Effective theory amplitudes: on-shell & effectively

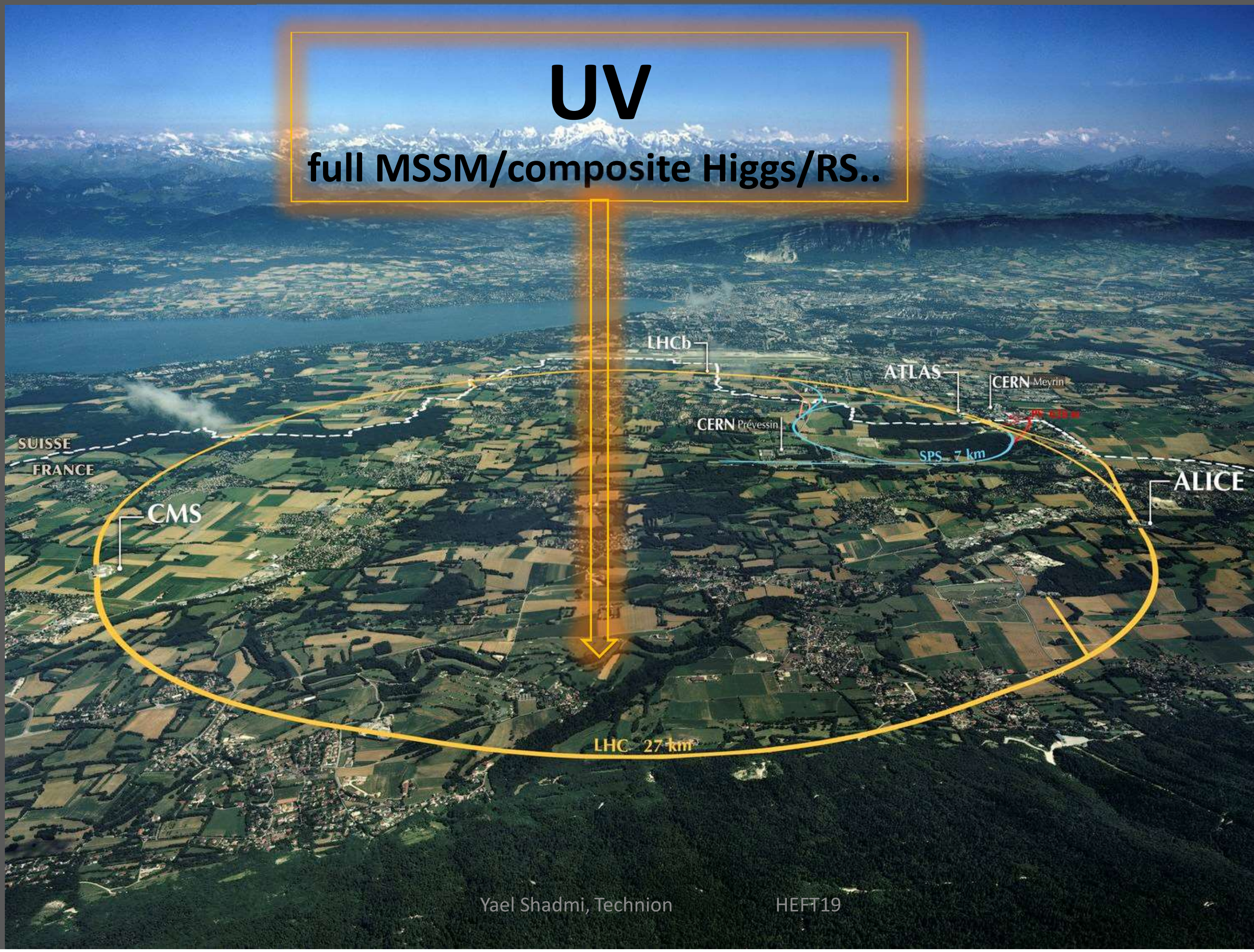
Yael Shadmi

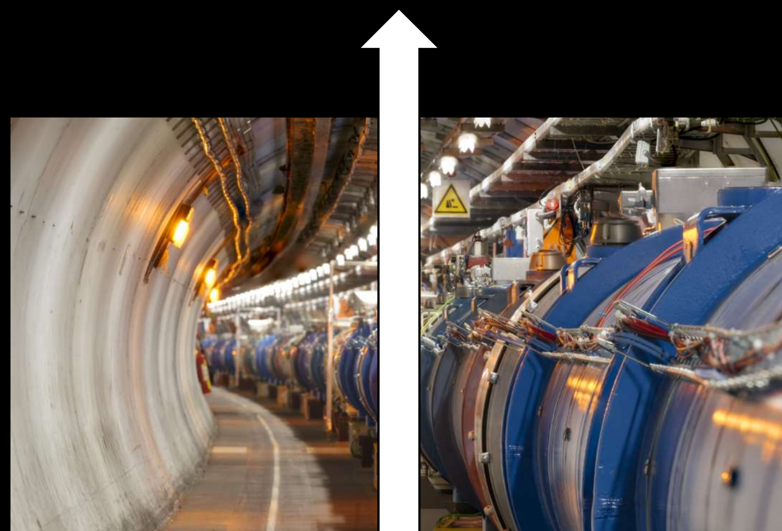
Technion

w/ Yaniv Weiss 1809.09644
+ Gauthier Durieux, Teppei Kitahara 190?.????

Effective field theory:

parametrizing our ignorance


UV
full MSSM/composite Higgs/RS..
LHCb
ATLAS
CERN Meyrin
CERN Prévessin
SPS 7 km
SUISSE
FRANCE
CMS
ALICE
LHC 27 km



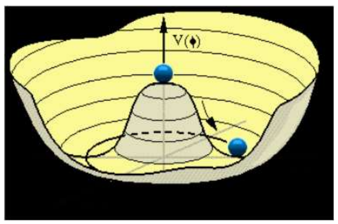
IR (~ -100m)

experiment must guide the way

EFT: a model independent & well-defined framework

Effective field theory:

parametrizing our **(considerable)** ignorance



$$V(\Phi) = -\frac{1}{2} m^2 \Phi^2 + \lambda \Phi^4$$

?? origin of this potential

Why mass-squared < 0? (dynamical mechanism?)

