \Rightarrow |1\rangle \propto |2\rangle \Rightarrow \propto |3\rangle \]

\( \rightarrow \) amplitude is a function of \( \langle 12 \rangle \), \( \langle 23 \rangle \), \( \langle 13 \rangle \) only
(or \([12], [13], [23]\) only)

completely determined by helicity weights: for all spins and helicities

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all 3-particles massless amplitudes:

for $\Sigma h_i \geq 0$, \[ g [12]^{n_{12}} [13]^{n_{13}} [23]^{n_{23}} \]

\[ n_{12} = h_1 + h_2 - h_3, \ldots \]

\[ \text{dim}(g) = 1 - \Sigma h_i \]
On-shell methods for calculating scattering amplitudes:

structure of amplitudes constrained by

✓ 1. Lorentz (little group)
2. unitarity: particularly simple for tree level amplitudes
2) unitarity & locality:

@ tree-level: amplitudes can have at most single particle-poles:

\[
\begin{align*}
\sim & \quad \frac{1}{p^2}
\end{align*}
\]

bootstrap: use 3-point amplitudes to construct 4-point amplitudes
or, infer form of (n+1) amplitude from collinear & soft limits:

\[ A_{n+1}(p_1, p_2, \ldots, p_n, p_{n+1}) \sim \text{Split}(p_n, p_{n+1}) \, A_n(p_1, p_2, \ldots, p_{n-1}, p) \]

for collinear n, n+1

\[ A_{n+1}(p_1, p_2, \ldots, p_n, p_{n+1}) \sim \text{Soft}(p_{n+1}) \, A_n(p_1, p_2, \ldots, p_n) \]

for soft n+1
Apply these tools to EFT:

not for the first time.. Cohen Elvang Kiermaier

space of possible EFTs Cheung Kampf Novotny Shen Trnka

RGE mixing (or lack thereof) of dim-6 operators Cheung Shen

orthogonality of SM and EFT amplitudes Azatov Contino Machado Riva
Apply these tools to EFT:

YS & Weiss 1809.09644

but how about calculating EFT amplitudes directly?
no reference to EFT Lagrangian

try to base the program of EFT – experimental measurements entirely in terms of physical amplitudes

& if you are interested in the EFT Lagrangian: infer it from amplitudes
→ new method for calculating the number of independent operators extremely simple!

see also talk by Jing Shu 1902.07204
example: \( \text{SM} + h \) \quad \text{gauge singlet spin-0 (Higgs or something new)}

consider just scalar + gluon sector

saw

\[
\mathcal{M}(h; g^{a+}(p_1)g^{b+}(p_2)) = \delta^{ab} \frac{c_{g g}}{\Lambda} [12]^2 \quad \text{[} \mathcal{L} \ni \frac{c}{\Lambda} h F^\mu\nu F_{\mu\nu} \text{]}
\]

\[
M(h; 1^+, 2^+, 3^+) = [12][13][23] F(s_{12}, s_{13}, s_{23})
\]
\[\text{dim}=-1\]

factorizable + non-factorizable parts
non-factorizable:

\[ A(h; 1^+, 2^+, 3^+) = [12][13][23] \ F(s_{12}, s_{13}, s_{23}) \]

- with \( f^{ab} \): \( F(s_{12}, s_{13}, s_{23}) \) symmetric polynomial of \( s_{ij} \)
  subject to constraint: \( s_{12} + s_{13} + s_{23} = m^2 \)

- with \( d^{abc} \): \( F(s_{12}, s_{13}, s_{23}) \) polynomial of \( s_{ij} \)
  antisymmetric under \( i \leftrightarrow j \)
\[ M \left( h; g^{a+}(p_1)g^{b+}(p_2)g^{c+}(p_3) \right) = \frac{[12][13][23]}{\Lambda} f^{abc} \left( -i \frac{m^4 g_s c_5^{hgg}}{s_{12}s_{13}s_{23}} + \frac{a_7}{\Lambda^2} \right) \]

\[ + \frac{a_{11}}{\Lambda^6} \left( s_{12}s_{23} + s_{13}s_{23} + s_{12}s_{13} \right) + \frac{a_{13}}{\Lambda^8} s_{12}s_{13}s_{23} \]

\[ + d^{abc} \frac{a_{13}'}{\Lambda^8} (s_{12} - s_{13}) (s_{12} - s_{23}) (s_{13} - s_{23}) \]

unknown parameters -- Wilson coefficients of (independent) EFT operators

on top of dim-5 operator that saw before:

\( f^{abc} \): one operator at dim-7, one at dim-11, one at dim-13
\( d^{abc} \): one operator at dim-13
(+ + −)
actorizable

\[
\mathcal{M}(h; g^{a+}(p_1)g^{b+}(p_2)g^{c-}(p_3)) = \frac{[12]^3}{[13][23]} \frac{1}{\Lambda} \left[ f^{abc} \left( i g_s c_5^{hgg} - \frac{i c_5^{hgg} c_6^{ggg}}{\Lambda^2} \right) \frac{s_{23}s_{13}}{s_{12}} + \frac{b_9}{\Lambda^4} s_{13}s_{23} 
+ \frac{b_{11}}{\Lambda^6} s_{12}s_{13}s_{23} + \frac{b_{13}}{\Lambda^8} s_{12}s_{23} + \frac{b'_{13}}{\Lambda^8} s_{13}s_{23}s_{12} \right) + d^{abc} s_{13}s_{23}(s_{13} - s_{23}) \left( \frac{b'_{11}}{\Lambda^6} + \frac{b''_{13}}{\Lambda^8} s_{12} \right) \right],
\]

on top of dim-5 operator that saw before:

\( f^{abc} \): contribution from \( tr(G^3) \) operator
+ one operator at dim-9, one at dim-11, two at dim-13

\( d^{abc} \): one operator at dim-11, one at dim-13

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so:
1. calculated helicity amplitudes relevant for LHC production & decay of new (massive) spin-0 resonance (or Higgs) at LHC

\[ gg \rightarrow h \rightarrow gg, ggg \]

2. can infer EFT Lagrangian and calculate number of independent operators very simply
to do this:

positive helicity gluon: \[ G_{SD}^{\mu\nu} = \frac{1}{2} \left( G^{\mu\nu} + \tilde{G}^{\mu\nu} \right) \]

negative helicity gluon: \[ G_{ASD}^{\mu\nu} = \frac{1}{2} \left( G^{\mu\nu} - \tilde{G}^{\mu\nu} \right) \]

\[ \tilde{G}^{\mu\nu} = \frac{i}{2} \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma} \]

\[ M(h; + + +): h \ G_{SD}^2 ; \ h \ G_{SD}^3 \ + \more \ derivatives \]

\[ M(h; + + -): h \ G_{SD}^2 \ G_{ASD} \ + \ more \ derivatives \]
did not have to worry about:
operator redundancy
gauge redundancy (or symmetry)

these just follow

<table>
<thead>
<tr>
<th>Mass dimension</th>
<th>Operators $\mathcal{M}(+++)$</th>
<th>Operators $\mathcal{M}(++-)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>7</td>
<td>$h G_{SD}^3 [1, f^{abc}]$</td>
<td>—</td>
</tr>
<tr>
<td>9</td>
<td>—</td>
<td>$\mathcal{D}^2 G_{SD}^2 G_{ASD} h [1, f^{abc}]$</td>
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<td>11</td>
<td>$\mathcal{D}^4 G_{SD}^3 h [1, f^{abc}]$</td>
<td>$\mathcal{D}^4 G_{SD}^2 G_{ASD} h [1, f^{abc}; 1, d^{abc}]$</td>
</tr>
<tr>
<td>13</td>
<td>$\mathcal{D}^6 G_{SD}^3 h [1, f^{abc}; 1, d^{abc}]$</td>
<td>$\mathcal{D}^6 G_{SD}^2 G_{ASD} h [2, f^{abc}; 1, d^{abc}]$</td>
</tr>
</tbody>
</table>
• check: derived using Mathematica notebook of 1512.03433