What sets its value?

What protects it from quantum corrections ? Naturalness?

particularly promising: Higgs, W, Z, top couplings

?clues on the mechanism of *electroweak* ***gauge symmetry*** *breaking*

On-shell methods for calculating scattering amplitudes:

a vast (and growing) toolbox

→ simplify the calculation of amplitudes

in some cases: guess the form of the amplitude

starting from three-point amplitudes: bootstrap to higher-point, higher-loop amplitudes

HERE: apply to EFT amplitudes

particularly useful:

- 1st step in EFT calculation: EFT Lagrangian

  (modulo field redefinitions; equations of motion, integration by parts)

- Higher-dim: many fields/derivatives: complicated Feynman rules

OUTLINE

On-shell methods (lightening review)

Putting these together in simple EFT examples

- SM + new (massive) scalar: scalar+gluon amplitudes
- SM + new (massive) vector: vector+gluon amplitudes

Effective field theory:

parametrizing our ignorance

Wilson coefficients

$$L = L_{renormalizable} + \Sigma \frac{c_i}{\Lambda^{d-4}} O_i^{(d)}$$

scale

with no assumptions about UV theory:

a **complete** set of ***independent*** operators

compare to experiment → measure c_i

finding a complete set of independent operators:

polynomials of operators subject to a set of constraints
equations of motion (EOM), integration by parts (IBP)
→ mathematical problem: Hilbert series
lots of progress in recent years using sophisticated methods

Alonso Jenkins Manohar Trott
Henning Lu Melia Murayama
Lehman Martin

…

source of problem:

no one has ever seen a Lagrangian in the lab

(or anywhere else apart from physicists writings)

experiments see particles ≠ fields

so let’s abandon Lagrangians and turn to amplitudes..

on-shell methods for calculating scattering amplitudes:

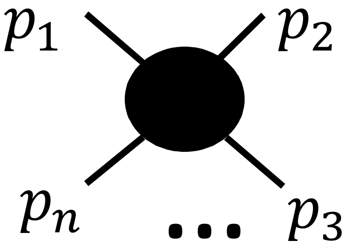
structure of amplitudes constrained by

1. Lorentz (little group)
2. unitarity: particularly simple for tree level amplitudes

output: gauge symmetry, full structure of amplitude etc

1) Lorentz symmetry:

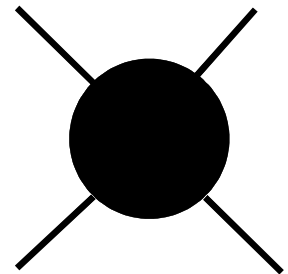
- ***spinor helicity variables***
- ***little group scaling***
- ***complex momenta***



writing amplitudes:

scalar amplitude:

$$A = A(p_1, p_2, p_3, p_4)$$



$+$ Lorentz: $A = A(p_i \cdot p_j)$

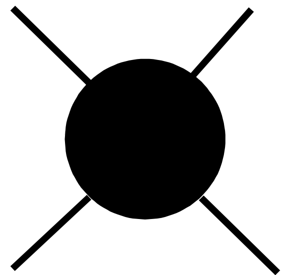
go to **basic building blocks**:

(lightlike) 4-momenta (spin 1) → **spin-1/2 massless spinors**

→ $|\mathrm{p}> = u_+(p)$ (dotted) $|\mathrm{p}] = u_-(p)$ (undotted)

writing amplitudes:

p



Parke Taylor
eg Mangano Parke Phys Repts

2 massless spinors |p>, |p]

$$p^{\alpha\dot{\alpha}} = |p\rangle[p| \,, \quad p_{\dot{\alpha}\alpha} = |p]\langle p|$$

$$p^{\alpha\dot{\alpha}} \equiv \sigma_{\mu}^{\alpha\dot{\alpha}} p^{\mu} \,, \quad p_{\dot{\alpha}\alpha} \equiv \bar{\sigma}^{\mu}_{\dot{\alpha}\alpha} p_{\mu} \,,$$

or translated to Dirac spinor notation: $u_{+}(p)\,\bar{u}_{+}(p) + u_{-}(p)\,\bar{u}_{-}(p)$

writing amplitudes:

a few more necessities:

$|p> = |p]^*$ for real momenta (not for long..)

$<qp> \equiv <q|p>$ $[qp] \equiv [qp]$ (both antisymmetric under $q \leftrightarrow p$)

$<qp>[pq] = 2p \cdot q = s_{pq}$

so: express amplitudes using massless spinor products

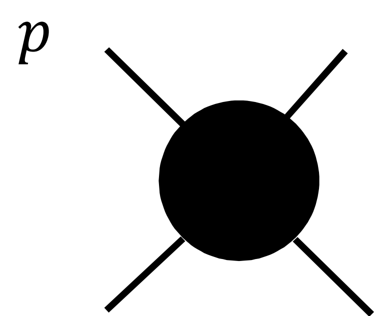
$<p_i p_j>$ $[p_i p_j]$

the spinor-helicity basis

what is this good for?

spinors carry *more* symmetry information:

$$p^{\alpha\dot{\alpha}} = |p\rangle[p|\,, \quad p_{\dot{\alpha}\alpha} = |p]\langle p|$$



rescale by c c^{-1} $c = e^{i\varphi}$ U(1)

and momentum stays the same

→ the little group of p !

$p = (p, 0,0, p)$: LG = SO(2) √

|p> carries LG weight +1; |p] carries LG weight -1

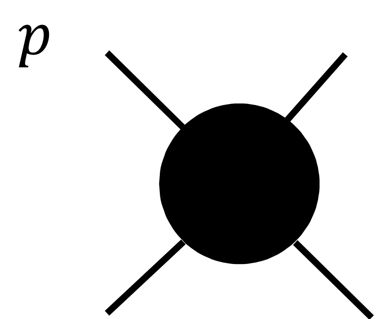
what’s this good for?

scalar amplitudes: not much?

but for particles with spin: need external polarizations

fermion amplitudes:

$$\boxed{\begin{array}{ll} \text{negative helicity:} & |p\rangle = u_-(p) \\ \text{positive helicity:} & |p] = u_+(p) \end{array}}$$



what's this good for?

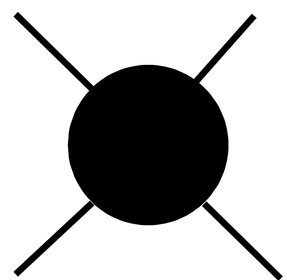
scalar amplitudes: not much?

but for particles with spin: need external polarizations

fermion amplitudes:

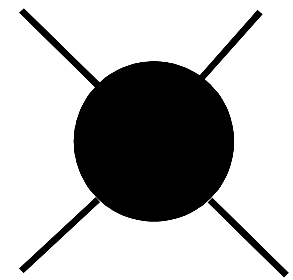
LG weight = -1	negative helicity:	|p> $= u_-(p)$
LG weight = $+1$	positive helicity:	|p] $= u_+(p)$

p



vector amplitudes: need polarization vectors:

$\boxed{\varepsilon^{\pm}(\mathrm{p})}$ p



can be written in terms of 2 spinors, eg

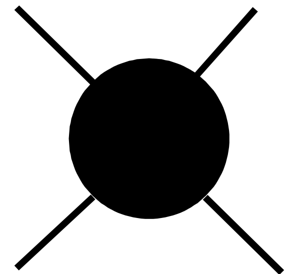
$$\varepsilon^{-}(\mathrm{p}) = \sqrt{2}\,\frac{|p\rangle[r|}{[pr]}$$

reference momentum:
choice doesn't affect any physical amplitude by virtue of gauge invariance

vector amplitudes: need polarization vectors:

LG weight = $+2$
LG weight = -2

$\boxed{\varepsilon^{\pm}(\mathrm{p})}$ p

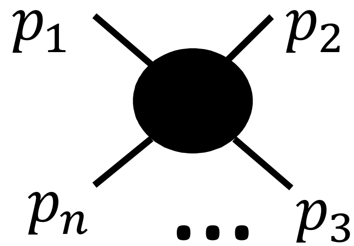


can be written in terms of 2 spinors, eg

$$\varepsilon^{-}(\mathrm{p}) = \sqrt{2}\frac{|p\rangle[r|}{[pr]}$$

reference momentum:
choice doesn't affect any physical amplitude by virtue of gauge invariance

assembling the pieces



function of: $\langle p_i p_j \rangle \equiv \langle ij \rangle$, $[ij]$, $(p_i \cdot p_j)$

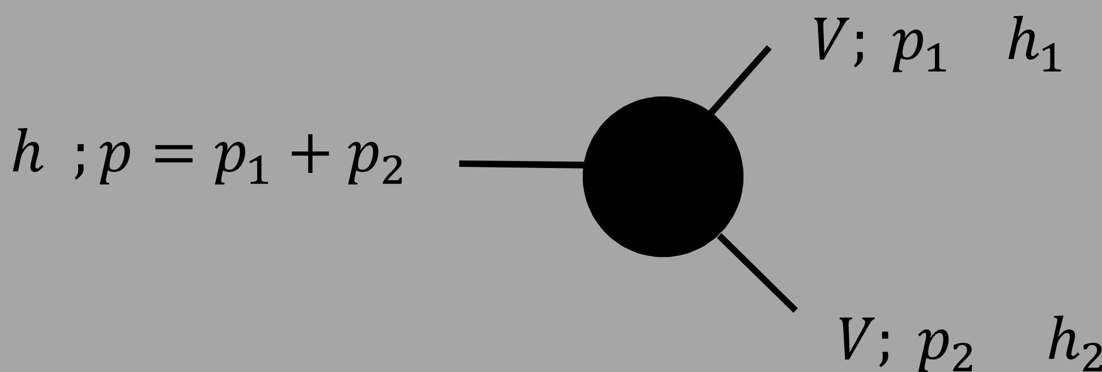
helicity weights: under LG(p_i):

i=scalar: weight 0

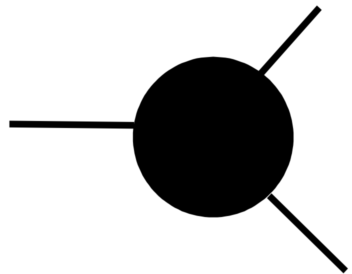
i=fermion of helicity $\mathrm{h} = \pm\frac{1}{2}$: weight h

i=vector of helicity $\mathrm{h} = \pm 1$: weight h

example: scalar decay to 2 (massless) vectors @ tree level:



$V;\ p_1 \quad h_1$

$h\ ; p = p_1 + p_2$ 

$V;\ p_2 \quad h_2$

$[12] \quad \& \quad < 12 >$

$$\boxed{\mathrm{A} = [12]^n\, F(s_{12} = m^2)}$$

LG: $n = h_1 \quad n = h_2 \quad \Rightarrow \quad h_1 = h_2$

angular momentum is conserved..