Henning, Lu, Melia, Murayama, 2, 84, 30, 993, 560, 15456, 11962, 261485, ..: Higher dimension operators in the SM EFT

• these coefficients will enter higher point amplitudes
→ given by factorization
to reproduce EFT Lagrangian up to eg, dim-7: (h + gluons only)

\[ G^2, \ G^3, \ hG^2, \ hG^3, \ h^2 G^2, \ h^3 G^2: \]

would need: ggg, hgg, hggg, hhgg, hhhgg (calculated all but last)
example: \( \text{SM} + Z' \) new gauge singlet, spin-1

\[
\mathcal{M}\left(Z'; g^a_{-(p_1)}; g^b_{-(p_2)}; g^c_{+(p_3)}\right) = d^{abc}_{\langle 12 \rangle^2} \times \left[ [34]^2 \tilde{f}^+_{-4}(1; 2) + [13] [23] \langle 14 \rangle \langle 24 \rangle \tilde{f}^+_6(1; 2) + [34] ([31] \langle 14 \rangle - [32] \langle 24 \rangle) \tilde{f}^-_5(1; 2) \right]
\]

\[
+ f^{abc}_{\langle 12 \rangle^2} \times \left[ [34]^2 \tilde{f}^-_{-4}(1; 2) + [13] [23] \langle 14 \rangle \langle 24 \rangle \tilde{f}^-_6(1; 2) + [34] ([31] \langle 14 \rangle - [32] \langle 24 \rangle) \tilde{f}^+_5(1; 2) \right]
\]

\[
\tilde{f}^+_{-4}(1; 2) = \frac{d_8}{\Lambda^4} + \frac{d^{(1)}_{10}}{\Lambda^6} s_{12} + \frac{d^{(2)}_{12} s_{12}^2 + d^{(2)}_{12} s_{13} s_{23}}{\Lambda^8}, \quad \tilde{f}^-_{-4}(1; 2) = (s_{23} - s_{13}) \left( \frac{d^{(3)}_{10}}{\Lambda^6} + \frac{d^{(4)}_{12}}{\Lambda^8} s_{12} \right),
\]

\[
\tilde{f}^+_{-5}(1; 2) = \frac{m d^{(2)}_{10}}{\Lambda^6} + \frac{m d^{(3)}_{12}}{\Lambda^8} s_{12}, \quad \tilde{f}^-_{-5}(1; 2) = (s_{13} - s_{23}) \frac{m d^{(5)}_{12}}{\Lambda^8},
\]

\[
\tilde{f}^+_{-6}(1; 2) = \frac{m^2 s_{12} d^{(6)}_{12}}{\Lambda^8}, \quad \tilde{f}^-_{-6}(1; 2) = 0.
\]

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example: $\text{SM} + Z'$ new gauge singlet, spin-1

$$\mathcal{M}(Z'; g^a_{-}(p_1); g^b_{-}(p_2); g^c_{+}(p_3))$$

$$= d^{abc} \langle 12 \rangle^2 \times \left[ [34]^2 \tilde{f}^+_4(1; 2) + [13] [23] \langle 14 \rangle \langle 24 \rangle \tilde{f}^+_6(1; 2) + [34] ([31] \langle 14 \rangle - [32] \langle 24 \rangle) \tilde{f}^-_5(1; 2) \right]$$

$$+ f^{abc} \langle 12 \rangle^2 \times \left[ [34]^2 \tilde{f}^-_4(1; 2) + [13] [23] \langle 14 \rangle \langle 24 \rangle \tilde{f}^-_6(1; 2) + [34] ([31] \langle 14 \rangle - [32] \langle 24 \rangle) \tilde{f}^+_5(1; 2) \right]$$

$$\tilde{f}^+_4(1; 2) = \frac{d_8}{\Lambda^4} + \frac{d^{(1)}_{10}}{\Lambda^6} s_{12} + \frac{d^{(1)}_{12} s_{12}^2 + d^{(2)}_{12} s_{13} s_{23}}{\Lambda^8}, \quad \tilde{f}^-_4(1; 2) = (s_{23} - s_{13}) \left( \frac{d^{(3)}_{10}}{\Lambda^6} + \frac{d^{(4)}_{12}}{\Lambda^8} s_{12} \right),$$