$h_1 = \ h_2 = + \ \Rightarrow \ n = 2$

$\dim(A) = 4 - 3 = 1 \ \Rightarrow \ \dim(F) = -1$

$$\boxed{\mathrm{A} = \frac{[12]^2}{\Lambda}}$$

nothing at the renormalizable level

$$\boxed{\mathrm{A} = \frac{[12]^2}{\Lambda}}$$

and indeed $\mathcal{L} \ni \frac{c}{\Lambda} h\, F^{\mu\nu}\, F_{\mu\nu}$

- our first EFT ``calculation"
  (admittedly a very simple example anyway..)
- no general function of s_{ij} : either zero or m^2 → constant
  general feature of 3-point amplitudes → completely determined

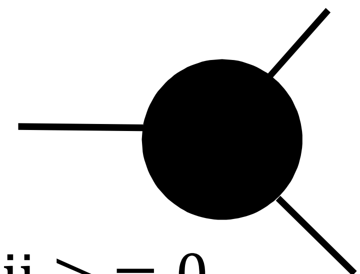
$$\boxed{\mathrm{A} = \frac{[12]^2}{\Lambda}}$$

- massless h limit? 3-point amplitude unphysical
- too bad, because 3-point amplitudes are handy, massless limits are handy..
- so: **complex momenta:** $p_1 + p_2 + p_3 = 0 \;\; \& \;\; p_i^2 = 0$

3-particle amplitudes (complex momenta)

the real power of the spinor formalism

all massless:



$$<\mathrm{ij}>\cdot[\mathrm{ji}] = s_{ij} = 0 \quad \Longrightarrow [\mathrm{ji}] = 0 \quad or \quad <\mathrm{ij}> = 0$$

say $< 12 > \neq 0 \quad [12] = 0 \Longrightarrow |1] \propto |2] \Longrightarrow \propto \; |3]$

→ amplitude is a function of $< 12 >, \; < 23 >, \; < 13 >$ only
(or $[12], [13], [23]$ only)

completely determined by helicity weights: for all spins and helicities

all 3-particles massless amplitudes:

for $\Sigma h_i \geq 0$, $$g \ [12]^{n12} \ [13]^{n13} \ [23]^{n23}$$

$$n12 = h_1 + h_2 - h_3 \ , ..$$

$$\dim(g) = 1 - \Sigma h_i$$

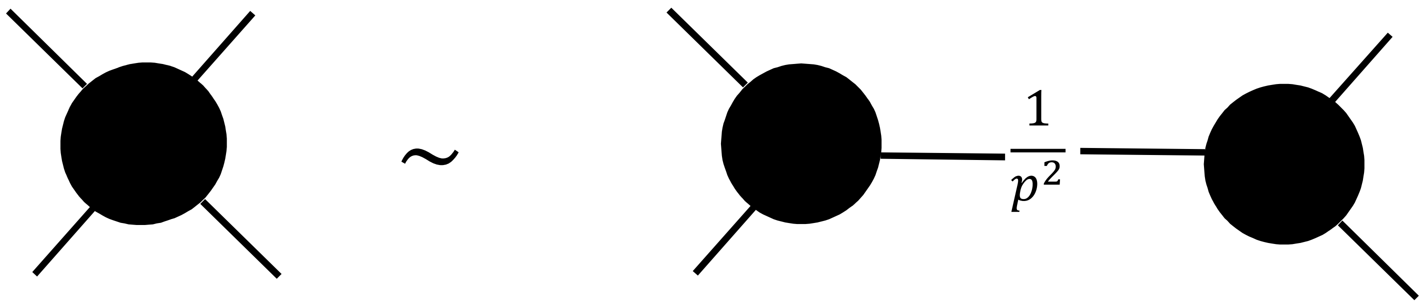
On-shell methods for calculating scattering amplitudes:

structure of amplitudes constrained by

√ 1. Lorentz (little group)
2. unitarity: particularly simple for tree level amplitudes

2) unitarity & locality:

@ tree-level: amplitudes can have at most single particle-poles:



bootstrap: use 3-point amplitudes to construct 4-point amplitudes

Britto Cachazo Feng Witten

…

or, infer form of (n+1) amplitude from collinear & soft limits:

$$A_{n+1}(p_1, p_2, .., p_n\ , p_{n+1}) \sim Split(p_n, p_{n+1})\ A_n(p_1, p_2, .., p_{n-1}, p)$$

for collinear n, n+1

Bern Dixon Kosower

$$A_{n+1}(p_1, p_2, .., \ p_n, p_{n+1}) \sim Soft(p_{n+1})\ A_n(p_1, p_2, .., p_n)$$

for soft n+1

universal

Apply these tools to EFT:

not for the first time.. Cohen Elvang Kiermaier

space of possible EFTs Cheung Kampf Novotny Shen Trnka

RGE mixing (or lack thereof) of dim-6 operators Cheung Shen

orthogonality of SM and EFT amplitudes Azatov Contino Machado Riva

Apply these tools to EFT: YS & Weiss 1809.09644

but how about calculating EFT amplitudes *directly*?
no reference to EFT Lagrangian

try to base the program of EFT – experimental measurements entirely in terms of physical amplitudes

& if you *are* interested in the EFT Lagrangian: infer it from amplitudes
→ new method for calculating the number of independent operators
extremely simple!

see also talk by Jing Shu
1902.07204

example: SM + h gauge singlet spin-0 (Higgs or something new)

consider just scalar + gluon sector

saw

$$\mathcal{M}\Big(h; g^{a+}(p_1) g^{b+}(p_2)\Big) = \delta^{ab} \frac{c_5^{hgg}}{\Lambda} [12]^2 \qquad [\, \mathcal{L} \ni \frac{c}{\Lambda} h\, F^{\mu\nu} F_{\mu\nu} \,]$$

$$M(\mathrm{h}; 1^+, 2^+, 3^+) = [12][13][23]\, F(s_{12}, s_{13}, s_{23})$$

dim=-1

factorizable + non-factorizable parts

non-factorizable:

$$\mathrm{A}(\mathrm{h}; 1^+, 2^+, 3^+) = [12][13][23]\, F(s_{12}, s_{13}, s_{23})$$

- with f^{ab} : $F(s_{12}, s_{13}, s_{23})$ symmetric polynomial of s_{ij}
  subject to constraint: $s_{12} + s_{13} + s_{23} = m^2$

- with d^{abc}: $F(s_{12}, s_{13}, s_{23})$ polynomial of s_{ij}
  antisymmetric under $\mathrm{i} \leftrightarrow j$

$$\mathcal{M}\Big(h; g^{a+}(p_1)g^{b+}(p_2)g^{c+}(p_3)\Big) = \frac{[12]\,[13]\,[23]}{\Lambda}\Bigg[f^{abc}\Bigg(-i\,\underline{\frac{m^4\,g_s\,c_5^{hgg}}{s_{12}s_{13}s_{23}}} + \frac{a_7}{\Lambda^2}$$

factorizable

$$+\frac{a_{11}}{\Lambda^6}(s_{12}s_{23}+s_{13}s_{23}+s_{12}s_{13}) + \frac{a_{13}}{\Lambda^8}s_{12}s_{13}s_{23}\Bigg)$$

$$+\,d^{abc}\,\frac{a'_{13}}{\Lambda^8}\;(s_{12}-s_{13})\,(s_{12}-s_{23})\,(s_{13}-s_{23})\Bigg]\,,$$

unknown parameters -- Wilson coefficients of (independent) EFT operators

on top of dim-5 operator that saw before:

f^{abc}: one operator at dim-7, one at dim-11, one at dim-13
d^{abc}: one operator at dim-13

$(+ \; + \; -)$

factorizable

$$\mathcal{M}\Big(h; g^{a+}(p_1) g^{b+}(p_2) g^{c-}(p_3)\Big) = \frac{[12]^3}{[13]\,[23]}\,\frac{1}{\Lambda}\Bigg[f^{abc}\Bigg(i g_s\, c_5^{hgg} - \frac{i c_5^{hgg}\, c_6^{ggg}}{\Lambda^2}\,\frac{s_{23}s_{13}}{s_{12}} + \frac{b_9}{\Lambda^4}\,s_{13}s_{23}$$
$$+\frac{b_{11}}{\Lambda^6}\,s_{12}s_{13}s_{23} + \frac{b_{13}}{\Lambda^8}\,s_{13}^2 s_{23}^2 + \frac{b'_{13}}{\Lambda^8}\,s_{13}s_{23}s_{12}^2\Bigg) + d^{abc}\,s_{13}s_{23}(s_{13}-s_{23})\left(\frac{b'_{11}}{\Lambda^6} + \frac{b''_{13}}{\Lambda^8}\,s_{12}\right)\Bigg],$$

on top of dim-5 operator that saw before:

f^{abc}: contribution from $tr(G^3)$ operator
+ one operator at dim-9, one at dim-11, two at dim-13
d^{abc}: one operator at dim-11, one at dim-13

so:

1. calculated helicity amplitudes relevant for LHC production & decay of new (massive) spin-0 resonance (or Higgs) at LHC

$$gg \to h \to gg, ggg$$

2. can infer EFT Lagrangian and calculate number of independent operators very simply

to do this:

positive helicity gluon: $G^{\mu\nu}_{\rm SD} = \frac{1}{2}\left(G^{\mu\nu} + \tilde{G}^{\mu\nu}\right)$

$$\tilde{G}^{\mu\nu} = \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}G_{\rho\sigma}$$

negative helicity gluon: $G^{\mu\nu}_{\rm ASD} = \frac{1}{2}\left(G^{\mu\nu} - \tilde{G}^{\mu\nu}\right)$

→ $M(h;\ +\ +\ +)$: $h\, G_{SD}^2$; $h\, G_{SD}^3$ + more derivatives

$M(h;\ +\ +-)$: $h\, G_{SD}^2\, G_{ASD}$ + more derivatives

Mass dimension	Operators	
	$\mathcal{M}(+++)$	$\mathcal{M}(++-)$
5	—	—
7	$h\, G_{\rm SD}^3\ [1, f^{abc}]$	—
9	—	$\mathcal{D}^2\, G_{\rm SD}^2\, G_{\rm ASD}\, h\ [1, f^{abc}]$
11	$\mathcal{D}^4\, G_{\rm SD}^3\, h\ [1, f^{abc}]$	$\mathcal{D}^4\, G_{\rm SD}^2\, G_{\rm ASD}\, h\ [1, f^{abc}; 1, d^{abc}]$
13	$\mathcal{D}^6\, G_{\rm SD}^3\, h\ [1, f^{abc}; 1, d^{abc}]$	$\mathcal{D}^6\, G_{\rm SD}^2\, G_{\rm ASD}\, h\ [2, f^{abc}; 1, d^{abc}]$

did not have to worry about:

operator redundancy

gauge redundancy (or symmetry)

these just follow

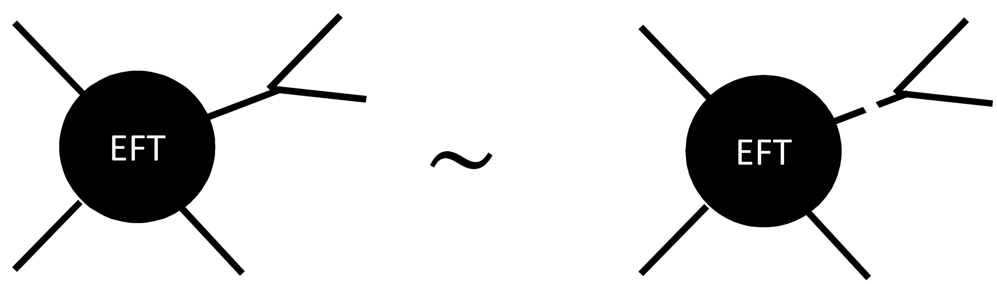
Mass dimension	Operators	
	$\mathcal{M}(+++)$	$\mathcal{M}(++-)$
5	—	—
7	$h\,G_{\rm SD}^3\ [1, f^{abc}]$	—
9	—	$\mathcal{D}^2\,G_{\rm SD}^2\,G_{\rm ASD}\,h\ [1, f^{abc}]$
11	$\mathcal{D}^4\,G_{\rm SD}^3\,h\ [1, f^{abc}]$	$\mathcal{D}^4\,G_{\rm SD}^2\,G_{\rm ASD}\,h\ [1, f^{abc}; 1, d^{abc}]$
13	$\mathcal{D}^6\,G_{\rm SD}^3\,h\ [1, f^{abc}; 1, d^{abc}]$	$\mathcal{D}^6\,G_{\rm SD}^2\,G_{\rm ASD}\,h\ [2, f^{abc}; 1, d^{abc}]$