$$\tilde{f}^+_5(1; 2) = \frac{m d^{(2)}_{10}}{\Lambda^6} + \frac{m d^{(3)}_{12}}{\Lambda^8} s_{12}, \quad \tilde{f}^-_5(1; 2) = (s_{13} - s_{23}) \frac{m d^{(5)}_{12}}{\Lambda^8},$$

$$\tilde{f}^+_6(1; 2) = \frac{m^2 s_{12} d^{(6)}_{12}}{\Lambda^8}, \quad \tilde{f}^-_6(1; 2) = 0,$$

(3.19)
example: $\text{SM} + Z'$  new gauge singlet, spin-1

\[ M(Z'; g^{a-}(p_1); g^{b-}(p_2); g^{c+}(p_3)) \]

\[
= d^{abc}_{\mu} \langle 12 \rangle^2 \times \left[ [34]^2 \tilde{f}^+_{-4}(1; 2) + [13][23] \langle 14 \rangle \langle 24 \rangle \tilde{f}^+_{-6}(1; 2) + [34] \langle [31] \langle 14 \rangle - [32] \langle 24 \rangle \rangle \tilde{f}^+_{-5}(1; 2) \right] \\
+ f^{abc}_{\mu} \langle 12 \rangle^2 \times \left[ [34]^2 \tilde{f}^-_{-4}(1; 2) + [13][23] \langle 14 \rangle \langle 24 \rangle \tilde{f}^-_{-6}(1; 2) + [34] \langle [31] \langle 14 \rangle - [32] \langle 24 \rangle \rangle \tilde{f}^-_{-5}(1; 2) \right]
\]

\[
\tilde{f}^+_{-4}(1; 2) = \frac{d_8}{\Lambda^4} + \frac{d^{(1)}_{10}}{\Lambda^6} s_{12} + \frac{d^{(1)}_{12} s_{12}^2 + d^{(2)}_{12} s_{13} s_{23}}{\Lambda^8}, \quad \tilde{f}^-_{-4}(1; 2) = (s_{23} - s_{13}) \left( \frac{d^{(3)}_{10}}{\Lambda^6} + \frac{d^{(4)}_{12}}{\Lambda^8} s_{12} \right),
\]

\[
\tilde{f}^+_{-5}(1; 2) = \frac{m d^{(2)}_{10}}{\Lambda^6} + \frac{m d^{(3)}_{12}}{\Lambda^8} s_{12}, \quad \tilde{f}^-_{-5}(1; 2) = (s_{13} - s_{23}) \frac{m d^{(5)}_{12}}{\Lambda^8},
\]

\[
\tilde{f}^+_{-6}(1; 2) = \frac{m^2 s_{12} d^{(6)}_{12}}{\Lambda^8}, \quad \tilde{f}^-_{-6}(1; 2) = 0.
\]
example: SM + $Z'$ new gauge singlet, spin-1

$$\mathcal{M}(Z'; g^{a-}(p_1); g^{b-}(p_2); g^{c+}(p_3))$$

$$= d^{abc} \langle 12 \rangle^2 \times \left[ [34]^2 \tilde{f}_+^-(1; 2) + [13] [23] \langle 14 \rangle \langle 24 \rangle \tilde{f}_-^-^+(1; 2) + [34] ([31] \langle 14 \rangle - [32] \langle 24 \rangle) \tilde{f}_-^-^-(1; 2) \right]$$

$$+ f^{abc} \langle 12 \rangle^2 \times \left[ [34]^2 \tilde{f}_-^-^+(1; 2) + [13] [23] \langle 14 \rangle \langle 24 \rangle \tilde{f}_-^-^+(1; 2) + [34] ([31] \langle 14 \rangle - [32] \langle 24 \rangle) \tilde{f}_-^-^+(1; 2) \right]$$

massive $Z'$!
this expression contains all three polarizations
massive particle polarizations