- check: derived using Mathematica notebook of 1512.03433

Henning, Lu, Melia, Murayama, 2, 84, 30, 993, 560, 15456, 11962, 261485, ..: Higher dimension operators in the SM EFT

- these coefficients will enter higher point amplitudes

→ given by factorization



- to reproduce EFT Lagrangian up to eg, dim-7: (h + gluons only)

$G^2,\ G^3,\ hG^2,\ hG^3,\ h^2G^2,\ h^3G^2$:

would need: ggg, hgg, hggg, hhgg, hhhgg (calculated all but last)

example: SM + Z' new gauge singlet, spin-1

$$\mathcal{M}\Big(Z'; g^{a-}(p_1); g^{b-}(p_2); g^{c+}(p_3)\Big) \tag{3.18}$$

$$= d^{abc}\,\langle 12\rangle^2 \times \Big[[34]^2\,\tilde{f}^+_{-4}(1;2) + [13]\,[23]\,\langle 14\rangle\,\langle 24\rangle\,\tilde{f}^+_{-6}(1;2) + [34]\,([31]\,\langle 14\rangle - [32]\,\langle 24\rangle)\tilde{f}^-_{-5}(1;2)\Big]$$

$$+ f^{abc}\,\langle 12\rangle^2 \times \Big[[34]^2\,\tilde{f}^-_{-4}(1;2) + [13]\,[23]\,\langle 14\rangle\,\langle 24\rangle\,\tilde{f}^-_{-6}(1;2) + [34]\,([31]\,\langle 14\rangle - [32]\,\langle 24\rangle)\tilde{f}^+_{-5}(1;2)\Big]$$

$$\tilde{f}^+_{-4}(1;2) = \frac{d_8}{\Lambda^4} + \frac{d^{(1)}_{10}}{\Lambda^6}\,s_{12} + \frac{d^{(1)}_{12}\,s^2_{12} + d^{(2)}_{12}\,s_{13}s_{23}}{\Lambda^8}\,, \qquad \tilde{f}^-_{-4}(1;2) = (s_{23} - s_{13})\Big(\frac{d^{(3)}_{10}}{\Lambda^6} + \frac{d^{(4)}_{12}}{\Lambda^8}\,s_{12}\Big)\,,$$

$$\tilde{f}^+_{-5}(1;2) = \frac{m\,d^{(2)}_{10}}{\Lambda^6} + \frac{m\,d^{(3)}_{12}}{\Lambda^8}\,s_{12}\,, \qquad \tilde{f}^-_{-5}(1;2) = (s_{13} - s_{23})\frac{m\,d^{(5)}_{12}}{\Lambda^8}\,,$$

$$\tilde{f}^+_{-6}(1;2) = \frac{m^2\,s_{12}\,d^{(6)}_{12}}{\Lambda^8}\,, \quad \tilde{f}^-_{-6}(1;2) = 0\,, \tag{3.19}$$

example: SM + Z' new gauge singlet, spin-1

$$\mathcal{M}\Big(Z'; g^{a-}(p_1); g^{b-}(p_2); g^{c+}(p_3)\Big)$$

no factorizable part: no 3-point amplitude

$$= d^{abc}\,\langle 12\rangle^2 \times \Big[[34]^2\,\tilde{f}^{+}_{-4}(1;2) + [13]\,[23]\,\langle 14\rangle\,\langle 24\rangle\,\tilde{f}^{+}_{-6}(1;2) + [34]\,([31]\,\langle 14\rangle - [32]\,\langle 24\rangle)\tilde{f}^{-}_{-5}(1;2) \Big]$$

$$+ f^{abc}\,\langle 12\rangle^2 \times \Big[[34]^2\,\tilde{f}^{-}_{-4}(1;2) + [13]\,[23]\,\langle 14\rangle\,\langle 24\rangle\,\tilde{f}^{-}_{-6}(1;2) + [34]\,([31]\,\langle 14\rangle - [32]\,\langle 24\rangle)\tilde{f}^{+}_{-5}(1;2) \Big]$$

$$\tilde{f}^{+}_{-4}(1;2) = \frac{d_8}{\Lambda^4} + \frac{d^{(1)}_{10}}{\Lambda^6}\,s_{12} + \frac{d^{(1)}_{12}\,s^2_{12} + d^{(2)}_{12}\,s_{13}s_{23}}{\Lambda^8}\,, \qquad \tilde{f}^{-}_{-4}(1;2) = (s_{23} - s_{13})\left(\frac{d^{(3)}_{10}}{\Lambda^6} + \frac{d^{(4)}_{12}}{\Lambda^8}\,s_{12}\right),$$

$$\tilde{f}^{+}_{-5}(1;2) = \frac{m\,d^{(2)}_{10}}{\Lambda^6} + \frac{m\,d^{(3)}_{12}}{\Lambda^8}\,s_{12}\,, \qquad \tilde{f}^{-}_{-5}(1;2) = (s_{13} - s_{23})\frac{m\,d^{(5)}_{12}}{\Lambda^8}\,,$$

$$\tilde{f}^{+}_{-6}(1;2) = \frac{m^2\,s_{12}\,d^{(6)}_{12}}{\Lambda^8}\,, \quad \tilde{f}^{-}_{-6}(1;2) = 0\,, \qquad (3.19)$$

example: SM + Z' new gauge singlet, spin-1

$$\mathcal{M}\Big(Z'; g^{a-}(p_1); g^{b-}(p_2); g^{c+}(p_3)\Big) \tag{3.18}$$

$$= d^{abc}\,\langle 12\rangle^2 \times \Big[[34]^2\,\tilde{f}^+_{-4}(1;2) + [13]\,[23]\,\langle 14\rangle\,\langle 24\rangle\,\tilde{f}^+_{-6}(1;2) + [34]\,([31]\,\langle 14\rangle - [32]\,\langle 24\rangle)\tilde{f}^-_{-5}(1;2) \Big]$$