• just as with spinor products, work with basic building blocks (as in ``tensor method” for SU(N)
  build any spin j rep as product of spin $\frac{1}{2}$
  (need symmetric combinations only; antisym=singlet)

• spin-1 object: $O_{ij}$, $i, j = 1, 2$
  $[ j = 1: \uparrow\uparrow, (\uparrow\downarrow+\downarrow\uparrow), \downarrow\downarrow]$
massive momentum: need two massless spinors:

\[ p^{\alpha \dot{\alpha}} = \lambda_I^\alpha \tilde{\lambda}^{\dot{\alpha} I} \equiv \left| p_I \right| | \bar{p}^I \rangle, \quad p_{\dot{\alpha} \alpha} = -\tilde{\lambda}_{\dot{\alpha} I} \lambda_I^\alpha \equiv -\left| p_I \right| \langle \bar{p}^I | \]

\[ I = 1,2 \]

can rescale: \[ U_I^{-1}, \quad U_I^j \] with \( p \) invariant

= little group, \( SU(2) \)
massive momentum: 

\[ p, \{IJ\} \]

factor of \[ |p^I| |p^J| \] for polarization with indices symmetrized

[ gives \((..)_{\alpha\beta}\) with \(\alpha\beta\) symmetrized ]

use bolded notation: \[ |\mathbf{p}| |\mathbf{p}| \]

using this formulation: general expressions for three-particle amplitudes for all masses & spins
massive momentum:

back to the amplitude we had before:

\[
\mathcal{M}\left(Z', g^{a-}(p_1); g^{b-}(p_2); g^{c+}(p_3)\right) \\
= d^{abc} \, (12)^2 \times \left[ [34]^2 \tilde{f}^+_{-4}(1; 2) + [13] [23] \langle 14 \rangle \langle 24 \rangle \tilde{f}^+_{-6}(1; 2) + [34] ([31] \langle 14 \rangle - [32] \langle 24 \rangle) \tilde{f}^-_{-5}(1; 2) \right] \\
+ f^{abc} \, (12)^2 \times \left[ [34]^2 \tilde{f}^-_{-4}(1; 2) + [13] [23] \langle 14 \rangle \langle 24 \rangle \tilde{f}^-_{-6}(1; 2) + [34] ([31] \langle 14 \rangle - [32] \langle 24 \rangle) \tilde{f}^+_{-5}(1; 2) \right]
\]
**high energy limit:** splits into $+,-,||$ polarizations
``simply” unbold massive spinors..

- match to EFT Lagrangian for transverse polarizations:

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<td>$G^3_{SD} Z'_{SD} [1, d^{abc}]$</td>
<td>$G^2_{ASD} G_{SD} Z'_{SD} [1, d^{abc}]$</td>
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• longitudinal polarization: same as scalar + 3 gluons
• see Higgs mechanism at amplitude level!

``IR (massive) amplitude unifies UV massless amplitudes”

Arkani-Hamed Huang Huang
To conclude:

EFT will play a key role in the coming years (until we find new particles at the LHC)

On-shell methods can be used to infer the structure of EFT amplitudes

Simple-minded way to calculate the number of EFT operators

Amplitudes with massive gauge-bosons: manifest Higgs mechanism at amplitude level

?? new insights into EWSB
THANK YOU!
example: three spin-1:

[bonus: gauge symmetry structure]
++ +: [12][13][23] but need dim=1: nothing at renormalizable level

[ EFT: \( \frac{[12][13][23]}{\Lambda^2} \) ]
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with extra quantum number, eg, index $a$

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\[ \Sigma h_i = 1 \quad \text{dim}(\text{coupling}) = 0 \]

antisymmetric in $1 \leftrightarrow 2$: no 3-point photon amplitude!

with extra quantum number, eg, index $a$

\[ f_{abc} \frac{[12]^3}{[13][23]} \]

completely antisymmetric
Example application: **4 spin-1:**

consistent poles: \[ f^{ade} f^{bce} + \cdots = 0 \]  

*Jacobi Identity*