$$+ f^{abc}\,\langle 12\rangle^2 \times \Big[[34]^2\,\tilde{f}^-_{-4}(1;2) + [13]\,[23]\,\langle 14\rangle\,\langle 24\rangle\,\tilde{f}^-_{-6}(1;2) + [34]\,([31]\,\langle 14\rangle - [32]\,\langle 24\rangle)\tilde{f}^+_{-5}(1;2) \Big]$$

$$\tilde{f}^+_{-4}(1;2) = \frac{d_8}{\Lambda^4} + \frac{d^{(1)}_{10}}{\Lambda^6} s_{12} + \frac{d^{(1)}_{12}\,s^2_{12} + d^{(2)}_{12}\,s_{13}s_{23}}{\Lambda^8}\,, \qquad \tilde{f}^-_{-4}(1;2) = (s_{23} - s_{13})\left(\frac{d^{(3)}_{10}}{\Lambda^6} + \frac{d^{(4)}_{12}}{\Lambda^8} s_{12}\right),$$

$$\tilde{f}^+_{-5}(1;2) = \frac{m\,d^{(2)}_{10}}{\Lambda^6} + \frac{m\,d^{(3)}_{12}}{\Lambda^8} s_{12}\,, \qquad \tilde{f}^-_{-5}(1;2) = (s_{13} - s_{23})\frac{m\,d^{(5)}_{12}}{\Lambda^8}\,,$$

$$\tilde{f}^+_{-6}(1;2) = \frac{m^2\,s_{12}\,d^{(6)}_{12}}{\Lambda^8}\,, \quad \tilde{f}^-_{-6}(1;2) = 0\,, \tag{3.19}$$

example: SM + Z' new gauge singlet, spin-1

$$\mathcal{M}\Big(Z'; g^{a-}(p_1); g^{b-}(p_2); g^{c+}(p_3)\Big) \tag{3.18}$$

$$= d^{abc}\, \langle 12\rangle^2 \times \left[[34]^2\, \tilde{f}^{+}_{-4}(1;2) + [13]\,[23]\,\langle 14\rangle\,\langle 24\rangle\, \tilde{f}^{+}_{-6}(1;2) + [34]\,([31]\,\langle 14\rangle - [32]\,\langle 24\rangle)\tilde{f}^{-}_{-5}(1;2)\right]$$

$$+ f^{abc}\, \langle 12\rangle^2 \times \left[[34]^2\, \tilde{f}^{-}_{-4}(1;2) + [13]\,[23]\,\langle 14\rangle\,\langle 24\rangle\, \tilde{f}^{-}_{-6}(1;2) + [34]\,([31]\,\langle 14\rangle - [32]\,\langle 24\rangle)\tilde{f}^{+}_{-5}(1;2)\right]$$

massive Z' !
this expression contains all three polarizations

massive particle polarizations

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Conde Marzola

- just as with spinor products, work with basic building blocks (as in ``tensor method" for SU(N)

  build any spin j rep as product of spin ½

  (need symmetric combinations only; antisym=singlet)

- spin-1 object: $O_{\{ij\}}$, $i, j = 1{,}2$

  [$j = 1$: ↑↑ , (↑↓+↓↑), ↓↓]

massive momentum: need two massless spinors:

$$p^{\alpha\dot{\alpha}} = \boldsymbol{\lambda}_I^{\alpha}\tilde{\boldsymbol{\lambda}}^{\dot{\alpha}I} \equiv |\boldsymbol{p}_I\rangle[\boldsymbol{p}^I|\,, \quad p_{\dot{\alpha}\alpha} = -\tilde{\boldsymbol{\lambda}}_{\dot{\alpha}I}\boldsymbol{\lambda}_{\alpha}^{I} \equiv -|\boldsymbol{p}_I]\langle\boldsymbol{p}^I|$$

$I = 1,2$

can rescale: U_I^{-1} U_J^I with p invariant

= little group, SU(2)

massive momentum:

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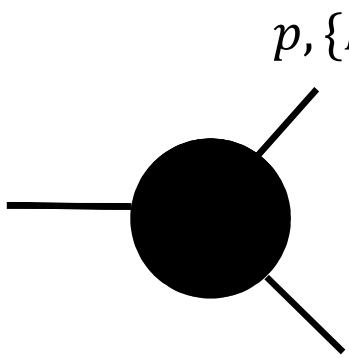


factor of $|p^I]|p^J]$ for polarization

with indices symmetrized

[gives $(..)_{\alpha\beta}$ with $\alpha\beta$ symmetrized]

use bolded notation: $|\boldsymbol{p}]|\boldsymbol{p}]$

using this formulation: general expressions for three-particle amplitudes for all masses & spins

massive momentum:

back to the amplitude we had before:

$$\mathcal{M}\Big(Z'; g^{a-}(p_1); g^{b-}(p_2); g^{c+}(p_3)\Big) \tag{3.18}$$

$$= d^{abc}\,\langle 12\rangle^2 \times \Bigg[\,[34]^2\,\tilde{f}^{+}_{-4}(1;2) + [13]\,[23]\,\langle 14\rangle\,\langle 24\rangle\,\tilde{f}^{+}_{-6}(1;2) + [34]\,([31]\,\langle 14\rangle - [32]\,\langle 24\rangle)\tilde{f}^{-}_{-5}(1;2)\Bigg]$$

$$+ f^{abc}\,\langle 12\rangle^2 \times \Bigg[\,[34]^2\,\tilde{f}^{-}_{-4}(1;2) + [13]\,[23]\,\langle 14\rangle\,\langle 24\rangle\,\tilde{f}^{-}_{-6}(1;2) + [34]\,([31]\,\langle 14\rangle - [32]\,\langle 24\rangle)\tilde{f}^{+}_{-5}(1;2)\Bigg]$$

high energy limit: splits into $+, -,$ $||$ polarizations

``simply" unbold massive spinors..

- match to EFT Lagrangian for transverse polarizations:

Mass dimension	Operators	
	$\mathcal{M}(+++)$	$\mathcal{M}(--+)$
8	$G^3_{\rm SD}\, Z'_{\rm SD}\, [1, d^{abc}]$	$G^2_{\rm ASD}\, G_{\rm SD}\, Z'_{\rm SD}\, [1, d^{abc}]$
10	$\mathcal{D}^2\, G^3_{\rm SD}\, Z'_{\rm SD}\, [1, d^{abc}; 1, f^{abc}]$ $\mathcal{D}^2\, G^3_{\rm SD}\, Z'_{\rm ASD}\, [1, d^{abc}]$	$\mathcal{D}^2\, G^2_{\rm ASD}\, G_{\rm SD}\, Z'_{\rm ASD}\, [1, d^{abc}]$ $\mathcal{D}^2\, G^2_{\rm ASD}\, G_{\rm SD}\, Z'_{\rm SD}\, [1, d^{abc}; 1, f^{abc}]$
12	$\mathcal{D}^4\, G^3_{\rm SD}\, Z'_{\rm SD}\, [1, d^{abc}]$	$\mathcal{D}^4\, G^2_{\rm ASD}\, G_{\rm SD}\, Z'_{\rm ASD}\, [1, d^{abc}; 1, f^{abc}]$ $\mathcal{D}^4\, G^2_{\rm ASD}\, G_{\rm SD}\, Z'_{\rm SD}\, [2, d^{abc}; 1, f^{abc}]$

- longitudinal polarization: same as scalar + 3 gluons
- see Higgs mechanism at amplitude level !

``IR (massive) amplitude unifies UV massless amplitudes"

Arkani-Hamed Huang Huang

To conclude:

EFT will play a key role in the coming years (until we find new particles at the LHC)

On-shell methods can be used to infer the structure of EFT amplitudes

Simple-minded way to calculate the number of EFT operators

Amplitudes with massive gauge-bosons: manifest Higgs mechanism at amplitude level

?? new insights into EWSB

THANK YOU!

example: three spin-1:

[bonus: gauge symmetry structure]

$++\,+$: [12][13][23] but need dim=1: nothing at renormalizable level

[EFT: $\frac{[12][13][23]}{\Lambda^2}$]

$++\,+$: [12][13][23] but need dim=1: nothing at renormalizable level

[EFT: $\frac{[12][13][23]}{\Lambda^2}$]

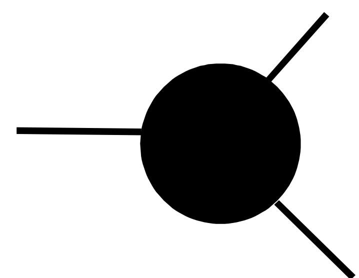
$$++-: \qquad \frac{[12]^3}{[13][23]}$$

antisymmetric in $1 \leftrightarrow 2$: *no 3-point photon amplitude!*

with extra quantum number, eg, index *a*

$$f^{abc}\frac{[12]^3}{[13][23]}$$

completely antisymmetric



$++\,+$: $[12][13][23]$ but need dim=1: nothing at renormalizable level

[EFT: $\frac{[12][13][23]}{\Lambda^2}$]

$$\boxed{\begin{gathered}\Sigma h_i = 3 \\ \dim(coupling) = -2\end{gathered}}$$

$++\,-$: $$\frac{[12]^3}{[13][23]}$$

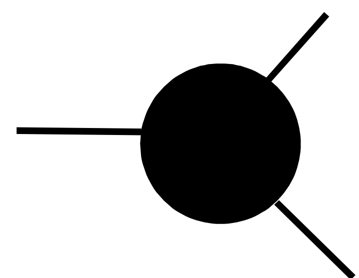
$$\boxed{\begin{gathered}\Sigma h_i = 1 \\ \dim(coupling) = 0\end{gathered}}$$

antisymmetric in $1 \leftrightarrow 2$: *no 3-point photon amplitude!*

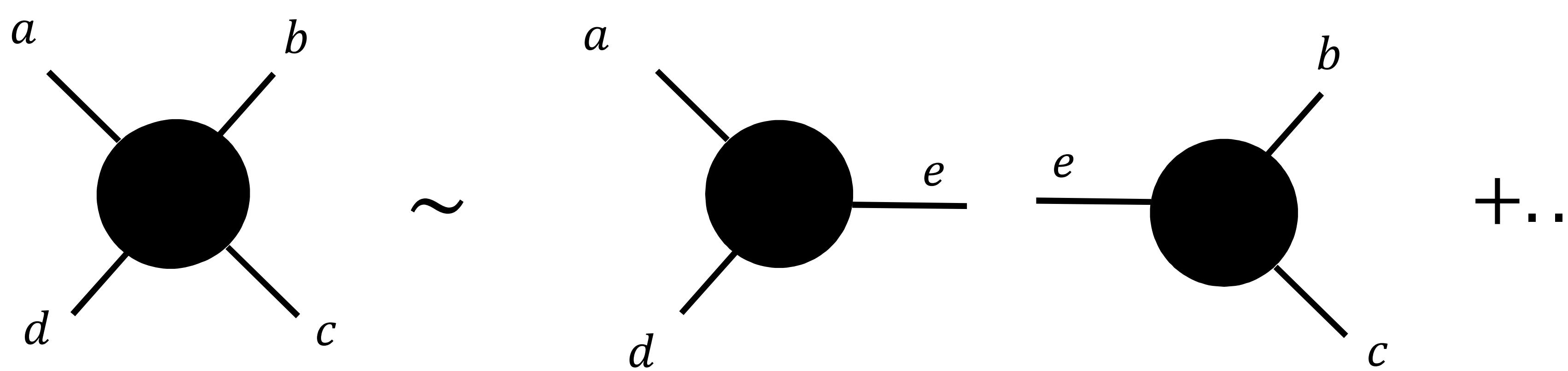
with extra quantum number, eg, index *a*

$$f^{abc}\frac{[12]^3}{[13][23]}$$

completely antisymmetric



Example application: **4 spin-1:**



consistent poles: $f^{ade}\, f^{bce} + \cdots = 0$ *Jacobi Identity*